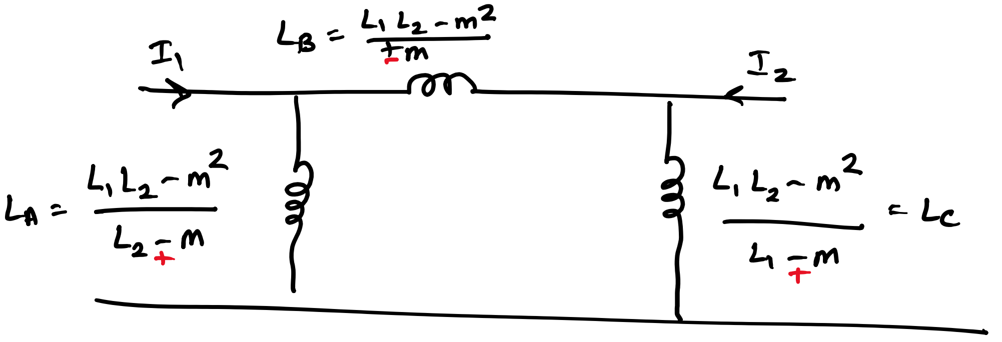
π Equivalent



$\Rightarrow$ $L_A = 10\,mH$, $L_B = 5\,mH$, $L_C = -20\,mH$

L_1, L_2 + m

let $L_1L_2 - m^2 = x$

$$\Rightarrow L_A = \frac{x}{L_2 - m} = 10\,mH \qquad \Rightarrow L_2 - m = \frac{x}{10\,mH}$$

$$L_B = \frac{x}{m} = 5\,mH \qquad \Rightarrow m = \frac{x}{5\,mH}$$

$$L_C = \frac{x}{L_1 - m} = -20\,mH \qquad L_1 - m = \frac{x}{-20\,mH}$$

$$\Rightarrow L_2 - \frac{x}{5\,mH} = \frac{x}{10\,mH} \qquad \Rightarrow L_2 = \frac{x}{10\,mH} + \frac{x}{5\,mH}$$

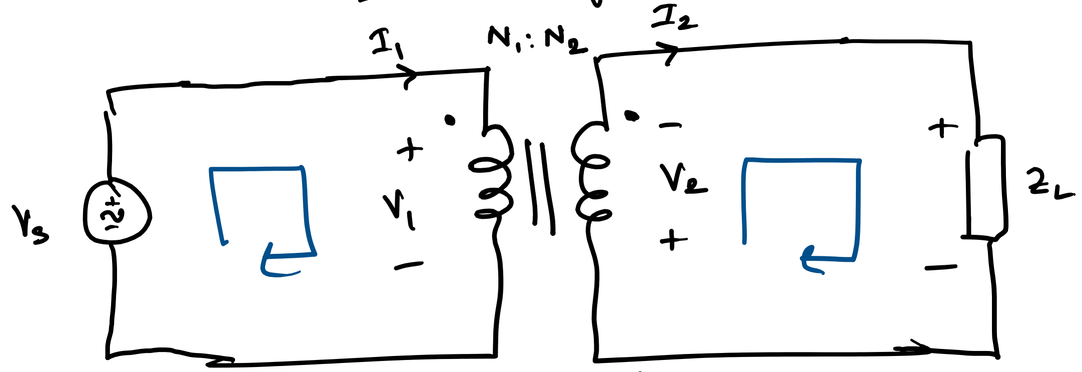
$$\text{my } L_1 = \frac{-x}{20\,mH} + \frac{x}{5\,mH}$$

$$L_1L_2 - m^2 = x$$

(Solve for x. Remember $L_1L_2 > m^2$)

$$k = \frac{m}{\sqrt{L_1L_2}} \leq 1$$

Ideal Transformer



M — opposite

$\frac{N_2}{N_1} = a =$ turns ratio

$k = 1$, $\quad L_1 \to \infty$, $\quad L_2 \to \infty$, $\quad M = \sqrt{L_1 L_2} \to \infty$

$L_1 \propto N_1^2 \qquad L_2 \propto N_2^2 \qquad \frac{L_2}{L_1} = \left(\frac{N_2}{N_1}\right)^2 = a^2$

In this case $\quad V_s = V_1$,

Loop 1: $\quad +V_1 - j\omega L_1 I_1 + j\omega M I_2 = 0$

$$V_1 = j\omega L_1 I_1 - j\omega M I_2 \quad \text{—— (1)}$$

Loop 2: $\quad -j\omega L_2 I_2 + j\omega M I_1 - I_2 Z_L = 0 \quad$ —— (2)

$\frac{(2)}{I_1} \Rightarrow \quad j\omega M + (-j\omega L_2 - Z_L)\left(\frac{I_2}{I_1}\right) = 0$

$$\Rightarrow \frac{I_2}{I_1} = \frac{j\omega M}{Z_L + j\omega L_2} \quad \text{—— (3)}$$

$\frac{(1)}{I_1} \Rightarrow \quad \frac{V_1}{I_1} = j\omega L_1 - j\omega M\left(\frac{I_2}{I_1}\right) \quad$ —— (4)

Sub (3) in (4) $\quad Z_{in} = j\omega L_1 - \frac{j\omega M \times j\omega M}{Z_L + j\omega L_2} = j\omega L + \frac{\omega^2 M^2}{Z_L + j\omega L_2}$

$M^2 = L_1 L_2$, $\quad L_2 = a^2 L_1$

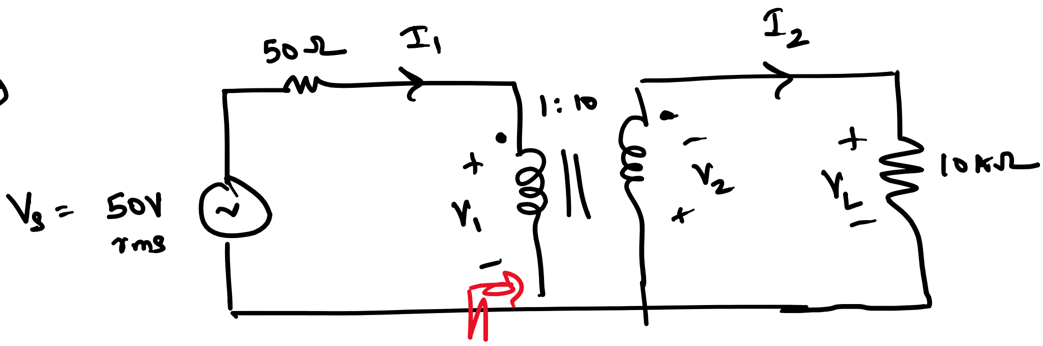
$$\Rightarrow Z_{in} = j\omega L_1 + \frac{\omega^2 L_1 L_2}{Z_L + j\omega L_2} = \frac{j\omega L_1 Z_L - \cancel{\omega^2 L_1 L_2} + \cancel{\omega^2 L_1 L_2}}{Z_L + j\omega L_2}$$

$$\Rightarrow \quad Z_{in} = \frac{j\omega L_1 Z_L}{Z_L + j\omega L_2} = \frac{j\omega L_1 Z_L}{Z_L + j\omega a^2 L_1} \times \frac{\frac{1}{j\omega L_1}}{\frac{1}{j\omega L_1}}$$

$$\Rightarrow \quad Z_{in} = \frac{Z_L}{a^2 + \frac{Z_L}{j\omega L_1}}$$

$$\boxed{\lim_{Z_1 \to \infty} Z_{in} = \frac{Z_L}{a^2}}$$

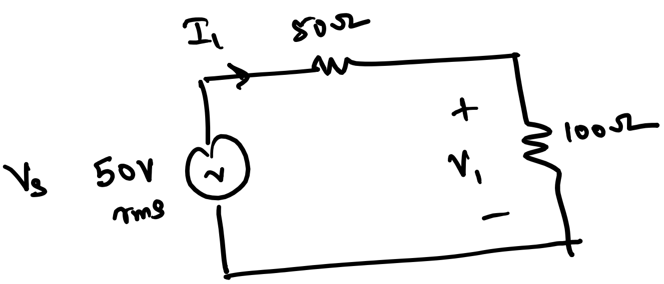
→



find I_1, I_2, V_1, V_L

$a = 10$

$$Z_{in} = \frac{Z_L}{a^2} = \frac{100\cancel{00}}{1\cancel{00}} = 100\Omega$$



$$I_1 = \frac{50}{50+100} = \frac{50}{150} = \frac{1}{3} \text{ A rms}$$

$$V_1 = \frac{50 \times 100}{50+100} = \frac{\cancel{50} \times 100}{\cancel{150}\,3} = 33.33 \text{ V rms}$$

Contd from earlier

$$\frac{I_2}{I_1} = \frac{j\omega M}{Z_L + j\omega L_2} = \frac{j\omega\sqrt{L_1 L_2}}{Z_L + j\omega L_2}$$

$$\approx \frac{j\omega\sqrt{L_1 L_2}}{j\omega L_2} = \sqrt{\frac{L_1}{L_2}} = \sqrt{\frac{1}{a^2}} = \frac{1}{a}$$

$$\Rightarrow \boxed{\frac{I_2}{I_1} = \frac{1}{a}}$$

Contd from previous problem

$$I_1 = \frac{1}{3} A \text{ rms}$$

$$I_2 = I_1 \times \frac{1}{a} = \frac{1}{3} \times \frac{1}{10} = \frac{1}{30} A \text{ rms}$$

Contd

$$\frac{V_1}{I_1} = Z_{in} \Rightarrow V_1 = I_1 Z_{in}$$

$$-V_2 - I_2 Z_L = 0 \Rightarrow V_2 = -I_2 Z_L$$

$$\frac{V_2}{V_1} = -\frac{I_2 Z_L}{I_1 Z_{in}} = -\frac{1}{a} \times a^2 = -a$$

$$\boxed{\frac{V_2}{V_1} = -a}$$

contd problem

$$V_L = -V_2 = -(-a \times V_1) = +10 \times 33.33$$

$$= 333.33 \text{ V rms}$$

Power across primary.

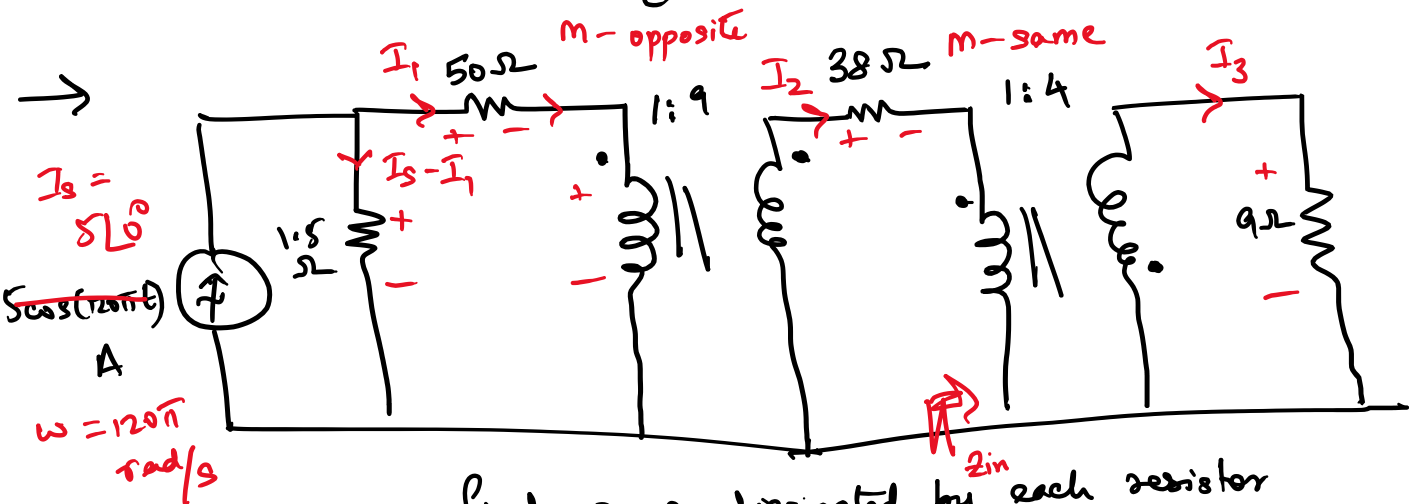
$$P_p = \mathrm{Re}\{V_{1rms} \, I_{1rms}^{*}\}$$

$$= \mathrm{Re}\left\{33.33 \times \frac{1}{3}\right\} = 11.11 \text{ W}$$

Power across secondary

$$P_s = \mathrm{Re}\{V_{2rms} \, I_{2rms}^{*}\}$$

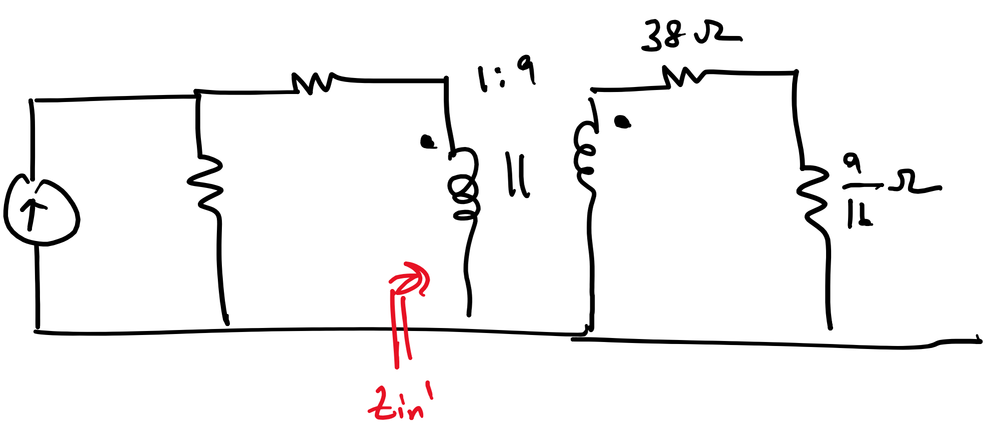
$$= \mathrm{Re}\left\{333.33 \times \frac{1}{30}\right\} = 11.11 \text{W}$$



find power dissipated by each resistor

Stage 1 : Find Zin

$$Z_{in} = \frac{Z_L}{a^2} = \frac{9}{4^2} = \frac{9}{16} \, \Omega$$



Stage 2

$$Z_{in}' = \frac{Z_L'}{a^2} = \frac{38 + \frac{9}{16}}{81}$$