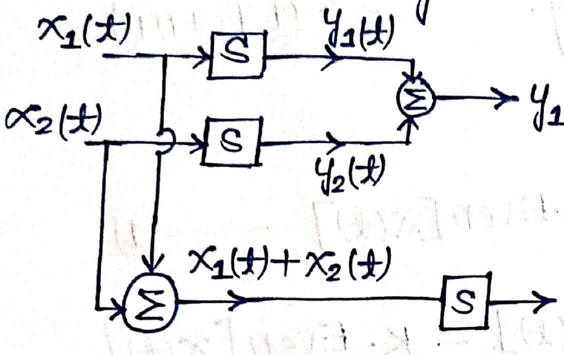
S&S ASSIGNMENT-2 SOLUTIONS

SOL (1):

(a) $y(t) = \text{Real}[x(t)] = \dfrac{x(t)+x^*(t)}{2}$

⇒ For additivity –



$\rightarrow y_1(t)+y_2(t) = \text{Real}[x_1(t)] + \text{Real}[x_2(t)]$

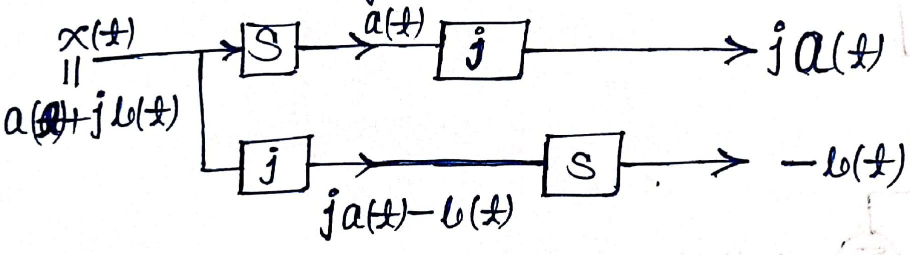
$= \dfrac{x_1(t)+x_1^*(t)}{2} + \dfrac{x_2(t)+x_2^*(t)}{2}$ —— (1)

$\rightarrow \text{Real}[x_1(t)+x_2(t)]$

$= \dfrac{x_1(t)+x_2(t) + x_1(t)^* + x_2^*(t)}{2}$ —— (2)

By (1) & (2), This is follow additivity. → (1 Points)

⇒ For Homogenity –



$\rightarrow ja(t)$ —— (1)

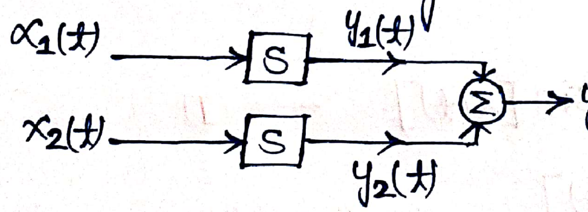
$\rightarrow -b(t)$ —— (2)

By (1) & (2), This is not follow Homogenity. → (1 Points)

Hence Non linear System.

(b) $y(t) = \text{Even}[x(t)] = \dfrac{x(t)+x(-t)}{2}$

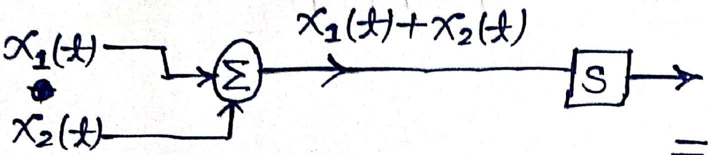
⇒ For additivity –



$\rightarrow y_1(t)+y_2(t) = \text{Even}[x_1(t)] + \text{Even}[x_2(t)]$

$= \dfrac{x_1(t)+x_1(-t)}{2} + \dfrac{x_2(t)+x_2(-t)}{2}$

—— (1)

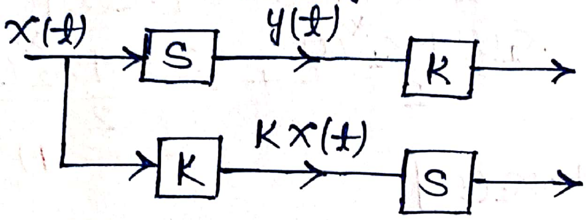


$\rightarrow \text{Even}[x_1(t)+x_2(t)]$

$$= \frac{x_1(t)+x_2(t)+x_1(-t)+x_2(-t)}{2} \quad \text{—— (2)}$$

By (1) & (2), This is follow additivity. → (1 POINT)

⇒ For Homogenity—



$\rightarrow K\,y(t) = K\cdot \text{Even}[x(t)]$ —— (1)

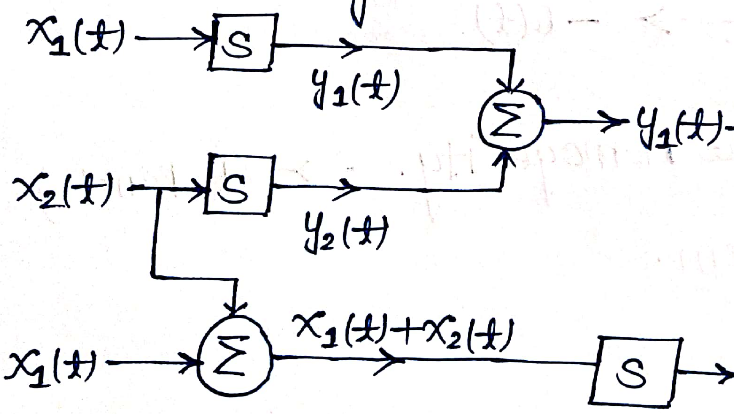
$\rightarrow \text{Even}[Kx(t)] = K\cdot \text{Even}[x(t)]$ —— (2)

By (1) & (2), This is follow Homogenity. → (1 POINT)

Hence Linear System.

(C) $y(t) = \sin[x(t)]$

⇒ For additivity—

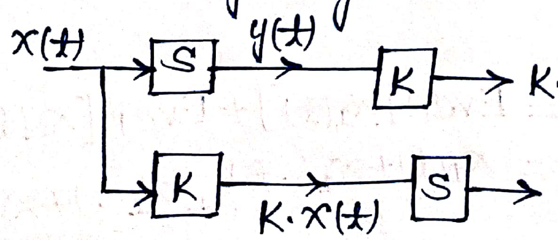


$\rightarrow y_1(t)+y_2(t) = \sin[x_1(t)]+\sin[x_2(t)]$ —— (1)

$\rightarrow \sin[x_1(t)+x_2(t)]$ —— (2)

By (1) & (2), This is not follow additivity.

⇒ For Homogenity—



$\rightarrow K\cdot y(t) = K\cdot \sin[x(t)]$ —— (1)

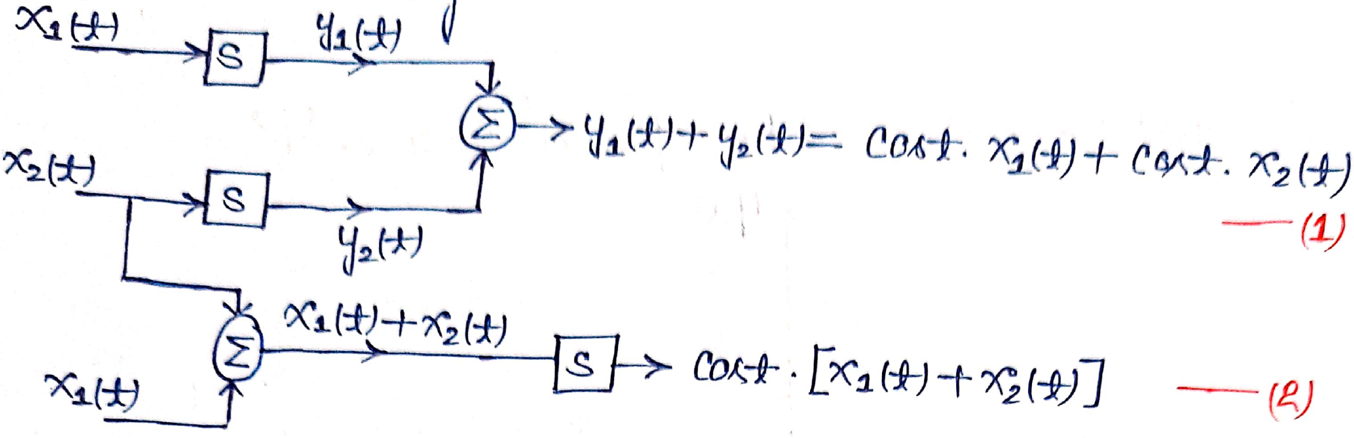
$\rightarrow \sin[K\cdot x(t)]$ —— (2)

By (1) & (2), This is not follow Homogenity.

Hence Non Linear System.

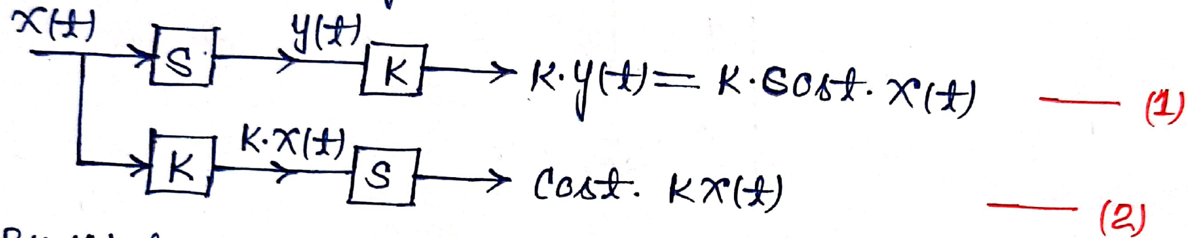
(d) $y(t) = \cos t \cdot x(t)$

⇒ For additivity –



By (1) & (2), This is follow additivity. → (1 Point)

⇒ For Homogenity –

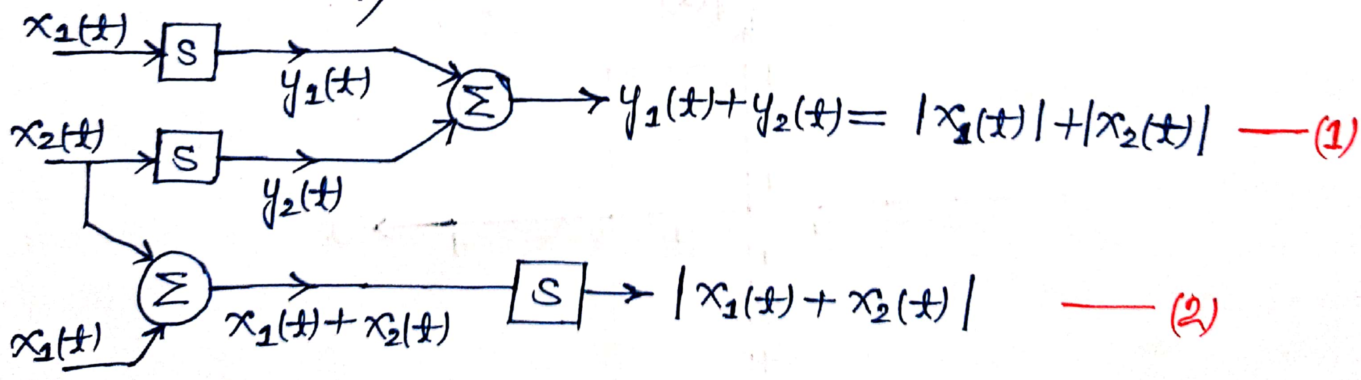


By (1) & (2), This is follow Homogenity. → (1 Point)

Hence Linear System.

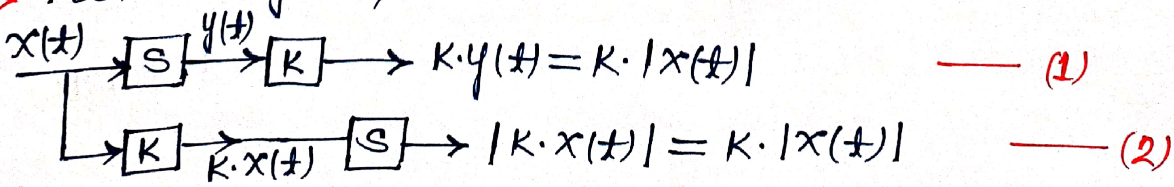
(e) $y(t) = |x(t)|$

⇒ For additivity –



By (1) & (2), This is not follow additivity. (1 Point)

⇒ For Homogenity –

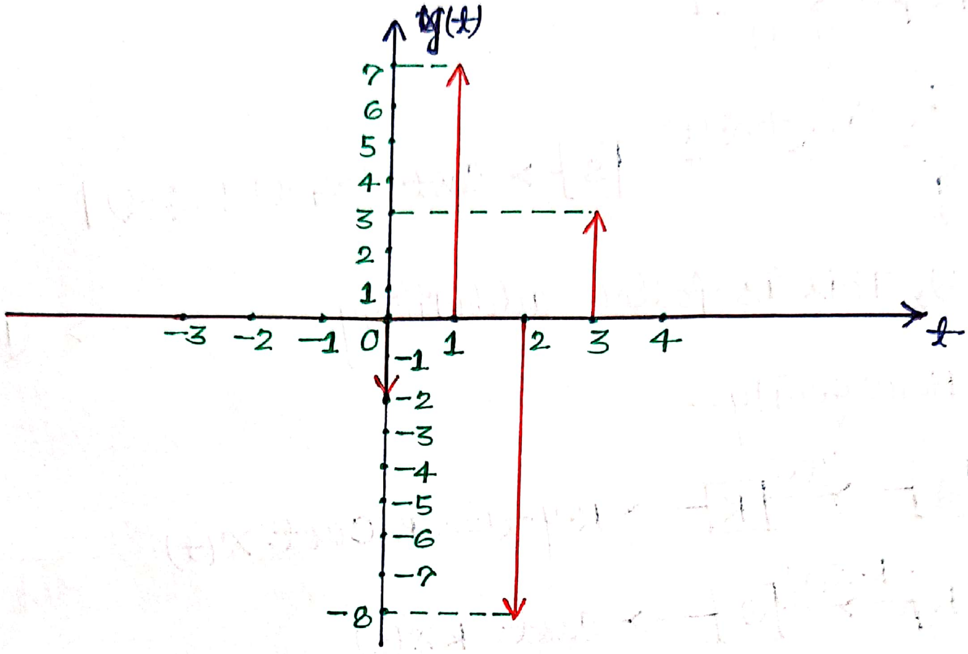


By (1) & (2), This is follow Homogenity. → (1 Point)

Hence Non-Linear System.

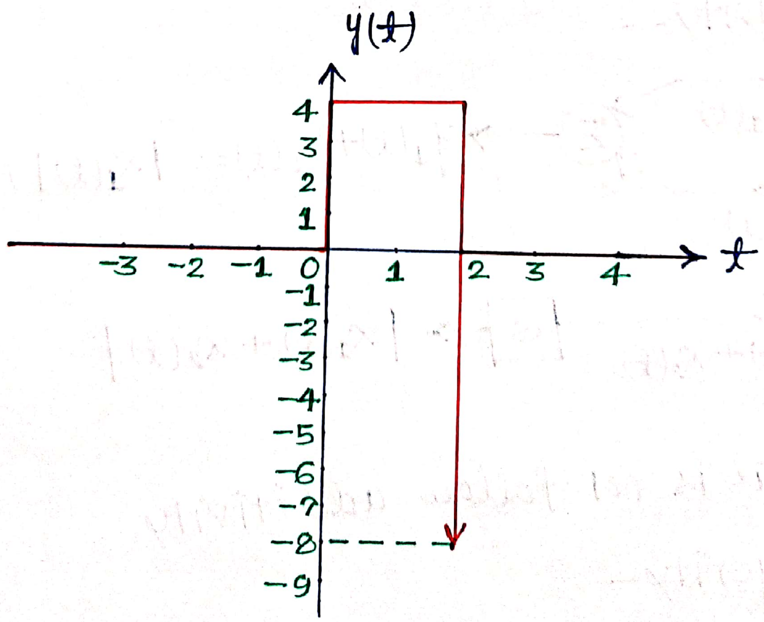
SOL(2): $y(t) = \frac{d}{dt} x(t)$ = Slope of '$x(t)$' w.r.t. 't'

(a)



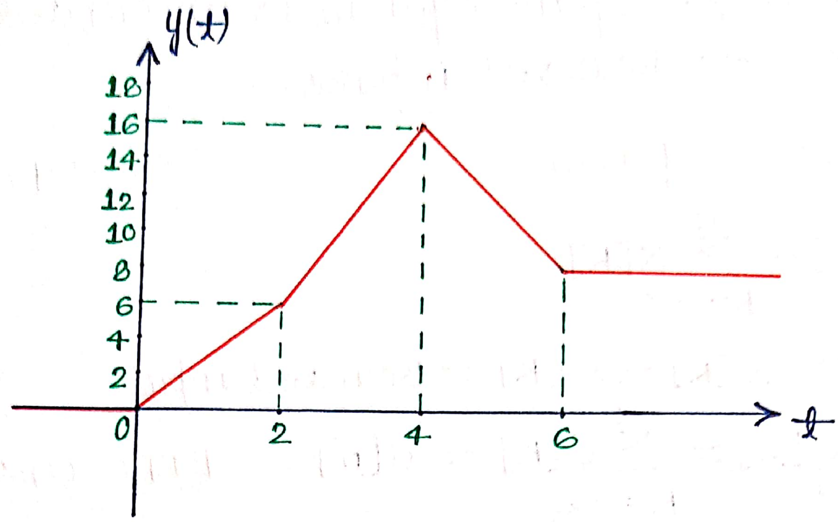
→ (5 Points)

(b)



→ (5 Points)

SOL(3): $y(t) = \int_{-\infty}^{t} x(t) \cdot dt$ = area of signal '$x(t)$' w.r.t. 't'



SOL(4):

(a) $y(t) = t \cdot x(t)$

Here output of system is unbounded/infinite at time $t = \infty$ for bounded inputs

Unstable system

→ (2 Points)

(b) $y(t) = x(t)/t$

Here output of system is unbounded/infinite at time $t = 0$ for bounded inputs.

Unstable System

→ (2 Points)

(c) $y(t) = \int_{-\infty}^{t} \cos(\tau) \cdot x(\tau)\, d\tau$

Put $x(\tau) = \cos(\tau)$ = Bounded input

$\therefore y(t) = \int_{-\infty}^{t} \cos(\tau) \cdot \cos(\tau)\, d\tau = \int_{-\infty}^{t} \cos^2(\tau)\, d\tau$

Unbounded

$$= \int_{-\infty}^{t} \left(\frac{1 + \cos(2\tau)}{2}\right) d\tau = \frac{1}{2}\left\{ \underbrace{[\tau]_{-\infty}^{t}}_{\text{Unbounded}} + \underbrace{\left[\frac{\sin(2\tau)}{2}\right]_{-\infty}^{t}}_{\text{Bounded}} \right\}$$

Unstable System → (2 Points)

(d) $y[n] = n^2 x[n]$

Here output of the system is unbounded/infinite at $n=\infty$ for bounded inputs.

Unstable System → (2 Points)

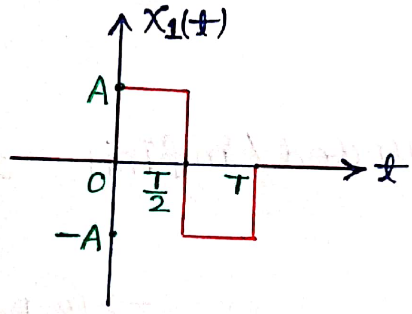
(e) $y[n] = \sum_{K=-\infty}^{n} x[K]$

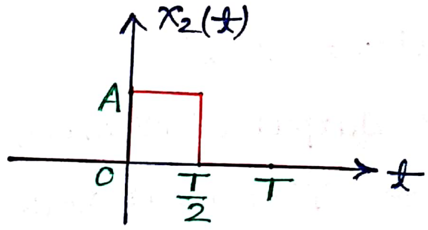
Put $x[K] = \delta[K]$ = Bounded input

$\therefore y[n] = \sum_{K=-\infty}^{\infty} \delta[K] = u[n]$ BIBO Unstable System

→ (2 Points)

SOL(5):





$x_2(t) = A\,u(t) - A\,u(t-\frac{T}{2})$

$x_1(t) = A\,u(t) - 2A\,u(t-\frac{T}{2}) + A\,u(t-T)$

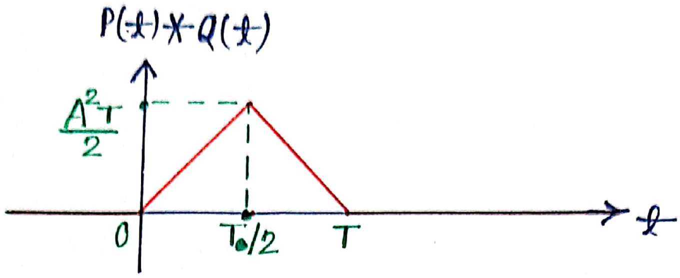
$x_1(t) * x_2(t) = \left\{A\,u(t) - 2A\,u(t-\frac{T}{2}) + A\,u(t-T)\right\} * \left\{A\,u(t) - A\,u(t-\frac{T}{2})\right\}$

→ Distributive Property, $x_1(t) * [x_2(t) + x_3(t)] = x_1(t) * x_2(t) + x_1(t) * x_3(t)$

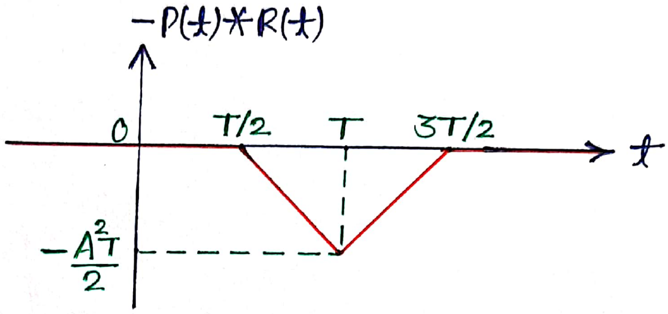
→ Step Response Property, $y(t) = x(t) * u(t) = \int_{-\infty}^{t} x(t)\,dt$

$\therefore x_1(t) * x_2(t) = \left\{A\,u(t) - 2A\,u(t-\frac{T}{2}) + A\,u(t-T)\right\} * A\,u(t)$
$\quad - \left\{A\,u(t) - 2A\,u(t-\frac{T}{2}) + A\,u(t-T)\right\} * A\,u(t-\frac{T}{2})$
$= P(t) * Q(t) - P(t) * R(t)$ ——— (1)

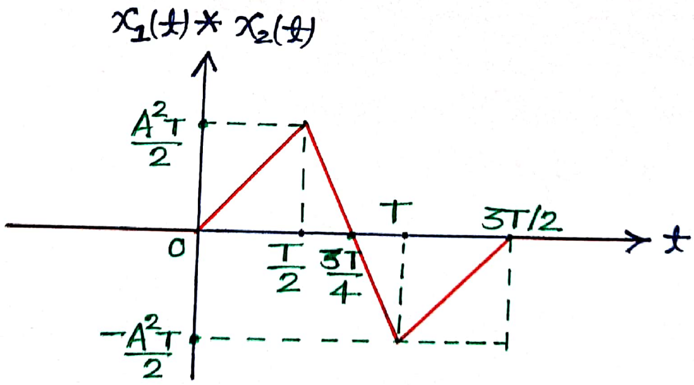
$$\therefore\ P(t) * Q(t) = A^2 r(t) - 2A^2 r\left(t - \frac{T}{2}\right) + A^2 r(t - T)$$



$$\therefore\ -P(t) * R(t) = -A^2 r(t - T/2) + 2A^2 r(t - T) - A^2 r(t - 3T/2)$$



$$\therefore\ x_1(t) * x_2(t) = P(t) * Q(t) - P(t) * R(t) \qquad \{\text{By } eq^n (1)\}$$



$\rightarrow$ (5 Points)