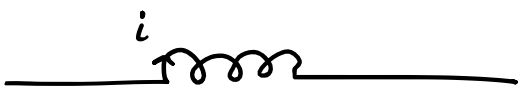
Lecture 5 / Chapter 13

Mutual Inductance

Types of problems

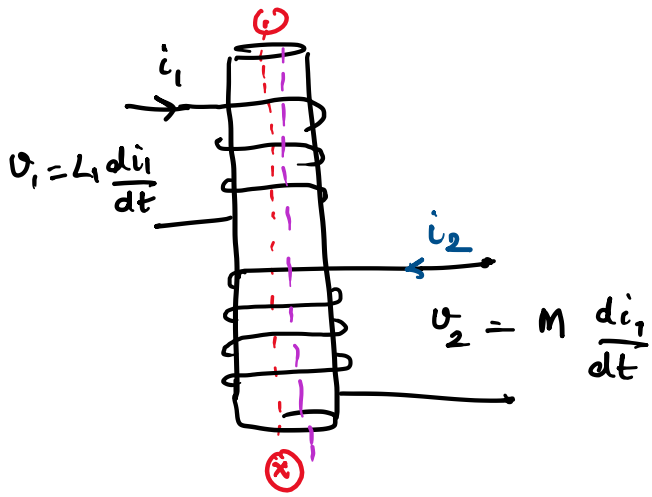
- Mesh analysis with coupled coils
- Linear transformers
- Ideal transformers

X nodal analysis



$$\phi = L\,i$$

$$v = \frac{d\phi}{dt} = L\frac{di}{dt}$$



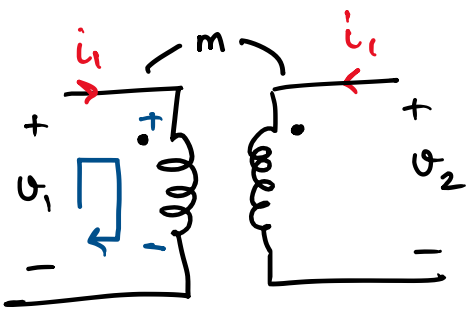
$L_1 \rightarrow$ self inductance (H)

$M \rightarrow$ mutual inductance (H)

$$v_1 = +L_1\frac{di_1}{dt} \pm M\frac{di_2}{dt}$$

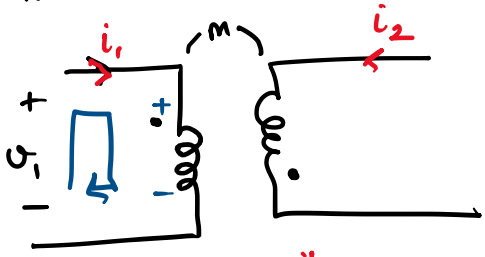
$$v_2 = \pm M\frac{di_1}{dt} + L_2\frac{di_2}{dt}$$

Dot convention → Shorthand to indicate how wires are wound



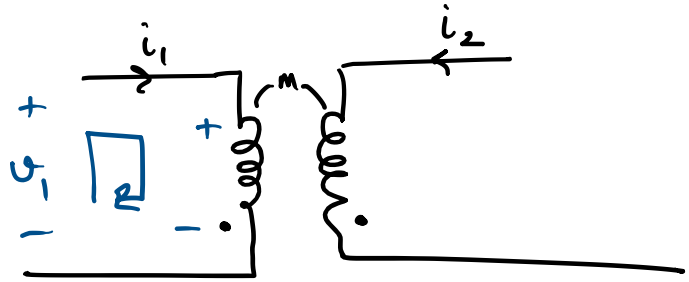
M – same

$$+v_1 - L_1\frac{di_1}{dt} - M\frac{di_2}{dt} = 0$$



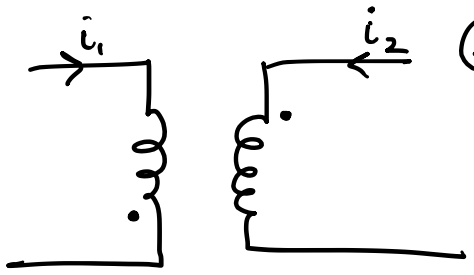
M – opposite

$$+v_1 - L_1\frac{di_1}{dt} + M\frac{di_2}{dt} = 0$$



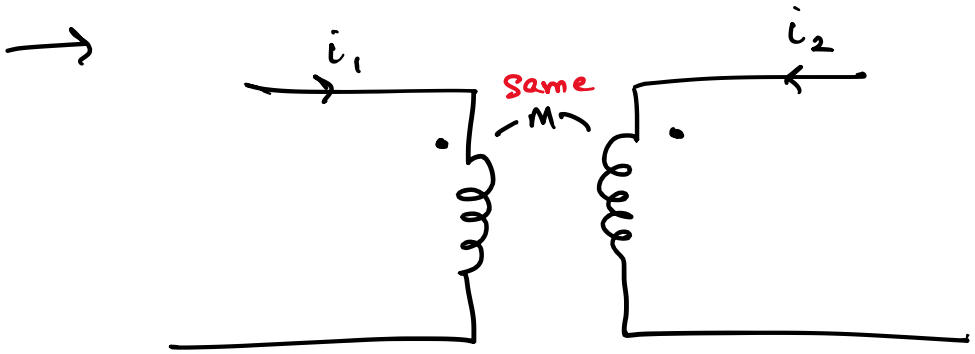
M – same

$$+v_1 - L_1\frac{di_1}{dt} - M\frac{di_2}{dt} = 0$$



M – opposite

$$v_1 - L_1\frac{di_1}{dt} + M\frac{di_2}{dt} = 0$$



$L_1 = 10\,mH$, $L_2 = 5\,mH$, $M = 1\,mH$

(a) find v_1 if $i_1 = 0$, $i_2 = 5\cos 8t$ A

$$v_1 - L_1\frac{di_1}{dt} - M\frac{di_2}{dt} = 0$$

$$v_1 = M\frac{di_2}{dt} = 1\times10^{-3}\frac{d}{dt}(5\cos 8t) \quad V$$

$$= -40\sin(8t)\ mV$$

(b) find v_2 if $i_1 = 3\sin 100t$ A, $i_2 = 0$

$$v_2 - L_2\frac{di_2}{dt} - M\frac{di_1}{dt} = 0$$

$$\Rightarrow \quad v_2 = M\frac{di_1}{dt} = 1\times10^{-3}\frac{d}{dt}(3\sin 100t) \quad V$$

$$= 300\cos(100t)\ mV$$

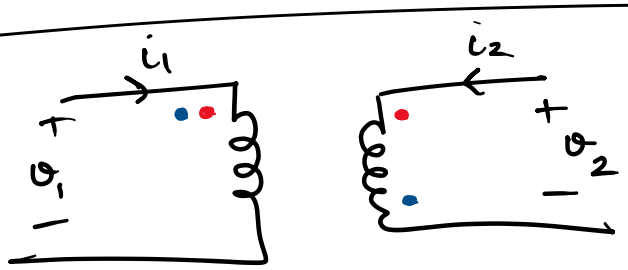
(c) find v_2 if $i_1 = 5\cos(8t - 40°)$ A ←

$i_2 = 4\sin 8t$ A

$$v_2 - L_2\frac{di_2}{dt} - m\frac{di_1}{dt} = 0$$

$$\Rightarrow v_2 = 5\times10^{-3}\times4\times8\cos(8t) + 1\times10^{-3}\times5\times8\sin(8t-40°)$$

$$= 160\cos(8t) + 40\sin(8t - 40°)\ mV$$



case 1 / case 2

$$v_1 - L_1\frac{di_1}{dt} \mp m\frac{di_2}{dt} = 0$$

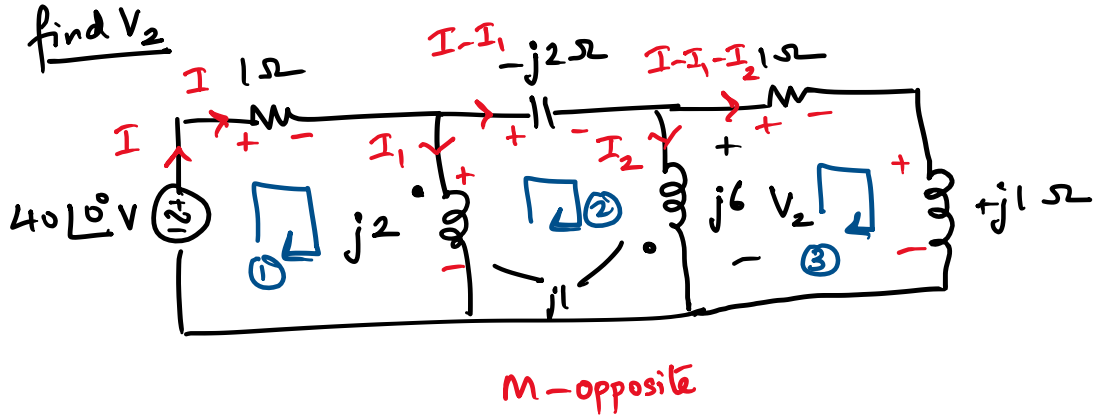
$$v_2 \mp m\frac{di_1}{dt} - L_2\frac{di_2}{dt} = 0$$

Suppose excitation is sinusoidal → Phasors

$$V_1 - j\omega L_1 I_1 \mp j\omega m I_2 = 0$$

$$V_2 \mp j\omega m I_1 - j\omega L_2 I_2 = 0$$

→ find V_2



Loop 1

$$+40 - I\times 1 - I_1\times j2 + I_2\times j1 = 0$$

$$\Rightarrow \quad I\times 1 + I_1\times j2 + I_2(-j1) = 40 \quad \text{—— (1)}$$

Loop 2

$$+I_1\times j2 - j1\times I_2 - (-j2)(I-I_1) - j6(I_2) + j1\times I_1 = 0$$

$$\Rightarrow \quad I(+j2) + I_1(+j1) + I_2(-j7) = 0 \quad \text{—— (2)}$$

Loop 3

$$+I_2(j6) - j1\times I_1 - 1(I-I_1-I_2) - 1j(I-I_1-I_2) = 0$$

$$\Rightarrow (-1-1j)I + (+1)I_1 + (+1+j7)I_2 = 0 \quad \text{—— (3)}$$

Use Cramer's rule to solve for I, I_1, I_2

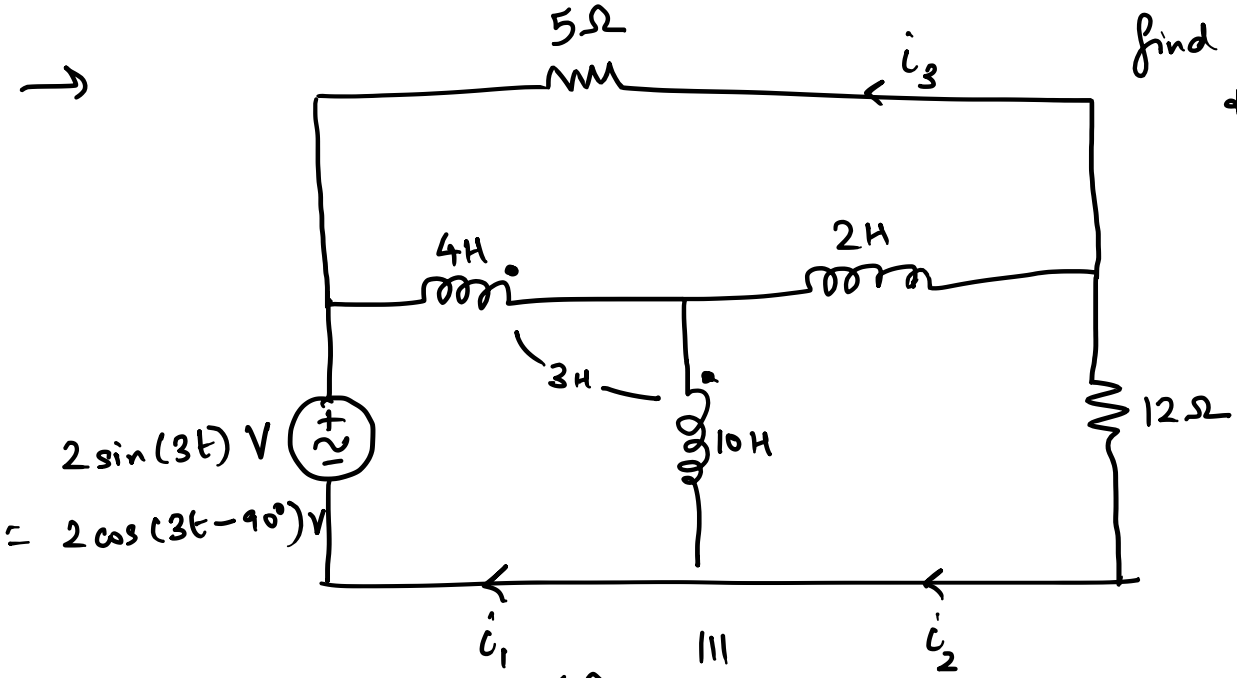
$$V_2 = j6\,I_2 - j1\,I_1$$

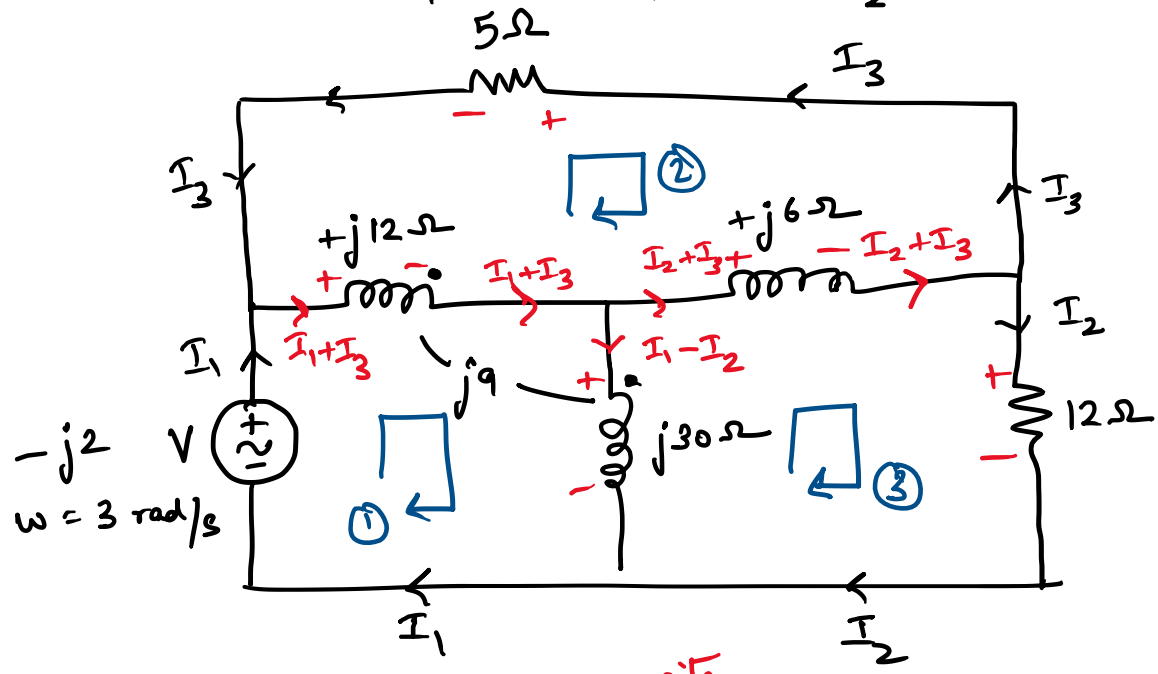
$$I_1 = 15.5586 + j2.7034 \text{ A}$$

$$I_2 = 0.9379 - j12.7948 \text{ A}$$

$$V_2 = 79.1724 - j9.9310 \text{ V}$$

find i_1, i_2 & i_3





M — opposite

Loop 1

$$+(-j2) - j12(I_1+I_3) + j9(I_1-I_2) \underbrace{-j30(I_1-I_2) + j9(I_1+I_3) = 0}$$

$$I_1(-j24) + I_2(j21) + I_3(-j3) = j2$$

$$\Rightarrow I_1(-24) + I_2(21) + I_3(-3) = 2 \quad \text{—— ①}$$

Loop 2

$$+5I_3 + j6(I_2+I_3) + \overbrace{j12(I_1+I_3) - j9(I_1-I_2)} = 0$$

$$\Rightarrow I_1(j3) + I_2(j15) + I_3(5+j18) = 0 \quad \text{—— ②}$$

Loop 3

$$\underbrace{+j30(I_1-I_2) - j9(I_1+I_3)} \quad -j6(I_2+I_3) - 12I_2 = 0$$

$$I_1(j21) + I_2(-12-j36) + I_3(-j15) = 0 \quad \text{—— ③}$$

Solving using Cramer's rule

$$I_1 = -0.0865 - j0.1183 \text{ A} = 0.1465 \angle -126.166^\circ \text{ A}$$

$$I_2 = -0.0054 - j0.1187 \text{ A} = 0.1189 \angle -92.62^\circ \text{ A}$$

$$I_3 = -0.0130 + j0.1150 \text{ A} = 0.1158 \angle 96.46^\circ \text{ A}$$

$$\therefore \quad i_1(t) = 0.1465 \cos(3t - 126.166^\circ) \text{ A}$$

$$i_2(t) = 0.1189 \cos(3t - 92.62^\circ) \text{ A}$$

$$i_3(t) = 0.1158 \cos(3t + 96.46^\circ) \text{ A}$$