ECE250: Signals and Systems
Practice Sheet 4

1. (CO3) Let $x[n]$ be a discrete-time periodic signal with period N and Fourier series representation as given below:

$$x[n] = \sum_{k=<N>} a_k e^{jk(2\pi/N)n} \tag{1}$$

Derive the expressions for the Fourier series coefficients of the following signals in terms of the coefficients a_k.

   (a) $x[n] - x[n-1]$

   (b) $x^*[-n]$, where $x^*[n]$ denotes the complex conjugate of the signal $x[n]$.

2. (CO3) Let $x_1(t)$ be a continuous-time periodic signal with fundamental frequency ω_1 and Fourier coefficients a_k. Given that

$$x_2(t) = x_1(1-t) + x_1(t-1), \tag{2}$$

how is the fundamental frequency ω_2 of the $x_2(t)$ related to ω_1? Also, find a relationship between the Fourier series coefficients b_k of $x_2(t)$ and the coefficients a_k?

3. (CO3) Given the following information about a signal $x(t)$, determine all possible Fourier series representations for $x(t)$:

   - $x(t)$ is real and even.
   - $x(t)$ is periodic with time period T=4, and has Fourier series coefficients a_k.
   - $a_k = 0$ for $|k| > 1$ and $k = 0$.
   - $\int_{-2}^{2} |x(t)|^2 \, dt = 4$

4. (CO3) Consider a signal $x(t)$ with Fourier series coefficients a_k. Determine the Fourier series coefficients of the following signals:

   (a) $g(t) = x(t - t_0) + x(t + t_0)$

   (b) $\frac{d^2 g(t)}{dt^2}$

5. (CO3) Consider a discrete time signal $x[n]$ with fundamental time period $N = 1$ and Fourier series coefficients a_k, and an LTI system with impulse response $h[n]$ with Fourier series coefficients b_k. Given

$$a_k = \begin{cases} (\frac{1}{2})^k, & \text{for } k \geq 0 \\ 0, & \text{for } k < 0 \end{cases} \tag{3}$$

$$b_k = \begin{cases} k, & \text{for } |k| \leq 3 \\ 0, & \text{for } |k| > 3 \end{cases} \tag{4}$$

determine the Fourier series representation of the output if $x[n]$ is passed as input to the system.

6. (CO3) When the impulse train

$$x[n] = \sum_{k=-\infty}^{\infty} \delta(n - 4k) \tag{5}$$

is the input to a particular LTI system with frequency response $H(e^{j\omega})$, the output of the system is found to be

$$y[n] = \cos\left(\frac{5\pi}{2}n + \frac{\pi}{4}\right) \tag{6}$$

Determine the values of $H(e^{jk\pi/2})$ for k=0,1,2 and 3.

7. (CO3) Consider the following discrete-time signals with the fundamental period of 6:

$$x[n] = 1 - \cos\left(\frac{2\pi}{6}n\right) \tag{7}$$

$$y[n] = sin\left(\frac{2\pi}{6}n + \frac{\pi}{4}\right) \tag{8}$$

$$z[n] = x[n]y[n] \tag{9}$$

(a) Determine the Fourier series coefficients of $x[n]$.

(b) Determine Fourier series coefficients of $y[n]$.

(c) Using the results of (a) and (b), along with the multiplicative property of the discrete-time Fourier series, determine the Fourier series coefficients of $z[n]$.

(a) Determine the Fourier series coefficients of $z[n]$ using direct evaluation, and compare your results with part (c).