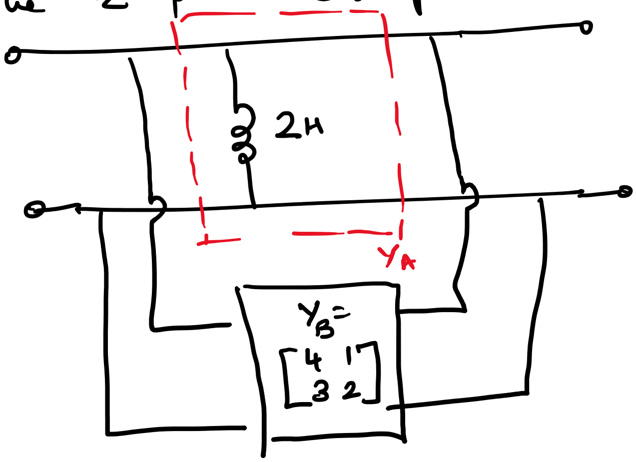
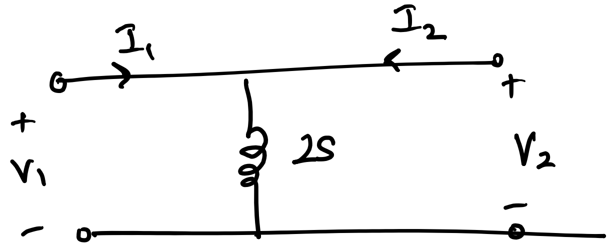
→ Find the Z parameters of the network shown below



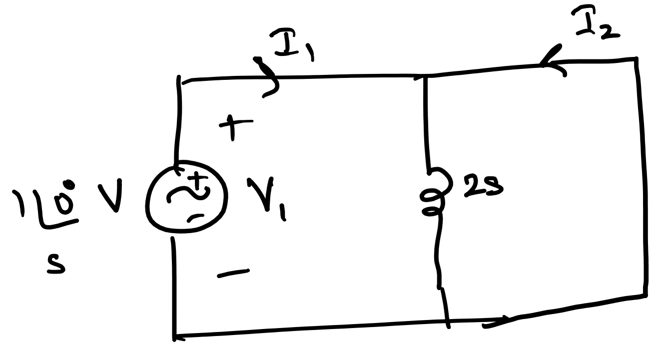
Step.1 Find Y_A parameters



$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Step.2 Find Y_{11}, Y_{21} by making $V_2 = 0$ (short port 2)

$V_1 = 1V$



$V_1 = 1V$, $I_1 = \frac{V_1}{Z} = \frac{1}{0} = \infty$

$I_2 = -I_1$

$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \infty$, $Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = -\infty$

Step.3 Find Y_{12}, Y_{22}

Since it is reciprocal $Y_{12} = Y_{21} = -\infty$

$Y_{22} = Y_{11} = +\infty$

$$[Y]_A = \begin{bmatrix} \infty & -\infty \\ -\infty & \infty \end{bmatrix}$$

Step.4 Total $[Y] = [Y]_A + [Y]_B$ since they are in parallel

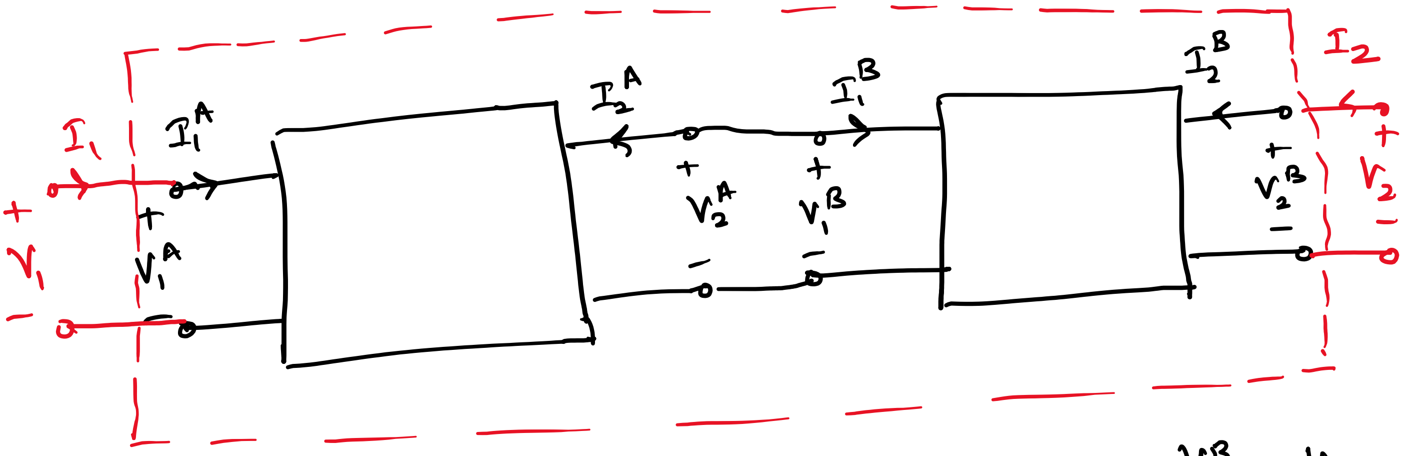
$$[Y] = \begin{bmatrix} \infty+4 & -\infty+1 \\ -\infty+3 & \infty+2 \end{bmatrix} = \begin{bmatrix} \infty & -\infty \\ -\infty & \infty \end{bmatrix}$$

Step.5 Find $[Z]$ parameters by converting from $[Y]$

$Z_{11} = \dfrac{Y_{22}}{\Delta y}$ = undefined $\quad Z_{12} = \dfrac{-Y_{12}}{\Delta y}$

$Z_{21} = \dfrac{-Y_{21}}{\Delta y} \quad Z_{22} = \dfrac{Y_{11}}{\Delta y}$

Cascade Connection



$V_1 = V_1^A$

$I_1 = I_1^A$

$V_2^A = V_1^B$

$-I_2^A = I_1^B$

$V_2^B = V_2$

$I_2^B = I_2$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} V_1^A \\ I_1^A \end{bmatrix} = \begin{bmatrix} t_A \end{bmatrix} \begin{bmatrix} V_2^A \\ -I_2^A \end{bmatrix}$$

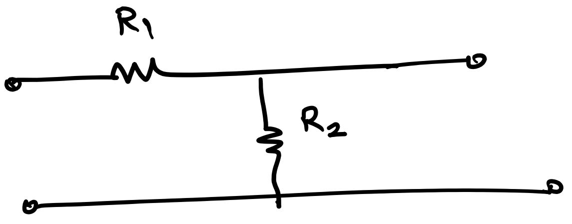
$$= \begin{bmatrix} t_A \end{bmatrix} \begin{bmatrix} t_B \end{bmatrix} \begin{bmatrix} V_2^B \\ -I_2^B \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} t_A \end{bmatrix} \begin{bmatrix} t_B \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

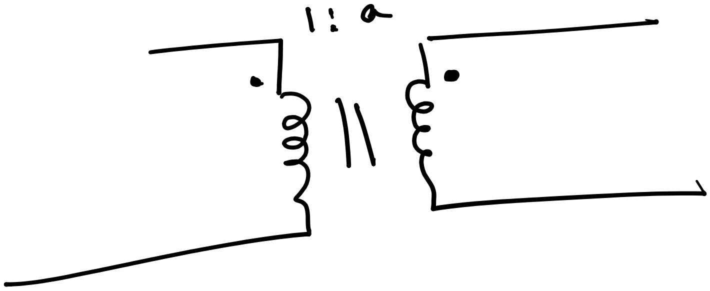
" Matrix multiplication & not element-wise multiplication"

→ Find $[t]$ for the following ckt

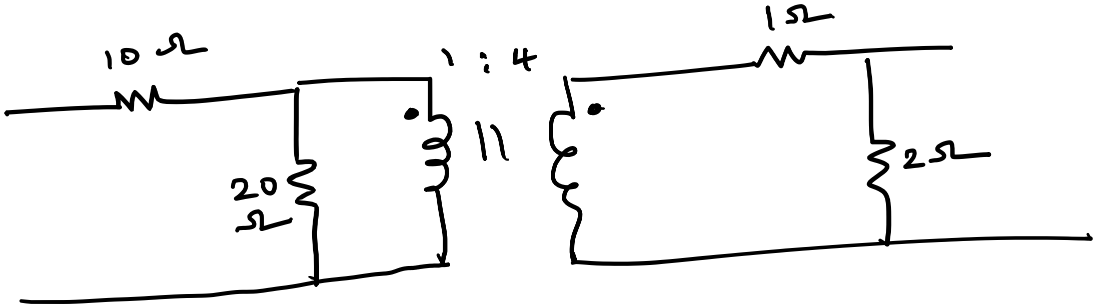
(a)



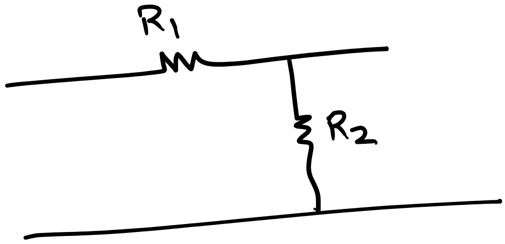
b)



c)

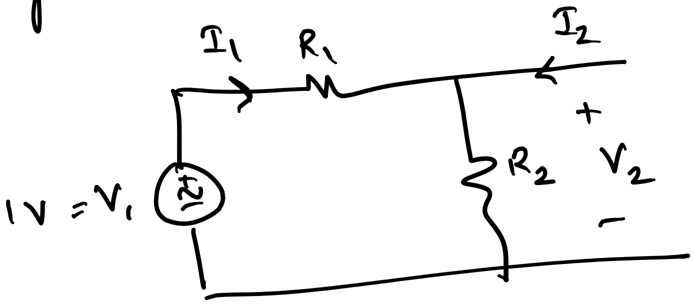


(a)



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

to find t_{11}, t_{21}, $I_2 = 0$, $V_1 = 1V$

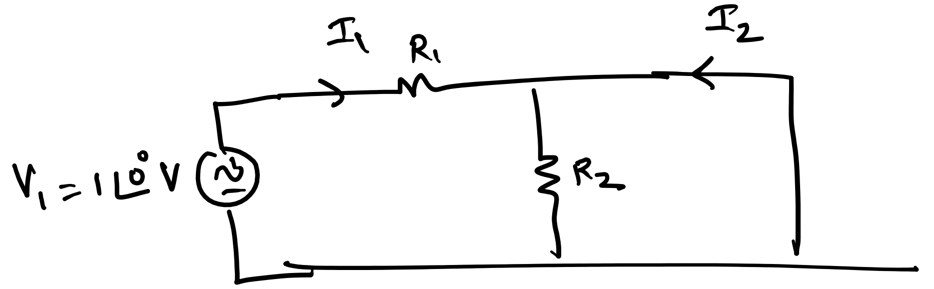


$$I_1 = \frac{V_1}{R_1 + R_2}, \quad \frac{V_2}{V_1} = \frac{R_2}{R_1 + R_2}$$

$$\Rightarrow \quad t_{11} = \left.\frac{V_1}{V_2}\right|_{-I_2=0} = \frac{R_1 + R_2}{R_2} = 1 + \frac{R_1}{R_2}$$

$$t_{21} = \left.\frac{I_1}{V_2}\right|_{-I_2=0} = \frac{I_1}{V_1} \times \frac{V_1}{V_2} = \frac{1}{R_1 + R_2} \times \frac{R_1 + R_2}{R_2} = \frac{1}{R_2}$$

to find t_{12}, t_{22}, $V_2 = 0$ (short port 2), $V_1 = 1V$



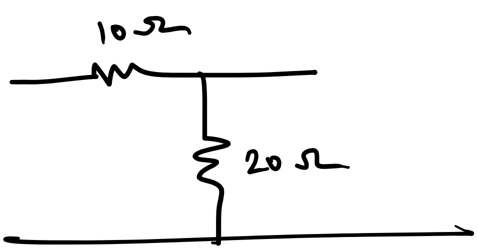
$I_1 = \frac{1}{R_1}$ $\quad I_2 = -I_1 = -\frac{1}{R_1}$

$$t_{12} = \left.\frac{V_1}{-I_2}\right|_{V_2=0} = \frac{1}{-(-1/R_1)} = +R_1$$

$$t_{22} = \left.\frac{I_1}{-I_2}\right|_{V_2=0} = +1$$

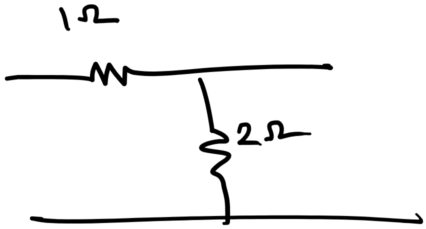
$$[t] = \begin{bmatrix} 1 + \frac{R_1}{R_2} & R_1 \\ \frac{1}{R_2} & 1 \end{bmatrix}$$

$\therefore$



$$\equiv \begin{bmatrix} 1 + 1/2 & 10 \\ \frac{1}{20} & 1 \end{bmatrix}$$

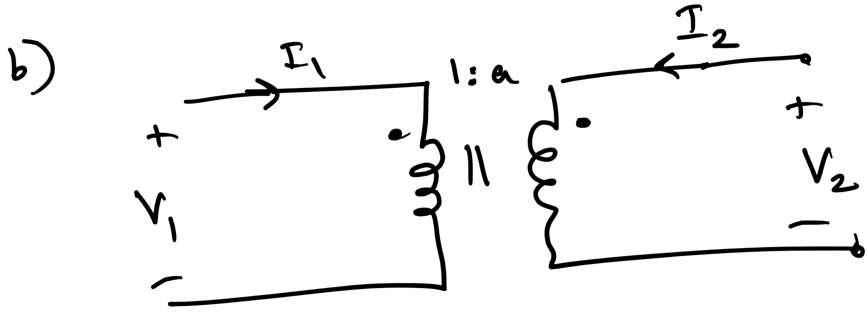
$$= \begin{bmatrix} 1.5 & 10 \\ 0.05 & 1 \end{bmatrix}$$



$$\equiv \begin{bmatrix} 1 + 1/2 & 1 \\ \frac{1}{2} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1.5 & 1 \\ 0.5 & 1 \end{bmatrix}$$

b)

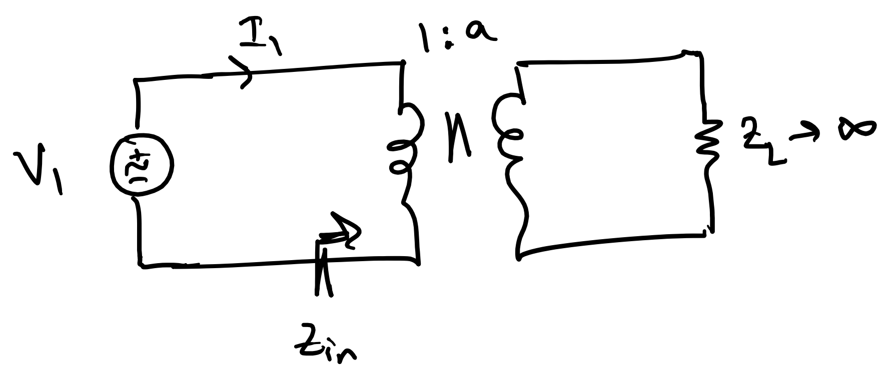


Ideal transformer

$$\frac{V_2}{V_1} = -a \qquad \frac{I_2}{I_1} = \frac{-1}{a}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

to find t_{11}, t_{21}, $I_2 = 0$ (open port 2), $V_1 = 1\text{ V}$



$$Z_{in} = \frac{Z_L}{a^2} \Rightarrow \quad Z_{in} \to \infty$$

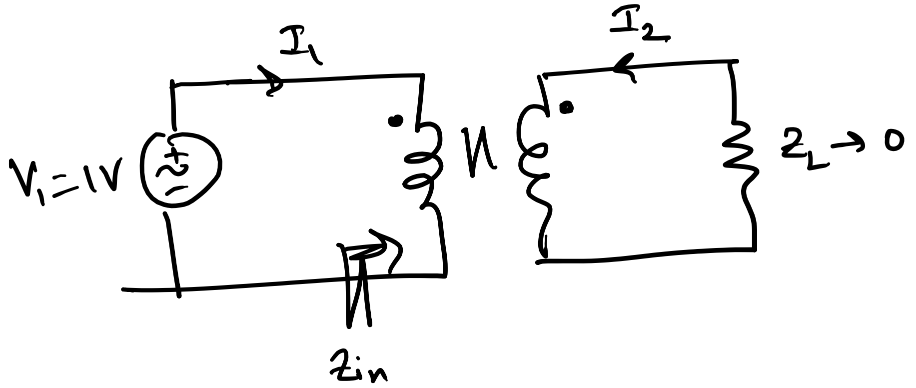
$$\therefore \; I_1 = \frac{V_1}{Z_{in}} = \lim_{Z_{in} \to \infty} \frac{V_1}{Z_{in}} = 0$$

$$V_2 = -aV_1 = -a$$

$$t_{11} = \left.\frac{V_1}{V_2}\right|_{-I_2=0} = \frac{1}{-a}$$

$$t_{21} = \left.\frac{I_1}{V_2}\right|_{-I_2=0} = \frac{0}{-a} = 0$$

to find t_{12} & t_{22} $\quad V_2 = 0$ (short port 2), $V_1 = 1V$



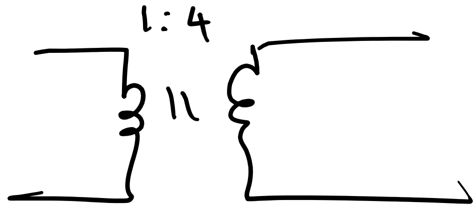
$$Z_{in} = \frac{Z_L}{a^2} = 0$$

$$\Rightarrow I_1 = \frac{V_1}{Z_{in}} = \frac{1}{0} = \infty \quad , \quad I_2 = -\frac{I_1}{a} = -\infty$$

$$t_{12} = \left.\frac{V_1}{-I_2}\right|_{V_2=0} = \frac{1}{+\infty} = 0$$

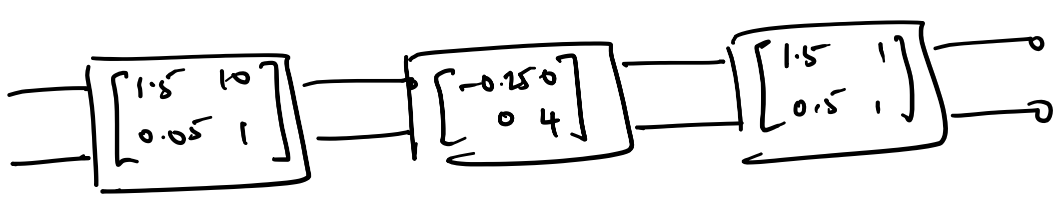
$$t_{22} = \left.\frac{I_1}{-I_2}\right|_{V_2=0} = a$$

$$[t] = \begin{bmatrix} -\frac{1}{a} & 0 \\ 0 & a \end{bmatrix}$$



$$= \begin{bmatrix} -0.25 & 0 \\ 0 & 4 \end{bmatrix}$$

(c)



$$\begin{bmatrix} 1.5 & 10 \\ 0.05 & 1 \end{bmatrix} \begin{bmatrix} -0.25 & 0 \\ 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -1.5 \times 0.25 + 10 \times 0 & 1.5 \times 0 + 10 \times 4 \\ -0.05 \times 0.25 + 1 \times 0 & 0.05 \times 0 + 1 \times 4 \end{bmatrix}$$

$$= \begin{bmatrix} -0.375 & 40 \\ -0.0125 & 4 \end{bmatrix}$$

$$\begin{bmatrix} -0.375 & 40 \\ -0.0125 & 4 \end{bmatrix} \begin{bmatrix} 1.5 & 1 \\ 0.5 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 19.4375 & 39.625 \\ 1.98125 & 3.9875 \end{bmatrix}$$