ECE250: Signals and Systems
Practice Sheet 5

1. (CO4)Suppose $g(t) = x(t)\cos t$ and the Fourier transform of $g(t)$ is

$$G(j\omega) = \begin{cases} 1 & for \quad |\omega| \leq 2 \\ 0 & for \quad otherwise \end{cases} \tag{1}$$

   (a) Determine $x(t)$.

   (b) Specify the Fourier transform $X_1(j\omega)$ of a signal $x_1(t)$ such that

$$g(t) = x_1(t)\cos\left(\frac{2}{3}t\right) \tag{2}$$

2. (CO4)A causal and stable LTI system S has the frequency response

$$H(j\omega) = \frac{j\omega + 4}{6 - \omega^2 + 5j\omega} \tag{3}$$

   (a) Determine a differential equation relating the input $x(t)$ and output $y(t)$ of S.

   (b) Determine the impulse response $h(t)$ of S.

   (c) What is the output of S when the input is

$$x(t) = e^{-4t}u(t) - te^{-4t}u(t)? \tag{4}$$

3. (CO4)Consider a discrete-time LTI system with impulse response

$$h[n] = \left[\left(\frac{1}{2}\right)^n \cos\left(\frac{\pi n}{2}\right)\right]u[n] \tag{5}$$

   Use Fourier transforms to determine the response to each of the following input signals:

   (a) $x[n] = (\frac{1}{2})^n u[n]$

   (b) $x[n] = \cos(\pi n/2)$

4. (CO4)Consider a causal LTI system described by the difference equation

$$y[n] + \frac{1}{2}y[n-1] = x[n] \tag{6}$$

   (a) Determine the frequency response $H(e^{j\omega})$ of this system

   (b) What is the response of the system to the following inputs?

      i. $x[n] = (1/2)^n u[n]$

      ii. $x[n] = \delta[n] + 1/2\delta[n-1]$

   (c) Find the response to the inputs with the following Fourier transforms:

i. $X(e^{j\omega}) = \frac{1-1/4e^{-j\omega}}{1+1/2e^{-j\omega}}$

ii. $X(e^{j\omega}) = 1 + 2e^{-3j\omega}$

5. (CO4)Compute the Fourier transform of the following signals:

   (a) $x[n] = 2^n sin(\frac{\pi}{4}n)u[-n]$

   (b) $x[n] = x[n-6]$, and $x[n] = u[n] - u[n-5]$ for $0 \leq n \leq 5$

   (c) $x[n] = \frac{1}{2}^{|n|} cos(\frac{\pi}{8}(n-1))$

6. (CO4)Let

$$g(t) = x(t)cos^2(t) * \frac{sint}{\pi t} \tag{7}$$

   Assuming that $x(t)$ is real and $X(j\omega) = 0$ for $|\omega| \geq 1$, show that there exists an LTI system S such that

$$x(t) \xrightarrow{S} g(t) \tag{8}$$