(1.) $x_1(t) = e^{-2t}u(t)$; $x_2(t) = e^{-3t}u(t)$

$$X_1(s) = \frac{1}{s+2}, \quad Re\{s\} > -2$$

$$X_2(s) = \frac{1}{s+3}; \quad Re(s) > -3.$$

using time shifting property

$$x_1(t-2) \longleftrightarrow e^{-2s}X_1(s) = \frac{e^{-2s}}{s+2}, \quad Re(s) > -2$$

$$x_2(-t+3) \longleftrightarrow e^{-3s}X_2(-s) = \frac{\cancel{e^{-3s}}}{\cancel{(s+2)}} \quad \frac{e^{-3s}}{3-s} \quad Re(s) > -3$$

$$\left[Y(s) = \frac{e^{-2s}}{(s+2)} \cdot \frac{e^{-3s}}{(-s+3)} \right] \text{Ans}$$

(2.) Take laplace transform of eqn.

$$Y(s)\left[s^3 + (1+\alpha)s^2 + \alpha(\alpha+1)s + \alpha^2\right] = X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s^3 + (1+\alpha)s^2 + \alpha(\alpha+1)s + \alpha^2}$$

(a.) Take laplace transform

$$G(s) = sH(s) + H(s)$$

substituting $H(s)$

$$G(s) = \frac{(s+1)}{s^3 + (1+\alpha)s^2 + \alpha(\alpha+1)s + \alpha^2} = \frac{1}{s^2 + \alpha s + \alpha^2}$$

$\therefore$ $G(s)$ has 2 poles.

(b.) $H(s) = \frac{1}{(s+1)(s^2+\alpha s+\alpha^2)}$

poles are at -1, $\alpha\left(-\frac{1}{2} \pm j\frac{\sqrt{3}}{2}\right)$

for stable system, real part of pole should be less than zero. So $\alpha > 0$.

(3.) (a.) ~~$X(j\omega)$~~ $X(j\omega) = 0$ for $|\omega| > 4000\pi$

$\therefore$ Nyquist rate for this signal is $\omega_N = 2(4000\pi)$

$\boxed{\omega_N = 8000\pi}$

(b.) $X(j\omega)$ is rectangular pulse for which $X(j\omega) = 0$ for $|\omega| > 4000\pi$ $\therefore$ Nquist rate $\omega_N = 8000\pi$

(c.)

(3.) From Nyquist theorem, we know that sampling frequency in this case must be atleast $\omega_s = 2000\pi$

In other words, sampling period should be at most $T = 2\pi/(\omega_s) = 1\times10^{-3}$. Clearly only (a) & (c) satisfy condition.

4. (a) $X(s) = \int_{-\infty}^{\infty} e^{-5t} u(t-1) e^{-st} dt$

$$= \int_{1}^{\infty} e^{-(5+s)t} dt$$

$$= \frac{e^{-(5+s)}}{(s+5)} \quad ; \text{ ROC: } Re(s) > -5$$

(b) $g(t) = A e^{-5t} u(-t-t_0)$ has laplace transform

$$G(s) = \frac{A e^{(s+5)t_0}}{(s+5)}$$

ROC is given as $Re(s) < -5$

$\therefore A = 1 ; \quad t_0 = -1$

5.

$$X(s) = \frac{4}{s+4} - \frac{2}{s+3}$$

taking inverse laplace transform

$$x(t) = 4e^{-4t}u(t) - 2e^{-3t}u(t)$$

(6.) (a.) Taking fourier transform of both equations & eliminating $W(e^{j\omega})$

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{3 - \frac{1}{2}e^{-j\omega}}{(1 - \frac{1}{2}e^{-j\omega})(1 - \frac{1}{4}e^{-j\omega})}$$

Taking inverse fourier transform of partial fraction expansion of the above expression

$$h(n) = 4\left(\frac{1}{2}\right)^n u(n) - \left(\frac{1}{4}\right)^n u(n)$$

(b.) $$H(e^{j\omega}) = \frac{3 - \frac{1}{2}e^{-j\omega}}{(1 - \frac{1}{2}e^{-j\omega})(1 - \frac{1}{4}e^{-j\omega})} = \frac{Y(e^{j\omega})}{X(e^{j\omega})}$$

$$y(n) - \frac{3}{4}y(n-1) + \frac{1}{8}y(n-2) = 3x(n) - \frac{1}{2}x(n-1)$$