Practice Sheet-2 Solutions.

(1) $x[n]$ exists during $-3 \le n \le 3$.

$$x[n] = \left\{ \begin{array}{ccccccc} x(-3) & x(-2) & x(-1) & x(0) & x(1) & x(2) & x(3) \\ n=-3 & & & \uparrow & & & \\ & & & n=0 & & & n=3 \end{array} \right.$$

$x[n+4] \Rightarrow$ shift leftwards.

$x[n+4]$ exists $n=-7$ to $n=-1$.

$x[n+4]$ exists between $\underline{\underline{-7}} \le n \le -1$

(2) (a) $\cos\left(\frac{\pi}{2}n\right)\cos\left(\frac{\pi}{8}n\right) = x[n]$

The period of $\cos\left(\frac{\pi}{2}n\right) = 4 = T_1$ because $\cos\left(\frac{\pi}{2}n + 2\pi\right) = \cos\left(\frac{\pi}{2}n\right)$

The period of $\cos\left(\frac{\pi}{8}n\right) = 16 = T_2$ " $\cos\left(\frac{\pi n}{8} + 2\pi\right) = \cos\left(\frac{\pi n}{8}\right)$

$\therefore$ Taking LCM of T_1 & T_2 $= \underline{\underline{16}}$.

$\therefore$ the function is $\boxed{\text{periodic}}$

b) $x[n] = \cos\left(\frac{\pi}{6}n^2\right)$

To be periodic, $f(n+T) = f(n)$

$\therefore \cos\left(\frac{\pi}{6}\cdot(n+T)^2\right) = \cos\left(\frac{\pi}{6}\cdot n^2\right)$.

$\therefore$ For different values of T, its periodic with period $= 6$.

$\boxed{\text{Periodic}}$

(c) $x[n] = \sin\left(\frac{4\pi}{7}n + 1\right)$

To determine $f(n+7) = f(n)$

For a function to be periodic, the argument must increase by a constant amount each period.

Here argument increases by $4\pi/7 \times n$ times, which means it will repeat after $7/4\pi$ units of n.

$\therefore$ $K \times 7/4\pi \Rightarrow$ integer.

$\therefore$ $\frac{7K}{4\pi}$ to be a integer, $7K$ must be a multiple of 4π, which is not possible as $7K$ & 4π are non-divisable.

$\boxed{\text{Not periodic}}$

(3) Even part $x_e[n] = \frac{\delta(n) + \delta(-n)}{2} = \delta(n)$

$x_o[n] = \frac{\delta(n) - \delta(-n)}{2} = 0.$

(4)

(a) $x(t) = e^{(1+j)t}$.

$x(t)$ is a complex exponential multiplied by another exponential. $\therefore$ its $\boxed{\text{not periodic}}$

$\boxed{\text{Non-periodic}}$

(2)

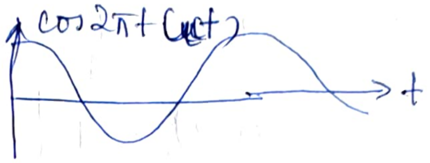
b) $x(t) = 2\cos(10t+1) - \sin(4t-1)$.

Period of first term is $t = 2\pi/10 = \pi/5$

Period of second term is $t = 2\pi/4 = \pi/2$.

$\boxed{\text{Periodic}}$

c) $x(t) = \cos(2\pi t)\, u(t)$



$\boxed{\text{Not periodic}}$

d) $x(t) = \sin(2\pi/3)\, t$.

$\sin(2\pi/3 + 2\pi)t = \sin(2\pi/3)$ ⇒ sine function is a periodic function with periode 2π.

$\therefore\ T = 3/2\pi$.

(5) $T = 0.2$

$$\omega_0 = \frac{2\pi}{0.2} = 10\pi \text{ rad/sec}.$$

(6). $y[n] - y[n-1] + y[n-2] = x[n] - x[n-1]$

$x(n) = \delta(n)$ & $y(n) = 0$ for $n<0$.

$y[n] = x[n] - x[n-1] + y[n-1] - y[n-2]$

$y[0] = x[0] - x[-1] + y[-1] - y[-2] = \underline{\underline{1}}$ $\boxed{y[0] = 1}$

$$y[1] = x(1) - x(0) + y(0) - y(-1)$$

$$= 0 - 1 + 1 - 0 = \underline{0}$$

$$\boxed{y(1) = 0}$$

$$y[2] = x(2) - x(1) + y(1) - y[0]$$

$$= 0 - 0 + 0 - 1$$

$$= \boxed{y[2] = -1}$$

(7)

a) $y(t) = x\sin t.$

$y(\pi) = x(0)$

$y(-\pi) = -x(0) \longrightarrow$ O/p depends on future input.

$\therefore$ $\boxed{\text{Non - causal}}$

b) $y(t) = x(1-t) + x(t-3)$

$\cancel{y(-1) = y(0) + x(-2)}$

$y(-1) = x(0) + x(-4) \longrightarrow$ o/p depends on future input

$\boxed{\text{Non - causal}}$

c) $y(t) = [\cos(3t)]\, x(t)$

$y(1) = [\cos(3)]\, x(1) \longrightarrow$ o/p does not depend on future i/p

$\boxed{\text{Causal}}$

(d) $y[n] = x[4n+1]$

$y[$[redacted]$] = x[5] \rightarrow$ $\boxed{\text{Non - Causal}}$

(8)

a) $x[4n]$

$x[-3] = 3$ $\quad$ $x[1] = 1$

$x[-2] = 2$ $\quad$ $x[2] = 2$

$x[-1] = 1$ $\quad$ $x[3] = 3$

$x[0] = 0$

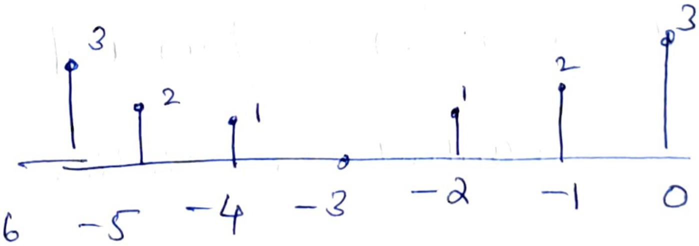
$x[4n] = 0$ when $n = \underline{\underline{0}}$ (All other values not known).

b) $x[4n+3]$

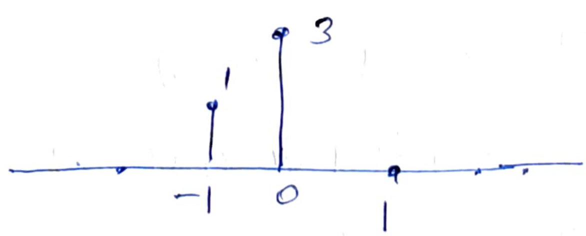
(1) Shift

(2) Scale.

$x[n+3] =$



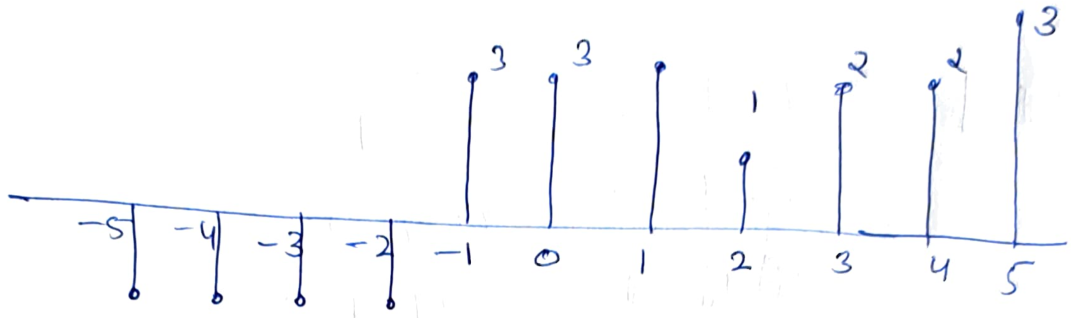
$x[4n+3]$



c) $x[n-2] + y[n+1]$

$x[n-2] + y[n+1]$

$= \{1, -1, -1, -1, 3, 3, 3, 1, 2, 2, 3\}$.



(9)

a) $y(t) = \begin{cases} 0, & t<0 \\ x(t) + x(t-3); & t \geq 0 \end{cases}$

$y(t) = x(t) + x(t-3) \Rightarrow$ Causal.

$y(t) = x(t) + x(t-3)$ is bounded at any value of Constant $\Rightarrow$ Stable.

Linear $\rightarrow$ Additive property.

$y(t) = x(t) + x(t-3)$

$a y_1(t) = a x_1(t) + a x_1(t-3)$

$b y_2(t) = b x_2(t) + b x_2(t-3)$

$\boxed{\text{Linear}}$

Scaling ppty.

$a y(t) = a x(t)$
$- a x(t-3)$

(4)

b) $y(t) = x(t/5)$ $\Rightarrow$ Linear, Stable.

c) $y(t) = x(t-2) + x(2-t)$ $\rightarrow$ Linear, Stable, Not causal.

d) $y(t) = e^{x(t)}$ $\rightarrow$ Causal & Stable.

e) $y(t) = \frac{d}{dt} x(t)$ $\Rightarrow$ Linear, Causal, Time invariant.

(16)

a) $y[n] = x[-n]$ $\Rightarrow$ Linear, Stable (Not causal)

b) $y[n] = x[n-1] - 4x[n-5]$ $\Rightarrow$ Linear, Causal, Stable.

c) $y[n] = n x[n]$ $\Rightarrow$ Causal, Linear.

d) $y[n] = x[2n+1]$ $\Rightarrow$ Linear, Stable (Not causal)

e) $y[n] = \begin{cases} x[n], & n \geq 1 \\ 0, & n = 0 \\ x[n], & n \leq -1 \end{cases}$ $\rightarrow$ Linear, Causal, Stable.

(11) $y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau = \int_{-\infty}^{t} \delta(\tau+2) - \delta(\tau-2)\,dt$

$$= \begin{cases} 0 & t < -2 \\ 1 & -2 \le t \le 2 \\ 0 & t > 2. \end{cases}$$

$\therefore\ E_\infty = \int_{-2}^{2} dt = \underline{\underline{4}}$

(12) $y[n] + 4y[n-1] = x[n] + x^2[n]$

Square $\Rightarrow$ Non-linear.

For causal :-

$y(n) = 0$ for $n < 0$. $\quad x(n) = y(n)$

$x(n) = 1$ for $n \ge 1$

$y[n] = x[n] + x^2[n] - 4y[n-1]$

$y(0) = x[0] + x^2[0] - 4y[-1] = 1 + 1 - 0$

$= \underline{2}$

$y(1) = x[1] + x^2[1] - 4y[0] = 1 + 1 - 8$

$= \underline{\underline{-6}}$

$y(2) = x(2) + x^2(2) - 4y(1) = 1 + 1 + 24$

$= \underline{\underline{26}}$

~~y(n) is increasing~~ $\rightarrow$ $\boxed{\text{Causal}}$

$$y(n)\Big|_{n\to\infty} = \infty .$$

For bounded input, we get unbounded o/p. So unstable system.