Quiz - 7

$$x(t) = e^{-3|t-1|} \sin 2t$$

$$X(j\omega) = \int_{-\infty}^{\infty} e^{-3|t-1|} \left(\frac{e^{2tj} - e^{-2tj}}{2j}\right) e^{-j\omega t}\, dt$$

$$= \underbrace{\frac{1}{2j} \int_{-\infty}^{1} e^{3(t-1)} \left(e^{2jt} - e^{-2jt}\right) e^{-j\omega t}\, dt}_{\text{I}} + \underbrace{\frac{1}{2j} \int_{1}^{\infty} e^{-3(t-1)} \left(e^{2jt} - e^{-2jt}\right) e^{-j\omega t}\, dt}_{\text{II}}$$

I →

$$\frac{1}{2j} \int_{-\infty}^{1} e^{3(t-1)} \left(e^{2jt} - e^{-2jt}\right) e^{-j\omega t}\, dt$$

$$= \frac{e^{-3}}{2j} \int_{-\infty}^{1} \left[e^{3t - j\omega t + 2jt} - e^{3t - j\omega t - 2jt}\right] dt$$

$$= \frac{e^{-3}}{2j} \left[\frac{e^{(3 - j\omega + 2j)t}}{3 - j\omega + 2j} - \frac{e^{(3 - j\omega - 2j)t}}{3 - j\omega - 2j}\right]\Bigg|_{-\infty}^{1}$$

$$= \frac{e^{-3}}{2j} \left[\frac{e^{(3 - j\omega + 2j)}}{3 - j\omega + 2j} - \frac{e^{(3 - j\omega - 2j)}}{3 - j\omega - 2j}\right] \longrightarrow (4 \text{ Point})$$

II →

$$\frac{1}{2j} \int_{1}^{\infty} e^{-3(t-1)} \left(e^{2jt} - e^{-2jt}\right) e^{-j\omega t}\, dt$$

$$= \frac{e^{3}}{2j} \int_{1}^{\infty} \left[e^{(-3t + 2jt - j\omega t)} - e^{(-3t - 2jt - j\omega t)}\right] dt$$

$$= \frac{e^3}{2j}\left[\frac{e^{(-3-jw+2j)t}}{(-3-jw+2j)} - \frac{e^{(-3-2j-jw)t}}{(-3-2j-jw)}\right]\Bigg|_1^{\infty}$$

$$= \frac{e^3}{2j}\left[-\frac{e^{(-3-jw+2j)}}{(-3-jw+2j)} + \frac{e^{(-3-2j-jw)}}{(-3-2j-jw)}\right] \longrightarrow \text{(4 Point)}$$

$$X(jw) = \text{I} + \text{II}$$

$$= \frac{e^{-3}}{2j}\left[\frac{e^{3-jw+2j}}{3-jw+2j} - \frac{e^{3-jw-2j}}{3-jw-2j}\right] +$$

$$\frac{e^3}{2j}\left[-\frac{e^{-3-jw+2j}}{(-3-jw+2j)} + \frac{e^{-3-jw-2j}}{(-3-jw-2j)}\right]$$

$$= \frac{1}{2j}\left[\frac{e^{(-jw+2j)}}{3-jw+2j} - \frac{e^{(-jw-2j)}}{3-jw-2j}\right] + \frac{1}{2j}\left[-\frac{e^{(-jw+2j)}}{(-3-jw+2j)} + \frac{e^{(-jw-2j)}}{(-3-jw-2j)}\right]$$

$$= \frac{1}{2j}\left[e^{j(2-w)}\left[\frac{1}{3-jw+2j} - \frac{1}{(-3-jw+2j)}\right] + e^{-j(2+w)}\left[\frac{1}{-3-2j-jw} - \frac{1}{3-jw-2j}\right]\right]$$

$$= \frac{1}{2j}\left[e^{j(2-w)}\left[\frac{-3-jw+2j+3+jw-2j}{[3+j(2-w)][-3+j(2-w)]}\right] + e^{-j(2+w)}\left[\frac{3-jw-2j+3+2j+jw}{[-3-j(2+w)][3-j(2+w)]}\right]\right]$$

$$= \frac{1}{2j}\left[\frac{6e^{j(2-w)}}{9+(2-w)^2} - \frac{6e^{-j(2+w)}}{9+(2+w)^2}\right] \longrightarrow \text{(2 Point)}$$