QUIZ–6 SOLUTION

SOL: Given that –

$$x[n] \underset{IFS}{\overset{FS}{\rightleftharpoons}} a_K$$
$$y[n] \underset{IFS}{\overset{FS}{\rightleftharpoons}} b_K$$

where, $x[n]$ & $y[n]$ are periodic signals with period N having fundamental frequency $\omega_0 = \frac{2\pi}{N}$

a_K & b_K are periodic with period N.

Let, $z[n] = x[n] \cdot y[n]$

$$= \sum_{K=\langle N \rangle} a_K \cdot e^{jK\omega_0 n} \cdot \sum_{K=\langle N \rangle} b_K \cdot e^{jK\omega_0 n}$$

$$= \sum_{K=\langle N \rangle} a_K b_K \; e^{j2K\omega_0 n}$$

$$= \sum_{K=\langle N \rangle} a_K b_K \; e^{jK\left(\frac{2\pi}{N/2}\right)n}$$

which shows that $z[n]$ is also periodic with period N (having fundamental period $N/2$). → (1 POINT)

We have to prove that –

$$\begin{pmatrix}\text{Multiplication in the} \\ \text{time domain}\end{pmatrix} \underset{IFS}{\overset{FS}{\rightleftharpoons}} \begin{pmatrix}\text{Convolution in the} \\ \text{frequency domain}\end{pmatrix}$$

$$z[n] = x[n] \cdot y[n] \underset{IFS}{\overset{FS}{\rightleftharpoons}} \sum_{l=\langle N \rangle} a_l \cdot b_{K-l} = C_K$$

Here $z[n] \underset{IFS}{\overset{FS}{\rightleftharpoons}} C_K$

$$\therefore \; C_K = \frac{1}{N} \sum_{n=\langle N \rangle} z[n] \cdot e^{-jK\omega_0 n}$$

$$= \frac{1}{N} \sum_{n=\langle N \rangle} x[n] \cdot y[n] \cdot e^{-jk\omega_0 n}$$

$$= \frac{1}{N} \sum_{n=\langle N \rangle} \sum_{l=\langle N \rangle} a_l \cdot e^{jl\omega_0 n} \sum_{m=\langle N \rangle} b_m \cdot e^{jm\omega_0 n} \cdot e^{-jk\omega_0 n}$$

$$= \frac{1}{N} \sum_{l=\langle N \rangle} \sum_{m=\langle N \rangle} a_l \cdot b_m \sum_{n=\langle N \rangle} e^{j(l+m-k)\omega_0 n}$$

$$= \begin{cases} \sum_{l=\langle N \rangle} \sum_{m=\langle N \rangle} a_l b_{k-l} = \sum_{l=\langle N \rangle} a_l b_{k-l} & l+m-k=0 \\ & m=(k-l) \\ 0 & \text{Otherwise} \end{cases}$$

→ (9 POINTS)