CTD Tutorial 7

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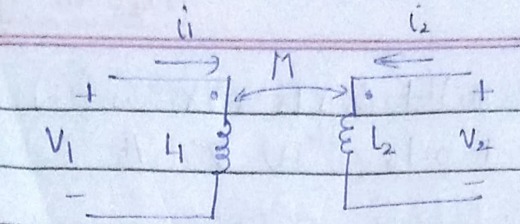
Soln)

(1)



From figure, it can be said that both the current i_1 and i_2 are entering the dotted terminal of coils, so the mutual inductance is added to self inductance.

⇒ (a)
$$V_1 = L\frac{di_1}{dt} + M\frac{di_2}{dt}$$

now, $i_1 = 0$, $i_2 = 5\cos 8t$ A, $L_1 = 10$ mH, $L_2 = 5$ mH, $M = 1$ mH

$$\Rightarrow V_1 = 0 + M\frac{d}{dt}(5\cos 8t)$$

$$= 1\times10^{-3} \times 8 \times (-5\sin 8t)$$

$$= -0.04 \sin(8t)\ V \quad \text{Ans}$$

(b) $i_1 = 3\sin 100t$ A, $i_2 = 0$ A

$$\Rightarrow V_2 = M\frac{di_1}{dt} + L_2\frac{di_2}{dt}$$

$$= M\frac{d}{dt} 3\sin(100t)$$

$$= 1\times10^{-3} \times 100 \times 3\cos(100t)$$

$$= 0.3\cos(100t)\ V \quad \text{Ans}$$

(c) $i_1 = 5\cos(8t-40)$ A and $i_2 = 4\sin 8t$ A

$$V_2 = M\frac{di_1}{dt} + L_2\frac{di_2}{dt}$$

$$= M\frac{d}{dt}(5\cos(8t-40^\circ)) + L_2\frac{d}{dt}(4\sin(8t))$$

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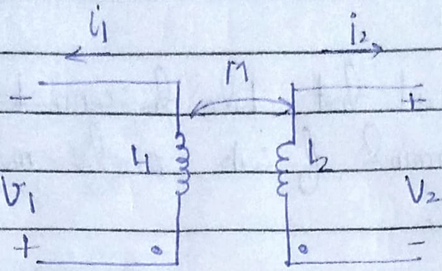
$= 10^{-3} \times 5 \times -8 \sin(8t - 40°) + 50 \times 10^{-3} \times 4 \times 8 \cos(8t)$

$= -0.04 \sin(8t - 40°) + 0.16 \cos(8t)$ V Ans

Soln(4)



From figure it can be said that both currents i_1 and i_2 are both entering the dotted terminals of their coils, so the mutual inductance is added to self inductance.

$L_1 = 0.5 L_2 = 1mH$, $M = 0.85\sqrt{L_1 L_2}$, $v_2(t) = ?$

(a) $i_2 = 0$ and $i_1 = 5e^{-t}$ mA

$V_2 = -M \frac{di_1}{dt} - L_2 \frac{di_2}{dt}$

$\Rightarrow V_2 = -M \cdot \frac{d}{dt}(5 \times 10^{-3} e^{-t})$

$= -0.85\sqrt{L_1 L_2} \frac{d}{dt}(5 \times 10^{-3} e^{-t})$

$= -0.85 \times \sqrt{2 \times 10^{-3} \times 1 \times 10^{-3}} \times -1 \times 5 \times e^{-t} \times 10^{-3}$

$= 6 \times 10^{-6} \times e^{-t}$ V

$\Rightarrow V_2 = 6e^{-t}$ μV Ans

(b) $i_2 = 0$ and $i_1 = 5\cos 10t$ mA

$V_2 = -M \frac{di_1}{dt} - L_2 \frac{di_2}{dt}$

$$\Rightarrow V_2 = -M \frac{d}{dt}\left(10^{-3} \times 5 \cos(10t)\right)$$

$$= -0.85\sqrt{L_1 L_2}\, \frac{d}{dt}\left(10^{-3} \times 5 \cos(10t)\right)$$

$$= -0.85\sqrt{2\times10^{-3} \times 1 \times 10^{-3}} \times 5 \times 10^{-3} \times 10 - \sin(10t)$$

$$= 60 \times 10^{-6} \sin(10t)\ V$$

$$\Rightarrow V_2 = 60 \sin(10t)\ \mu V \quad \text{Ans}$$

(c) $i_2 = 5 \cos 70t$ mA and $i_1 = 0.5\, i_2$

$$V_2 = -M\frac{di_1}{dt} - L_2\frac{di_2}{dt}$$

$$= -0.85\sqrt{2\times10^{-3} \times 1\times10^{-3}}\ \frac{d}{dt}\left(2.5\times10^{-3}\cos(70t)\right) -$$

$$2\times10^{-3}\ \frac{d}{dt}\left(5\times10^{-3}\cos(70t)\right)$$

$$= -0.85\sqrt{2\times10^{-3}\times1\times10^{-3}} \times 2.5\times10^{-3}\times 70 \times (-\sin(70t)) -$$

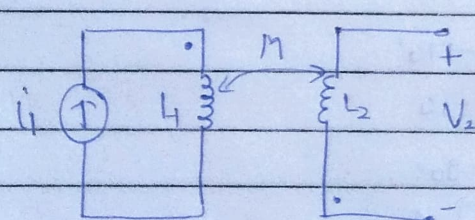
$$2\times10^{-3} \times 5 \times 70 \times 10^{-3} \times \sin(70t)$$

$$\Rightarrow V_2 = 210\times10^{-6}\sin(70t) + 700\times10^{-6}\sin(70t)$$

$$= 910\times10^{-6}\sin(70t)\ V$$

$$\Rightarrow V_2 = 910 \sin(70t)\ \mu A \quad \text{Ans}$$

Soln (6)



$i_1 = 5 \sin(100t - 80°)$ mA

$L_1 = 1H$

$L_2 = 2H$

$V_2 = 250 \sin(100t - 80°)$ mV

$M = ?$

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From figure, the current in the left loop is entering the dotted terminal, so the positive voltage reference of the mutual inductance of the second coil should be at the dotted terminal of the second coil opposite to V_2. Thus, mutual inductance is negative.

$$V_2 = L\frac{di_2}{dt} - M\frac{di_1}{dt}$$

$$V_2 = -M\frac{di_1}{dt} \qquad (\because i_2 = 0)$$

$$= -M\frac{d}{dt}(5\times10^{-3}\sin(100t - 80^\circ))$$

$$= -M\times5\times10^{-3}\times100\cos(100t - 80^\circ) \quad \text{— (1)}$$

$$\sin(\omega t - \phi) = \cos(\omega t - \phi - 90^\circ)$$

$$V_2 = 250\times10^{-3}\sin(100t - 80^\circ)$$

$$= 250\times10^{-3}\cos(100t - 170^\circ) \quad \text{— (2)}$$

Equating (1) and (2):

$$0.25\cos(100t - 170^\circ) = -0.5M\cos(100t - 80^\circ)$$

$$\Rightarrow \quad 0.25\angle{-170^\circ} = M\times -0.5\angle{-80^\circ}$$

now, solving for M,

$$M = \frac{0.25\angle{-170^\circ}}{-0.5\angle{-80^\circ}}$$

$$= -0.5\angle{-90^\circ}$$

$$= \boxed{j\,0.5H} \text{ Ans}$$