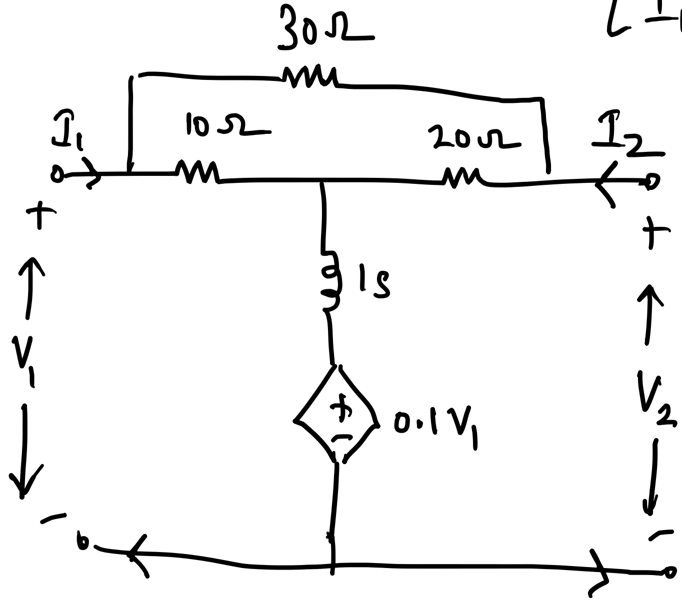
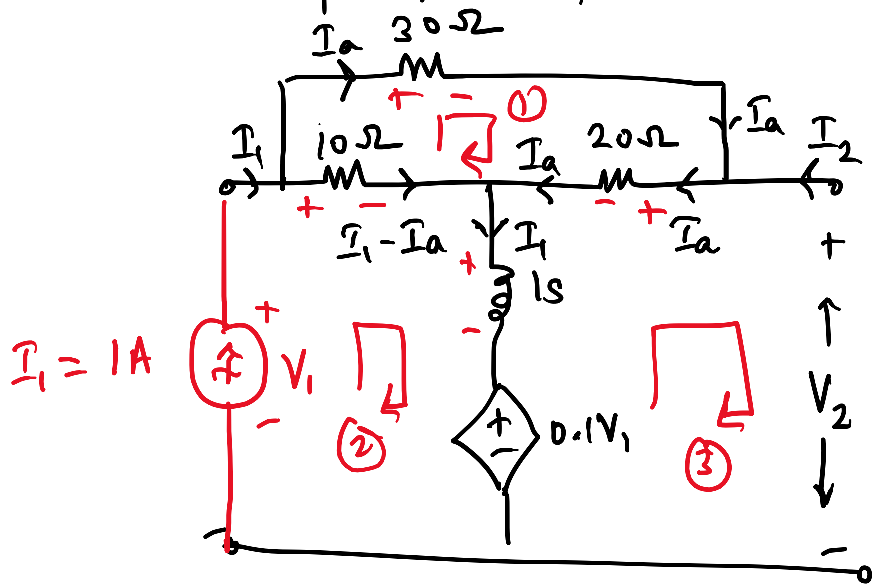
→ Find [t] for

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$



Step.1 Open port 2 / $I_2 = 0$ to find A + C



Loop 1

$$-30 I_a - 20 I_a + 10(1 - I_a) = 0$$

$$60 I_a = 10 \Rightarrow I_a = \frac{1}{6} A$$

Loop 2

$$+V_1 - 10\left(1 - \frac{1}{6}\right) - 1 \times 1s - 0.1 V_1 = 0$$

$$\Rightarrow 0.9 V_1 = \frac{50}{6} + s \Rightarrow V_1 = \frac{\frac{50}{6} + s}{0.9}$$

Loop 3

$$+0.1V_1 + 1\times Is + 20\times\frac{1}{6} - V_2 = 0$$

$$\Rightarrow V_2 = \frac{\cancel{0.1}\times\left(\frac{50}{6}+s\right)}{\cancel{0.9}\ 9} + s + \frac{20}{6}$$

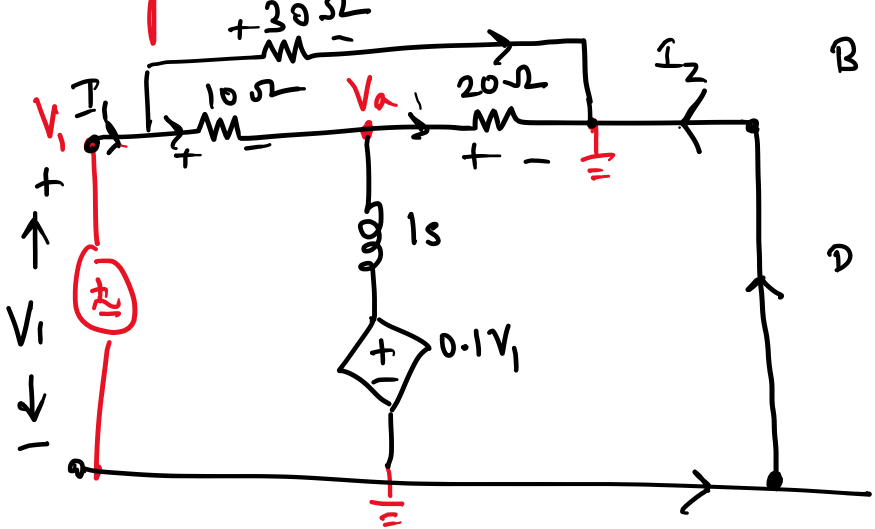
$$= \frac{50}{54} + \frac{s}{9} + s + \frac{20}{6} = \frac{10s}{9} + \frac{50+180}{54}$$

$$= \frac{10s}{9} + \frac{230}{54}$$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{\frac{50}{6}+s}{0.9} \div \frac{10s}{9} + \frac{230}{54} \quad (V/V)$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = 1 \div \frac{10s}{9} + \frac{230}{54} \quad (\Omega^{-1})$$

Step 3 To find B & D, $V_2 = 0$ (short port 2)



$$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0}$$

$$D = \left.\frac{I_1}{-I_2}\right|_{V_2=0}$$

Nodal analysis at node a

$$\frac{1-V_a}{10} = \frac{V_a - 0.1\times 1}{1s} + \frac{V_a - 0}{20}$$

$$\Rightarrow 0.1 + \frac{0.1}{s} = V_a\left(\frac{1}{20} + \frac{1}{10} + \frac{1}{s}\right)$$

$$\Rightarrow 0.1\left(\frac{s+1}{\cancel{s}}\right) = V_a\left(\frac{s + 2s + 20}{20\cancel{s}}\right)$$

$$\Rightarrow V_a = \frac{0.1\times(s+1)\times 20}{(3s+20)} = \frac{2(s+1)}{(3s+20)}$$

$$\Rightarrow I_1 = \frac{V_1 - 0}{30} + \frac{V_1 - V_a}{10}$$

$$= \frac{1}{30} + \frac{1}{10} - \frac{2}{10}\left(\frac{s+1}{3s+20}\right)$$

$$I_1 = \frac{4}{30} - \frac{2}{10}\left(\frac{s+1}{3s+20}\right)$$

$$= \frac{4 - 6s - 6}{30(3s+20)} = \frac{-2-6s}{30(3s+20)}$$

$$I_2 = -\left(\frac{V_a}{20} + \frac{V_1}{30}\right) = -\left(\frac{1}{30} + \frac{1}{10}\frac{(s+1)}{(3s+20)}\right)$$

$$= -\frac{1 + 3s + 3}{30(3s+20)} = -\frac{(4+3s)}{30(3s+20)}$$

$$\rightarrow \; C = \left.\frac{V_1}{-I_2}\right|_{V_2=0}$$

$$= \frac{1}{} \div \frac{4+3s}{30(3s+20)} = \frac{30(3s+20)}{4+3s} \; (\Omega)$$

$$D = \left.\frac{+I_1}{-I_2}\right|_{V_2=0}$$

$$= \frac{-2-6s}{30(3s+20)} \div \frac{4+3s}{30(3s+20)} = \frac{-2-6s}{4+3s} \; (A/A)$$

Table 17.1 (pg 709)   Convert one set of parameters to another

→ Convert z parameters to Y parameters

$$V_1 = z_{11} I_1 + z_{12} I_2 \quad \text{—— (1)}$$

$$V_2 = z_{21} I_1 + z_{22} I_2 \quad \text{—— (2)}$$

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \quad \text{—— (3)}$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2 \quad \text{—— (4)}$$

$\frac{(1)}{z_{11}}$:

$$I_1 = +\frac{1}{z_{11}} V_1 - \frac{z_{12}}{z_{11}} I_2 \quad \text{—— (5)}$$

$\frac{(2)}{z_{22}}$:

$$I_2 = +\frac{1}{z_{22}} V_2 - \frac{z_{21}}{z_{22}} I_1 \quad \text{—— (6)}$$

Sub (6) in (5)

$$I_1 = \frac{1}{z_{11}} V_1 - \frac{z_{12}}{z_{11}} \left(\frac{1}{z_{22}} V_2 - \frac{z_{21}}{z_{22}} I_1 \right)$$

$$\Rightarrow I_1 \left(1 - \frac{z_{12} z_{21}}{z_{11} z_{22}} \right) = \frac{1}{z_{11}} V_1 - \frac{z_{12}}{z_{11} z_{22}} V_2$$

$$\Rightarrow I_1 = \frac{\cancel{z_{11}} z_{22} \; V_1}{\cancel{z_{11}} \left(z_\Delta \right)} - \frac{z_{12} \cancel{z_{11}} \cancel{z_{22}}}{\cancel{z_{11}} \cancel{z_{22}} \, z_\Delta} V_1$$

$$z_\Delta = z_{11} z_{22} - z_{12} z_{21}$$

$$\Rightarrow \quad I_1 = \frac{z_{22}}{z_\Delta} V_1 - \frac{z_{12}}{z_\Delta} V_2 \quad \text{—— (7)}$$

Compare (7) & (3), we get

$$Y_{11} = \frac{z_{22}}{z_\Delta}, \qquad Y_{12} = \frac{-z_{12}}{z_\Delta}$$

Sub (7) in (6)

$$I_2 = \frac{1}{z_{22}} V_2 - \frac{z_{21}}{z_{22}} \left(\frac{z_{22}}{z_\Delta} V_1 - \frac{z_{12}}{z_\Delta} V_2 \right)$$

$$= \frac{-z_{21} \cancel{z_{22}}}{\cancel{z_{22}} z_\Delta} V_1 + \left(\frac{1}{z_{22}} + \frac{z_{21} z_{12}}{z_\Delta z_{22}} \right) V_2$$

$$= \frac{-z_{21}}{z_\Delta} V_1 + \left(\frac{z_\Delta + z_{21} z_{12}}{z_{22} z_\Delta} \right) V_2$$

$$I_2 = \frac{-z_{21}}{z_\Delta} V_1 + \frac{z_{11} \cancel{z_{22}}}{\cancel{z_{22}} z_\Delta} V_2 \quad \text{—— (8)}$$

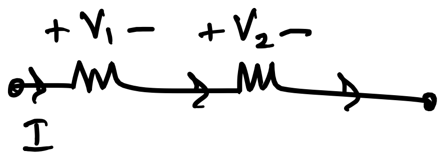
Compare (4) & (8)

$$Y_{21} = \frac{-z_{21}}{z_\Delta}, \qquad Y_{22} = \frac{z_{11}}{z_\Delta}$$

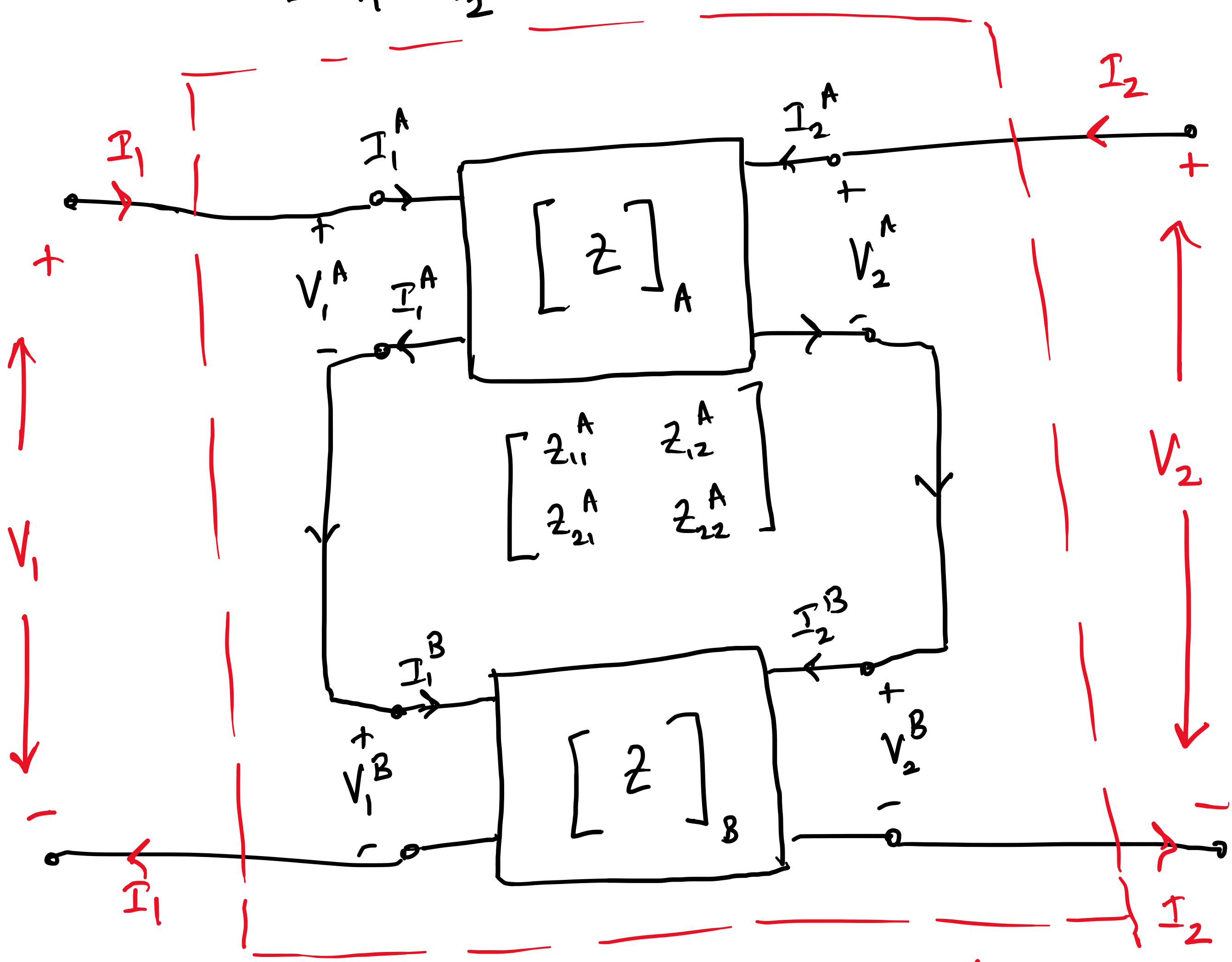
III Connections between two 2port networks

1. Series "z"
2. Parallel "y"
3. Cascade "ABCD/t"

1. Series Connection



$$V = V_1 + V_2$$



$$V_1 = V_1^A + V_1^B \qquad V_2 = V_2^A + V_2^B$$

$$I_1 = I_1^A = I_1^B \qquad I_2 = I_2^A = I_2^B$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} V_1^A + V_1^B \\ V_2^A + V_2^B \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$V_1^A + V_1^B = z_{11} I_1 + z_{12} I_2 \quad \text{--- (1)}$$

But $V_1^A = z_{11}^A I_1^A + z_{12}^A I_2^A$

$V_1^B = z_{11}^B I_1^B + z_{12}^B I_2^B$

$$V_1^A + V_1^B = (z_{11}^A + z_{11}^B) I_1 + (z_{12}^A + z_{12}^B) I_2 \quad \text{--- (2)}$$

$\therefore \quad z_{11} = z_{11}^A + z_{11}^B \qquad z_{12} = z_{12}^A + z_{12}^B$

Illy $\quad z_{21} = z_{21}^A + z_{21}^B \qquad z_{22} = z_{22}^A + z_{22}^B$

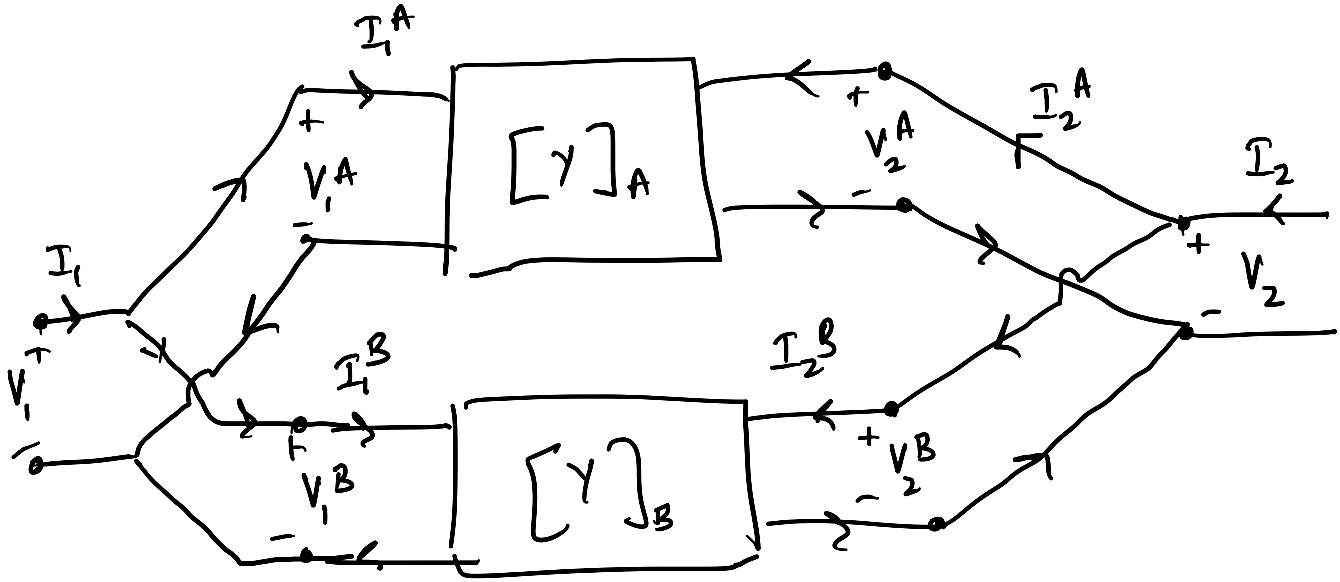
$\rightarrow \quad [z]_A = \begin{bmatrix} 2s & 0 \\ 5 & 3+4s \end{bmatrix}, \quad [z]_B = \begin{bmatrix} 3 & 2/s \\ \infty & 3 \end{bmatrix}$

find $[z]_A + [z]_B$

<u>Soln</u>

$$[z] = \begin{bmatrix} 2s+3 & 2/s \\ \infty & 6+4s \end{bmatrix}$$

(2) Parallel Connection of 2 port networks



$$V_1 = V_1^A = V_1^B \qquad V_2 = V_2^A = V_2^B$$

$$I_1 = I_1^A + I_1^B \qquad I_2 = I_2^A + I_2^B$$

$$[Y] = [Y]_A + [Y]_B$$

$$\Rightarrow \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} Y_{11}^A + Y_{11}^B & Y_{12}^A + Y_{12}^B \\ Y_{21}^A + Y_{21}^B & Y_{22}^A + Y_{22}^B \end{bmatrix}$$