Q1)

(a) $$\varepsilon_r(\omega) = 1 + \frac{Nq^2}{m\epsilon_0} \sum_j \frac{f_j}{\omega_j^2 - \omega^2 + i\omega\gamma_j}$$

Now given that all the electrons have the same natural frequency and damping constant.
let's denote the natural frequency as ω_k and damping constant as γ_k
and $\sum_j f_j = Z$ (Total electrons)

$$\varepsilon_r(\omega) = 1 + \frac{Nq^2}{m\epsilon_0} \sum_j \frac{f_j}{\omega_k^2 - \omega^2 + i\omega\gamma_k}$$

$$\varepsilon_r(\omega) = 1 + \frac{NZq^2}{m\epsilon_0} \frac{1}{(\omega_k^2 - \omega^2 + i\omega\gamma_k)}$$

(b) Here $N = 1$.

$$\varepsilon_r(\omega) = 1 + \frac{Zq^2}{m\epsilon_0(\omega_k^2 - \omega^2 + i\omega\gamma_k)}$$

$$= 1 + \frac{Zq^2 (\omega_k^2 - \omega^2 - i\omega\gamma_k)}{m\epsilon_0 (\omega_k^2 - \omega^2 + i\omega\gamma_k)(\omega_k^2 - \omega^2 - i\omega\gamma_k)}$$

$$\varepsilon_r(\omega) = 1 + \frac{Zq^2 (\omega_k^2 - \omega^2 - i\omega\gamma_k)}{m\epsilon_0 ((\omega_k^2 - \omega^2)^2 + \omega^2\gamma_k^2)}$$

$$Re(\varepsilon_r(\omega)) = 1 + \frac{Zq^2 (\omega_k^2 - \omega^2)}{m\epsilon_0 ((\omega_k^2 - \omega^2)^2 + \omega^2\gamma_k^2)}$$

$$Im(\varepsilon_r(\omega)) = \frac{-\omega\gamma_k Zq^2}{m\epsilon_0 ((\omega_k^2 - \omega^2)^2 + \omega^2\gamma_k^2)}$$

(c) $$\varepsilon_r(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$$

By looking at the above equation we can assume that $Im(\varepsilon_r(\omega)) = 0$ so $\gamma_k = 0$ and By looking at the $Re(\varepsilon_r(\omega))$, we can conclude that ω_k is also 0. This can happen only in a lossless metal where the damping will be zero and for free electrons (because its a metal) the $\omega_k \approx 0$.

$$\varepsilon_r(\omega) = 1 + \frac{Nq^2}{m\epsilon_0} \sum_j \frac{f_j}{\omega_j^2 - \omega^2 + i\omega\gamma_j}$$

$$= 1 + \frac{Nq^2}{m\epsilon_0}\left(-\frac{1}{\omega^2}\right) \quad \because \sum_j f_j = 1$$

$$= 1 - \frac{Nq^2}{m\epsilon_0 \omega^2}$$

$$\omega_p = \sqrt{\frac{Nq^2}{m\epsilon_0}}$$

$$\varepsilon_r(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$$

Q3)

(a) (iii) solenoidal

(b) (ii), (iii)

(c) (iii)

Q2)

$$\vec{E} = 4\exp(i(5\times10^6 t - kz))\,\hat{x}$$

(a) The given electric field is along $\hat{x}$ axis. The polarization of the given EM wave is along x-axis.

(b) The direction of wave propagation is along positive z-axis.

(c) wave number is given by

$$k = \frac{\omega}{c} = \frac{5\times10^6}{3\times10^8} = 0.0167 \text{ rad/m}$$

(d) we know that

$$\vec{k} = \vec{E}\times\vec{B}$$

$$\hat{z} = \hat{x}\times\vec{B}$$

So direction of $\vec{B}$ is along y axis.

magnitude of $\vec{B}$ is given by

$$\frac{|\vec{E}|}{|\vec{B}|} = 3\times10^8$$

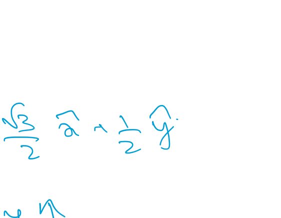
$$|\vec{B}| = \frac{4}{3\times10^8}$$

$$\vec{B} = \frac{4}{3\times10^8}\exp(i(5\times10^6 t - kz))\,\hat{y}$$

Q4)

(a) $$\vec{k} = \sqrt{3}\,\hat{x} + \hat{y}$$

Unit vector along $\vec{k} = \frac{\sqrt{3}}{2}\hat{x} + \frac{1}{2}\hat{y}$



$$\theta = \tan^{-1}\left(\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) = 30^\circ$$

The direction of propagation of wave is 30° with respect to x axis

(b) $$k = \sqrt{k_x^2 + k_y^2} = \frac{\omega}{c}$$

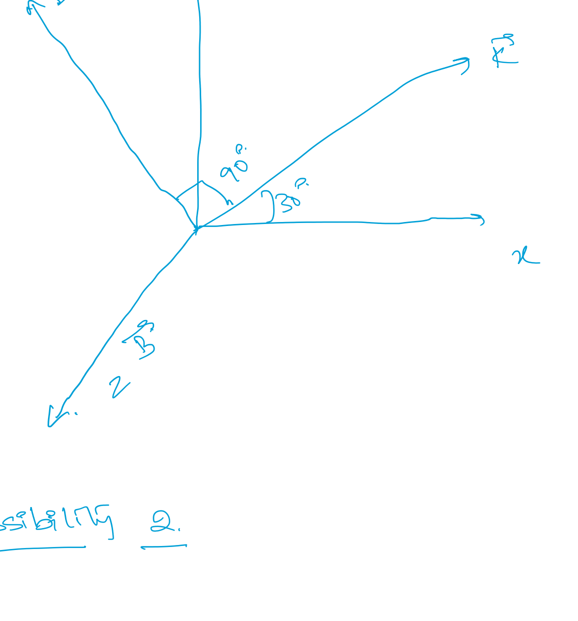
$$\sqrt{3+1} = \frac{\omega}{3\times10^8}$$

$$\omega = 6\times10^8 \text{ rad/sec.}$$

(c) We know that $\vec{E}, \vec{B}, \vec{k}$ are mutually perpendicular to each other and related with

$$\vec{k} = \vec{E}\times\vec{B}$$

possibility 1



possibility 2