ECE250: Signals and Systems
Practice Sheet 3

1. (CO2) Let $x(t) = u(t-3) - u(t-5)$ and $h(t) = e^{-3t}u(t)$.

   (a) Compute $y(t) = x(t) * h(t)$.
   (b) Compute $g(t) = (\frac{dx(t)}{dt}) * h(t)$.
   (c) How is g(t) relaled to y(t)?

2. (CO2) For each of the following input-output relationships, determine whether the corresponding system is linear, time-invariant, or both.
   (a) $y(t) = t^2 x(t-1)$
   (b) $y(t) = x(t^2)$
   (c) $y[n] = x^2[n-2]$
   (d) $y[n] = x[n+1] - x[n-1]$

3. (CO2) Consider a system with input x[n] and output y[n]. This system is obtained through a series interconnection of a system S_1 followed by a system S_2. The input-output relationships for S_1 and S_2 are

   $S_1 : \ y_1[n] = 2x_1[n] + 4x_1[n-1],$

   $S_2 : \ y_2[n] = x_2[n-2] + 0.5x_2[n-3]$

   where $x_1[n]$ and $x_2[n]$ denote inpµt signals.

   (a) Determine the input-output relationship for system S.
   (b) Does the input-output relationship of system S change if the order in which S_1 and S_2 are connected in series are reversed?

4. (CO2) Consider an input $x[n]$ and a unit impulse response $h[n]$ given by

   $x[n] = \left(\frac{1}{2}\right)^{n-2} u[n-2],$
   $h[n] = u[n+2]$

   Determine and plot the output y[n]=x[n]*h[n]

5. (CO2) Consider the cascade of the following two systems S_1 & S_2, as depicted in Figure 1:

   S_1 : Causal LTI

   $w[n] = \frac{1}{2}w[n-1] + x[n]$

   S_2 : Causal LTI

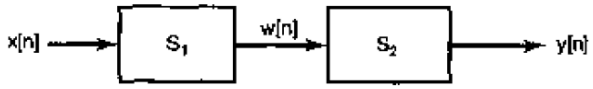

Figure 1: System for Problem 5

$y[n] = \alpha y[n-1] + \beta w[n]$

The difference equation relating x[n] and y[n] is:

$y[n] = -\frac{1}{8}y[n-2] + \frac{3}{4}y[n-1] + x[n]$

Determine the value of α and β.

6. (CO2) Let x(t) be a continuous time signal $x(t) = \delta(t)+\delta(t-1)+\delta(t-2)$. Signal x (t) is applied as input to a system whose impulse response is $h(t) = 2u(t)-u(t-1)-u(t-2)$. The output of the system is y(t), then find the value of y(t) at t = 2.

7. (CO2) Which of the following statements is correct for the given system?

$y[n] = x^2[n] + \frac{1}{x^2[n-1]}$

   (a) The given system is linear, non-causal, and shift variant.
   (b) The system is non-linear, causal, and shift-invariant.
   (c) The system is non-linear, causal, and shift-variant.
   (d) The system is linear, non-causal, and shift-invariant.

8. (CO2) A discrete-time system has an impulse response

   (a) $h[n] = a^n u[n+2]$
   (b) $h[n] = n\cos(\frac{\pi}{4}n)u[n]$

   Is this system BIBO stable, causal, and memoryless?

9. (CO2) Consider a discrete-time system S_1 with impulse response $h[n] = (\frac{1}{5})^n u[n]$.

   (a) Find the integer A such that $h[n] - Ah[n-1] = \delta[n]$.

   (b) Using the result from part (a), determine the impulse response g[n] of an LTI system S_2 which is the inverse system of S_1.

10. Consider the cascade of two LTI systems as in below Fig. 2 , where

$$h_1[n] = \sin 8n \tag{1}$$

and

$$h_2[n] = a^n u[n], |a| < 1 \tag{2}$$

and where the input is

$$x[n] = \delta[n] - a\delta[n-1] \tag{3}$$

Determine the output y[n].

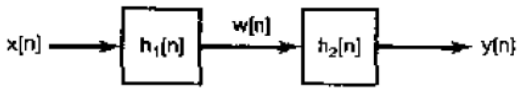

Figure 2: System for Problem 10