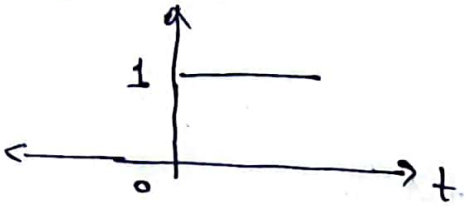
Solution

Practice sheet 1 (Signal and Systems)

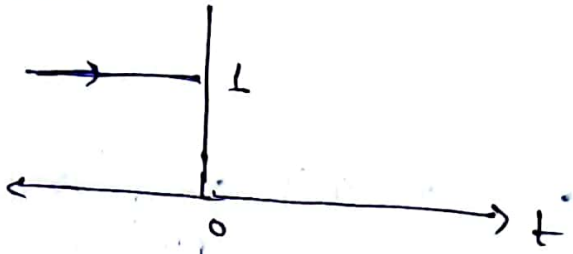
Soln 1:-

(a) $U(-t+3)$

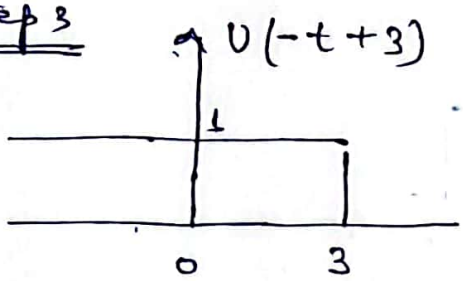
Step 1 $U(t)$



Step 2 $U(-t)$

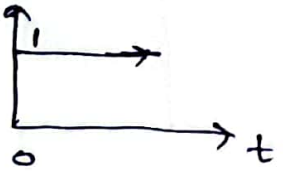


Step 3



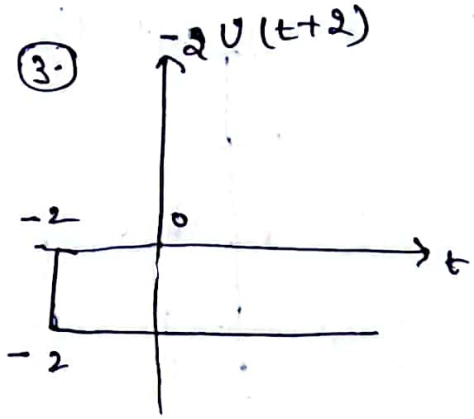
(b) $-2\,U(t+3)$

(1) $U(t)$



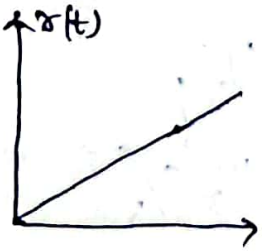
(2) $U(t+3)$

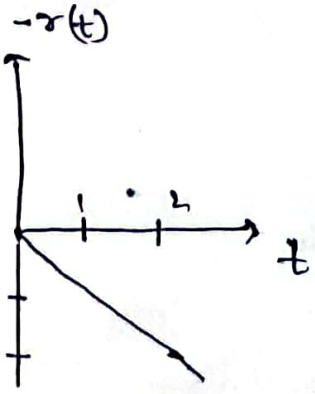
(3) $-2\,U(t+2)$

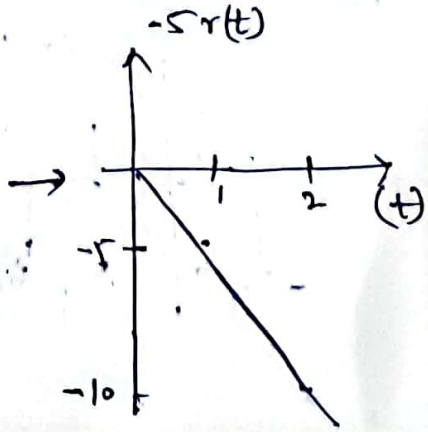


(c) $-5\,r(t)$

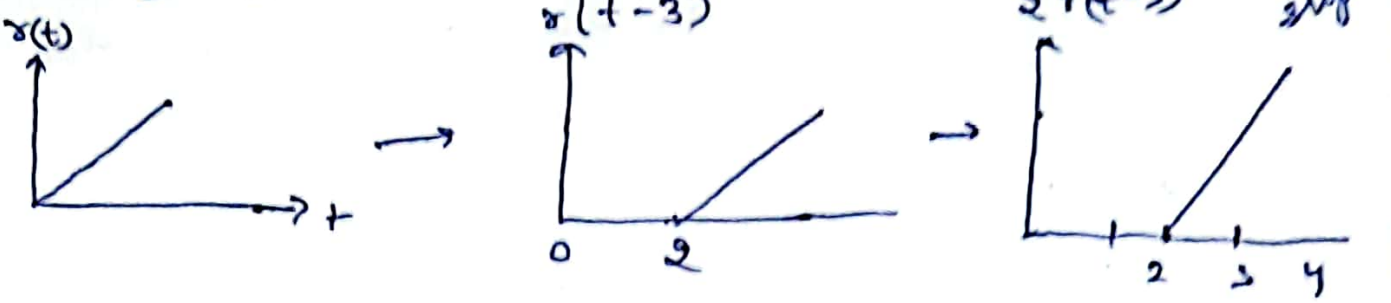
$x(t) = -5r(t)$





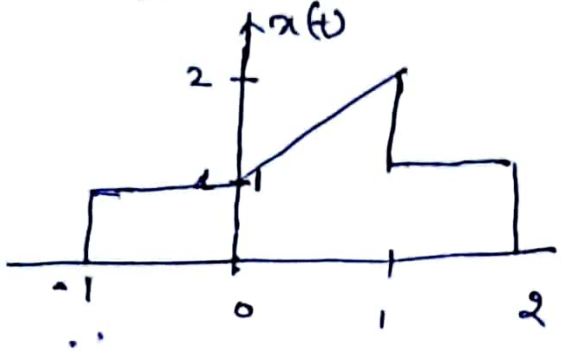


(d) $2r(t-3)$

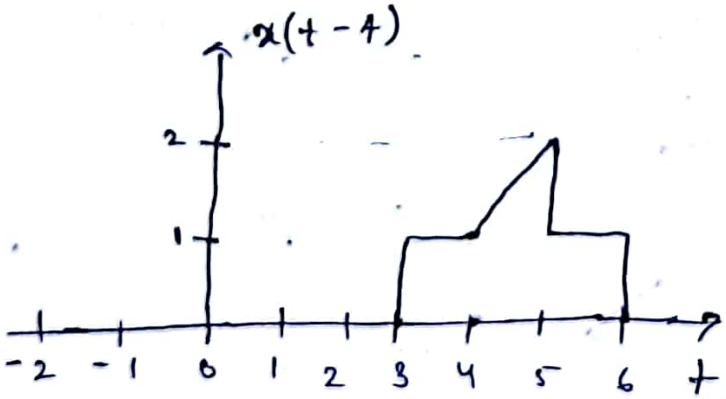


Soln 2

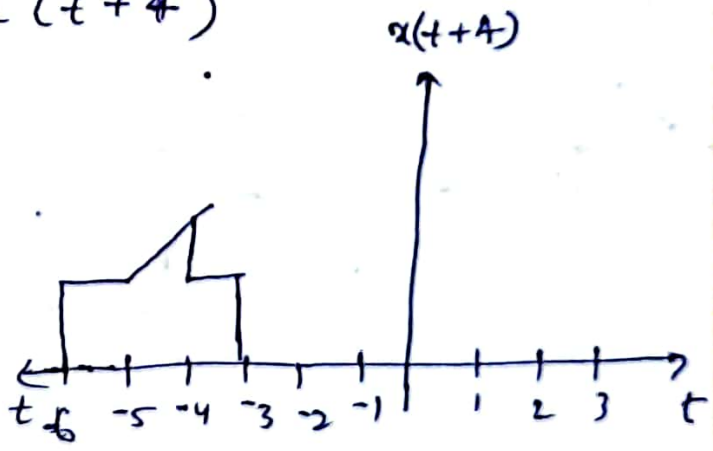
(a) ~~x(t)~~ $x(t-4)$ and $x(t+4)$



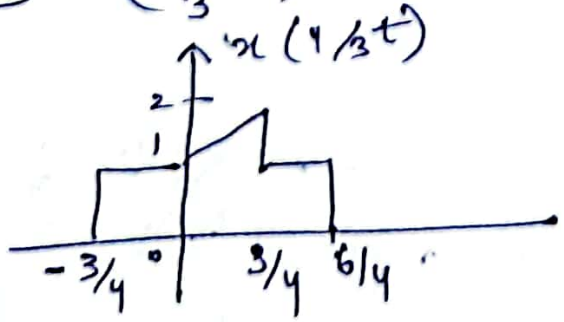
(i) $x(t-4)$



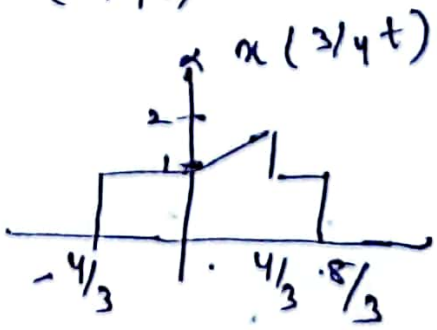
(ii) $x(t+4)$



(b) $x\left(\frac{4}{3}t\right)$

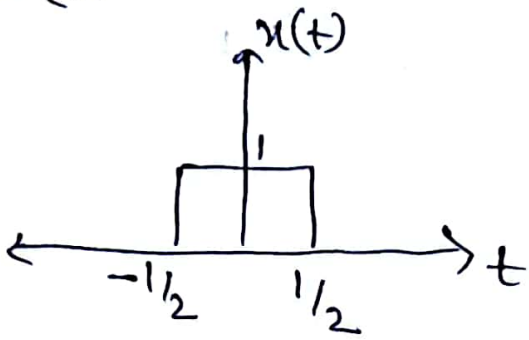


$x(3/4t)$

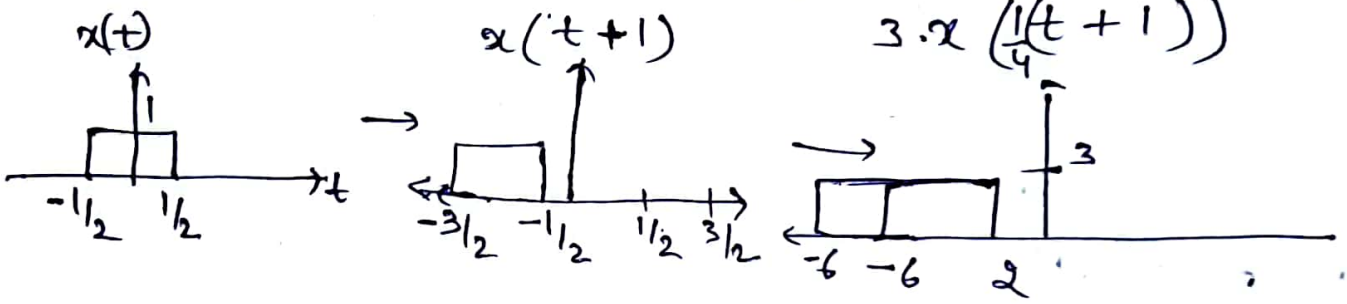


(3.) $x(t) = \begin{cases} 1 & \text{for } |t| \le 1/2 \\ 0 & \text{for } |t| > 1/2 \end{cases}$

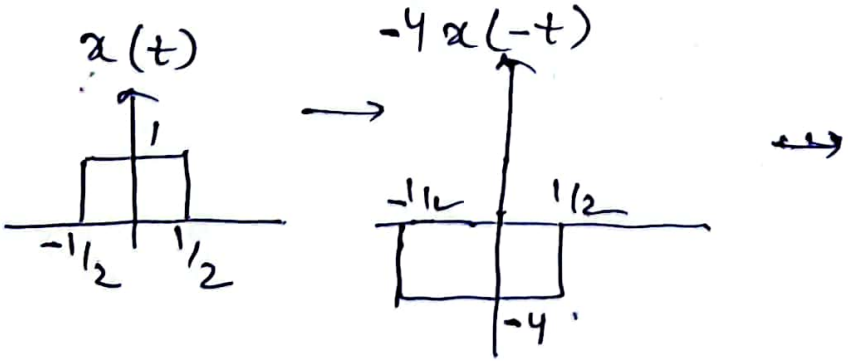
(a.) $x(t)$



(b.) $3\,x\left(\frac{t+1}{4}\right) \Rightarrow 3\,x\left(\frac{t}{4}+\frac{1}{4}\right)$

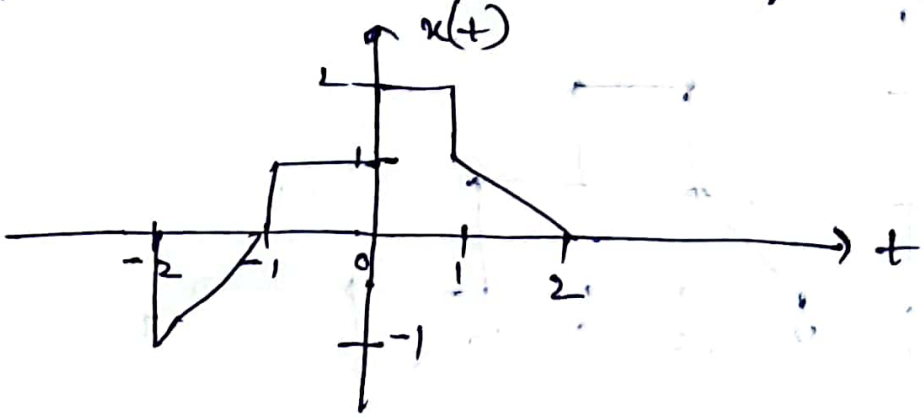


(c.) $-4\,x(-t)$

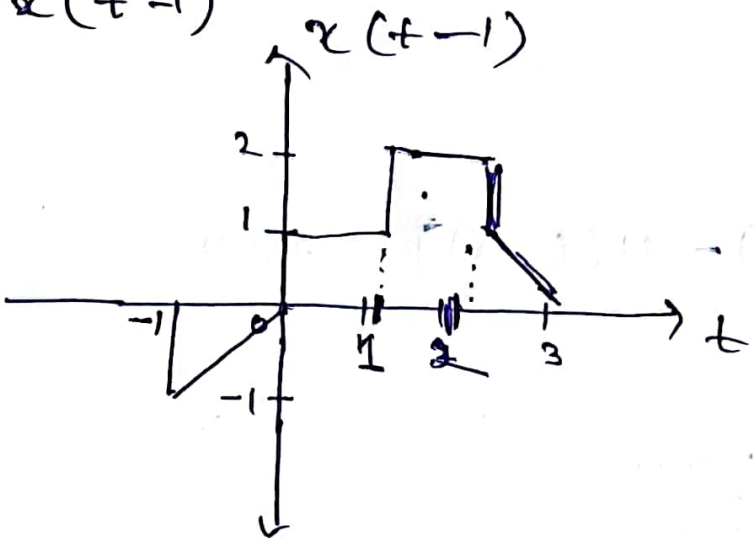


(d.)

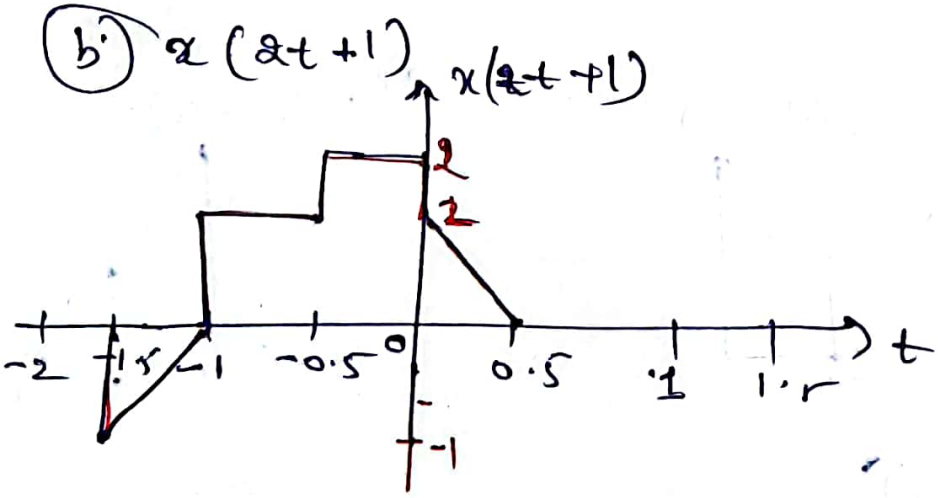
(4.) $x(t)$ is a continuous time signal



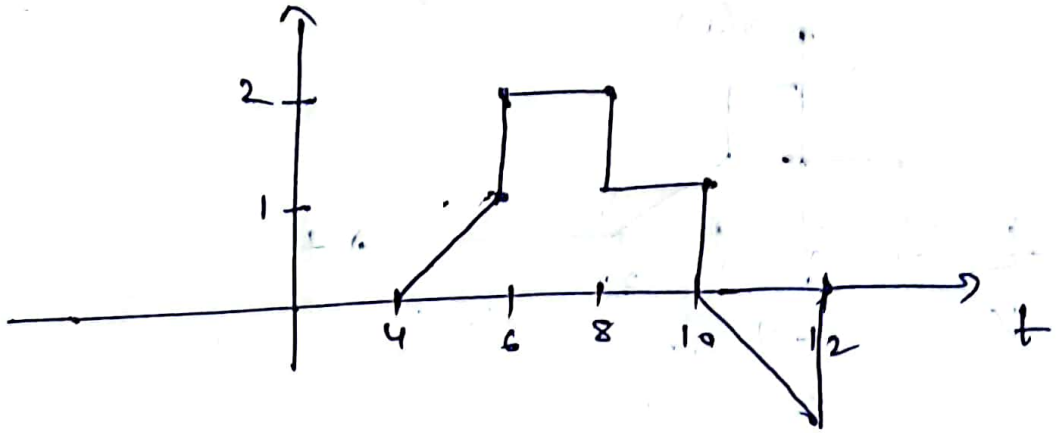
(a) $x(t-1)$



(b) $x(2t+1)$



(c) $x(4-t/2)$

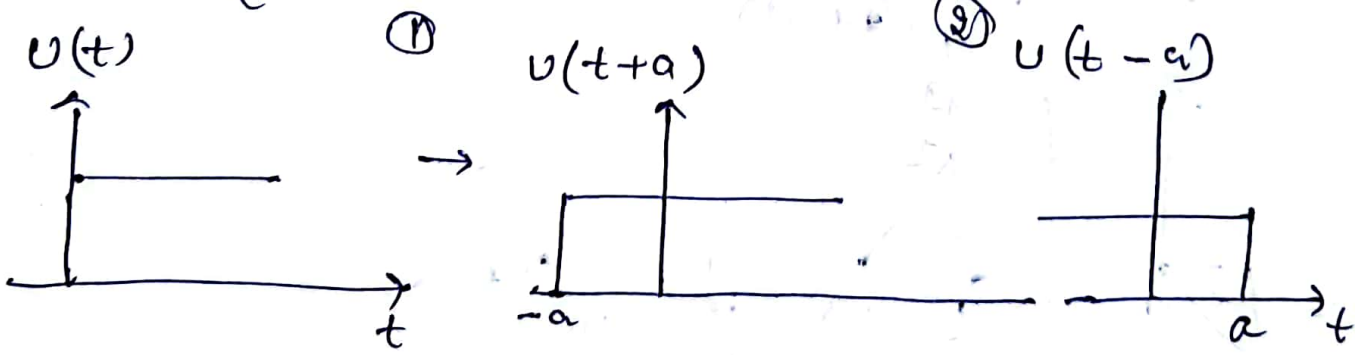


(5)

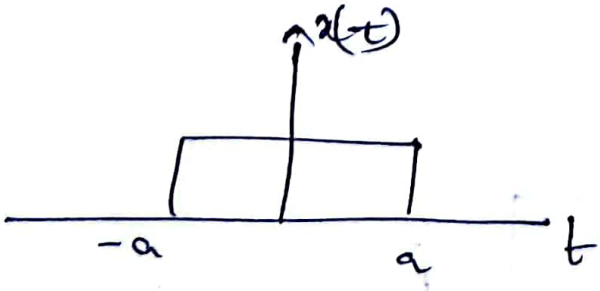
(a) $x(t) = A[u(t+a) - u(t-a)] \quad a>0$

we know that

$$u(t) \begin{cases} = 1 & t \geq 0 \\ = 0 & t < 0 \end{cases}$$



(3) $[u(t+a) - u(t-a)]$



finite energy

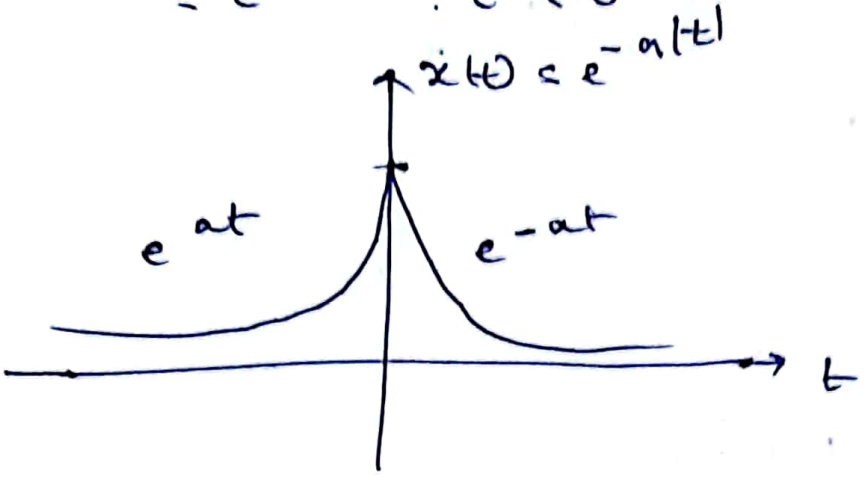
↳ Energy signal

(b) $x(t) = e^{-a|t|}$ for $a > 0$

$= e^{-at} \quad t > 0$

$= e^{+at} \quad t < 0$

Energy signal



(6.) Determine for energy signal or power signal

(a) $x(t) = \sin^2 \omega_0 t$

$\cos 2A = 1 - 2\sin^2 A$

$\sin^2 \omega_0 t = \left[\frac{1 - \cos 2\omega_0 t}{2}\right]$

For Power signal $= T$

$$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T} |x^2(t)|^2 dt$$

$$= \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |\sin^2(\omega_0 t)|^2 dt$$

$$= \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \sin^4(\omega_0 t)$$

$\because \sin^4 \omega t = \frac{1}{8}\left[3 - 4\cos 2\omega_0 t + \cos 4\omega t\right]$

$$\therefore P = \lim_{T\to\infty} \frac{1}{2T}\left[\int_{-T}^{T} 3/8 - \int_{-T}^{T} \frac{4}{8}\cos 2\omega t + \int_{-T}^{T} 1/8 \cos 4\omega t\, dt\right]$$

$$P = \lim_{T\to\infty} \frac{1}{2T}\left[\frac{3}{8}\right][T]_{-T}^{T}$$

$$\boxed{P = 3/8}$$

(b) $x(t) = t\,u(t)$

$$P = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T}[(t\cdot u(t)]^2\, dt$$

$$= \lim_{T\to\infty} \frac{1}{2T}\int_{0}^{T} t^2\, dt$$

$$= \lim_{T\to\infty} \frac{1}{2T}\cdot\frac{t^3}{3}\Big|_0^T \Rightarrow \frac{1}{2T}\cdot\frac{T^3}{3} = \frac{T^2}{6}$$

$$\boxed{P = \infty}$$

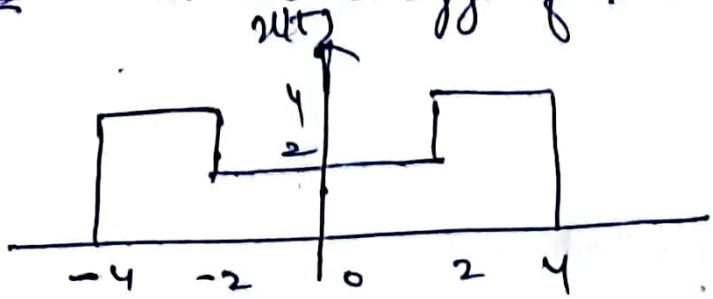
$E = \infty$

Neither energy nor power.

(c) $x(t) = e^{j[(3t + \pi/2)]}$

Power signal

Q7. Find the energy of the signal —



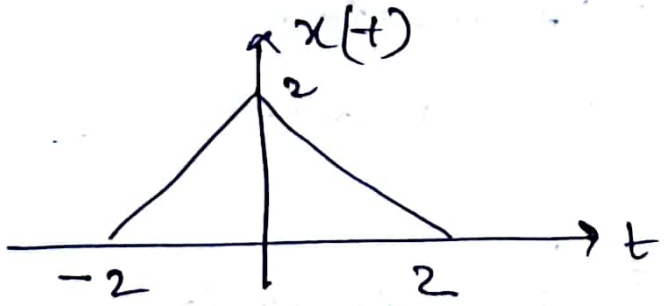
(a)

(a) $x(t) = \int_{-4}^{-2} [4]^2 \cdot dt + \int_{-2}^{2} [2]^2 \cdot dt + \int_{2}^{4} [4]^2 \cdot dt$

$= 16[-4-(-2)] + 4[2-(-2)] + 16[16-4]$

$= 80$ joule.

(b)



$E = \int_{-2}^{0} [t+2]^2 dt + \int_{0}^{3} [2-t]^2 dt$

$= \int_{-2}^{0} (t^2 + 2t + 4)\, dt + \int_{0}^{3} (4 + t^2 - 2t)\, dt$

$= \left[\frac{t^3}{3} + \frac{4t^2}{2} + 4t\right]_{-2}^{0} + \left[4t + \frac{t^3}{3} - \frac{4t^2}{2}\right]_{0}^{3}$

$= \frac{16}{3}$ Jowles.

Q8. Periodic or not ?

(a) $e^{j5\pi t}$

$\Rightarrow \cos 5\pi t + j \sin 5\pi t$

fundamental time period $= T$

$$T = \frac{2\pi}{\omega}$$

$$T = \frac{2\pi}{5\pi} = 2/5 \quad \text{Periodic}$$

(b) $\sin(10\pi t) + \cos(20\pi t)$

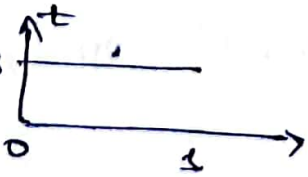
$$T_1 = \frac{2\pi}{\omega}$$

$$T_1 = \frac{2\pi}{10\pi} = \frac{1}{5} \; ; \; T_2 = \frac{2\pi}{\omega} = \frac{2\pi}{20\pi} = \frac{1}{10}.$$

$$T = \frac{LCM(T_1, T_2)}{HCF(T_1, T_2)} \quad \begin{matrix} \rightarrow \text{numerator} \\ \rightarrow \text{denominator} \end{matrix}$$

$$= \frac{LCM(1, 1)}{HCF(5, 10)} = \frac{1}{5} \quad \text{periodic}$$

(c) $3u(t) + 2\sin(2t)$

$u(t) =$ 

$\rightarrow$ Aperiodic

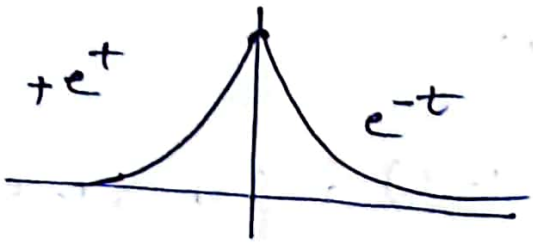
$2\sin(2t) \rightarrow$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi \} \text{ irrational.}$$

Aperiodic —

(d) $e^{-|t|}$

Aperiodic



(9.) Discrete signals are periodic or not ?

(a) $\sin(0.02\pi n)$

$\Omega = 0.02\pi$ $\quad$ $n \rightarrow$ positive integer number.

$N = \frac{2\pi m}{\omega} = \frac{2\pi \cdot 1}{0.02\pi} \leq \frac{100}{}$

Periodic signal

$\Rightarrow \boxed{\frac{\Omega_0}{2\pi} = \frac{m}{N}}$

$\rightarrow$ a rational number.

(b) $\cos\left(\frac{\pi}{2} + 0.3n\right)$

$0.3n = .2\pi f$

$f_0 = \frac{0.3}{2\pi} = \frac{3}{20\pi} \rightarrow$ not rational number.

(10.) Given

$$x(t) = 0, \quad t < 3$$

Value of t for which signal is guaranteed to be 0.

(a) $x(1-t)$

$$x(1-t) = \begin{cases} 0 & ; 1-t < 3 \\ 0 & , -2 < t \end{cases}$$

for $t > -2$ signal is zero.

(b) $x(3t)$

$$x(t) = \begin{cases} 0 & 3t < 3 \\ & t < 1 \end{cases}$$

for $t > 1$ $\quad x(t) = 0$

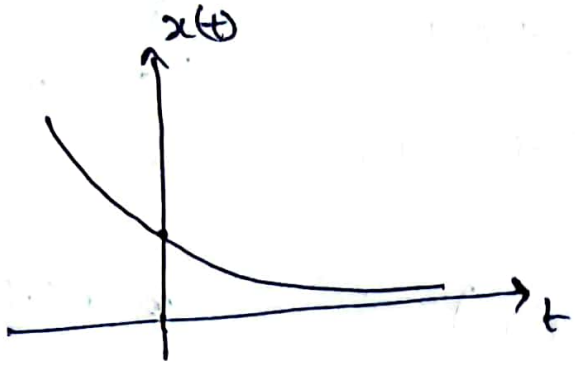
(c) $x(2 - t/3)$

$$x(t) = \begin{cases} 0 & , 2 - t/3 < 0 \\ & 2 < t/3 \end{cases}$$

$\boxed{t > 6}$

$\hookrightarrow x(t) = 0$

11. (a) $x(t) = e^{-3t}$ Signal is even or odd.



Neither even nor odd.

$x(-t) = e^{3t}$

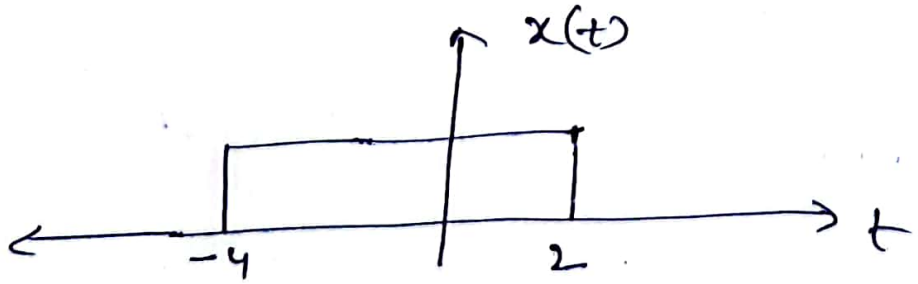
(1) $x(-t) \neq x(t)$ → Not even

(2) $x(-t) \neq -x(t)$ → Not odd.

(b) $x(t) = 3e^{j4\pi t}$

Similarly, $x(t)$ neither even nor odd

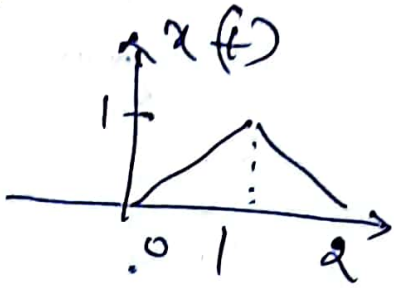
(c) $x(t) = [u(t+4) - u(t-2)$



Not symmetric about the origin → Neither even nor odd.

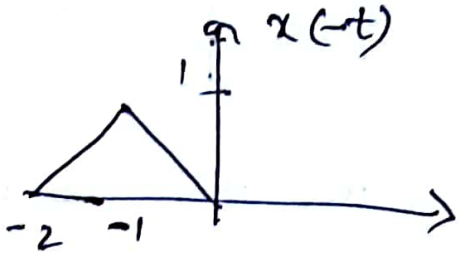
Q12. Sketch the even & odd components

$$x(t) = \begin{cases} t & 0 \le t \le 1 \\ 2-t & 1 \le t \le 2 \end{cases}$$



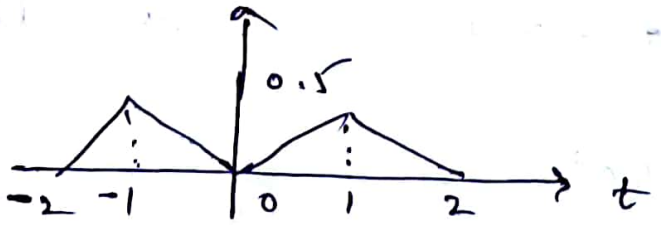
$$x_e(t) = \frac{1}{2}\left(x(t) + x(-t)\right)$$

$$x_o(t) = \frac{1}{2}\left(x(t) - x(-t)\right)$$



Even part

$x_e(t) =$



Odd part

$x_o(t) =$