S&S ASSIGNMENT-1 SOLUTIONS

SOL(1): Given $x[n] = \{1, 2, \underset{\uparrow}{3}, 4, 5\}$ (4X2 POINTS)

(a) $y[n] = x[-2n]$

$= \{5, \underset{\uparrow}{3}, 1\}$

(b) $y[n] = x[-n+1]$

$= \{5, \underset{\uparrow}{4}, 3, 2, 1\}$

(c) $y[n] = x\left[\frac{2n}{3}\right]$

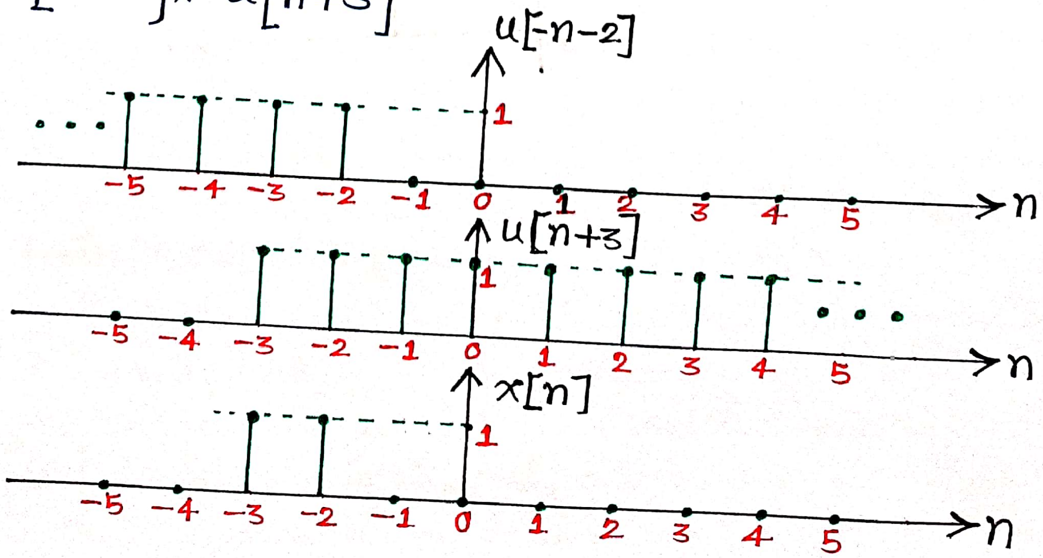
$= \{1, 0, 0, \underset{\uparrow}{3}, 0, 0, 5\}$

(d) $y[n] = x[2n-1]$

$= \{\underset{\uparrow}{2}, 4\}$

SOL(2): (4X5 POINTS)

(a) $x[n] = u[-n-2] \times u[n+3]$

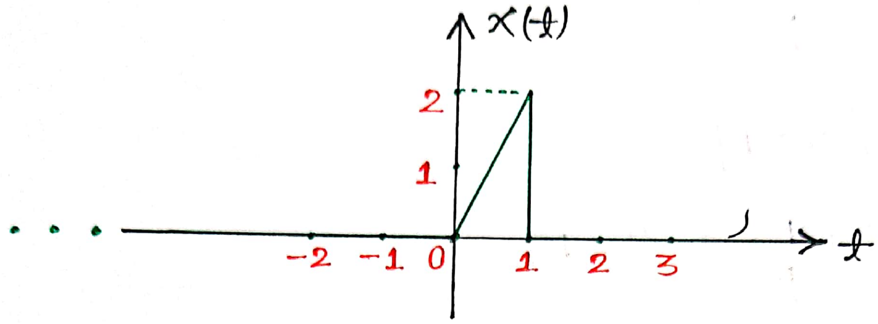


(b) $x(t) = 2t\left[u(t) - u(t-1)\right]$

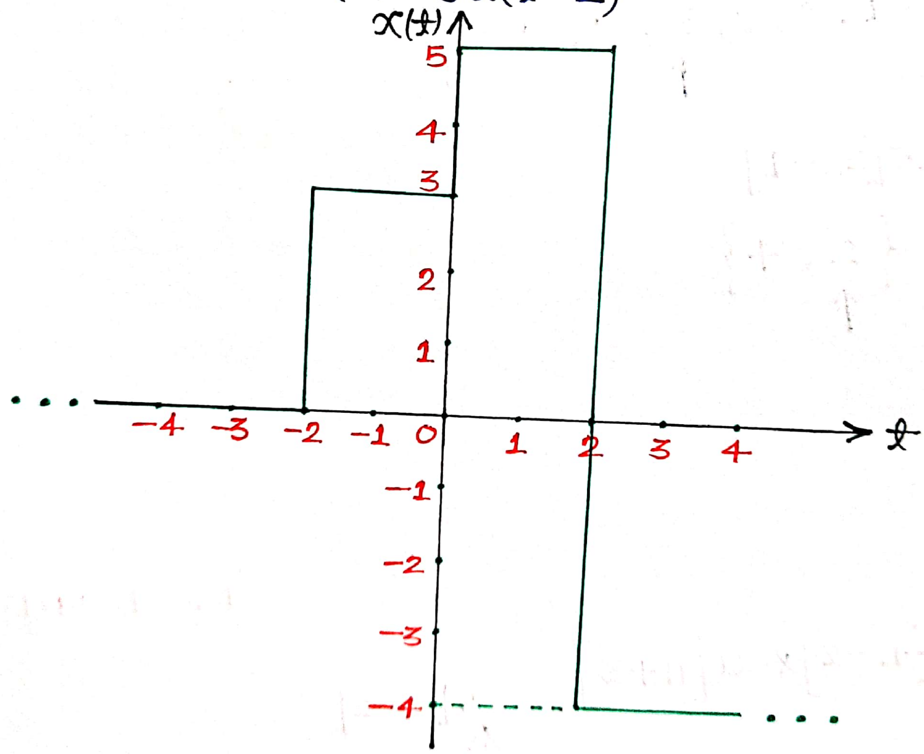
$= 2t\,u(t) - 2t\,u(t-1)$

$= 2t\,u(t) - 2(t-1)\,u(t-1) - 2\,u(t-1)$

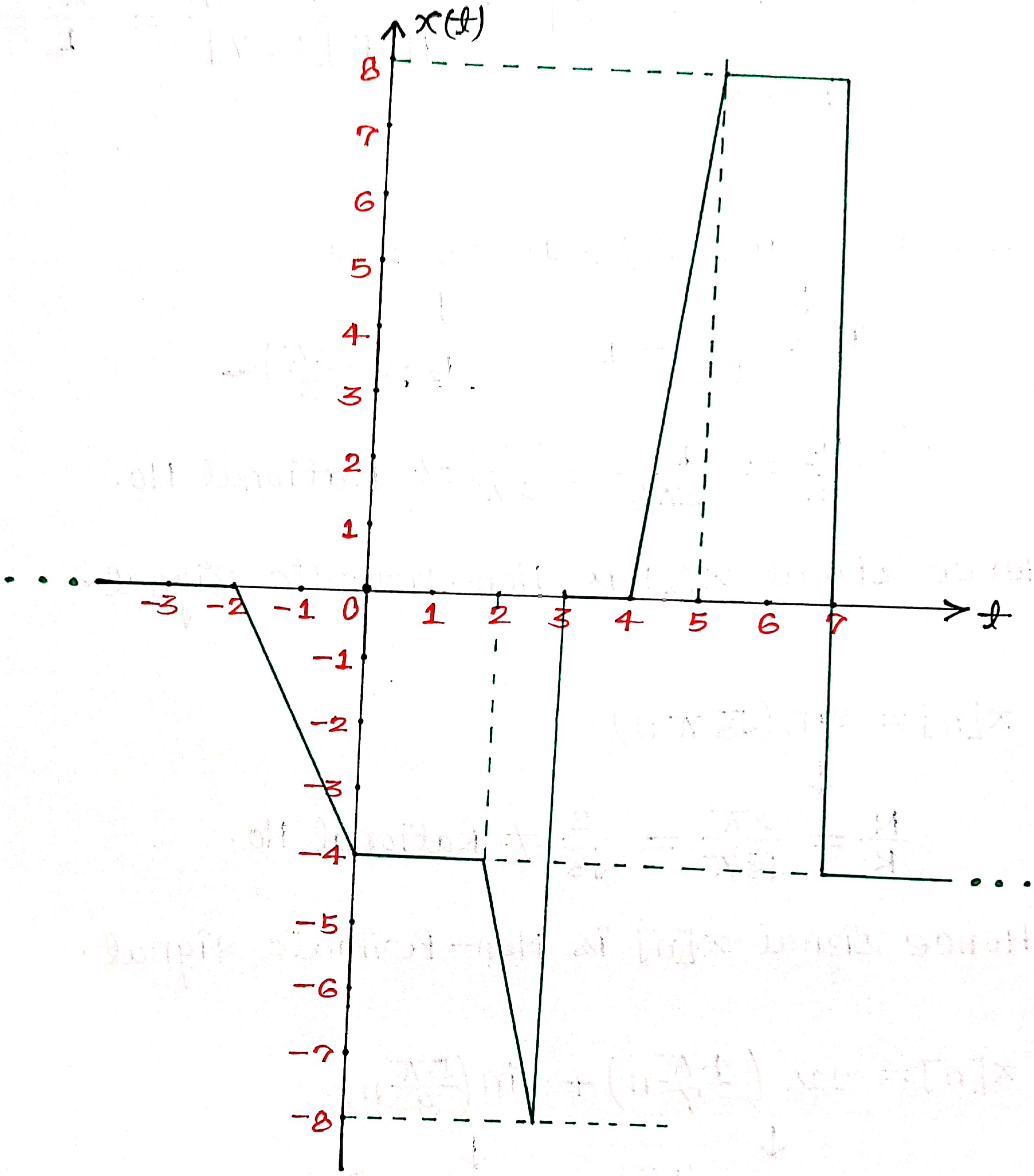
$= 2\,r(t) - 2\,r(t-1) - 2\,u(t-1)$



(C) $x(t) = 3\,u(t+2) + 2\,u(t) - 9\,u(t-2)$



(d) $x(t) = -2r(t+2) + 2r(t) - 4r(t-2) + 4r(t-3) + 8u(t-3)$
$+ 8r(t-4) - 8r(t-5) - 12u(t-7)$



SOL(3):

(4X2 POINTS)

(a) $x(t) = 8\sin(4\pi t) + 7\cos(7\pi t)$

$$T_1 = \left(\frac{2\pi}{4\pi}\right) = \left(\frac{1}{2}\right) \qquad T_2 = \left(\frac{2\pi}{7\pi}\right) = \left(\frac{2}{7}\right)$$

$$\therefore \left(\frac{T_1}{T_2}\right) = \left(\frac{7}{4}\right) = \text{Rational No.}$$

Hence signal $x(t)$ is Periodic signal with FTP,

$$T_0 = LCM\left[\frac{1}{2}, \frac{2}{7}\right] = \frac{LCM[1,2]}{HCF[2,7]} = \frac{2}{1} = 2$$

$\therefore T_0 = 2$

(b) $x(t) = 26\sin(2\pi t) + 15\cos(3t)$

$$T_1 = \left(\frac{2\pi}{2\pi}\right) = 1 \qquad T_2 = \left(\frac{2\pi}{3}\right)$$

$$\therefore \frac{T_1}{T_2} = \frac{1}{2\pi/3} = \left(\frac{3}{2\pi}\right) \neq \text{Rational No.}$$

Hence signal $x(t)$ is Non-Periodic signal.

(c) $x[n] = \sin(\sqrt{3}\pi n)$

$$\frac{N}{K} = \frac{2\pi}{\sqrt{3}\pi} = \frac{2}{\sqrt{3}} \neq \text{Rational No.}$$

Hence signal $x[n]$ is Non-Periodic signal.

(d) $x[n] = \cos\left(\frac{4\pi}{7}n\right) + \sin\left(\frac{5\pi}{9}n\right)$

$$\omega_1 = \left(\frac{4\pi}{7}\right) \qquad \omega_2 = \left(\frac{5\pi}{9}\right)$$

$$N_1 = \left(\frac{2\pi}{\omega_1}\right)K_1 \qquad N_2 = \left(\frac{2\pi}{\omega_2}\right)K_2$$

$$= \left(\frac{2\pi}{4\pi/7}\right)K_1 \qquad = \frac{2\pi}{(5\pi/9)} K_2$$

$N_1 = 7$ for $K_1 = 2$ $\qquad$ $N_2 = 18$ for $K_2 = 5$

$\therefore$ FTP of $x[n] = N = LCM(N_1, N_2) = LCM(7, 18)$

$= 126$

SOL(4):

(4×3 POINTS)

(a) $x[n] = \{1+j,\ \underset{\uparrow}{1-j},\ -1,\ 2\}$

$\therefore$ Energy of signal $x[n] = E = \sum_{n=-\infty}^{\infty} |x[n]|^2$

$= \left(\sqrt{1^2+1^2}\right)^2 + \left(\sqrt{1^2+(-1)^2}\right)^2 + (-1)^2 + (2)^2$

$= 9$ Unit

(b) $x[n] = \left(\frac{1}{3}\right)^n u[n] = \{\underset{\uparrow}{1},\ \frac{1}{3},\ \frac{1}{9},\ \ldots\}$

$\therefore E = \sum_{n=-\infty}^{\infty} |x[n]|^2$

$= 1 + \frac{1}{3^2} + \frac{1}{3^4} + \ldots$

$= \frac{1}{1-(1/3^2)} = \frac{9}{8}$ Unit

(c) $x[n] = 3^{(-2n-3j)} \cdot u[n]$

$= 3^{-2n} \cdot 3^{-3j}\ u[n] = \left(\frac{1}{9}\right)^n u[n] \cdot 3^{-3j}$

$\therefore E = \sum_{n=-\infty}^{\infty} |x[n]|^2 = \sum_{n=-\infty}^{\infty} \left|\left(\frac{1}{9}\right)^n u[n]\right|^2 \times |3^{-3j}|^2$

$= \sum_{n=-\infty}^{\infty} \left|\left(\frac{1}{9}\right)^n u[n]\right|^2 \times 1$

$= \left(\frac{1}{9}\right)^0 + \left(\frac{1}{9}\right)^2 + \left(\frac{1}{9}\right)^4 + \ldots$

$= \left(\frac{81}{80}\right)$ Unit

$\because a^n = e^{n\log a}$

$a^j = e^{j\log a}$

$\therefore (3^{-3})^j = e^{j\log 3^{-3}}$

Hence –

$|(3^{-3})^j| = |e^{j\log 3^{-3}}|$

$= 1$

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(d) $x(t) = e^{-a|t|}, \quad a>0$

$$= \begin{cases} e^{at}, & t<0 \\ e^{-at}, & t>0 \end{cases}$$

$$= e^{at}u(-t) + e^{-at}u(t)$$

$$\therefore E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$= \int_{-\infty}^{\infty} |e^{at} \cdot u(-t)|^2 dt + \int_{-\infty}^{\infty} |e^{-at} \cdot u(t)|^2 dt$$

$$= \int_{-\infty}^{0} e^{2at} \cdot dt + \int_{0}^{\infty} e^{-2at} \cdot dt$$

$$= \frac{1}{2a} + \frac{1}{2a} = \left(\frac{1}{a}\right) \text{ Unit}$$

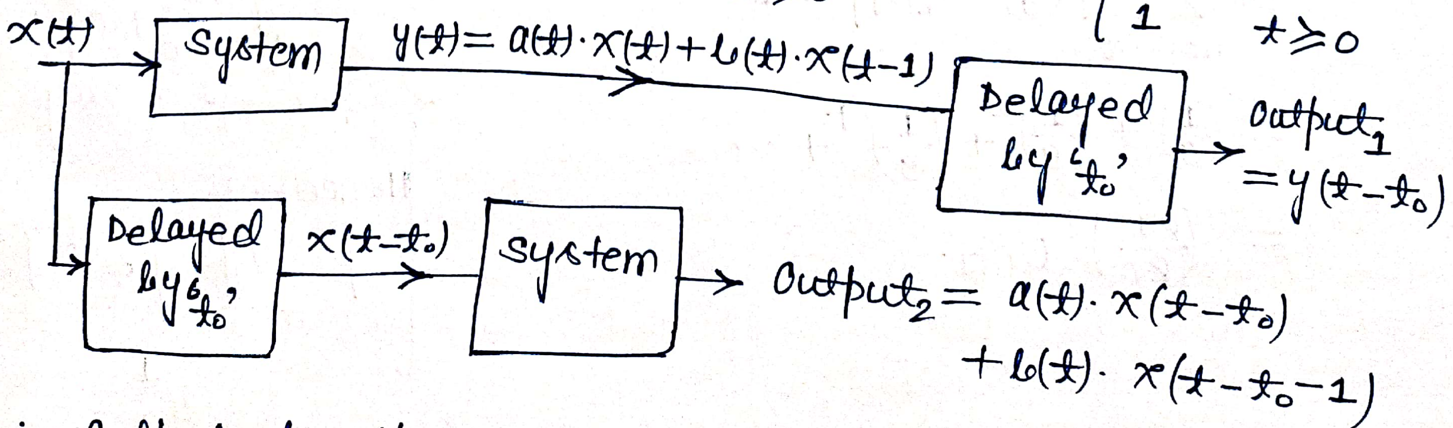
SOL(5):

(2X2 POINTS)

(a) $y(t) = x(t)$ for $t<0$ & $y(t) = x(t-1)$ for $t \geq 0$

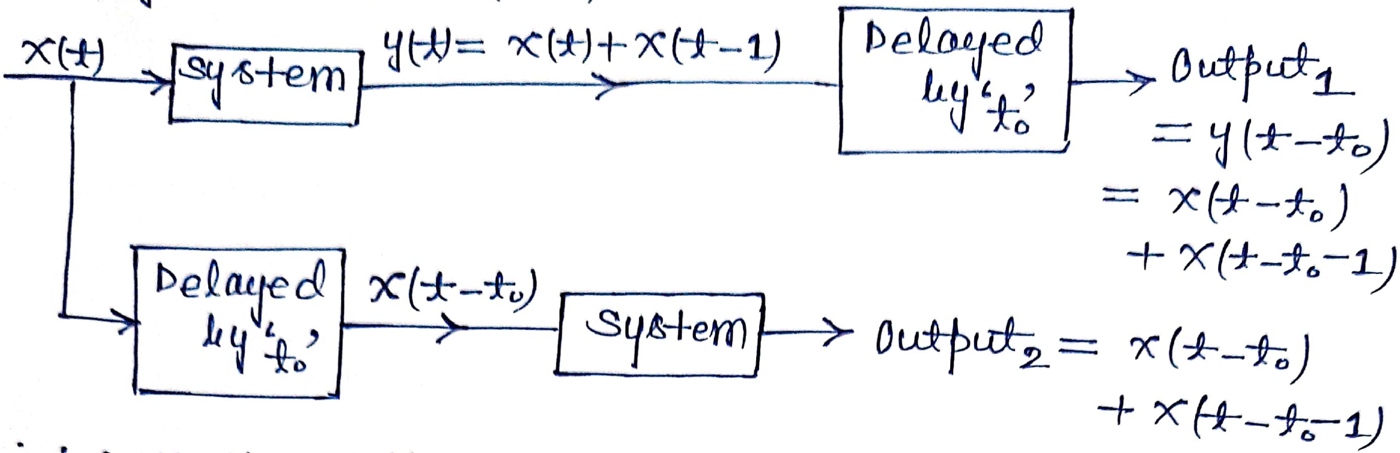
$$y(t) = a(t) \cdot x(t) + b(t) \cdot x(t-1)$$

where, $a(t) = \begin{cases} 1 & t<0 \\ 0 & t \geq 0 \end{cases}$ & $b(t) = \begin{cases} 0 & t<0 \\ 1 & t \geq 0 \end{cases}$



$\therefore$ Output$_1$ & Output$_2$ are not same Hence Time Variant

(b) $y(t) = x(t) + x(t-1)$



$\because$ Both the output are same hence Time Invariant

SOL(6):

(2X3 POINTS)

(a) $N = 7$

$$\therefore P = \frac{1}{N}\sum_{n=0}^{6} |x[n]|^2$$

$$= \frac{1}{7}\left[3 \times 2^2 + 2 \times 1^2 + 2 \times 0\right] = 2 \text{ Unit}$$

(b) $$P = \frac{1}{T_0}\left[\int_0^{T_0/2} |x(t)|^2 dt + \int_{T_0/2}^{T} |x(t)|^2 dt\right]$$

$$= \frac{2}{T_0}\int_0^{T_0/2} |x(t)|^2 dt$$

$$= \frac{2}{T_0}\int_0^{T_0/2} \left(\frac{2A_0}{T_0}\right)^2 t^2 dt$$

$$= \left(\frac{8A_0^2}{T_0^3}\right)\left[\frac{t^3}{3}\right]_0^{T_0/2} = \left(\frac{8A_0^2}{T_0^3}\right)\left(\frac{1}{3}\right)\left(\frac{T_0^3}{8}\right) = \left(\frac{A_0^2}{3}\right) \text{ Unit}$$