MODELLING OF FLYBACK CONVERTER USING STATE SPACE AVERAGING TECHNIQUE

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***Abstract*—In 1960s demand made by space programs led to the development of power supplies that are highly reliable, efficient, light weight and small in size. The innovative ideas of the engineer's usher in the era of modern power electronics and switch mode power supplies came into existence. Design and optimization of dc-dc converter which offers high efficiency, small converters with isolation transformers can have multiple outputs of various magnitudes and polarities. The regulated power supply of this type has a wide application such as digital systems, in TVs instrumentation system, in industry automation etc., where in a low voltage, high current power supply with low output ripple and fast transient response are essential.**

This paper gives the methodology to model flyback converter (24V dc - 5V dc) operating under continuous conduction mode by using state space averaging technique which linearizes the system and simplifies the designing procedure.

***Index Terms*—Flyback converter, small signal analysis, state space averaging, continuous conduction mode.**

I. INTRODUCTION

Switched-mode converters are nonlinear variable-structure systems [1],[2]. Up to half the switching frequency, the dynamics of a switched-mode converter may be accurately captured using proper averaging methods. The resulting transfer functions are linear time-invariant (LTI) models of the system dynamics at a certain operating point [1],[4]. The load of the converter is not necessarily known, when the converter is designed, produced, and sold. Therefore, it may be most convenient to define the set of transfer functions in unterminated mode, i.e., excluding the effect of impedance-type load from the transfer functions.

Modelling is the representation of the physical behavior of any circuit by the mathematical means [2],[4]. The parameters which decide the behavior of a circuit are mainly the current through its elements and the voltage across them. The simplified model yields physical insight, allowing engineer to design system to operate in specified manner. Approximations are made to neglect the small quantities [3],[7],[5] but later the model can be redefined to account for these approximations.

A. Average Value Modelling

The averaged-value modelling, wherein the effects of fast switching are averaged over a switching interval, is most frequently applied when investigating power-electronics-based systems [5],[6],[8]. Continuous large-signal models are typically non-linear and can be linearized around a desired operating point. Averaged models of dc-dc converters offer several advantages over the switching models. [3],[7] These advantages are:

(i) Straightforward approach in determining local transfer-functions
(ii) Faster simulation of large-signal system-level transients
(iii) Use of general-purpose simulators to linearize converters for designing the feedback controllers.

A typical switched-inductive dc-dc converter can operate in two modes. One is the Continuous Conduction Mode (CCM) in which inductive current never falls to zero, and the second mode is Discontinuous Conduction Mode (DCM) allowing inductive current to become zero for a portion of switching period [1],[2]. The DCM typically occur at light loads and differs from CCM [14] since this mode results into three different switched networks over one switching cycle (as opposed to two switched networks in the case of CCM operation). Numerous methods have been developed for the average value modelling of PWM dc-dc converters in DCM such as reduced-order state-space averaging, reduced-order averaged-switch modelling, equivalent duty ratio models, loss-free resistor model, full-order averaged-switch modelling, and full-order state-space averaging [10],[11].

Analytic averaging, is based on so-called small-ripple approximation. Most of the previous works on averaging methods were derived for a specific ideal topology. In addition, derivation of state-space average-value model, the equivalent series resistance (ESR) [1] of circuit components are often neglected and the state variables are considered as linear segments. Such assumptions result in inaccuracy of the corresponding time constants as well as the waveforms. If the losses due to the switch and/or active elements are taken into account, whereby the linear shape of the current waveform would change into exponential form [12],[13], the analytically derived models would become significantly more complicated and challenging. The analytic derivation also becomes more complicated when the number of energy storage elements (inductors and capacitors) is high [9].

B. Parametric Average Value Modelling

Parametric average-value modelling methodology [3] has been successfully demonstrated for synchronous machine-converter systems. The effect of parasitic included in the detailed model becomes automatically included in the numerically constructed parametric functions [7], which are then used for the state-variable-based average-value models. This approach also reduces the effort of the model developer and avoids many complicated analytical derivations. This method has been extended to the PWM dc-dc converters in based on corrected full-order averaged models proposed for circuit averaging and state-space averaging that very accurately capture the high-frequency dynamics of fast state variables.

II. Small Signal AC Model of Flyback Converter Under CCM

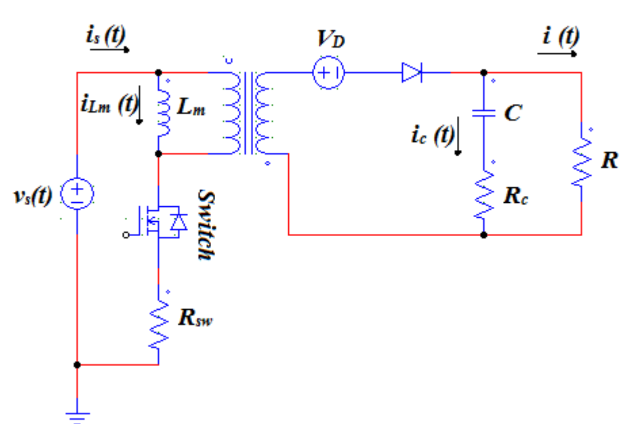

Fig. 1. Second order flyback converter with parasitic

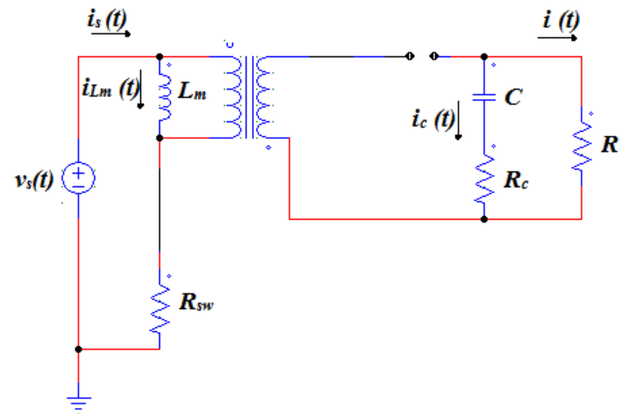

Fig. 2. Circuit under sub-interval 1

A. Sub-interval 1:

During the sub-interval 1 [15], when the MOSFET conducts and the diode is off, the circuit reduces to Fig. 2 . For this interval, the inductor voltage $v_L(t)$, capacitor current $i_c(t)$, converter output voltage $v(t)$ and converter input current $i_s(t)$ are

$$v_L(t) = v_s(t) - R_{sw} i_s(t) \tag{1}$$

$$i_c(t) = -\frac{v_c(t)}{R + R_c} \tag{2}$$

$$v(t) = \frac{v_c(t) R}{R + R_c} \tag{3}$$

$$i_s(t) = i_{L_m}(t) \tag{4}$$

B. Sub-interval 2:

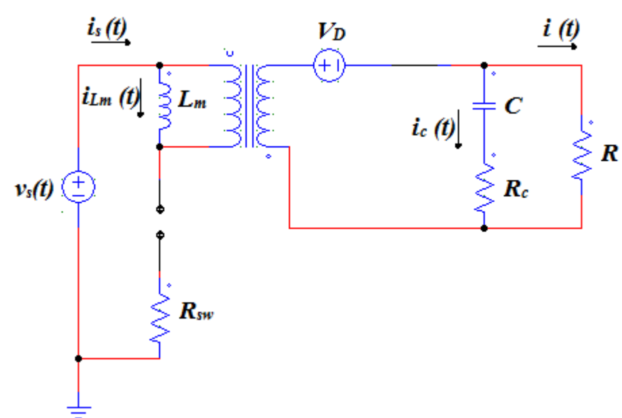

Fig. 3. Cuircuit under sub-interval 2

During the sub-interval 2 [15], the MOSFET is off and diode conducts, as shown in the circuit of Fig. 3. For this interval, the inductor voltage $v_L(t)$, capacitor current $i_c(t)$, converter output voltage $v(t)$ and converter input current $i_s(t)$ are

$$
\begin{aligned}
v_L(t) &= (v_c(t) - R_c i_c(t) - V_d)n && (5)\\
i_c(t) &= -\left[i_{L_m}(t)n + \frac{v(t)}{R}\right] && (6)\\
v(t) &= v_c(t) - R_c i_c(t) && (7)\\
i_s(t) &= 0 && (8)
\end{aligned}
$$

where n is $\frac{N_1}{N_2}$

C. Averaging parameters:

The average inductor voltage now can be found by averaging the sub-intervals over one complete switching period T_s [3]. This leads to the following equation for the average inductor current

$$
\begin{aligned}
L_m\frac{d[i_{L_m}(t)]_{T_s}}{dt} &= [v_s(t)]_{T_s}d_1(t) - R_{sw}[i_s(t)]_{T_s}d_1(t)\\
&+ [v_c(t)]_{T_s}nd_2(t) - [i_c(t)]_{T_s}R_cnd_2(t)\\
&- V_dnd_2(t) \qquad (9)
\end{aligned}
$$

The average capacitor current now can be found by averaging the sub-intervals over one switching period, which results in the following equation:

$$
\begin{aligned}
C\frac{d[v_c(t)]_{T_s}}{dt} &= -\frac{[v_c(t)]_{T_s}}{R+R_c}d_1(t) - [i_{L_m}(t)]_{T_s}nd_2(t)\\
&- \frac{[v(t)]_{T_s}}{R}d_2(t) \qquad (10)
\end{aligned}
$$

Similarly, the converter output voltage and input current can be given by:

$$
\begin{aligned}
[v(t)]_{T_s} =& \frac{[v_c(t)]_{T_s}R}{R+R_c}d_1(t) + [v_c(t)]_{T_s}d_2(t)\\
&- [i_c(t)]_{T_s}R_cd_2(t) \qquad (11)\\
[i_s(t)]_{T_s} =& [i_{L_m}(t)]_{T_s}d_1(t) \qquad (12)
\end{aligned}
$$

where $d_1(t)$ and $d_2(t)$ are the duty cycles of the respective sub-intervals.

The above equations are nonlinear differential equations. To construct the converter small-signal ac model, the equations are perturbed and linearized. Assumption made that the converter input voltage $v_s(t)$ and duty cycle $d_1(t)$ can be expressed as quiescent values plus small ac variations, as follows:

$$
\begin{aligned}
[v_s(t)]_{T_s} &= V_s + \widehat{v_s}(t) && (13)\\
d_1(t) &= D_1 + \widehat{d_1}(t) && (14)\\
d_2(t) &= D_2 - \widehat{d_1}(t) && (15)
\end{aligned}
$$

In response to these inputs, after all transients have decayed, the averaged converter parameters can be expressed as quiescent values and small ac variations.

$$
\begin{aligned}
[i_s(t)]_{T_s} &= I_s + \widehat{i_s}(t) && (16)\\
[i_{L_m}(t)]_{T_s} &= I_{L_m} + \widehat{i_{L_m}}(t) && (17)\\
[i_c(t)]_{T_s} &= I_c + \widehat{i_c}(t) && (18)\\
[v_c(t)]_{T_s} &= V_c + \widehat{v_c}(t) && (19)\\
[v(t)]_{T_s} &= V + \widehat{v}(t) && (20)
\end{aligned}
$$

After applying these perturbations to equations (9) (10) (11) and (12), the obtained equations will have three terms dc terms, first order ac terms (linear) and second order ac terms (non-linear). The dc term contains no time varying quantities. The first order ac terms are linear functions of the ac variations in the circuit, while the second order ac terms are functions of the products of the ac variations. Assumption is made that the ac variations are small in magnitude compared to the dc quiescent values. If the small signal assumptions are satisfied, then the second-order terms are much smaller in magnitude than the first-order terms and hence are neglected. The dc terms must satisfy:

$$0 = V_sD_1 - R_{sw}I_sD_1 + V_cnD_2 - I_cR_cnD_2 - V_dnD_2 \qquad (21)$$

$$0 = -\frac{V_cD_1}{R+R_c} - I_{L_m}nD_2 - \frac{VD_2}{R} \qquad (22)$$

$$V = \frac{V_cD_1R}{R+R_c} + V_cD_2 - I_cR_cD_2 \qquad (23)$$

$$I_s = I_{L_m}D_1 \qquad (24)$$

The first order ac terms must satisfy:

$$
\begin{aligned}
L_m\left[\frac{d\widehat{i_{L_m}}(t)}{dt}\right] &= V_s\widehat{d_2}(t) + \widehat{v_s}(t)D_1 - R_{sw}I_s\widehat{d_1}(t)\\
&- R_{sw}\widehat{i_s}(t)D_1 - V_cn\widehat{d_1}(t) + \widehat{v_c}(t)nD_2\\
&+ I_cR_cn\widehat{d_1}(t) - \widehat{i_c}(t)R_cnD_2 + V_dn\widehat{d_1}(t) \qquad (25)
\end{aligned}
$$

$$
\begin{aligned}
C\left[\frac{d\widehat{v_c}(t)}{dt}\right] &= \frac{V_c\widehat{d_1}(t)}{R+R_c} - \frac{\widehat{v_c}(t)D_1}{R+R_c} + I_{L_m}n\widehat{d_1}(t)\\
&- \widehat{i_{L_m}}(t)nD_2 + \frac{V\widehat{d_1}(t)}{R} - \frac{\widehat{v}(t)D_2}{R} \qquad (26)\\
\widehat{v}(t) &= \frac{V_c\widehat{d_1}(t)R}{R+R_c} + \frac{\widehat{v_c}(t)D_1R}{R+R_c} - V_c\widehat{d_1}(t)\\
&+ \widehat{v_c}(t)D_2 + I_cR_c\widehat{d_1}(t) - \widehat{i_c}(t)R_cD_2 \qquad (27)\\
\widehat{i_s}(t) &= I_{L_m}\widehat{d_1}(t) + \widehat{i_{L_m}}(t)D_1 \qquad (28)
\end{aligned}
$$

These linear equations represent the low frequency ac variations in the converter parameters; they are the magnetizing current through the inductor, voltage across the capacitor, output voltage and the input current.

III. State Space Averaging

Now applying the state-space averaging method to the second order flyback converter shown in Fig. 1 . The independent state variables as usual are the inductor current $i_{L_m}(t)$ and the capacitor voltage $v_c(t)$, which form the state vector.

$$x(t) = \begin{bmatrix} i_{L_m}(t)\\ v_c(t) \end{bmatrix} \qquad (29)$$

The input vector becomes the input voltage and $v_s(t)$ the independent source that is the diode voltage drop V_d. Therefore, the input vector is

$$u(t) = \begin{bmatrix} v_s(t) \\ V_d \end{bmatrix} \tag{30}$$

To model the converter input port and output port, find the converter output voltage $v(t)$. To calculate this dependent voltage, it should be included as a output vector $y(t)$ as

$$y(t) = v(t) \tag{31}$$

Considering the state equations obtained from the sub-interval 1 (equations (1)-(4)) and averaging those equations:

$$\begin{bmatrix} \frac{di_{L_m}(t)}{dt} \\ \frac{dv_c(t)}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{R_{sw}}{L_m} & 0 \\ 0 & -\frac{1}{RC+R_c} \end{bmatrix} \begin{bmatrix} i_{L_m}(t) \\ v_c(t) \end{bmatrix} + \begin{bmatrix} -\frac{1}{L_m} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} v_s(t) \\ V_d \end{bmatrix} \tag{32}$$

$$v(t) = \begin{bmatrix} 0 & \frac{R}{R+R_c} \end{bmatrix} \begin{bmatrix} i_{L_m}(t) \\ v_c(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \end{bmatrix} \begin{bmatrix} v_s(t) \\ V_d \end{bmatrix} \tag{33}$$

Similarly, when state euations of sub-interval 2 (equations (5)-(8)) are considered and averaged:

$$\begin{bmatrix} \frac{di_{L_m}(t)}{dt} \\ \frac{dv_c(t)}{dt} \end{bmatrix} = \begin{bmatrix} \frac{n^2 R_c R}{RL_m - R_c L_m} & \frac{nR}{RL_m - R_c L_m} \\ -\frac{nR}{RC - R_c C} & -\frac{1}{RC+R_c C} \end{bmatrix} \begin{bmatrix} i_{L_m}(t) \\ v_c(t) \end{bmatrix} + \begin{bmatrix} 0 & -\frac{n}{L_m} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} v_s(t) \\ V_d \end{bmatrix} \tag{34}$$

$$v(t) = \begin{bmatrix} \frac{nRR_c}{R-R_c} & \frac{R}{R-R_c} \end{bmatrix} \begin{bmatrix} i_{L_m}(t) \\ v_c(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \end{bmatrix} \begin{bmatrix} v_s(t) \\ V_d \end{bmatrix} \tag{35}$$

Using the general notations of state space averaging and combining the above mentioned matrices to obtain linear continuous system:

$$\dot{x} = (A_1 d + A_2 d')x + (B_1 d + B_2 d')v_s \tag{36}$$

$$y = (C_1{}^T d + C_2{}^T d')x \tag{37}$$

where $d' = 1 - d$ and d is duty cycle for ON period.

Applying perturbation to consider line voltage variations and thus the variations in input and output vectors.

$$v_s = V_s + \widetilde{v_s} \tag{38}$$

$$x = X + \widetilde{x} \tag{39}$$

$$y = Y + \widetilde{y} \tag{40}$$

$$d = D + \widetilde{d} \tag{41}$$

Now, the perturbed equations gets altered as follows:

$$\begin{aligned} \dot{\widetilde{x}} = \; & AX + BV_s + A\widetilde{x} + B\widetilde{v_s} \\ & +[(A_1 - A_2)X + (B_1 - B_2)V_s]\widetilde{d} \\ & +[(A_1 - A_2)\widetilde{x} + (B_1 - B_2)\widetilde{v_s}]\widetilde{d} \end{aligned} \tag{42}$$

$$\begin{aligned} Y + \widetilde{y} = \; & C^T X + C^T \widetilde{x} + (C_1{}^T - C_2{}^T)X\widetilde{d} \\ & +(C_1{}^T - C_2{}^T)\widetilde{x}\widetilde{d} \end{aligned} \tag{43}$$

After neglecting the small signals, the duty ratio modulation to the state variable or output transfer functions can be directly obtained as:

$$\frac{\widetilde{x}}{\widetilde{d}} = (sI - A^{-1})[(A_1 - A_2)X + (B_1 - B_2)V_s] \tag{44}$$

$$\begin{aligned} \frac{\widetilde{y}}{\widetilde{d}} = \; & C^T(sI - A^{-1})[(A_1 - A_2)X + (B_1 - B_2)V_s] \\ & +(C_1{}^T - C_2{}^T)X \end{aligned} \tag{45}$$

These are the required transfer functions to design the flyback converter.

IV. Simulation And Conclusion

Specifications of the flyback converter being designed is as follows:

$$\begin{aligned} V_s &= 24V \pm 10\% \\ V_o &= 5V \\ I_o &= 5A \\ V_{rip} &= 1\% \text{ of } V_o \\ P_{out} &= 25W \\ \eta &= 80\% \\ f_{sw} &= 25kHz \\ D_{max} &= 45\% \end{aligned}$$

The transfer functions given by equations (44) and (45) are considering the parasitic values and all components of the circuit. The following assumptions [3] are made while designing:

(i) R_c is very small when compared with R. Therefore, neglect it.
(ii) V_d is also small compared to $v_s(t)$. Thus, neglecting V_d.

By making these assumptions in the matrices of section [III], with the specifications as above and using Matlab tool obtained Bode plots are as follows:

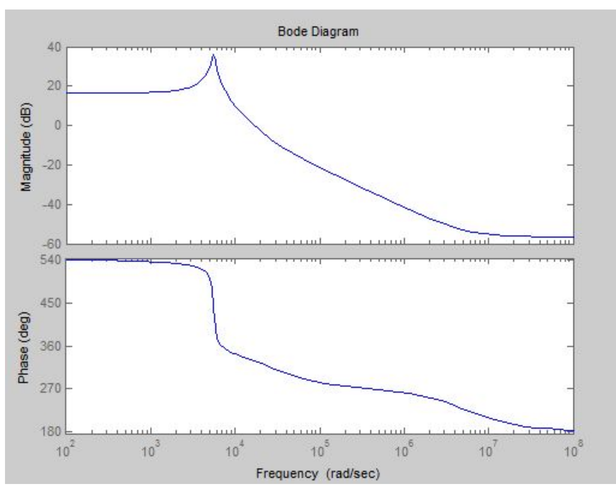

Fig. 4. Bode plot for the second order flyback converter

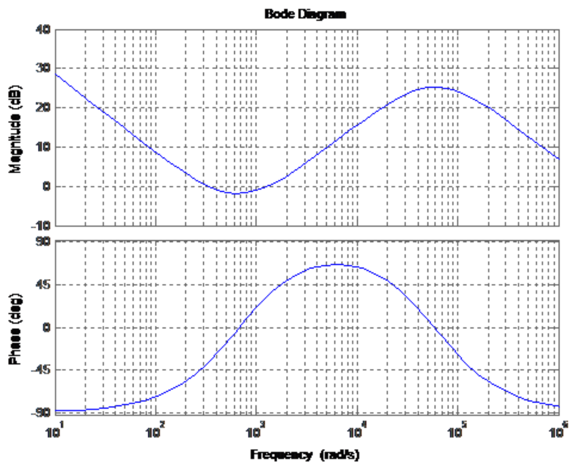

Fig. 5. Bode plot for the Type III error amplifier

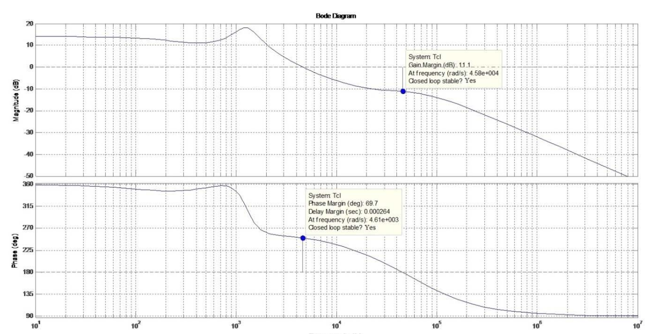

Fig. 6. Bode plot of the closed loop plant with controller

The gain margin and phase margin of the designed flyback converter are tabulated below saying the system is stable as shown in Fig. 6.

Gain margin	Phase margin	Stability condition
$-11.1dB$	69.7°	Stable

An accurate model of the flyback converter in the presence of parasitic elements is achieved in this paper. It is clear in the above table that the model designed is stable.

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