QUIZ-8 SOLUTION

SOL(1):

Given that –

$$x(t) = 1 + \sin(\omega_0 t) + 2\cos(\omega_0 t) + \cos\left(2\omega_0 t + \frac{\pi}{4}\right)$$
$$\rightleftharpoons X(j\omega)$$

As we know that – $1 \rightleftharpoons 2\pi\cdot\delta(\omega)$ —— (1)

(1 POINT)

$\therefore e^{j\omega_0 t} \rightleftharpoons 2\pi\,\delta(\omega-\omega_0)$ {By Frequency shifting —— (2)

$\therefore e^{-j\omega_0 t} \rightleftharpoons 2\pi\,\delta(\omega+\omega_0)$ {By Frequency shifting —— (3)

Let $y(t) = \sin(\omega_0 t) \rightleftharpoons Y(j\omega)$

$$= \left(\frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j}\right) = \frac{1}{2j}\left(e^{j\omega_0 t} - e^{-j\omega_0 t}\right)$$

By using eq^n(2) & (3) & Property of Linearity,

$$\therefore Y(j\omega) = \frac{1}{2j}\left\{2\pi\,\delta(\omega-\omega_0) - 2\pi\,\delta(\omega+\omega_0)\right\}$$

$$= \frac{\pi}{j}\left\{\delta(\omega-\omega_0) - \delta(\omega+\omega_0)\right\}$$

$$= \pi j\left\{\delta(\omega+\omega_0) - \delta(\omega-\omega_0)\right\} \quad \text{—— (4)}$$

(1 POINT)

Let $g(t) = 2\cos(\omega_0 t) \rightleftharpoons G(j\omega)$

$$= 2\left(\frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}\right) = \left(e^{j\omega_0 t} + e^{-j\omega_0 t}\right)$$

By using eq^n(2), (3) and Linearity property,

$$\therefore G(j\omega) = \left\{2\pi\,\delta(\omega-\omega_0) + 2\pi\,\delta(\omega+\omega_0)\right\}$$

$$= 2\pi\left\{\delta(\omega+\omega_0) + \delta(\omega-\omega_0)\right\} \quad \text{—— (5)}$$

(1 POINT)

Let $z(t) = \cos\left(2\omega_0 t + \frac{\pi}{4}\right) \Longleftrightarrow Z(j\omega)$

$z(t) = \cos(2\omega_0 t)\cdot\cos\left(\frac{\pi}{4}\right) - \sin(2\omega_0 t)\cdot\sin\left(\frac{\pi}{4}\right)$

$z(t) = \frac{1}{\sqrt{2}}\left\{\cos(2\omega_0 t) - \sin(2\omega_0 t)\right\}$

$\therefore Z(j\omega) = \frac{1}{\sqrt{2}}\Big\{\left[\pi\delta(\omega+2\omega_0) + \pi\delta(\omega-2\omega_0)\right]$

$\qquad - \left[\pi j\delta(\omega+2\omega_0) - \pi j\delta(\omega-2\omega_0)\right]\Big\}$

$\{$By using eq^n (4), (5)$\}$

$$= \frac{\pi}{\sqrt{2}}\left\{(1-j)\,\delta(\omega+2\omega_0) + (1+j)\,\delta(\omega-2\omega_0)\right\} \quad \text{—— (6)}$$

(1 POINT)

Now, By using eq^n (1), eq^n (4), eq^n (5) & eq^n (6), we get —

$\therefore X(j\omega) = 2\pi\delta(\omega) + Y(j\omega) + G(j\omega) + Z(j\omega)$

$$= \Big[2\pi\delta(\omega) + \pi j\left\{\delta(\omega+\omega_0) - \delta(\omega-\omega_0)\right\}$$

$$+ 2\pi\left\{\delta(\omega+\omega_0) - \delta(\omega-\omega_0)\right\}$$

$$+ \frac{\pi}{\sqrt{2}}\left\{(1-j)\,\delta(\omega+2\omega_0) + (1+j)\,\delta(\omega-2\omega_0)\right\}\Big]$$

—— (7)

(1 POINT)

SOL(2): Given $x[n] = (n-1)\left(\frac{1}{3}\right)^{|n|} \longleftrightarrow X(e^{j\omega})$

Let, $y[n] = \left(\frac{1}{3}\right)^{|n|} \longleftrightarrow Y(e^{j\omega})$

$$\therefore Y(e^{j\omega}) = \sum_{n=-\infty}^{\infty} y[n] \cdot e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} \left(\frac{1}{3}\right)^{|n|} \cdot e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{-1} \left(\frac{1}{3}\right)^{-n} \cdot e^{-j\omega n} + \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^{n} \cdot e^{-j\omega n} + 1 - 1$$

$$= \sum_{n=-\infty}^{0} \left(\frac{1}{3} e^{j\omega}\right)^{-n} + \sum_{n=0}^{\infty} \left(\frac{1}{3} \cdot e^{-j\omega}\right)^{n} - 1$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{3} e^{j\omega}\right)^{n} + \sum_{n=0}^{\infty} \left(\frac{1}{3} e^{-j\omega}\right)^{n} - 1$$

$$= \frac{1}{1 - \frac{1}{3} \cdot e^{j\omega}} + \frac{1}{1 - \frac{1}{3} \cdot e^{-j\omega}} - 1$$

$$= \frac{1 - (1/3)^2}{1 - \frac{2}{3} \cdot \cos\omega + \left(\frac{1}{3}\right)^2} = \frac{8}{9 - 6\cos\omega + 1}$$

$$= \left(\frac{4}{5 - 3 \cdot \cos\omega}\right) \text{ —— (1) } \quad \text{(2 POINTS)}$$

Let $z[n] = n \cdot \left(\frac{1}{3}\right)^{|n|} = n \cdot y[n] \rightleftharpoons Z(e^{j\omega})$

By using the property— Differentiation in Frequency

$$\therefore Z(e^{j\omega}) = j \cdot \frac{d}{d\omega} Y(e^{j\omega}) = j \frac{d}{d\omega} \left\{ \frac{4}{5 - 3 \cdot \cos\omega} \right\}$$

$$= -j \frac{12 \sin\omega}{(5 - 3\cos\omega)^2} \text{ —— (2) } \quad \text{(2 POINTS)}$$

Now, $x[n] = (n-1)\left(\frac{1}{3}\right)^{|n|}$

$x[n] = n\left(\frac{1}{3}\right)^{|n|} - \left(\frac{1}{3}\right)^{|n|}$

$x[n] = -y[n] + z[n]$

$\therefore\ X(e^{j\omega}) = -Y(e^{j\omega}) + Z(e^{j\omega})$

{By using the property of Linearity

$$X(e^{j\omega}) = \left\{ j\frac{(-12\sin\omega)}{(5-3\cos\omega)^2} - \frac{4}{(5-3\cos\omega)} \right\}$$

{By using $eq^n(1)$ & $eq^n(2)$

(1 POINTS)