Tutorial 11

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Soln 3) By observing waveform :-

$$f(t) = -f(-t)$$
$$f(t) = -f(t - T/2)$$

Hence, wave is odd and half wave symmetric

$$T = 10ms$$

$$w_0 = \frac{2\pi}{10 \times 10^{-3}}$$

$$a_0 = 0$$
$$a_n = 0$$

Hence,

$$a_0 = 0, \quad a_1 = 0$$

=) formula for b_n for n-odd is given by:-

$$b_n = \frac{8}{T}\int_0^{T/4} f(t)\sin(n w_0 t)\,dt$$

$$= \frac{8}{10 \times 10^{-3}}\int_0^{1\times 10^{-3}} (10)\sin\left(\frac{2\pi n t}{10 \times 10^{-3}}\right)dt$$

$$= 800\int_0^{1\times 10^{-3}} (10)\sin\left(\frac{2\pi n t}{10 \times 10^{-3}}\right)dt$$

$$= 8000\; \frac{-\cos(200 n\pi t)}{200 n\pi}\Bigg|_0^{1\times 10^{-3}}$$

$$= \frac{-40}{n\pi}\left(\cos(0.2 n\pi) - 1\right)$$

Substitute in b_n,

$b_1 = 2.432$, $b_3 = 5.556$, $b_5 = 5.093$, $b_7 = 2.381$

$b_9 = 0.2702$

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b_n for n-even - $b_n = 0$

$\Rightarrow b_2 = b_4 = b_6 = b_8 = 0$ ~~ft~~

Soln 4) From figure: $T = \pi/5$

$\omega_0 = 2\pi/T$

$\omega_0 = 10$ rad/s

Equation for a_0 is given by:-

$$a_0 = \frac{1}{T}\int_0^T f(t)\,dt$$

$$= 5/\pi \int_0^{\pi/10} 12\,dt$$

$$= \frac{60}{\pi}(t)_0^{\pi/10}$$

$$= 6$$

Equation for a_n is given by:-

$$a_n = 2/T \int_0^T f(t)\cos(n\omega_0 t)\,dt$$

$$= 10/\pi \int_0^{\pi/10} 12\cos(n\omega_0 t)\,dt$$

$$= \frac{120}{\pi}\left(\frac{\sin(n\omega_0 t)}{n\omega_0}\right)_0^{\pi/10}$$

$$= \frac{120}{\pi}\left[\frac{\sin(n\pi)}{10n} - \sin(0)\right]$$

$$= 0$$

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Equation for b_n is given by :-

$$b_n = \frac{2}{T}\int_0^T f(t)\sin(n\omega_0 t)\,dt$$

$$= \frac{10}{2}\int_0^{2/10} 12\sin(n\omega_0 t)\,dt$$

$$= \frac{-120}{2}\left(\frac{\cos(n\omega_0 t)}{n\omega_0}\right)_0^{2/10}$$

$$= \frac{-120}{2}\left(\frac{\cos(n\pi)}{10n} - \frac{\cos(0)}{10n}\right)$$

$$= -\frac{120}{2}\left[\frac{(-1)^n - 1}{10n}\right]$$

for odd numbers b_n is :

$$b_n = \frac{-120}{2}\left(\frac{-1-1}{20n}\right)$$

$$= 24/\pi n$$

for even numbers b_n is,

$$b_n = \frac{-120}{2}\left(\frac{1-1}{10n}\right) = 0$$

Therefore,

$$b_n = \sum_{n=1\ \text{for odd}}^{\infty} 24/n\pi$$

Equation for $v_s(t)$.

$$v_s(t) = 6 + \sum_{n=1\ \text{for odd}}^{\infty} 24/n\pi \sin(\omega_0 n t)$$

$$= 6 + \sum_{n=1\ \text{for odd}}^{\infty} \frac{24}{n\pi}\sin(10\pi t)$$

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→ Rewriting the n^{th} harmonic –

$$V_s(t) = \frac{24}{n\pi} \angle -90°$$

$$= -j24/n\pi$$

→ Now, impedance of circuit (from figure)

$$Z_n = 4 + j(10n)(2)$$
$$= 4 + j20n$$

→ Calculating, the forced response –

$$I_{fn} = \frac{V_{sn}}{Z_n} = \frac{-j24/n\pi}{4 + j20n}$$

$$= -\frac{6j}{n\pi(1+j5n)}$$

→ Transform from frequency to time domain –

$$i_{fn} = \frac{6}{n\pi\sqrt{1+25n^2}} \cos(10nt - 90° - \tan^{-1}(5n))$$

$$= \frac{6}{n\pi\sqrt{1+25n^2}} \sin(10nt - \tan^{-1}(5n))$$

$$= \frac{6}{n\pi\sqrt{1+25n^2}} [\sin(10nt)\cos(-\tan^{-1}(5n)) - \cos(10nt)\sin(\tan^{-1}(5n))]$$

$$= \frac{6}{\pi(1+25n^2)} \left(\frac{\sin(10nt)}{n} - 5\cos(10nt)\right)$$

→ Calculating response of DC component,

$$I = 6/4 = 1.5\,A$$

→ The full forced response is :-

$$i_F(t) = 1.5 + \sum_{n=1 \text{ for n odd}}^{\infty} \frac{6}{\pi(1+25n^2)}\left(\frac{\sin(10t)}{n} - 5\cos(10nt)\right]$$

→ Calculating the natural response :-

$$\frac{I_F}{V_s} = \frac{1}{4+2s}$$

$$\Rightarrow i_n(t) = Ae^{-2t}$$

→ The complete response is given by :-

$$i(t) = i_n(t) + i_F(t)$$

$$= Ae^{-2t} + 1.5 + \sum_{n=1}^{\infty} \frac{6}{\pi(1+25n^2)}\left(\frac{\sin(10nt)}{n} - 5\cos(10nt)\right)$$

⇒ Calculate A at $t = 0$,

$$i(0) = Ae^{-2(0)} + 1.5 + \sum_{n=1}^{\infty} \frac{6}{\pi(1+25n^2)}\left(\frac{\sin(10n(0))}{n} - 5\cos(10n(0))\right)$$

$$A = -1.5 + \frac{30}{\pi}\sum_{n=1}^{\infty} \frac{1}{1+25n^2}$$

$$A = -1.5 + \frac{30}{\pi}(0.01)$$

$$A = -1.4$$

Thus,

$$i(t) = -1.4e^{-2t} + 1.5 + \sum_{n=1}^{\infty} \frac{6}{\pi(1+25n^2)}\left(\frac{\sin(10nt)}{n} - 5\cos(10nt)\right)$$

A