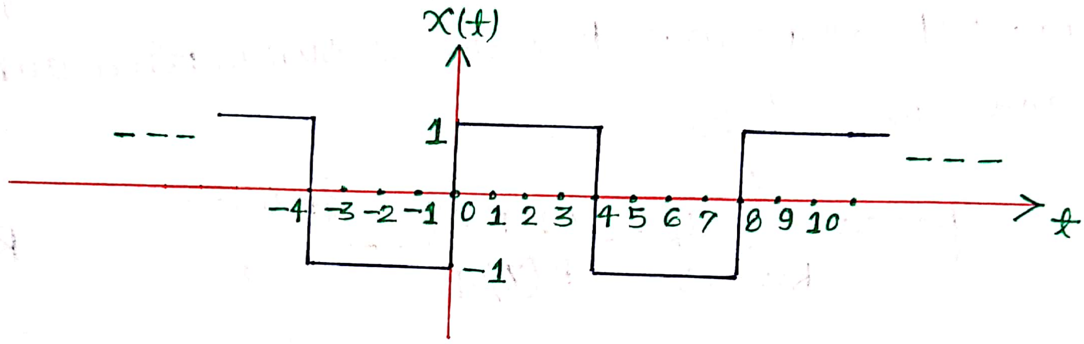
Quiz-5 solution

SOL (1): Given input signal with period $T = 8$,

$$x(t) = \begin{cases} 1, & 0 \leq t < 4 \\ -1, & 4 \leq t < 8 \end{cases}$$



Clearly, $x(t)$ is real and odd, Hence Fourier Series coefficient of $x(t)$ i.e. a_K is purely imaginary and odd.

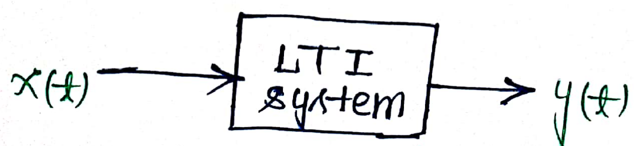
Therefore, $a_0 = 0$ → (1 Point)

Now, $$a_K = \frac{1}{8}\int_0^8 x(t)\cdot e^{-j(2\pi/8)Kt}\cdot dt$$

$$a_K = \frac{1}{8}\int_0^4 1\cdot e^{-j(2\pi/8)Kt}\cdot dt + \frac{1}{8}\int_4^8 (-1)\cdot e^{-j(2\pi/8)Kt}\cdot dt$$

$$a_K = \frac{1}{j\pi K}\left[1 - e^{-j\pi K}\right]$$

$$a_K = \begin{cases} 0, & K \in \text{Even} \\ \dfrac{2}{j\pi K}, & K \in \text{Odd} \end{cases}$$ → (4 Point)



$$H(j\omega) = \frac{\sin(4\omega)}{\omega}$$

By using the property, the output —

$$y(t) = \sum_{K=-\infty}^{\infty} a_K \cdot H(jK\omega_0) \cdot e^{jK\omega_0 t}$$ → (1 Point)

$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{8} = \frac{\pi}{4}$$

Since a_K is non-zero only for odd value of K, we need to evaluate the above summation only for odd value of K.

$$y(t) = \sum_{K=-\infty}^{\infty} a_K \cdot \frac{\sin(K\pi)}{K(\pi/4)} \cdot e^{jK\omega_0 t}, \quad K \in \text{odd}$$

$$y(t) = 0, \quad K \in \text{Even}$$

$$y(t) = \sum_{K=-\infty}^{\infty} \left(\frac{2}{j\pi K}\right) \cdot \frac{\sin(K\pi)}{K(\pi/4)} \cdot e^{jK\omega_0 t}, \quad K \in \text{odd}$$

$$y(t) = 0, \quad K \in \text{Even}$$

$$y(t) = 0, \quad K \in \text{Even}$$

$$y(t) = 0, \quad K \in \text{odd}$$

$$\therefore y(t) = 0, \quad \forall K$$ → (4 Point)