Endsem, Winter 2022.

1. (a) Just like electric monopoles (i.e. electric charges), there will be magnetic charges. Let the magnetic charge density be ρ_m.

Also, these magnetic charges can flow. So, there'll be magnetic current density $\vec{J}_m$.

$$\therefore \vec{\nabla} \times \vec{E} = -\vec{J}_m - \frac{\partial \vec{B}}{\partial t} \quad \text{———} \quad (1)$$

[one can use $+\vec{J}_m$ here, marks will be given]

$$\vec{\nabla} \times \vec{B} = \mu_0 \left[\vec{J} + \frac{\partial \vec{D}}{\partial t} \right] \quad \text{———} \quad (2)$$

$$\vec{\nabla} \cdot \vec{E} = \rho/\varepsilon \quad \text{———} \quad (3)$$

$$\vec{\nabla} \cdot \vec{B} = \rho_m \quad \text{———} \quad (4)$$

[There are 2 marks for each of the equations. As an example, if someone identifies only eqn. (2) and (3), 4 marks will be given.

* In all cases, marks deduction for wrong vector notation will be

applicable].

(b) In traditional EM framework an electric current gives rise to a magnetic vector potential. Similarly, in this case, $\vec{J}_m$ would give rise to an electric vector potential $(\vec{F})$.

Further due to the magnetic charge density ρ_m, there would be a magnetic scalar potential, ϕ_m.

$$\therefore \quad \vec{E} = -\vec{\nabla}\phi - \frac{\partial \vec{A}}{\partial t} \text{ and } \vec{B} = \vec{\nabla} \times \vec{A}$$

[$\phi \rightarrow$ Electric scalar potential

$\vec{A} \rightarrow$ Magnetic vector potential]

[If someone has mentioned these two along with the equation, 2 marks will be given.

* without the equation, no marks]

$$\vec{B} = -\vec{\nabla}\phi_m - \frac{\partial \vec{F}}{\partial t} \text{ and } \vec{E} = -\vec{\nabla} \times \vec{F}$$

[For all 4 potentials, 4 marks

if the equations are mentioned]

** No credit for mentioning only one potential. At least one set of scalar and vector potentials has to be mentioned.

Q2. $f(x,y,z) = x^2 y^3 z^4$

$$\vec{\nabla} f = \hat{x}\frac{\partial f}{\partial x} + \hat{y}\frac{\partial f}{\partial y} + \hat{z}\frac{\partial f}{\partial z}$$

$$= 2xy^3z^4\,\hat{x} + 3x^2y^2z^4\,\hat{y} + 4x^2y^3z^3\,\hat{z}$$

[If solution is correct, 2 marks otherwise 0]

Q3. $$\vec{\nabla} \times \vec{f} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 & 3xz^2 & -2xz \end{vmatrix}$$

$$= \hat{x}\left[0 - \frac{\partial}{\partial z}(3xz^2)\right] - \hat{y}\left[\frac{\partial}{\partial x}(-2xz) - 0\right] + \hat{z}\left[\frac{\partial}{\partial x}(3xz^2) - 0\right]$$

$= -6xz\,\hat{x} + 2z\,\hat{y} + 3z^2\,\hat{z}$

[marking scheme is similar to Q2 i.e. no step marking]

Q4. If a certain volume contains charge Q and a current I goes out of it,

$I = -\frac{dQ}{dt}$ [outgoing current must be equal to rate of decrease of charge]

[This reasoning is important. If missing, deduct $\frac{1}{2}$ mark]

$$\oint \vec{J} \cdot \vec{ds} = -\frac{d}{dt}\int \rho\, dv$$

[If the circle is missing deduct $\frac{1}{2}$ mark]

$$\Rightarrow \int (\vec{\nabla}\cdot\vec{J})\, dv = -\int \frac{\partial \rho}{\partial t}\, dv$$

$$\Rightarrow \boxed{\vec{\nabla}\cdot\vec{J} + \frac{\partial \rho}{\partial t} = 0.}$$

It represents law of conservation of charge.

This is key concept.

Misidentification

Q5. $$\Gamma = \frac{\sqrt{\mu_0/\varepsilon_2} - \sqrt{\mu_0/\varepsilon_1}}{\sqrt{\mu_0/\varepsilon_2} + \sqrt{\mu_0/\varepsilon_1}}$$

[wave goes from med. 1 to med. 2]

$$= \frac{\sqrt{\varepsilon_1} - \sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}} = \boxed{\frac{n_1 - n_2}{n_1 + n_2}}$$

of med. 1 an 2 would lead to zero.

[one can directly start here, no marks will be deducted]

$$T = \frac{2n_2}{n_1 + n_2} = \frac{2\sqrt{\mu_0/\varepsilon_2}}{\sqrt{\mu_0/\varepsilon_1} + \sqrt{\mu_0/\varepsilon_2}}$$

$$= \frac{2\sqrt{\varepsilon_1}}{\sqrt{\varepsilon_1}+\sqrt{\varepsilon_2}} = \boxed{\frac{2n_1}{n_1+n_2}}$$

Again one can directly start here, no marks will be deducted.

$n_1 = 1$, $n_2 = 1.5$

$$\therefore \Gamma = \frac{1-1.5}{1+1.5} = -\frac{.5}{2.5}$$

$$= -\frac{1}{5} = -0.2$$

[2 marks for correct answer]

$$T = \frac{2}{1+1.5} = \frac{2}{2.5} = \frac{4}{5} = 0.8$$

[2 marks for correct answer]

Q6. $L = 227\,nH/m$, $C = 90{\cdot}9\,pF/m$

$f = 14\pi \times 10^{8}\,Hz$

(a) $Z_0 = \sqrt{L/C} = \sqrt{\dfrac{227 \times 10^{-9}}{90{\cdot}9 \times 10^{-12}}}$ Ohm

$= 49{\cdot}97$ ohm

must be mentioned at least at the last line. Deduct 1 mark otherwise.

(b) $v = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{227 \times 10^{-9} \times 90{\cdot}9 \times 10^{-12}}}$ m/s

$= 2{\cdot}2 \times 10^{8}$ m/s

without unit at the final answer deduct 1.

(c) $\omega = 2\pi f = 28\pi^{2} \times 10^{8}$ rad/s

This is crucial, misidentification of $14\pi \times 10^{8}$ Hz with ω yields to zero.

$$v = \frac{\omega}{\beta}$$

$$\Rightarrow \beta = \frac{\omega}{v} = \frac{28\pi^2 \times 10^8}{2.2 \times 10^8}$$

$$= 125.6131 \text{ or } 12.727\pi^2$$

[full-marks will be given without the unit in this case]

(d) $Z_L = 30\,\Omega$, $Z_0 \approx 50\,\Omega$ [If someone proceeds with $49.97\,\Omega$, that should be fine too, check if the final answer is correct (should be close to the approxi- - mation)]

$$\therefore \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$= \frac{30 - 50}{30 + 50}$$

$= -\frac{20}{80} = -\frac{1}{4} = -0.25$

'−' sign must be present, zero otherwise.

(e) $VSWR = \frac{1+|\Gamma|}{1-|\Gamma|}$

$= \frac{1+0.25}{1-0.25} = \frac{1.25}{0.75}$

$= 1.67$

[If '−' wasn't written in part (e) but this part is correct, do not deduct marks in this part]

Q7. $f = 3\,GHz = 3\times10^9\,Hz$

$\boxed{\omega = 2\pi f = 6\pi\times10^9\,rad/s}$

[Again this is crucial, if 3×10^9 Hz is used as ω, no marks in part (a). But 50% marks will be given in part (b) and (c), provided they are correct.

(a) $\varepsilon_{eff} = \varepsilon\left(1 - i\frac{\sigma}{\omega\varepsilon}\right)$

$= \varepsilon - i\frac{\sigma}{\omega} = 24\times10^{-12} - i\frac{7.2\times10^{4}}{6\pi\times10^{9}}$

$= \left(24\times10^{-12} - i\,3.82\times10^{-6}\right)$

(b) $E_x/H_y = E/H = \sqrt{\frac{\mu_0}{\varepsilon_{eff}}}$

$= \sqrt{\frac{4\pi\times10^{-7}}{24\times10^{-12} - i\,3.82\times10^{-6}}}$ ohm

0.5 marks for writing till this point

$= \sqrt{4\pi\times10^{-7}\left(1.645 + i\,2.618\times10^{5}\right)}\ \Omega$

$\approx \sqrt{i\times4\pi\times10^{-2}\times2.618}\ \Omega$

$\boxed{= \sqrt{0.329\,i}\ \Omega}$ ← 2 marks if someone came till this.

$= (0.4056 + i\,0.4056)\ \Omega$

*** Imp: Here $E/H = E_x/H_y$

$\neq \left|\sqrt{\frac{\mu_0}{\varepsilon_{eff}}}\right|$

So, simply calculating directly mod $\left(\sqrt{\frac{\mu_0}{\varepsilon_{eff}}}\right)$ won't fetch any marks.

(c) $k = k' - ik'' = \omega\sqrt{\mu_0 \varepsilon_{eff}}$

$= 6\pi \times 10^9 \sqrt{4\pi \times 10^{-7} \times (24 \times 10^{-12} - i\,3.82 \times 10^{-6})}$

Someone writes this correctly
⇓
0.5 marks
(But if the ω is incorrect and the calculation is left here, 0 marks)

$\approx 6\pi \times 10^9 \sqrt{-i\,4\pi \times 3.82 \times 10^{-13}}$

$= 6\pi \times 10^9 (1.549 - i\,1.549) \times 10^{-6}$

$= 6\pi \times 10^3 \,(1.549 - i\,1.549)$

Full marks if anyone reaches till this point

$$= 1.885 \times 10^{4} (1.549 - i\, 1.549)$$

$$= (2.92 - i\, 2.92) \times 10^{4}$$