# Mid-semester examination: Fields & Waves (ECE230), Winter 2022

Total: 25 points

**Submission format: **ALL** the answers must be **scanned to a single pdf** and submitted through classroom. Any late submission will be strictly ignored. Please avoid sending any email to TAs or instructor with such requests.

****No communication through messaging services/email/phone call with your friends is allowed. If such a case is detected, it will be dealt as per the dishonesty policy of the institute.**

**Vector quantities must be represented with an overhead arrow, vector calculus operators must be written correctly, dot and cross products must be written properly. In case of any of these mistake -1 will be awarded for each count of mistake. If you have n number of such mistakes in a single question, -n will awarded upto the total marks in the question.

NO credit will be given to the answers that do not accompany proper explanation.
Any case of copying/cheating will be dealt as per institute guidelines.

Q1. A certain material has a conductivity of $5 \times 10^6 s/m$ and a permittivity of $5F/m$. A total charge of $1C$ is distributed uniformly in the bulk of a sphere (of radius of $5m$) made of this material. How much would be the surface charge density after $2\mu s$? ($1\mu s = 10^{-6}S$) [6 points]

Q2. Prove that $\vec{\nabla} \times (\vec{\nabla}\phi) = 0$ [3 points]

Q3. You must have heard that lightning prefers to strike the sharper metals more readily. The reason is, the electric fields are more intense near the sharp parts of a metallic object. So, the air molecules near the sharper parts become ionized more readily by the intense electric field existing between the object and the clouds. This ionized path provides a conducting channel for the current to flow between the sharp part of the metallic object and the cloud. This is what you see as a lightning-strike. But what causes the electric field to be more intense near the sharp parts of the metal? Walk through the following steps to answer this:

($a$) Imagine two charged metal spheres, one bigger than the other. Also, they are placed at sufficiently large distance so that the charge distribution on one sphere has negligible effect on the charge distribution of the other. The two are connected through a metal wire. If all the metals are perfect conductors and the potential of the wire is $V$, compare the total charges residing on each sphere.

($b$) Compare the surface charge densities on the spheres.

($c$) Apply Gauss's law to qualitatively compare the electric fields on each sphere.

($d$) Now, try to reason why the electric field is more intense near the sharper parts of an arbitrarily shaped object.

$4 \times 2 = 8$ points

Q4. Evaluate $\vec{\nabla}.\vec{E}$ for an inhomogeneous medium with a volume charge density $\rho$. [4 points]

Q5. Consider two magnetic fields given by ($a$)$\vec{B} = 3x^2y - 4xz + 5z^2$ and ($b$) $\vec{B}$ =constant. Comment on these magnetic fields. [2+2=4 points]