Review

$\rightarrow \quad F(s) = 7 - \dfrac{s+\frac{1}{s}}{s^2+3s+1}$

$$G(s) = \frac{s+\frac{1}{s}}{s^2+3s+1} = \frac{s^2+1}{s(s^2+3s+1)}$$

Residue

$$s^2+3s+1=0$$

$$s = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-3 \pm \sqrt{9-4}}{2}$$

$$s = \frac{-3+\sqrt{5}}{2}, \quad \frac{-3-\sqrt{5}}{2}$$

$$G(s) = \frac{s^2+1}{s\left(s+\frac{3-\sqrt{5}}{2}\right)\left(s+\frac{3+\sqrt{5}}{2}\right)} = \frac{A}{s} + \frac{B}{\left(s+\frac{3-\sqrt{5}}{2}\right)} + \frac{C}{\left(s+\frac{3+\sqrt{5}}{2}\right)} \quad -(1)$$

(1) $\rightarrow$

$$\Rightarrow s^2+1 = A\left(s+\frac{3-\sqrt{5}}{2}\right)\left(s+\frac{3+\sqrt{5}}{2}\right) + Bs\left(s+\frac{3+\sqrt{5}}{2}\right) + Cs\left(s+\frac{3-\sqrt{5}}{2}\right)$$

put $s=0$

(2): $1 = A\left(\frac{3-\sqrt{5}}{2}\right)\left(\frac{3+\sqrt{5}}{2}\right) + B(0) + C(0)$

$\Rightarrow 1 = A\left(\frac{9-5}{4}\right) \Rightarrow A = 1$

put $s = \dfrac{-3+\sqrt{5}}{2}$

(3): $\left(\frac{-3+\sqrt{5}}{2}\right)^2 + 1 = A(0) + B\left(\frac{-3+\sqrt{5}}{2}\right)\left(\frac{-3+\sqrt{5}}{2} + \frac{3+\sqrt{5}}{2}\right) + C(0)$

$\frac{14-6\sqrt{5}}{4} + 1 = B\left(\frac{-3+\sqrt{5}}{2}\right)(\sqrt{5})$

$$\frac{14-6\sqrt{5}}{4}+1 = B\left(\frac{-3+\sqrt{5}}{2}\right)(\sqrt{5})$$ ②

$$\Rightarrow \frac{18-6\sqrt{5}}{\cancel{4}\,2} = \frac{B\,(-3\sqrt{5}+5)}{\cancel{2}\,1}$$

$$\Rightarrow \boxed{B = \frac{18-6\sqrt{5}}{2(5-3\sqrt{5})}}$$

Compare coeffs of s^2

Ⓔ : $1 = A+8+C \Rightarrow C = 1-(A+B)$

$$\Rightarrow \boxed{C = 1-1-B = -B}$$

$$\Rightarrow F(s) = 7 + \frac{1}{s} + \frac{\frac{18-6\sqrt{5}}{2(5-3\sqrt{5})}}{\left(s+\frac{3-\sqrt{5}}{2}\right)} + \frac{-\left(\frac{18-6\sqrt{5}}{2(5-3\sqrt{5})}\right)}{\left(s+\frac{3+\sqrt{5}}{2}\right)}$$

$\therefore$

$$f(t) = 7\delta(t) + u(t) + B e^{-\left(\frac{3-\sqrt{5}}{2}\right)t} u(t) - C e^{-\left(\frac{3+\sqrt{5}}{2}\right)t} u(t)$$

$$\rightarrow \quad F(s) = \frac{8}{s^3+8s^2+21s+18}$$

$$= \frac{8}{(s+2)(s^2+6s+9)} = \frac{8}{(s+2)(s+3)^2}$$

$$\frac{8}{(s+2)(s+3)^2} = \frac{A}{(s+2)} + \frac{Bs+C}{(s+3)^2}$$

$$\left[\frac{8}{(s+2)(s+3)^2} = \frac{A}{s+2} + \frac{Bs+C}{(s+3)^2}\right] \times (s+2)(s+3)^2$$

$$\Rightarrow 8 = A(s+3)^2 + Bs(s+2) + C(s+2) \quad \text{—— } (1)$$

put $s = -2$

$$\Rightarrow 8 = A(1)^2 \Rightarrow A = 8$$

Compare coeff of s^2

$$0 = A + B \Rightarrow B = -A = -8$$

compare coeff of s^0

$$8 = 9A + 2C \Rightarrow C = \frac{8-9A}{2} = \frac{8-9\times 8}{2}$$

$$= -\frac{64}{2} = -32$$

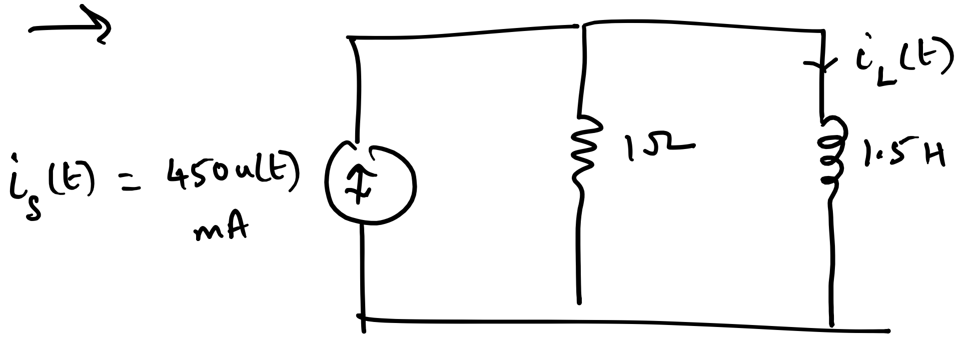
$$F(s) = \frac{8}{s+2} + \frac{-8s-32}{(s+3)^2}$$

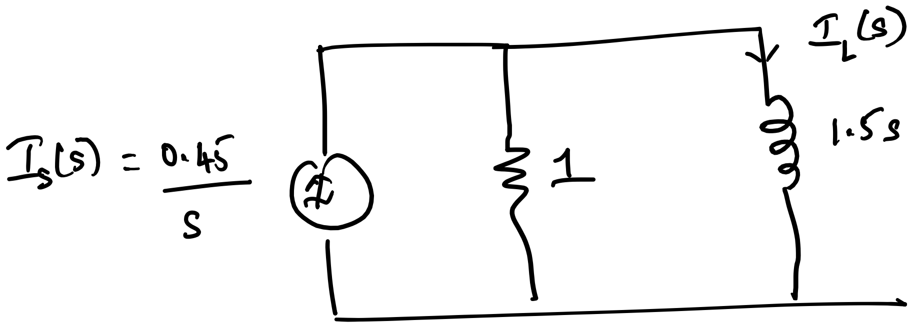
$$= \frac{8}{s+2} + \frac{-8(s+3-3)-32}{(s+3)^2}$$

$$= \frac{8}{s+2} + \frac{-8}{s+3} + \frac{+24-32}{(s+3)^2} \qquad \frac{-8}{(s+3)^2}$$

$$f(t) = 8e^{-2t}u(t) - 8e^{-3t}u(t) - 8te^{-3t}u(t)$$

Assume there are no initial conditions





Using current division

$$I_L(s) = \frac{I_s(s) \times 1}{1 + 1.5s} = \frac{\frac{0.45}{} \times \frac{2}{3}}{s\left(1 + \frac{3}{2}s\right) \times \frac{2}{3}}$$

$$\left[= \frac{0.3}{s\left(s+\frac{2}{3}\right)} = \frac{A}{s} + \frac{B}{s+\frac{2}{3}} \right] \times s\left(s+\frac{2}{3}\right)$$

$$\Rightarrow 0.3 = A\left(s+\frac{2}{3}\right) + Bs$$

put $s = 0$

$$\Rightarrow 0.3 = A\left(\frac{2}{3}\right) \Rightarrow A = \frac{0.9}{2} = 0.45$$

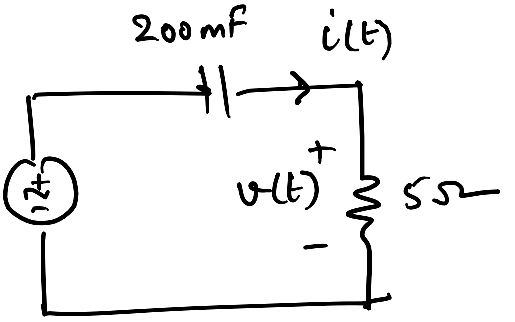
compare coeff of s^1

$$0 = A + B \Rightarrow B = -A = -0.45$$

$$I_L(s) = \frac{0.45}{s} + \frac{-0.45}{s+\frac{2}{3}}$$

$$\Rightarrow i_L(t) = 0.45\,u(t) - 0.45\,e^{-\frac{2t}{3}}\,u(t)$$

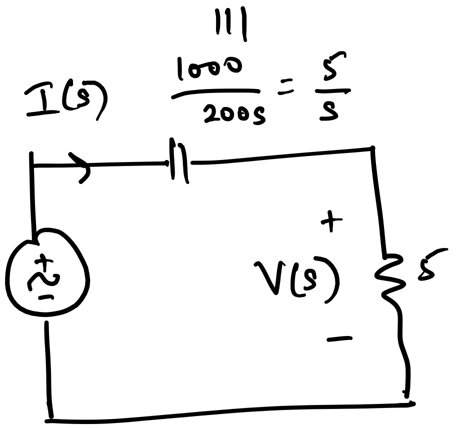
$\rightarrow$



$v_s(t) = 2e^{-t}u(t)$ V

Ignore initial conditions & find $i(t)$ & $v(t)$

$V_s(s) = \frac{2}{s+1}$



$\frac{1000}{200s} = \frac{5}{s}$

$$I(s) = \frac{V_s(s)}{5+\frac{5}{s}} = \frac{\frac{2}{s+1}}{5+\frac{5}{s}}$$

$$\Rightarrow I(s) = \frac{2}{s+1} \div 5\left(1+\frac{1}{s}\right)$$

$$= \frac{2}{s+1} \times \frac{s}{5(s+1)} = \frac{2s}{5(s+1)^2}$$

$$= \frac{\frac{2}{5}(s+1-1)}{(s+1)^2} = \frac{\frac{2}{5}}{(s+1)} - \frac{\frac{2}{5}}{(s+1)^2}$$

$$\Rightarrow i(t) = \frac{2}{5}e^{-t}u(t) - \frac{2}{5}te^{-t}u(t) \quad \text{A}$$

Initial conditions of L & C

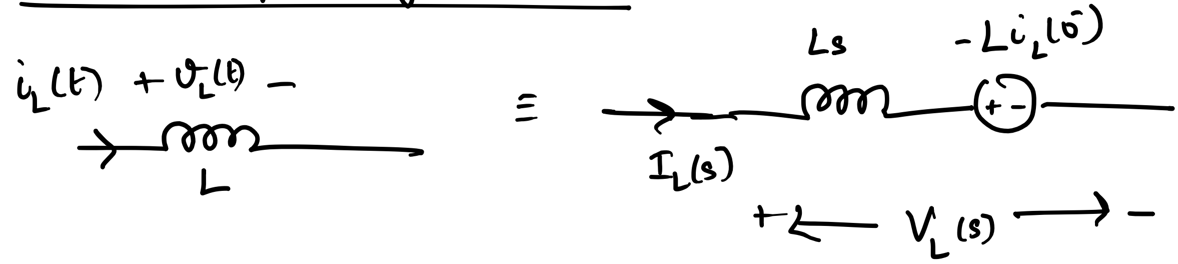
$$\mathcal{L}T\left[v_L = L\frac{di}{dt} \right]$$

$$\Rightarrow V_L(s) = L\left[s I_L(s) - i_L(0^-) \right]$$

↑ initial condition of inductor

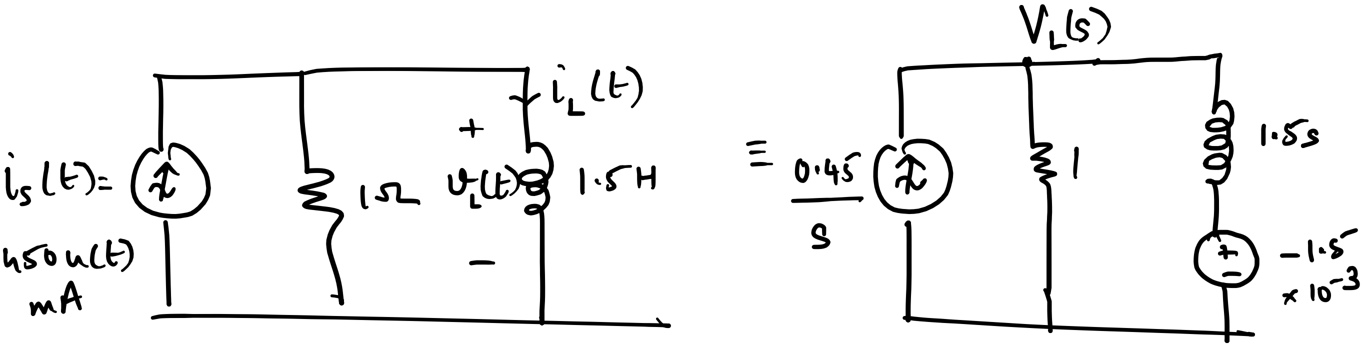
$$\Rightarrow V_L(s) = Ls\, I_L(s) - L i_L(0^-)$$

Redraw in frequency domain



Redo previous problem

with $i_L(0^-) = 1\,mA$



Response — Forced response from forcing excitation $(i_s(t))$ + Natural response from initial conditions $(i_L(0^-))$

$$\frac{0.45}{s} = \frac{V_L}{1} + \frac{V_L + 0.015}{\frac{3}{2}s}$$

Please solve for V_L

$$\Rightarrow \frac{0.45}{s} = V_L\left(1 + \frac{2}{3s}\right) + \frac{\cancel{2\times 0.015}\ \cancel{0.003}\ 0.001}{\cancel{3}s}$$

$$\Rightarrow \frac{0.45}{\cancel{s}} - \frac{0.001}{\cancel{s}} = V_L\left(\frac{3s+2}{3\cancel{s}}\right)$$

$$\Rightarrow V_L = \frac{(0.45 - 0.001)\times 3}{3s+2} = \frac{(0.45-0.001)}{\left(s+\frac{2}{3}\right)}$$

$$\Rightarrow v_L(t) = 0.449\, e^{-2t/3}\, u(t)\ \text{V} \quad \text{— Complete response}$$