Lecture 8 - Two Port Networks

Chapter 17 of textbook

→ Two port network parameters @ low frequencies

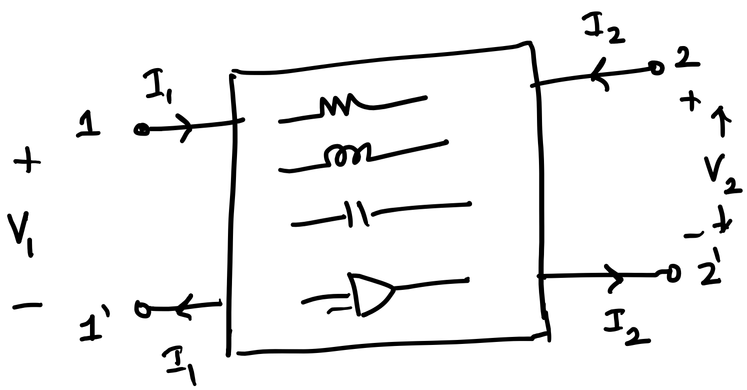
Z parameters ⇒ Impedance parameters

Y " ⇒ Admittance parameters

h " ⇒ hybrid parameters

t / ABCD " ⇒ transmission parameters

Two port network



Z parameters

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

} Frequency Domain (no time domain)

⇓

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \qquad Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \qquad Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

→ find $[Z]$

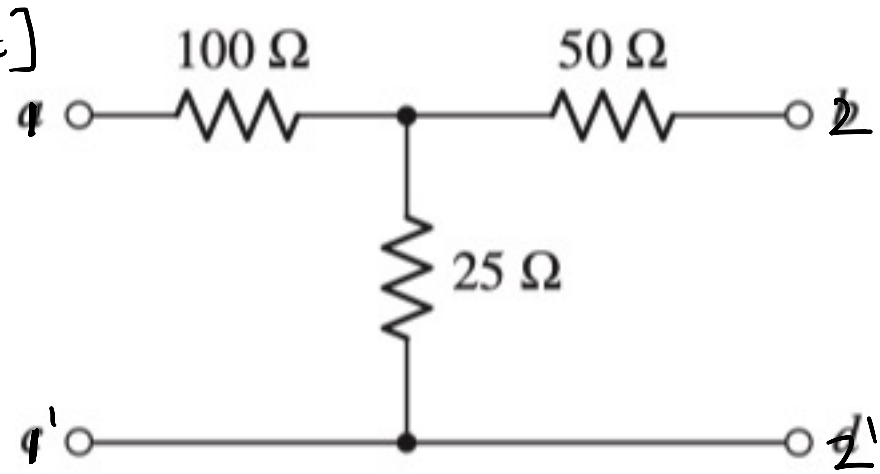
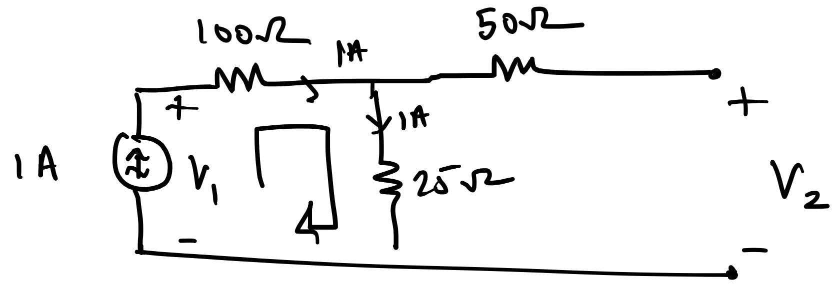


FIGURE 17.56

Soln: (I) $I_2 = 0$, $I_1 = 1\ A$



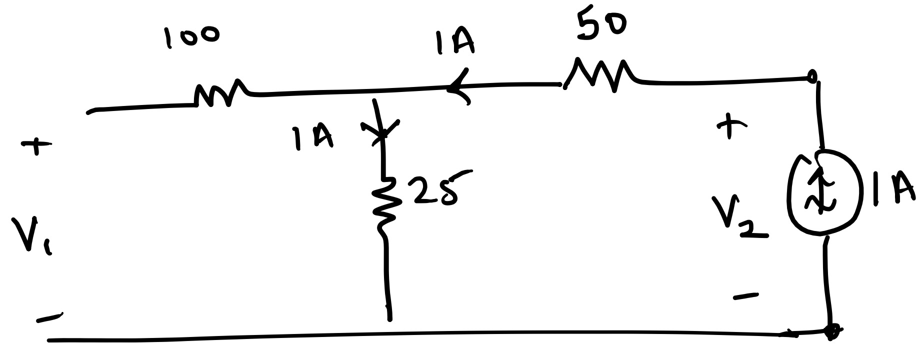
$$V_2 = 25\,\Omega \times 1\,A = 25\ V$$

$$+V_1 - 100 \times 1 - 25 \times 1 \Rightarrow V_1 = 125\ V$$

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \left.\frac{125\ V}{1\ A}\right|_{I_2=0} = 125\,\Omega$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \left.\frac{25\ V}{1\ A}\right|_{I_2=0} = 25\,\Omega$$

Step. 2 $\quad I_1 = 0 \;,\; I_2 = 1A$



$$V_1 = 1 \times 25 = 25V$$

$$V_2 = 50 \times 1 + 25 \times 1 = 75V$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \left.\frac{25V}{1A}\right|_{I_1=0} = 25\Omega$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = 75\Omega$$

Notes

→ All Z parameters have unit of Ω

→ $\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [Z] \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$ → source

response

$I_2 \to$ open, $I_1 \to$ source

$I_1 \to$ open, $I_2 \to$ source

→ [Z] also called open circuit parameters

Y parameters / Short circuit parameters

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

$$\Downarrow$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2 = 0} \qquad Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1 = 0}$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2 = 0} \qquad Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1 = 0}$$

Notes

→ All Y parameters have $1/\Omega$ or Ω^{-1}

→ $\left.\begin{matrix} Y_{11} \\ Y_{21} \end{matrix}\right\}$ $V_2 = 0$ (short port 2), $V_1 = 1V$

$\left.\begin{matrix} Y_{12} \\ Y_{22} \end{matrix}\right\}$ $V_1 = 0$ (short port 1), $V_2 = 1V$

48. Find **y**, **z**, and **h** for both of the two-ports shown in Fig. 17.63. If any parameter is infinite, skip that parameter set.

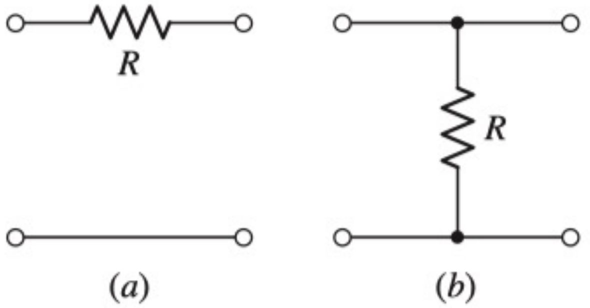
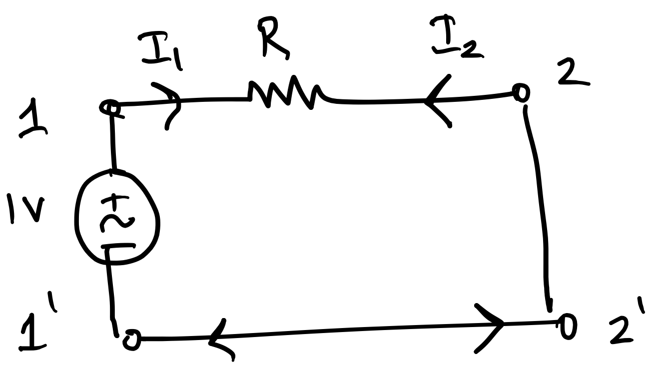


FIGURE 17.63



<u>Step. 1</u>

$$V_2 = 0 \text{ (short)}, \quad V_1 = 1 \text{ V}$$

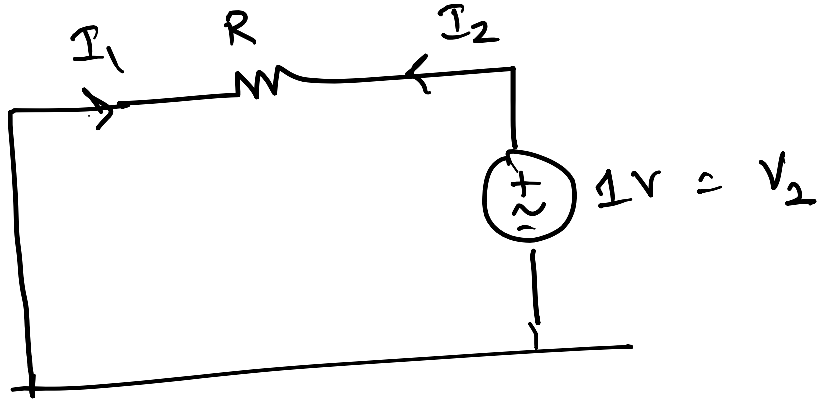
$$+1\text{V} - I_1 R = 0 \quad \Rightarrow I_1 = \frac{1}{R}$$

$$I_2 = -I_1 = \frac{-1}{R}$$

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \frac{\frac{1}{R}}{1} = \frac{1}{R} \quad (\Omega^{-1})$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = \frac{\frac{-1}{R}}{1} = \frac{-1}{R} \quad (\Omega^{-1})$$

Step. 2 $\quad V_1 = 0, \quad V_2 = 1V$



Reciprocal network

$$Y_{21} = Y_{12}, \quad Y_{11} = Y_{22}$$

$$[Y] = \begin{bmatrix} 1/R & -1/R \\ -1/R & 1/R \end{bmatrix}$$

Hybrid Parameters

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

$$\Downarrow$$

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

Integrated Electronics

$$h_{11} = \frac{V_1}{I_1}\bigg|_{V_2=0} \qquad (\Omega)$$

$$h_{21} = \frac{I_2}{I_1}\bigg|_{V_2=0} \qquad (A/A)$$

short port 2

+

$I_1 = 1A$

$$h_{12} = \frac{V_1}{V_2}\bigg|_{I_1=0} \qquad (V/V)$$

$$h_{22} = \frac{I_2}{V_2}\bigg|_{I_1=0} \qquad (\Omega^{-1})$$

open port 1

+

$V_2 = 1V$

→ Obtain h parameters

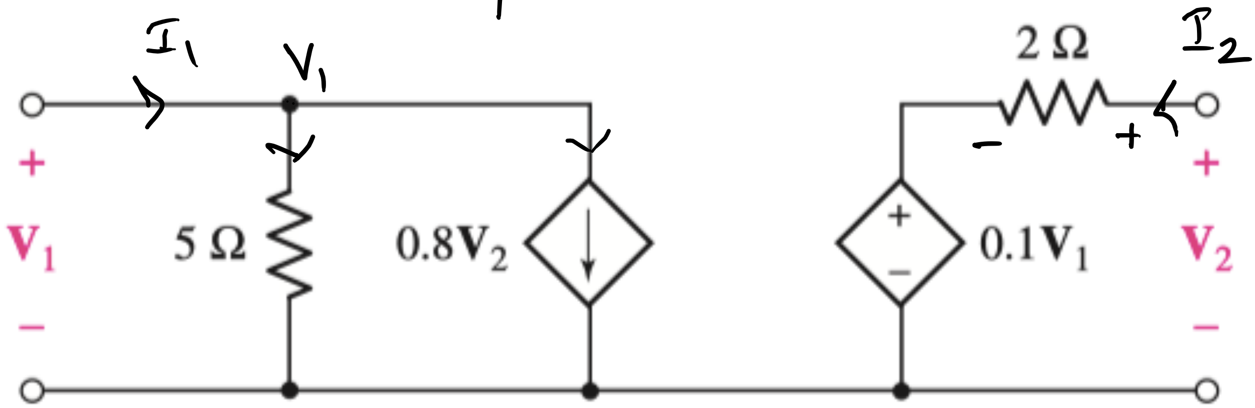


FIGURE 17.58

$$I_1 = \frac{V_1}{5} + 0.8\,V_2 \quad \text{—— (1)}$$

$$+V_2 - 2I_2 - 0.1V_1 = 0 \quad \text{—— (2)}$$

$$I_1 = 0.2V_1 + 0.8\,V_2 \quad \text{—— (3)}$$

$$I_2 = \frac{-0.1V_1}{2} + \frac{1}{2}V_2$$

$$\Rightarrow \quad I_2 = -0.05V_1 + 0.5\,V_2 \quad \text{—— (4)}$$

⇒ Y parameters

$$\left.\begin{aligned} I_1 &= Y_{11}V_1 + Y_{12}V_2 \\ I_2 &= Y_{21}V_1 + Y_{22}V_2 \end{aligned}\right\} \quad (5)$$

Comparing (5) with (3) + (4), we get

$$Y = \begin{bmatrix} 0.2 & 0.8 \\ -0.05 & 0.5 \end{bmatrix}$$

$$I_1 = 0.2 V_1 + 0.8 V_2$$

$$I_2 = -0.05 V_1 + 0.5 V_2$$

$$0.2 V_1 = I_1 - 0.8 V_2$$

$$V_1 = \frac{1}{0.2} I_1 - \frac{0.8}{0.2} V_2$$

$$V_1 = 5 I_1 - 4 V_2 \longrightarrow (6)$$

$$I_2 = -0.05 (5 I_1 - 4 V_2) + 0.5 V_2$$

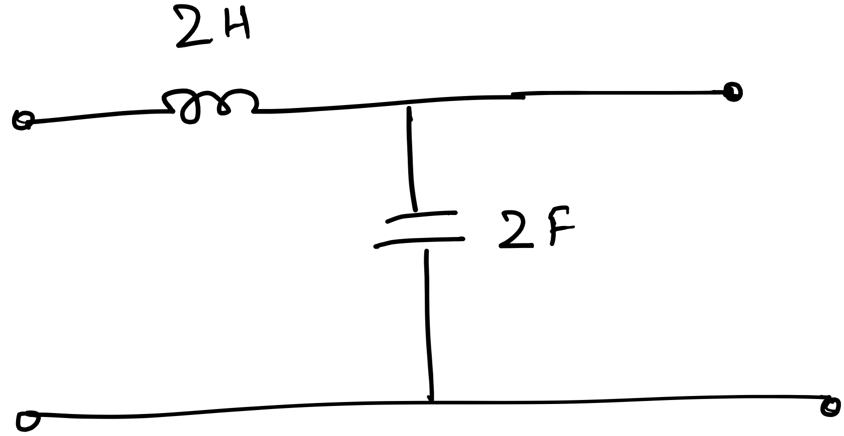
$$\Rightarrow I_2 = -0.25 I_1 + 0.7 V_2 \quad \text{---} (7)$$

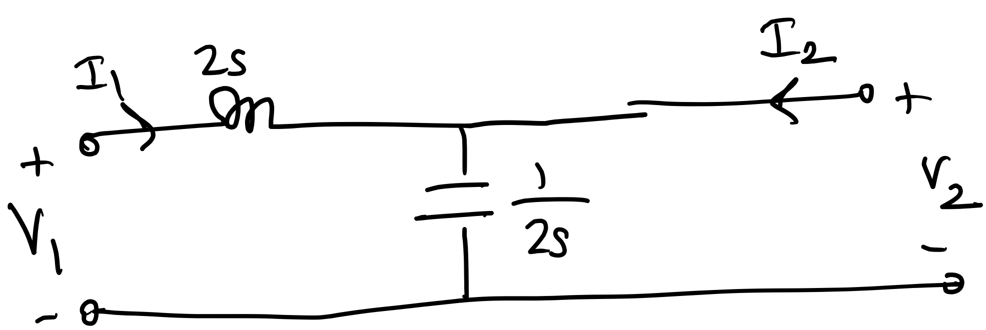
$$\left.\begin{aligned} V_1 &= h_{11} I_1 + h_{12} V_2 \\ I_2 &= h_{21} I_1 + h_{22} V_2 \end{aligned}\right\} (8)$$

Compare (8) with (6) & (7)

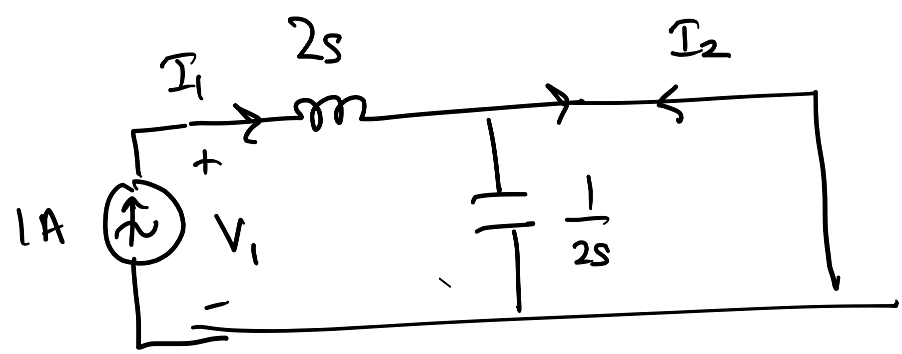
$$h = \begin{bmatrix} 5\,\Omega & -4\ V/V \\ -0.25\ A/A & 0.7\,\Omega^{-1} \end{bmatrix}$$

→ Find h parameters for the following circuit





Step 1 Short port 2 & $I_1 = 1A$ ($1\angle 0^\circ$ A)
↑
AC phasor



$$+V_1 - 1\times 2s - 0 = 0$$

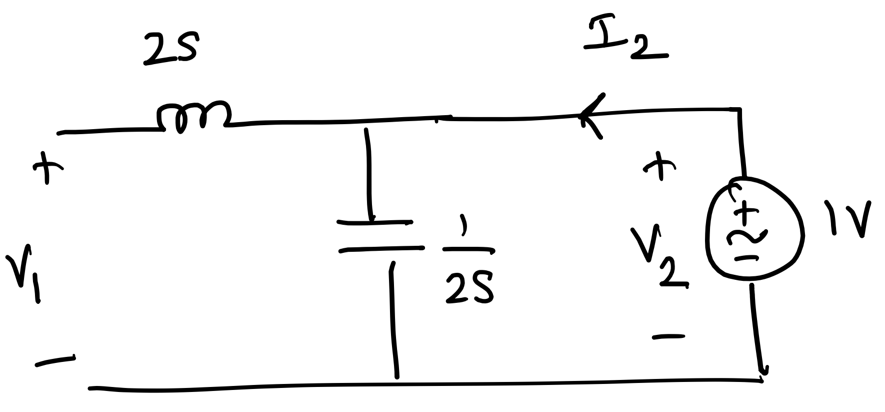
$$V_1 = 2s$$

$$I_2 = -I_1 = -1\text{ A}$$

$$\Rightarrow h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = \frac{2s}{1} = 2s \ \Omega$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = -1 \ A/A$$

<u>Step. 2</u> Open port 1 & $V_2 = 1V$



$$I_2 = \frac{V_2}{(1/2s)} = \frac{1}{(1/2s)} = 2s$$

$$V_1 = I_2 \times \frac{1}{2s} = 2s \times \frac{1}{2s} = 1 \ V$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = 1 \ V/V$$

$$h_{22} = I_2/V_2 \,\Big|_{I_1=0} = \frac{2s}{1} \ \Omega^{-1}$$

Transmission Parameters

$$V_1 = t_{11} V_2 - t_{12} I_2$$

$$I_1 = t_{21} V_2 - t_{22} I_2$$

$$\Downarrow$$

response

input

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$= \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$\begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

<u>Units</u> : $\begin{bmatrix} V/V & \Omega \\ \Omega^{-1} & A/A \end{bmatrix}$