Tutorial 3

Q1)

$$\frac{s}{s^2+7s+10}$$

$$= \frac{s}{(s+5)(s+2)}$$

$$= \frac{s}{5(1+s/5)\,2(1+s/2)}$$

$$= \frac{0.1(s)}{(1+s/5)(1+s/2)}$$

Put ; $s = j\omega$

$$\Rightarrow 0.1\left[\frac{j\omega}{[1+j(\omega/5)][1+j(\omega/2)]}\right]$$

Magnitude :- $20\log_{10}(0.1) + 20\log_{10}|j\omega| - 20\log_{10}|1+j\omega/5|$
$- 20\log_{10}|1+j\omega/2|$

$$\Rightarrow -20 + 20\log_{10}\omega - 20\log_{10}\left(\sqrt{1+\omega^2/25}\right) - 20\log_{10}\left(\sqrt{1+\omega^2/4}\right)$$

Angle :- $90° - \tan^{-1}(\omega/5) - \tan^{-1}(\omega/2)$

Now; at $\omega = 1$:-

Magnitude $\Rightarrow -20 + 20\log_{10}(1) - 20\log_{10}\sqrt{1+1/25} - 20\log_{10}\sqrt{1+1/4}$

$= -20 + 0 + 0 + 0$

$\Rightarrow \boxed{-20\,dB}$

Angle : $90^\circ - \tan^{-1}(1/5) - \tan^{-1}(1/2)$

$= 90^\circ - 11.30 - 26.56$

$= \underline{52.14^\circ}$

at $\underline{\omega = 10}$;

Magnitude $\Rightarrow -20 + 20\log_{10}(10) - 20\log_{10}\left(\sqrt{1+100/25}\right) - 20\log_{10}\left(\sqrt{1+100/4}\right)$

$= -20 + 20 - 6.96 - 13.97$

$= -20.93$ dB.

Angle : $90^\circ - \tan^{-1}(10/5) - \tan^{-1}(10/2)$

$= 90^\circ - \tan^{-1}(2) - \tan^{-1}(5)$

$= 90^\circ - 63.43 - 78.69$

$= \underline{-52.12^\circ}$

at $\underline{\omega = 100}$;

Magnitude $\Rightarrow -20 + 20\log_{10}(100) - 20\log_{10}\left(\sqrt{1+10^4/25}\right) - 20\log_{10}\left(\sqrt{1+10^4/4}\right)$

$= -20 + 40 - 20\log_{10}(20) - 20\log_{10}(50)$

$= 20 - 26.02 - 33.97$

$= -39.99$ dB $\approx \underline{-40}$ dB

Angle : $90^\circ - \tan^{-1}(100/5) - \tan^{-1}(100/2)$

$\Rightarrow 90^\circ - \tan^{-1}(20) - \tan^{-1}(50)$

$= 90^\circ - 87.13 - 88.85$

$= \underline{-85.98^\circ}$

$\omega = 1000$;

Magnitude ⇒ $-20 + 20\log_{10}(1000) - 20\log_{10}(\sqrt{1+10^6/25}) - 20\log_{10}(\sqrt{1+10^6/4})$

$= -20 + 60 - 20\log_{10}(200) - 20\log_{10}(500)$

$= 40 - 46.02 - 53.97$

$= -59.99 \simeq \underline{-60 \text{ dB}}$.

Angle ⇒

$90^\circ - \tan^{-1}(1000/5) - \tan^{-1}(1000/2)$

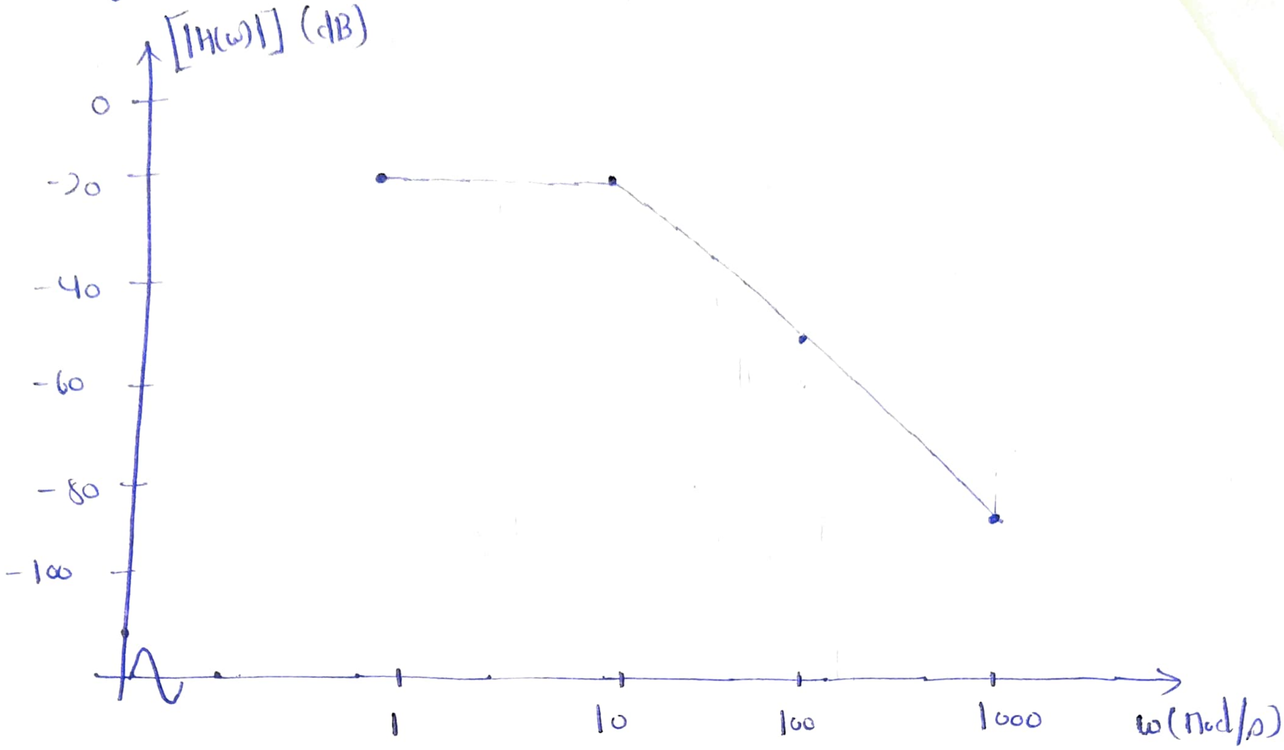
⇒ $90^\circ - 89.71 - 89.88$

$= -89.59^\circ \simeq \underline{-90^\circ}$

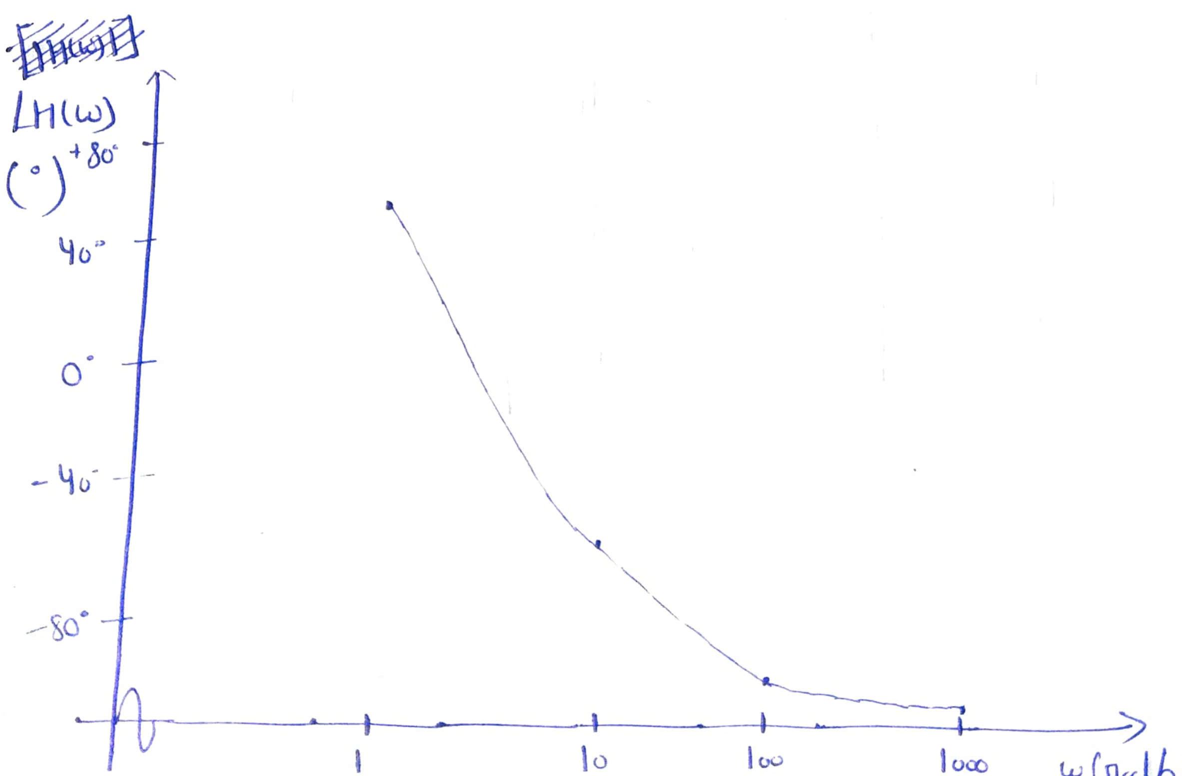
⇒ Table :-

ω	$\lvert H(\omega)\rvert$	$\angle H(\omega)$
1	−20 dB	52.14°
10	−20.93 dB	−52.12°
100	−40 dB	−85.90°
1000	−60 dB	−90°

Magnitude Response :-



Phase Response :-



$$\frac{S+1}{S(S+2)^2}$$ ← Q.2)

$$= \frac{S+1}{S(S+2)(S+2)}$$

$$= \frac{S+1}{4 \cdot S(1+S/2)(1+S/2)}$$

$$= 0.25\left[\frac{S+1}{S(1+S/2)(1+S/2)}\right]$$

$$= 0.25\left[\frac{j\omega+1}{j\omega(1+j\omega/2)(1+j\omega/2)}\right]$$

⇒ Magnitude :-

$$20\log_{10}(0.25) + 20\log_{10}\sqrt{1+\omega^2} - 20\log_{10}\omega - 20\log_{10}\left(\sqrt{1+\omega^2/4}\right) - 20\log_{10}\left(\sqrt{1+\omega^2/4}\right)$$

$$\Rightarrow 20\log_{10}(0.25) + 20\log_{10}\sqrt{1+\omega^2} - 20\log_{10}\omega - 2\left(20\log_{10}\sqrt{1+\omega^2/4}\right)$$

$$= -12.04 + 20\log_{10}\sqrt{1+\omega^2} - 20\log_{10}\omega - 40\log_{10}\sqrt{1+\omega^2/4}$$

Angle :- $0 + \tan^{-1}\omega - 90^\circ - 2\tan^{-1}(\omega/2)$

$$\Rightarrow \tan^{-1}\omega - 90^\circ - 2\tan^{-1}(\omega/2)$$

Now, at $\omega = 1$:-

magnitude $-12.04 + 20\log_{10}\sqrt{1+1} - 20\log_{10}(1) - 2\left(20\log_{10}\sqrt{1+1/4}\right)$

$$= -12.04 + 3.010 - 0 - (40 \times 0) \approx -9.04\,dB$$

angle :- $\tan^{-1}(1) - 90 - 2\tan^{-1}(0.5)$

$= 45° - 90° - 53.13°$

$= -98.13°$

at $\omega = 10$;

magnitude :-

$= 20\log_{10}(0.25) + 20\log_{10}\sqrt{100+1} - 20\log_{10}(10) - 40\log_{10}\left(\sqrt{1+100/4}\right)$

$= -12.04 + 20 - 20 - 56.59$

$= -68.63 \text{ dB}$

Angle :- $\tan^{-1}(10) - 90 - 2\tan^{-1}(5)$

$= 84.28 - 90 - 2 \times 78.69$

$= -163.38°$

at $\omega = 100$;

magnitude :-

$20\log_{10}(0.25) + 20\log_{10}\sqrt{10^4+1} - 20\log_{10}(10^2) - 40\log_{10}\left(\sqrt{1+10^4/4}\right)$

$= -12.04 + 40 - 40 - 67.95$

$= -79.99 \approx -80 \text{ dB}$

angle :- $\tan^{-1}(100) - 90° - 2\tan^{-1}(50)$

$= 89.42 - 90° - 177.78$

$= -178.28°$

$\omega = 1000$:

Magnitude:-

$-12.04 + 20\log_{10}\sqrt{10^6+1} - 20\log_{10} 10^3 - 40\log_{10}\left(\sqrt{10^6/4}+1\right)$

$= -12.04 + 60 - 60 - 107.95$

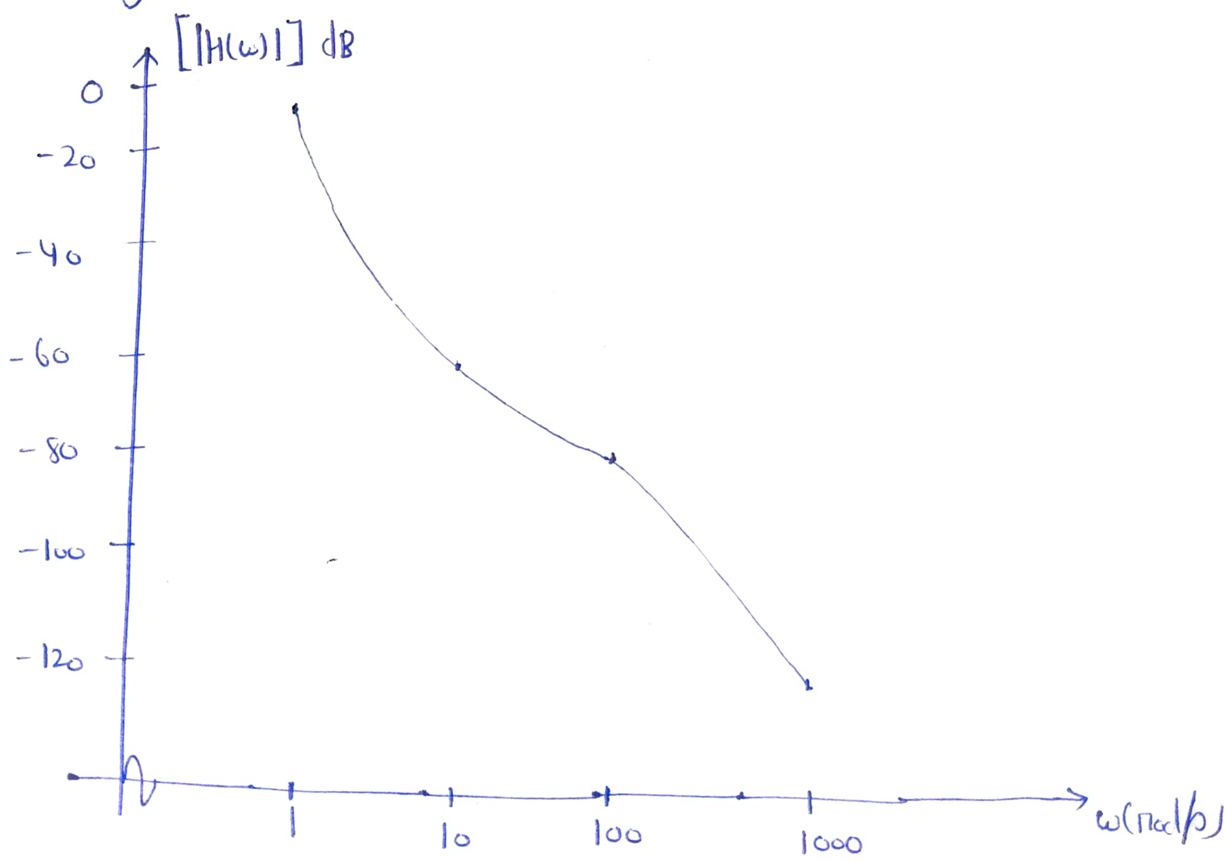
$= -199.99 \simeq -120\ dB$

Angle $= \tan^{-1}(1000) - 90 - 2\tan^{-1}(500)$

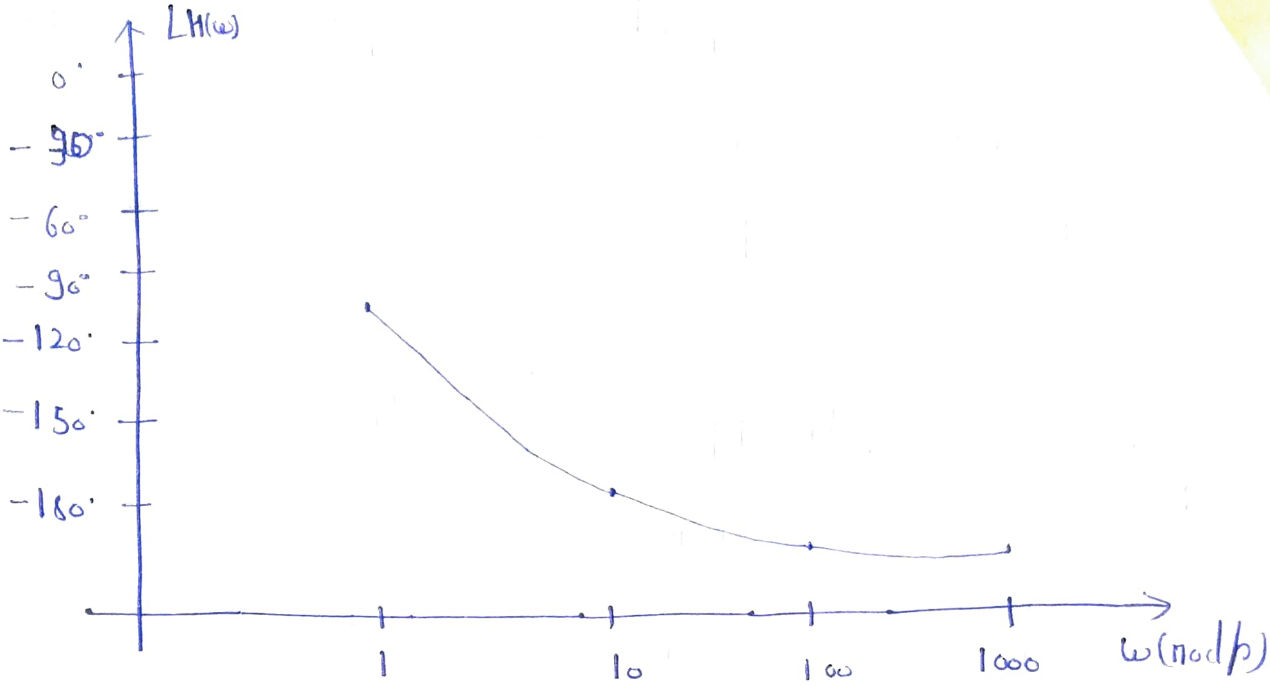
$= -179.83°$

ω	$[\lvert H(\omega)\rvert]$	$\angle H(\omega)$
1	-9.04 dB	$-98.13°$
10	-68.63 dB	$-163.38°$
100	-80 dB	$-178.28°$
1000	-120 dB	$-179.83°$

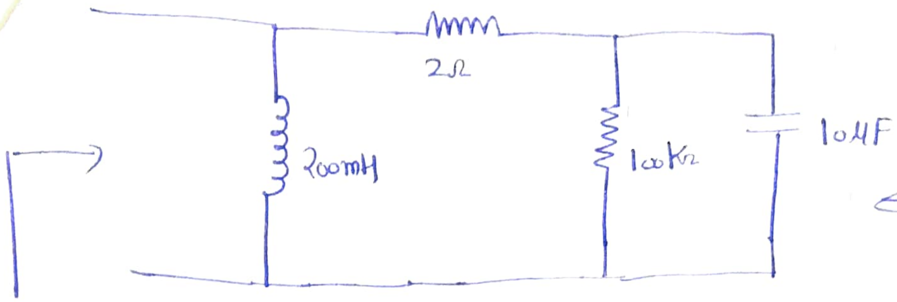
Magnitude response :-



Phase Response:



Q.3



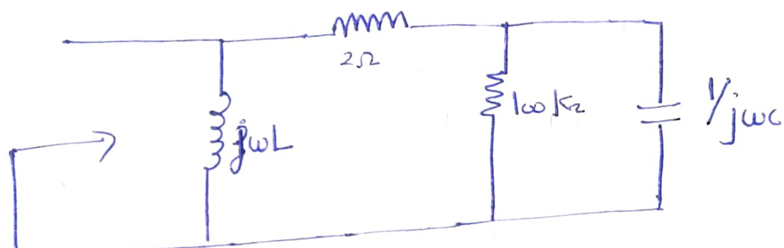
admittance

(i) find admittance ?

(ii) find ω when it will be in resonant frequency.

(iii) find ω when $|Y(\omega)| = \frac{1}{\sqrt{2}} |Y(\omega)|_{max}$.

Soln)



y(s)

$X_L = j\omega L$

$L = 200\text{ mH} = 200 \times 10^{-3}\text{ H}$

$X_L = j \times \omega \times 200 \times 10^{-3}$

$= 0.2 j\omega\ \Omega$

$C = 10\mu F = 10 \times 10^{-6}\text{ F}$

$X_c = \frac{1}{j\omega c} = \frac{1}{j \times \omega \times 10 \times 10^{-6}} = \frac{1}{j\omega \times 10^{-5}} = \frac{-j10^5}{\omega}\ \Omega$

Admittance = 1/impedance

$\text{Impedance} = (j0.2\omega) \,||\, \left(2 + \left(100\times10^3 \,||\, \frac{-j100\times10^3}{\omega}\right)\right)$

$= (j0.2\omega) \,||\, \left(2 + \left(\frac{100\times10^3 \times \frac{-j100\times10^3}{\omega}}{100\times10^3 + \frac{-j100\times10^3}{\omega}}\right)\right)$

$$\Rightarrow (j0.2\omega) \parallel \left(2 + \left(\frac{-j10^{10}}{100\times10^{3}\,\omega - j100\times10^{3}}\right)\right)$$

$$= j0.2\omega \parallel \left(\frac{200\times10^{3}\,\omega - j200\times10^{3} - j10^{10}}{100\times10^{3}\,\omega - j100\times10^{3}}\right)$$

$$= \frac{(j0.2\omega) \times \dfrac{200\times10^{3}\,\omega - j200\times10^{3} - j10^{10}}{100\times10^{3}\,\omega - j100\times10^{3}}}{(j0.2\omega) + \dfrac{200\times10^{3}\,\omega - j10^{10}}{100\times10^{3}\,\omega - j100\times10^{3}}}$$

$$= \frac{j40\times10^{3}\,\omega^{2} + 2\times10^{9}\,\omega}{j20\times10^{3}\,\omega^{2} + 20\times10^{3}\,\omega + 200\times10^{3}\,\omega - j10^{10}}$$

$$= \frac{j40\times10^{3}\,\omega^{2} + 2\times10^{9}\,\omega}{j20\times10^{3}\,\omega^{2} + 220\times10^{3}\,\omega - j10^{10}}$$

$$= \frac{2\times10^{9}\,\omega + j40\times10^{3}\,\omega^{2}}{220\times10^{3}\,\omega + j(20\times10^{3}\,\omega^{2} - 10^{10})}$$

now, admittance = 1/impedance

$$= \frac{220\times10^{3}\,\omega + j(20\times10^{3}\,\omega^{2} - 10^{10})}{2\times10^{9}\,\omega + j40\times10^{3}\,\omega^{2}}.$$

$\Rightarrow$ <u>Rationalise</u> it:-

$$\Rightarrow \frac{220\times10^{3}\,\omega + j(20\times10^{3}\,\omega^{2} - 10^{10})}{2\times10^{9}\,\omega + j(40\times10^{3}\,\omega^{2})} \times \frac{2\times10^{9}\,\omega - j40\times10^{3}\,\omega^{2}}{2\times10^{9}\,\omega - j40\times10^{3}\,\omega^{2}}$$

$$\frac{4\times10^{14} - j(8.8\times10^{9}\,\omega^3) + j(4\times10^{13}\,\omega^3 - 2\times10^{19}\,\omega) + (8\times10^{8}\,\omega^4 - 4\times10^{14}\,\omega^2)}{(2\times10^{19}\,\omega)^2 + (40\times10^{3}\,\omega^2)^2}$$

$=$ (Neglected)

$$\cancel{-j8.8\times10^{9}\,\omega^3} + j4\times10^{13}\,\omega^3 - 2\times10^{19}\,\omega = 0$$

(Becaus it is 10^9 and other term is 10^{13})

$$\Rightarrow \quad j4\times10^{13}\,\omega^3 - 2\times10^{19}\,\omega = 0$$

$$\Rightarrow \quad 4\times10^{13}\,\omega^2 = 2\times10^{19}$$

$$\Rightarrow \quad \omega^2 = \frac{2\times10^{6}}{4}$$

$$\Rightarrow \quad \omega = \sqrt{0.5\times10^{6}} = \underline{707.1 \text{ rad/s}} \quad \text{Ans.}$$