Quiz - 9

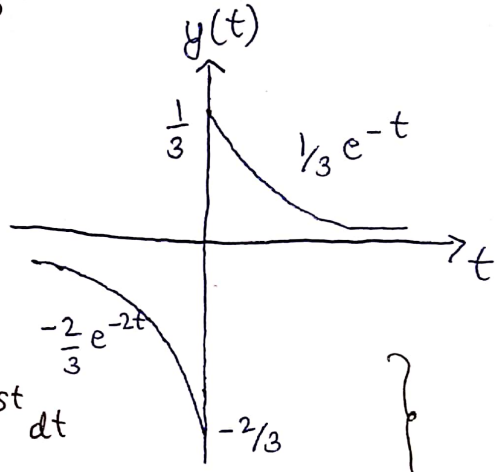
Total Marks - 10

(1.) Given that

$$X(s) = \frac{s+2}{s-2}$$

$$x(t) = 0 \qquad t > 0$$

$$y(t) = -\frac{2}{3} e^{2t} u(-t) + \frac{1}{3} e^{-t} u(t)$$



(a) $H(s)$

$$Y(s) = \int_{-\infty}^{\infty} y(t)\, e^{-st}\, dt$$

$$= \int_{-\infty}^{\infty} \left(-\frac{2}{3}\left[e^{2t} u(-t)\right] + \frac{1}{3}\left[e^{-t} u(t)\right] \right) e^{-st}\, dt$$

$$= -\frac{2}{3} \int_{-\infty}^{\infty} e^{-(s-2)t} u(-t)\, dt + \frac{1}{3} \int_{-\infty}^{\infty} e^{-(s+1)t} u(t)\, dt$$

$$= -\frac{2}{3} \int_{-\infty}^{0} e^{-(s-2)t}\, dt + \frac{1}{3} \int_{0}^{\infty} e^{-(s+1)t}\, dt$$

$$= -\frac{2}{3} \left. \frac{e^{-(s-2)t}}{-(s-2)} \right|_{-\infty}^{0} + \frac{1}{3} \left. \frac{e^{-(s+1)t}}{-(s+1)} \right|_{0}^{\infty}$$

$$= -\frac{2}{3} \left[\frac{1-0}{-(s-2)} \right] + \frac{1}{3} \left[\frac{0-1}{-(s+1)} \right]$$

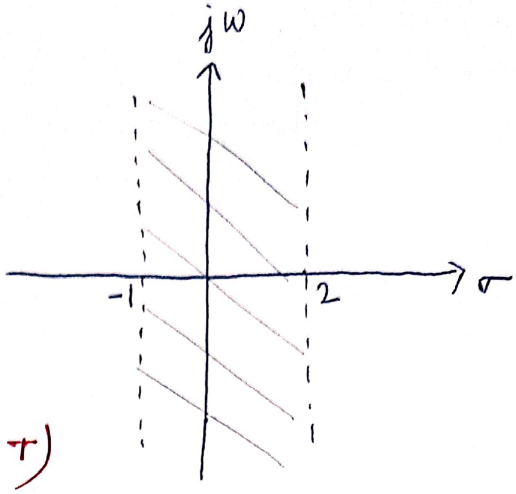
$$= -\frac{2}{3} \left[\frac{-1}{s-2} \right] + \frac{1}{3} \left[\frac{1}{s+1} \right]$$

$$= \frac{2/3}{s-2} + \frac{1/3}{s+1}$$

$$= \frac{\frac{2}{3}s + \frac{2}{3} + \frac{1}{3}s - \frac{2}{3}}{(s-2)(s+1)}$$

$$= \frac{s}{(s-2)(s+1)}$$

→ (2 POINT)



ROC is $-1 < \mathrm{Re}\{s\} < 2$

→ 0.5 Marks

Since $x(t)$ is right sided signal

Hence ROC for $X(s)$ is $\mathrm{Re}\{s\} < 2$

Now,

$$H(s) = \frac{Y(s)}{H(s)} = \left(\frac{s}{(s-2)(s+1)}\right) \Big/ \left(\frac{(s+2)}{(s-2)}\right)$$

$$\boxed{H(s) = \frac{s}{(s+2)(s+1)}}$$

→ (1 POINT)

Now, ROC of $Y(s)$ has to be intersection of ROC of $X(s)$ and $H(s)$. Hence $H(s) \rightarrow$ ROC is $\mathrm{Re}\{s\} > -1$

(→ 0.5 marks)

(b) $$H(s) = \frac{2}{s+2} - \frac{1}{s+1}$$

Using property,

$$h(t) = 2e^{-2t}u(t) - e^{-t}u(t)$$

→ 3 marks.

(c) $e^{-3t}\,u(t)$ $\quad -\infty < t < \infty$

$$X(s) = \frac{1}{s+3}$$

ROC: $Re\{s\} > -3$ → (0.5 POINT)

$$Y(s) = H(s) \cdot X(s)$$

$$= \left[\frac{s}{(s+2)(s+1)}\right]\frac{1}{(s+3)}$$

$$= \frac{s}{(s+1)(s+2)(s+3)}$$

$$= \frac{A}{(s+1)} + \frac{B}{(s+2)} + \frac{C}{(s+3)}$$

0.5 marks

$$s = A(s+2)(s+3) + B(s+1)(s+3) + C(s+1)(s+2)$$

$s = -1$
$-1 = 2A$
$A = -\frac{1}{2}$

$s = -2$
$-2 = -B$
$B = 2$

$s = -3$
$-3 = 2C$
$C = -\frac{3}{2}$

$$Y(s) = \frac{-1/2}{s+1} + \frac{2}{s+2} + \frac{-3/2}{s+3}$$

ROC: $Re\{s\} > -1$ → (0.5 POINT)

$$y(t) = -\frac{1}{2}e^{-t}u(t) + 2e^{-2t}u(t) - \frac{3}{2}e^{-3t}u(t)$$

1.5 mark