Review

Time Domain $\longrightarrow$ Phasor / Frequency Domain

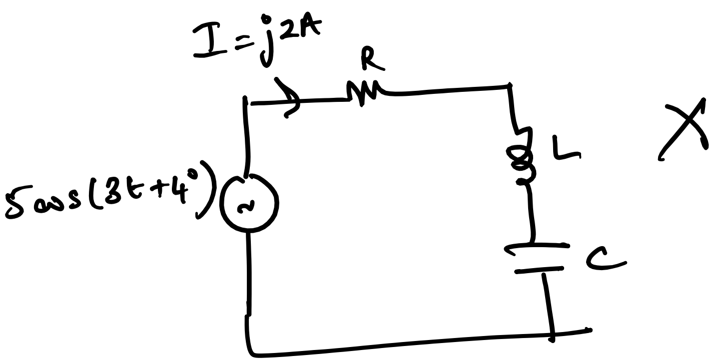
$A\cos(\omega t+\phi) \longrightarrow A\angle\phi, \quad Ae^{j\phi}$

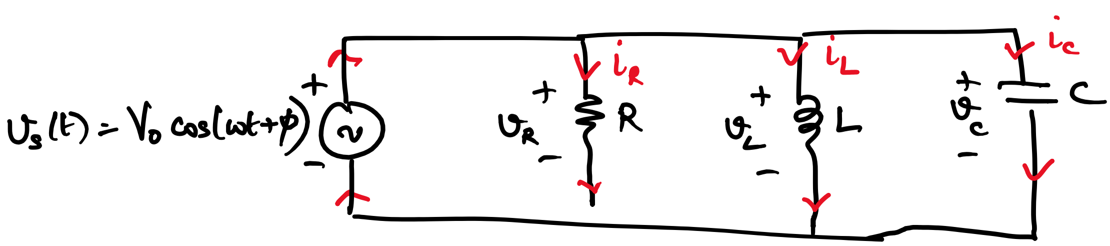
Real valued time varying function

$A\cos\phi + jA\sin\phi$ $\Big\}$ complex number

$+j\sin(5t+30^\circ)$ ✗

$5\sin((3+j4)t+3+4j)$ ✗





$$v_R = v_s(t) = V_0\cos(\omega t+\phi)$$

$$i_R = \frac{v_R(t)}{R} = \frac{V_0}{R}\cos(\omega t+\phi)$$

$$v_L(t) = L\frac{di_L}{dt} \Rightarrow i_L(t) = \frac{1}{L}\int_0^t v_L(t)\,dt \quad [\text{No initial conditions}]$$

$$= \frac{V_0}{L}\int_0^t \cos(\omega t+\phi)\,dt = \frac{V_0}{\omega L}[\sin(\omega t+\phi)]$$

$$= \frac{V_0}{\omega L}\cos(\omega t+\phi-\pi/2)$$

$$i_c = C\frac{dv_c}{dt} = C\left[\frac{d}{dt}\left(V_0\cos(\omega t+\phi)\right)\right]$$

$$= -CV_0\omega\left[\sin(\omega t+\phi)\right] = +V_0 C\omega \sin(\omega t+\phi+\pi)$$

$$= V_0 C\omega \cos(\omega t+\phi+\pi/2)$$

TD	Phasor
$i_R(t) = \frac{V_0}{R}\cos(\omega t+\phi)$	$I_R = \frac{V_0}{R}\angle\phi$
$i_L(t) = \frac{V_0}{\omega L}\cos(\omega t+\phi-\frac{\pi}{2})$	$I_L = \frac{V_0}{\omega L}\angle\phi-\pi/2$
$i_c(t) = V_0 C\omega\cos(\omega t+\phi+\frac{\pi}{2})$	$I_c = V_0\omega C\angle\phi+\pi/2$
$V_S(t) = V_0\cos(\omega t+\phi)$	$V_S = V_0\angle\phi$

Observations

1) R, v_R, i_R → are in phase
2) L : $v_L \xrightarrow{\text{leads}} i_L$
3) C : $i_c \xrightarrow{\text{leads}} v_c$

Impedance / Admittance

→ Only in Phasor / Frequency domain

→ Unit Ω : Z Impedance Ohms

$\mho$: $Y = \frac{1}{Z}$ Admittance mhos

→ $Z(\omega) = \frac{V(\omega)}{I(\omega)} = \underset{\text{Resistance }(\Omega)}{R} + j\underset{\text{Reactance }(\Omega)}{X}$

Resistance :

$$V_R = V_0 \angle \phi$$

$$I_R = \frac{V_0}{R} \angle \phi$$

$$Z_R = \frac{V_R}{I_R} = \frac{V_0 \angle \phi}{\left(\frac{V_0}{R}\right) \angle \phi} = R$$

$$Y_R = \frac{1}{R}$$

Inductance :

$$V_L = V_0 \angle \phi$$

$$I_L = \frac{V_0}{\omega L} \angle \phi - \pi/2$$

$$Z_L = \frac{V_0 \angle \phi}{\frac{V_0}{\omega L} \angle \phi - \pi/2} = \omega L \angle \pi/2 = \omega L \left(e^{j\pi/2}\right)$$

$$= \omega L \left(\cos\frac{\pi}{2} + j\sin\frac{\pi}{2}\right) = j\omega L$$

$$Y_L = \frac{1}{j\omega L}$$

Capacitance :

$$V_c = V_0 \angle \phi$$

$$I_c = V_0 \omega C \angle \phi + \pi/2$$

$$Z_c = \frac{V_c}{I_c} = \frac{1}{j\omega C}$$

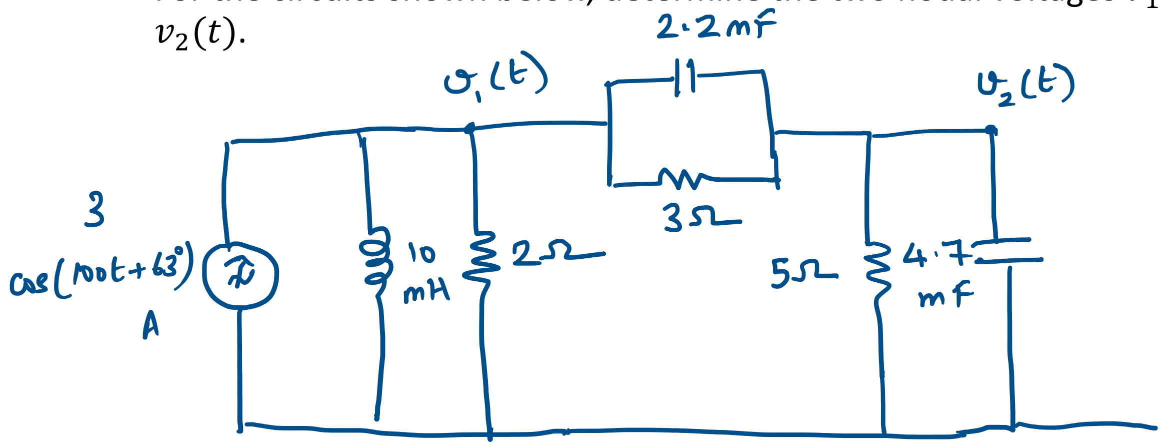
$$Y_c = j\omega C$$

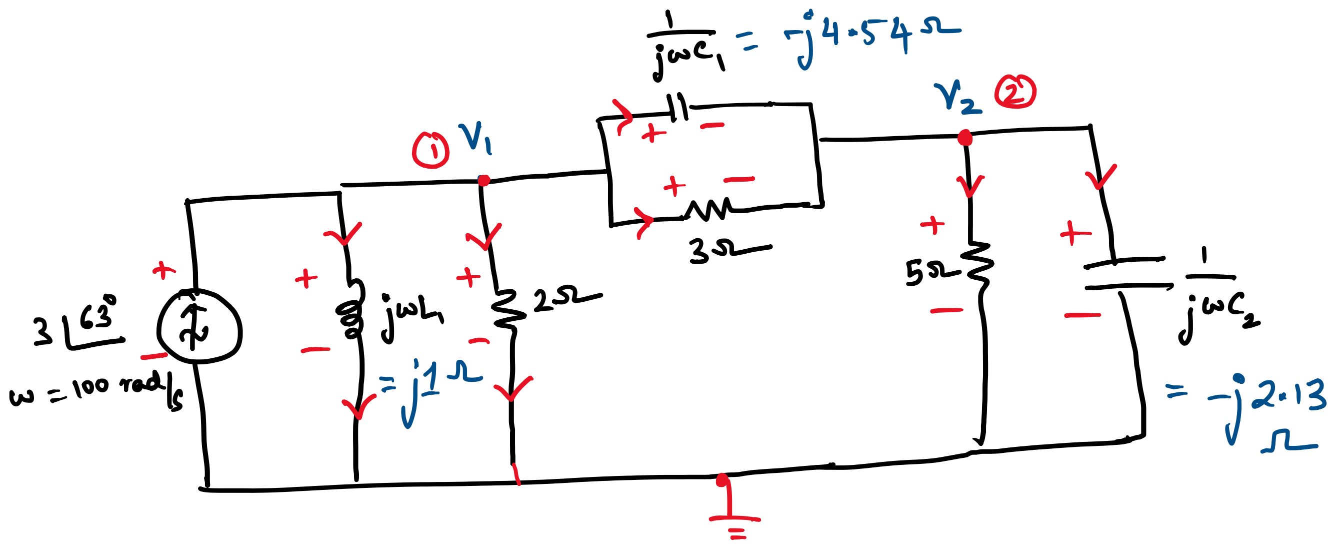
$$Y = G + jB$$

Conductance ($\mho$)

Susceptance ($\mho$)

- For the circuits shown below, determine the two nodal voltages $v_1(t)$ and $v_2(t)$.





Node 1

$$3e^{j63°} = \frac{V_1 - 0}{j\omega L} + \frac{V_1 - 0}{2} + \frac{V_1 - V_2}{(1/j\omega C_1)} + \frac{V_1 - V_2}{3}$$

$$V_1\left(\frac{1}{j\omega L} + \frac{1}{2} + j\omega C_1 + \frac{1}{3}\right) + V_2\left(-j\omega C_1 - \frac{1}{3}\right) = 3e^{j63°} \quad \text{— (1)}$$

$$A_1 V_1 + B_1 V_2 = C_1$$

Node 2

$$\frac{V_1 - V_2}{(1/j\omega C_1)} + \frac{V_1 - V_2}{3} = \frac{V_2 - 0}{5} + \frac{V_2 - 0}{(1/j\omega C_2)}$$

$$\Rightarrow V_1\left(j\omega C_1 + \frac{1}{3}\right) + V_2\left(-j\omega C_1 - \frac{1}{3} - \frac{1}{5} - j\omega C_2\right) = 0 \quad \text{— (2)}$$

$$A_2 V_1 + B_2 V_2 = C_2$$

2

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$$A_1 V_1 + B_1 V_2 = C_1$$

$$A_2 V_1 + B_2 V_2 = C_2$$

$$= \begin{bmatrix} 0.633 - j0.78 & 0.33 + j0.22 \\ -0.33 - j0.22 & -0.53 - j0.69 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1.36 + j\,2.67 \\ 0 \end{bmatrix}$$

<u>Cramer's Rule</u>

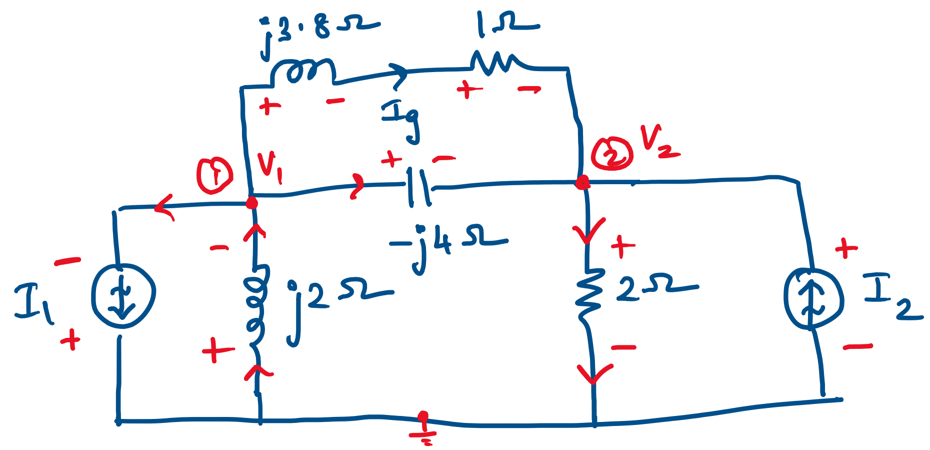
$$V_1 = \frac{\begin{vmatrix} C_1 & B_1 \\ C_2 & B_2 \end{vmatrix}}{\begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix}} = V_1 \angle \phi_1 = 3.18 \angle 124^\circ$$

$$V_2 = \frac{\begin{vmatrix} A_1 & C_1 \\ A_2 & C_2 \end{vmatrix}}{\begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix}} = V_2 \angle \varphi_2 = 1.46 \angle -74.6^\circ$$

$$v_1(t) = V_1 \cos(100t + \varphi_1)\ V = 3.18 \cos(100t + 124^\circ)\ V$$

$$v_2(t) = V_2 \cos(100t + \varphi_2)\ V = 1.46 \cos(100t - 74.6^\circ)\ V$$

- Determine I_g in the circuit if $I_1 = 5\angle -18°A$ and $I_2 = 2\angle 5°A$.



Node 1

$$\frac{0 - V_1}{j2} = I_1 + \frac{V_1 - V_2}{-j4} + \frac{V_1 - V_2}{j3.8 + 1}$$

$$\Rightarrow V_1\left[\frac{-1}{j2} + \frac{1}{j4} \quad \frac{-1}{1+j3.8}\right] + V_2\left[\frac{-1}{j4} + \frac{1}{1+j3.8}\right] = I_1 \quad -(1)$$

Node 2

$$I_2 + \frac{V_1 - V_2}{1+j3.8} + \frac{V_1 - V_2}{-j4} = \frac{V_2 - 0}{2}$$

$$V_1\left(\frac{1}{1+j3.8} + \frac{1}{-j4}\right) + V_2\left(\frac{-1}{1+j3.8} + \frac{1}{j4} - \frac{1}{2}\right) = -I_2 \quad -(2)$$

$$\Rightarrow \begin{bmatrix} -0.0648+j\,0.4961, & 0.0648+j\,0.0039 \\ 0.0648+j\,0.0039, & -0.5648-j\,0.0039 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 4.7553 - j\,1.5451 \\ -1.9924 - j\,0.1743 \end{bmatrix}$$

Solving using Cramer's Rule, we get

$$V_1 = -4.1698 - j8.6298 \text{ V}$$

$$V_2 = 3.1040 - j0.7311 \text{ V}$$

$$I_g = -2.4151 + j1.2786 \text{ A}$$