Q1)

given $\sigma = 5\times10^{6}$

$\varepsilon = 5F/m$

Total charge in the bulk = 1C

radius of the sphere = 5 m.

Volume of the sphere = $\frac{4}{3}\pi r^3$ $= \frac{4}{3}\pi\times125$

volume charge density decay as

$$\rho = \rho_0 \, e^{-\frac{\sigma}{\varepsilon}t}.$$

The decayed charge goes to the surface.

$$\rho_{2\mu s} = \frac{3}{4\pi\times125} e^{-\frac{5\times10^{6}}{5}\times2\times10^{-6}}$$

$$\rho_{2\mu s} = \frac{3}{500\pi} e^{-2} \quad C/m^3$$

Total charge decayed $= (1C - \rho_{2\mu s}\times\frac{4}{3}\pi(5)^3)$

$= (1 - e^{-2})$

Surface charge density $= \frac{1-e^{-2} \text{ (Charge)}}{4\pi r^2 \text{ (Area)}}$

$$= \frac{(1-e^{-2})}{100\pi} \quad C/m^2$$

Q2.

Let $\phi(x,y,z)$ be a scalar-valued function.

$$\therefore \vec{\nabla}\phi(x,y,z) = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}$$

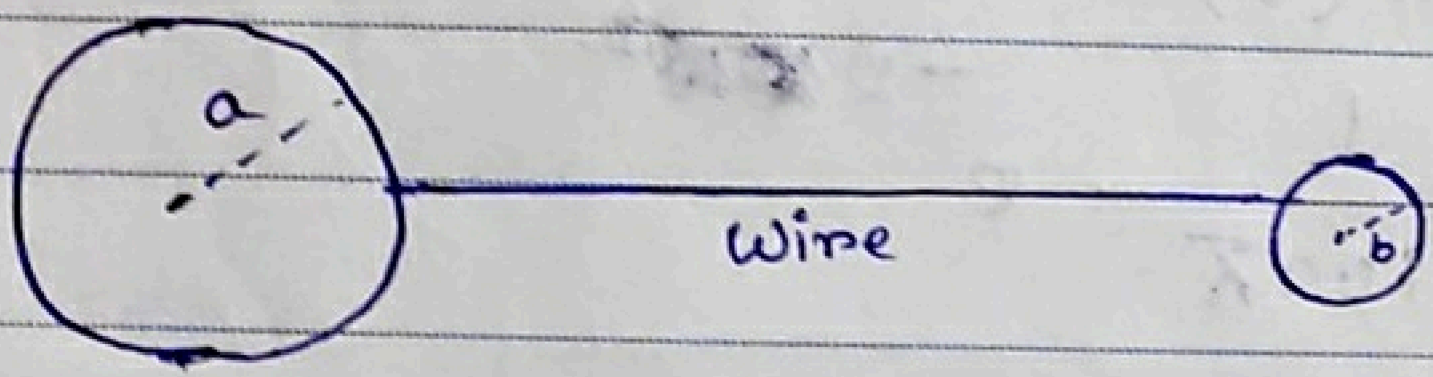
Now, $$\vec{\nabla}\times(\vec{\nabla}\phi) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial\phi}{\partial x} & \frac{\partial\phi}{\partial y} & \frac{\partial\phi}{\partial z} \end{vmatrix}$$

$$= \hat{i}\left(\frac{\partial^2\phi}{\partial y\partial z} - \frac{\partial^2\phi}{\partial z\,\partial y}\right) - \hat{j}\left(\frac{\partial^2\phi}{\partial x\partial z} - \frac{\partial^2\phi}{\partial z\partial x}\right) + \hat{k}\left(\frac{\partial^2\phi}{\partial x\,\partial y} - \frac{\partial^2\phi}{\partial y\,\partial x}\right)$$

$$\therefore \boxed{\vec{\nabla}\times(\vec{\nabla}\phi) = 0}$$

(Proved)

Q3. (a)



Let us assume, the larger sphere (radius a) Carries charge Q and the Smaller sphere (radius b) Carries charge q.

$\therefore$ The potential ~~for~~ of large sphere, $\phi_1 = \frac{Q}{4\pi\epsilon_0 a}$

The potential of small sphere, $\phi_2 = \frac{q}{4\pi\epsilon_0 b}$

Since, the two Conducting sphere are Connected by a Conductor, they form an equipotential ~~surface~~ and are

Subject Date

thus at the same voltage, V, relative to "infinity".

$$\therefore \quad \phi_1 = \phi_2$$

$$\Rightarrow \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{a} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{b}$$

$$\Rightarrow \frac{Q}{a} = \frac{q}{b}$$

$$\Rightarrow \boxed{\frac{Q}{q} = \frac{a}{b}} \quad \cdots \quad (1)$$

Equation (1) shows the comparison between the total charges residing on each sphere. Since $a > b$, the charge Q on larger sphere will be greater than the charge q on smaller sphere.

(b) ~~we know that surface charge density~~

Surface charge density of larger sphere, $\sigma_a = \frac{Q}{4\pi a^2}$

Surface charge density of smaller sphere, $\sigma_b = \frac{q}{4\pi b^2}$

$$\therefore \quad \frac{\sigma_a}{\sigma_b} = \frac{\frac{Q}{4\pi a^2}}{\frac{q}{4\pi b^2}}$$

$$= \frac{Q}{a^2} \times \frac{b^2}{q}$$

$$= \frac{Q}{q} \times \frac{b^2}{a^2}$$

$$= \frac{a}{b} \times \frac{b^2}{a^2} \quad \left[\because \frac{Q}{q} = \frac{a}{b}, \text{ from equation (1)}\right]$$

$$\therefore \quad \boxed{\frac{\sigma_a}{\sigma_b} = \frac{b}{a}} \quad \dots \quad (2)$$

Equation (2) compares the surface charge densities on the two sphere. Since, $a > b$, the charge density of ~~larger~~ small sphere (radius b) becomes higher compared to large sphere (radius a).

(c) According to Gauss's law, electric field just outside the surface of a conductor is proportional to the local surface charge density (σ) and can be written as $E = \frac{\sigma}{\epsilon_0}$

$\therefore$ For larger sphere, $E_a = \frac{\sigma_a}{\epsilon_0}$

For smaller sphere, $E_b = \frac{\sigma_b}{\epsilon_0}$

Subject   Date

$$\therefore \frac{E_a}{E_b} = \frac{\sigma_a/\epsilon_0}{\sigma_b/\epsilon_0}$$

$$= \frac{\sigma_a}{\sigma_b}$$

$$\therefore \boxed{\frac{E_a}{E_b} = \frac{b}{a}}$$ [using Equation (2)]

$$\text{.....(3)}$$

Equation (3) compares the electric fields of the two sphere. Since, a>b, the field ~~at~~ is higher at the surface of small sphere.

(d) As shown in equation (3), the electric fields are inversely proportional to the radius of the conductor. If charges are deposited on a conducting object that is not a sphere, the charges will not distribute uniformly. Instead, there will be higher charge density near the parts of the object where radius of curvature is small (similar to small sphere as discussed in the example by equation 2).

As a consequence of these high concentration of charges near sharper parts of ~~the~~ the object, The electric field becomes more intense in these regions.

Q4. For inhomogeneous medium, ϵ is dependent on the space coordinate.

we know that,

$$\vec{\nabla} \cdot \vec{D} = \rho$$

$$\Rightarrow \vec{\nabla} \cdot (\epsilon \vec{E}) = \rho \quad \text{— (1)}$$

From vector identity we can write

$$\vec{\nabla} \cdot (K\vec{A}) = K(\vec{\nabla} \cdot \vec{A}) + \vec{A} \cdot (\vec{\nabla} K)$$

Since in equation(1), ϵ is not constant,

$$\epsilon(\vec{\nabla} \cdot \vec{E}) + \vec{E} \cdot (\vec{\nabla} \epsilon) = \rho$$

$$\Rightarrow \boxed{\vec{\nabla} \cdot \vec{E} = \frac{\rho - \vec{E} \cdot (\vec{\nabla} \epsilon)}{\epsilon}}$$

Subject Date

Q5. Given,

(a) $\vec{B} = 3x^2y - 4xz + 5z^2$

$$\vec{\nabla}\cdot\vec{B} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\cdot\left(3x^2y\,\hat{i} + (-4xz)\hat{j} + (5z^2)\hat{k}\right)$$

$$= \frac{\partial}{\partial x}(3x^2y) + \frac{\partial}{\partial y}(-4xz) + \frac{\partial}{\partial z}(5z^2)$$

$$= 6xy + 10z$$

$\because \vec{\nabla}\cdot\vec{B} \neq 0 \Rightarrow$ The given field does not satisfy Maxwell's equation. So, this field does not exist.

(b) $\vec{B} =$ Constant

$\therefore \vec{\nabla}\cdot\vec{B} = 0. \Rightarrow$ The given field is a solenoidal field and satisfy Maxwell's equation.