Quiz 1: Fields & Waves (ECE230), Winter 2024

Duration: 1hr 10min, Total: 25 points (Attempt all questions)

Feb 13, 2024

**Vector quantities must be represented with an overhead arrow, vector calculus operators must be written correctly, dot and cross products must be written properly. In case of any of these mistake -1 will be awarded in each question with such mistake.

NO credit will be given to the answers that do not accompany proper explanation.

Any case of copying, cheating will be dealt as per institute guidelines.

Q1. For a vector field $\vec{A}$, Prove that $\vec{\nabla}.\left(\vec{\nabla}\times\vec{A}\right)=0$.

4 points

Q2. Consider a hollow metal sphere of radius R. Some positive charge of amount Q is put at certain point on the surface of the sphere. Calculate and plot the magnitude of the electric field as a function of r, where r is the distance from the center of the sphere.

$4+4=8$ points

Q3. Calculate gradient of the function $F(x,y,z)=e^{z}ln(x)ln(y)$.

2 points

Q5. Calculate curl of the function $\vec{A}(x,y,z)=\hat{x}yz+\hat{y}4xy+\hat{z}y$.

3 points

Q6. Consider an electrostatic potential $\phi=\frac{1}{x}+\frac{3}{y^2}-\frac{2}{z}$. Calculate:

(i) the electric field ($\vec{E}$)

(ii) the volume charge density ρ.

$2+2=4$ points

Q7. Choose correct option(s). If multiple options are correct, you need to choose all of them for full point (the questions carry 1point each).

a) Electrostatic field is: (i) conservative, (ii) irrotational, (iii) solenoidal, (iv) all of the previous options

b) A certain vector field has both zero divergence and curl. Which one could be a possible candidate: (i) Electrostatic field inside a uniformly charged dielectric sphere, (ii) Electrostatic field inside a charge-free dielectric sphere, (iii) Electrodynamic field inside a uniformly charged dielectric sphere, (iv) Electrodynamic field inside a charge-free dielectric sphere.

c) Consider a charge-free interface between two media with permittivities ϵ_1 and ϵ_2. E_{n1} and E_{n2} are the normal components of the electric fields and E_{t1} and E_{t2} are the tangential components at the interface: (i) $E_{t1}=E_{t2}$, (ii) $E_{n1}=E_{n2}$, (iii) $\epsilon_1 E_{n1}=\epsilon_2 E_{n2}$, (iv) $\epsilon_1 E_{t1}=\epsilon_2 E_{t2}$.

d) Which expression of divergence theorem is correct: (i) $\oint_V(\vec{\nabla}.\vec{E})\mathrm{dv}=\oint_S\vec{E}.\vec{dS}$, (ii) $\oint_V(\vec{\nabla}.\vec{E})\mathrm{dv}=\int_S\vec{E}.\vec{dS}$, (iii) $\int_V(\vec{\nabla}.\vec{E})\mathrm{dv}=\oint_S\vec{E}.\vec{dS}$, (iv) $\int_V(\vec{\nabla}.\vec{E})\mathrm{dv}=\int_S\vec{E}.\vec{dS}$

4 points

Mid-semester examination: Fields & Waves (ECE230), Winter 2024

Total: 20 points

Time: 1hr

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Q1. A certain material has a conductivity of $5 \times 10^6 s/m$ and a permittivity of $5F/m$. A total charge of $1C$ is distributed uniformly throughout the volume of a sphere (of radius of $5m$) made of this material. How much would be the surface charge density after $2\mu s$? ($1\mu s = 10^{-6}S$) [5 points]

Q2. An electron has a charge-to-mass ratio of $1.759 \times 10^{11} C/kg$. If the electron is moving with a speed of $3.518 \times 10^7 m/s$ perpendicular to a uniform magnetic field of 1 Tesla, what is the radius of the cyclotron motion? [4 points]

Q3. Consider an electrostatic potential $\phi(r) = \frac{k}{r^4}$ over free space. Calculate the energy density of the field. [3 points]

Q4. Prove that $\vec{\nabla} \times (\vec{\nabla}\phi) = 0$. [3 points]

Q5. Evaluate the following integrals:

(a) $\int_{-1}^{+1} (r^2 + 2)\, \vec{\nabla}.\left(\frac{\hat{r}}{r^2}\right) dv$

(b) $\int_{1}^{\infty} (r^2 + 2)\, \vec{\nabla}.\left(\frac{\hat{r}}{r^2}\right) dv$ [2.5 + 2.5 = 5 points]

$\int_{-1}^{1} (r^2+2)\, \vec{\nabla} \cdot \left(\frac{r}{r^2}\right) dv$

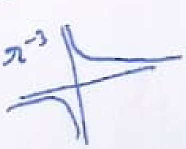
$\frac{-2}{r^3} \times r^2 \quad +2 \times \frac{-2}{r^3}$

$\phi(r) = \frac{k}{r^4}$ $\quad$ ka

$\left(\frac{-2}{r} - \frac{4}{r^3}\right)$

r^{-2}

$-2r^{-3}$

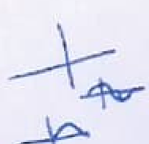


$\left[-2 \ln r + \frac{2}{r^2}\right]_1^{\infty}$

$\frac{-1}{r^3}$

$\left(\frac{1}{r} + \frac{2}{r^3}\right)_{-1}^{1}$

$(0 + 2) - [\infty + 0]$



$\ln r +$

$\ln |r|$

A

Quiz 2: Fields & Waves (ECE230), Winter 2024

Duration: 1hr 10 mins, Total: 20 points (Attempt all questions)

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Q1. Consider a very thin straight, wire of infinite length and oriented along z-direction carries a constant current I_0. Calculate the magnetic field.

[2 marks]

Q2. For a certain electromagnetic field, the scalar potential (ϕ) and vector potential (A) are given by:

$$\phi = (1/r^2)\exp(i\omega t)$$

$$\vec{A} = (\hat{r}/r)\exp(i\omega t)$$

Calculate the electric and magnetic fields.

$(4+4) = 8$ points

Q3. Under certain conditions, the relative permittivity of a metal can be written as: $\epsilon_r(\omega) = \left(1 - \frac{\omega_p^2}{\omega^2}\right)$. Plot $\epsilon_r(\omega)$ as a function of frequency. All the important points must be shown on the graph for full credit.

4 points

Q4. Consider the expression of the electric field of a uniform plane wave traveling in vacuum:

$\vec{E} = 4\hat{x}\exp\left[i\left(5\times10^6 t - kz\right)\right]$

(a) What is the direction of polarization of this EM wave?

(b) Which direction is the wave traveling?

(c) How much is the value of k?

(d) Write down the vector expression of the magnetic field associated with this wave.

$1+1+2+2 = 6$ points

End-sem: Fields & Waves (ECE230), Winter 2024

Total marks: 37, Time: 2hrs

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Q1. Derive the equation of continuity. What physical law does this equation represent? [2 + 1 = 3 points]

Q2. Consider a lossless transmission line has a distributed inductance of $227nH/m$ and distributed capacitance of $90.9pF/m$ and has been connected to a signal generator of frequency $14\pi \times 10^8$ Hz.

(a) How much is the characteristic impedance of the line?

(b) What would be the speed of the voltage wave propagating in this line?

(c) What would be the propagation constant of the wave?

(d) If the transmission line is terminated with a load $Z_L = 30\Omega$, how much would be voltage reflection coefficient at the load?

(e) Under the same condition as part (*d*), what is the VSWR?

[2 + 2 + 2 + 2 + 2 = 10 points]

Q3. Consider a uniform, x−polarized plane wave of frequency $3GHz$ is propagating along z−direction in a nonmagnetic medium ($\mu_0 = 4\pi \times 10^{-7} H/m$) with conductivity $7.2 \times 10^4 S/m$ and permittivity $21 \times 10^{-12} F/m$.

(a) Write down the effective permittivity of the medium.

(b) How much is the ratio $\frac{E}{H}$ for this wave?

(c) How much is the propagation constant of the EM wave?

[2 + 2 + 2 = 6 points]

Q4. Suppose, you are shining red light from a glass slab (refractive index= 1.5) to air. [4 points]

(a) Calculate the reflection coefficient at the interface.

(b) Does the electric field flips sign on reflection?

(c) How much is the transmission coefficient is?

(d) Calculate the SWR.

Q5. Choose correct options (if multiple options are correct you must choose all of them to get any credit)

(a) Consider an interface between two dielectric media with permeabilities μ_1 and μ_2. B_{n1} and B_{n2} are the normal components of the magnetic fields and B_{t1} and B_{t2} are the tangential components at the interface: (i) $B_{t1} = B_{t2}$, (ii) $\mu_1 B_{n1} = \mu B_{2n2}$, ($iii$) $H_{n1} = H_{n2}$, (iv) $H_{t1} = H_{t2}$. [1 point]

(b) Magnetostatic field is: (i) conservative, (ii) irrotational, (iii) solenoidal, (iv) all of the previous options. [1 point]

(c) Which expression of divergence theorem is correct: (i) $\oint_V (\vec{\nabla}.\vec{E})dv = \oint_S \vec{E}.d\vec{S}$, ($ii$) $\oint_V (\vec{\nabla}.\vec{E})dv = \int_S \vec{E}.d\vec{S}$, ($iii$) $\int_V (\vec{\nabla}.\vec{E})dv = \oint_S \vec{E}.d\vec{S}$, ($iv$) $\int_V (\vec{\nabla}.\vec{E})dv = \int_S \vec{E}.d\vec{S}$ [1 point]

(d) Consider a charge-free interface between two media with permittivities ϵ_1 and ϵ_2. E_{n1} and E_{n2} are the normal components of the electric fields and E_{t1} and E_{t2} are the tangential components at the interface: (i) $E_{t1} = E_{t2}$, (ii) $E_{n1} = E_{n2}$, (iii) $\epsilon_1 E_{n1} = \epsilon_2 E_{n2}$, ($iv$) $\epsilon_1 E_{t1} = \epsilon_2 E_{t2}$. [1 point]

Q6. A thin uniformly charged spherical shell of radius R carries a charge Q. Calculate:

(a) Energy density inside and outside the shell. [1 + 1 = 2 points]

(b) Total energy. [3 points]

Q7. The electrostatic potential for certain charge distribution is given by: $V = A\frac{e^{-\lambda r}}{r}$, where A and λ are constants. Calculate the electric field and volume charge density. [2 + 3 = 5 points]