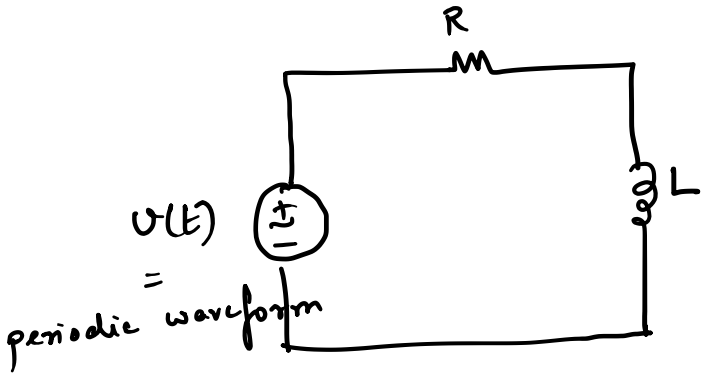
Review



$$v(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)$$

$$\omega_0 = \frac{2\pi}{T}, \quad T = \text{time periodic}$$

$\forall$ periodic waveforms

$$a_0 = \frac{1}{T}\int_0^T v(t)\,dt$$

$$a_n = \frac{2}{T}\int_0^T v(t)\cos(n\omega_0 t)\,dt$$

$$b_n = \frac{2}{T}\int_0^T v(t)\sin(n\omega_0 t)\,dt$$

Chapter 18, Pg 738 [9]-[11]

<u>To make it faster</u> Refer to Table 18.1 Pg 747

① $v(t)$ — even, $b_n = 0 \quad \forall n$

— odd $\quad a_n = 0 \quad \forall n$

② half symmetric

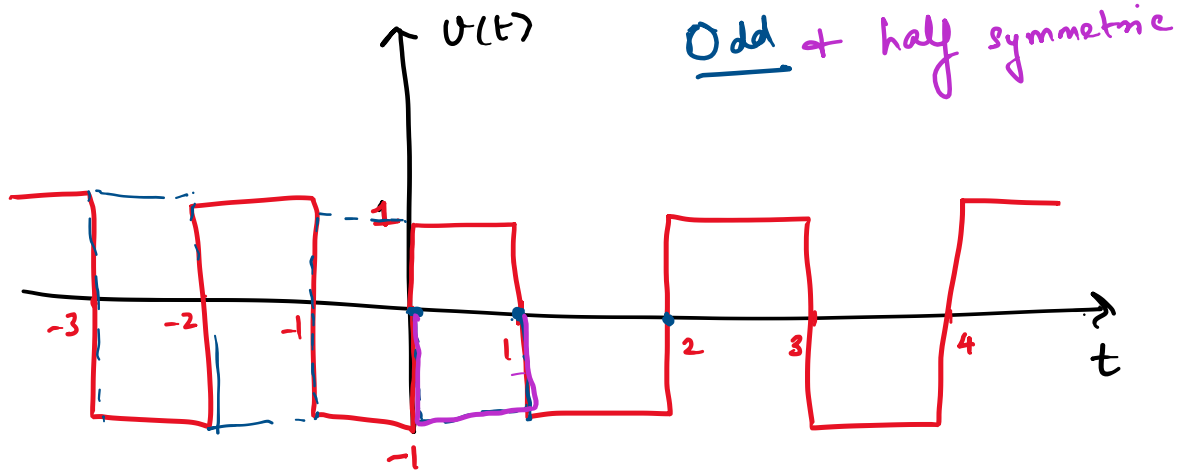
$$f(t) = -f\left(t - \frac{T}{2}\right) = -f\left(t + \frac{T}{2}\right)$$

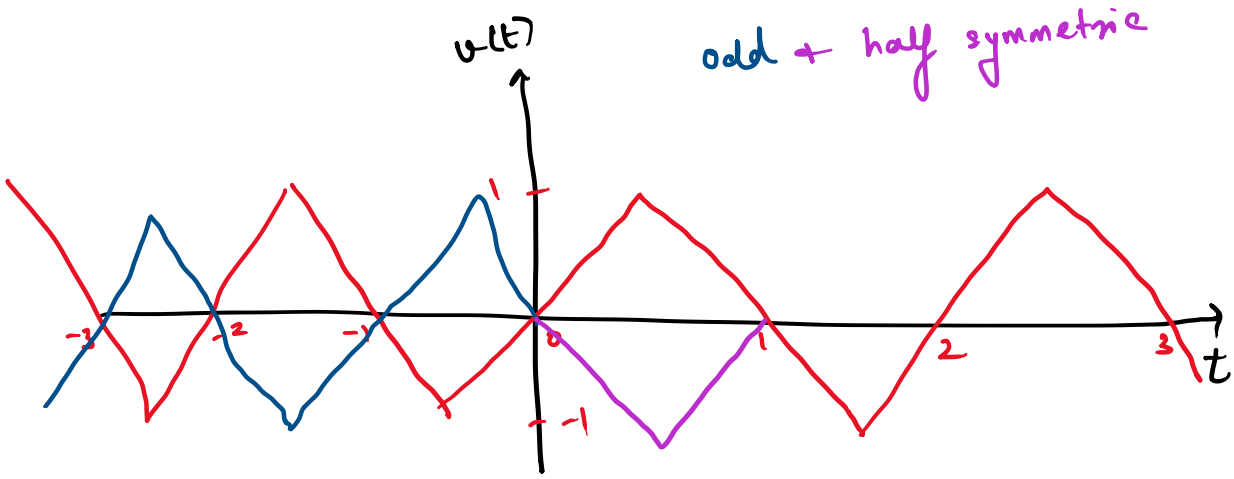
$$a_n = \begin{cases} \frac{4}{T}\int_0^{T/2} v(t)\cos(n\omega_0 t), & n \text{ odd} \\ 0, & n \text{ even} \end{cases}$$

$$b_n = \begin{cases} \frac{4}{T}\int_0^{T/2} v(t)\sin(n\omega_0 t), & n \text{ odd} \\ 0, & n \text{ even} \end{cases}$$

- Even + half symmetric
- Odd + half symmetric

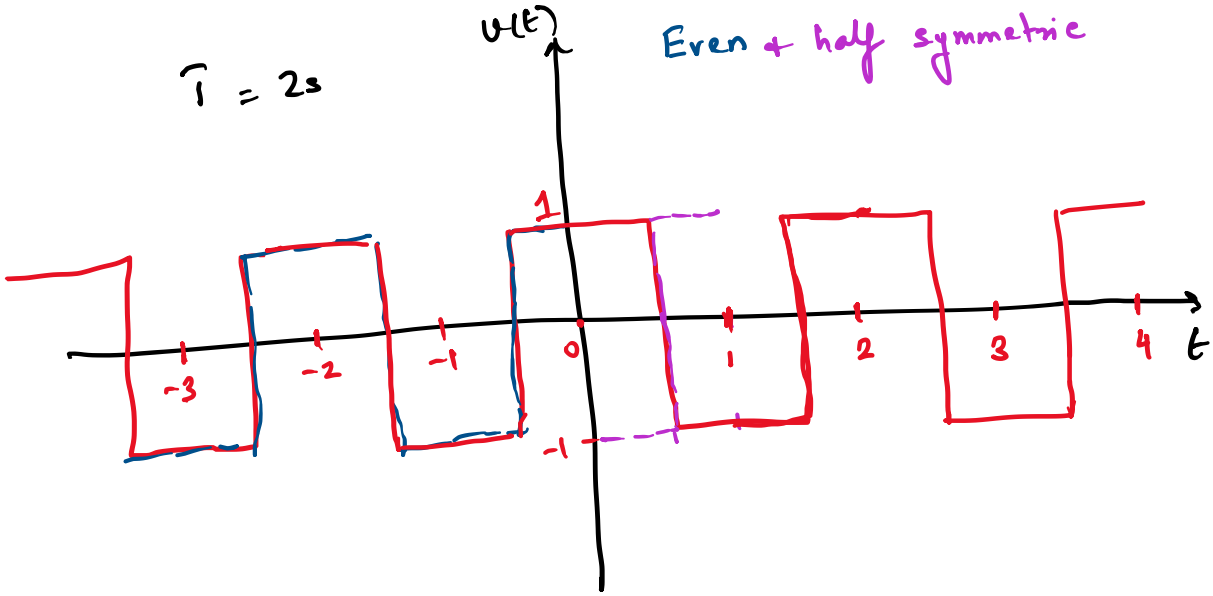
→





$T = 2s$

Even + half symmetric

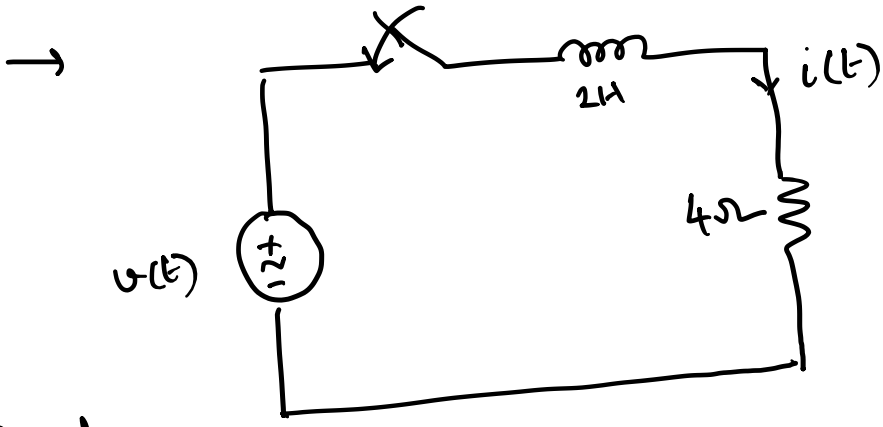


Complete Response of circuit

- Transient Response or Natural Response
  - $e^{-\sigma t}$
  - initial conditions of L + C
- Steady state response or forced response
  - DC / $\cos(n\omega_0 t)$ / $\sin(n\omega_0 t)$
  - forcing functions - DC, AC, periodic waveform, $f(t)u(t)$

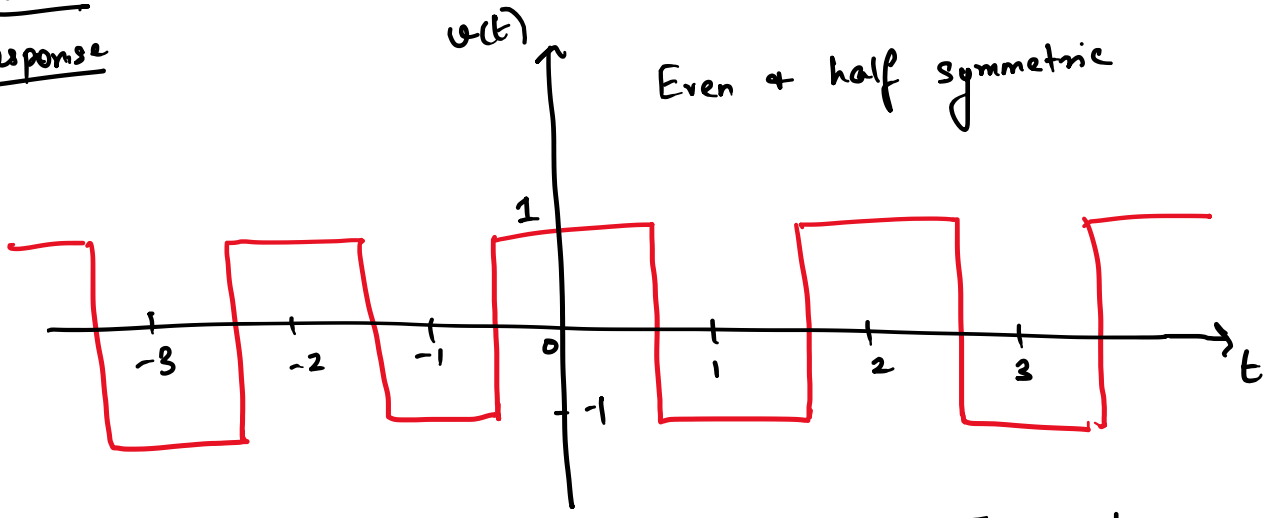
Natural Response

- find the # L + C & initial conditions
- $A e^{-\sigma_1 t}$ $\quad \sigma_1 \rightarrow$ each L or C with an initial condition



$i(0) = 1\,A$

Forced Response



Even & half symmetric

$T = 2s$, $\omega_0 = \frac{2\pi}{T} = \pi$ rad/s

even & half symmetric

Table 18.1

$$a_n = \frac{8}{T}\int_0^{T/4} v(t)\cos(n\omega_0 t)\,dt \quad \begin{cases} n \text{ odd} \\ n \text{ even} \end{cases}$$

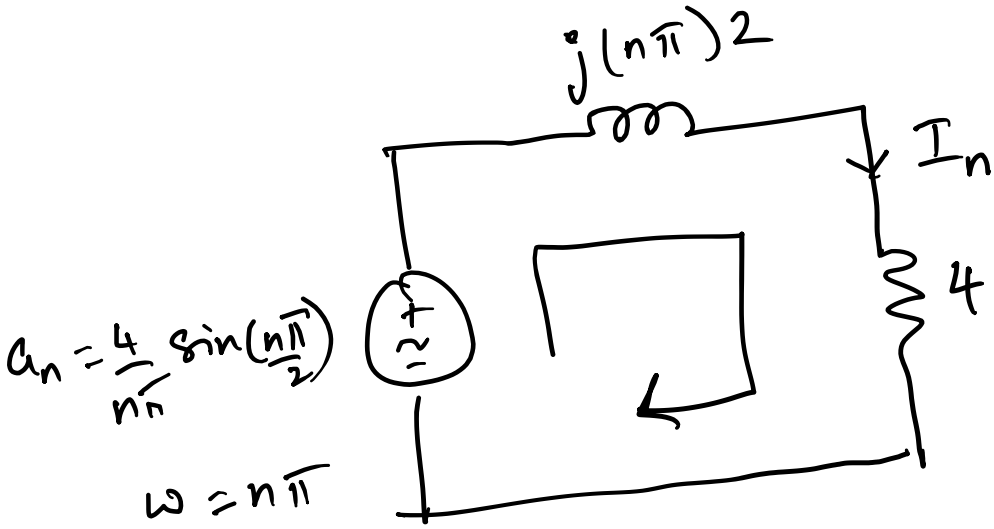
$$0$$

$$b_n = 0 \quad \forall n$$

$$a_n = \frac{8}{2}\int_0^{1/2} 1\cos(n\pi t)\,dt = 4\left[\frac{\sin(n\pi t)}{n\pi}\right]_0^{1/2}$$

$$= \frac{4}{n\pi}\left[\sin\left(\frac{n\pi}{2}\right) - 0\right] = \frac{4}{n\pi}\sin\left(\frac{n\pi}{2}\right)$$

$$a_n = \begin{cases} \dfrac{4}{n\pi}\sin\left(\dfrac{n\pi}{2}\right), & n \text{ odd} \\ 0, & n \text{ even} \end{cases}$$



$$I_n = \frac{a_n}{j2n\pi + 4} = \frac{\frac{4}{n\pi}\sin\left(\frac{n\pi}{2}\right)}{4 + j2n\pi}$$

$$= \frac{\frac{4}{n\pi}\sin\left(\frac{n\pi}{2}\right)\angle 0^\circ}{\sqrt{4^2 + (4n^2\pi^2)}\,\angle \tan^{-1}\left(\frac{2n\pi}{4}\right)}$$

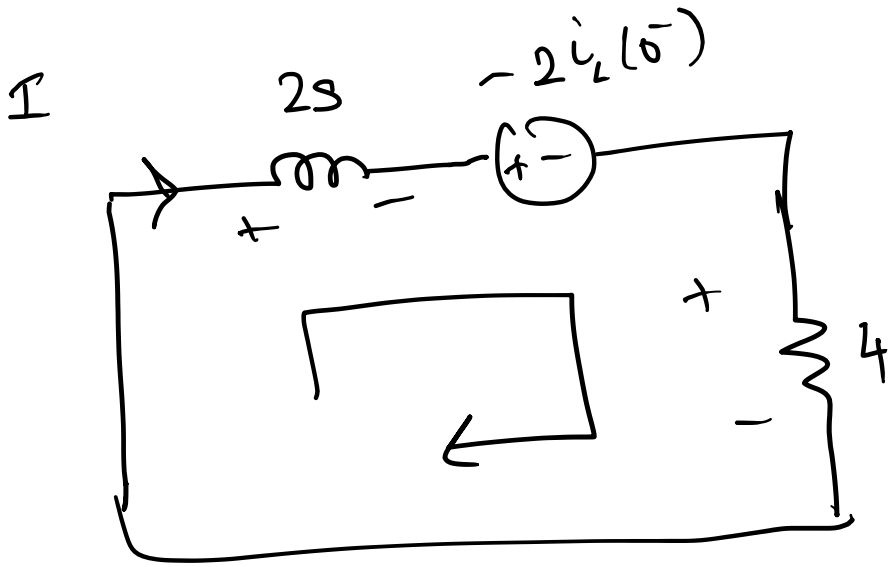
$$= \frac{4\sin(n\pi/2)}{n\pi \times 4\sqrt{4 + n^2\pi^2}}\,\angle -\tan^{-1}(n\pi/2)$$

$$I_n = \frac{1}{n\pi} \frac{\sin\left(\frac{n\pi}{2}\right)}{\sqrt{4+n^2\pi^2}} \angle -\tan^{-1}\left(\frac{n\pi}{2}\right)$$

$$i_n(t) = \frac{\sin\left(\frac{n\pi}{2}\right)}{n\pi\sqrt{4+n^2\pi^2}} \cos\left(n\pi t - \tan^{-1}\left(\frac{n\pi}{2}\right)\right)$$

$$i_f(t) = \sum_{n=1,3,5\ldots} \frac{\sin\left(\frac{n\pi}{2}\right)}{n\pi\sqrt{4+n^2\pi^2}} \cos\left(n\pi t - \tan^{-1}\left(\frac{n\pi}{2}\right)\right) \text{ A}$$

<u>Natural Response</u> (make forcing function $= 0$)



$$-2s \times I - \left(-2i_L(0^-)\right) - 4I = 0$$

$$2i_L(0^-) = I(-2s-4)$$

$$I = \frac{-\cancel{2}\, i_L(0^-)}{\cancel{2}\,(s+2)} = \frac{A}{s+2}$$

where $-i_L(0^-) = A$

$$i_{natural}(t) = Ae^{-2t}$$

Complete Response

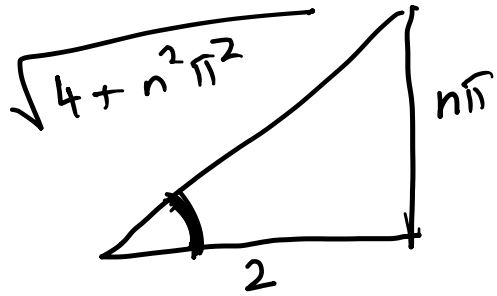
$$i(t) = i_{natural}(t) + i_f(t)$$

$$= Ae^{-2t} + \sum_{n=1,3,5\ldots} \frac{\sin\left(\frac{n\pi}{2}\right)}{n\pi\sqrt{4+n^2\pi^2}} \cos\left(n\pi t - \tan^{-1}\left(\frac{n\pi}{2}\right)\right)$$

unknown

$$i(0) = 1\,A$$

$$\Rightarrow 1 = A + \sum_{n=1,3,5\ldots} \frac{\sin\left(\frac{n\pi}{2}\right)}{n\pi\sqrt{4+n^2\pi^2}} \cos\left(-\tan^{-1}\left(\frac{n\pi}{2}\right)\right)$$



$$\cos\left(-\tan^{-1}\left(\frac{n\pi}{2}\right)\right) = \cos\left(\tan^{-1}\left(\frac{n\pi}{2}\right)\right)$$

$$= \frac{2}{\sqrt{4+n^2\pi^2}}$$

$$\Rightarrow 1 = A + \sum_{n=1,3,5\ldots} \frac{\sin\left(\frac{n\pi}{2}\right)}{n\pi\sqrt{4+n^2\pi^2}} \times \frac{2}{\sqrt{4+n^2\pi^2}}$$

$$1 = A + \sum_{n=1,3,5\ldots} \frac{2\sin\left(\frac{n\pi}{2}\right)}{n\pi\left(4+n^2\pi^2\right)}$$

just do $n=1$ because $n=3,5,7\ldots$ will be very small

$$1 = A + \frac{2\times 1}{\pi\left(4+\pi^2\right)} = A + \frac{2}{\pi\left(4+\pi^2\right)}$$

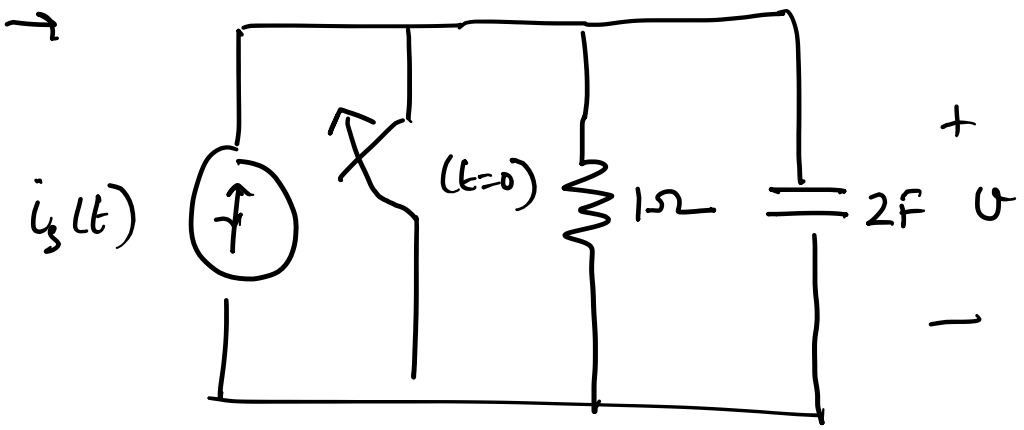
$$\Rightarrow A = 1 - \frac{2/\pi}{(4+\pi^2)} = 0.954$$

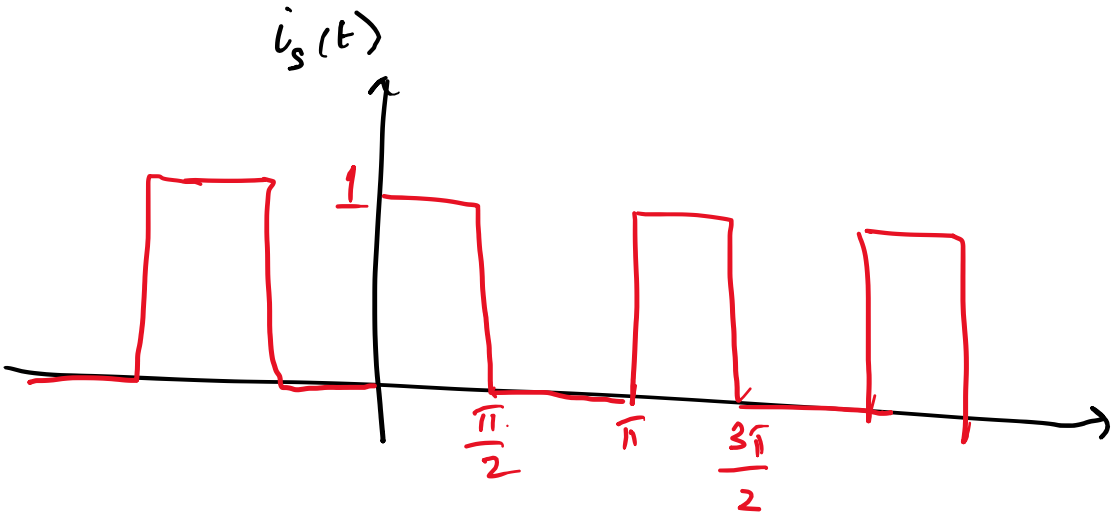
Complete Response

$$i(t) = 0.954\,e^{-2t} + \sum_{n=1,3,5,\ldots} \frac{\sin\left(\frac{n\pi}{2}\right)}{n\pi\sqrt{4+n^2\pi^2}} \cos\left(n\pi t - \tan^{-1}\left(\frac{n\pi}{2}\right)\right)$$

A

$v(0) = 0\ V$



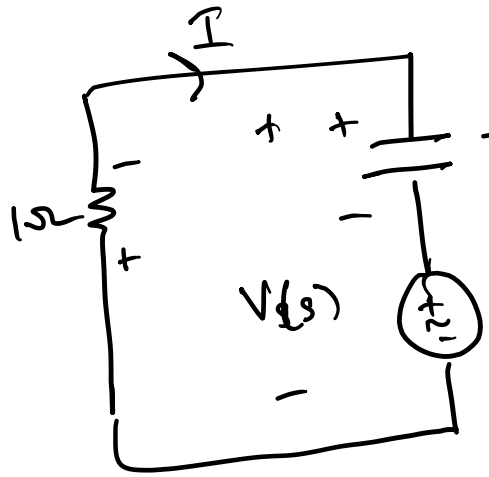


(previous class)

Forced Response

$$v(t)_{forced} = 5 + \sum_{n=1,3,5,\ldots} \frac{40/\pi}{\sqrt{1+4n^2}} \sin\left(nt + \tan^{-1}\left(\frac{1}{2n}\right)\right)\ V$$

Natural Response (forcing function = 0)



$$\frac{v_c(0^-)}{s} = \frac{A}{s}$$

$$\boxed{\frac{\frac{A}{2}}{s(s+\frac{1}{2})} = \frac{B}{s} + \frac{C}{(s+\frac{1}{2})}}$$

$$-1 \times I - \frac{1}{2s} \times I - \frac{A}{s} = 0$$

$$-I\left(1 + \frac{1}{2s}\right) = \frac{A}{s} \Rightarrow I = \frac{A/s}{1+\frac{1}{2s}} \times \frac{s}{s}$$

$$I = \frac{A}{s + 1/2}$$

$$V = \frac{1}{2s} \times I + \frac{A}{s} = \frac{A/2}{s(s+\frac{1}{2})} + \frac{A}{s}$$

$$= \frac{B}{s} + \frac{C}{s+\frac{1}{2}} + \frac{A}{s}$$

$$\frac{A}{2} = B\left(s+\frac{1}{2}\right) + Cs$$

put $s = 0 \Rightarrow \frac{A}{2} = \frac{B}{2} + C(0) \Rightarrow B = A$

compare coeff of s $\quad 0 = B + C \Rightarrow C = -B = -A$

$$V(s) = \frac{A}{s} + \frac{-A}{s+\frac{1}{2}} + \frac{A}{s} = \frac{2A}{s} - \frac{A}{s+\frac{1}{2}}$$

$$\underset{\text{natural}}{v(t)} = 2A\,u(t) - Ae^{-\frac{1}{2}t}\,u(t)$$

$$= A\left(2 - e^{-\frac{1}{2}t}\right)u(t)$$

Complete Response

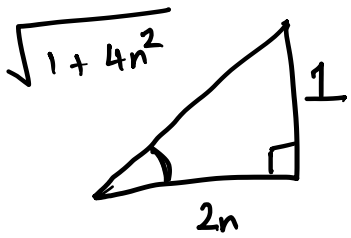
$$v(t) = v_{\text{natural}}(t) + v_{\text{forced}}(t)$$

$$= A\left(2 - e^{-\frac{1}{2}t}\right)u(t) + 5 + \sum_{n=1,3,5\ldots} \frac{40/\pi}{\sqrt{1+4n^2}} \sin\left(nt + \tan^{-1}\left(\frac{1}{2n}\right)\right)$$

unknown

Use $v(0) = 0$

$$\Rightarrow 0 = A(2-1) + 5 + \sum_{n=1,3,5\ldots} \frac{40/\pi}{\sqrt{1+4n^2}} \sin\left(\tan^{-1}\left(\frac{1}{2n}\right)\right)$$



$$\sin\left(\tan^{-1}\left(\frac{1}{2n}\right)\right) = \frac{1}{\sqrt{1+4n^2}}$$

$$0 = A + 5 + \sum_{n=1,3,5\ldots} \frac{40/\pi}{\sqrt{1+4n^2}} \times \frac{1}{\sqrt{1+4n^2}}$$

$$0 = A + 5 + \sum_{n=1,3,5 \ldots} \frac{40/\pi}{1+4n^2}$$

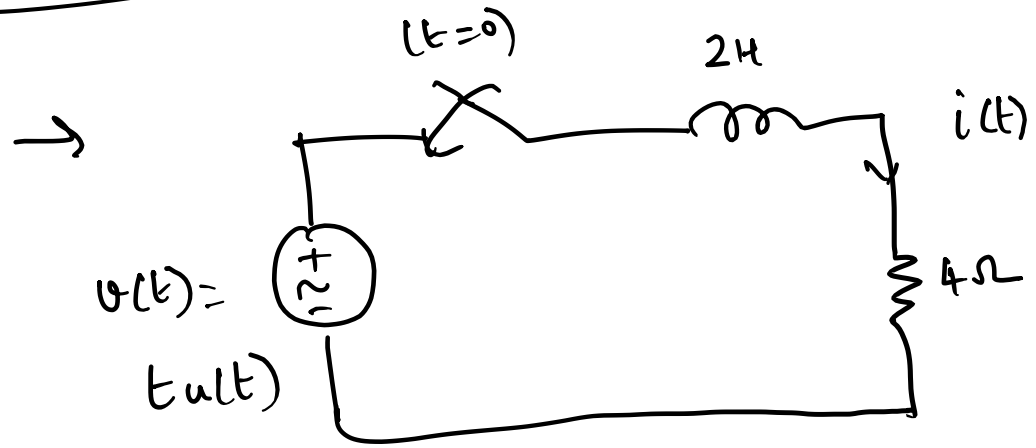
take $n=1$

$$0 = A + 5 + \frac{40/\pi}{1+4} = A + 5 + 8/\pi$$

$$A = -5 - \frac{8}{\pi} = -7.54$$

Complete Response

$$v(t) = -7.54\left(2 - e^{-\frac{1}{2}t}\right)u(t) + 5 + \sum_{n=1,3,5 \ldots} \frac{40/\pi}{\sqrt{1+4n^2}} \sin\left(nt - \tan^{-1}\left(\frac{1}{2n}\right)\right)$$



$i(0) = 1A$

① Forced Response , ② Natural Response

③ Complete Response with unknown A

④ Find A using $i(0) = 1A$

⑤ ~~Find~~

(1st) Complete Response with known A