Tutorial 5 Solutions

Q1) Sketch the magnitude of :-

(a) $\dfrac{4}{s^3+7s^2+12s}$

Soln: First of all bring the equation to ZPK form.

$\Rightarrow$ $$H(s) = \frac{4}{s^3+7s^2+12s}$$

$$= \frac{4}{s(s^2+7s+12)} = \frac{4}{s(s+3)(s+4)}$$

now factor out 3 and 4 to convert into ZPK form

$$\Rightarrow \frac{4}{12 \times s\,(s/3+1)(1+s/4)}$$

$$= \frac{0.33}{s(1+s/3)(1+s/4)}$$

Put $s = j\omega$:-

$$\Rightarrow H(j\omega) = \frac{0.33}{j\omega(1+j\omega/3)(1+j\omega/4)}$$

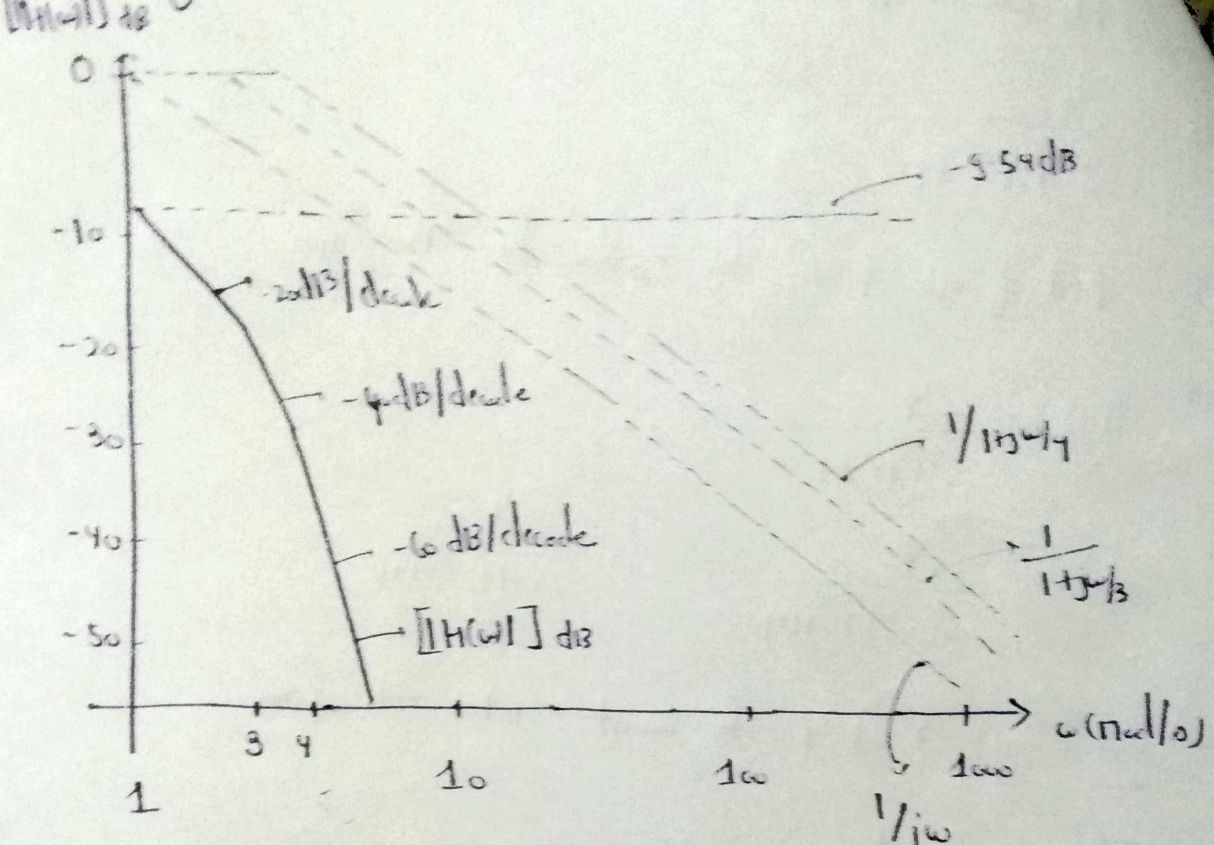
$\Rightarrow$ Magnitude of transfer function in dB is $[|H(\omega)|]$ dB

$$\Rightarrow H_{dB} = 20\log_{10}|H(j\omega)|$$

$$= 20\log_{10}\left|\frac{0.33}{j\omega \times (1+j\omega/3)(1+j\omega/4)}\right|$$

$= 20\log 0.333 - 20\log|j\omega| - 20\log|(1+j\omega/3)| - 2$

Now: $20\log 0.33 = -9.54\ dB$



Q1) (b) $\frac{s+300}{s(5s+8)}$

Soln $H(s) = \frac{s+300}{s(5s+8)}$

→ factor out 300 and 5 to reach ZPK form.

$$H(s) = \frac{300\ (1+s/300)}{8s\,(1+s/1.6)}$$

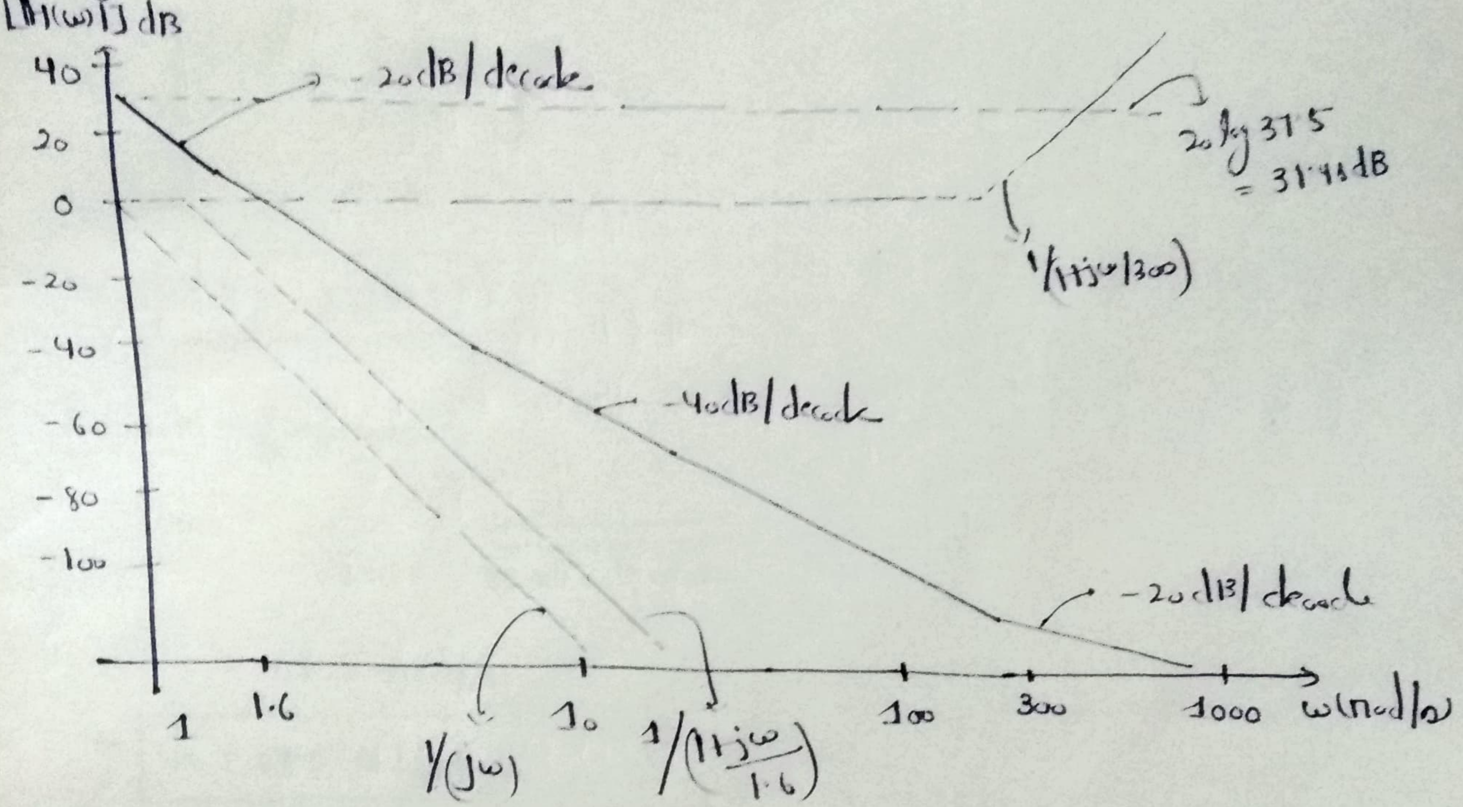
$$H(j\omega) = 37.5\ \frac{1+j\omega/300}{j\omega(1+j\omega/1.6)}$$

$$|H|_{dB} = 20 \log |H(j\omega)|$$

$$= 20 \log \left| 31.5 \frac{(1 + j\omega/300)}{j\omega(1 + j\omega/1.6)} \right|$$

$$= 20 \log 31.5 + 20 \log |1 + j\omega/300| - 20 \log |j\omega| - 20 \log |1 + j\omega/1.6|$$

$$20 \log 31.5 = 31.48 \text{ dB}$$



Circuit Component values -

$R = 1000\Omega$

$C = 0.022 F$

Q2) $L = 0.012 H$

Damping Coefficient :-

$\alpha = 1/2RC$

$= \frac{1}{2 \times 1000 \times 0.022}$

$= 0.023\ s^{-1}$

$\Rightarrow \boxed{\alpha = 0.023\ s^{-1}}$

Resonant frequency -

$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.022 \times 0.012}}$

$= 61.5\ rad/s$

$\Rightarrow \boxed{\omega_0 = 61.5\ rad/s}$

Damping factor :-

$\zeta = \alpha/\omega_0 = \frac{0.023}{61.55}$

$= 0.37 \times 10^{-3}$

$\Rightarrow \boxed{\zeta = 0.37 \times 10^{-3}}$

Frequency :- $f_0 = \frac{\omega_0}{2\pi} = \frac{61.55}{2\pi}$

$= \boxed{9.8\ Hz}$

→ Natural resonant frequency : (ω_d)

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

$$= \sqrt{(61.55)^2 - (0.025)^2}$$

$$= 61.55 \text{ rad/s}$$

$$\Rightarrow \boxed{\omega_d = 61.55 \text{ rad/s}}$$

(b) Reactance of the inductor -

$$X_L(\omega) = j\omega L$$

$$= j0.012\omega \,\Omega$$

Reactance of Capacitor :-

$$X_c(\omega) = 1/j\omega c$$

$$= -j/0.022\omega$$

$$= \frac{-j45.45}{\omega} \,\Omega$$

⇒ Equivalent impedance (Z_{eq})

$$\Rightarrow Z_{eq} = R \| X_L \| X_c$$

$$= 1000 \| j0.012\omega \| \frac{-j45.45}{\omega}$$

$$= 1000 \| \frac{j0.012\omega \times \frac{-j45.45}{\omega}}{j0.012\omega + \left(\frac{-j45.45}{\omega}\right)}$$

$$= 1000 \| \frac{0.5454}{j0.012\omega + j\left(\frac{-45.45}{\omega}\right)}$$

now, multiply numerator and denominator by ω.

$$\Rightarrow Z_{eq} = 1000 \| \frac{0.5454\omega}{j0.012\omega^2 - j45.45}$$

$$\frac{1000 \times \dfrac{0.5454\omega}{j0.012\omega^2 - j45.45}}{1000 + \dfrac{0.5454\omega}{j0.012\omega^2 - j45.45}}$$

$$= \frac{545.4\omega}{j12\omega^2 - j45450 + 0.545\omega}$$

Multiply numerator and denominator by $\frac{1}{j545.4}$ to get,

$$Z_{eq} = \frac{-j\omega}{0.022\omega^2 - j0.001\omega - 83.33}\ \Omega$$

Now, voltage $(v) = I Z_{eq}$

$$= 1\angle 0^\circ \times \frac{-j\omega}{0.022\omega^2 - j0.001\omega - 83.33}$$

$$= \frac{-j\omega}{0.022\omega^2 - j0.001\omega - 83.33}\ V$$

Now, current I_{LC} can be found using current division rule.

$$I_{LC} = I \cdot \frac{R}{R + (X_L \| X_C)}$$

$$= 1\angle 0^\circ \times \frac{1000}{1000 + \dfrac{0.5454\omega}{j0.012\omega^2 - j45.45}}$$

$$= \frac{j12\omega^2 - j45450}{j12\omega^2 - j45450 + 0.545\omega}$$

Multiply numerator and denominator by $1/j12$,

$$\Rightarrow I_{LC} = \frac{\omega^2 - 3787.5}{\omega^2 - j0.0454\omega - 3787.5} \text{ A}$$

Now, current I_L can be found using current division rule,

$$I_L = I_{LC} \frac{X_C}{X_C + X_L}$$

$$= \frac{j12\omega^2 - j45450}{j12\omega^2 - j45450 + 0.545\omega} \times \frac{\frac{-j45.45}{\omega}}{j0.012\omega - \frac{j45.45}{\omega}}$$

$$= \frac{\left(j12\omega^2 - j45450\right) \times \frac{1000}{1000}}{j12\omega^2 - j45450 + 0.545\omega} \times \frac{-j45.45}{j0.012\omega^2 - j45.45}$$

$$= \frac{-j45450}{j12\omega^2 - j45450 + 0.545\omega}$$

multiply numerator and denominator by $1/j12$,

$$I_L = \frac{-3787.5}{\omega^2 - j0.0454\omega - 3787.5}$$

Now, current I_C can be found using current division rule,

$$I_C = I_{LC} \frac{X_L}{X_C + X_L}$$

$$= \frac{j12\omega^2 - j45450}{j12\omega^2 - j45450 + 0.545\omega} \times \frac{j0.012\omega}{j0.012\omega - \frac{j45.45}{\omega}}$$

$$= \frac{j12\omega^2 - j45450 \times \left(\frac{1000}{1000}\right)}{j12\omega^2 - j45450 + 0.545\omega} \times \frac{j0.012\omega}{\left(j0.012\omega - \frac{j45.45}{\omega}\right) \times \frac{\omega}{\omega}}$$

$$\frac{j12\omega^2}{j12\omega^2 - j45450 + 0.545\omega}$$

Now multiply numerator and denominator by $\frac{1}{j12}$, we get,

$$I_c = \frac{\omega^2}{\omega^2 - j0.454\omega - 3787.5} \; A$$

The voltage V at $\omega = \omega_0$ is,

$$v(\omega_0) = \frac{-j61.55}{0.022(61.55)^2 - j0.001 \times 61.55 - 83.33}$$

$$\simeq 1000 \angle 0^\circ$$

→ I and V are verified to be in phase (without approximation)

(c) The current I_L at $\omega = \omega_0$ is,

$$I_L = \frac{-3787.5}{61.55^2 - j0.0454(61.55) - 3787.5}$$

$$\simeq -2000 \; A$$

The current I_c at $\omega = \omega_0$ is,

$$I_c = \frac{61.55^2}{61.55^2 - j0.0454(61.55) - 3787.5}$$

$$\simeq 2000 \; A$$

therefore $I_L = -I_c$ at $\omega = \omega_0$.