Path planning of Robots

Suppose the robot is in $[x, y, \theta]$ location currently, and it wants to go to $[x', y', \theta']$. Note that all the coordinates and orientations are measured concerning a global coordinate frame. Using coordinated geometry, we can come up with two possible ways to move our robot from the current location to the desired location.

Direct route

From the coordinate information given we can decompose the path needed to a pure rotation $\Delta\theta_1$, pure translation Δx, followed by another pure rotation $\Delta\theta_2$ in that order.

The angle of the first rotation, $\Delta\theta_1$, Δx, and $\Delta\theta_2$ are computed as

$$\Delta\theta_1 = \operatorname{atan}\left(\frac{y' - y}{x' - x}\right) - \theta, \quad \Delta x = \sqrt{(x' - x)^2 + (y' - y)^2}, \qquad \Delta\theta_2 = \theta' - \operatorname{atan}\left(\frac{y' - y}{x' - x}\right)$$

Notes:

- If you are only interested in reaching $[x', y']$ coordinate without concerning the orientation, then you can disregard the final rotation, $\Delta\theta_2$.
- Positive rotations are in the counterclockwise direction.

Square shaped or Perpendicular route

In this method, the path follows the direction of coordinates axes to form a square-edge-like path in parallel to the coordinate axes. Reaching $[x', y', \theta']$ can be decomposed into a pure rotation $\Delta\theta_1$, pure translation Δx_1, pure rotation $\Delta\theta_2$, pure translation Δx_2, and pure rotation $\Delta\theta_3$

First pure rotation, $\Delta\theta_1$ aligns the robot parallel to the +X axis.

$$\Delta\theta_1 = -\theta$$

The first pure translation follows the X-axis toward the target coordinate

$$\Delta x_1 = x' - x$$

Second pure rotation, $\Delta\theta_2$ aligns the robot parallel to the +Y axis.

$$\Delta\theta_2 = +90^\circ$$

The second pure translation follows the Y-axis toward the target coordinate

$$\Delta x_2 = y' - y$$

Third pure rotation re-orients the robot the destination orientation

$$\Delta\theta_3 = \theta'$$

Notes:

- If you are only interested in reaching $[x', y']$ coordinate without concerning the orientation, then you can disregard the final rotation, $\Delta\theta_3$.
- Δx_1 and Δx_2 can be negative, which means the robot goes in reverse.