Tutorial 4

Q1) (a) $R = 1\,k\Omega$ $\quad \zeta = ?$

$C = 10\,mF$ $\quad Q_0 = ?$

$L = 1\,H$

now,

$$\zeta = \frac{\alpha}{\omega_0} = \frac{1}{2Q_0}$$

$$Q_{op} = R \cdot \sqrt{\frac{C}{L}}$$

$$Q_0 = 10^3 \times \sqrt{\frac{10 \times 10^{-3}}{1}}$$

$$= \boxed{100} \text{ A}$$

$$\zeta = \frac{1}{2 \times 100} = \boxed{0.005} \text{ A}$$

(b) $R = 1\,\Omega$

$C = 10\,mF$

$L = 1\,H$

$$Q_0 = 1 \times \sqrt{\frac{10 \times 10^{-3}}{1}}$$

$$= \boxed{0.1} \text{ Ans}$$

$$\zeta = \frac{1}{2 \times 0.1} = \boxed{5} \text{ Ans}$$

(c) $R = 1\,k\Omega$

$C = 1\,F$

$L = 1\,H$

$$Q_0 = 10^3 \sqrt{1/1}$$

$$= \boxed{1000} \text{ A}$$

$$\zeta = \frac{1}{2 \times 1000} = \boxed{5 \times 10^{-4}} \text{ Ans}$$

(d) $R = 1\Omega$

$C = 1F$

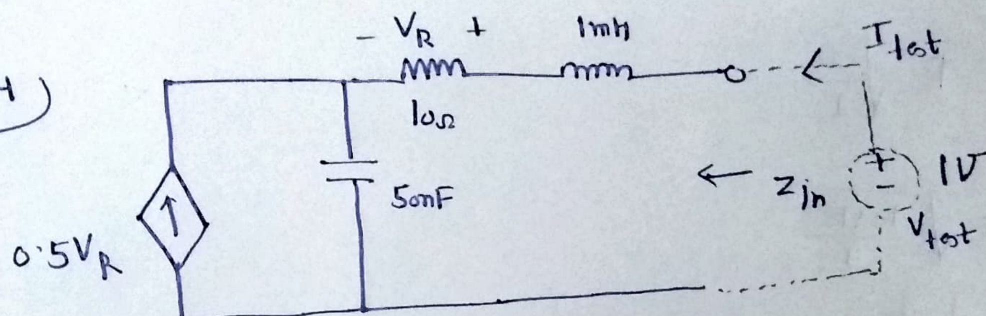
$L = 1H$

$$Q_0 = 1 \cdot \sqrt{1/1}$$

$$= \boxed{1} \text{ Ans}$$

$$\zeta = 1/2\times1 = \boxed{0.5} \text{ Ans}$$

Q4)



$Z_{in}(s) = ?$

a) $\omega_0 = ?$

b) $Q_0 = ?$

Soln $L = 1mH = 10^{-3}$ H

$$X_L = j\omega L$$

$$= j \times 10^{-3} \omega \ \Omega$$

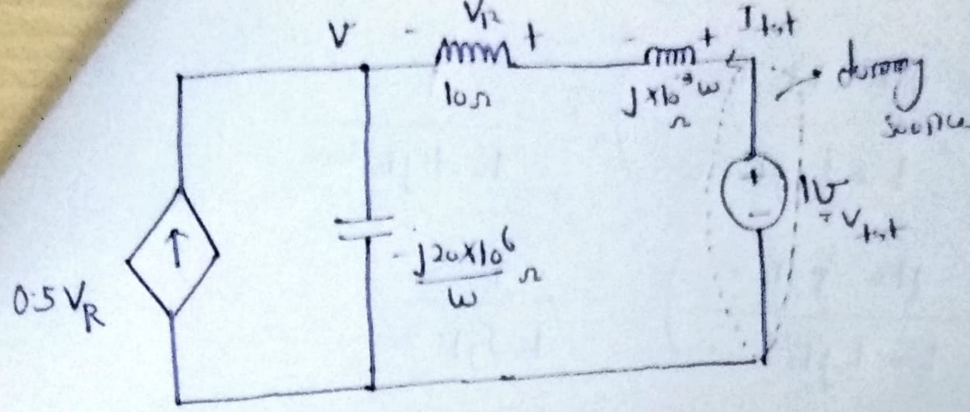
$$C = 50nF = 50 \times 10^{-9} F$$

$$X_C = 1/j\omega C$$

$$= \frac{1}{j50 \times 10^{-9}\omega} = \frac{-j20 \times 10^{6}}{\omega} \Omega$$

now, apply dummy voltage ; $V_{test} = 1V$

⇒ Need drawing circuit in frequency domain with applied changes



perform KCL at node voltage V.

$$\Rightarrow \quad 0.5V_R + \frac{1-V}{10+j10^{-3}\omega} = \frac{V}{-\frac{j20\times10^6}{\omega}} \quad \cdots (1)$$

and;

$$V_R = 10\, I_{test} \quad \cdots (2)$$

and,

$$V = (0.5V_R + I_{test}) \times -\frac{j20\times10^6}{\omega}$$

$$= (0.5\times10\, I_{test} + I_{test}) \times -\frac{j20\times10^6}{\omega}$$

$$= 6\, I_{test} \times -\frac{j20\times10^6}{\omega}$$

$$= I_{test} \times \frac{-j120\times10^6}{\omega} \quad \cdots (3)$$

Substitute the result of (3) and (2) in (1).

$$\Rightarrow \quad 0.5\times10\, I_{test} + \frac{1}{10+j10^{-3}\omega} - \frac{I_{test}\times-\frac{j20\times10^6}{\omega}}{10+j10^{-3}\omega} = \frac{I_{test}\times\frac{-j120\times10^6}{\omega}}{-\frac{j20\times10^6}{\omega}}$$

$$\Rightarrow \quad 5I_{test} + \frac{1}{10+j10^{-3}\omega} + \frac{6\,I_{test}\times j20\times10^6}{10\omega+j10^{-3}\omega^2} = 6\,I_{test}$$

$$\Rightarrow I_{test}\left(5-6+\frac{j120\times10^6}{10\omega+j10^{-3}\omega^2}\right)=\frac{-1}{10+j10^{-3}\omega}$$

$$\Rightarrow I_{test}\left(-1+\frac{j120\times10^6}{10\omega+j10^{-3}\omega^2}\right)=\frac{-1}{10+j10^{-3}\omega}.$$

$$I_{test}\left(\frac{-10\omega-j10^{-3}\omega^2+j120\times10^6}{\omega(10+j10^{-3}\omega)}\right)=\frac{-1}{10+j10^{-3}\omega}$$

$$\Rightarrow \frac{-10\omega-j10^{-3}\omega^2+j120\times10^6}{\omega(10+j10^{-3}\omega)}\times(-10+j(-10^{-3})\omega)=\frac{1}{I_{test}}$$

$$\Rightarrow \frac{-10\omega-j10^{-3}\omega^2+j120\times10^6}{-\omega}=\frac{1}{I_{test}}$$

$$\Rightarrow Z_{in}=1/I_{test}\quad\left(\because Z_{in}=V_{test}/I_{test}\ ;\ V_{test}=1V\right)$$

$$\Rightarrow \frac{-10\omega-j10^{-3}\omega^2+j120\times10^6}{\omega}=Z_{in}$$

$$\Rightarrow Z_{in}=10+j10^{-3}\omega-\frac{j120\times10^6}{\omega}$$

$$=10+j\left(10^{-3}\omega-\frac{120\times10^6}{\omega}\right)$$

to calculate the ω_0, we equate the imaginary part to 0

$$\Rightarrow 10^{-3}\omega_0-\frac{120\times10^6}{\omega_0}=0$$

$$\Rightarrow \boxed{\omega_0=346.41\times10^3\ \text{rad/s}}$$ ← Ans.

The quality factor Q_0 for a series RLC circuit is :-

$$Q_0 = \frac{\omega_0 L}{R}$$

$$= \frac{346.41 \times 10^3 \times 1 \times 10^{-3}}{10}$$

$$= \boxed{34.64} \text{ Ans}$$