Bode Plots → semilog

→ Draw magnitude + phase response of

$$H(s) = \frac{1}{1 + s/a} \qquad a = \text{real constant}$$

(i) $s = j\omega$

$$H(j\omega) = \frac{1}{1 + j(\omega/a)}$$ → complex function of ω

Magnitude Response Phase Response

(ii) $$|H(\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{a}\right)^2}}, \quad \angle H(\omega) = 0^\circ - \tan^{-1}\left(\frac{\omega}{a}\right)$$

$$[|H(\omega)|] = 20\log_{10}\left(\left|\frac{1}{\sqrt{1 + \omega^2/a^2}}\right|\right)$$

ω	$\lvert H(\omega)\rvert$	$[\lvert H(\omega)\rvert]$	$\angle H(\omega)$
$\omega = a/100$	1	0	$\sim 0^\circ$
$\omega = a/10$	1	0	-5.7°
$\omega = a$	$1/\sqrt{2}$	-3	-45°
$\omega = 10a$	0.1	-20	-85°
$\omega = 100a$	0.01	-40	-90°

log scales

1. $\omega = a/100$

$$|H(\omega)| = \frac{1}{\sqrt{1 + (\omega/a)^2}} = \frac{1}{\sqrt{1 + \left(\frac{a/100}{a}\right)^2}} = \frac{1}{\sqrt{1 + \left(\frac{1}{100}\right)^2}}$$

$$= \frac{1}{\sqrt{1 + 0.0001}} \approx 1$$

$$[|H(\omega)|] = 20\log_{10}(1) = 20\log_{10}(10^0) = 0$$

2. $\omega = a/10$

$$|H(\omega)| = \left| \frac{1}{\sqrt{1 + \left(\frac{\omega}{a}\right)^2}} \right| = \frac{1}{\sqrt{1 + \left(\frac{1}{10}\right)^2}} \approx 1$$

$$[|H(\omega)|] = 20\log_{10}(1) = 0$$

$$\angle H(\omega) = -\tan^{-1}\left(\frac{1}{10}\right) = -5.7^\circ$$

3. $\boxed{\omega = a} \rightarrow$ -3 dB frequency

$$|H(\omega)| = \frac{1}{\sqrt{1+1}} = \frac{1}{\sqrt{2}} = 0.707$$

$$[|H(\omega)|] = 20\log_{10}(0.707) = \boxed{-3}$$

$$\angle H(\omega) = -\tan^{-1}\left(\frac{\omega}{a}\right) = -\tan^{-1}(1) = -45^\circ$$

4. $\omega = 10a$

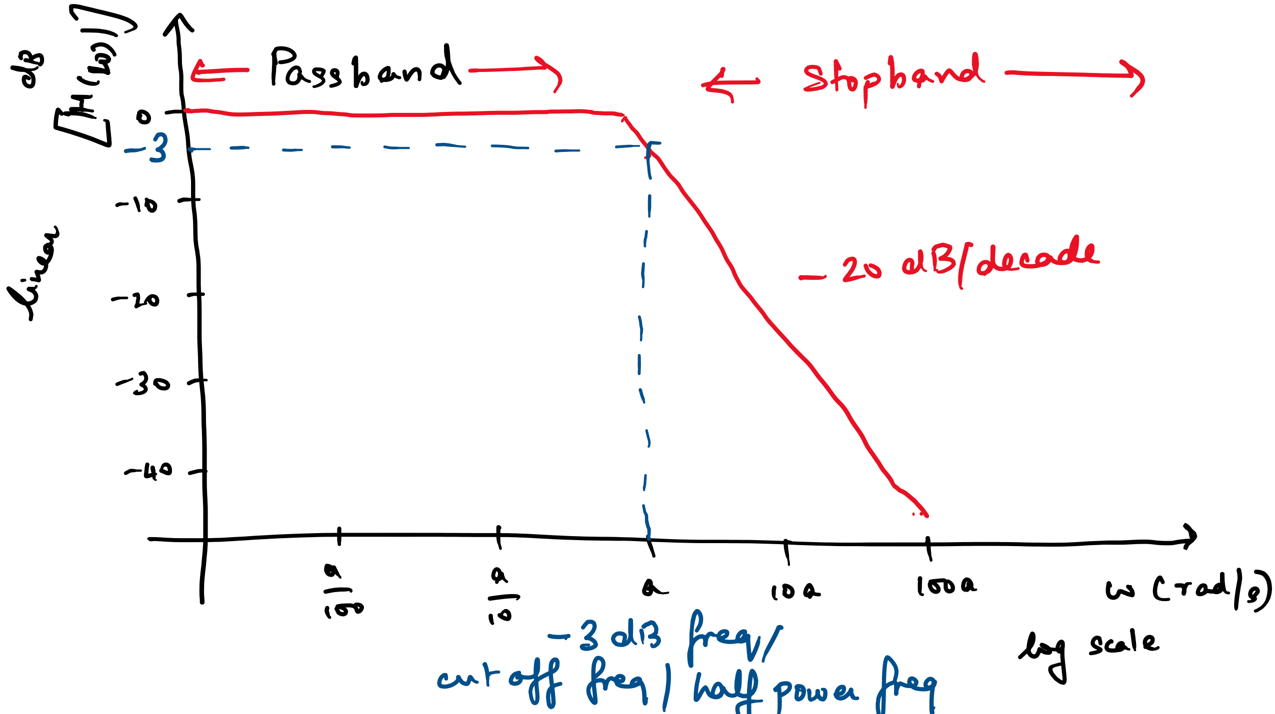
$$|H(\omega)| = \frac{1}{\sqrt{1 + (10)^2}} \approx \frac{1}{10} = 0.1$$

$$[|H(\omega)|] = 20\log_{10}(10^{-1}) = -20$$

$$\angle H(\omega) = -\tan^{-1}(10) = -85^\circ$$

5. $\omega = 100a$

Please work it out



$$H(s) = \frac{S_{out}(s)}{S_{in}(s)}$$

$$20 \log_{10} (10^{-1}) = 20 \times -1 = -20 \text{ dB}$$

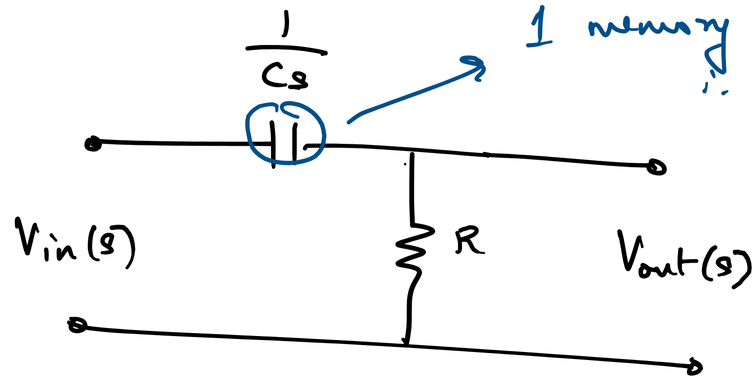
Passive First order low pass filter

$$\left| \frac{V_{out}(a)}{V_{in}(a)} \right| = \frac{1}{\sqrt{2}}$$

$$\left| \frac{V_{out}(a)}{V_{in}(a)} \right|^2 = \frac{1}{2}$$

$$\frac{P_{out}(a)}{P_{in}(a)} = \frac{1}{2}$$

$\rightarrow$



1 memory element

∴ First order

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} =$$

$$V_{out}(s) = V_{in}\left[\frac{R}{R + \frac{1}{Cs}}\right] \Rightarrow \frac{V_{out}}{V_{in}} = \frac{R}{R + \frac{1}{Cs}}$$

$$= \frac{RCs}{RCs + 1} = \frac{s}{s + \frac{1}{RC}}$$

$\rightarrow$ First order

$$H(s) = \frac{s}{s + \frac{1}{RC}} \Rightarrow H(\omega) = \frac{j\omega}{j\omega + \frac{1}{RC}}$$

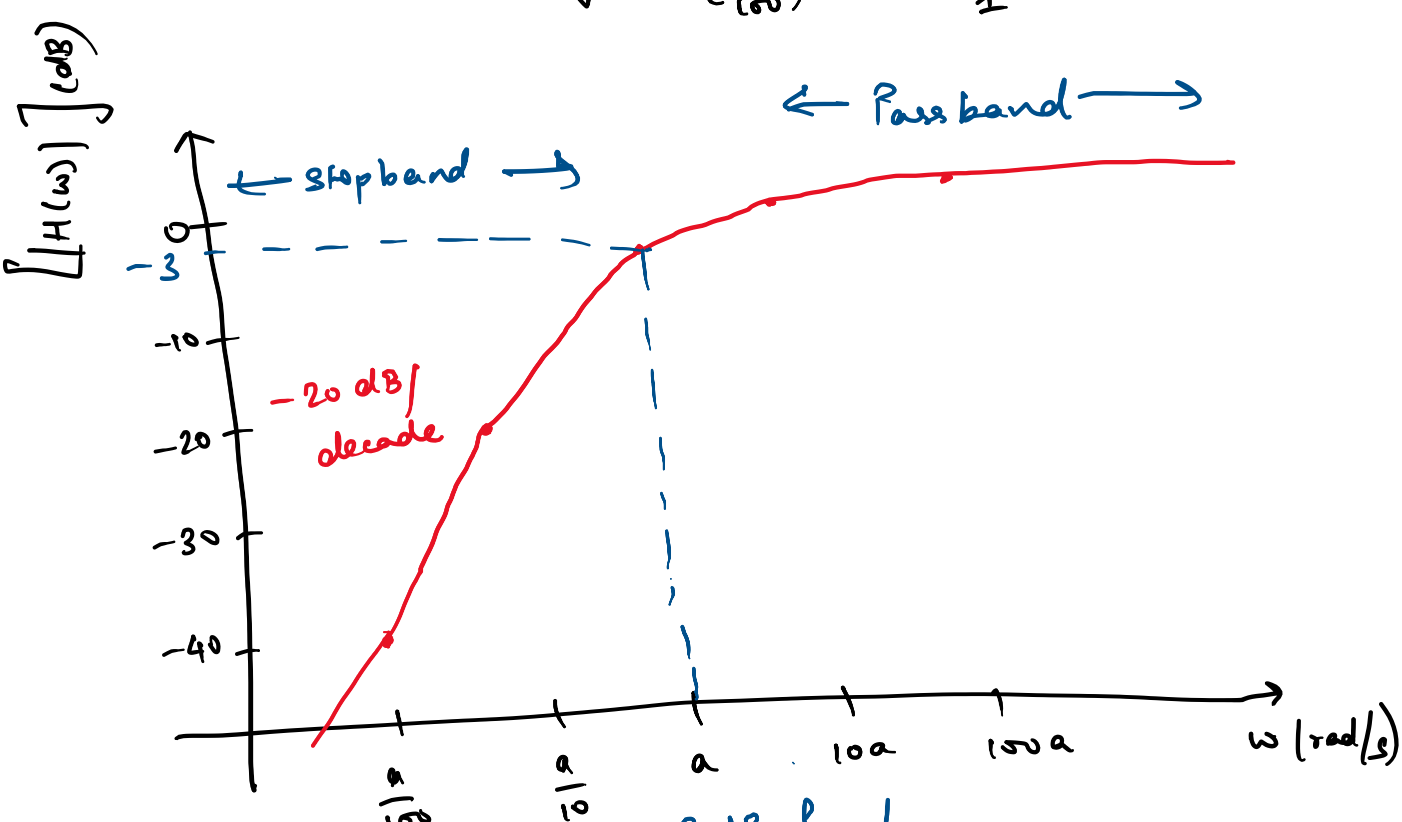
$$|H(\omega)| = \frac{\omega}{\sqrt{\omega^2 + \left(\frac{1}{RC}\right)^2}} = \frac{\omega}{\sqrt{\omega^2 + a^2}} \quad \left(\text{where } a = \frac{1}{RC}\right)$$

$$= \frac{\omega/a}{\sqrt{1 + \left(\frac{\omega}{a}\right)^2}} \Rightarrow \left[|H(\omega)|\right] = 20\log_{10}\left[\frac{\omega/a}{\sqrt{1 + \left(\frac{\omega}{a}\right)^2}}\right]$$

$$\angle H(\omega) = 90^\circ - \tan^{-1}\left(\frac{\omega}{a}\right)$$

ω	$\lvert H(\omega)\rvert$	$\lfloor \lvert H(\omega)\rvert \rceil$	$\angle H(\omega)$
$\frac{a}{100}$	0.01	-40	90°
$\frac{a}{10}$	0.1	-20	85°
a	$\frac{1}{\sqrt{2}}$	-3	45°
$10a$	1	-0	5°
$100a$	1	0	0°

$$H\left(\frac{a}{100}\right) = \frac{1/100}{\sqrt{1+\left(\frac{1}{100}\right)^2}} \simeq \frac{\frac{1}{100}}{1} = 0.01$$



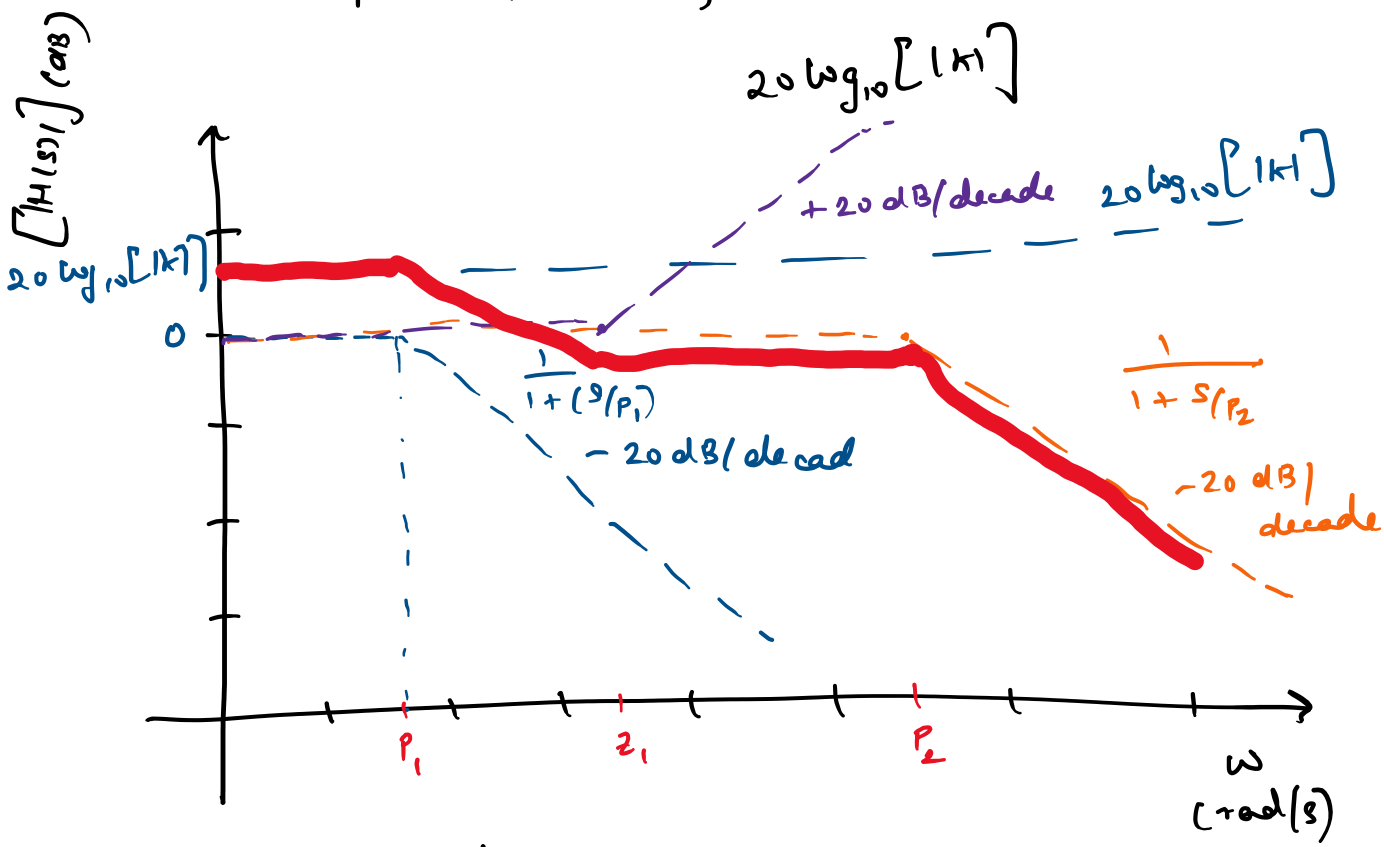
-3 dB freq/ cutoff / $\frac{1}{2}$ power freq

Passive First order high pass filter

(Bonus) Bode Plots with Asymptotic Method

$$H(s) = \frac{k\left(1+\frac{s}{z_1}\right)}{\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)}$$

$p_1 < z_1 < p_2$, $k > 1$



Active 2nd order somewhat lpf.

$\rightarrow \quad H(s) = \dfrac{0.1\left(1+\frac{s}{10}\right)}{\left(1+\frac{s}{1000}\right)\left(1+\frac{s}{10000}\right)}$

$20\log_{10}(0.1) = [-20]$