ECE250: Signals and Systems
Practice Sheet 7

1. (CO5) Consider the rectangular signal

$$x[n] = \begin{cases} 1, & 0 \leq n \leq 5 \\ 0, & otherwise \end{cases}$$

Let

$$g[n] = x[n] - x[n-1]$$

(a) Find the signal $g[n]$ and directly evaluate its $z - transform$.
(b) Noting that

$$x[n] = \sum_{k=-\infty}^{n} g[k]$$

Determine the $z - transform$ of $x[n]$.

2. (CO5) Let $x[n]$ be an absolutely summable signal with rational $z - transform\ X(z)$. If X(z) is known to be have a pole at $z = 1/2$, could $x[n]$ be

(a) a finite-duration signal ?
(b) a left-sided signal ?
(c) a right-sided signal ?
(d) a two-sided signal ?

3. (CO5) Consider the signal

$$x[n] = \begin{cases} (1/3)^n cos(\frac{\pi}{4}n), & n \leq 0 \\ 0, & n > 0 \end{cases}$$

Determine the poles and ROC for $X(z)$.

4. (CO4) A signal $x[n]$ with Fourier transform $X(e^{j\omega})$ has the property that

$$(x[n] \sum_{k=-\infty}^{\infty} \delta[n-3k]) * (\frac{sin \frac{\pi}{3} n}{\frac{\pi}{3} n}) = x[n]$$

For what values of ω is it guaranteed that $X(e^{j\omega}) = 0$?

5. (CO3) A signal $x(t)$ with Fourier transform $X(j\omega)$ undergoes impulse-train sampling to generate

$$x_p(t) = \sum_{n=-\infty}^{\infty} x(nT)\delta(t - nT)$$

where $T = 10^{-4}$. For each of the following sets of constraints on $x(t)$ and/or $X(j\omega)$, does the sampling theorem guarantee that $x(t)$ can be recovered exactly from $x_p(t)$?

(a) $X(j\omega) = 0$ for $|\omega| > 5000\pi$
(b) $X(j\omega) * X(j\omega) = 0$ for $|\omega| > 15000\pi$

6. (CO3,CO4) Consider a real, odd and periodic signal $x(t)$ whose Fourier series representation may be expressed as

$$x(t) = \sum_{k=0}^{5} (\tfrac{1}{2})^k sin(k\pi t)$$

Let $\hat{x}(t)$ represent the signal obtained by performing impulse-train sampling on $x(t)$ using a sampling period $T = 0.2$.

(a) Does aliasing occur when this impulse-train sampling is performed on $x(t)$?

(b) If $\hat{x}(t)$ is passed through an ideal low-pass filter with cutoff frequency π/t and pass band gain T, determine the Fourier series representation of the output signal g(t).