Review

Time Domain

$$v(t) = V_0 \cos(\omega t + \theta) \quad V$$

$$i(t) = I_0 \cos(\omega t + \phi) \quad A$$

- instantaneous power

$$p(t) = v(t)\, i(t) \quad \text{Watts}$$

Phasor / Freq Domain

$$V(\omega) = V_0 \angle \theta$$

$$I(\omega) = I_0 \angle \phi$$

$$\cancel{P(\omega) = V(\omega)\, I(\omega)}$$

$$= \frac{1}{2} V_0 I_0 \cos(\theta - \phi) + \frac{1}{2} V_0 I_0 \cos(2\omega t + \theta + \phi) \quad \text{Watts}$$

- $\langle P_{avg} \rangle = \frac{1}{T} \int_0^T p(t)\, dt$ — Time averaged power

Complex Power $S(\omega) = \frac{1}{2} V I^* \quad VA$

$$\underbrace{S(\omega)}_{VA} = \underbrace{\langle P_{avg} \rangle}_{\text{Watts}} + j \underbrace{\text{Reactive Power}}_{VAR}$$

$$\langle P_{avg} \rangle = \frac{1}{2} V_0 I_0 \cos(\theta - \phi)$$

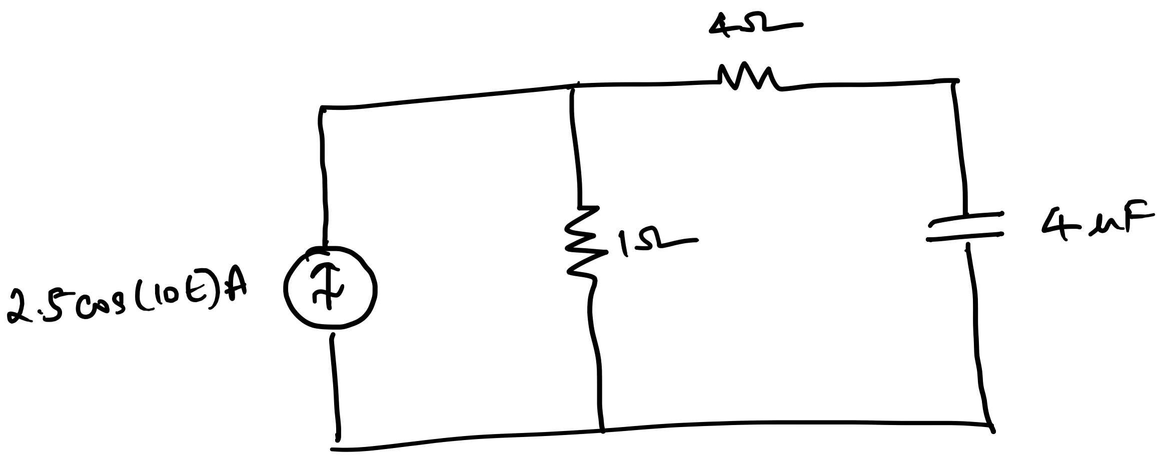
$\| S(\omega) \| \longrightarrow$ apparent power

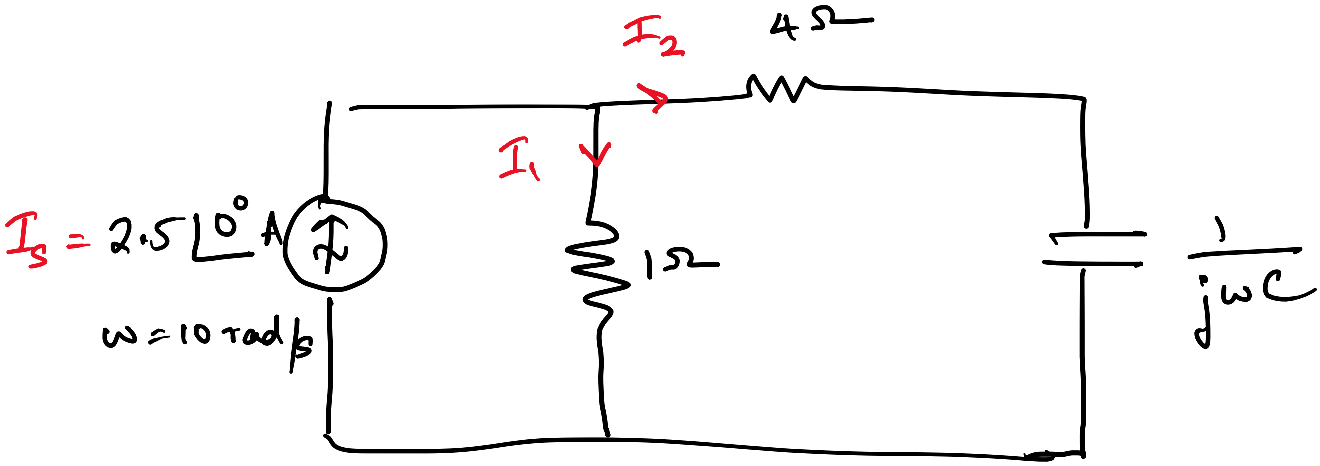
$$\text{power factor} = \cos(\theta - \phi) = \frac{\langle P_{avg} \rangle}{\| S \|}$$

$I \xrightarrow{\text{lead}} V$ leading pf capacitive

$I \xleftarrow{\text{lags}} V$ lagging pf inductive

- Calculate the power absorbed by each element shown in the circuit at $t = 0, 10, 20\ ms$





$$I_1 = \frac{I_s \times \left(4 + \frac{1}{j\omega C}\right)}{1 + 4 + \frac{1}{j\omega C}} = 2.5\angle 0^\circ\ A$$

$$i_1(t) = 2.5\cos(10t)\ A$$

$$I_2 = \frac{I_s \times 1}{1 + 4 + \frac{1}{j\omega C}} = 100\ \mu A\angle -90^\circ$$

$$i_2(t) = 100\cos(10t - 90^\circ)\ \mu A$$

③

$$S_{1\Omega} = \frac{1}{2} V_{1\Omega} I_1^*$$

$$= \frac{1}{2} (I_1 \times 1) I_1^* = \frac{1}{2} |I_1|^2$$

$$V_{1\Omega} = I_1 \times 1$$

$$\Rightarrow v_{1\Omega}(t) = 2.5 \cos(10t + 0^\circ) \quad V$$

$$P_{1\Omega}(t) = v_{1\Omega}(t)\, i_1(t)$$

$$= 3.125 + 3.125 \cos(20t) \quad W$$

$$P_{1\Omega}(t=0) = 6.25 \text{ W}$$

$$P_{1\Omega}(t=10\text{ms}) = 6.1877 \text{ W}$$

$$P_{1\Omega}(t=20\text{ms}) = 6.0033 \text{ W}$$

$$V_{4\Omega} = I_2 \times 4 = j\, 4 \times 10^{-4} \quad V$$

$$v_{4\Omega} = 4 \times 10^{-4} \cos(10t + 90^\circ) \; V$$

$$P_{4\Omega}(t) = v_{4\Omega}(t)\, i_2(t) = 2 \times 10^{-8} + 2 \times 10^{-8} \cos(20t) \quad W$$

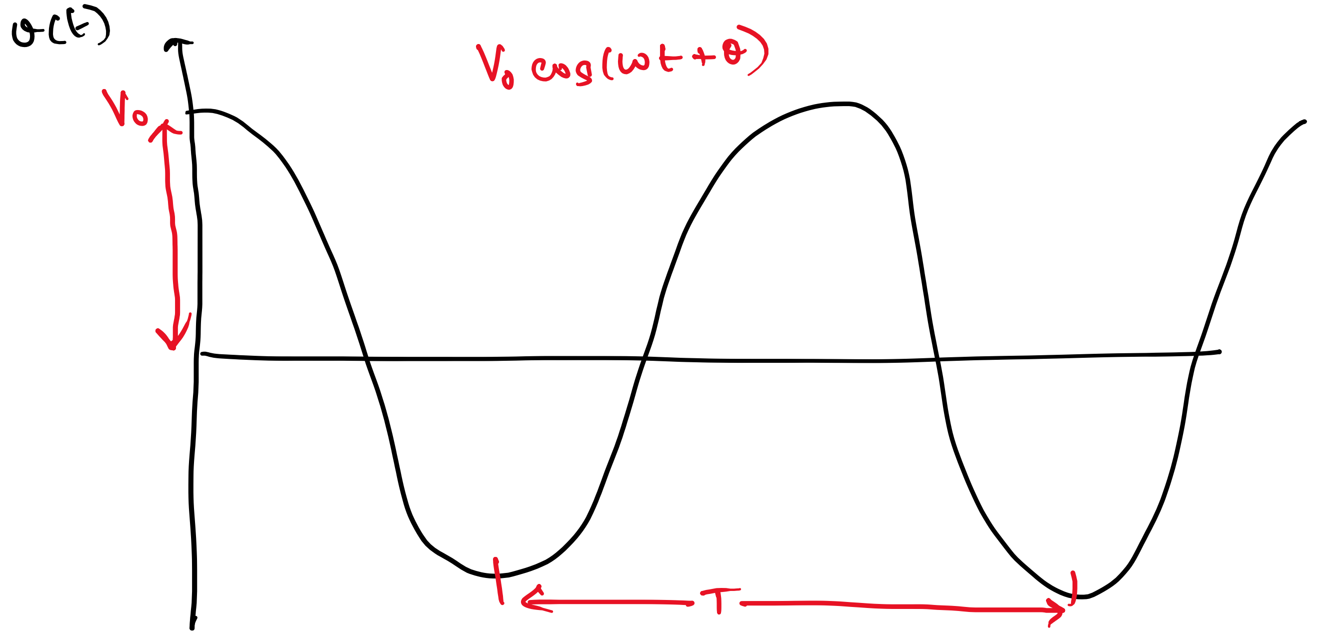
$$P_{4\Omega}(t=0, 10\text{ms}, 20\text{ms}) = [0.4,\; 0.396,\; 0.3842] \times 10^{-7} \quad W$$

$$V_c = I_2 \times \frac{1}{j\omega C} = 2.5 - j\,0.005 \; V = 2.5\angle 0^\circ \; V$$

$$v_c(t) = 2.5 \cos(10t) \; V$$

$$P_c(t) = 1.25 \times 10^{-4} \cos(20t - 90^\circ) \quad W$$

Sinusoids



$$\omega = \frac{2\pi}{T}$$

Root mean square

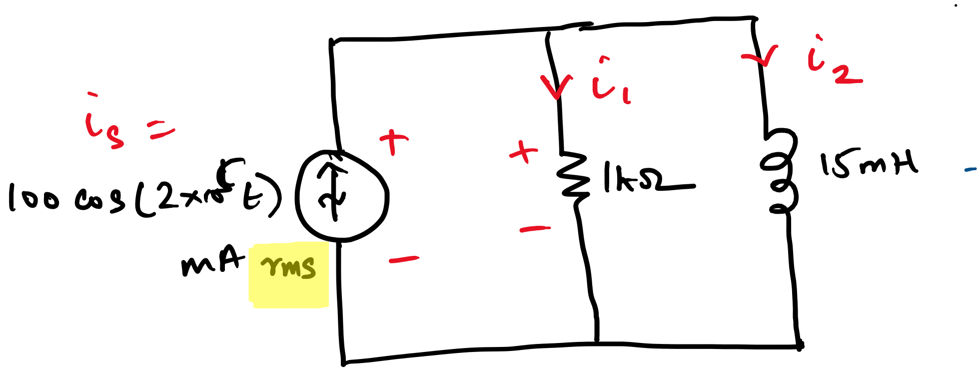
$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T (v(t))^2 \, dt}$$

$$= \sqrt{\frac{1}{T}\int_0^T V_0^2 \cos^2(\omega t + \theta) \, dt} = \frac{V_0}{\sqrt{2}}$$

$$2\cos^2\theta = \cos(2\theta) + 1$$ (please check)

$$= \sqrt{\frac{V_0^2}{2T}\int_0^T \left[1 + \cos(2\omega t + 2\theta)\right] dt} = \sqrt{\frac{V_0^2}{2\cancel{T}} \times \cancel{T} + 0} = \frac{V_0}{\sqrt{2}}$$

- Three elements are connected in parallel: 1 kΩ resistor, a 15 mH inductor and a $100\cos(2\times 10^5 t) mA$ rms sinusoidal source. Determine the power being absorbed by each element at $t = 10\mu s$.



$$I_{rms} = 100\times 10^{-3} \text{ A}$$

$$I_s = I_{rms}\sqrt{2}$$

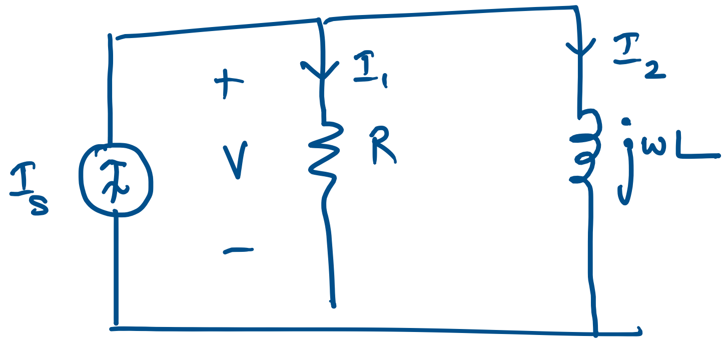
$$\langle P_{avg}\rangle = \frac{1}{2} V_0 I_0 \cos(\theta - \phi)$$

$$= \left(\frac{V_0}{\sqrt{2}}\right)\times\left(\frac{I_0}{\sqrt{2}}\right)\cos(\theta-\phi)$$

$$= V_{0,rms}\times I_{0,rms}\cos(\theta-\phi)$$

$$S = V_{rms} I_{rms}^{*}$$

$$I_{1_{rms}} = \frac{I_{s_{rms}} \times j\omega L}{1000 + j\omega L} \qquad I_{2_{rms}} = \frac{I_{s_{rms}}\times 1000}{1000 + j\omega L}$$



$$I_{1_{rms}} = \frac{I_s \times j\omega L}{R + j\omega L} = 0.095 \angle 18.43^\circ \text{ A}$$

$$I_{2_{rms}} = \frac{I_s \times R}{R + j\omega L} = 0.032 \angle -71.56^\circ \text{ A}$$

$$V_{rms} = I_{1rms} \times R = 94.86 \angle 18.43^\circ \text{ V}$$

$$P_R(t) = 2 \times 94.86 \cos(2\times10^5 t + 18.43^\circ) \times 0.095 \cos(2\times10^5 t + 18.43^\circ)$$

$$= 9.0117 + 9.0117\cos(4\times10^5 t + 36.86^\circ) \text{ W}$$

$$P_R(t = 1\times10^{-6}) = 9.0117 + 9.0117\cos\left(4\times10^{-1} + 36.86 \times \frac{\pi}{180}\right)$$

$$= 13.53 \text{ W}$$

$$P_L(t) = 2 \times 94.86 \cos(2\times10^5 t + 18.43^\circ) \times 0.032 \cos(2\times10^{-5} t - 71.56^\circ)$$

$$= 3.0355 \cos(4\times10^5 t - 53.13^\circ) \text{ W}$$

$$P_L(t = 1\times10^{-6}) = 3.0355 \cos\left(0.4 - 53.13 \times \frac{\pi}{180}\right)$$

$$= 2.6232 \text{ W}$$

$$V_{1000\Omega_{rms}} = I_{1_{rms}} \times 1000$$

$$S_{1000} = V_{1000\Omega\, rms} \, I_{1rms}^{*}$$

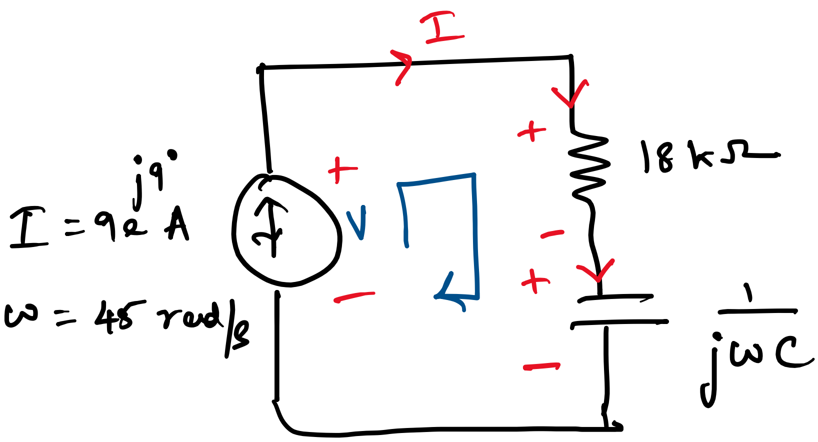
$$V_{L\,rms} = I_{2\,rms} \times j\omega L$$

$$S_L = V_{L\,rms} \times I_{2\,rms}^{*}$$

$$pf_L = \frac{Re\{S_L\}}{\|S_L\|} = 0 \quad \text{lagging}$$

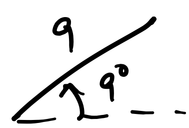
$$pf_R = \frac{Re\{S_{1k\Omega}\}}{\|S_{1k\Omega}\|} = 1 \quad \text{inphase}$$

- The phasor current $I = 9e^{j9^\circ}$ A corresponding to a sinusoidal source operating at 45 rad/s is applied to a series combination of $18k\Omega$ resistor and a $1\mu F$ capacitor. Obtain an expression for (a) instantaneous power, (b) complex power provided by the source, (c) time averaged power absorbed by the combined load, (d) reactive power absorbed by the load, (e) apparent power, (e) power factor (also mention if it is lagging or leading).



$9\angle 9^\circ$

$9e^{j9^\circ}$



$$+V - I \times 18000 - I \times \left(\frac{1}{j\omega C}\right) = 0$$

$$\Rightarrow \quad V = I\left(18000 + \frac{1}{j\omega C}\right)$$

$$= 9\angle 9^\circ \left(18000 + \frac{1}{j\omega C}\right) \quad V$$

(b) $S = \frac{1}{2} V I^* = \langle P_{avg} \rangle + j$ Reactive Power

$$= (7.29 - j9) \times 10^5 \text{ VA}$$

(c) $\Rightarrow \langle P_{avg} \rangle = 7.29 \times 10^5$ W

(d) Reactive Power $= -9 \times 10^5$ VAR

(e) $\| S \| = 1.158 \times 10^6$ VA

(f) $pf = \dfrac{\text{Real}\{S\}}{\| S \|} = \dfrac{7.29 \times 10^5}{1.158 \times 10^6} = 0.6294$

leading

$i(t) = 9 \cos(45t + 9^\circ)$ A

$v(t) = 2.57 \times 10^5 \cos(45t - 42^\circ)$ A

(a) $p(t) = v(t)\, i(t) =$

$= 7.29 \times 10^5 + 1.158 \times 10^6 \cos(90t - 33^\circ)$ W