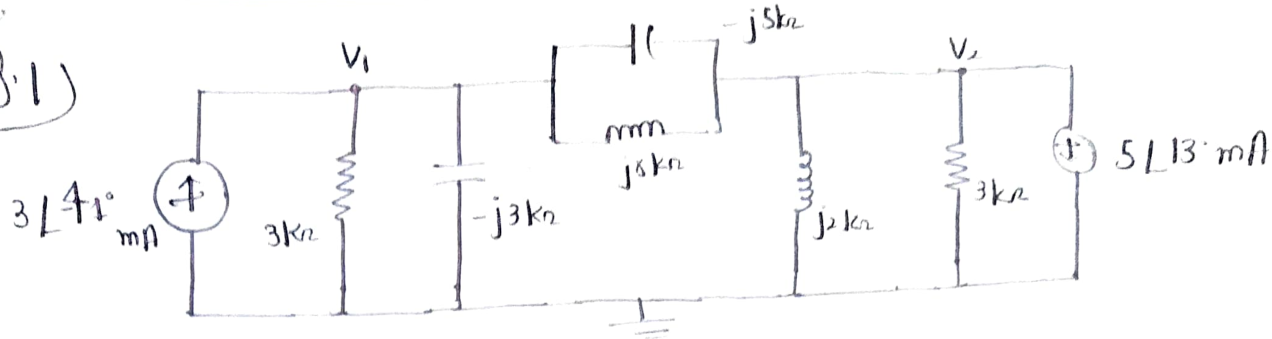
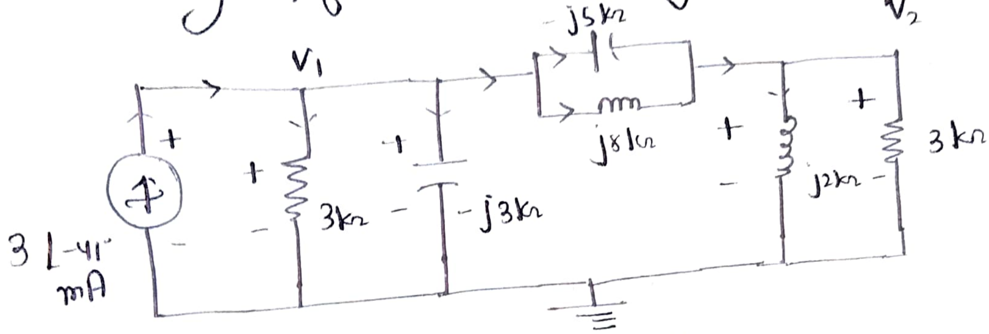
Q.1)



⇒ Considering left hand source only :-



⇒ writing KCL equation at node V_1 :-

$$3\times10^{-3}\angle-41^\circ = \frac{V_1}{3000} + \frac{V_1}{-j3000} + \frac{V_1 - V_2}{-j5000} + \frac{V_1 - V_2}{j8000}$$

$$\Rightarrow 3\times10^{-3}\angle-41^\circ = V_1\left(\frac{1}{3000} + \frac{1}{-j3000} + \frac{1}{-j5000} + \frac{1}{j8000}\right)$$

$$+ V_2\left(\frac{1}{j5000} - \frac{1}{j8000}\right)$$

$$\Rightarrow V_1\left(\frac{1}{3} + j0.4083\right) + V_2\left(-j7.5\times10^{-2}\right) = 3\angle-41^\circ \quad \text{—— (1)}$$

$$A_1V_1 \quad + \quad B_1V_2 \quad = \quad C_1$$

$$\left(\because 3\angle-41^\circ \Rightarrow 3\cos(-41) + 3j\sin(-41) = 2.26 + j(-1.96)\right)$$

⇒ writing KCL equation at node V_2 :-

$$\Rightarrow \frac{V_1 - V_2}{-j5000} + \frac{V_1 - V_2}{j8000} = \frac{V_2}{j2000} + \frac{V_2}{3000} = 0$$

$$\Rightarrow V_1\left(\frac{1}{-j5000} + \frac{1}{j8000}\right) + V_2\left(\frac{1}{j5000} - \frac{1}{j8000} - \frac{1}{j2000} - \frac{1}{3000}\right) = 0$$

$$\Rightarrow V_1\left(-j7.5\times10^{-2}\right) + V_2\left(\frac{1}{3} - j0.425\right) = 0 \quad \text{—— (2)}$$

$$A_2V_1 \quad + \quad B_2V_2 \quad = \quad C_2$$

⇒ Solving ① and ② using Cramer's rule -

$$V_1 = \frac{\begin{vmatrix} C_1 & B_1 \\ C_2 & B_2 \end{vmatrix}}{\begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix}} = \frac{\begin{vmatrix} 2.26 - j1.96 & -j7.5\times10^{-2} \\ 0 & \frac{1}{3} - j0.425 \end{vmatrix}}{\begin{vmatrix} \frac{1}{3} + j0.4083 & -j7.5\times10^{-2} \\ -j7.5\times10^{-2} & \frac{1}{3} - j0.425 \end{vmatrix}}$$

Solving it :-

$$\boxed{V_1 = -0.618 - j5.573\ V} \quad \text{Ans}$$

$$V_2 = \frac{\begin{vmatrix} A_1 & C_1 \\ A_2 & C_2 \end{vmatrix}}{\begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix}} = \frac{\begin{vmatrix} \frac{1}{3} + j0.4083 & 2.26 - j1.96 \\ -j7.5\times10^{-2} & 0 \end{vmatrix}}{\begin{vmatrix} \frac{1}{3} + j0.4083 & -j7.5\times10^{-2} \\ -j7.5\times10^{-2} & \frac{1}{3} - j0.425 \end{vmatrix}}$$

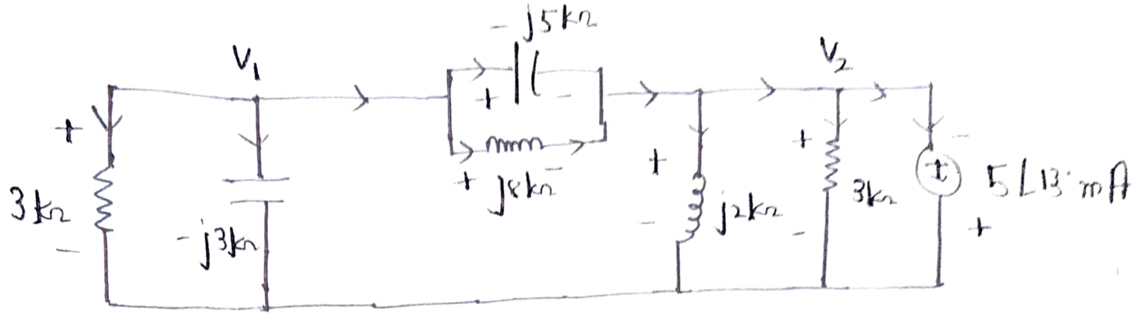
Solving it :-

$$V_2 = \boxed{0.496 + j0.595\ V} \quad \text{Ans}$$

Thus, due to left hand source, the nodal voltage V_1 comes out to be $-0.618 - j5.573\ V$ and the nodal voltage V_2 comes out to be $0.496 + j0.595\ V$

⇒ Now, the circuit needs to be solved for the contribution of right hand side source.

⇒ Redrawing the circuit :-



Applying KCL at node V1 :-

$$\Rightarrow \frac{V_1}{3000} + \frac{V_1}{-j3k} + \frac{V_1 - V_2}{-j5k} + \frac{V_1 - V_2}{j8k} = 0$$

$$\Rightarrow V_1\left(\frac{1}{3000} + \frac{1}{-j3k} + \frac{1}{-j5k} + \frac{1}{j8k}\right) + V_2\left(\frac{1}{j5k} - \frac{1}{j8k}\right) = 0$$

$$\Rightarrow V_1\left(\frac{1}{3} + j0.4083\right) + V_2\left(-j0.075\right) = 0 \quad \text{— (3)}$$

$$A_3 V_1 + B_3 V_2 = C_3$$

Applying KCL at node V_2 :-

$$\Rightarrow \frac{V_1 - V_2}{-j5k} + \frac{V_1 - V_2}{j8k} = \frac{V_2}{j2k} + \frac{V_2}{3k} + 5\angle 13°$$

$$\Rightarrow \frac{V_1 - V_2}{-j5k} + \frac{V_1 - V_2}{j8k} - \frac{V_2}{j2k} - \frac{V_2}{3k} - 5\angle 13° = 0$$

$$\Rightarrow V_1\left(\frac{1}{-j5k} + \frac{1}{j8k}\right) + V_2\left(\frac{1}{j5k} - \frac{1}{j8k} - \frac{1}{j2k} - \frac{1}{3k}\right) = 5\angle 13°$$

$$\Rightarrow V_1\left(-j0.075\right) + V_2\left(\frac{1}{3} - j0.425\right) = -5\angle 13° \quad \text{— (4)}$$

$$A_4 V_1 + B_4 V_2 = C_4$$

⇒ Solving ③ and ④ using cramer's rule :-

$$5\angle 13^\circ = 5\cos(13^\circ) + 5 \cdot j\sin(13^\circ)$$

$$= 4.87 + j1.12$$

$$V_1 = \frac{\begin{vmatrix} C_3 & B_3 \\ C_4 & B_4 \end{vmatrix}}{\begin{vmatrix} A_3 & B_3 \\ A_4 & B_4 \end{vmatrix}} = \frac{\begin{vmatrix} 0 & -j0.075 \\ -4.87 - j1.12 & \frac{1}{3} - j0.425 \end{vmatrix}}{\begin{vmatrix} \frac{1}{3} + j0.4083 & -j0.075 \\ -j0.075 & \frac{1}{3} - j0.425 \end{vmatrix}}$$

Solving it :-

$$\boxed{V_1 = -0.315 - j1.253\ \text{V}}\ \text{Ans}$$

$$V_2 = \frac{\begin{vmatrix} A_3 & C_3 \\ A_4 & C_4 \end{vmatrix}}{\begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix}} = \frac{\begin{vmatrix} \frac{1}{3} + j0.4083 & 0 \\ -j0.075 & -4.87 - j1.12 \end{vmatrix}}{\begin{vmatrix} \frac{1}{3} + j0.4083 & -j0.075 \\ -j0.075 & \frac{1}{3} - j0.425 \end{vmatrix}}$$

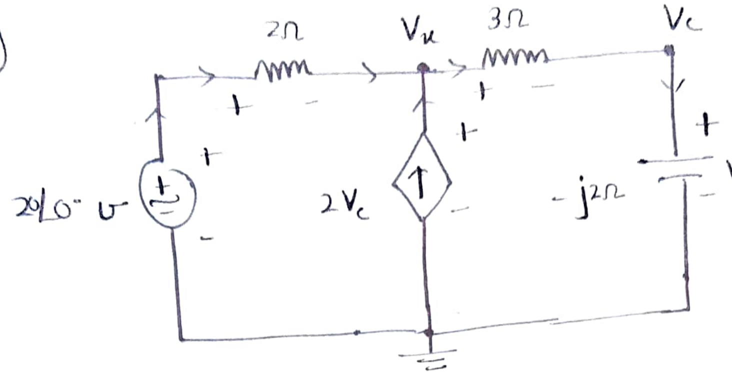
Solving it :-

$$\boxed{V_2 = -3.85 - j8.22\ \text{V}}\ \text{Ans}$$

Thus, due to right hand source, the nodal voltage V_1 comes out to be $-0.315 - j1.253\ \text{V}$ and the nodal voltage V_2 comes out to be $-3.85 - j8.22\ \text{V}$.

Determine the average power supplied by dependent source.

Q5)



Average power supplied by dependent source = ?

→ Identify nodes :- V_x, V_c, ground.

⇒ Apply KCL at node V_x :-

$$\Rightarrow \frac{20 - V_x}{2} + 2V_c = \frac{V_x - V_c}{3}$$

$$\Rightarrow \frac{20}{2} - \frac{V_x}{2} + 2V_c + \frac{V_c}{3} - \frac{V_x}{3} = 0$$

$$\Rightarrow 10 - \frac{V_x}{2} + 2V_c + \frac{V_c}{3} - \frac{V_x}{3} = 0$$

$$\Rightarrow V_c\left(2 + \frac{1}{3}\right) + V_x\left(-\frac{1}{2} - \frac{1}{3}\right) = -10$$

$$\Rightarrow V_x\left(-5/6\right) + V_c\left(7/3\right) = -10$$

multiply by -6 on both sides.

$$\Rightarrow 5V_x - 14V_c = 60 \quad \text{—— (1)}$$

$A_1V_x + B_1V_c = C_1$

$$\left[\begin{array}{l} \because A_1 = 5 \\ B_1 = -14 \\ C_1 = 60 \end{array}\right]$$

⇒ Apply KCL at node V_c :-

$$\Rightarrow \frac{V_c}{-j2} = \frac{V_x - V_c}{3}$$

$$\Rightarrow V_c\left(-\frac{1}{j2}\right) + \frac{V_c}{3} - \frac{V_x}{3} = 0$$

$$\Rightarrow V_c\left(-\frac{1}{j2}+\frac{1}{3}\right)+V_x\left(-\frac{1}{3}\right)=0$$

$$\Rightarrow V_c\left(\frac{3-j2}{-j6}\right)+V_x\left(-\frac{1}{3}\right)=0$$

$\Rightarrow$ multiply by $-j6$ on both side

$$\Rightarrow V_c(3-j2)+V_x(j2)=0$$

$$\Rightarrow \underline{V_x(j2)+(3-j2)V_c}=0 \quad \text{——} \quad (2)$$

$$A_2V_x+B_2V_c=C_2$$

$$\left[\begin{array}{l} \because A_2=j2 \\ B_2=3-j2 \\ C_2=0 \end{array}\right]$$

now; $V_x = \dfrac{\begin{vmatrix} C_1 & B_1 \\ C_2 & B_2 \end{vmatrix}}{\begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix}} \Rightarrow \dfrac{\begin{vmatrix} 60 & -14 \\ 0 & 3-j2 \end{vmatrix}}{\begin{vmatrix} 5 & -14 \\ j2 & 3-j2 \end{vmatrix}}$

solving by Cramer's rule

$$\Rightarrow \frac{60(3-j2)-0}{5(3-j2)-(j2)(-14)} = \frac{180-120j}{(15-10j)-(-28j)}$$

$$\Rightarrow \frac{180-120j}{15-10j+28j} = \frac{180-120j}{15+18j}$$

$$\Rightarrow \frac{180-120j}{15+18j}\times\frac{15-18j}{15-18j}$$ (By Rationalizing & solving)

$$\Rightarrow 0.983-j\,9.17$$

Convert into polar form:-

$$\text{magnitude} = \sqrt{(0.983)^2 + (-9.17)^2}$$

$$= 9.223$$

$$\text{Angle} = \tan^{-1}\left(\frac{-9.17}{0.983}\right) = -83.88°$$

$$\Rightarrow 9.223 \angle -83.88° \, V \equiv 9.223 \cos(\omega t - 83.88°) \, V$$

Equivalent to

now, solving for V_c:-

$$V_c = \frac{\begin{vmatrix} A_1 & C_1 \\ A_2 & C_2 \end{vmatrix}}{\begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix}} = \frac{\begin{vmatrix} 5 & 60 \\ j2 & 0 \end{vmatrix}}{\begin{vmatrix} 5 & -14 \\ j2 & 3-j2 \end{vmatrix}}$$

$$\Rightarrow \frac{0 - 60(j2)}{5(3-j2) - (j2)(-14)} = \frac{-j120}{15 + 18j}$$

$$\Rightarrow \frac{j120}{15+18j} \times \frac{15-18j}{15-18j} \Rightarrow -3.93 - j\,3.27$$

Convert into polar form

$$\text{magnitude} = \sqrt{(-3.93)^2 + (-3.27)^2} = 5.122$$

$$\text{angle} = \tan^{-1}\left(\frac{3.27}{3.93}\right) = -140.2°$$

$$\Rightarrow 5.122 \angle -140.2° = 5.122 \cos(\omega t - 140.2°) \, V$$

⇒ Power associated with dependent source:-

$$P_{2V_c} = -\frac{1}{2}(9.223)(2 \times 5.122)\cos(-83.88^\circ + 140.2^\circ)$$

$$= -26.22\,W$$

⇒ power supplied by dependent source = $\boxed{26.22\,W}$ Ans

Q.6) (a) $V = 240\angle 243^\circ$ V rms

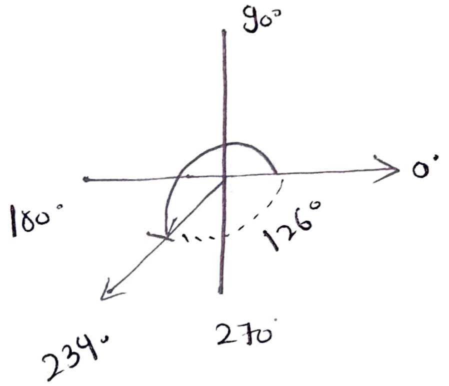
$I = 3\angle 9^\circ$ A rms

Phase angle difference between voltage and current = ?

$$\Rightarrow 243^\circ - 9^\circ = 234^\circ$$

$$= -360^\circ + 234^\circ \text{ (OR) } -198^\circ$$

$$= \boxed{-126^\circ}$$



⇒ Voltage lags current by $\boxed{126^\circ}$ Ans

(b) power factor of load is 0.55 lagging.

Power factor = $\cos\underbrace{(\theta - \phi)}$

↓

phase angle difference b/w voltage and current

if $\theta > \phi$ (voltage lead)

$\theta < \phi$ (voltage lag).

$\Rightarrow \cos(\theta - \phi) = 0.55$

$\Rightarrow \theta - \phi = \cos^{-1}(0.55)$

$= 56.6°$

$\Rightarrow$ Voltage leads current by $\boxed{56.6°}$ Ans

(c) power factor of load is 0.685 leading

$\Rightarrow \cos(\theta - \phi) = -0.685$

$\theta - \phi = \cos^{-1}(0.685)$

$= -46.76°$

$\Rightarrow$ Voltage lags current by $\boxed{46.76°}$ Ans

(d) Capacitive load

Draws 100 W average power

500 VA apparent power

$\Rightarrow \theta - \phi = \cos^{-1}\left(\dfrac{\text{Average power}}{\text{Apparent power}}\right)$

$\Rightarrow \theta - \phi = \cos^{-1}\left(\dfrac{-100}{500}\right) = -78.46°$

$\Rightarrow$ Voltage lags current by $\boxed{78.46°}$ Ans