Initial Condition of C

1

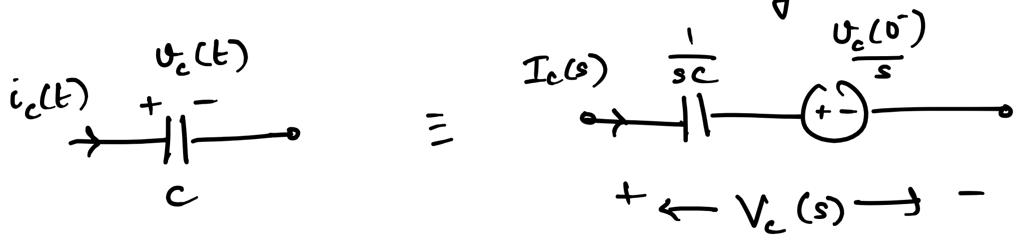
$$\mathcal{LT}\{i_c(t)\} = \mathcal{LT}\left\{C\,\frac{dv_c}{dt}\right\}$$

$$\Rightarrow \quad I_c(s) = C\left[sV_c(s) - v_c(0^-)\right]$$

$$sCV_c(s) = I_c(s) + Cv_c(0^-) \qquad \div\, sC$$

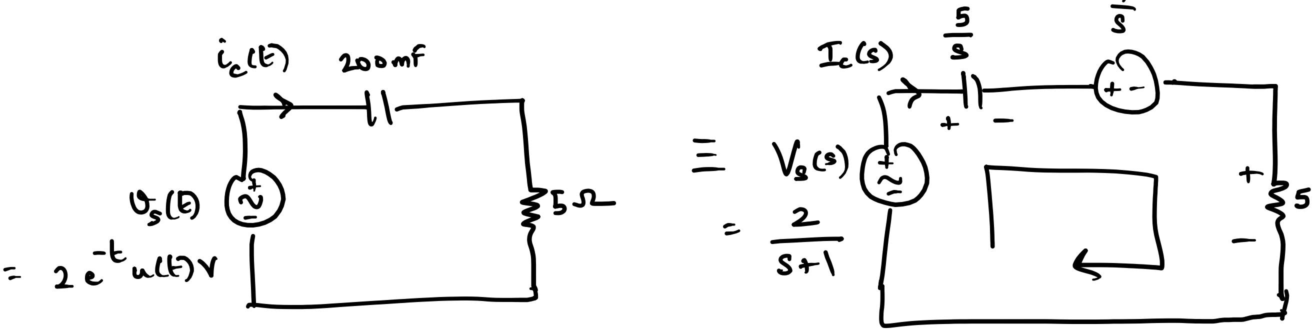
$$\Rightarrow \quad V_c(s) = \frac{I_c(s)}{sC} + \frac{v_c(0^-)}{s}$$

$\Downarrow$ voltage source



Redo the previous problem

$$v_c(0^-) = 1\text{ V}$$



$$+V_s(s) - I_c(s)\times\frac{5}{s} - \frac{1}{s} - I_c(s)\times 5 = 0$$

$$\Rightarrow \quad I_c(s)\left(\frac{5}{s} + 5\right) = \frac{2}{s+1} - \frac{1}{s}$$

$$\Rightarrow \quad I_c(s)\times 5\left(\frac{1+s}{s}\right) = \frac{2}{s+1} - \frac{1}{s}$$

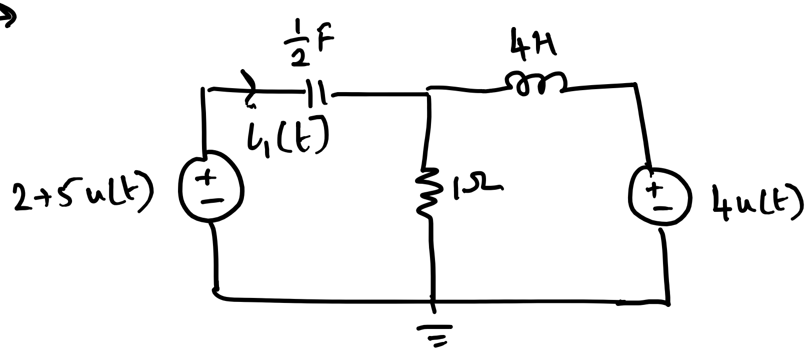
$$\Rightarrow \quad I_c(s) = \frac{2/5\; s}{(s+1)^2} - \frac{1/5}{(s+1)} = \frac{\frac{2}{5}(s+1-1)}{(s+1)^2} - \frac{1/5}{s+1}$$

2

$$\Rightarrow I_c(s) = \frac{2/5}{s+1} - \frac{2/5}{(s+1)^2} - \frac{1/5}{(s+1)}$$

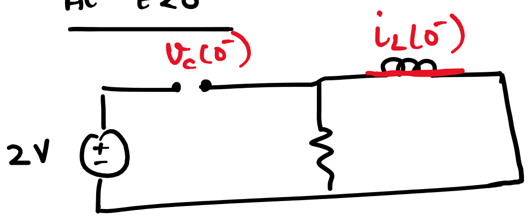
$$= \frac{1/5}{s+1} - \frac{2/5}{(s+1)^2}$$

$$\Rightarrow i_c(t) = \frac{1}{5} e^{-t} u(t) - \frac{2}{5} t e^{-t} u(t) \quad A$$



Yes there are initial conditions

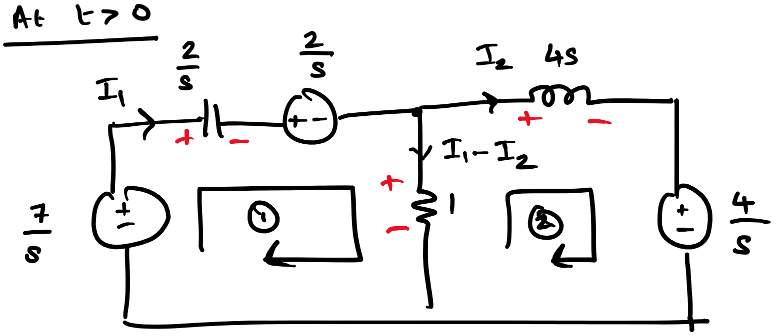
At $t<0$



$i_L(0^-) = 0\ A$

$v_c(0^-) = +2\ V$

At $t>0$



mesh (1)

$$+\frac{7}{s} - I_1 \times \frac{2}{s} - \frac{2}{s} - (I_1 - I_2)1 = 0$$

$$I_1\left(\frac{2}{s}+1\right) + I_2(-1) = \frac{5}{s} \quad \text{— (1)}$$

Mesh (2)

$$+1(I_1 - I_2) - 4s \times I_2 - \frac{4}{s} = 0$$

$$I_1(1) + I_2(-1-4s) = \frac{4}{s} \quad \text{— (2)}$$

$$(1) \times (4s+1) : \left[I_1\left(\frac{2}{s}+1\right) - I_2 = \frac{5}{s} \right] \times (4s+1)$$

$$I_2(1) - I_2(4s+1) = \frac{4}{s}$$

$$I_1\left(\frac{2}{s}+1\right)(4s+1) - I_2(4s+1) = \frac{5}{s}(4s+1)$$

$$(-)\; I_1(1) \overset{(+)}{-} I_2(4s+1) \overset{(-)}{=} \frac{4}{s}$$

$$I_1\left[\left(\frac{2}{s}+1\right)(4s+1) - 1\right] = \frac{5}{s}(4s+1) - \frac{4}{s}$$

$$\Rightarrow \quad I_1\left(8 + 4s + \frac{2}{s} + 1 - 1\right) = 20 + \frac{1}{s}$$

$$\Rightarrow \quad I_1 = \frac{20 + 1/s}{4s + \frac{2}{s} + 8} = \frac{20s+1}{4s^2+8s+2}$$

$$I_1 = \frac{20\left(s+\frac{1}{20}\right)}{4\left(s^2+2s+\frac{1}{2}\right)} = \frac{5\left(s+\frac{1}{20}\right)}{s^2+2s+\frac{1}{2}}$$

Roots of $s^2+2s+\frac{1}{2} = 0$

$$s = \frac{-2 \pm \sqrt{4 - 4\times\frac{1}{2}}}{2} = \frac{-2\pm\sqrt{2}}{2} = -1 \pm \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad I_1(s) = \frac{5\left(s+\frac{1}{20}\right)}{\left(s+1-\frac{1}{\sqrt{2}}\right)\left(s+1+\frac{1}{\sqrt{2}}\right)} = \frac{A}{\left(s+1-\frac{1}{\sqrt{2}}\right)} + \frac{B}{\left(s+1+\frac{1}{\sqrt{2}}\right)}$$

$$\Rightarrow \quad 5\left(s+\frac{1}{20}\right) = A\left(s+1+\frac{1}{\sqrt{2}}\right) + B\left(s+1-\frac{1}{\sqrt{2}}\right)$$

let $s = -1-\frac{1}{\sqrt{2}}$

$$\Rightarrow \quad 5\left(-1-\frac{1}{\sqrt{2}}+\frac{1}{20}\right) = B\left(-1-\frac{1}{\sqrt{2}}+1-\frac{1}{\sqrt{2}}\right) = -B\left(\frac{2}{\sqrt{2}}\right)$$

$$\Rightarrow \quad B = -5.86$$

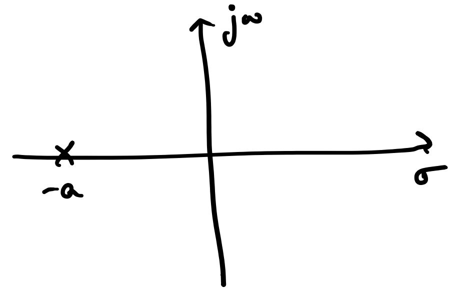
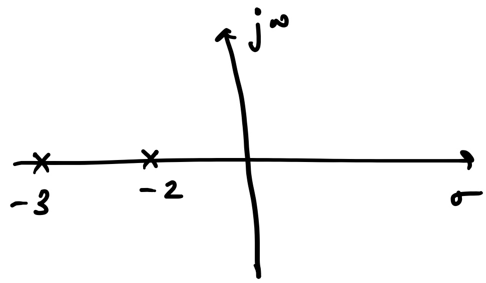
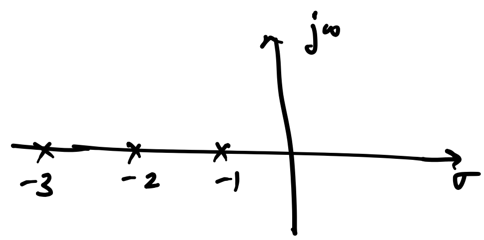
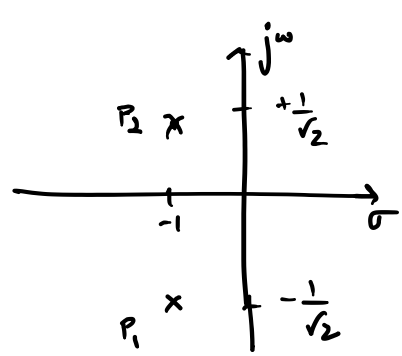
Compare coeff of s

$$5 = A + B \quad \Rightarrow \quad A = 5 - B = 10.86$$

$\Rightarrow I_1(s) = \dfrac{10.86}{s+0.29} + \dfrac{-5.86}{s+1.7}$ (5)

$\Rightarrow i_1(t) = 10.86\, e^{-0.29t}\, u(t) - 5.86\, e^{-1.7t}\, u(t)$ A

Stability Criterion

$f(t)$	$F(s)$	Complex s plane
$e^{-at}\,u(t)$	$\frac{1}{s+a}$	 stable
$3e^{-2t}u(t) + 4e^{-3t}u(t)$	$\frac{3}{s+2} + \frac{4}{s+3}$	
	$\frac{8}{(s+1)(s+2)(s+3)}$	 Stable
	$\frac{8}{\left(s+1+\frac{1}{\sqrt{2}}\right)\left(s+1-\frac{1}{\sqrt{2}}\right)}$	Poles: $-1-\frac{1}{\sqrt{2}}$, $-1+\frac{1}{\sqrt{2}}$ All left half plane so stable
$A e^{-t} e^{-\frac{1}{\sqrt{2}}jt} u(t) +$ $B e^{-t} e^{+\frac{1}{\sqrt{2}}jt} u(t)$	$\frac{8}{\left(s+1+\frac{1}{\sqrt{2}}j\right)\left(s+1-\frac{1}{\sqrt{2}}j\right)}$ $P_1 = -1-\frac{1}{\sqrt{2}}j$ $P_2 : -1+\frac{1}{\sqrt{2}}j$	

Summarize

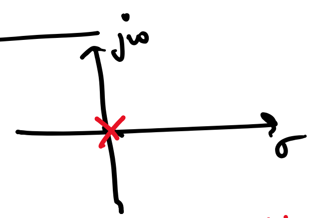
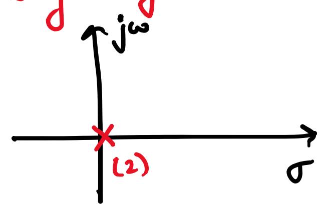
$$F(s) = k \frac{(s-z_1)(s-z_2)\cdots(s-z_n)}{(s-p_1)(s-p_2)\cdots(s-p_m)}, \quad m>n$$

$z_1, z_2, z_3 \cdots z_n \rightarrow$ complex frequencies called **zeros**

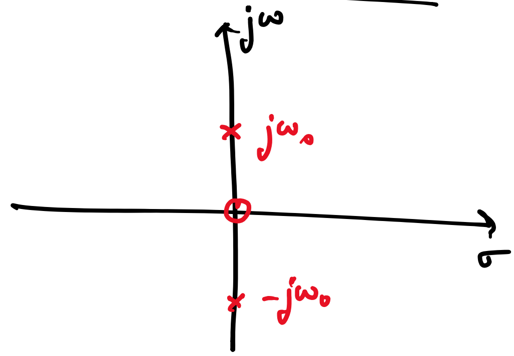
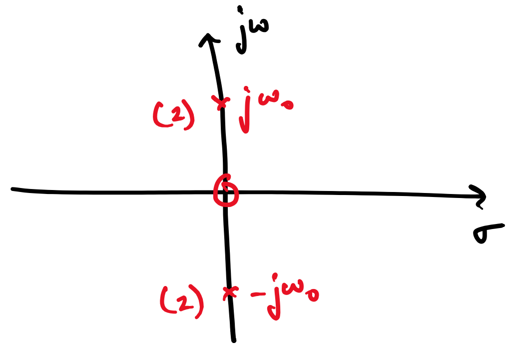
$p_1, p_2, \ldots, p_m \longrightarrow$ complex frequencies are called **poles**

$k =$ Scaling constant

(1) If all poles lie on the left half s plane, then system is stable.

$f(t)$	$F(s)$	
$u(t)$	$\frac{1}{s}$	 Marginally stable
$t\,u(t)$	$\frac{1}{s^2}$	 Unstable

(2) 1^{st} pole @ $s=0$ marginaly stable but higher order/repeated poles are unstable

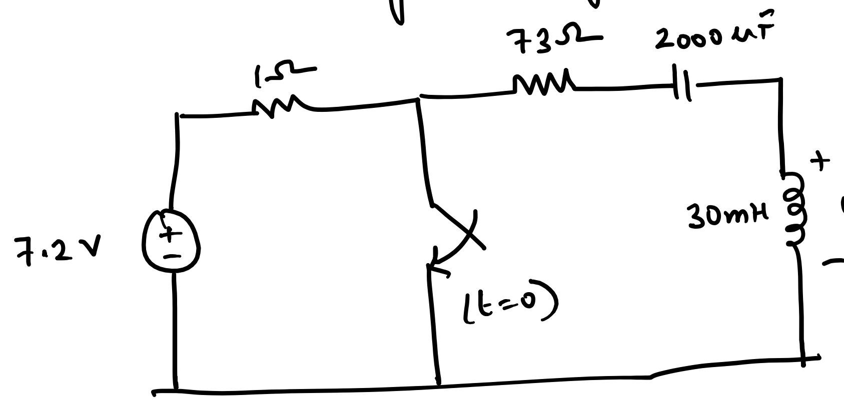
$f(t)$	$F(s)$	Complex s plane
$\cos(\omega_0 t)\,u(t)$	$\dfrac{s}{s^2+\omega_0^2} = \dfrac{s}{(s+j\omega_0)(s-j\omega_0)}$	 Marginally stable
$\dfrac{t}{2\omega_0}\sin(\omega_0 t)\,u(t)$	$\dfrac{s}{(s^2+\omega_0^2)^2}$	 Unstable

$$\boxed{t f(t) \;—\; -\frac{d}{ds}F(s)}$$

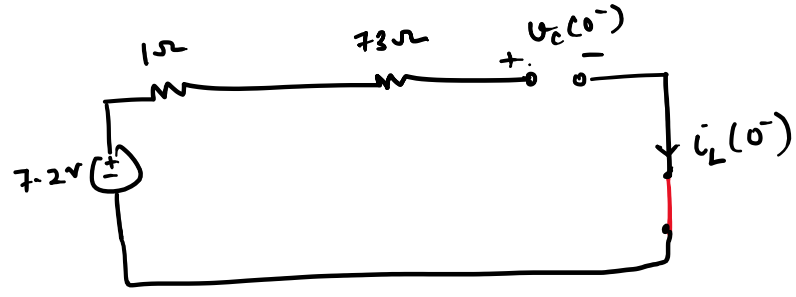
- 1^{st} order conjugate pair of poles is marginally stable but higher order repeated poles unstable

(9)

→ Find $v(t \geq 0)$ for the following circuit



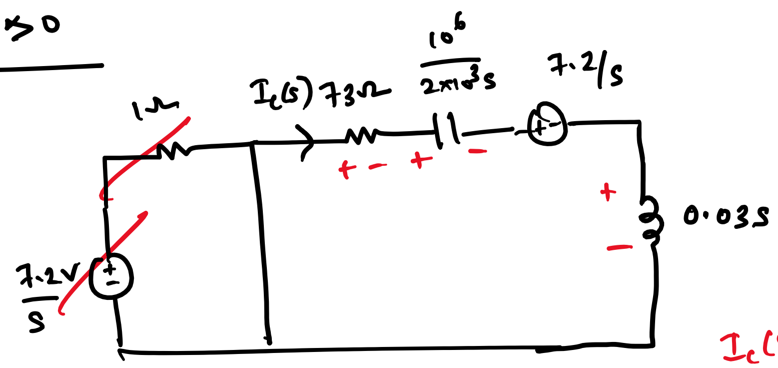
At t<0



$v_c(0^-) = 7.2V$

$i_L(0^-) = 0$ A

At t>0



$I_c(s) = \dfrac{-7.2/s}{73 + \frac{500}{s} + 0.03s}$

$I_c(s)\left(-73 - \frac{500}{s} - 0.03s\right) - 7.2/s = 0 \Rightarrow$

$$I_c(s) = \frac{-7.2}{0.03s^2 + 73s + 500} = \frac{-7.2/0.03}{s^2 + \frac{73}{0.03}s + \frac{500}{0.03}}$$

$$= \frac{-240}{(s+2426.1)(s+6.9)} = \frac{A}{(s+2426.1)} + \frac{B}{(s+6.9)}$$

$$\Rightarrow -240 = A(s+6.9) + B(s+2426.1)$$

Put $s = -6.9$

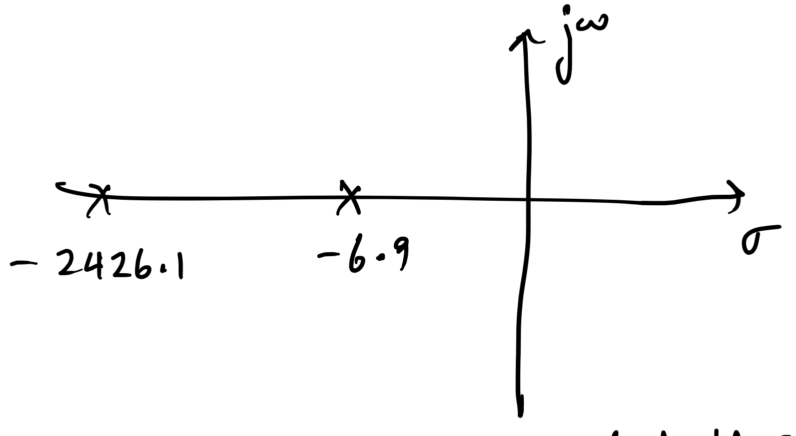
$\Rightarrow -240 = A(0) + B(-6.9 + 2426.1)$

$\Rightarrow B = -0.0989$

Compare coeff of s^1

$0 = A + B \Rightarrow A = -B = +0.0989$

$$\Rightarrow I_c(s) = \frac{0.0989}{(s + 2426.1)} + \frac{-0.0989}{(s + 6.9)}$$



Poles are in the left half s plane.
So system is stable

$$i_c(t) = 0.0989\, e^{-2426.1t}\, u(t) - 0.0989\, e^{-6.9t}\, u(t)\ A.$$