(b) Given that – $x(t) = \begin{cases} t, & 0 \leq t \leq 1 \\ 2-t, & 1 \leq t \leq 2 \end{cases}$

consider the signal $x_1(t) = t\,[u(t) - u(t-1)]$

$\therefore\ x(t) = x_1(t) + x_1(-t+2)$ —— (1)

we know that – $u(t) \rightleftharpoons 1/s$, $\mathrm{Re}\{s\} > 0$

$u(t-1) \rightleftharpoons e^{-s}/s$, $\mathrm{Re}\{s\} > 0$

$\therefore\ [u(t) - u(t-1)] \rightleftharpoons \dfrac{(1 - e^{-s})}{s}$, entire s-plane

Using the differentiation in s-domain property, we have –

$$t \cdot [u(t) - u(t-1)] \rightleftharpoons \frac{d}{ds}\left[\frac{1 - e^{-s}}{s}\right]$$

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~~Sol 1~~ $x_1(t) \rightleftharpoons -\left(\frac{se^{-s}-1+e^{-s}}{s^2}\right)$ ROC: entire s-plane

Using the time-scaling property, we obtain—

$$x_1(-t) \rightleftharpoons -\left(\frac{-se^{s}-1+e^{s}}{s^2}\right) \quad \text{ROC: entire s-plane}$$

Using the time shift property, we obtain—

$$x_1(-t+2) \rightleftharpoons -e^{-2s}\left(\frac{-se^{s}-1+e^{s}}{s^2}\right)$$

ROC: entire s-plane

Therefore,

$$x(t) = x_1(t) + x_1(-t+2)$$

$$\therefore X(s) = -\left(\frac{se^{-s}-1+e^{-s}}{s^2}\right) + e^{-2s}\left(\frac{-se^{s}-1+e^{s}}{s^2}\right) \rightarrow \text{(3 POINT)}$$

ROC: entire s-plane

→ (1 POINT)

SOL (2): Given that— $x(t) \rightleftharpoons X(S)$

From given facts (1) & (2), we know that $X(s)$ is of the form,

$$X(S) = \frac{A}{(S+a)(S+b)}$$

From given fact (3), one of the poles of $X(s)$ is $(-1+j)$. Since $x(t)$ is real, then the poles of $X(S)$ must occur in conjugate reciprocal pairs. Therefore—

$$a = (1-j)$$

$$b = (1+j)$$

$$\therefore X(S) = \frac{A}{(S+1-j)(S+1+j)}$$

From given fact (5), $X(0) = 8$

$$\frac{A}{(0+1-j)(0+1+j)} = 8$$

$$\therefore A = 16$$

Now, $$X(s) = \frac{16}{(s+1-j)(s+1+j)} = \frac{16}{s^2+2s+2}$$ → (2 POINT)

There are two possible case for ROC of $X(s)$.
Either $Re\{s\} < -1$ or $Re\{s\} > -1$.

From given fact (4), $e^{2t}x(t)$ is not absolutely integrable.

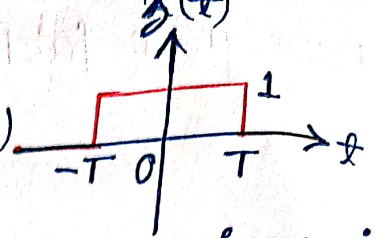
$$e^{2t} \cdot x(t) \Longleftrightarrow X(s-2)$$

$\therefore$ The ROC of $X(s-2)$ is shifted by 2 to the right. Since it is given that $e^{2t} \cdot x(t)$ is not absolutely integrable, [the ROC of $X(s-2)$ should not include the $j\omega$-axis]. This is possible only ROC is [for stable $X(s)$],

$$Re\{s\} > -1$$ → (2 POINT)

SOL(3):

Let $z(t) = \begin{cases} 1, & -T \leq t \leq T \\ 0, & \text{Otherwise} \end{cases} \rightleftharpoons Z(j\omega)$



$$\therefore Z(j\omega) = \int_{-\infty}^{\infty} z(t) \cdot e^{-j\omega t} \cdot dt = \int_{-T}^{T} e^{-j\omega t} \cdot dt = \left(\frac{e^{-j\omega T} - e^{j\omega T}}{-j\omega} \right)$$

$$= \frac{2}{\omega} \cdot \sin(\omega T) = 2T \, Sa(\omega T) \quad \text{— (1)}$$

By using the property of duality,

$$z(t) = \begin{cases} 1, & |t| \leq T \\ 0, & \text{otherwise} \end{cases} \rightleftharpoons Z(j\omega) = 2T \, Sa(\omega T) \quad \text{— (2)}$$

$t \to (-\omega)$

$\omega \to t$

$$\frac{2T \, Sa(Tt)}{2\pi} \rightleftharpoons \begin{cases} 1, & |\omega| \leq T \\ 0, & \text{Otherwise} \end{cases} \quad \text{— (3)}$$

By using the above eqn (3), we can write –

$$\because Y(j\omega) = \begin{cases} 2, & |\omega| \leq 2 \\ 0, & \text{Otherwise} \end{cases} \quad \text{(Given)}$$

$$\therefore y(t) = 2 \times \frac{2}{\pi} \times Sa(2t) = \frac{4}{\pi} Sa(2t)$$

$$= \frac{4}{\pi} \cdot \frac{\sin(2t)}{(2t)} = \frac{2}{\pi} \cdot \frac{\sin(2t)}{t} \quad \text{— (4)}$$

$$\because y(t) = x(t) \cdot \cos t \quad \text{(Given)}$$

$$\therefore x(t) = y(t) / \cos t$$

$$= \left(\frac{2}{\pi} \cdot \frac{\sin(2t)}{t} \right) / (\cos t) = \frac{2}{\pi} \cdot \frac{2 \sin t \cdot \cos t}{t \cdot \cos t}$$

$$= \left(\frac{4}{\pi} \right) \cdot \left(\frac{\sin t}{t} \right) \quad \text{— (5)}$$

(4 POINTS)

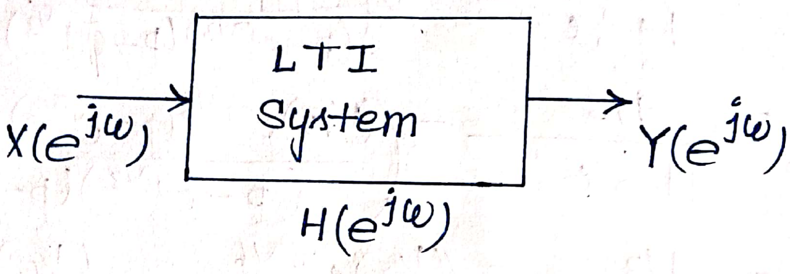
SOL(4):

(a) The frequency response of the overall system is—

$$H(e^{j\omega}) = H_1(e^{j\omega}) \times H_2(e^{j\omega})$$

$$H(e^{j\omega}) = \frac{(2-e^{-j\omega})}{(1+\frac{1}{2}e^{-j\omega})} \times \frac{1}{(1-\frac{1}{2}e^{-j\omega}+\frac{1}{4}e^{-j2\omega})}$$

$$H(e^{j\omega}) = \frac{(2-e^{-j\omega})}{(1+\frac{1}{8}e^{-j3\omega})}$$



$$\therefore\ H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{(2-e^{-j\omega})}{(1+\frac{1}{8}e^{-j3\omega})}$$

$$Y(e^{j\omega})\left[1+\frac{1}{8}e^{-j3\omega}\right] = X(e^{j\omega})\left[2-e^{-j\omega}\right]$$

Taking the inverse discrete time fourier transform, we obtain the difference equation—

$$y[n] + \frac{1}{8}y[n-3] = 2x[n] - x[n-1]$$

$\longrightarrow$ (2 POINT)

(b) Impulse response of the overall system,

$$H(e^{j\omega}) = \left(\frac{2-e^{-j\omega}}{1+e^{-j\omega}/2}\right) \times \left(\frac{1}{1-e^{-j\omega}/2+e^{-2j\omega}/4}\right)$$

Let $e^{-j\omega} \rightarrow p$

$$\Rightarrow H(e^{j\omega}) = \frac{4-2p}{2+p} \times \frac{4}{4-2p+p^2}$$

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$$\Rightarrow H(e^{j\omega}) = \frac{8(2-p)}{(2+p)(4-2p+p^2)} \times \frac{3}{3}$$

$$= \frac{8}{3}\left(\frac{-3p+6}{(p+2)(p^2-2p+4)}\right)$$

$$= \frac{8}{3}\left(\frac{(p^2-2p+4)-(p^2+p-2)}{(p+2)(p^2-2p+4)}\right)$$

$$= \frac{8}{3}\left(\frac{1}{p+2} - \frac{p-1}{p^2-2p+4}\right)$$

$$= \frac{4/3}{1+\frac{1}{2}e^{-j\omega}} - \frac{8}{3}\left(\frac{p-1}{(p-e^{j\pi/3})(p-e^{-j\pi/3})}\right)$$

$$= \frac{4/3}{1+\frac{1}{2}e^{-j\omega}} - \frac{8}{3}\left(\frac{(1/\sqrt{3})e^{-j\pi/6}(p-e^{j\pi/3})+(1/\sqrt{3})e^{j\pi/6}(p-e^{-j\pi/3})}{(p-e^{j\pi/3})(p-e^{-j\pi/3})}\right)$$

$$= \frac{4/3}{1+\frac{1}{2}e^{-j\omega}} - \left(\frac{(1/\sqrt{3})e^{-j\pi/6}}{p-e^{-j\pi/3}} + \frac{(1/\sqrt{3})e^{j\pi/6}}{p-e^{j\pi/3}}\right) \times \frac{8}{3}$$

$$= \frac{4/3}{1+\frac{1}{2}e^{-j\omega}} + \left(\frac{1}{\sqrt{3}}\,\frac{e^{j\pi/6}}{e^{-j\omega}-e^{j\pi/3}} + \frac{1}{\sqrt{3}}\,\frac{e^{-j\pi/6}}{e^{-j\omega}-e^{-j\pi/3}}\right) \times \frac{8}{3}$$

$$= \frac{4/3}{1+\frac{1}{2}e^{-j\omega}} + \frac{(1+j\sqrt{3})/3}{1-\frac{1}{2}e^{+j2\pi/3}e^{-j\omega}} + \frac{(1-j\sqrt{3})/3}{1+\frac{1}{2}e^{j\pi/3}e^{-j\omega}}$$

$\Rightarrow$ Taking Inverse Fourier Transform :

$$h[n] = \frac{4}{3}\left(\frac{-1}{2}\right)^n u[n] + \frac{1+j\sqrt{3}}{3}\left(\frac{1}{2}e^{j2\pi/3}\right)^n u[n] + \frac{1-j\sqrt{3}}{3}\left(\frac{-1}{2}e^{j\pi/3}\right)^n \times u[n]$$

$\rightarrow$ (2 POINT)