ECE250: Signals and Systems
Practice Sheet 6

1. (CO4) Consider a signal $y(t)$ which is related to two signals $x_1(t)$ and $x_2(t)$ by

$$y(t) = x_1(t-2) * x_2(-t+3)$$

where

$$x_1(t) = e^{-2t}u(t) \text{ and } x_2(t) = e^{-3t}u(t)$$

Given that

$$e^{-at}u(t) \longleftrightarrow \tfrac{1}{s+a}, \qquad Re\{s\} > -a$$

use properties of the Laplace transform to determine the Laplace transform $Y(s)$ of $y(t)$.

2. (CO4) A causal LTI system S with impulse response $h(t)$ has its input $x(t)$ and the output $y(t)$ related through a linear constant-coefficient differential equation of the form

$$\tfrac{d^3y(t)}{dt^3} + (1+\alpha)\tfrac{d^2y(t)}{dt^2} + \alpha(\alpha+1)\tfrac{dy(t)}{dt} + \alpha^2 y(t) = x(t)$$

(a) If

$$g(t) = \tfrac{dh(t)}{dt} + h(t)$$

how many poles does $G(s)$ have ?

(b) For what real values of the parameter α is system S to be stable ?

3. (CO4) A continuous-time signal $x(t)$ is obtained at the output of an ideal low pass filter with cut off frequency $\omega_c = 1000\pi$. If impulse-train sampling is performed on $x(t)$, which of the following sampling periods would guarantee that $x(t)$ can be recovered from its sampled version using an appropriate low pass filter?
(a) $T = 0.5 \times 10^{-3}$
(b) $T = 2 \times 10^{-3}$
(c) $T = 10^{-4}$

4. (CO4) Consider the signal

$$x(t) = e^{-5t}u(t-1)$$

and denote its Laplace transform $X(s)$

(a) Evaluate $X(s)$ using general equation of Laplace transform and specify its region of convergence.

(b) Determine the values of finite numbers A and t_0, such that Laplace transform $G(s)$ of

$$g(t) = Ae^{-5t}u(-t - t_0)$$

has the same algebraic form as $X(s)$. What is the region of convergence corresponding to $G(s)$?

5. (C05) Given that

$$e^{-at}u(t) \longleftrightarrow \frac{1}{s+a} \qquad \mathfrak{Re}\{s\} > \mathfrak{Re}\{-a\} \tag{1}$$

determine the inverse Laplace transform of

$$X(s) = \frac{2(s+2)}{s^2 + 7s + 12} \qquad \mathfrak{Re}\{s\} > -3 \tag{2}$$

6. (C05) We are given a discrete-time linear. time-invariant,causal system with input denoted by $x[n]$ and output by $y[n]$. This system is specified by the following pair of difference equations, involving an intermediate signal $w[n]$:

$$y[n] + \frac{1}{4}y[n-1] + w[n] + \frac{1}{2}w[n-1] = \frac{2}{3}x[n], \tag{3}$$

$$y[n] - \frac{5}{4}y[n-1] + 2w[n] - 2w[n-1] = -\frac{5}{3}x[n]. \tag{4}$$

(a) Find the frequency response and the unit sample response of the system

(b) Find a single difference equation relating $x[n]$ and $y[n]$ for the system.