Quiz 3 (30 marks)
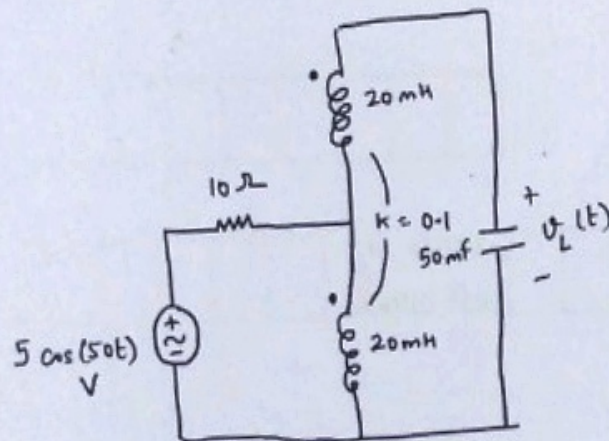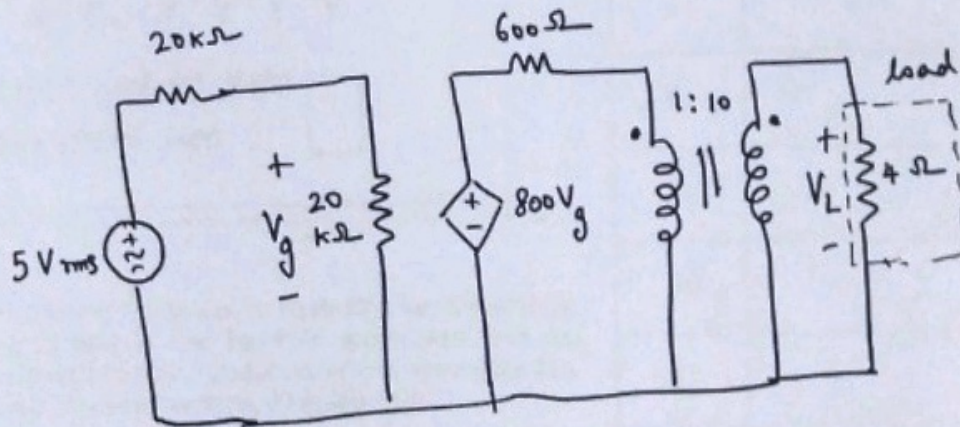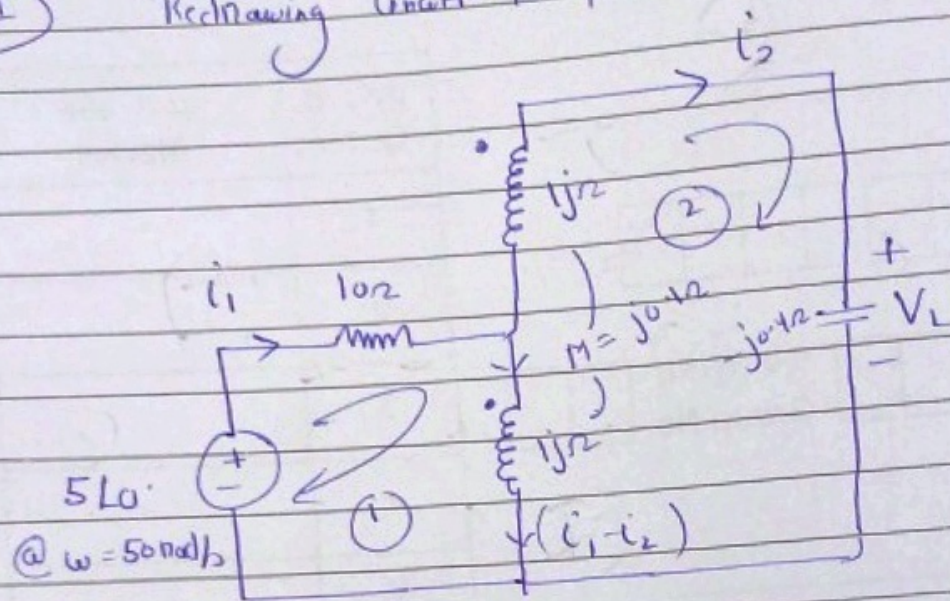
1. **(15 marks)** For the following circuit, write the mesh equations (in phasor domain) and steps required to compute the load voltage V_L. Detailed calculations can be omitted since they are not required.

2. **(15 marks)** In the following circuit, find the time averaged power delivered to the load.

1

Soln 1) Redrawing circuit in phasor domain:-

$$\Rightarrow \quad X_L = j\omega L$$

$$X_{20mH} = j \times 50 \times 20 \times 10^{-3}$$
$$= 1j\Omega$$

$$\Rightarrow \quad X_c = \frac{1}{j\omega c} = -j/\omega c \qquad ①$$
$$= \frac{-j}{50 \times 50 \times 10^{-3}} = -j0.4\Omega$$

$$k = M/\sqrt{4_1 4_2}$$
$$M = k\sqrt{4_1 4_2}$$
$$= 0.1 \times \sqrt{20 \times 20 \times 10^{-6}}$$
$$= 0.1 \times \sqrt{400 \times 10^{-6}}$$
$$= 0.1 \times \sqrt{4 \times 10^{-4}}$$
$$= 0.1 \times 2 \times 10^{-2}$$
$$= 2 \times 10^{-3} = 2mH.$$
$$X_{2mH} = j\omega M = j \times 50 \times 2 \times 10^{-3} = j0.1\Omega$$

2

let current in loop ① is i_1 and in loop ② is i_2 :-

⇒ Mesh equation for loop ① :-

$$5\angle 0 - 10(i_1) - 1j(i_1 - i_2) + j0.1(i_2) = 0 \quad —①$$

③

⇒ Mesh equation for loop ② :-

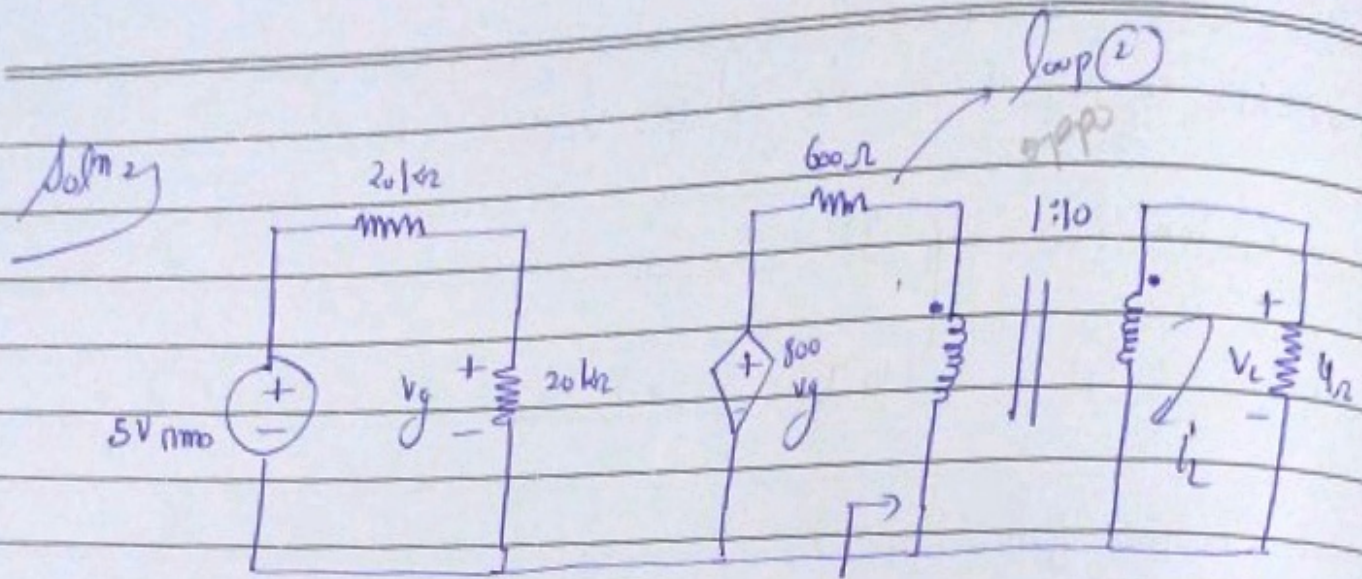$$-(-j0.4)(i_2) + 1j(i_1 - i_2) - j(0.1)(i_2) - 1j(i_2) + j(0.1)(i_1 - i_2) = 0$$

③ —②

By solving equation ① and ②, we can find i_1 and i_2

$$V_L = i_2 \times (-j0.4\,\Omega). \quad —③ = V_L \angle 0 \quad ③$$

Now, from putting the value of i_2 in ③, we can find V_L and then convert it into time domain.
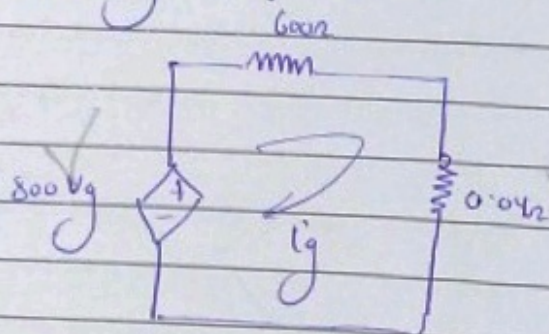
$$V_L(t) = V_L \cos(50t + \theta) \; V \quad ✓$$

Soln 2)

$$V_g = \frac{5 \times 20k}{20k + 20k}$$

(∵ a = 10)

$$\Rightarrow \frac{5 \times 20k}{40k}$$

③

$$= 5/2 = 2.5 \, V \, rms$$

Z_{in}

$$Z_{in} = \frac{Z_L}{a^2} = \frac{4}{10^2} = \frac{4}{100} = 0.04 \, \Omega$$

⇒ redrawing loop ① :-

③

$$8\omega \times V_g = 8\omega \times 2.5$$
$$= 2000 \, V \, rms$$

③

$$I_g = \frac{2000}{600 + 0.04} \Rightarrow \frac{2000}{600.04} = 3.33 \, A \, rms$$

4

$$\frac{i_L}{i_g} = \frac{1}{a} \quad (\because M \text{ is opposite})$$

$$\Rightarrow \quad \frac{i_L}{3.33} = \frac{1}{10} \qquad \textcircled{3}$$

$$\Rightarrow \quad i_L = 0.33 \text{ A } \underline{\text{rms}}$$

$$P_{load} = (i_L)^2 \times 4\Omega$$
$$= (0.33)^2 \times 4\Omega$$
$$= 0.4356 \text{ W } \Omega \underline{\qquad} \qquad \textcircled{3}$$

$\hookrightarrow$ time avg.d power