Tutorial 8

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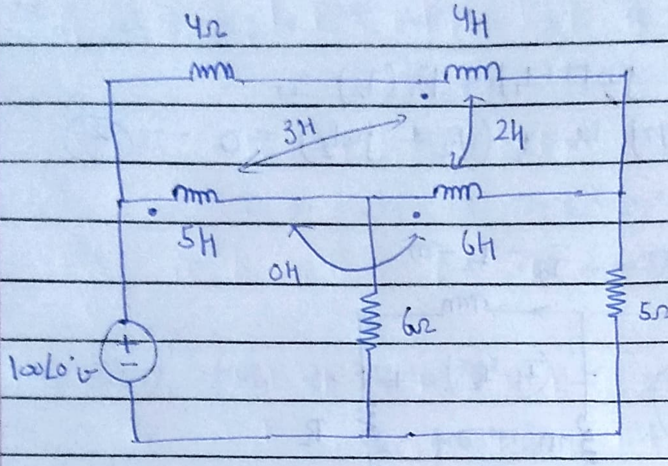
Soln (1)



(a) Write equation in terms of $I_1(j\omega)$, $I_2(j\omega)$, $I_3(j\omega)$:-

$$X_C = 1/j\omega C \ \Omega$$
$$X_L = j\omega L \ \Omega$$

Now, reactance of the 5H inductor is,
$$X_{5H} = j5\omega$$

Reactance of the 4H inductor is,
$$X_{4H} = j4\omega$$

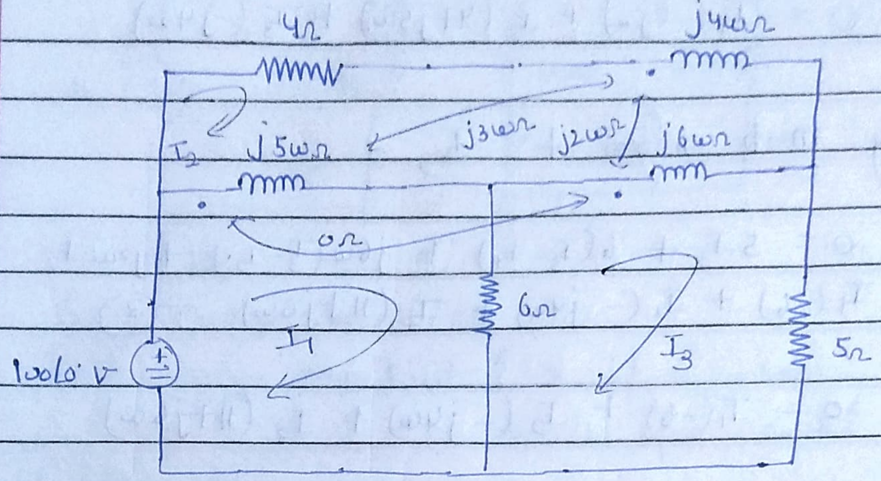
Reactance of the 6H inductor is,
$$X_{6H} = j6\omega$$

Reactance of the 3H mutual inductance is,
$$X_{3H} = j3\omega$$

Reactance of the 2H mutual inductance is,
$$X_{2H} = j2\omega$$

Reactance of the 0H mutual inductance is,
$$X_{0H} = 0$$

⇒ Redrawing the circuit in the phasor domain :



From figure, the currents $I_1 - I_2$, I_2 and $I_3 - I_2$ are all entering the dotted terminal of their coils, so the mutual inductance is added to the self-inductance when these current direction are used

→ Using mesh analysis at I_1 :-

$$100\angle 0° = j5\omega(I_1 - I_2) + j3\omega I_2 + 6(I_1 - I_3)$$

$$= I_1(6+j5\omega) + I_2(-j2\omega) + I_3(-6) \quad (1)$$

$$\Rightarrow \quad 100 = I_1(6+j5\omega) + I_2(-j2\omega) - 6I_3$$

→ Using mesh analysis at I_2 :-

$$0 = 4I_2 + j4\omega I_2 + j2\omega(I_3 - I_2) + j3\omega(I_1 - I_2) - (j6\omega(I_3 - I_2) + j2\omega I_2) - (j5\omega(I_1 - I_2) + j3\omega I_2)$$

$$= I_1(j3\omega - j5\omega) + I_2(4 + j4\omega - j2\omega - j3\omega + j6\omega - j2\omega + j5\omega - j3\omega) + I_3(j2\omega - j6\omega)$$

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$= I_1(-j2\omega) + I_2(4+j5\omega) + I_3(j4\omega)$ — (2)

$0 = I_1(-2j\omega) + I_2(4+j5\omega) + I_3(-j4\omega)$

→ Using mesh analysis at I_3,

$0 = 5I_3 + 6(I_3 - I_1) + j6\omega(I_3 - I_2) + j2\omega I_2$

$\Rightarrow I_1(-6) + I_2(-j4\omega) + I_3(11+j6\omega)$ — (3)

$0 = I_1(-6) + I_2(-j4\omega) + I_3(11+j6\omega)$

(b) now, to find $I_3(j\omega)$ substitute in (1), (2) and (3) by $\omega = 2$ rad/s,

first, substitute in (1)

$100 = I_1(6+j5\times2) + I_2(-j2\times2) + I_3(-6)$

$= I_1(6+j10) + I_2(-j4) + I_3(-6)$ — (4)

substitute in (2)

$0 = I_1(-j2\times2) + I_2(4+j5\times2) + I_3(-j4\times2)$

$\Rightarrow I_1(-j4) + I_2(4+j10) + I_3(-j8)$ — (5)

substitute in (3),

$0 = I_1(-6) + I_2(-j4\times2) + I_3(11+j6\times2)$

$= I_1(-6) + I_2(-j8) + I_3(11+j12)$ — (6)

(b) By solving (4), (5), (6) for I_1, I_2 and I_3

$I_3 = 2.52 - j3.5$

$= 4.3\angle -54.2^\circ$ A Ans

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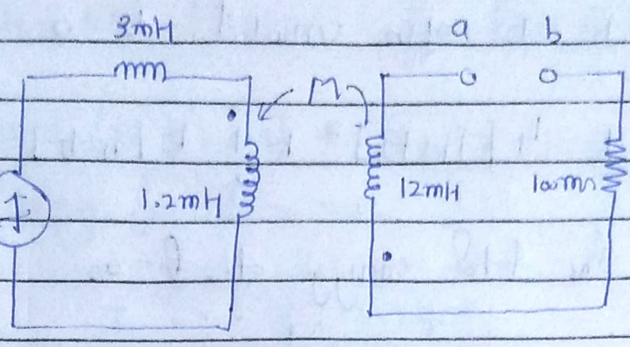
Soln 2)

(a)



$k = 0.75$

$i_s = 5 \cos 200t$ mA

total energy stored at $t = 0$ and $t = 5$ms.

a-b is open circuited

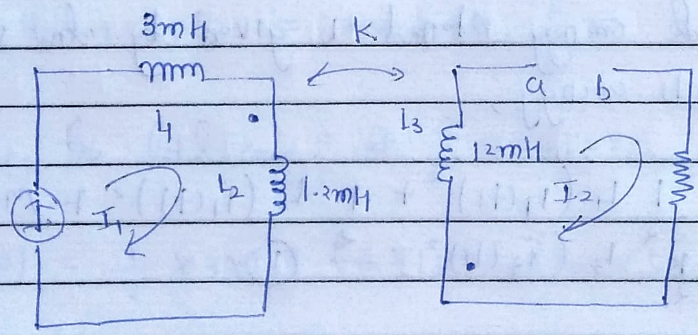
$$k = \frac{M}{\sqrt{L_1 L_2}}$$

now, $M = k\sqrt{L_1 L_2}$

$= 0.75\sqrt{1.2 \times 10^{-3} \times 12 \times 10^{-3}}$

$= 2.85 \times 10^{-3}$ H

from figure, the two currents i_1 and i_2 are entering the dotted terminals of their coils, so the mutual energy is added to self inductance energy.



⇒ The total energy stored is given by the summation of all coils energy :-

$$w(t) = \frac{1}{2} L_1 [i(t)]^2 + \frac{1}{2} L_2 [i_1(t)]^2 + M [i_1(t)] [i_2(t)]$$

$$+ \frac{1}{2} L_3 [i_2(t)]^2$$

Since a-b is open circuited $i_2 = 0$

$$w(t) = \frac{1}{2} L_1 [i_1(t)]^2 + \frac{1}{2} L_2 [i_1(t)]^2$$

⇒ Calculating the total energy at $t = 0s$

$$w(0\,ms) = \frac{1}{2} \times 3 \times 10^{-3} [5 \times 10^{-3} \cos(200 \times 0)]^2 + \frac{1}{2} \times 1.2 \times 10^{-3} [5 \times 10^{-3} \cos(200 \times 0)]^2$$

$$= 52.5\,nJ \quad A$$

⇒ Calculating the total energy at $t = 5 \times 10^{-3}\,s$

$$w(5\,ms) = \frac{1}{2} \times 3 \times 10^{-3} [5 \times 10^{-3} \cos(200 \times 5 \times 10^{-3})]^2 + \frac{1}{2} \times 1.2 \times 10^{-3} [5 \times 10^{-3} \cos(200 \times 5 \times 10^{-3})]^2$$

$$= 15.32 \times 10^{-9}\,J \quad A$$

$$= 15.32\,nJ \quad B$$

(b) → The total energy stored is given by the summation of all coils energy;

$$w(t) = \frac{1}{2} L_1 (i_1(t))^2 + \frac{1}{2} L_2 (i_1(t))^2 + M (i_1(t)\, i_2(t)) + \frac{1}{2} L_3 (i_3(t))^2 \quad — (1)$$

since, a-b is short circuited, the value of i_2 needed to be determined using mesh analysis at I_2.

$$0 = j12 \times 10^{-3} \omega I_2 + 100 \times 10^{-3} I_2 + j\,2.85 \times 10^{-3} \omega I_1$$

$$= I_2 (j12 \times 10^{-3} \times 200 + 100 \times 10^{-3}) + I_1 (2.85 \times 10^{-3} \times 200)$$

$$= I_2 (j2.4 + 0.1) + I_1 (j0.57)$$

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$\Rightarrow$ But $I_1 = 5\times10^{-3}\cos(200t)$ for $t=0$, $I_1 = 5\times10^{-3}$,

$$0 = I_2(j2.4+0.1) + 5\times10^{-3}\times(j0.57)$$

now, solving for I_2:-

$$I_2 = \frac{-j2.85\times10^{-3}}{0.1+j2.4}$$

$$= 1.1865\times10^{-3}\angle -177.6^\circ \text{ A}$$

$$\Rightarrow i_2(t) = 1.1865\cos(200t - 177.6) \text{ A}$$

$\rightarrow$ Substitute by the values of i_1 and i_2 in (1) to calculate the total energy stored.

$\rightarrow$ now, calculating the total energy at $t=0$.

$$w(0\,ms) = \frac{1}{2}\times3\times10^{-3}(5\times10^{-3})^2 + \frac{1}{2}\times1.2\times10^{-3}(5\times10^{-3})^2$$
$$+ 2.85\times10^{-3}(5\times10^{-3})(1.1865\times10^{-3}\cos(0-177.6))$$
$$+ \frac{1}{2}\times12\times10^{-3}(1.1865\times10^{-3}(0-177.6))^2$$

$$= 4.404\times10^{-8}\text{J} = 44.04\,nJ$$ B

$\rightarrow$ Calculating the total energy at $t = 5\times10^{-3}$ s,

$$w(5ms) = \frac{1}{2}\times3\times10^{-3}(5\times10^{-3}\cos(200\times5\times10^{-3})]^2$$
$$+ \frac{1}{2}\times1.2\times10^{-3}(5\times10^{-3}\cos(200\times5\times10^{-3})]^2$$
$$+ 2.85\times10^{-3}[5\times10^{-3}][1.1865\times10^{-3}\cos(200\times5\times10^{-3} - 177.6\times\pi/180)]$$
$$+ \tfrac{1}{2}\times12\times10^{-3}(1.1865\times10^{-3}\cos(200\times5\times10^{-3} - 177.6\times\pi/180))^2$$

$$= 8.945\,nJ$$ B

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Soln 3) $Z_{in} = Z_{11} + \frac{\omega^2 M^2}{Z_{22}}$ — (1)

and, $Z_{11} = R_1 + jL_1\omega$
$= (10 + j2 \times 10^{-6} \times 10)$
$= 10 + j20 \times 10^{-6}\ \Omega$ — (2)

and,
$Z_{22} = R_2 + j\omega L_2 + Z_L$
$= 1 + j1 \times 10^{-6} \times 10 + Z_L$
$= 1 + j10^{-5} + Z_L\ \Omega$ — (3)

→ Substitute by (2) and (3) in (1) and the value of ω and M to get the expression for the input impedance in terms of Z_L

$$Z_{in}(Z_L) = 10 + j20 \times 10^{-6} + \frac{10^2(500 \times 10^{-9})^2}{1 + j10^{-5} + Z_L}$$

→ now, Calculating Z_{in} when $Z_L = 1\Omega$

(a) $Z_{in}(1\Omega) = 10 + j20 \times 10^{-6} + \frac{10^2(500 \times 10^{-9})^2}{1 + j10^{-5} + 1}$

$= 10 + j2 \times 10^{-5}\ \Omega$ Ans

(b) Z_{in} when, $Z_L = j\Omega$

$Z_{in}(j\Omega) = 10 + j20 \times 10^{-6} + \frac{10^2(500 \times 10^{-9})^2}{1 + j10^{-5} + j}$

$= 10 + j1.999 \times 10^{-5}\ \Omega$ Ans

(c) Z_{in} when, $Z_L = -j\Omega$

$$Z_{in}(-j\Omega) = 10 + j20\times10^{-6} + \frac{10^{2}(500\times10^{-9})^{2}}{1+j10^{-5}+(j)}$$

$$= 10 + j2\times10^{-5}\,\Omega$$

(d) Z_{in} when, $Z_L = 5\angle33^\circ\,\Omega$

$$Z_{in}(5\angle33^\circ) = 10 + j20\times10^{-6} + \frac{10^{2}(500\times10^{-9})^{2}}{1+j10^{-5}+5\angle33}$$

$$= 10 + j1.999\times10^{-5}\,\Omega$$