# Practice Sheet 7

(1)

$$x[n] = \begin{cases} 1 & , \quad 0 \le n \le 5 \\ 0 & , \quad \text{ow} \end{cases}$$

$$g[n] = x[n] - x[n-1]$$

(a)

$$g[n] = \delta[n] - \delta[n-6]$$

$$\therefore \; G(z) = 1 - z^{-6} \qquad |z| > 0.$$

(b) $$x[n] = \sum_{k=-\infty}^{n} g[k] \xleftrightarrow{z} X(z) = \frac{1}{1-z^{-1}} G(z)$$

For $|z| < 1$

$$\therefore \; X(z) = \frac{1 - z^{-6}}{1 - z^{-1}} , \quad |z| > 0.$$

The ROC is $|z| > 0$ because $x[n]$ is a finite length signal.

(2)

(a) NO, finite length signal ROC is the entire z-plane

$\therefore$ No poles in the finite z-plane for a finite length signal.

So this is not the case for given problem.

(b) No. Here signal is absolutely summable, the ROC must include unit circle.

Signal has a pole at $z = \frac{1}{2}$, ROC can never be of the form $0 \leq |z| < r_0$. So signal can not be left sided.

(c) YES. Since the signal is absolutely summable, the ROC must include the unit circle.

Since it is given that the signal has a pole at $z = \frac{1}{2}$, a valid ROC for this signal would be $|z| > \frac{1}{2}$. So this would correspond to a right sided signal.

(d) Yes, since the signal is absolutely summable. the ROC must include the unit circle. Clearly, we can define an ROC which is a ring in 2D plane and includes the unit circle. The signal is two sided.

(3)

$$x(n) = \left(\frac{1}{3}\right)^n \cos\left(\frac{\pi}{4} n\right) \text{ for } n \leq 0$$

$$\therefore \quad X(z) = \sum_{n=-\infty}^{0} \left(\frac{1}{3}\right)^n \cos\left(\frac{\pi}{4} n\right) z^{-n}$$

$$= \frac{1}{2}\sum_{n=-\infty}^{0}\left(\frac{1}{3}\right)^{n} e^{jn\pi/4} z^{-n} + \left(\frac{1}{2}\right)\sum_{n=-\infty}^{0}\left(\frac{1}{3}\right)^{n} e^{-jn\pi/4} z^{-n}$$

$$= \frac{1}{2}\frac{1}{1-3e^{j\pi/4}z} + \frac{1}{2}\frac{1}{1-3e^{j\pi/4}z}, \quad |z| < \frac{1}{3}$$

$$= \frac{1}{2}\frac{1}{1-3e^{-j\pi/4}z} + \frac{1}{2}\frac{1}{1-3e^{j\pi/4}z} \text{ for}$$

Poles at $z = \frac{1}{3}e^{j\pi/4}$ and $z = \frac{1}{3}e^{-j\pi/4}$.

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(4) Let

$$y[n] = x[n]\sum_{k=-\infty}^{\infty}\delta[n-3k]$$

Then

$$Y(e^{j\omega}) = \frac{1}{3}\sum_{k=0}^{3} X\left(e^{j(\omega - 2\pi k/3)}\right)$$

Here kindly note $\frac{\sin(\pi n/3)}{\pi n/3}$ is the impulse response of an ideal LPF with cut off frequency $\pi/3$ and pass band freq is 3.

∴ It is required when $y[n]$ passed through this filter should yield $x[n]$.

Not to overlap. $X(e^{j\omega}) = 0$ for $\pi/3 \le |\omega| \le \pi$

(5) (b) If $x(j\omega) = 0$ for $|\omega| > \omega_1$

then $x(j\omega) * x(j\omega) = 0$ for $|\omega| > 2\omega_1$

$\therefore$ $x(j\omega) = 0$ for $|\omega| > 7500\pi$.

Nyquist rate is $NR = 2 \times 7500\pi = 15000\pi$

In order to be able to retrieve $x(t)$ from $x_p(t)$

Maximum sampling period time $= \dfrac{2\pi}{15000\pi}$

$= 1.33 \times 10^{-4}$ sec

Sampling period $T = 10^{-4}$ sec $< T_{max}$.

(5) (a) Nyquist rate in this case is $2 \times 5000\pi = 10000\pi$.

So for proper recovering of $x(t)$,

sampling period must be atmost

$T_{max} = \dfrac{2\pi}{10000\pi} = 2 \times 10^{-4}$ sec

$\therefore$ Sampling period is $10^{-4}$ sec $< T_{max}$.

$\therefore$ $x(t)$ is recovered.

(6)

(a) Yes, Aliasing will occur in this case,

Consider the sinusoidal signal. x(t) for k=5.

$$y(t) = \left(\tfrac{1}{2}\right)^5 \sin(5\pi t)$$

If x(t) sampled at T=0.2, then we will always be sampling y(t) at exactly zero crossings.

∴ Sinusoidal y(t) of freq. $5\pi$ is aliased into sinusoidal freq zero of sampled signal.

(b)

Since aliasing resulted in the loss of signal $\left(\tfrac{1}{2}\right)^5 \sin(5\pi t)$, the output is

$$x_0(t) = \sum_{k=0}^{4} \left(\tfrac{1}{2}\right)^k \sin(k\pi t)$$

The fourier series representation

$$x_0(t) = \sum_{k=-4}^{4} a_k e^{j(k\pi/t)}$$

where $a_k = \begin{cases} 0 & k=0 \\ -j\left(\tfrac{1}{2}\right)^{k+1}, & 1 \le k \le 4 \\ j\left(\tfrac{1}{2}\right)^{-k+1}, & -4 \le k \le -1 \end{cases}$