Given— $y[n] = x_1[n] * x_2[n] * x_3[n]$

where, $x_1[n] = (0.5)^n u[n]$

$x_2[n] = u[n+3]$

$x_3[n] = \delta[n] - \delta[n-1]$

(a) $x_1[n] * x_2[n] = \sum\limits_{k=-\infty}^{\infty} x_1[k] \cdot x_2[n-k] = \sum\limits_{k=-\infty}^{\infty} (0.5)^k u[k] \cdot u[n+3-k]$

$u[k] = 0$, $k < 0$

$u[n+3-k] = 0$, $k > (n+3)$

$\therefore y_1[n] = x_1[n] * x_2[n] = \sum\limits_{k=0}^{n+3} (0.5)^k$, $n \geqslant -3$

$= \dfrac{1[1-(0.5)^{n+3+1}]}{[1-(0.5)]}$, $n \geqslant -3$

$= 2[1-(0.5)^{n+4}]$, $n \geqslant -3$

→(1 Point)

$\therefore y_1[n] = x_1[n] * x_2[n] = 0$, $n < -3$

→ (1 Point)

(b) $y[n] = \{\text{Result of Part (a)}\} * x_3[n] = y_1[n] * x_3[n]$

$= y_1[n] * \{\delta[n] - \delta[n-1]\} = y_1[n] - y_1[n-1]$

$\therefore y_1[n] = \begin{cases} 2[1-(0.5)^{n+4}] & , \quad n \geqslant -3 \\ \\ 0 & , \quad n < -3 \end{cases}$

$\therefore y_1[n-1] = \begin{cases} 2[1-(0.5)^{n+3}] & , \quad n \geqslant -2 \\ \\ 0 & , \quad n < -2 \end{cases}$

$\therefore y[n] = \begin{cases} (0.5)^{n+3} & , \quad n \geqslant -2 \\ 2[1-(0.5)^{n+4}] & , \quad -3 \leqslant n < -2 \\ 0 & , \quad n < -3 \end{cases}$

→ (2 Points)

(c) $\quad x_2[n] * x_3[n] = u[n+3] * \{\delta[n] - \delta[n-1]\}$

$\qquad\qquad y_2[n] = u[n+3] - u[n-1+3]$

$\qquad \therefore y_2[n] = u[n+3] - u[n+2] \qquad\qquad \rightarrow (2 \text{ Points})$

(d) $\quad y[n] = \{\text{Result of Part(c)}\} * x_1[n]$

$\qquad = x_1[n] * \{\text{Result of Part(c)}\} = x_1[n] * y_2[n]$

$\qquad = \sum_{k=-\infty}^{\infty} x_1[k] \cdot y_2[n-k] = \sum_{k=-\infty}^{\infty} (0.5)^k u[k] u[n+3-k]$

$$\qquad\qquad\qquad\qquad - \sum_{k=-\infty}^{\infty} (0.5)^k u[k] \cdot u[n+2-k]$$

$\qquad u[k] = 0 \qquad , \; k < 0 \qquad = y_3[n] - y_4[n]$

$\qquad u[n+3-k] = 0 \;, \; k > (n+3)$

$\qquad u[n+2-k] = 0 \;, \; k > (n+2)$

$$\therefore y_3[n] = \begin{cases} \sum_{k=0}^{n+3} (0.5)^k & , \quad n \geq -3 \\ \\ 0 & , \quad n < -3 \end{cases}$$

$$= \begin{cases} 2\left[1 - (0.5)^{n+4}\right] & , \quad n \geq -3 \\ \\ 0 & , \quad n < -3 \end{cases}$$

$$\therefore y_4[n] = \begin{cases} \sum_{k=0}^{n+2} (0.5)^k & , \quad n \geq -2 \\ \\ 0 & , \quad n < -2 \end{cases}$$

$$= \begin{cases} 2\left[1 - (0.5)^{n+3}\right] & , \quad n \geq -2 \\ \\ 0 & , \quad n < -2 \end{cases}$$

$$\therefore y[n] = y_3[n] - y_4[n]$$

$$= \begin{cases} 2[1-(0.5)^{n+4}] - 2[1-(0.5)^{n+3}] & , n \geqslant -2 \\ 2[1-(0.5)^{n+4}] & , -3 \leqslant n < -2 \\ 0 & , n < -3 \end{cases}$$

$$y[n] = \begin{cases} (0.5)^{n+3} & , n \geqslant -2 \\ 2[1-(0.5)^{n+4}] & , -3 \leqslant n < -2 \\ 0 & , n < -3 \end{cases}$$

$\rightarrow$ (2 Points)

(e) Here result of Part (b) & Part (d) are same. Hence satisfy the property of Associativity.

$\rightarrow$ (2 Points)