

Quiz 1: Fields & Waves (ECE230), Winter 2025

Duration: 1hr 15min, Total: 25 points (Attempt all questions)

Feb 2, 2025

**Vector quantities must be represented with an overhead arrow, vector calculus operators must be written correctly, dot and cross products, closed and open integrals must be written properly. In case of any of these mistake -1 will be awarded in each question with such mistake.

NO credit will be given to the answers that do not accompany proper explanation.

Any case of copying cheating will be dealt as per institute guidelines.

Q1. The electrostatic potential of some charge-distribution is given by:

$$V(r) = A \frac{e^{-\lambda r}}{r}$$

where, A and λ are constants. Find the electric field $\vec{E}(r)$ and the charge density $\rho(r)$. [2 + 3 = 5]

Q2. Calculate curl of the function $\vec{A}(x, y, z) = \hat{x}yz + \hat{y}4xy + \hat{z}y$. [3]

Q3. Consider an infinitely long, thin wire, oriented along z -direction. Assume that it carries a uniform line-charge density λ .

a) Calculate the electric field at a point P with cylindrical coordinates (ρ, ϕ, z) .

b) How much work would you need to do if you want to move a charge $+q$ from $\rho = a$ to $\rho = b$? [3 + 3 = 6]

Q4. An infinite plane carries a surface charge density σ .

(a) Draw the geometry and direction of \vec{E} .

(b) Calculate the electric field. [1 + 3 = 4]

Q5. A total charge Q is distributed uniformly over a sphere of radius R . How much work would you need to do to move a charge q from $r = a$ to $r = b$? Here, $a, b < R$. [4]

Q6. Consider two infinite parallel plates kept at $x = 0$ and $x = d$, respectively. The first plate is connected to $5V$ and the second one to $-5V$. Calculate the potential distribution between them. [3]

Quiz 2: Fields & Waves (ECE230), Winter 2025

Duration: 1hr 10 mins, Total: 20 points (Attempt all questions)

Feb 17, 2025

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Q1. Consider an electrostatic potential $\phi(r) = \frac{k}{r^1}$ over free space. Calculate the energy density of the field. [3 points]

Q2. Evaluate the following integrals:

(a) $\int_{-1}^{+1} (r^2 + 2) \vec{\nabla} \cdot \left(\frac{\hat{r}}{r^2} \right) dv$

(b) $\int_1^{\infty} (r^2 + 2) \vec{\nabla} \cdot \left(\frac{\hat{r}}{r^2} \right) dv$

[1.5 + 1.5 = 3 points]

Q3. Choose correct options (if multiple options are correct you must choose all of them to get any credit):

(a) Which expression of divergence theorem is correct: (i) $\oint_V (\vec{\nabla} \cdot \vec{E}) dv = \oint_S \vec{E} \cdot d\vec{S}$, (ii) $\oint_V \phi (\vec{\nabla} \cdot \vec{E}) dv = \oint_S \vec{E} \cdot d\vec{S}$, (iii) $\oint_V (\vec{\nabla} \cdot \vec{E}) dv = \oint_S \phi \vec{E} \cdot d\vec{S}$,

(iv) $\int_V (\vec{\nabla} \cdot \vec{E}) dv = \int_S \vec{E} \cdot d\vec{S}$

[1 point]

(b) Consider a charge-free interface between two media with permittivities ϵ_1 and ϵ_2 . E_{n1} and E_{n2} are the normal components of the electric fields and E_{t1} and E_{t2} are the tangential components at the interface: (i) $E_{t1} = E_{t2}$, (ii) $E_{n1} = E_{n2}$, (iii) $\epsilon_1 E_{n1} = \epsilon_2 E_{n2}$, (iv) $\epsilon_1 E_{t1} = \epsilon_2 E_{t2}$.

Q4. A thin uniformly charged spherical shell of radius R carries a charge Q . Calculate: [1 point]

(a) Energy density inside and outside the shell.

[1 + 1 = 2 points]

(b) Total energy.

[3 points]

Q5. An infinite plane carries a uniform surface charge σ . Find its electric field.

[3 points]

Q6. A primitive model for an atom consists of a point nucleus $+q$ surrounded by a uniformly charged spherical cloud $-q$ of radius a . Calculate the atomic polarizability of such an atom (recall: $\vec{p} = \alpha \vec{E}$, α being the polarizability). You can assume that the spherical shape of the electron cloud remains undistorted. [4 points]

Quiz 3: Fields & Waves (ECE230), Winter 2025

Total marks: 20. Duration 1hr 15mins

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- Q1. Under certain condition, the relative permittivity of a material can be written as: $\epsilon_r(\omega) = \left(1 - \frac{\omega_p^2}{\omega^2}\right)$. Plot $\epsilon_r(\omega)$ as a function of frequency. You need to clearly show all the critical values in this plot. [3] points
- Q2. Consider the expression of the electric field of a uniform plane wave traveling in vacuum:
 $\vec{E} = 4\hat{x} \exp[i(5 \times 10^6 t - kz)]$
- What is the direction of polarization of this EM wave?
 - Which direction is the wave traveling?
 - How much is the value of k ?
 - What is the wavelength?
- [1 + 1 + 1 + 1 = 4] points
- Q3. You have studied Gauss's law already and this is one of the Maxwell's equations. Consider an inhomogeneous medium i.e. a medium where the permittivity is a function of (x, y, z) . How much is $\vec{\nabla} \cdot \vec{E}$ for such a medium? [3] points
- Q4. Suppose, you wake up one fine morning and find a magnetic monopole on your breakfast table. As a student of F&W, it is your responsibility to save EM theory by modifying Maxwell's equations. How are you going to modify 'em? What is your thought process behind those modifications? [10] points

Quiz 4: Fields & Waves (ECE230), Winter 2025

Total marks: 15. Duration 50 mins.

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Q1. (a) What is the unit of the ratio of E to H in a medium? Explain your answer (without explanation there's no marks).

(b) Write down the above ratio for a medium with a finite conductivity σ .

(c) Will E and H be in the same phase for the medium in part (b)?

(d) Suppose σ is very high within certain frequency range of interest. What would be the phase angle between E and H ?

[1 + 2 + 1 + 3] = 7 points

Q2. In free space:

$$\vec{E} = 4\hat{x} \exp [i(5 \times 10^6 t - kz)]$$

Calculate the displacement current.

2 points

Q3. Consider two uniform plane electromagnetic waves, of unequal amplitudes and same frequencies, propagating in opposite directions ($+z$ and $-z$). Obtain an expression for the resultant electric field and plot the amplitude of the total e-field as a function of z . [Hint: you may want to forget about the time variation part here and consider only the frequency domain field. Then try to put the total field in a form: $re^{i\theta}$]

[4 + 2] = 6 points