

Mutual inductance between rectangular circuits

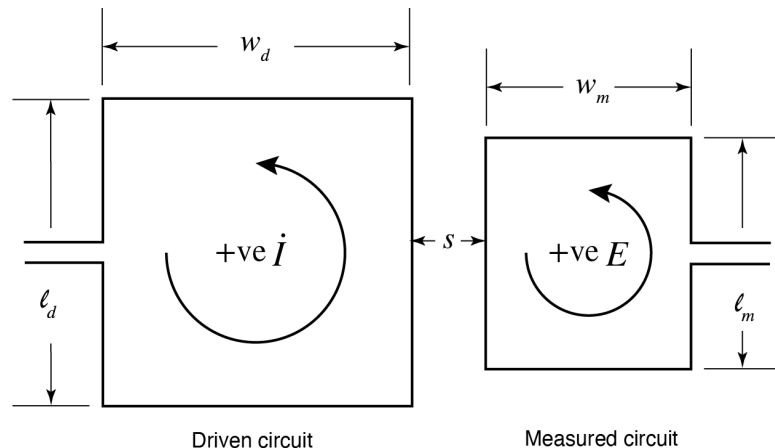
This notebook is part of the supplemental material for the paper *An inductive field component of the electric field that corresponds to Faraday's law*. The enclosed source code is used to find numerical solutions of electromagnetic force (EMF) due to mutual inductance between two rectangular wire circuits. This is done for the circuits in both a coplanar and co-axial arrangement, the calculations being performed for a set of horizontal and vertical separations, respectively. Results are found using Faraday's law, the full electric field acceleration term, the static field term, and the dynamic field term. The calculations are done in a set of Mathematica modules in this notebook. The output of the calculations are two tables in the section *Comparison of generated EMFs*. If your the goal is simply to evaluate different circuit geometries and separations, this can be achieved by modifying parameters in that section and evaluating the notebook.

Note that in two cases the symbols used here differ from the paper. The circuit height is given by h in the paper and by script l here. The vertical separation is v in the paper and is h here.

IMPORTANT: When making changes to the source code it is suggested evaluating the whole notebook twice. Changes in initialization cells are sometimes into picked up on the first notebook evaluation.

Symbol definitions

The layout of the circuits and the symbol definitions are shown below. The driven circuit has a changing current applied to it while the induced voltage is calculated for the measured circuit. As noted above, in the paper the height of the circuit is h and here it is l . Also the vertical separation for the co-axial case is v in the paper and is h here.



Equations for EMF using Faraday's law

The change electromagnetic flux through a point in space due to a current flowing in a wire of length dl is

$$\frac{dB}{dt} = \frac{\mu_0}{4\pi} \frac{\dot{\mathbf{I}} d\ell \times \mathbf{r}}{|\mathbf{r}|^3} \quad (1)$$

The total EMF will be the sum of all of individual EMF's generated by each straight wire segment of the driven circuit on the measured circuit.

$$\varepsilon = \sum_{i=1}^4 -\frac{\mu_0}{4\pi} \int_0^{\ell_{de}^i} \int_0^{h_m} \int_0^{w_m} \left[\frac{\dot{\mathbf{I}} \times \vec{\mathbf{r}}}{|\vec{\mathbf{r}}|^3} \right] dw_s dh_s dt_d \quad (2)$$

where the dimensions are from the circuit diagram above. This is the integral over the area of the measured circuit for all points along the driven wire. The distance between a point on a drive wire segment and a point within the measured circuit is

$$\vec{\mathbf{r}} = [x_m - m_{x_d}s_d - b_{x_d}] \vec{\mathbf{i}} + [y_m - m_{y_d}s_d - b_{y_d}] \vec{\mathbf{j}} + [h] \vec{\mathbf{k}} \quad (3)$$

The current vector parallel to the point on the driven wire.

$$\vec{\mathbf{I}} = \dot{I} [m_{x_m} \vec{\mathbf{i}} + m_{y_m} \vec{\mathbf{j}}] \quad (4)$$

Theory and equations for inductive field

For equations using the acceleration terms the EMF is the sum of the forces from one segment of driven wire on a segment of the measured wire. For the dynamic acceleration term

$$\varepsilon = \sum_{i=1}^4 \sum_{j=1}^4 \frac{-1}{4\pi c^2 \epsilon_0} \int_0^{\ell_{de}^i} \int_0^{\ell_{me}^j} \frac{\ddot{\mathbf{I}}}{\mathbf{r}_{ij}} \cdot [m_{x_m} \vec{\mathbf{i}} + m_{y_m} \vec{\mathbf{j}}] dt_m dt_d \quad (5)$$

For the standard derivation the acceleration term is replaced by the double cross product term.

$$\varepsilon = \sum_{i=1}^4 \sum_{j=1}^4 \frac{1}{4\pi c^2 \epsilon_0} \int_0^{\ell_{de}^i} \int_0^{\ell_{me}^j} \left[-\frac{\vec{\mathbf{r}} \times (\vec{\mathbf{r}} \times \ddot{\mathbf{I}})}{\mathbf{r}_{ij}} \right] \cdot [m_{x_m} \vec{\mathbf{i}} + m_{y_m} \vec{\mathbf{j}}] dt_m dt_d \quad (6)$$

The value of the vector \mathbf{r} is found from

$$\vec{\mathbf{r}}_{ij} = [m_{x_m}^j t_d + b_{x_m}^j - m_{x_d}^i t_d - b_{x_d}^i] \vec{\mathbf{i}} + [m_{y_m}^j t_d + b_{y_m}^j - m_{y_d}^i t_d - b_{y_d}^i] \vec{\mathbf{j}} + h \vec{\mathbf{k}} \quad (7)$$

Modules overview

The numerical solutions were arrived at using a set of Mathematica modules, each one responsible for

either part of the calculation or combining the results. They are arranged rather like classes in object oriented languages.

The most likely use of this notebook is to replicate the results of the paper for the same or different geometries. This is most easily done by copying the section *Comparison of generated EMFs* and modifying geometries and separations.

Aside from inspecting the source code, a deeper understanding of the modules can come from browsing the unit tests, their input, and their output.

Comparison of generated EMFs

The table below shows the numerical results of the EMF generated using Faraday's law, the full acceleration term, the static term, and the dynamic term. The EMF for the Faraday and dynamic acceleration terms are equal or differ in the last few least significant digits. The full acceleration term is consistently close to zero, and the static term is the negative of the Faraday and dynamic terms.

If you wish to try other geometries, you can copy this section and change the arguments in the call to **ComparisonOfEmfForRectangularCircuits**, or just do so below and then reevaluate the notebook.

```
comparisionNear = ComparisonOfEmfForRectangularCircuits[
  {0.005, 0.010, 0.025, 0.05, 0.075, 0.1}, (* The list of separations,
    in meters, for co-planar and co-axial separations *)
  {0.1, 0.1}, (* The width and length(height in the paper)
    of the driven circuit, in meters *)
  {0.05, 0.04}, (* The width and length(height in the paper)
    of the measured circuit, in meters *)
  Automatic
];
```

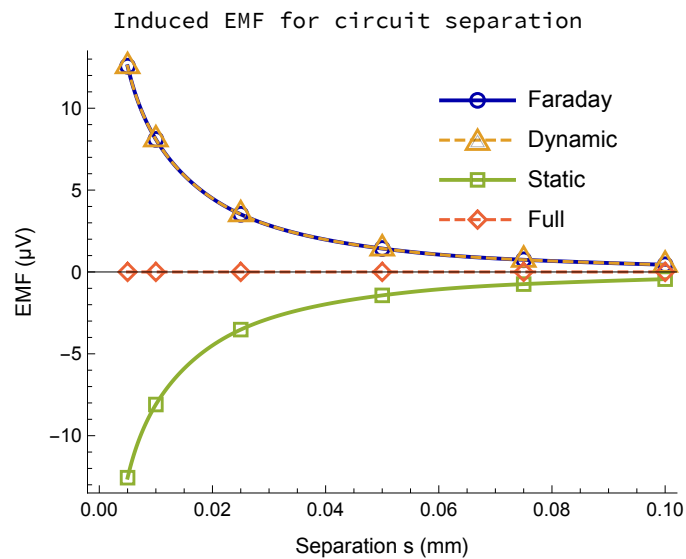
```
comparisionNear["writeResultsToTable"][]
```

| | | Coplanar (μV) | | | | | Coaxial (μV) | | | |
|----------------------|--|----------------------------|-----------------------------|-----------------|------------------|--|---------------------------|----------------------------|-----------------|------------------|
| separation n (mm) | | Faraday | Full accel | Static accel | Dynamic accel | | Faraday | Full accel | Static accel | Dynamic accel |
| 5. | | 12.5657 | $-0.0001049 \times 10^{-7}$ | -12.5657 | 12.5657 | | -24.4164 | 5.08175×10^{-7} | 24.4164 | -24.4164 |
| 10. | | 8.09668 | -1.79483×10^{-7} | -8.09663 | 8.09663 | | -23.152 | 4.14725×10^{-7} | 23.152 | -23.152 |
| 25. | | 3.52078 | -5.56233×10^{-8} | -3.52078 | 3.52078 | | -17.0874 | 1.35166×10^{-6} | 17.0874 | -17.0874 |
| 50. | | 1.43025 | -3.55023×10^{-8} | -1.43025 | 1.43025 | | -8.6877 | 5.63091×10^{-8} | 8.6877 | -8.6877 |
| 75. | | 0.742114 | 1.13269×10^{-8} | -0.742114 | 0.742114 | | -4.47633 | -4.54098×10^{-9} | 4.47633 | -4.47633 |
| 100. | | 0.439113 | -3.62991×10^{-9} | -0.439113 | 0.439113 | | -2.47621 | -4.04461×10^{-10} | 2.47621 | -2.47621 |

```
comparisonGraphics =
```

```
GraphicsForComparisonOfEmfForRectangularCircuits[comparisionNear];
```

```
comparisonGraphics["plotEmfsVsSeparation"][]
```



An interesting aspect of the conventional acceleration term is that the sum of EMF's around a loop of wire is always zero, within numerical accuracy. This is true whether it is a driven wire loop on a single measured wire, or a single driven wire on a measured wire loop. In either case the sum of EMF is zero.

The i index refers to the driven and j to the measured wires. The wire with index 1 is on the right and increases counter-clockwise.

`comparisionNear["writeWireToWireEmfsToTable"][0.005]`

| Full accel term EMF's (μV) | | | | | | | Static term EMF's (μV) | | | | | | | Dynamic term EMF's (μV) | | | | | |
|-----------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|--|-------------------------------|----------------------|-----------------|-----------------|-----------------|----------------------|--|--------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| | i=1 | 2 | 3 | 4 | Su m | | | i=1 | 2 | 3 | 4 | Su m | | | i=1 | 2 | 3 | 4 | Su m |
| j=1 | -5. 27 45 | 1.4 17 6 | 2.4 39 4 | 1.4 17 6 | 0 | | j=1 | 1.1 86 6 | 1.4 17 6 | -0. 09 23 | 1.4 17 6 | 3.9 28 8 | | j=1 | -6. 46 05 | 0 | 2.5 31 7 | 0 | -3. 92 88 |
| 2 | -1. 33 92 | -1. 64 39 | 0.4 79 6 | 2.5 03 5 | 0 | | 2 | -1. 33 92 | 4.9 80 8 | 0.4 79 6 | -2. 31 67 | 1.8 04 5 | | 2 | 0 62 46 | -6. 0 | 0 20 2 | 4.8 80 45 | -1. |
| 3 | 7.9 52 9 | -2. 27 72 | -3. 39 85 | -2. 27 72 | -0. 00 01 | | 3 | -1 5.8 12 2 | -2. 27 72 | 0.2 63 2 | -2. 27 72 | -2 0.1 03 4 | | 3 | 23. 76 51 | 0 | -3. 66 17 | 0 | 20. 10 34 |
| 4 | -1. 33 92 | 2.5 03 5 | 0.4 79 6 | -1. 64 39 | 0 | | 4 | -1. 33 92 | -2. 31 67 | 0.4 79 6 | 4.9 80 8 | 1.8 04 5 | | 4 | 0 20 2 | 4.8 | 0 | -6. 62 46 | -1. |
| Su m | 0 | 0 | 0 | 0 | 0 | | | -1 7.3 04 6 | 1.8 04 5 | 1.1 3 | 1.8 04 5 | -1 2.5 65 7 | | | 17. 30 46 | -1. 80 45 | -1. 13 | -1. 80 45 | 12. 56 57 |

Precision and convergence of values

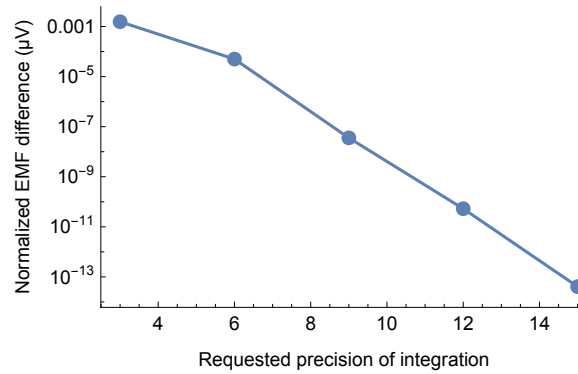
It is shown above that the The table below shows the numerical results of the EMF generated using Faraday's law, the full acceleration term, the static term, and the dynamic term. The EMF for the Faraday and dynamic acceleration terms are equal or differ in the last few least significant digits. The full acceleration term is consistently close to zero, and the static term is the negative of the Faraday and dynamic terms.

If you wish to try other geometries, you can copy this section and change the arguments in the call to **ComparisonOfEmfForRectangularCircuits**, or just do so below and then reevaluate the notebook.

```
precisionAnalysis = AnalysisOfPrecisionForRectangularCircuits[
  0.010, (* Separation *)
  {0.1, 0.1}, (* The width and length
    (height in the paper) of the driven circuit, in meters *)
  {0.05, 0.04}, (* The width and length(height in the paper)
    of the measured circuit, in meters *)
  {3, 6, 9, 12, 15} (* The precision values *)
];
```

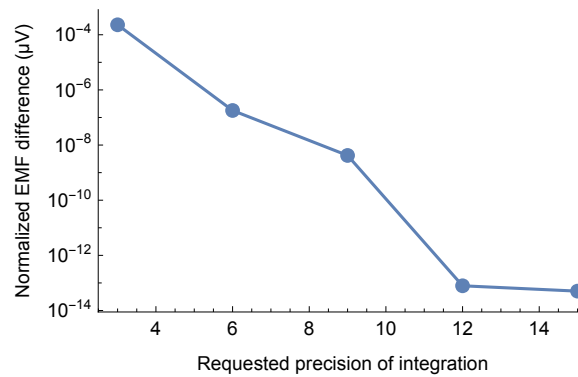
```
graphics = GraphicsForPrecisionForRectangularCircuits[precisionAnalysis];
graphics["plotAbsFaradayDiffDynamic"][]
```

Difference of Faraday to dynamic acceleration equation for requested precision



```
graphics["plotAbsFullDiffZero"][]
```

Difference of full Feynman acceleration EMF to zero for requested precision



```
precisionAnalysis["writeResultsToTable"][]
```

| | Faraday | Difference between Faraday and dynamic accel | | Full accel | Difference of full accel |
|--|---------|--|--|------------|--------------------------|
| | 8.09835 | 1.55×10^{-3} | | -0.000230 | 2.3×10^{-4} |
| | 8.09668 | 5.01×10^{-5} | | -0.000000 | 1.79×10^{-7} |
| | 8.09663 | 3.56×10^{-8} | | 0.000000 | 4.2×10^{-9} |
| | 8.09663 | 5.29×10^{-11} | | 0.000000 | 7.88×10^{-14} |
| | 8.09663 | 4.09×10^{-14} | | 0.000000 | 5.04×10^{-14} |

Common functionality

This is the common base 'class' for the Faraday and inductive field modules. It mostly has a helper function for the constructing geometry of the circuits.

```
BaseObject[] := Module[
{
```

```

(* public variables *)
this = <|>,

(* public functions *)
wiresFromDimensions, wiresFromCorners, log, makePublic,

(* internal variables *)
wireList, addCorner, diCorner, subs, argList, iCorner,
},

wiresFromDimensions[circuitDimensions_, offset_ : {0, 0, 0}] := (
  wiresFromCorners[
    {
      {circuitDimensions[[1]]/2, -circuitDimensions[[2]]/2, 0.0},
      {circuitDimensions[[1]]/2, circuitDimensions[[2]]/2, 0.0},
      {-circuitDimensions[[1]]/2, circuitDimensions[[2]]/2, 0.0},
      {-circuitDimensions[[1]]/2, -circuitDimensions[[2]]/2, 0.0}
    },
    offset,
    True]
  );

wiresFromCorners[
  wireCorners_ : {_?NumericQ..},
  offsetCornersBy_ : {_?NumericQ..},
  isClosed_?BooleanQ
] := (
  wireList = {};
  addCorner[start_, end_] := AppendTo[wireList,
    <| "start" → wireCorners[[start]] + offsetCornersBy,
      "end" → wireCorners[[end]] + offsetCornersBy,
      "wireId" → start|>];
  For[iCorner = 1, iCorner < Length[wireCorners], ++iCorner,
    addCorner[iCorner, iCorner + 1];
  ];
  If[isClosed, addCorner[iCorner, 1]];
  wireList
  );

(* Assign a value to this. The argument is a list of the form
   { string, value, string value,...}
  *)
makePublic[thisArg_, stringList_] := (

```

```

    subs = Partition[stringList, 2];
    argList = thisArg;
    Scan[(argList = AssociateTo[argList, #[[1]] -> #[[2]]]) &, subs];
    argList
  );

  log[name_, value_] := Print[name <> ": " <> TextString[value]];
  this = makePublic[this, {
    "wiresFromDimensions", wiresFromDimensions,
    "wiresFromCorners", wiresFromCorners,
    "log", log,
    "makePublic", makePublic}];
  this
];

```

EMF from Faraday's law

This module calculates the induced EMF from Faraday's law.

```

FaradayEmfForRectangularCircuits[precision_] := Module[
{
  (* public values *)
  this = <||>,

  (* constants *)
  e0, c, mu0,

  (* public functions *)
  faradaysEmfWireToWireSet, faradaysEmfWireSetToWireSet,
  faradayEmfForWireOnWireSetsAtOffsets, wiresFromCorners,

  (* local values *)
  drivenWireSet, measuredWireSetAtOffset, wireEmf,
  wireList, iCorner, addCorner, faradayEmfTotal, faradayEmfData,
  faradayEmfFromWires, srcWireStartFdy, srcWireVecFdy, srcWireLengthFdy,
  srcWireNormalizedFdy, dIdtVecFdy, msrXWireStartFdy, msrXWireEndFdy,
  msrYWireStartFdy, msrYWireEndFdy, msrXWireVecFdy, msrXWireLengthFdy,
  msrXWireNormalizedFdy, msrYWireVecFdy, msrYWireLengthFdy,
  msrYWireNormalizedFdy, srcWirePosFdy, msrCircuitPosFdy, msrXYDotFdy, log
},
  this = BaseObject[];
  log = this["log"];

```



```

(* Constants *)
e0 = 8.8541878128*^-12;
c = 299792458;
mu0 = 1/(e0 c^2); (* 1.2566370621219*^-6 *)

(* Calculate the EMF from Faraday's
   law for one driven wire on the measured wire loop. *)
faradaysEmfWireToWireSet[
  dIdt_,
  srcWire_,
  msrWireSet_
] := (
  srcWireStartFdy = srcWire["start"];
  srcWireVecFdy = srcWire["end"] - srcWire["start"];
  srcWireLengthFdy = Norm[srcWireVecFdy];
  srcWireNormalizedFdy = Normalize[srcWireVecFdy];
  dIdtVecFdy = dIdt srcWireNormalizedFdy;

  (* This algorithm assumes that the the measured wire set is a closed
     rectangle. The first two sides are used as perpendicular vectors. Start
     by verifying this. *)
  msrXWireStartFdy = msrWireSet[[1]]["start"];
  msrXWireEndFdy = msrWireSet[[1]]["end"];
  msrYWireStartFdy = msrWireSet[[2]]["start"];
  msrYWireEndFdy = msrWireSet[[2]]["end"];

  msrXWireVecFdy = msrXWireEndFdy - msrXWireStartFdy;
  msrXWireLengthFdy = Norm[msrXWireVecFdy];
  msrXWireNormalizedFdy = Normalize[msrXWireVecFdy];

  msrYWireVecFdy = msrYWireEndFdy - msrYWireStartFdy;
  msrYWireLengthFdy = Norm[msrYWireVecFdy];
  msrYWireNormalizedFdy = Normalize[msrYWireVecFdy];

  msrXYDotFdy = Dot[msrXWireVecFdy, msrYWireVecFdy]/msrXWireLengthFdy;
  If[Abs[msrXYDotFdy] > 0.0001,
    Throw["Error in faradaysEmfWireToWireSet: Wires not perpendicular"]];

  -mu0/(4*Pi) *
  NIntegrate[
    srcWirePosFdy =
      srcWireStartFdy + srcWireLinearPosFdy srcWireNormalizedFdy;
    msrCircuitPosFdy = msrXWireStartFdy + xmFdy msrXWireNormalizedFdy +

```

```

        ymFdy msrYWireNormalizedFdy;
    Dot[
        Cross[dIdtVecFdy, msrCircuitPosFdy - srcWirePosFdy],
        {0, 0, 1}
    ] / Norm[msrCircuitPosFdy - srcWirePosFdy]^3,
    {srcWireLinearPosFdy, 0, srcWireLengthFdy},
    {xmFdy, 0, msrXWireLengthFdy},
    {ymFdy, 0, msrYWireLengthFdy},
    Method -> "LocalAdaptive",
    PrecisionGoal -> precision
    ]
);

(* Calculate the EMF from Faraday's law
for the driven wire loop on the measured wire loop. *)
faradaysEmfWireSetToWireSet[
    dIdt_,
    srcWireSet_,
    msrWireSet_
] := (
    If[srcWireSet // Length != 4,
        Throw["Error in faradaysEmfWireSetToWireSet: srcWireSet size incorrect,
            expected 4 but found "<> TextString[srcWireSet // Length]]];

    faradayEmfTotal = 0.0;
    faradayEmfData = Map[
        Function[
            {drivenWire},
            faradayEmfFromWires =
                faradaysEmfWireToWireSet[dIdt, drivenWire, msrWireSet];
            faradayEmfTotal += faradayEmfFromWires;
            <|"drivenWireId" -> drivenWire["wireId"], "emfs" -> faradayEmfFromWires|>
        ],
        srcWireSet
    ];

    <| "emfTotal" -> faradayEmfTotal, "drivenWireEmfs" -> faradayEmfData |>
);

(* Calculate the EMF from Faraday's law for the driven on
measured wire loops for a set of horizontal loop separations. *)
faradayEmfForWireOnWireSetsAtOffsets[
    dIdt_?NumericQ,

```

```

drivenWireCorners_,
measuredWireCorners_,
measuredXOffsets_
]
:= (
  drivenWireSet =
    this["wiresFromCorners"][drivenWireCorners, {0, 0, 0}, True];
  Map[
    Function[
      {xOffset},
      measuredWireSetAtOffset = this["wiresFromCorners"][
        measuredWireCorners,
        {xOffset, 0, 0},
        True];

      wireEmf = faradaysEmfWireSetToWireSet[
        dIdt,
        drivenWireSet,
        measuredWireSetAtOffset
      ];
      {xOffset, wireEmf["emfTotal"]}
    ],
    measuredXOffsets]
);

this = this["makePublic"][this, {
  "faradaysEmfWireToWireSet", faradaysEmfWireToWireSet,
  "faradaysEmfWireSetToWireSet", faradaysEmfWireSetToWireSet,
  "faradayEmfForWireOnWireSetsAtOffsets",
  faradayEmfForWireOnWireSetsAtOffsets
}];
this
]

```

EMF from inductive field

This module calculates the induced EMF from the full, static, and dynamic acceleration terms.

```

InductiveFieldEmfForRectangularCircuits[precision_] := Module[
{
  (* public values *)
  this = <||>,

```

```

(* constants *)
e0, c, mu0,

(* public variables *)
fullAccelTermCoeff, staticAccelTermCoeff, dynamicAccelTermCoeff,

(* public functions *)
setFullAccelTermCoeff, setStaticAccelTermCoeff, setDynamicAccelTermCoeff,
accelProduct, emfAtPoint, emfFromWireToWire, inlineEmfForWires,
inlineEmfForWireSetsOnWire, inlineEmfForWireOnWireSets,
inlineEmfForWireSetsOnWireSets, inlineEmfForWireOnWireSetsAtOffsets,

(* local values *)
rMsrDrv, drvLen, zeroVec, mDrv, msrLen, mMsr, dIdtVec,
rrVec, drvWireVec, drvWireLength, drvWireNormalized, drvWirePos,
msrWireLength, msrWireNormalized, msrWirePos, msrWireVec, emfResult,
xyEmf, emfVec, inlineEmf, emfTotal, drivenWireEmfs, wireEmf,
measuredWireEmfs, emfTotalForSets, drivenWireSet, wireList,
addCorner, measuredWireSetAtOffset, iCorner, drvWireStart,
msrCircuitPos, msrXWireStart, msrXWireEnd, msrYWireStart,
msrYWireEnd, msrXWireVec, msrXWireLength, msrXWireNormalized,
msrYWireVec, msrYWireLength, msrYWireNormalized, rUnit
},
this = BaseObject[];

(* Constants *)
e0 = 8.8541878128*^-12;
c = 299 792 458;
mu0 = 1 / (e0 c^2); (* 1.2566370621219*^-6 *)

fullAccelTermCoeff = 1;
staticAccelTermCoeff = 0;
dynamicAccelTermCoeff = 0;

(* These three functions allow for switching between the full,
static, or dynamicfield terms. *)
setFullAccelTermCoeff[coeff_] := fullAccelTermCoeff = coeff;
setStaticAccelTermCoeff[coeff_] := staticAccelTermCoeff = coeff;
setDynamicAccelTermCoeff[coeff_] := dynamicAccelTermCoeff = coeff;

accelProduct[accel_, r_] := (
  rUnit = Normalize[r];

```

```

fullAccelTermCoeff Cross[rUnit, Cross[rUnit, accel] ]
+ (staticAccelTermCoeff Dot[rUnit, accel] rUnit)
- dynamicAccelTermCoeff accel
);

emfAtPoint[dIdtVec_, rDrv_, rMsr_] := (
  rMsrDrv = rMsr - rDrv;
  accelProduct[dIdtVec, rMsrDrv] / (4 Pi e0 Norm[rMsrDrv] c^2)
);

(*****
  Rectangular wire related functions
  *****)

(* This function calculates the
  EMF for one      to another given the wire ends *)
emfFromWireToWire[
  dIdt_?NumericQ,
  drvWireStart_?(VectorQ[#, NumericQ] &),
  drvWireEnd_?(VectorQ[#, NumericQ] &),
  msrWireStart_?(VectorQ[#, NumericQ] &),
  msrWireEnd_?(VectorQ[#, NumericQ] &)
]
:= (
  (* Calculate the slope m of the wires and the wire lengths *)
  drvLen = Norm[drvWireEnd - drvWireStart];
  mDrv = (drvWireEnd - drvWireStart) / drvLen;
  msrLen = Norm[msrWireEnd - msrWireStart];
  mMsr = (msrWireEnd - msrWireStart) / msrLen;
  dIdtVec = dIdt mDrv;
  zeroVec = {0, 0, 0};

  emfResult = NIntegrate[
    rrVec = mMsr sMsr + msrWireStart - mDrv sDrv - drvWireStart;
    Dot[emfAtPoint[dIdtVec, rrVec, zeroVec], mMsr],
    {sMsr, 0, msrLen},
    {sDrv, 0, drvLen},
    Method -> "LocalAdaptive",
    PrecisionGoal -> precision
  ]
);

(* This function calculates the inline emf for two wire sets. *)

```

```

inlineEmfForWires[
  dIdt_?NumericQ,
  drivenWire_,
  measuredWire_
]
:= (
  emfFromWireToWire[
    dIdt,
    drivenWire["start"], drivenWire["end"],
    measuredWire["start"], measuredWire["end"]]
  );

(* This function calculates the inline emf for two wire sets. *)
inlineEmfForWireSetsOnWire[
  dIdt_?NumericQ,
  drivenWireSets_,
  measuredWire_
]
:= (
  emfTotal = 0.0;
  drivenWireEmfs = Map[
    Function[
      {drivenWire},
      wireEmf = inlineEmfForWires[
        dIdt,
        drivenWire,
        measuredWire
      ];
      emfTotal += wireEmf;
      <| "drivenWireId" → drivenWire["wireId"], "emf" → wireEmf |>
    ],
    drivenWireSets];
  <| "emfTotal" → emfTotal, "drivenWireEmfs" → drivenWireEmfs |>
);

(* This function calculates the inline emf for two wire sets. *)
inlineEmfForWireOnWireSets[
  dIdt_?NumericQ,
  drivenWire_,
  measuredWireSets_
]
:= (
  emfTotal = 0.0;

```

```

drivenWireEmfs = Map[
  Function[
    {measuredWire},
    wireEmf = inlineEmfForWires[
      dIdt,
      drivenWire,
      measuredWire
    ];
    emfTotal += wireEmf;
    <| "measuredWireId" → measuredWire["wireId"], "emf" → wireEmf |>
  ],
  measuredWireSets];
<| "emfTotal" → emfTotal, "drivenWireEmfs" → drivenWireEmfs |>
);

(* This function calculates the inline emf for two sets of wires. *)
inlineEmfForWireSetsOnWireSets[
  dIdt_?NumericQ,
  drivenWireSets_,
  measuredWireSets_
]
:= (
  emfTotalForSets = 0.0;
  measuredWireEmfs = Map[
    Function[
      {measuredWire},
      wireEmf = inlineEmfForWireSetsOnWire[
        dIdt,
        drivenWireSets,
        measuredWire
      ];
      emfTotalForSets += wireEmf["emfTotal"];
      <| "measuredWireId" → measuredWire["wireId"],
        "measuredWireEmfs" → wireEmf |>
    ],
    measuredWireSets];
  <| "emfTotal" → emfTotalForSets, "individualWireEmfs" → measuredWireEmfs |>
);

(* This function calculates the inline emf for two sets of wires. *)
inlineEmfForWireOnWireSetsAtOffsets[
  dIdt_?NumericQ,
  drivenWireCorners_,

```

```

    measuredWireCorners_,
    measuredXOffsets_
]
:= (
  drivenWireSet =
    this["wiresFromCorners"][drivenWireCorners, {0, 0, 0}, True];
  Map[
    Function[
      {xOffset},
      measuredWireSetAtOffset = this["wiresFromCorners"][
        measuredWireCorners,
        {xOffset, 0, 0},
        True];

      wireEmf = inlineEmfForWireSetsOnWireSets[
        dIdt,
        drivenWireSet,
        measuredWireSetAtOffset
      ];
      {xOffset, wireEmf["emfTotal"]}
    ],
    measuredXOffsets]
);

this = this["makePublic"][this, {
  "setFullAccelTermCoeff", setFullAccelTermCoeff,
  "setStaticAccelTermCoeff", setStaticAccelTermCoeff,
  "setDynamicAccelTermCoeff", setDynamicAccelTermCoeff,
  "accelProduct", accelProduct,
  "emfAtPoint", emfAtPoint,
  "emfFromWireToWire", emfFromWireToWire,
  "inlineEmfForWires", inlineEmfForWires,
  "inlineEmfForWireSetsOnWire", inlineEmfForWireSetsOnWire,
  "inlineEmfForWireOnWireSets", inlineEmfForWireOnWireSets,
  "inlineEmfForWireSetsOnWireSets", inlineEmfForWireSetsOnWireSets,
  "inlineEmfForWireOnWireSetsAtOffsets",
  inlineEmfForWireOnWireSetsAtOffsets,
  "fullAccelTermCoeff", fullAccelTermCoeff,
  "staticAccelTermCoeff", staticAccelTermCoeff,
  "dynamicAccelTermCoeff", dynamicAccelTermCoeff
}];
this
]

```


Module for comparison of EMFs generated by Faradays law and electric field terms

The calculation and tabulation of all results is done with this class.

```
ComparisonOfEmfForRectangularCircuits[
  separations_,
  drvDimension_,
  msrDimension_,
  precision_
] := Module[
{
  (* public values *)
  this = <||>,

  (* functions *)
  generateAllResults, writeResultsToTable, writeWireToWireEmfsToTable,
  getFaradayEmfCoplanar, getFaradayEmfCoaxial, getStaticAccelEmfCoplanar,
  getStaticAccelEmfCoaxial, getDynamicAccelEmfCoplanar,
  getDynamicAccelEmfCoaxial, getFullAccelEmfCoplanar,
  getFullAccelEmfCoaxial, findMsrLoopEmfForSingleSrcWire,
  findMsrWireEmfForSrcLoop, generateFaraday, generateFullAccel,
  generateStaticAccel, generateDynamicAccel, circuitAtSeparation,

  (* constants *)
  dIdt = 1000.0,

  (* local values *)
  faraday, eFieldFull, eFieldStatic, eFieldDynamic, faradayEmfCoplanar,
  fullAccelEmfCoplanar, staticAccelEmfCoplanar, dynamicAccelEmfCoplanar,
  faradayEmfCoaxial, fullAccelEmfCoaxial, staticAccelEmfCoaxial,
  dynamicAccelEmfCoaxial, resultsArray, srcWires, msrWires,
  msrOffsetX, emf, first, eField, emfs10nMsr, circuit, row, sumFullEmf,
  sumStaticEmf, sumDynamicEmf, prec = 6, msrWireEmfs, fullEmfs,
  staticEmfs, dynamicEmfs, inlineEmf, micro = 1.0*^6, milli = 1.0*^3
},
  this = BaseObject[];
  faradayEmfCoplanar = {};
  staticAccelEmfCoplanar = {};
  dynamicAccelEmfCoplanar = {};
  fullAccelEmfCoplanar = {};
  faradayEmfCoaxial = {};
```

```

staticAccelEmfCoaxial = {};
dynamicAccelEmfCoaxial = {};
fullAccelEmfCoaxial = {};

generateAllResults[] := (
  If[Length[fullAccelEmfCoplanar] == 0,
    (generateFaraday[];
     generateFullAccel[];
     generateStaticAccel[];
     generateDynamicAccel[];)
  ]
);

writeResultsToTable[] := (
  resultsArray = {
    {"", "", "      Coplanar ( $\mu$ V)", SpanFromLeft,
     SpanFromLeft, SpanFromLeft, "", "      Coaxial ( $\mu$ V)",
     SpanFromLeft, SpanFromLeft, SpanFromLeft},
    {"separation (mm)", "", "Faraday", "Full accel",
     "Static accel", "Dynamic accel", " ", "Faraday",
     "Full accel", "Static accel", "Dynamic accel"}
  };
  For[i = 1, i <= (separations // Length), i++,
    row = {milli separations[[i]]};
    row = Join[row, {" "}];
    row = Join[row, {NumberForm[faradayEmfCoplanar[[i]], prec],
     NumberForm[fullAccelEmfCoplanar[[i]], prec],
     NumberForm[staticAccelEmfCoplanar[[i]], prec],
     NumberForm[dynamicAccelEmfCoplanar[[i]], prec]}}];
    row = Join[row, {" "}];
    row = Join[row, {NumberForm[faradayEmfCoaxial[[i]], prec],
     NumberForm[fullAccelEmfCoaxial[[i]], prec],
     NumberForm[staticAccelEmfCoaxial[[i]], prec],
     NumberForm[dynamicAccelEmfCoaxial[[i]], prec]}}];
    AppendTo[resultsArray, row]
  ];
  TextGrid[resultsArray, Frame → All]
);

format[number_, decimalPrec_: 4] :=
  (TextString[Round[number 10^decimalPrec] / 10^decimalPrec]);

generateFaraday[] := (

```

```

If[Length[faradayEmfCoplanar] == 0,
(
  faraday = FaradayEmfForRectangularCircuits[precision];
  srcWires = faraday["wiresFromDimensions"][drvDimension];
  msrOffsetX = (drvDimension[[1]] + msrDimension[[1]])/2;

  Scan[
    (
      msrWires = faraday["wiresFromDimensions"][
        msrDimension, {msrOffsetX + #1, 0, 0}];
      emf = faraday["faradaysEmfWireSetToWireSet"][
        dIdt, srcWires, msrWires];
      AppendTo[faradayEmfCoplanar, micro emf["emfTotal"]] &,
      separations
    ];
  ];

  Scan[
    (
      msrWires = faraday["wiresFromDimensions"][msrDimension, {0, 0, #1}];
      emf =
        faraday["faradaysEmfWireSetToWireSet"][dIdt, srcWires, msrWires];
      AppendTo[faradayEmfCoaxial, micro emf["emfTotal"]] &,
      separations
    );
  ];
  {faradayEmfCoplanar, faradayEmfCoaxial}
);

generateFullAccel[] := (
  If[Length[fullAccelEmfCoplanar] == 0,
    (
      eFieldFull = InductiveFieldEmfForRectangularCircuits[precision];
      eFieldFull["setFullAccelTermCoeff"][1];
      eFieldFull["setStaticAccelTermCoeff"][0];
      eFieldFull["setDynamicAccelTermCoeff"][0];

      srcWires = eFieldFull["wiresFromDimensions"][drvDimension];
      msrOffsetX = (drvDimension[[1]] + msrDimension[[1]])/2;

      Scan[
        (
          msrWires = faraday["wiresFromDimensions"][

```

```

        msrDimension, {msrOffsetX + #1, 0, 0}];
    emf = eFieldFull["inlineEmfForWireSetsOnWireSets"][
        dIdt, srcWires, msrWires];
    AppendTo[fullAccelEmfCoplanar, micro emf["emfTotal"]]
    ) &,
    separations
];

Scan[
(
    msrWires = faraday["wiresFromDimensions"][msrDimension, {0, 0, #1}];
    emf = eFieldFull["inlineEmfForWireSetsOnWireSets"][
        dIdt, srcWires, msrWires];
    AppendTo[fullAccelEmfCoaxial, micro emf["emfTotal"]]) &,
    separations
];
);
{fullAccelEmfCoplanar, fullAccelEmfCoaxial}
);

generateStaticAccel[] := (
    If[Length[staticAccelEmfCoplanar] == 0,
    (
        eFieldStatic = InductiveFieldEmfForRectangularCircuits[precision];
        eFieldStatic["setFullAccelTermCoeff"][0];
        eFieldStatic["setStaticAccelTermCoeff"][1];
        eFieldStatic["setDynamicAccelTermCoeff"][0];

        srcWires = eFieldStatic["wiresFromDimensions"][drvDimension];
        msrOffsetX = (drvDimension[[1]] + msrDimension[[1]])/2;

        Scan[
            (
                msrWires = faraday["wiresFromDimensions"][
                    msrDimension, {msrOffsetX + #1, 0, 0}];
                emf = eFieldStatic["inlineEmfForWireSetsOnWireSets"][
                    dIdt, srcWires, msrWires];
                AppendTo[staticAccelEmfCoplanar, micro emf["emfTotal"]]
            ) &,
            separations
        ];
    )
];

```

```

Scan[
  (
    msrWires = faraday["wiresFromDimensions"][msrDimension, {0, 0, #1}];
    emf = eFieldStatic["inlineEmfForWireSetsOnWireSets"][
      dIdt, srcWires, msrWires];
    AppendTo[staticAccelEmfCoaxial, micro emf["emfTotal"]]) &,
  separations
];
)
];
{staticAccelEmfCoplanar, staticAccelEmfCoaxial}
);

generateDynamicAccel[] := (
  If[Length[dynamicAccelEmfCoplanar] == 0,
    (
      eFieldDynamic = InductiveFieldEmfForRectangularCircuits[precision];
      eFieldDynamic["setFullAccelTermCoeff"][0];
      eFieldDynamic["setStaticAccelTermCoeff"][0];
      eFieldDynamic["setDynamicAccelTermCoeff"][1];

      srcWires = eFieldDynamic["wiresFromDimensions"][drvDimension];
      msrOffsetX = (drvDimension[[1]] + msrDimension[[1]])/2;

      Scan[
        (
          msrWires = faraday["wiresFromDimensions"][
            msrDimension, {msrOffsetX + #1, 0, 0}];
          emf = eFieldDynamic["inlineEmfForWireSetsOnWireSets"][
            dIdt, srcWires, msrWires];
          AppendTo[dynamicAccelEmfCoplanar, micro emf["emfTotal"]]
        ) &,
        separations
      ];

      Scan[
        (
          msrWires = faraday["wiresFromDimensions"][msrDimension, {0, 0, #1}];
          emf = eFieldDynamic["inlineEmfForWireSetsOnWireSets"][
            dIdt, srcWires, msrWires];
          AppendTo[dynamicAccelEmfCoaxial, micro emf["emfTotal"]]) &,
        separations
      ];
    )
  ]
);

```

```

    )
  ];
  {dynamicAccelEmfCoplanar, dynamicAccelEmfCoaxial}
);

circuitAtSeparation[separation_, full_, static_, dynamic_] := (
  eField = InductiveFieldEmfForRectangularCircuits[precision];
  eField["setFullAccelTermCoeff"][full];
  eField["setStaticAccelTermCoeff"][static];
  eField["setDynamicAccelTermCoeff"][dynamic];

  srcWires = eField["wiresFromDimensions"][drvDimension];
  msrOffsetX = (drvDimension[[1]] + msrDimension[[1]])/2;

  msrWires = eField["wiresFromDimensions"][
    msrDimension, {msrOffsetX + separation, 0, 0}];
  <|"eField" → eField, "src" → srcWires, "msr" → msrWires|>
);

findMsrLoopEmfForSingleSrcWire[
  separation_, srcWireId_, full_, static_, dynamic_] := (
  circuit = circuitAtSeparation[separation, full, static, dynamic];
  Map[
    Function[{msrId},
      circuit["eField"]["inlineEmfForWires"][dIdt,
        circuit["src"][[srcWireId]], circuit["msr"][[msrId]]]
    ],
    Range[1, 4]
  ]
);

findMsrWireEmfForSrcLoop[separation_, msrId_, full_, static_, dynamic_] := (
  circuit = circuitAtSeparation[separation, full, static, dynamic];
  Map[
    Function[{srcWireId},
      circuit["eField"]["inlineEmfForWires"][
        dIdt, circuit["src"][[srcWireId]], circuit["msr"][[msrId]]]
    ],
    Range[1, 4]
  ]
);

writeWireToWireEmfsToTable[separation_] := (

```

```

resultsArray = {
  "      Full accel term EMF's ( $\mu$ V)", SpanFromLeft,
  SpanFromLeft, SpanFromLeft, SpanFromLeft, SpanFromLeft, "",
  "      Static term EMF's ( $\mu$ V)", SpanFromLeft, SpanFromLeft,
  SpanFromLeft, SpanFromLeft, SpanFromLeft, "",
  "      Dynamic term EMF's ( $\mu$ V)", SpanFromLeft,
  SpanFromLeft, SpanFromLeft, SpanFromLeft, SpanFromLeft},
{"", "i=1", "2", "3", "4", "Sum", " ", "", "i=1", "2",
  "3", "4", "Sum", " ", "", "i=1", "2", "3", "4", "Sum"}
};

sumFullEmf = {0, 0, 0, 0};
sumStaticEmf = {0, 0, 0, 0};
sumDynamicEmf = {0, 0, 0, 0};

For[j = 1, j ≤ 4, j++, (
  row = {If[j == 1, "j=1", TextString[j]]};
  fullEmfs = findMsrWireEmfForSrcLoop[separation, j, 1, 0, 0];
  For[i = 1, i ≤ 4, i++, (
    sumFullEmf[[i]] += micro fullEmfs[[i]];
    row = Join[row, {format[micro fullEmfs[[i]]}]}
  );
  row = Join[row,
    {format[micro Total[fullEmfs]], "", If[j == 1, "j=1", TextString[j]]}];

  staticEmfs = findMsrWireEmfForSrcLoop[separation, j, 0, 1, 0];
  For[i = 1, i ≤ 4, i++, (
    sumStaticEmf[[i]] += micro staticEmfs[[i]];
    row = Join[row, {format[micro staticEmfs[[i]]}]}
  );
  row = Join[row, {format[micro Total[staticEmfs]],
    "", If[j == 1, "j=1", TextString[j]]}];

  dynamicEmfs = findMsrWireEmfForSrcLoop[separation, j, 0, 0, 1];
  For[i = 1, i ≤ 4, i++, (
    sumDynamicEmf[[i]] += micro dynamicEmfs[[i]];
    row = Join[row, {format[micro dynamicEmfs[[i]]}]}
  );
  row = Join[row, {format[micro Total[dynamicEmfs]]}];

  AppendTo[resultsArray, row];
)
];

```

```

AppendTo[resultsArray, {
  "Sum", format[sumFullEmf[[1]], 1], format[sumFullEmf[[2]]], format[
    sumFullEmf[[3]]], format[sumFullEmf[[4]]], format[Total[sumFullEmf]],
  "", "", format[sumStaticEmf[[1]]], format[sumStaticEmf[[2]]],
  format[sumStaticEmf[[3]]], format[sumStaticEmf[[4]]],
  format[Total[sumStaticEmf]],
  "", "", format[sumDynamicEmf[[1]]], format[sumDynamicEmf[[2]]],
  format[sumDynamicEmf[[3]]], format[sumDynamicEmf[[4]]],
  format[Total[sumDynamicEmf]]
}];
TextGrid[resultsArray, Frame → All]
);

getFaradayEmfCoplanar[] := (faradayEmfCoplanar);
getFaradayEmfCoaxial[] := (faradayEmfCoaxial);
getStaticAccelEmfCoplanar[] := (staticAccelEmfCoplanar);
getStaticAccelEmfCoaxial[] := (staticAccelEmfCoaxial);
getDynamicAccelEmfCoplanar[] := (dynamicAccelEmfCoplanar);
getDynamicAccelEmfCoaxial[] := (dynamicAccelEmfCoaxial);
getFullAccelEmfCoplanar[] := (fullAccelEmfCoplanar);
getFullAccelEmfCoaxial[] := (fullAccelEmfCoaxial);

generateAllResults[];

this = this["makePublic"][this, {
  "writeResultsToTable", writeResultsToTable,
  "writeWireToWireEmfsToTable", writeWireToWireEmfsToTable,
  "findMsrLoopEmfForSingleSrcWire", findMsrLoopEmfForSingleSrcWire,
  "findMsrWireEmfForSrcLoop", findMsrWireEmfForSrcLoop,
  "circuitAtSeparation", circuitAtSeparation,
  "separations", separations,
  "getFaradayEmfCoplanar", getFaradayEmfCoplanar,
  "getFaradayEmfCoaxial", getFaradayEmfCoaxial,
  "getStaticAccelEmfCoplanar", getStaticAccelEmfCoplanar,
  "getStaticAccelEmfCoaxial", getStaticAccelEmfCoaxial,
  "getDynamicAccelEmfCoplanar", getDynamicAccelEmfCoplanar,
  "getDynamicAccelEmfCoaxial", getDynamicAccelEmfCoaxial,
  "getFullAccelEmfCoplanar", getFullAccelEmfCoplanar,
  "getFullAccelEmfCoaxial", getFullAccelEmfCoaxial,
  "generateAllResults", generateAllResults,
  "generateFaraday", generateFaraday,
  "generateFullAccel", generateFullAccel,
  "generateStaticAccel", generateStaticAccel,

```



```

    "generateDynamicAccel", generateDynamicAccel
  ]];
  this
];

```

```

GraphicsForComparisonOfEmfForRectangularCircuits[
  analysis_ (* instance of ComparisonOfEmfForRectangularCircuits *)
] := Module[
  {
    (* public values *)
    this = <|>,

    (* functions *)
    plotEmfsVsSeparation,

    (* constants *)

    (* local values *)
    separations, faradayEmfCoplanar, staticAccelEmfCoplanar,
    dynamicAccelEmfCoplanar, fullAccelEmfCoplanar, resultsArray, row,
    scale, dynScale, markers, dynMarker, micro = 1.0*^6, milli = 1.0*^3
  },

  this = BaseObject[];

  separations = analysis["separations"];
  faradayEmfCoplanar = analysis["getFaradayEmfCoplanar"][];
  staticAccelEmfCoplanar = analysis["getStaticAccelEmfCoplanar"][];
  dynamicAccelEmfCoplanar = analysis["getDynamicAccelEmfCoplanar"][];
  fullAccelEmfCoplanar = analysis["getFullAccelEmfCoplanar"][];

  format[number_, decimalPrec_: 4] :=
    (TextString[Round[number 10^decimalPrec] / 10^decimalPrec]);
  (* scale, dynScale, markers, dynMarker *)
  plotEmfsVsSeparation[] := (
    Print["Induced EMF for circuit separation"];

    {circle, uptriangle, diamond, square, downtriangle} =
      Charting`CommonDump`GraphicsOpenPlotMarkers[][[;; 5]];

    scale = 1.2;
    dynScale = 1.7;
    markers = {circle, square, diamond} /. {p : _Disk | _Polygon => {FaceForm[], p},

```

```

    Offset[{a_, b_}] := Offset[Scale[{a, b}]];
    dynMarker = {uptriangle} /. {p : _Disk | _Polygon := {FaceForm[], p},
    Offset[{a_, b_}] := Offset[dynScale[{a, b}]]};
    markers = Insert[markers, dynMarker[[1]], 2];

    ListPlot[
    {
    Transpose[{separations, faradayEmfCoplanar}],
    Transpose[{separations, dynamicAccelEmfCoplanar}],
    Transpose[{separations, staticAccelEmfCoplanar}],
    Transpose[{separations, fullAccelEmfCoplanar}]
    },
    ImageSize -> 350,
    Joined -> True,
    InterpolationOrder -> 3,
    PlotMarkers -> markers,
    Frame -> {{True, False}, {True, False}},
    BaseStyle -> {FontSize -> 10},
    PlotRange -> {Automatic, {-13.5, 13.5}},
    AspectRatio -> 0.75,
    PlotStyle -> {Directive[Darker[Blue], Thick], Dashed, Thick, Dashed},
    PlotLegends ->
    Placed[{"Faraday", "Dynamic", "Static", "Full"}, {0.75, 0.75}],
    FrameLabel -> {
    Style["Separation s (mm)", 11],
    Style["EMF ( $\mu$ V)", 11]
    }
    ];

    this = this["makePublic"][this, {
    "plotEmfsVsSeparation", plotEmfsVsSeparation
    }];
    this
];

```

The following is for debugging and development of the plot of EMF versus separation. Note the reduced precision requested.

```
(* comparisonNearTest = ComparisonOfEmfForRectangularCircuits[
  {0.005, 0.010, 0.025, 0.05, 0.075, 0.1}, (* The list of separations,
  in meters, for co-planar and co-axial separations *)
  {0.1, 0.1}, (* The width and length (height in the paper)
  of the driven circuit, in meters *)
  {0.05, 0.04}, (* The width and length (height in the paper)
  of the measured circuit, in meters *)
  4
];
comparisonGraphicsTest =
  GraphicsForComparisonOfEmfForRectangularCircuits[comparisonNearTest];
comparisonGraphicsTest["plotEmfsVsSeparation"][] *)
```

Module for analysis of precision and convergence

The calculation and tabulation of all results is done with this class.

```
AnalysisOfPrecisionForRectangularCircuits[
  separation_,
  drvDimension_,
  msrDimension_,
  precisions_
] := Module[
  {
    (* public values *)
    this = <||>,

    (* public functions *)
    generateEmfValues, writeResultsToTable, writeWireToWireEmfsToTable,
    findMsrLoopEmfForSingleSrcWire, findMsrWireEmfForSrcLoop, generateFaraday,
    generateFullAccel, generateStaticAccel, generateDynamicAccel,
    circuitAtSeparation, plotCharts, getFaradayEmfCoplanar,
    getDynamicAccelEmfCoplanar, getFullAccelEmfCoplanar,

    (* constants *)
    dIdt = 1000.0,

    (* local values *)
    faraday, eFieldFull, eFieldDynamic, faradayEmfCoplanar,
    fullAccelEmfCoplanar, dynamicAccelEmfCoplanar, resultsArray, srcWires,
    msrWires, msrOffsetX, emf, first, eField, emfs10nMsr, circuit, row, sumFullEmf,
    sumStaticEmf, sumDynamicEmf, precNum, precSci, msrWireEmfs, fullEmfs,
    staticEmfs, dynamicEmfs, inlineEmf, micro = 1.0*^6, milli = 1.0*^3
```

```

},

this = BaseObject[];
faradayEmfCoplanar = {};
dynamicAccelEmfCoplanar = {};
fullAccelEmfCoplanar = {};

generateEmfValues[] := (
  generateFaraday[];
  generateFullAccel[];
  generateDynamicAccel[];
);

writeResultsToTable[] := (
  If[Length[faradayEmfCoplanar] == 0, generateEmfValues[]];

  resultsArray = {
    {"precision", "", "Faraday",
      "Difference between Faraday and dynamic accel", "",
      "Full accel", "Difference of full accel from zero"}
  };
  precNum = 6;
  precSci = 3;
  For[i = 1, i <= (precisions // Length), i++,
    row = {precisions[[i]]};
    row = Join[row, {" "}];
    row =
      Join[row, {DecimalForm[faradayEmfCoplanar[[i]], precNum], ScientificForm[
        Abs[faradayEmfCoplanar[[i]] - dynamicAccelEmfCoplanar[[i]], precSci],
        " ", DecimalForm[fullAccelEmfCoplanar[[i]], {precNum, precNum}],
        ScientificForm[Abs[fullAccelEmfCoplanar[[i]]], precSci]}];
    AppendTo[resultsArray, row]
  ];
  TextGrid[resultsArray, Frame → All,
    ItemSize → {{10}, {2}, {10}, {10}, {2}, {10}, {10}}]
  );

format[number_, decimalPrec_: 4] :=
  (TextString[Round[number 10^decimalPrec] / 10^decimalPrec]);

generateFaraday[] := (
  Scan[

```

```

(
  faraday = FaradayEmfForRectangularCircuits[#1];
  srcWires = faraday["wiresFromDimensions"][drvDimension];
  msrOffsetX = (drvDimension[[1]] + msrDimension[[1]])/2;
  msrWires = faraday["wiresFromDimensions"][
    msrDimension, {msrOffsetX + separation, 0, 0}];
  emf = faraday["faradaysEmfWireSetToWireSet"][dIdt, srcWires, msrWires];
  msrWires = faraday["wiresFromDimensions"][
    msrDimension, {msrOffsetX + separation, 0, 0}];
  emf = faraday["faradaysEmfWireSetToWireSet"][dIdt, srcWires, msrWires];
  AppendTo[faradayEmfCoplanar, micro emf["emfTotal"]]
) &,
precisions
];

faradayEmfCoplanar
);

generateFullAccel[] := (

Scan[
  (
    eFieldFull = InductiveFieldEmfForRectangularCircuits[#1];
    eFieldFull["setFullAccelTermCoeff"][1];
    eFieldFull["setStaticAccelTermCoeff"][0];
    eFieldFull["setDynamicAccelTermCoeff"][0];

    srcWires = eFieldFull["wiresFromDimensions"][drvDimension];
    msrOffsetX = (drvDimension[[1]] + msrDimension[[1]])/2;
    msrWires = faraday["wiresFromDimensions"][
      msrDimension, {msrOffsetX + separation, 0, 0}];
    emf = eFieldFull["inlineEmfForWireSetsOnWireSets"][
      dIdt, srcWires, msrWires];
    AppendTo[fullAccelEmfCoplanar, micro emf["emfTotal"]]
  ) &,
precisions
];

fullAccelEmfCoplanar
);

generateDynamicAccel[] := (
  Scan[

```

```

(
  eFieldDynamic = InductiveFieldEmfForRectangularCircuits[#1];
  eFieldDynamic["setFullAccelTermCoeff"][0];
  eFieldDynamic["setStaticAccelTermCoeff"][0];
  eFieldDynamic["setDynamicAccelTermCoeff"][1];

  srcWires = eFieldDynamic["wiresFromDimensions"][drvDimension];
  msrOffsetX = (drvDimension[[1]] + msrDimension[[1]])/2;

  msrWires = faraday["wiresFromDimensions"][
    msrDimension, {msrOffsetX + separation, 0, 0}];
  emf = eFieldDynamic["inlineEmfForWireSetsOnWireSets"][
    dIdt, srcWires, msrWires];
  AppendTo[dynamicAccelEmfCoplanar, micro emf["emfTotal"]]
) &,
precisions
];

dynamicAccelEmfCoplanar
);

plotCharts[] := (
  If[Length[faradayEmfCoplanar] == 0, generateEmfValues[]];
  Grid[{{
    ListPlot[
      Transpose[
        {precisions, Abs[faradayEmfCoplanar - dynamicAccelEmfCoplanar]}],
      ImageSize → 600,
      Joined → True,
      PlotMarkers → {Automatic, 8},
      ScalingFunctions → "Log",
      Frame → {{True, False}, {True, False}},
      (*PlotLabel→"Difference Faraday to Dyn. Accel vs Precision",*)
      FrameLabel → {"Requested precision of integration",
        "Normalized EMF difference ( $\mu\text{V}$ )"}],
    },
  {
    ListPlot[
      Transpose[{precisions, Abs[fullAccelEmfCoplanar]}],
      ImageSize → Large,
      Joined → True,
      PlotMarkers → {Automatic, 8},
      ScalingFunctions → "Log",

```

```

        Frame → {{True, False}, {True, False}},
        (*PlotLabel→"Difference Full Accel to Zero vs Precision",*)
        FrameLabel → {"Requested precision of integration",
            "Normalized EMF difference ( $\mu\text{V}$ )"}]
    }]]
);

getFaradayEmfCoplanar[] := (faradayEmfCoplanar);
getDynamicAccelEmfCoplanar[] := (dynamicAccelEmfCoplanar);
getFullAccelEmfCoplanar[] := (fullAccelEmfCoplanar);

generateEmfValues[];

this = this["makePublic"][this, {
    "generateEmfValues", generateEmfValues,
    "writeResultsToTable", writeResultsToTable,
    "plotCharts", plotCharts,
    "writeWireToWireEmfsToTable", writeWireToWireEmfsToTable,
    "findMsrLoopEmfForSingleSrcWire", findMsrLoopEmfForSingleSrcWire,
    "findMsrWireEmfForSrcLoop", findMsrWireEmfForSrcLoop,
    "circuitAtSeparation", circuitAtSeparation,
    "generateFaraday", generateFaraday,
    "generateFullAccel", generateFullAccel,
    "generateDynamicAccel", generateDynamicAccel,
    "precisions", precisions,
    "getFaradayEmfCoplanar", getFaradayEmfCoplanar,
    "getDynamicAccelEmfCoplanar", getDynamicAccelEmfCoplanar,
    "getFullAccelEmfCoplanar", getFullAccelEmfCoplanar
}];
this
];

```

```

precisionAnalysis = AnalysisOfPrecisionForRectangularCircuits[
    0.010, (* Separation *)
    {0.1, 0.1}, (* The width and length
        (height in the paper) of the driven circuit, in meters *)
    {0.05, 0.04}, (* The width and length (height in the paper)
        of the measured circuit, in meters *)
    {3, 6, 9, 12, 15} (* The precision values *)
];

```

```

GraphicsForPrecisionForRectangularCircuits[
    analysis_ (* instance of AnalysisOfPrecisionForRectangularCircuits *)
] := Module[

```



```

    ItemSize → {{10}, {2}, {10}, {10}, {2}, {10}, {10}}]
  );

format[number_, decimalPrec_: 4] :=
  (TextString[Round[number 10^decimalPrec]/10^decimalPrec]);

plotAbsFaradayDiffDynamic[] := (
  Print["Difference of Faraday to dynamic
    acceleration equation for requested precision"];
  ListPlot[
    Transpose[
      {precisions, Abs[faradayEmfCoplanar - dynamicAccelEmfCoplanar]}],
    ImageSize → 300,
    Joined → True,
    PlotMarkers → {Automatic, 8},
    ScalingFunctions → "Log",
    Frame → {{True, False}, {True, False}},
    BaseStyle → {FontSize → 10},
    (*ImagePadding→{{100, None}, {None, None}}, *)
    (*PlotLabel→"Difference Faraday to Dyn. Accel vs Precision", *)
    FrameLabel → {
      Style["Requested precision of integration", 10],
      Style["Normalized EMF difference (μV)", 10]
    }
  ]
);

plotAbsFullDiffZero[] := (
  Print["Difference of full Feynman
    acceleration EMF to zero for requested precision"];
  ListPlot[
    Transpose[{precisions, Abs[fullAccelEmfCoplanar]}],
    ImageSize → 300,
    Joined → True,
    PlotMarkers → {Automatic, 8},
    ScalingFunctions → "Log",
    Frame → {{True, False}, {True, False}},
    BaseStyle → {FontSize → 10},
    (*PlotLabel→"Difference Full Accel to Zero vs Precision", *)
    FrameLabel → {
      Style["Requested precision of integration", 10],
      Style["Normalized EMF difference (μV)", 10]
    }
  ]
);

```

```

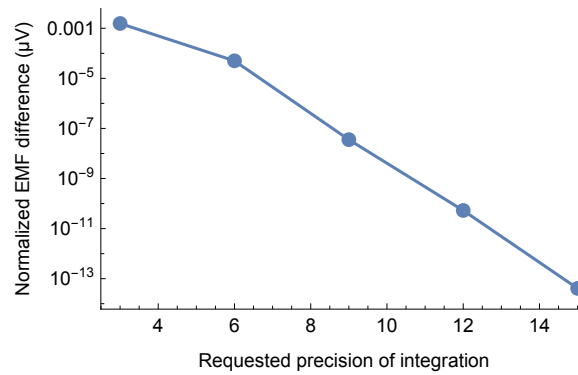
this = this["makePublic"][this, {
  "writeResultsToTable", writeResultsToTable,
  "plotAbsFaradayDiffDynamic", plotAbsFaradayDiffDynamic,
  "plotAbsFullDiffZero", plotAbsFullDiffZero,
  "writeWireToWireEmfsToTable", writeWireToWireEmfsToTable
}];
this
];

```

```
graphics = GraphicsForPrecisionForRectangularCircuits[precisionAnalysis];
```

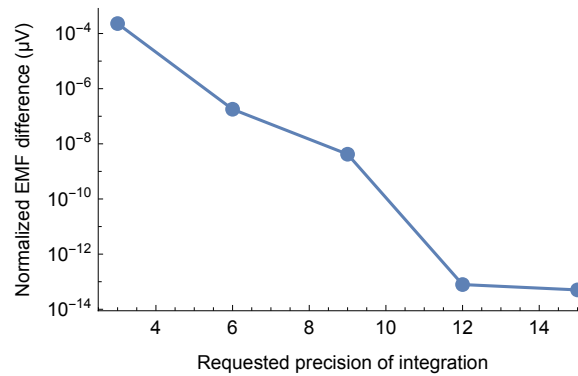
```
graphics["plotAbsFaradayDiffDynamic"][]
```

Difference of Faraday to dynamic acceleration equation for requested precision



```
graphics["plotAbsFullDiffZero"][]
```

Difference of full Feynman acceleration EMF to zero for requested precision



```
precisionAnalysis["writeResultsToTable"] []
```

| | Faraday | Difference between Faraday and dynamic accel | | Full accel | Difference of full accel |
|--|---------|--|--|------------|--------------------------|
| | 8.09835 | 1.55×10^{-3} | | -0.000230 | 2.3×10^{-4} |
| | 8.09668 | 5.01×10^{-5} | | -0.000000 | 1.79×10^{-7} |
| | 8.09663 | 3.56×10^{-8} | | 0.000000 | 4.2×10^{-9} |
| | 8.09663 | 5.29×10^{-11} | | 0.000000 | 7.88×10^{-14} |
| | 8.09663 | 4.09×10^{-14} | | 0.000000 | 5.04×10^{-14} |

Module for comparison of single wire EMFs generated on full circuits by Faradays law and electric field terms

One question that arose late was what the EMF was for a single finite length wire on a full measured circuit for Faraday's law and comparing that to the electric field terms. The question did not initially arise because the initial results compared full circuits found in real world examples.

The calculation and tabulation of all results is done with this class.

```
ComparisonOfSingleDrivenWireEmf[
  separations_,
  drvX_,
  drvWireWidth_,
  msrDimension_,
  precision_
] := Module[
{
  (* public values *)
  this = <||>,

  (* functions *)
  writeResultsToTable,
  writeWireToWireEmfsToTable, findMsrLoopEmfForSingleSrcWire,
  findMsrWireEmfForSrcLoop, generateFaraday, generateFullAccel,
  generateStaticAccel, generateDynamicAccel, circuitAtSeparation,

  (* constants *)
  dIdt = 1000.0,

  (* local values *)
  drvWire, log, i,
  faraday, eFieldFull, eFieldStatic, eFieldDynamic, faradayEmfCoplanar,
  fullAccelEmfCoplanar, staticAccelEmfCoplanar, dynamicAccelEmfCoplanar,
```

```

    faradayEmfCoaxial, fullAccelEmfCoaxial, staticAccelEmfCoaxial,
    dynamicAccelEmfCoaxial, resultsArray, srcWires, msrWires,
    msrOffsetX, emf, first, eField, emfs10nMsr, circuit, row, sumFullEmf,
    sumStaticEmf, sumDynamicEmf, prec = 6, msrWireEmfs, fullEmfs,
    staticEmfs, dynamicEmfs, inlineEmf, micro = 1.0*^6, milli = 1.0*^3
},
this = BaseObject[];
faradayEmfCoplanar = {};
staticAccelEmfCoplanar = {};
dynamicAccelEmfCoplanar = {};
fullAccelEmfCoplanar = {};
faradayEmfCoaxial = {};
staticAccelEmfCoaxial = {};
dynamicAccelEmfCoaxial = {};
fullAccelEmfCoaxial = {};
log = this["log"];

writeResultsToTable[] := (
    generateFaraday[];
    generateFullAccel[];
    generateStaticAccel[];
    generateDynamicAccel[];

    resultsArray = {
        {"", "", "          Coplanar ( $\mu$ V)", SpanFromLeft,
        SpanFromLeft, SpanFromLeft, "", "          Coaxial ( $\mu$ V)",
        SpanFromLeft, SpanFromLeft, SpanFromLeft},
        {"separation (mm)", "", "Faraday", "Full accel",
        "Static accel", "Dynamic accel", "    ", "Faraday",
        "Full accel", "Static accel", "Dynamic accel"}
    };
    For[i = 1, i <= (separations // Length), i++,
        row = {milli separations[[i]]};
        row = Join[row, {"    "}];
        row = Join[row, {NumberForm[faradayEmfCoplanar[[i]], prec],
        NumberForm[fullAccelEmfCoplanar[[i]], prec],
        NumberForm[staticAccelEmfCoplanar[[i]], prec],
        NumberForm[dynamicAccelEmfCoplanar[[i]], prec]}}];
        row = Join[row, {"    "}];
        row = Join[row, {NumberForm[faradayEmfCoaxial[[i]], prec],
        NumberForm[fullAccelEmfCoaxial[[i]], prec],
        NumberForm[staticAccelEmfCoaxial[[i]], prec],
        NumberForm[dynamicAccelEmfCoaxial[[i]], prec]}}];

```

```

AppendTo[resultsArray, row]
];
TextGrid[resultsArray, Frame → All]
);

format[number_, decimalPrec_: 4] :=
(TextString[Round[number 10^decimalPrec] / 10^decimalPrec]);

drvWire = <| "start" → {drvX, 0.5 drvWireWidth, 0.0},
  "end" → {drvX, -0.5 drvWireWidth, 0.0},
  "wireId" → 1|>;

generateFaraday[] := (
  If[Length[faradayEmfCoplanar] == 0,
    (
      faraday = FaradayEmfForRectangularCircuits[precision];

      Scan[
        (
          msrWires = faraday["wiresFromDimensions"][msrDimension, {#1, 0, 0}];
          emf = faraday["faradaysEmfWireToWireSet"][dIdt, drvWire, msrWires];
          AppendTo[faradayEmfCoplanar, micro emf] &,
          separations
        );
      ];

      Scan[
        (
          msrWires = faraday["wiresFromDimensions"][msrDimension, {0, 0, #1}];
          emf = faraday["faradaysEmfWireToWireSet"][dIdt, drvWire, msrWires];
          AppendTo[faradayEmfCoaxial, micro emf] &,
          separations
        );
      ];
      {faradayEmfCoplanar, faradayEmfCoaxial}
    );
  );
(*
generateFullAccel[] := (
  If[Length[fullAccelEmfCoplanar] == 0,
    (
      eFieldFull = InductiveFieldEmfForRectangularCircuits[precision];
      eFieldFull["setFullAccelTermCoeff"][1];
      eFieldFull["setStaticAccelTermCoeff"][0];

```

```

eFieldFull["setDynamicAccelTermCoeff"][0];

srcWires = eFieldFull["wiresFromDimensions"][drvDimension];
msrOffsetX = (drvDimension[[1]]+msrDimension[[1]])/2;

Scan[
(
  msrWires =
    faraday["wiresFromDimensions"][msrDimension, {msrOffsetX+#1,0,0}];
  emf = eFieldFull["inlineEmfForWireSetsOnWireSets"][
    dIdt,srcWires,msrWires];
  AppendTo[fullAccelEmfCoplanar,micro emf["emfTotal"]]
)&,
separations
];

Scan[
(
  msrWires = faraday["wiresFromDimensions"][msrDimension, {0,0,#1}];
  emf = eFieldFull["inlineEmfForWireSetsOnWireSets"][
    dIdt,srcWires,msrWires];
  AppendTo[fullAccelEmfCoaxial,micro emf["emfTotal"]])&,
separations
];
)
];
{fullAccelEmfCoplanar,fullAccelEmfCoaxial}
);
*)
generateStaticAccel[] := (
  If[Length[staticAccelEmfCoplanar] == 0,
  (
    eFieldStatic = InductiveFieldEmfForRectangularCircuits[precision];
    eFieldStatic["setFullAccelTermCoeff"][0];
    eFieldStatic["setStaticAccelTermCoeff"][1];
    eFieldStatic["setDynamicAccelTermCoeff"][0];

    Scan[
      (
        msrWires = faraday["wiresFromDimensions"][msrDimension, {#1, 0, 0}];
        emf =
          eFieldStatic["inlineEmfForWireOnWireSets"][dIdt, drvWire, msrWires];
        AppendTo[staticAccelEmfCoplanar, micro emf["emfTotal"]]

```

```

    ) &,
    separations
  ];

  Scan[
    (
      msrWires = faraday["wiresFromDimensions"][msrDimension, {0, 0, #1}];
      emf =
        eFieldStatic["inlineEmfForWireOnWireSets"][dIdt, drvWire, msrWires];
      AppendTo[staticAccelEmfCoaxial, micro emf["emfTotal"]]) &,
    separations
  ];
)
];
{staticAccelEmfCoplanar, staticAccelEmfCoaxial}
);

generateDynamicAccel[] := (
  If[Length[dynamicAccelEmfCoplanar] == 0,
    (
      eFieldDynamic = InductiveFieldEmfForRectangularCircuits[precision];
      eFieldDynamic["setFullAccelTermCoeff"][0];
      eFieldDynamic["setStaticAccelTermCoeff"][0];
      eFieldDynamic["setDynamicAccelTermCoeff"][1];

      Scan[
        (
          msrWires = faraday["wiresFromDimensions"][msrDimension, {#1, 0, 0}];
          emf = eFieldDynamic[
            "inlineEmfForWireOnWireSets"][dIdt, drvWire, msrWires];
          AppendTo[dynamicAccelEmfCoplanar, micro emf["emfTotal"]]
        ) &,
        separations
      ];

      Scan[
        (
          msrWires = faraday["wiresFromDimensions"][msrDimension, {0, 0, #1}];
          emf = eFieldDynamic[
            "inlineEmfForWireOnWireSets"][dIdt, drvWire, msrWires];
          AppendTo[dynamicAccelEmfCoaxial, micro emf["emfTotal"]]) &,
        separations
      ];
    )
  ];
)

```

```

    )
  ];
  {dynamicAccelEmfCoplanar, dynamicAccelEmfCoaxial}
);
(*
circuitAtSeparation[separation_, full_, static_,dynamic_] := (
  eField = InductiveFieldEmfForRectangularCircuits[precision];
  eField["setFullAccelTermCoeff"][full];
  eField["setStaticAccelTermCoeff"][static];
  eField["setDynamicAccelTermCoeff"][dynamic];

  srcWires = eField["wiresFromDimensions"][drvDimension];
  msrOffsetX = (drvDimension[[1]]+msrDimension[[1]])/2;

  msrWires =
    eField["wiresFromDimensions"][msrDimension, {msrOffsetX+separation,0,0}];
  <|"eField"→eField, "src"→srcWires,"msr"→ msrWires|>
);

findMsrLoopEmfForSingleSrcWire[
  separation_, srcWireId_, full_, static_,dynamic_] := (
  circuit = circuitAtSeparation[separation, full,static,dynamic];
  Map[
    Function[{msrId},
      circuit["eField"]["inlineEmfForWires"][dIdt,
        circuit["src"][[srcWireId]], circuit["msr"][[msrId]]]
    ],
    Range[1,4]
  ]
);

findMsrWireEmfForSrcLoop[separation_, msrId_, full_, static_,dynamic_] := (
  circuit = circuitAtSeparation[separation, full,static,dynamic];
  Map[
    Function[{srcWireId},
      circuit["eField"]["inlineEmfForWires"][
        dIdt,circuit["src"][[srcWireId]], circuit["msr"][[msrId]]]
    ],
    Range[1,4]
  ]
);

writeWireToWireEmfsToTable[separation_] :=(

```



```

resultsArray = {
  "      Full accel term EMF's ( $\mu$ V)", SpanFromLeft,
  SpanFromLeft, SpanFromLeft, SpanFromLeft, SpanFromLeft, "",
  "      Static term EMF's ( $\mu$ V)", SpanFromLeft, SpanFromLeft,
  SpanFromLeft, SpanFromLeft, SpanFromLeft, "",
  "      Dynamic term EMF's ( $\mu$ V)", SpanFromLeft,
  SpanFromLeft, SpanFromLeft, SpanFromLeft, SpanFromLeft},
{"", "i=1", "2", "3", "4", "Sum", " ", "", "i=1", "2", "3",
  "4", "Sum", " ", "", "i=1", "2", "3", "4", "Sum"}
};

sumFullEmf = {0,0,0,0};
sumStaticEmf = {0,0,0,0};
sumDynamicEmf = {0,0,0,0};

For[j=1, j≤4, j++, (
  row = {If[j==1, "j=1", TextString[j]]};
  fullEmfs = findMsrWireEmfForSrcLoop[separation, j, 1, 0, 0];
  For[i=1, i≤4, i++, (
    sumFullEmf[[i]] += micro fullEmfs[[i]];
    row = Join[row, {format[micro fullEmfs[[i]]]}]
  )];
  row = Join[row,
    {format[micro Total[fullEmfs]], "", If[j==1, "j=1", TextString[j]]}];

  staticEmfs = findMsrWireEmfForSrcLoop[separation, j, 0, 1, 0];
  For[i=1, i≤4, i++, (
    sumStaticEmf[[i]] += micro staticEmfs[[i]];
    row = Join[row, {format[micro staticEmfs[[i]]]}]
  )];
  row = Join[row,
    {format[micro Total[staticEmfs]], "", If[j==1, "j=1", TextString[j]]}];

  dynamicEmfs = findMsrWireEmfForSrcLoop[separation, j, 0, 0, 1];
  For[i=1, i≤4, i++, (
    sumDynamicEmf[[i]] += micro dynamicEmfs[[i]];
    row = Join[row, {format[micro dynamicEmfs[[i]]]}]
  )];
  row = Join[row, {format[micro Total[dynamicEmfs]]}];

  AppendTo[resultsArray, row];
)
];

```

```

AppendTo[resultsArray,{
  "Sum", format[sumFullEmf[[1]],1],format[sumFullEmf[[2]]],format[
    sumFullEmf[[3]]],format[sumFullEmf[[4]]],format[Total[sumFullEmf]],
  "", "",format[sumStaticEmf[[1]]],format[sumStaticEmf[[2]]],
  format[sumStaticEmf[[3]]],format[sumStaticEmf[[4]]],
  format[Total[sumStaticEmf]],
  "", "",format[sumDynamicEmf[[1]]],format[sumDynamicEmf[[2]]],
  format[sumDynamicEmf[[3]]],format[sumDynamicEmf[[4]]],
  format[Total[sumDynamicEmf]]
}];
TextGrid[resultsArray, Frame→All]
);
*)

this = this["makePublic"][this, {
  "writeResultsToTable", writeResultsToTable,
  "writeWireToWireEmfsToTable", writeWireToWireEmfsToTable,
  "findMsrLoopEmfForSingleSrcWire", findMsrLoopEmfForSingleSrcWire,
  "findMsrWireEmfForSrcLoop", findMsrWireEmfForSrcLoop,
  "circuitAtSeparation", circuitAtSeparation,
  "generateFaraday", generateFaraday,
  "generateFullAccel", generateFullAccel,
  "generateStaticAccel", generateStaticAccel,
  "generateDynamicAccel", generateDynamicAccel
}];
this
];

```

Unit tests for EMF from Faraday's law

This section contains the module for unit tests for EMF generated using Faraday's law. Any errors will be noted in the output with red text highlighting the expected and actual output.

```

UnitTestsForFaradayEmfForRectangularCircuits[
  verbose_: True, skipIntensiveTests_: False] := Module[
{
  (* public values *)
  this = <||>,

  (* functions *)
  EXPECTNEAR, emfById, skipTests, log,

  (* local values *)

```

```

numTestsPassed, numTestsFailed, numTestsSkipped, testNumber,
toUsed, actualTypeString, emfFdy, dIdtClosest, experimentalXList,
emfAtOffset59, wires21, srcWires, srcCorner1, srcCorner2, srcCorner3,
srcCorner4, srcCorners, msrCorner1, msrCorner2, msrCorner3, msrCorner4,
msrWiresClosest, msrWiresFarthest, msrCorners, emfFaradayAtClosest1,
emfFaradayAtClosest2, emfFaradayAtClosest3, emfFaradayAtClosest4,
faradayClosestSum, srcOffset, faradayClosestFull, msrOffsetClosest,
emfAtOffset7, msrOffsetFarthest, srcWires8, msrWires8, emf8,
srcWidth8, msrWidth8, sep8, msrWires9, emf9, msrWires10, emf10,
msrWires11, emf11, msrWires12, emf12, micro = 1000000, milli = 1000
},
this = UnitTestBaseObject[verbose, skipIntensiveTests];
EXPECTNEAR = this["EXPECTNEAR"];
skipTests = this["skipTests"];
log = this["log"];

dIdtClosest = 6110.3534985431;

emfFdy = FaradayEmfForRectangularCircuits[8];

srcCorner1 = {0.0, -0.0507425, 0.0};
srcCorner2 = {0.0, 0.0507425, 0.0};
srcCorner3 = {-0.101035, 0.0507425, 0.0};
srcCorner4 = {-0.101035, -0.0507425, 0.0};
srcCorners = {srcCorner1, srcCorner2, srcCorner3, srcCorner4};
srcOffset = ConstantArray[0.0, 3];

msrCorner1 = {0.0, 0.0195775, 0.0};
msrCorner2 = {0.0, -0.0195775, 0.0};
msrCorner3 = {0.050515, -0.0195775, 0.0};
msrCorner4 = {0.050515, 0.0195775, 0.0};
msrCorners = {msrCorner1, msrCorner2, msrCorner3, msrCorner4};
msrOffsetClosest = {0.000615, 0.0, 0.0};
msrOffsetFarthest = {0.039175, 0.0, 0.0};

srcWires = emfFdy["wiresFromCorners"][srcCorners, srcOffset, True];
msrWiresClosest =
  emfFdy["wiresFromCorners"][msrCorners, msrOffsetClosest, True];

msrWiresFarthest =
  emfFdy["wiresFromCorners"][msrCorners, msrOffsetFarthest, True];

experimentalXList = {0.000615, 0.000945, 0.002155, 0.003545, 0.005305, 0.006915,
  0.009085, 0.011315, 0.014395, 0.017285, 0.020965, 0.026325, 0.031905, 0.039175};

```

```

(* Test 1: faradaysEmfWireToWireSet from experimental data for near near *)
emfFaradayAtClosest1 = emfFdy["faradaysEmfWireToWireSet"] [
  dIdtClosest, srcWires[[1]], msrWiresClosest];
EXPECTNEAR[201.579, micro emfFaradayAtClosest1, 0.001];

(* Test 2: faradaysEmfWireToWireSet from experimental data for top near *)
emfFaradayAtClosest2 = emfFdy["faradaysEmfWireToWireSet"] [
  dIdtClosest, srcWires[[2]], msrWiresClosest];
EXPECTNEAR[-12.1687, micro emfFaradayAtClosest2, 0.001];

(* Test 3: faradaysEmfWireToWireSet from experimental data for top near *)
emfFaradayAtClosest3 = emfFdy["faradaysEmfWireToWireSet"] [
  dIdtClosest, srcWires[[3]], msrWiresClosest];
EXPECTNEAR[-7.24911, micro emfFaradayAtClosest3, 0.001];

(* Test 4: faradaysEmfWireToWireSet from experimental data for top near *)
emfFaradayAtClosest4 = emfFdy["faradaysEmfWireToWireSet"] [
  dIdtClosest, srcWires[[4]], msrWiresClosest];
EXPECTNEAR[-12.1687, micro emfFaradayAtClosest4, 0.001];

(* Test 5: faradaysEmfWireToWireSet from experimental data for top near *)
faradayClosestSum = emfFaradayAtClosest1 +
  emfFaradayAtClosest2 + emfFaradayAtClosest3 + emfFaradayAtClosest4;
EXPECTNEAR[169.992, micro faradayClosestSum, 0.001];

(* Test 6:
  faradaysEmfWireSetToWireSet from experimental data for closest separation *)
faradayClosestFull = emfFdy["faradaysEmfWireSetToWireSet"] [
  dIdtClosest, srcWires, msrWiresClosest];
EXPECTNEAR[169.992, micro faradayClosestFull["emfTotal"], 0.001];

(* Test 7: inlineEmfForWireOnWireSetsAtOffsets from experimental data *)
If[! skipTests[7],
  emfAtOffset7 = emfFdy["faradayEmfForWireOnWireSetsAtOffsets"] [
    dIdtClosest, srcCorners, msrCorners, experimentalXList];
  Print["emfsAtOffsets: " <> TextString[emfAtOffset7]];
  EXPECTNEAR[12.3345, micro emfAtOffset7[[14]][[2]], 0.0001];
];

(* Test 8: wireset with vertical displacement *)
srcWidth8 = 0.101035;
msrWidth8 = 0.050515;
sep8 = (srcWidth8 + msrWidth8) / 2 + msrOffsetClosest[[1]];

```

```

srcWires8 = emfFdy["wiresFromDimensions"][{srcWidth8, 2 (0.0507425)}];
msrWires8 =
  emfFdy["wiresFromDimensions"][{msrWidth8, 2 (0.0195775)}, {sep8, 0, 0}];

emf8 =
  emfFdy["faradaysEmfWireSetToWireSet"][dIdtClosest, srcWires8, msrWires8];
EXPECTNEAR[169.992, micro emf8["emfTotal"], 0.001];

(* Test 9: wireset with vertical displacement *)
msrWires9 =
  emfFdy["wiresFromDimensions"][{msrWidth8, 2 (0.0195775)}, {0, 0, 0.0}];
emf9 = emfFdy["faradaysEmfWireSetToWireSet"][
  dIdtClosest, srcWires8, msrWires9];
EXPECTNEAR[-148.064, micro emf9["emfTotal"], 0.001];

(* Test 10: wireset with vertical displacement *)
msrWires10 =
  emfFdy["wiresFromDimensions"][{msrWidth8, 2 (0.0195775)}, {0, 0, 0.01}];
emf10 = emfFdy["faradaysEmfWireSetToWireSet"][
  dIdtClosest, srcWires8, msrWires10];
EXPECTNEAR[-138.135, micro emf10["emfTotal"], 0.001];

(* Test 11: wireset with vertical displacement *)
msrWires11 =
  emfFdy["wiresFromDimensions"][{msrWidth8, 2 (0.0195775)}, {0, 0, 0.02}];
emf11 = emfFdy["faradaysEmfWireSetToWireSet"][
  dIdtClosest, srcWires8, msrWires11];
EXPECTNEAR[-115.356, micro emf11["emfTotal"], 0.001];

(* Test 12: wireset with vertical displacement *)
msrWires12 =
  emfFdy["wiresFromDimensions"][{msrWidth8, 2 (0.0195775)}, {0, 0, -0.02}];
emf12 = emfFdy["faradaysEmfWireSetToWireSet"][
  dIdtClosest, srcWires8, msrWires12];
EXPECTNEAR[-115.356, micro emf12["emfTotal"], 0.001];

this["printResults"][];

this = this["makePublic"][this, {
  }];
this
];
UnitTestsForFaradayEmfForRectangularCircuits[True, False];

```

```

Test: 1 Expected: 201.579 Actual: 201.578
Test: 2 Expected: -12.1687 Actual: -12.1687
Test: 3 Expected: -7.24911 Actual: -7.24911
Test: 4 Expected: -12.1687 Actual: -12.1687
Test: 5 Expected: 169.992 Actual: 169.992
Test: 6 Expected: 169.992 Actual: 169.992
emfsAtOffsets: {{0.000615, 0.000169992}, {0.000945, 0.000149907},
  {0.002155, 0.00011216}, {0.003545, 0.0000902518}, {0.005305, 0.0000733158},
  {0.006915, 0.0000627257}, {0.009085, 0.000052399}, {0.011315, 0.0000446053},
  {0.014395, 0.0000366718}, {0.017285, 0.0000311281}, {0.020965, 0.00002578},
  {0.026325, 0.0000201803}, {0.031905, 0.0000160765}, {0.039175, 0.0000123345}}
Test: 7 Expected: 12.3345 Actual: 12.3345
Test: 8 Expected: 169.992 Actual: 169.992
Test: 9 Expected: -148.064 Actual: -148.064
Test: 10 Expected: -138.135 Actual: -138.135
Test: 11 Expected: -115.356 Actual: -115.356
Test: 12 Expected: -115.356 Actual: -115.356
PASSED 12 tests

```

Unit tests for EMF from the inductive field

This section contains the module for unit tests for inductive field EMF. Any errors will be noted in the output with red text highlighting the expected and actual output.

```

UnitTestsForInductiveFieldEmfForRectangularCircuits[
  verbose_: True, skipIntensiveTests_: False] := Module[
  {
    (* public values *)
    this = <||>,

    (* functions *)
    EXPECTNEAR, emfById, skipTests, log,

    (* local values *)
    numTestsPassed, numTestsFailed, numTestsSkipped, testNumber, toUsed,
    actualTypeString, emf, r1, a1, v1, v2, v3, r4Drv, r4Msr, dIdt, eField0ldV4,
    eFieldV4, dIdt5, dIdtVec5, srcStart5, srcEnd5, msrPnt5, emf5, emf6, emf7,
    emf8, emf10, emf11, emf11ForIndex, emf12, emf13, emf14, emf15, emf16, emf17,
    emf18, emf19, emf20, emf28, emf29, emf30, emf31, emf36, emf37, emf38, emf39,
    emf44, emf46, emfAtOffset47, emf49, emf50, emf51, srcStart11, srcEnd11,
    msrPnt11, msrStart13, msrEnd13, srcStart15, srcEnd15, msrStart15, msrEnd15,

```

```

msrStart19, msrEnd19, srcCorner1, srcCorner2, srcCorner3, srcCorner4,
srcCorners, msrCorner1, msrCorner2, msrCorner3, msrCorner4, msrCorners,
msrOffsetClosest, msrOffsetFarthest, wires21, srcOffset, dIdtClosest,
srcWires, msrWiresClosest, msrWiresFarthest, experimentalXList,
emfAtOffset59, emfAtOffset51, rDrv67, rMsr67, emf41, emf42, emf43, emf48,
emfAtOffset49, emf54, emf55, emf56, emf57, emf58, emf59, emf60, emf61,
emf62, emf63, emf64, emf65, emf66, emf67, srcWidth68, msrWidth68, sep68,
srcWires68, msrWires68, emf68, msrWires69, emf69, msrWires70, emf70,
msrWires71, emf71, msrWires72, emf72, msrWires73, emf73, msrWires74, emf74,
msrWires75, emf75, msrWires76, emf76, srcWires77, msrWires77, dot77, sep77,
dIdt78, emf78, emf79, emf80, emf81, emf82, micro = 1000000, milli = 1000
},
this = UnitTestBaseObject[verbose, skipIntensiveTests];
EXPECTNEAR = this["EXPECTNEAR"];
skipTests = this["skipTests"];
log = this["log"];

dIdtClosest = 6110.3534985431;

(* Test 1*)
emf = InductiveFieldEmfForRectangularCircuits[6];
emf["setFullAccelTermCoeff"][1];
emf["setStaticAccelTermCoeff"][0];
emf["setDynamicAccelTermCoeff"][1];
(* Yes, both full and dynamic are 1 for these tests*)
r1 = {2, 3, 0}; a1 = {1, 1, 0};
v1 = emf["accelProduct"][a1, r1];
EXPECTNEAR[
  micro (Cross[Normalize[r1], Cross[Normalize[r1], a1]] - a1), micro v1, 0.01];

(* Test 2*)
v2 = emf["accelProduct"][{1, 0, 0}, {1, 1, 0}];
EXPECTNEAR[micro {-1.5, 0.5, 0}, micro v2, 0.00001];

(* Test 3*)
v3 = emf["accelProduct"][{1, 0, 0}, {0, 1, 0}];
EXPECTNEAR[micro {-2, 0, 0}, micro v3, 0.00001];

(* Test 4: compare emf to old method. It seems like the old version
  produced an EMF opposite of that calculated in the new code. *)
r4Drv = {0.000615, 0.0, 0};
r4Msr = {0.0, 0.0, 0};
dIdt = {dIdtClosest, 0, 0}; (* old used acceleration in x direction only*)

```

```

eFieldV4 = emf["emfAtPoint"][dIdt, r4Drv, r4Msr];
EXPECTNEAR[{-0.993553, 0.0, 0.0}, eFieldV4, 0.000001];

(* Tests 5-9: Get EMF at end points and middle point of Test 5 (because the
    test is currently failing). *)
dIdt5 = 1.0;
dIdtVec5 = {dIdt5, 0.0, 0.0};
srcStart5 = {-0.05, 0, 0};
srcEnd5 = {0.05, 0, 0};
msrPnt5 = {0.0, 0.002, 0};
emf5 = emf["emfAtPoint"][dIdtVec5, srcStart5, msrPnt5];
EXPECTNEAR[{-2.00159, 0.0798084, 0.0}, micro emf5, 0.000001];

emf6 = emf["emfAtPoint"][dIdtVec5, Mean[{srcStart5, srcEnd5}], msrPnt5];
EXPECTNEAR[{-100, 0, 0}, micro emf6, 0.000001];

emf7 = emf["emfAtPoint"][dIdtVec5, srcEnd5, msrPnt5];
EXPECTNEAR[{-2.00159, -0.0798084, 0.0}, micro emf7, 0.000001];
EXPECTNEAR[micro emf5[[1]], micro emf7[[1]], 0.000001];
EXPECTNEAR[micro emf5[[2]], -micro emf7[[2]], 0.000001];

(* Test 10: emfFromWireToWire *)
srcStart11 = {0.0, -0.05, 0.0};
srcEnd11 = {0.0, 0.05, 0.0};
msrPnt11 = {0.002, 0.0, 0.0};
msrStart13 = {-0.05, 0.002, 0.0};
msrEnd13 = {0.05, 0.002, 0.0};
emf10 = emf["emfFromWireToWire"][
    dIdt5,
    srcStart5, srcEnd5,
    msrStart13, msrEnd13];
EXPECTNEAR[-0.0921054, micro emf10, 0.000001];

(* Test 11: emfFromWireToWire perpendicular to Test 13 *)
srcStart15 = {srcStart5[[2]], srcStart5[[1]], 0.0};
srcEnd15 = {srcEnd5[[2]], srcEnd5[[1]], 0.0};
msrStart15 = {msrStart13[[2]], msrStart13[[1]], 0.0};
msrEnd15 = {msrEnd13[[2]], msrEnd13[[1]], 0.0};
emf11 = emf["emfFromWireToWire"][
    dIdt5,
    srcStart15, srcEnd15,
    msrStart15, msrEnd15];
EXPECTNEAR[-0.0921054, micro emf11, 0.000001];

```



```

(* Test 12: emfFromWireToWire as per
   Test 13 with measured wire at 30 degrees from parallel *)
msrStart19 = {-0.025, 0.001, 0.0};
msrEnd19 = {0.025, 0.001 + 0.05 Sin[30 Degree], 0.0};
emf12 = emf["emfFromWireToWire"][
  dIdt5,
  srcStart5, srcEnd5,
  msrStart19, msrEnd19];
EXPECTNEAR[-0.0313341, micro emf12, 0.000001];

srcCorner1 = {0.0, -0.0507425, 0.0};
srcCorner2 = {0.0, 0.0507425, 0.0};
srcCorner3 = {-0.101035, 0.0507425, 0.0};
srcCorner4 = {-0.101035, -0.0507425, 0.0};
srcCorners = {srcCorner1, srcCorner2, srcCorner3, srcCorner4};
srcOffset = ConstantArray[0.0, 3];

msrCorner1 = {0.0, 0.0195775, 0.0};
msrCorner2 = {0.0, -0.0195775, 0.0};
msrCorner3 = {0.050515, -0.0195775, 0.0};
msrCorner4 = {0.050515, 0.0195775, 0.0};
msrCorners = {msrCorner1, msrCorner2, msrCorner3, msrCorner4};
msrOffsetClosest = {0.000615, 0.0, 0.0};
msrOffsetFarthest = {0.039175, 0.0, 0.0};

(* Test 13-15: wiresFromCorners test for single wire. *)
wires21 = emf["wiresFromCorners"][{srcCorner1, srcCorner2}, srcOffset, False];
EXPECTNEAR[1, Length[wires21], 0];
EXPECTNEAR[1, wires21[[1]]["wireId"], 0];
EXPECTNEAR[{0.0, 0.0507425, 0.0}, wires21[[1]]["end"], 0];

(* Test 16-21: wiresFromCorners test for full driven wire loop. *)
srcWires = emf["wiresFromCorners"][srcCorners, srcOffset, True];
EXPECTNEAR[4, Length[srcWires], 0];
EXPECTNEAR[4, srcWires[[4]]["wireId"], 0];
EXPECTNEAR[srcCorner2, srcWires[[1]]["end"], 0];
EXPECTNEAR[srcCorner3, srcWires[[3]]["start"], 0];
EXPECTNEAR[srcCorner4, srcWires[[4]]["start"], 0];
EXPECTNEAR[srcCorner1, srcWires[[4]]["end"], 0];

(* Test 22-27: wireFromCorners test with an offset. *)
msrWiresClosest = emf["wiresFromCorners"][msrCorners, msrOffsetClosest, True];
EXPECTNEAR[4, Length[msrWiresClosest], 0];

```

```

EXPECTNEAR[4, msrWiresClosest[[4]]["wireId"], 0];
EXPECTNEAR[msrCorner2 + msrOffsetClosest, msrWiresClosest[[1]]["end"], 0];
EXPECTNEAR[msrCorner3 + msrOffsetClosest, msrWiresClosest[[3]]["start"], 0];
EXPECTNEAR[msrCorner4 + msrOffsetClosest, msrWiresClosest[[4]]["start"], 0];
EXPECTNEAR[msrCorner1 + msrOffsetClosest, msrWiresClosest[[4]]["end"], 0];

(* Test 28: inlineEmfForWires test for nearest wires at closest offset. *)
If[! skipTests[28],
  emf28 =
    emf["inlineEmfForWires"][dIdtClosest, srcWires[[1]], msrWiresClosest[[1]]];
  EXPECTNEAR[290.935, micro emf28, 0.01];
];

(* Test 29: inlineEmfForWires test for top
  driven and closest measured wires at closest offset. *)
emf29 = emf["inlineEmfForWires"][dIdtClosest,
  srcWires[[2]], msrWiresClosest[[1]]];
EXPECTNEAR[-13.3628, micro emf29, 0.01];

(* Test 30: inlineEmfForWires test for
  top driven and top measured wires at closest offset. *)
emf30 = emf["inlineEmfForWires"][dIdtClosest,
  srcWires[[2]], msrWiresClosest[[4]]];
EXPECTNEAR[-55.2659, micro emf30, 0.01];

emfById[emfSet_, id_] :=
  Select[emfSet["drivenWireEmfs"], #["drivenWireId"] == id &][[1]]["emf"];

(* Tests 31-35: inlineEmfForWireSetsOnWire
  test for all driven wires and closest measured wire. *)
emf31 = emf["inlineEmfForWireSetsOnWire"][dIdtClosest,
  srcWires, msrWiresClosest[[1]]];
EXPECTNEAR[290.935, micro emfById[emf31, 1], 0.001];
EXPECTNEAR[-13.3628, micro emfById[emf31, 2], 0.001];
EXPECTNEAR[-44.007, micro emfById[emf31, 3], 0.001];
EXPECTNEAR[-13.3628, micro emfById[emf31, 4], 0.001];
Print["Starting wire to wire for full and dynamic terms"];
EXPECTNEAR[220.202, micro emf31["emfTotal"], 0.001];

(* Test 36: inlineEmfForWireSetsOnWire
  test for all driven wires and closest measured wire. *)
emf36 = emf["inlineEmfForWireSetsOnWire"][dIdtClosest,
  srcWires, msrWiresClosest[[2]]];
EXPECTNEAR[-12.1687, micro emf36["emfTotal"], 0.001];

```

```

(* Test 37: inlineEmfForWireSetsOnWire
   test for all driven wires and closest measured wire. *)
emf37 = emf["inlineEmfForWireSetsOnWire"][dIdtClosest,
      srcWires, msrWiresClosest[[3]]];
EXPECTNEAR[-25.8672, micro emf37["emfTotal"], 0.001];

(* Test 38: inlineEmfForWireSetsOnWire
   test for all driven wires and closest measured wire. *)
emf38 = emf["inlineEmfForWireSetsOnWire"][dIdtClosest,
      srcWires, msrWiresClosest[[4]]];
EXPECTNEAR[-12.1687, micro emf38["emfTotal"], 0.001];

(* Test 39: inlineEmfForWireSetsOnWireSets *)
emf39 = emf["inlineEmfForWireSetsOnWireSets"][
      dIdtClosest, srcWires, msrWiresClosest];
EXPECTNEAR[169.998, micro emf39["emfTotal"], 0.001];

(* Test 40-45: wireFromCorners test with the furthest offset. *)
msrWiresFarthest =
  emf["wiresFromCorners"][msrCorners, msrOffsetFarthest, True];
EXPECTNEAR[4, Length[msrWiresFarthest], 0];
EXPECTNEAR[4, msrWiresFarthest[[4]]["wireId"], 0];
EXPECTNEAR[msrCorner2 + msrOffsetFarthest, msrWiresFarthest[[1]]["end"], 0];
EXPECTNEAR[msrCorner3 + msrOffsetFarthest, msrWiresFarthest[[3]]["start"], 0];
EXPECTNEAR[msrCorner4 + msrOffsetFarthest, msrWiresFarthest[[4]]["start"], 0];
EXPECTNEAR[msrCorner1 + msrOffsetFarthest, msrWiresFarthest[[4]]["end"], 0];

(* Test 46: inlineEmfForWireSetsOnWireSets *)
emf46 = emf["inlineEmfForWireSetsOnWireSets"][
      dIdtClosest, srcWires, msrWiresFarthest];
EXPECTNEAR[12.3345, micro emf46["emfTotal"], 0.001];

experimentalXList = {0.000615, 0.000945, 0.002155, 0.003545, 0.005305, 0.006915,
      0.009085, 0.011315, 0.014395, 0.017285, 0.020965, 0.026325, 0.031905, 0.039175};

(* Test 47: inlineEmfForWireOnWireSetsAtOffsets from experimental data *)
If[! skipTests[47],
  emfAtOffset47 = emf["inlineEmfForWireOnWireSetsAtOffsets"][
      dIdtClosest, srcCorners, msrCorners, experimentalXList];
  Print["emfAtOffset51: " <> TextString[emfAtOffset47]];
  EXPECTNEAR[12.3345, micro emfAtOffset47[[14]][[2]], 0.0001];
];

```

```

(* Test 48: emfAtPoint for comparison to rectangular wires *)
rDrv67 = {0, 0, 0};
rMsr67 = {0.021, 0.02, 0};
emf48 = emf["emfAtPoint"][{0, dIdtClosest, 0}, rDrv67, rMsr67];
EXPECTNEAR[{0.0105226, -0.0321189, 0.0}, emf48, 0.0001];

(* Test 49: inlineEmfForWireOnWireSetsAtOffsets from experimental data *)
If[! skipTests[49],
  emfAtOffset49 = emf["inlineEmfForWireOnWireSetsAtOffsets"][
    dIdtClosest, srcCorners, msrCorners, experimentalXList];
  Print["emfAtOffset49: " <> TextString[emfAtOffset49]];
  EXPECTNEAR[12.3345, micro emfAtOffset49[[14]][[2]], 0.0001];
];

(* Run tests for full and dynamic LW E-field terms individually. *)
emf["setFullAccelTermCoeff"][1];
emf["setStaticAccelTermCoeff"][0];
emf["setDynamicAccelTermCoeff"][0];

(* Tests 50-54 *)
emf50 = emf["inlineEmfForWireSetsOnWire"][
  dIdtClosest, srcWires, msrWiresClosest[[1]]];
Print["Front wire to all wires for existing double cross term only"];
EXPECTNEAR[47.8461, micro emfById[emf50, 1], 0.001];
EXPECTNEAR[-13.3628, micro emfById[emf50, 2], 0.001];
EXPECTNEAR[-21.1205, micro emfById[emf50, 3], 0.001];
EXPECTNEAR[-13.3628, micro emfById[emf50, 4], 0.001];
Print["Starting wire to wire for existing double cross term only"];
EXPECTNEAR[0, micro emf50["emfTotal"], 0.001];

(* Test 55 *)
emf55 = emf["inlineEmfForWireSetsOnWire"][
  dIdtClosest, srcWires, msrWiresClosest[[2]]];
EXPECTNEAR[0, micro emf55["emfTotal"], 0.001];

(* Test 56 *)
emf56 = emf["inlineEmfForWireSetsOnWire"][
  dIdtClosest, srcWires, msrWiresClosest[[3]]];
EXPECTNEAR[0, micro emf56["emfTotal"], 0.001];

(* Test 57 *)
emf57 = emf["inlineEmfForWireSetsOnWire"][
  dIdtClosest, srcWires, msrWiresClosest[[4]]];
EXPECTNEAR[0, micro emf57["emfTotal"], 0.001];

```

```

(* Test 58 *)
emf58 =
  emf50["emfTotal"] + emf55["emfTotal"] + emf56["emfTotal"] + emf57["emfTotal"];
EXPECTNEAR[0, micro emf58, 0.001];

(* Run tests for full and dynamic LW E-field terms individually. *)
emf["setFullAccelTermCoeff"][0];
emf["setStaticAccelTermCoeff"][0];
emf["setDynamicAccelTermCoeff"][1];

(* Tests 59-63 *)
emf59 = emf["inlineEmfForWireSetsOnWire"][
  dIdtClosest, srcWires, msrWiresClosest[[1]]];
EXPECTNEAR[243.083, micro emfById[emf59, 1], 0.001];
EXPECTNEAR[0, micro emfById[emf59, 2], 0.001];
EXPECTNEAR[-22.8865, micro emfById[emf59, 3], 0.001];
EXPECTNEAR[0, micro emfById[emf59, 4], 0.001];
Print["Starting wire to wire for dynamic a term only"];
EXPECTNEAR[220.196, micro emf59["emfTotal"], 0.001];

(* Test 64 *)
emf64 = emf["inlineEmfForWireSetsOnWire"][
  dIdtClosest, srcWires, msrWiresClosest[[2]]];
EXPECTNEAR[-12.1687, micro emf64["emfTotal"], 0.001];

(* Test 65 *)
emf65 = emf["inlineEmfForWireSetsOnWire"][
  dIdtClosest, srcWires, msrWiresClosest[[3]]];
EXPECTNEAR[-25.8672, micro emf65["emfTotal"], 0.001];

(* Test 66 *)
emf66 = emf["inlineEmfForWireSetsOnWire"][
  dIdtClosest, srcWires, msrWiresClosest[[4]]];
EXPECTNEAR[-12.1687, micro emf66["emfTotal"], 0.001];

(* Test 67 *)
emf67 =
  emf59["emfTotal"] + emf64["emfTotal"] + emf65["emfTotal"] + emf66["emfTotal"];
EXPECTNEAR[169.992, micro emf67, 0.001];

(* Test 68: wireset with vertical displacement *)
srcWidth68 = 0.101035;
msrWidth68 = 0.050515;

```

```

sep68 = (srcWidth68 + msrWidth68) / 2 + msrOffsetClosest[[1]];
srcWires68 = emf["wiresFromDimensions"][{srcWidth68, 2 (0.0507425)}];
msrWires68 =
  emf["wiresFromDimensions"][{msrWidth68, 2 (0.0195775)}, {sep68, 0, 0}];

emf["setFullAccelTermCoeff"][0];
emf["setStaticAccelTermCoeff"][0];
emf["setDynamicAccelTermCoeff"][1];

emf68 =
  emf["inlineEmfForWireSetsOnWireSets"][dIdtClosest, srcWires68, msrWires68];
EXPECTNEAR[169.992, micro emf68["emfTotal"], 0.001];

(* Test 69: wireset with vertical displacement *)
msrWires69 =
  emf["wiresFromDimensions"][{msrWidth68, 2 (0.0195775)}, {0, 0, 0.0}];
emf69 = emf["inlineEmfForWireSetsOnWireSets"][
  dIdtClosest, srcWires68, msrWires69];
EXPECTNEAR[-148.064, micro emf69["emfTotal"], 0.001];

(* Test 70: wireset with vertical displacement *)
msrWires70 =
  emf["wiresFromDimensions"][{msrWidth68, 2 (0.0195775)}, {0, 0, 0.01}];
emf70 = emf["inlineEmfForWireSetsOnWireSets"][
  dIdtClosest, srcWires68, msrWires70];
EXPECTNEAR[-138.135, micro emf70["emfTotal"], 0.001];

(* Test 71: wireset with vertical displacement *)
msrWires71 =
  emf["wiresFromDimensions"][{msrWidth68, 2 (0.0195775)}, {0, 0, 0.02}];
emf71 = emf["inlineEmfForWireSetsOnWireSets"][
  dIdtClosest, srcWires68, msrWires71];
EXPECTNEAR[-115.356, micro emf71["emfTotal"], 0.001];

(* Test 72: wireset with vertical displacement *)
msrWires72 =
  emf["wiresFromDimensions"][{msrWidth68, 2 (0.0195775)}, {0, 0, -0.02}];
emf72 = emf["inlineEmfForWireSetsOnWireSets"][
  dIdtClosest, srcWires68, msrWires72];
EXPECTNEAR[-115.356, micro emf72["emfTotal"], 0.001];

emf["setFullAccelTermCoeff"][1];
emf["setStaticAccelTermCoeff"][0];

```

```

emf["setDynamicAccelTermCoeff"][0];

(* Test 73: wireset with vertical displacement *)
msrWires73 =
  emf["wiresFromDimensions"][{msrWidth68, 2 (0.0195775)}, {0, 0, 0.0}];
emf73 = emf["inlineEmfForWireSetsOnWireSets"] [
  dIdtClosest, srcWires68, msrWires73];
EXPECTNEAR[0.0, micro emf73["emfTotal"], 0.001];

(* Test 74: wireset with vertical displacement *)
msrWires74 =
  emf["wiresFromDimensions"][{msrWidth68, 2 (0.0195775)}, {0, 0, 0.01}];
emf74 = emf["inlineEmfForWireSetsOnWireSets"] [
  dIdtClosest, srcWires68, msrWires74];
EXPECTNEAR[0.0, micro emf74["emfTotal"], 0.001];

(* Test 75: wireset with vertical displacement *)
msrWires75 =
  emf["wiresFromDimensions"][{msrWidth68, 2 (0.0195775)}, {0, 0, 0.02}];
emf75 = emf["inlineEmfForWireSetsOnWireSets"] [
  dIdtClosest, srcWires68, msrWires75];
EXPECTNEAR[0.0, micro emf75["emfTotal"], 0.001];

emf["setFullAccelTermCoeff"][0];
emf["setStaticAccelTermCoeff"][0];
emf["setDynamicAccelTermCoeff"][1];

(* Test 76: perpendicular wires *)
emf76 = emf["emfFromWireToWire"] [
  dIdtClosest, {1, 0, 0}, {2, 0, 0}, {2, 0.5, 0}, {2, 2, 0}];
EXPECTNEAR[0.0, micro emf76, 0.001];

(* Test 77: from comparison table non-zero perp wire issue *)
sep77 = 0.005;
srcWires77 = emf["wiresFromDimensions"][{0.1, 0.1}];
msrWires77 =
  emf["wiresFromDimensions"][{0.05, 0.04}, {(0.1 + 0.05) / 2 + sep77, 0, 0}];
dot77 = Dot[srcWires77[[1]]["end"] - srcWires77[[1]]["start"],
  msrWires77[[4]]["start"] - msrWires77[[4]]["end"]];
EXPECTNEAR[0.0, dot77, 0.00001];

(* Test 78: from comparison table non-
zero perp wire issue - emf being start/end - perp wires *)

```

```

dIdt78 = 1000.0;
emf78 = emf["emfFromWireToWire"][dIdt78, srcWires77[[1]]["end"],
  srcWires77[[1]]["start"], msrWires77[[4]]["start"], msrWires77[[4]]["end"]];
EXPECTNEAR[0.0, micro emf78, 0.001];

(* Test 79: from comparison table non-
  zero perp wire issue - emf being start/end - nearest wires *)
(*Print["srcWires77: "<>TextString[srcWires77]];
Print["msrWires77: "<>TextString[msrWires77]]; *)
emf79 = emf["emfFromWireToWire"][dIdt78, srcWires77[[1]]["end"],
  srcWires77[[1]]["start"], msrWires77[[3]]["end"], msrWires77[[3]]["start"]];
EXPECTNEAR[23.7651, micro emf79, 0.001];

(* Test 80: from comparison table non-
  zero perp wire issue - emf being start/end - nearest wires by index *)
emf80 = emf["inlineEmfForWires"][dIdt78, srcWires77[[1]], msrWires77[[3]]];
EXPECTNEAR[23.7651, micro emf80, 0.001];

(* Test 81: from comparison table non-
  zero perp wire issue - emf being start/end - nearest wires by index *)
emf81 = emf["inlineEmfForWires"][dIdt78, srcWires77[[1]], msrWires77[[1]]];
EXPECTNEAR[-6.46048, micro emf81, 0.001];

(* Test 82: from comparison table non-
  zero perp wire issue - emf being start/end - nearest wires by index *)
emf82 = emf["inlineEmfForWires"][dIdt78, srcWires77[[1]], msrWires77[[2]]];
EXPECTNEAR[0.0, micro emf82, 0.001];

this["printResults"][];

this = this["makePublic"][this, {
  }];
this
];
UnitTestsForInductiveFieldEmfForRectangularCircuits[True, True];
Test: 1 Expected: {-1230769., -846154., 0} Actual: {-1230769., -846154., 0}
Test: 2 Expected: {-1500000., 500000., 0} Actual: {-1500000, 500000, 0}
Test: 3 Expected: {-2000000, 0, 0} Actual: {-2000000, 0, 0}
Test: 4 Expected: {-0.993553, 0.0, 0.0} Actual: {-0.993553, 0.0, 0.0}
Test: 5 Expected: {-2.00159, 0.0798084, 0.0} Actual: {-2.00159, 0.0798084, 0.0}
Test: 6 Expected: {-100, 0, 0} Actual: {-100.0, 0.0, 0.0}
Test: 7 Expected: {-2.00159, -0.0798084, 0.0} Actual: {-2.00159, -0.0798084, 0.0}

```


Test: 8 Expected: -2.00159 Actual: -2.00159
 Test: 9 Expected: 0.0798084 Actual: 0.0798084
 Test: 10 Expected: -0.0921054 Actual: -0.0921054
 Test: 11 Expected: -0.0921054 Actual: -0.0921054
 Test: 12 Expected: -0.0313341 Actual: -0.0313341
 Test: 13 Expected: 1 Actual: 1
 Test: 14 Expected: 1 Actual: 1
 Test: 15 Expected: {0.0, 0.0507425, 0.0} Actual: {0.0, 0.0507425, 0.0}
 Test: 16 Expected: 4 Actual: 4
 Test: 17 Expected: 4 Actual: 4
 Test: 18 Expected: {0.0, 0.0507425, 0.0} Actual: {0.0, 0.0507425, 0.0}
 Test: 19 Expected: {-0.101035, 0.0507425, 0.0} Actual: {-0.101035, 0.0507425, 0.0}
 Test: 20 Expected: {-0.101035, -0.0507425, 0.0} Actual: {-0.101035, -0.0507425, 0.0}
 Test: 21 Expected: {0.0, -0.0507425, 0.0} Actual: {0.0, -0.0507425, 0.0}
 Test: 22 Expected: 4 Actual: 4
 Test: 23 Expected: 4 Actual: 4
 Test: 24 Expected: {0.000615, -0.0195775, 0.0} Actual: {0.000615, -0.0195775, 0.0}
 Test: 25 Expected: {0.05113, -0.0195775, 0.0} Actual: {0.05113, -0.0195775, 0.0}
 Test: 26 Expected: {0.05113, 0.0195775, 0.0} Actual: {0.05113, 0.0195775, 0.0}
 Test: 27 Expected: {0.000615, 0.0195775, 0.0} Actual: {0.000615, 0.0195775, 0.0}
 Skipping tests 28 to 28
 Test: 29 Expected: -13.3628 Actual: -13.3628
 Test: 30 Expected: -55.2659 Actual: -55.2659
 Test: 31 Expected: 290.935 Actual: 290.935
 Test: 32 Expected: -13.3628 Actual: -13.3628
 Test: 33 Expected: -44.007 Actual: -44.007
 Test: 34 Expected: -13.3628 Actual: -13.3628
 Starting wire to wire for full and dynamic terms
 Test: 35 Expected: 220.202 Actual: 220.202
 Test: 36 Expected: -12.1687 Actual: -12.1687
 Test: 37 Expected: -25.8672 Actual: -25.8672
 Test: 38 Expected: -12.1687 Actual: -12.1687
 Test: 39 Expected: 169.998 Actual: 169.998
 Test: 40 Expected: 4 Actual: 4
 Test: 41 Expected: 4 Actual: 4
 Test: 42 Expected: {0.039175, -0.0195775, 0.0} Actual: {0.039175, -0.0195775, 0.0}

Test: 43 Expected: {0.08969, -0.0195775, 0.0} Actual: {0.08969, -0.0195775, 0.0}

Test: 44 Expected: {0.08969, 0.0195775, 0.0} Actual: {0.08969, 0.0195775, 0.0}

Test: 45 Expected: {0.039175, 0.0195775, 0.0} Actual: {0.039175, 0.0195775, 0.0}

Test: 46 Expected: 12.3345 Actual: 12.3345

Skipping tests 47 to 47

Test: 48 Expected: {0.0105226, -0.0321189, 0.0} Actual: {0.0105226, -0.0321189, 0.0}

Skipping tests 49 to 49

Front wire to all wires for existing double cross term only

Test: 50 Expected: 47.8461 Actual: 47.8461

Test: 51 Expected: -13.3628 Actual: -13.3628

Test: 52 Expected: -21.1205 Actual: -21.1205

Test: 53 Expected: -13.3628 Actual: -13.3628

Starting wire to wire for existing double cross term only

Test: 54 Expected: 0 Actual: 0.00000517672

Test: 55 Expected: 0 Actual: 0.000000543189

Test: 56 Expected: 0 Actual: -0.000000222988

Test: 57 Expected: 0 Actual: 0.00000054319

Test: 58 Expected: 0 Actual: 0.00000604011

Test: 59 Expected: 243.083 Actual: 243.083

Test: 60 Expected: 0 Actual: 0.0

Test: 61 Expected: -22.8865 Actual: -22.8865

Test: 62 Expected: 0 Actual: 0.0

Starting wire to wire for dynamic a term only

Test: 63 Expected: 220.196 Actual: 220.196

Test: 64 Expected: -12.1687 Actual: -12.1687

Test: 65 Expected: -25.8672 Actual: -25.8672

Test: 66 Expected: -12.1687 Actual: -12.1687

Test: 67 Expected: 169.992 Actual: 169.992

Test: 68 Expected: 169.992 Actual: 169.992

Test: 69 Expected: -148.064 Actual: -148.064

Test: 70 Expected: -138.135 Actual: -138.135

Test: 71 Expected: -115.356 Actual: -115.356

Test: 72 Expected: -115.356 Actual: -115.356

Test: 73 Expected: 0.0 Actual: -0.00000103103

Test: 74 Expected: 0.0 Actual: -0.00000135603

Test: 75 Expected: 0.0 Actual: -0.000000259609

```

Test: 76 Expected: 0.0 Actual: 0.0
Test: 77 Expected: 0.0 Actual: 0.0
Test: 78 Expected: 0.0 Actual: 0.0
Test: 79 Expected: 23.7651 Actual: 23.7651
Test: 80 Expected: 23.7651 Actual: 23.7651
Test: 81 Expected: -6.46048 Actual: -6.46048
Test: 82 Expected: 0.0 Actual: 0.0
PASSED 79 tests

```

Unit tests for Comparison of Single Wire EMF

This section contains the module for unit tests for inductive field EMF. Any errors will be noted in the output with red text highlighting the expected and actual output.

```

UnitTestsForComparisonOfSingleDrivenWireEmf[
  verbose_: True, skipIntensiveTests_: False] := Module[
  {
    (* public values *)
    this = <||>,

    (* functions *)
    EXPECTNEAR, emfById, skipTests, log,

    (* local values *)
    numTestsPassed, numTestsFailed, numTestsSkipped, testNumber, toUsed,
    actualTypeString, micro = 1000000, milli = 1000, comparison1, separations,
    drvX, drvWidth, msrDimension, faradayEmfs1, dynamicAccelEmfs1, staticAccelEmfs1
  },
  this = UnitTestBaseObject[verbose, skipIntensiveTests];
  EXPECTNEAR = this["EXPECTNEAR"];
  skipTests = this["skipTests"];
  log = this["log"];

  separations = {0.005, 0.010, 0.025, 0.05, 0.075, 0.1};
  drvX = -0.025;
  drvWidth = 0.1;
  drvWidth = 10.0;
  msrDimension = {0.05, 0.04};

  (* Test 1*)
  comparison1 = ComparisonOfSingleDrivenWireEmf[
    separations, drvX, drvWidth, msrDimension, Automatic];

```

```

faradayEmfs1 = comparison1["generateFaraday"] [];
staticAccelEmfs1 = comparison1["generateStaticAccel"] [];
dynamicAccelEmfs1 = comparison1["generateDynamicAccel"] [];

log["faraday", faradayEmfs1];

log["static", staticAccelEmfs1];

log["dynamic", dynamicAccelEmfs1];

this["printResults"] [];

this = this["makePublic"][this, {
  }];
this
];
UnitTestsForComparisonOfSingleDrivenWireEmf[True, True];
faraday: {{-18.5374, -14.1606, -8.7616, -5.5442, -4.08539,
  -3.24265}, {-17.9468, -12.9073, -6.42553, -2.77178, -1.46986, -0.892068}}
static: {{19.2135, 14.3362, 8.79067, 5.54812, 4.08589,
  3.24288}, {18.4903, 13.0344, 6.43965, 2.77588, 1.4707, 0.892379}}
dynamic: {{-19.1828, -14.3333, -8.78822, -5.54479, -4.08583,
  -3.24265}, {-18.4601, -13.0316, -6.43724, -2.77243, -1.4708, -0.892396}}
PASSED 0 tests

```

Unit tests for the base/common object

These are the unit tests for the BaseObject module. Any errors will be noted in the output with red text highlighting the expected and actual output.

```

UnitTestBaseObject[verbose_> True, skipIntensiveTests_> False] := Module[
{
  (* public values *)
  this = <|>,

  (* functions *)
  EXPECTNEAR, emfById, skipTests, printResults,

  (* local values *)
  numTestsPassed, numTestsFailed, numTestsSkipped, testNumber,
  toUsed, actualTypeString, micro = 1000 000, milli = 1000
},

```

```

this = BaseObject[];
numTestsPassed = 0;
numTestsFailed = 0;
numTestsSkipped = 0;
testNumber = 1;

skipTests[from_, to_: 0] := (
  toUsed = to;
  If[toUsed == 0, toUsed = from];
  If[skipIntensiveTests,
    Print[
      "Skipping tests "<>TextString[from]<>" to "<>TextString[toUsed]];
    numTestsSkipped += toUsed - from + 1;
    testNumber += toUsed - from + 1;
  ];
  skipIntensiveTests
);

EXPECTNEAR[expected_, actual_, eps_] := (
  If[verbose,
    Print["Test: "<>TextString[testNumber]<>" Expected: "<>
      TextString[expected]<>" Actual: "<>TextString[actual]];
  ];
  testNumber++;

  actualTypeString = TextString[Head[actual]];

  If[!(actualTypeString == "List" || actualTypeString == "Real" ||
    actualTypeString == "Integer") || Max[Abs[expected - actual]] > eps,
    (
      Print[Style["Expected: "<>TextString[expected], Red]];
      Print[Style["Actual: "<>TextString[actual], Red]];
      Print[Style["Difference: "<>TextString[expected - actual], Red]];
      Print[Style["EPS: "<>TextString[eps], Red]];
      numTestsFailed++;
    ),
    (
      numTestsPassed++
    )
  ];
);

printResults[] := (

```

```
Print["PASSED " <> TextString[numTestsPassed] <> " tests"];
If[numTestsFailed > 0,
  Print["FAILED: " <> TextString[numTestsFailed]];
];
);

this = this["makePublic"][this,
  {"skipTests", skipTests, "EXPECTNEAR", EXPECTNEAR,
   "numTestsPassed", numTestsPassed, "numTestsFailed", numTestsFailed,
   "numTestsSkipped", numTestsSkipped, "printResults", printResults}];
this
];
```