End-sem: Fields & Waves (ECE230), Winter 2025

Total marks: 40, Time: 2hrs

**Vector quantities must be represented with an overhead arrow, vector calculus operators must be written correctly, dot and cross products must be written properly. In case of any of these mistake -1 will be awarded.

Q1. Derive the equation of continuity. What physical law does this equation represent? [2 + 1 = 3 points]

Q2. Consider a lossless transmission line has a distributed inductance of $227nH/m$ and distributed capacitance of $90.9pF/m$ and has been connected to a signal generator of frequency $14\pi \times 10^8$ Hz.
(a) How much is the characteristic impedance of the line?
(b) What would be the speed of the voltage wave propagating in this line?
(c) What would be the propagation constant of the wave?
(d) If the transmission line is terminated with a load $Z_L = 30\Omega$, how much would be voltage reflection coefficient at the load?
(e) Under the same condition as part (d), what is the VSWR?

[2 + 2 + 2 + 2 + 2] = 10 points

Q3. Consider a uniform, $x-$polarized plane wave of frequency $3GHz$ is propagating along $z-$direction in a nonmagnetic medium ($\mu_0 = 4\pi \times 10^{-7} H/m$) with conductivity $7.2 \times 10^4 S/m$ and permittivity $24 \times 10^{-12} F/m$.
(a) Write down the effective permittivity of the medium.
(b) How much is the ratio $\frac{E}{H}$ for this wave?
(c) How much is the propagation constant of the EM wave?

[2 + 2 + 2 = 6 points]

Q4. Suppose, you are shining red light from air to glass slab (refractive index= 1.5). [3 points]
(a) Calculate the reflection coefficient at the interface.
(b) Does the electric field flips sign on reflection?
(c) How much is the transmission coefficient is?

Q5. Choose correct options (if multiple options are correct you must choose all of them to get any credit)
(a) Consider an interface between two dielectric media with permeabilities μ_1 and μ_2. B_{n1} and B_{n2} are the normal components of the magnetic fields and B_{t1} and B_{t2} are the tangential components at the interface: (i) $B_{t1} = B_{t2}$, (ii) $\mu_1 B_{n1} = \mu B_{2n2}$, (iii) $H_{n1} = H_{n2}$, (iv) $H_{t1} = H_{t2}$. [1 point]
(b) Magnetostatic field is: (i) conservative, (ii) irrotational, (iii) solenoidal, (iv) all of the previous options. [1 point]
(c) Which expression of divergence theorem is correct: (i) $\oint\limits_V (\vec{\nabla}.\vec{E})\text{dv} = \oint\limits_S \vec{E}.\vec{dS}$, (ii) $\oint\limits_V (\vec{\nabla}.\vec{E})\text{dv} = \int\limits_S \vec{E}.\vec{dS}$, (iii) $\int\limits_V (\vec{\nabla}.\vec{E})\text{dv} = \oint\limits_S \vec{E}.\vec{dS}$, (iv) $\int\limits_V (\vec{\nabla}.\vec{E})\text{dv} = \int\limits_S \vec{E}.\vec{dS}$ [1 point]
(d) Consider a charge-free interface between two media with permittivities ϵ_1 and ϵ_2. E_{n1} and E_{n2} are the normal components of the electric fields and E_{t1} and E_{t2} are the tangential components at the interface: (i) $E_{t1} = E_{t2}$, (ii) $E_{n1} = E_{n2}$, (iii) $\epsilon_1 E_{n1} = \epsilon_2 E_{n2}$, (iv) $\epsilon_1 E_{t1} = \epsilon_2 E_{t2}$. [1 point]

Q6. For an electromagnetic field, the scalar potential (ϕ) and vector potential (A) are given by:

$$\phi = (1/r^2)\exp(i\omega t)$$

$$\vec{A} = (\hat{r}/r)\exp(i\omega t)$$

Calculate the electric field. [4 points]

Q7. The voltage in a transmission line is given by: $\vec{V} = [6\exp(ikz) + 3\exp(-ikz)]$. Calculate voltage and current SWR. [2 points]

Q8. Consider an electrostatic potential $\phi = \frac{1}{(x^2+y^2+z^2)^{1/4}}$. Calculate:
(i) the electric field $(\vec{E})$
(ii) the volume charge density ρ. [3 + 3 = 6 points]

Q9. Calculate curl of the function $\vec{A}(x, y, z) = \hat{x}yz + \hat{y}4xy + \hat{z}y$. [2 points]