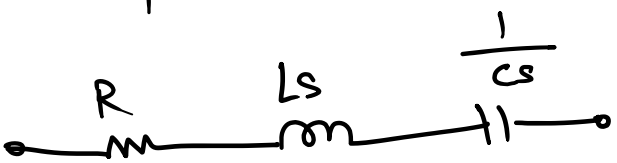
Lecture-3

Chapter 16 from Text book

Frequency Response

→ Laplace variable $s = j\omega$ (Note)



~~$S = j\omega t$~~

$S = j\omega$

→ Transfer function = $\frac{\text{Output response}}{\text{Input response}} = H(s)$

case 1:

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} \quad (V/V)$$

case 2:

$$H(s) = \frac{V_{out}(s)}{I_{in}(s)} \quad (\Omega)$$

case 3

$$H(s) = \frac{I_{out}(s)}{V_{in}(s)} \quad (\Omega^{-1})$$

case 4

$$H(s) = \frac{I_{out}(s)}{I_{in}(s)} \quad (A/A)$$

} linear scale

$[H(s)]$

1. $[H(s)] = 20\log_{10}\left|\frac{V_{out}(s)}{V_{in}(s)}\right|$ dB

2. $[H(s)] = 20\log_{10}\left|\frac{V_{out}(s)}{I_{in}(s)}\right|$ dBΩ

3. $[H(s)] = 20\log_{10}\left|\frac{I_{out}(s)}{I_{in}(s)}\right|$ dB

4. $[H(s)] = 20\log_{10}\left|\frac{I_{out}(s)}{V_{in}(s)}\right|$ dBΩ^{-1}

→ $H(s) = 3+j4$ → $[H(s)]$ ln

$H(s) = 25+j100$ → $[H(s)]$

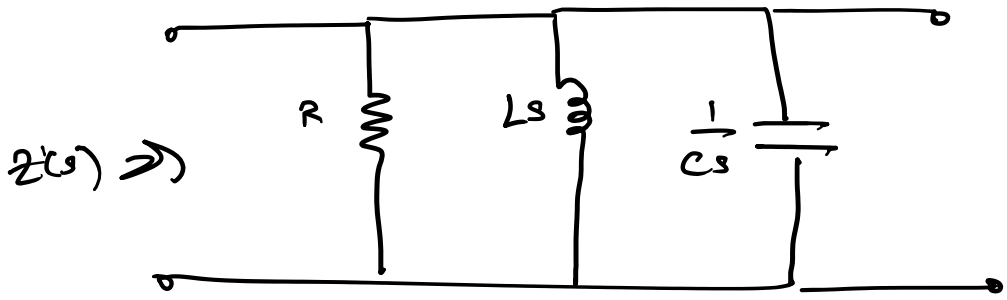
$|H(s)| = \sqrt{3^2+4^2} = 5$

$\log_{10} 5 = 0.69$

$20\log_{10} 5 = 20\times 0.69 = 13.8$ dB

→ Find the transfer function of the following ckt

(3)



$$H(s) = \frac{V_{out}(s)}{I_{in}(s)} = \frac{1}{Y(s)} = \frac{1}{\frac{1}{R} + \frac{1}{Ls} + Cs}$$

$$= \frac{1}{\frac{Ls + R + RLCs^2}{RLs}}$$

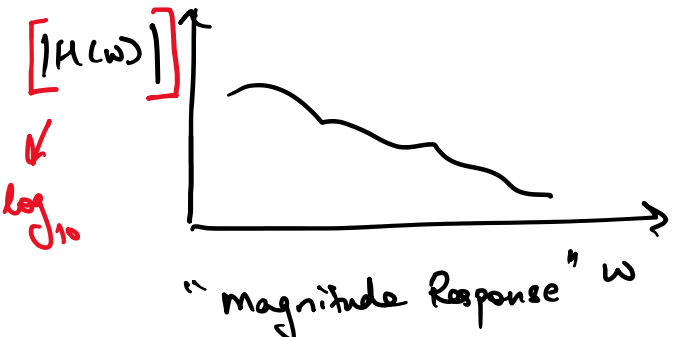
$$= \frac{RLs}{RLCs^2 + Ls + R} \times \frac{\frac{1}{RLC}}{\frac{1}{RLC}} = \frac{s/c}{s^2 + \frac{s}{RC} + \frac{1}{LC}} \quad (\Omega)$$

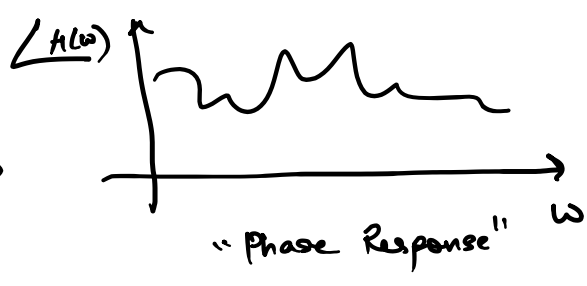
Frequency Response of ckt

$$H(s) = \frac{s/c}{s^2 + \frac{s}{RC} + \frac{1}{LC}}$$

$$H(\omega) = \frac{j\omega/c}{(j\omega)^2 + \frac{j\omega}{RC} + \frac{1}{LC}}$$

→ Complex function in frequency domain





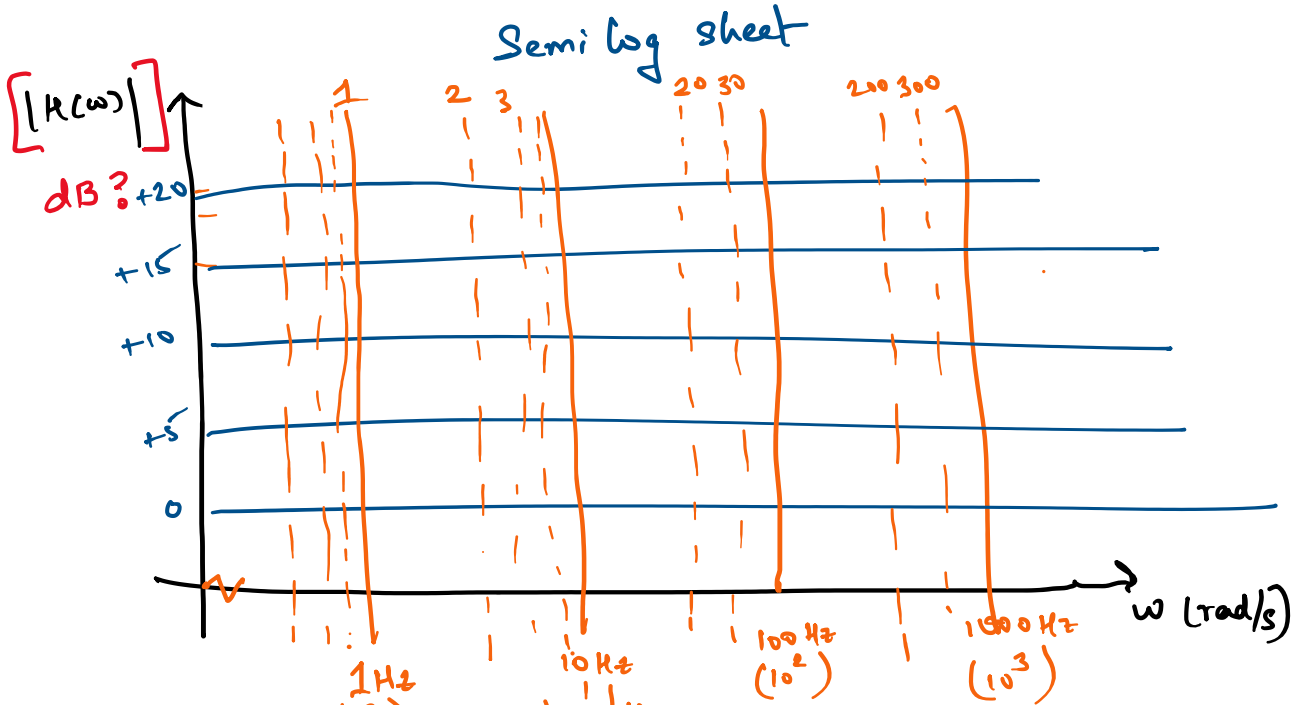
$$H(\omega) = \frac{j\omega/C}{\left(\frac{1}{LC} - \omega^2\right) + j\frac{\omega}{RC}}$$

$x = a + jb$

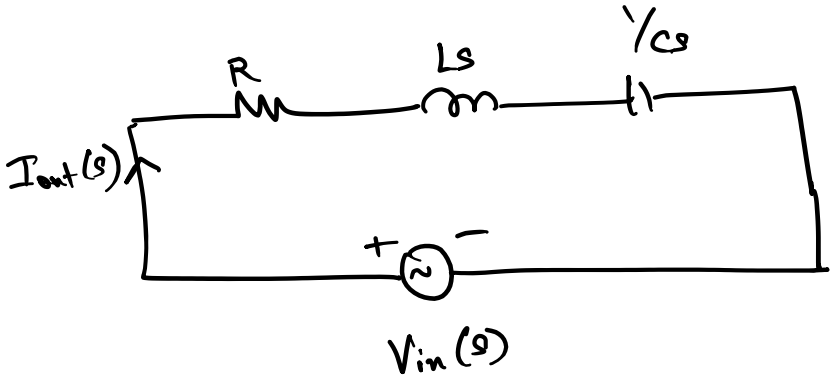
$|x| = \sqrt{a^2 + b^2}$

$\angle x = \tan^{-1}\left(\frac{b}{a}\right)$

$$|H(\omega)| = \frac{\omega/C}{\sqrt{\left(\frac{1}{LC} - \omega^2\right)^2 + \left(\frac{\omega}{RC}\right)^2}}$$



$$\angle H(\omega) = 90^\circ - \tan^{-1}\left(\frac{\omega/RC}{\frac{1}{LC} - \omega^2}\right)$$



$$H(s) = \frac{I_{out}(s)}{V_{in}(s)} = \frac{1}{Z(s)} = \frac{1}{R + Ls + \frac{1}{Cs}}$$

$$= \frac{Cs}{RCs + LCs^2 + 1} \times \frac{\frac{1}{LC}}{\frac{1}{LC}} = \frac{s/L}{s^2 + s\frac{R}{L} + \frac{1}{LC}}$$

→ Draw magnitude & phase response of (6)

$$H(s) = \frac{1}{1 + s/a}$$ a = real constant

(i) $s = j\omega$

$$H(j\omega) = \frac{1}{1 + j(\omega/a)}$$

(ii) $$|H(\omega)| = \frac{1}{\sqrt{1 + (\frac{\omega}{a})^2}}, \quad \angle H(\omega) = 0^\circ - \tan^{-1}\left(\frac{\omega}{a}\right)$$

$$[|H(\omega)|] = 20\log_{10}\left(\left|\frac{1}{\sqrt{1+\omega^2/a^2}}\right|\right)$$

ω	$\lvert H(\omega)\rvert$	$[\lvert H(\omega)\rvert]$	$\angle H(\omega)$
$\omega = a/100$	1	0	$\sim 0^\circ$
$\omega = a/10$	1	0	-5.7°
$\omega = a$	$1/\sqrt{2}$	-3	-45°
$\omega = 10a$	0.1	-20	-85°
$\omega = 100a$	0.01	-40	-90°

↑ Scale by

1. $\omega = a/100$

$$|H(\omega)| = \frac{1}{\sqrt{1 + (\omega/a)^2}} = \frac{1}{\sqrt{1 + (\frac{a/100}{a})^2}} = \frac{1}{\sqrt{1 + (\frac{1}{100})^2}}$$

$$= \frac{1}{\sqrt{1 + 0.0001}} \approx 1$$

$$[|H(\omega)|] = 20\log_{10}(1) = 20\log_{10}(10^0) = 0$$

$$\angle H(\omega) = -\tan^{-1}\left(\frac{\omega}{a}\right) = -\tan^{-1}\left(\frac{a/100}{a}\right) = \textcircled{7}$$

$$= -\tan^{-1}\left(\frac{1}{100}\right) = -0.573^\circ$$

2. $\omega = a/10$

$$|H(\omega)| = \left| \frac{1}{\sqrt{1 + \left(\frac{\omega}{a}\right)^2}} \right| = \frac{1}{\sqrt{1 + \left(\frac{1}{10}\right)^2}} \approx 1$$

$$[|H(\omega)|] = 20\log_{10}(1) = 0$$

$$\angle H(\omega) = -\tan^{-1}\left(\frac{1}{10}\right) = -5.7^\circ$$

3. $\omega = a$ → −3 dB frequency

$$|H(\omega)| = \frac{1}{\sqrt{1+1}} = \frac{1}{\sqrt{2}} = 0.707$$

$$[|H(\omega)|] = 20\log_{10}(0.707) = -3$$

$$\angle H(\omega) = -\tan^{-1}\left(\frac{\omega}{a}\right) = -\tan^{-1}(1) = -45^\circ$$

4. $\omega = 10a$

$$|H(\omega)| = \frac{1}{\sqrt{1+(10)^2}} \approx \frac{1}{10} = 0.1$$

$$[|H(\omega)|] = 20\log_{10}(10^{-1}) = -20$$

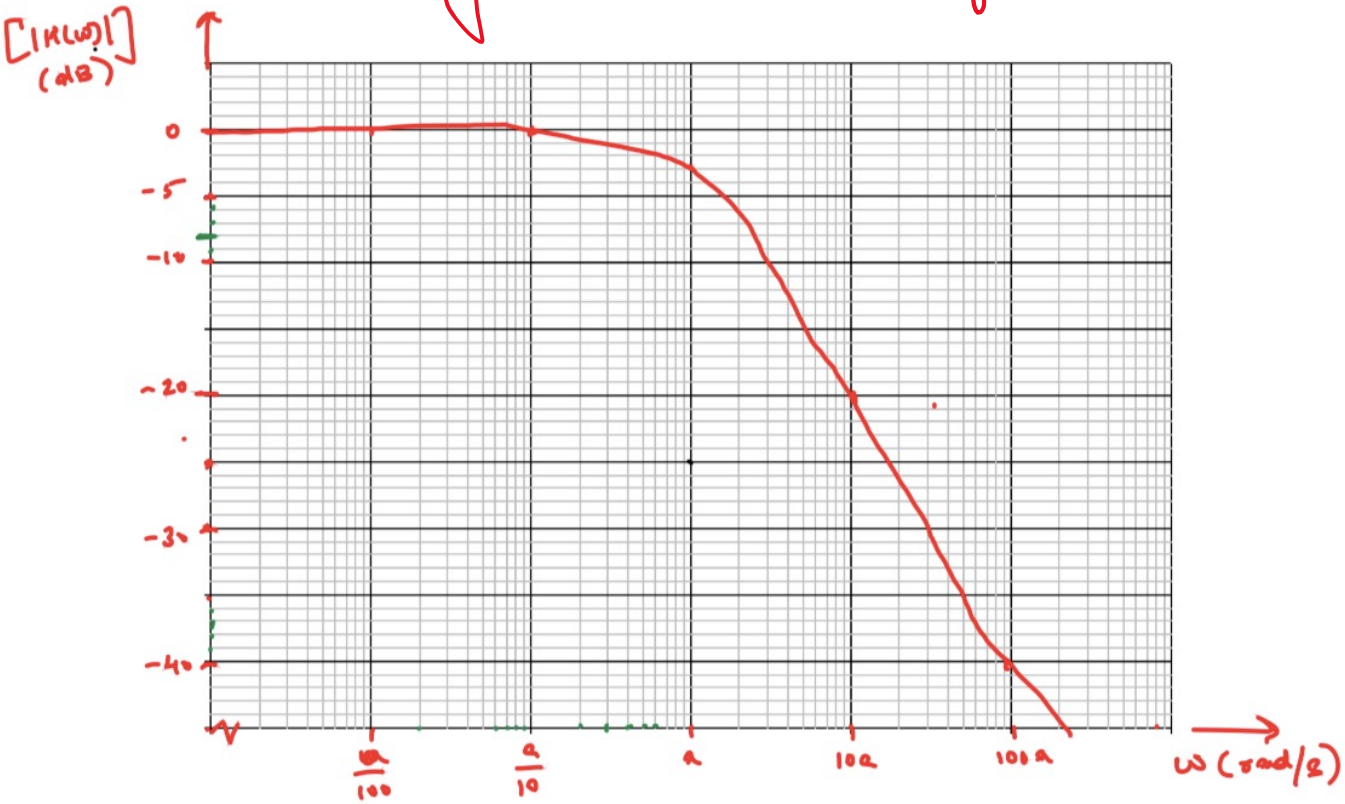
$$\angle H(\omega) = -\tan^{-1}(10) = -85^\circ$$

5. $\omega = 100a$

Please work it out

Bode Plot

Magnitude Response



Phase Response