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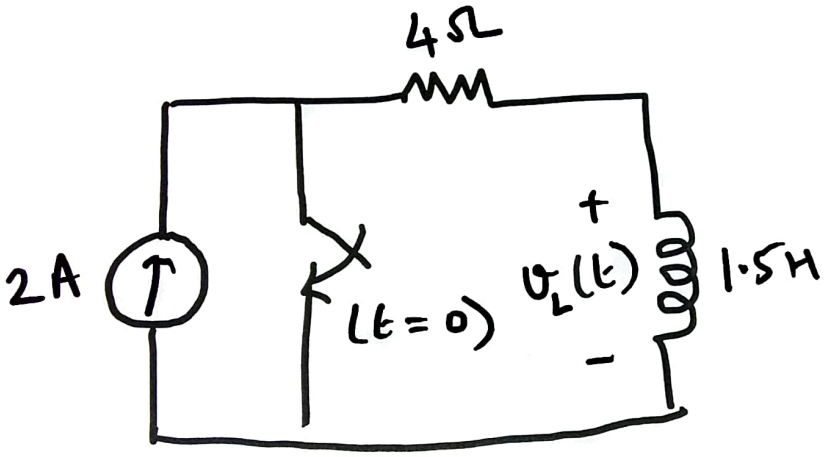
**ECE 215 – Circuit Theory and Devices**
**Quiz 4 (20 marks)**

1. (10 marks) Given the following function,

$$F(s) = \frac{3(s-1)}{(2s^2 + 24s + 70)(s+5)}$$

   (a) Find $f(t)$
   (b) Plot the poles and zeros on the complex s plane
   (c) Comment on the stability

2. (10 marks) For the following circuit, find $v_L(t)$ at t>0



1) (a)

$$F(s) = \frac{3(s-1)}{(2s^2+24s+70)(s+5)}$$

$$= \frac{3(s-1)}{2(s^2+12s+35)(s+5)} = \frac{3(s-1)}{2(s+5)(s+7)(s+5)}$$

$$= \frac{3/2\,(s-1)}{(s+5)^2(s+7)}$$

$$= \frac{1.5(s-1)}{(s+5)^2(s+7)} = \frac{A}{s+5} + \frac{B}{(s+5)^2} + \frac{C}{(s+7)} \quad -(6)$$

$$\Rightarrow \left(\frac{A}{s+5} + \frac{B}{(s+5)^2} + \frac{C}{s+7}\right) \times (s+5)^2(s+7)$$

$$\Rightarrow A(s+5)(s+7) + B(s+7) + C(s+5)^2$$

$$= A(s^2+12s+35) + Bs + 7B + Cs^2 + 10sC + 25C$$

$$= As^2 + 12As + 35A + Bs + 7B + Cs^2 + 10Cs + 25C$$

Compare the coefficients:-

$$\Rightarrow A + C = 0 \longrightarrow A = -C \quad -(1)$$

$$12A + B + 10C = 1.5 \quad -(2)$$

$$35A + 7B + 25C = -1.5 \quad -(3)$$

Substitute (1) in (2) and (3) :-

$$B - 2C = 1.5 \quad -(4)$$

$$7B - 10C = -1.5 \quad -(5)$$

Multiply (4) by 5 :-

$$\Rightarrow \quad 5B - 10C = 7.5$$
$$7B - 10C = -1.5$$
$$(-) \quad (+) \quad (+)$$
$$-2B = 9$$
$$B = -9/2 = \boxed{-4.5}$$

Put value of B in (5) :-

$$\Rightarrow \quad 7 \times (-4.5) - 10C = -1.5$$
$$C = \frac{7(-4.5) + 1.5}{10}$$
$$= \frac{-30}{10} = \boxed{-3}$$
$$A = -C \Rightarrow \boxed{A = 3}$$

Put all the values in (6)

$$\Rightarrow \quad \frac{3}{s+5} - \frac{4.5}{(s+5)^2} - \frac{3}{(s+1)}$$

$$\Rightarrow \quad 3e^{-5t}u(t) - 4.5te^{-5t}u(t) - 3e^{-1t}u(t)$$

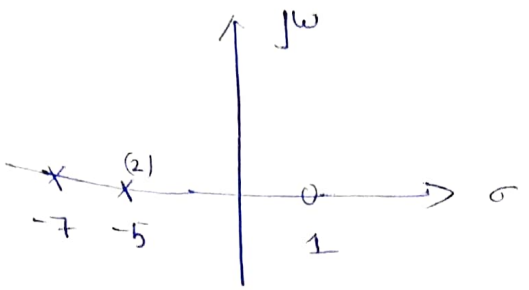
(b) Zeros :- $1.5s - 1.5 = 0$

$$\Rightarrow s = \frac{1.5}{1.5} = 1$$

Poles :- $(s+5)^2 = 0$

$$s = -5, -5$$

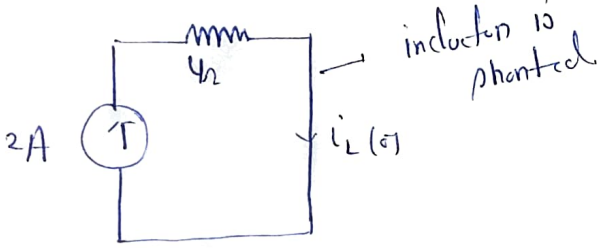
$$s+1 = 0 \Rightarrow s = -1$$




(c) The system is stable.

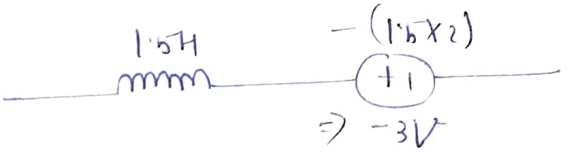
As all the pole lies in left hand side of s plane.

Soln 2) for $t<0$ :-

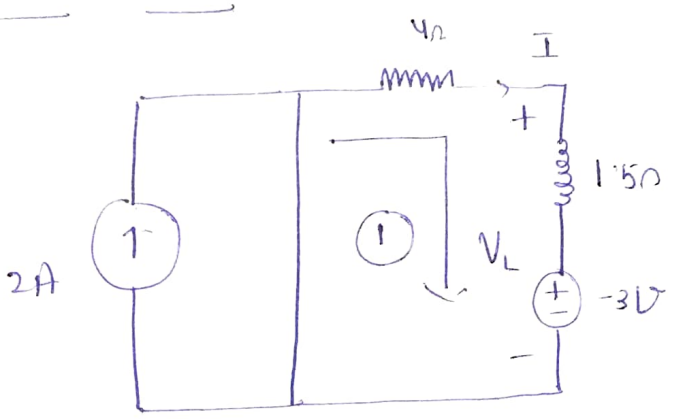



$i_L(0^-) = 2A$ (initial condition)

Hence, inductor will be replaced by :-




Circuit at $t>0$ :-



Apply KVL in loop ①

$$I\left(4+1.5s\right)+(-3)=0$$

$$I=\frac{3}{4+1.5s}=\frac{3/1.5}{s+\frac{4}{3}\times 2}$$

$$=\frac{2}{s+8/3}$$

$$V_L(s)=1.5sI+3$$

$$=3+\frac{2\times 1.5s}{s+8/s}$$

$$=3+\frac{3s}{s+8/3}$$

$$=3+\frac{3\left(s+\frac{8}{3}-\frac{8}{3}\right)}{s+8/3}$$

$$\Rightarrow 3+3-\frac{8}{s+8/3}$$

$$\Rightarrow 6-8/s+8/3$$

$$\boxed{V_L(t)\Rightarrow 6\delta(t)-8e^{-8/3t}u(t)}$$