Introduction to Electronics
Part 4: Non-Linear Element
L19: Non-Linear Analysis

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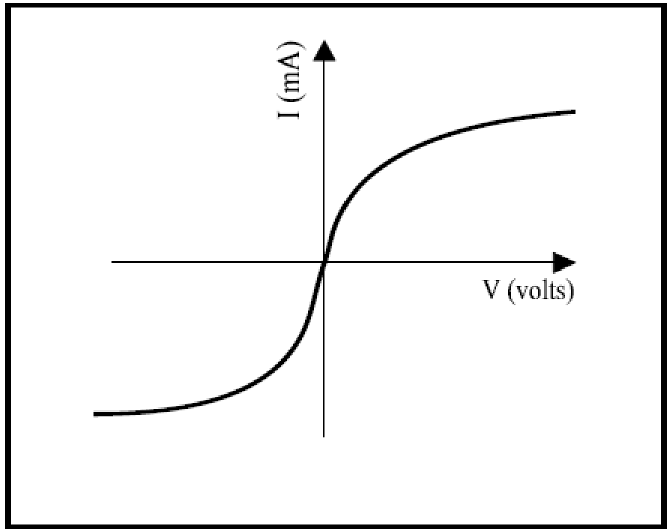
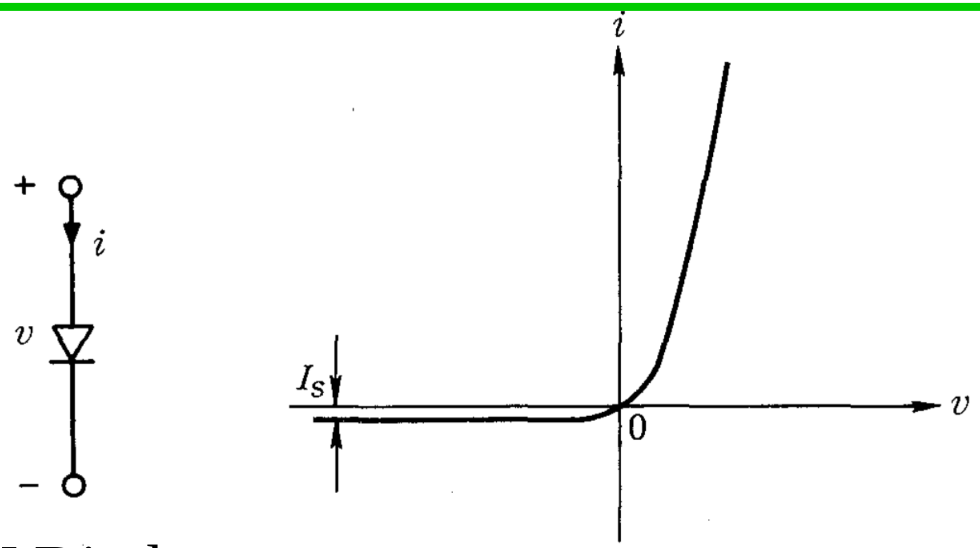
References

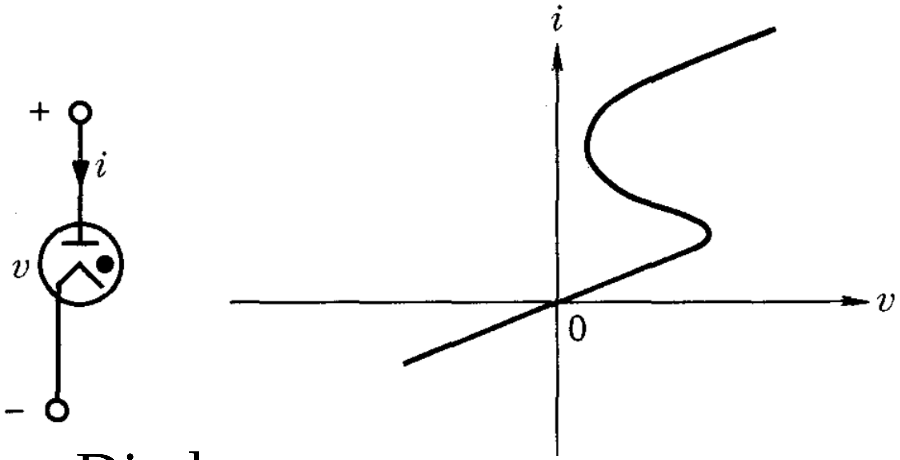
- To prepare these slides, materials from following books have been used.
  - Foundations of Analog and Digital Electronic Circuits by A. Agarwal and J. H. Lang, Elsevier



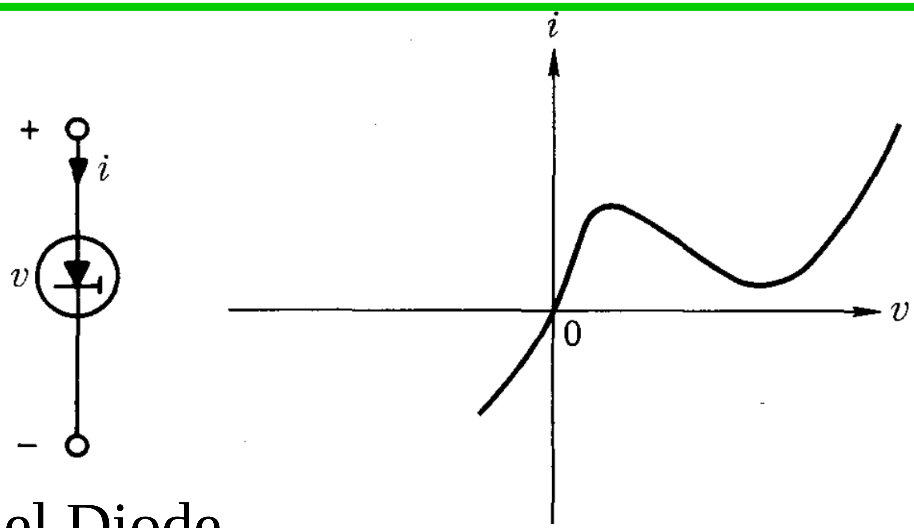
Bulb



PN Diode

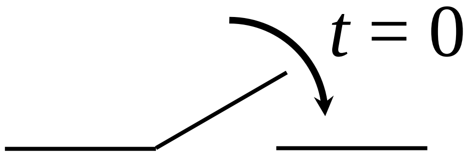


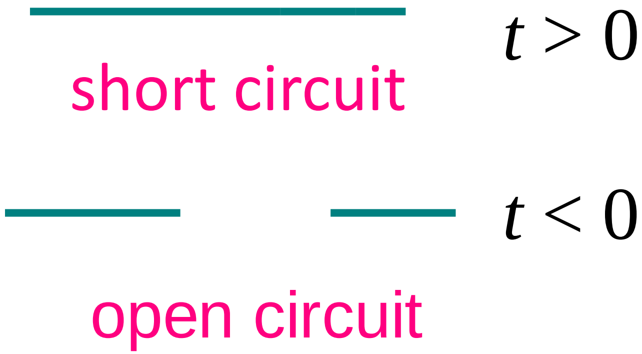
Gas Diode

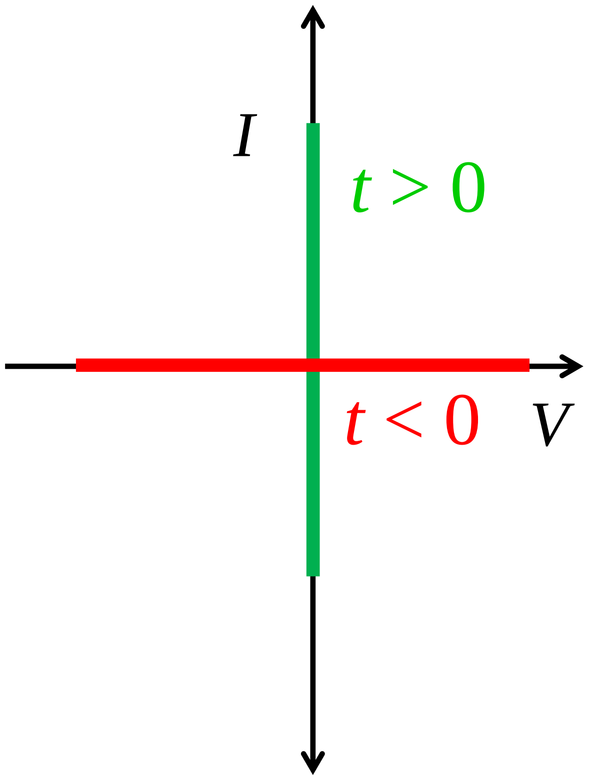


Tunnel Diode

Switch: Linear Time-Varying Resistor







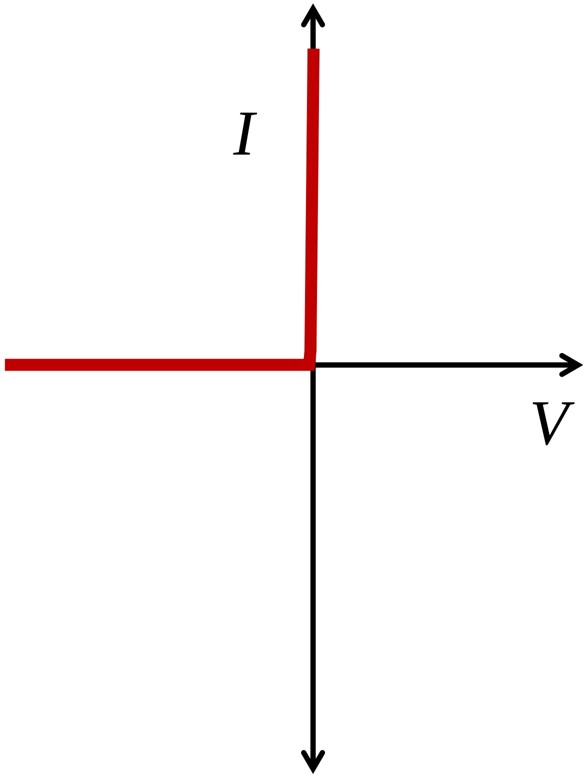
Unidirectional Device: Non-linear Time-invariant Resistor

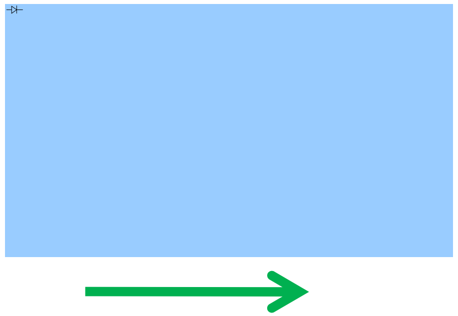
V > 0

short circuit

V < 0

open circuit





I-V Characteristics: Non-linear Behavior

Applied voltage $= v_D$

Diode current:

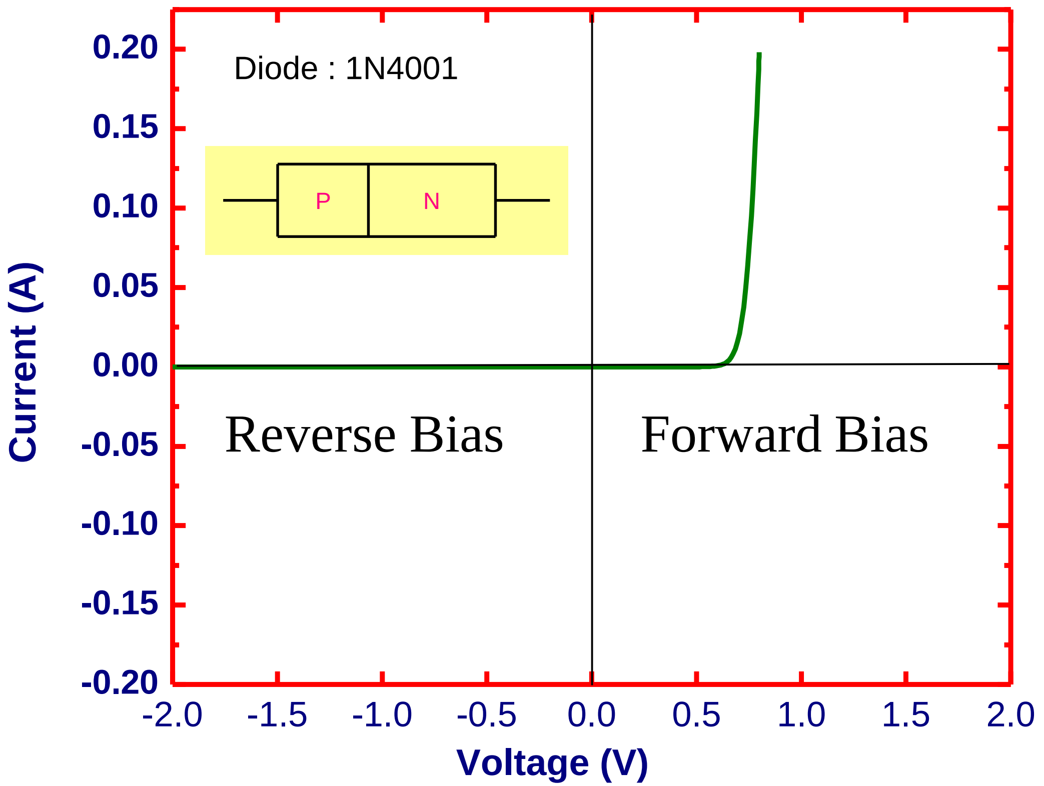
$$i_D = I_S\left(\exp\left(\frac{v_D}{nV_T}\right) - 1\right)$$

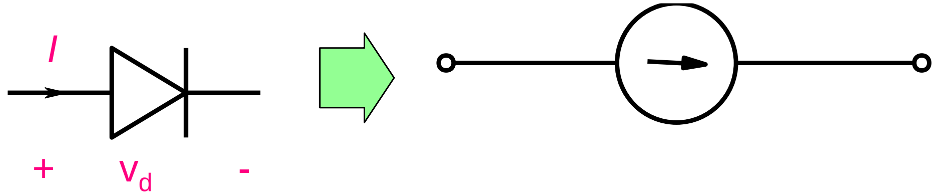
I_S : Reverse saturation current

n: ideality factor ($=1$ for ideal diodes)

$$V_T = \frac{kT}{q} \approx 26m\text{V at T} = 300\text{K}$$

- ❑ How to analyze circuits containing diodes?





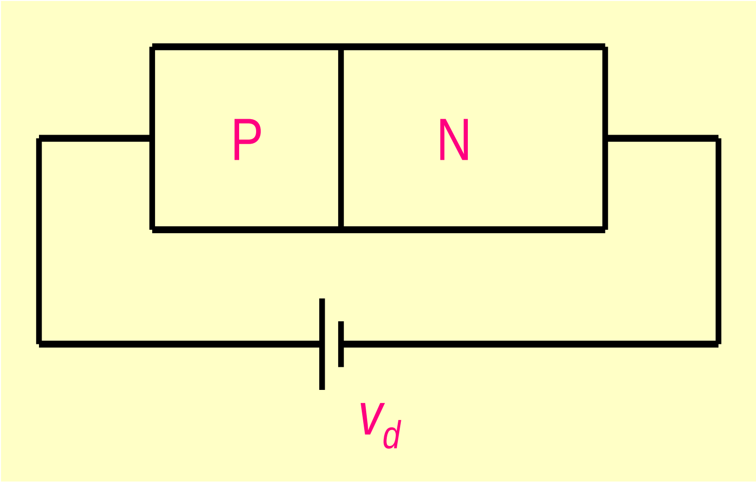
Forward and Reverse Bias

$$I_D = I_S\left(\exp\left(\frac{v_D}{V_T}\right) - 1\right)$$

- Forward Bias:

$$v_d >> V_T = 26mV$$

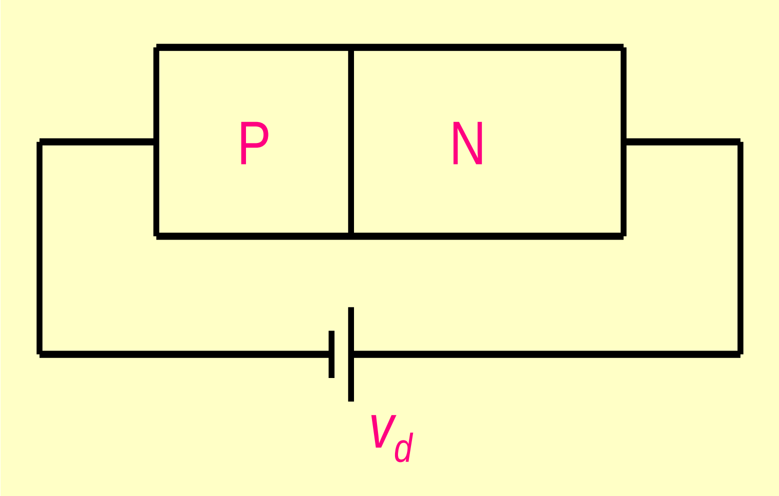
$$i_D \approx I_S \times \exp\left(\frac{v_d}{V_T}\right)$$



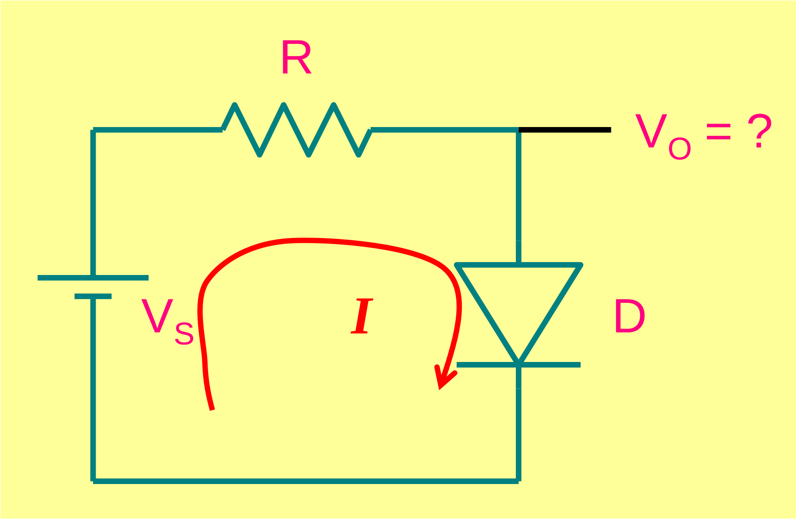
- Reverse Bias:

$$v_d = -v_R \qquad |v_R| >> V_T$$

$$i_D = I_S\left(\exp\left(-\frac{v_R}{V_T}\right) - 1\right) \approx -I_S$$



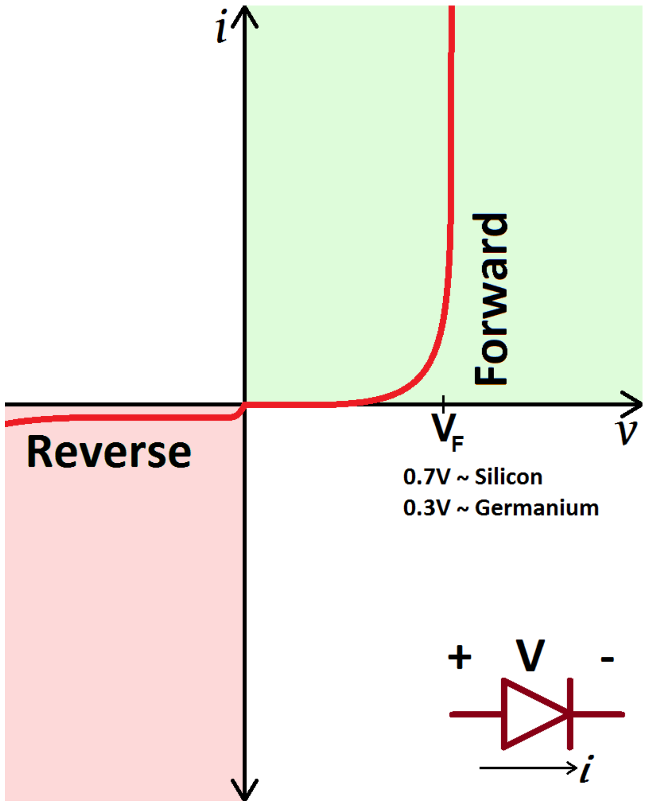
Method of Approximation



$$V_S = R\,I + V_D \qquad I = I_S\left(e^{\frac{V_D}{V_T}} - 1\right)$$

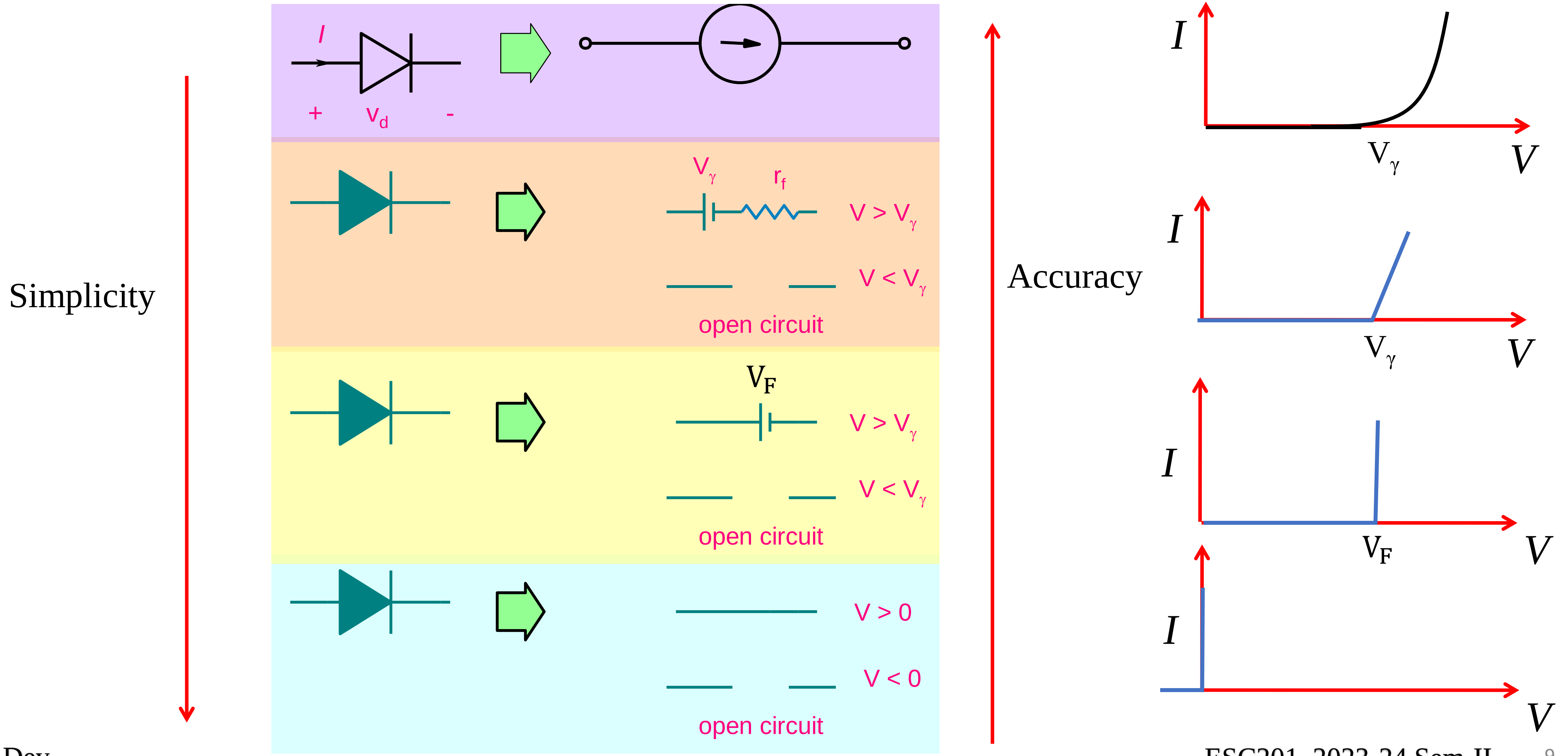
2 equations, 2 variables

$$V_S = RI_S\left(e^{\frac{V_D}{V_T}} - 1\right) + V_D$$



- Non-linear equation: How to solve?
  - Numerical methods, graphical method, analytical method, etc.
- We can however approximate its behavior with piecewise linear one
  - I-V graph is approximated by joining two or more straight lines

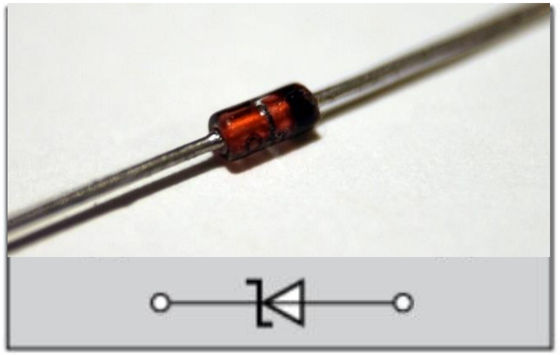
Diode: Approximate I-V Models

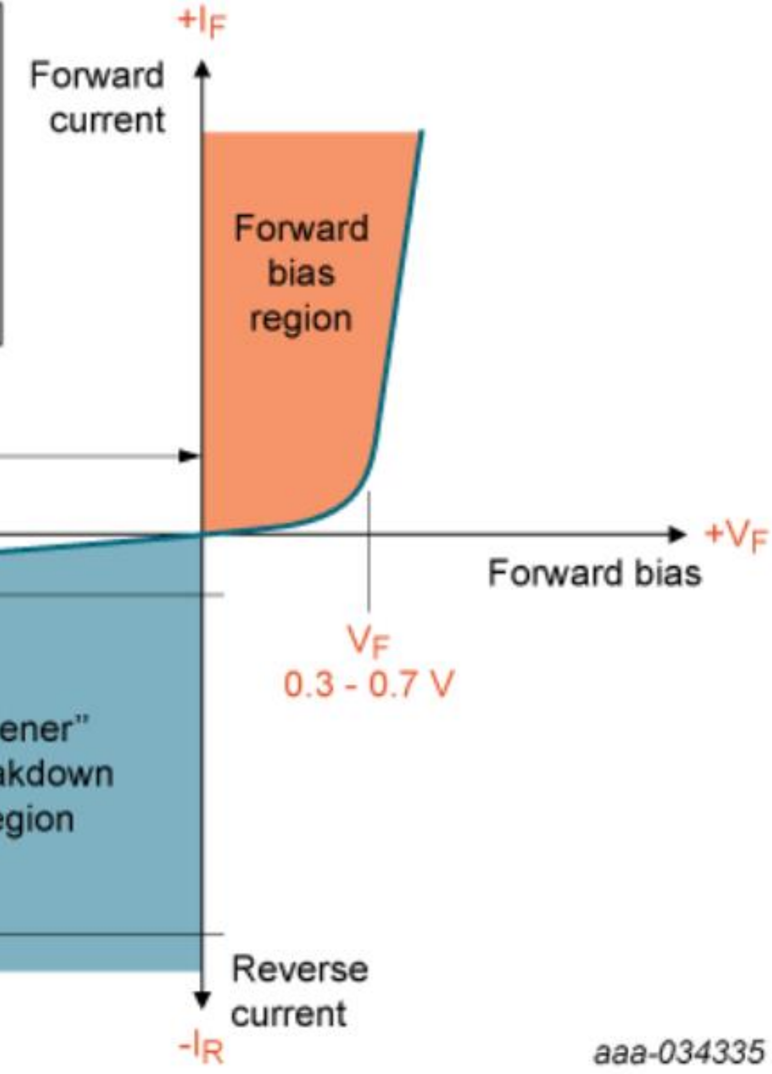


Self-Consistent Analysis

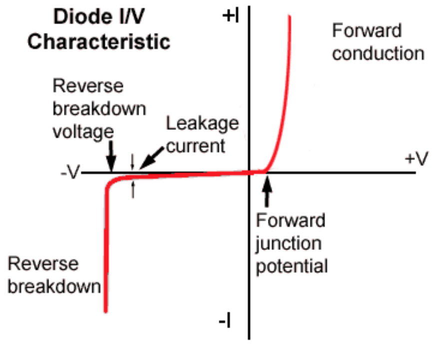
- How to know in which state diode is?
  - Easier if the voltage is known.
  - Otherwise
  - Analyze circuit assuming diode is forward biased
    - Check assumption ($I > 0$?)
  - Analyze circuit assuming diode is reverse biased
    - Check assumption ($V < 0$?)
  - Select the consistent one.
- What if 2 diodes: 4 possible circuits, only 1 will be valid
- N diodes => 2^N circuits, only one will be valid

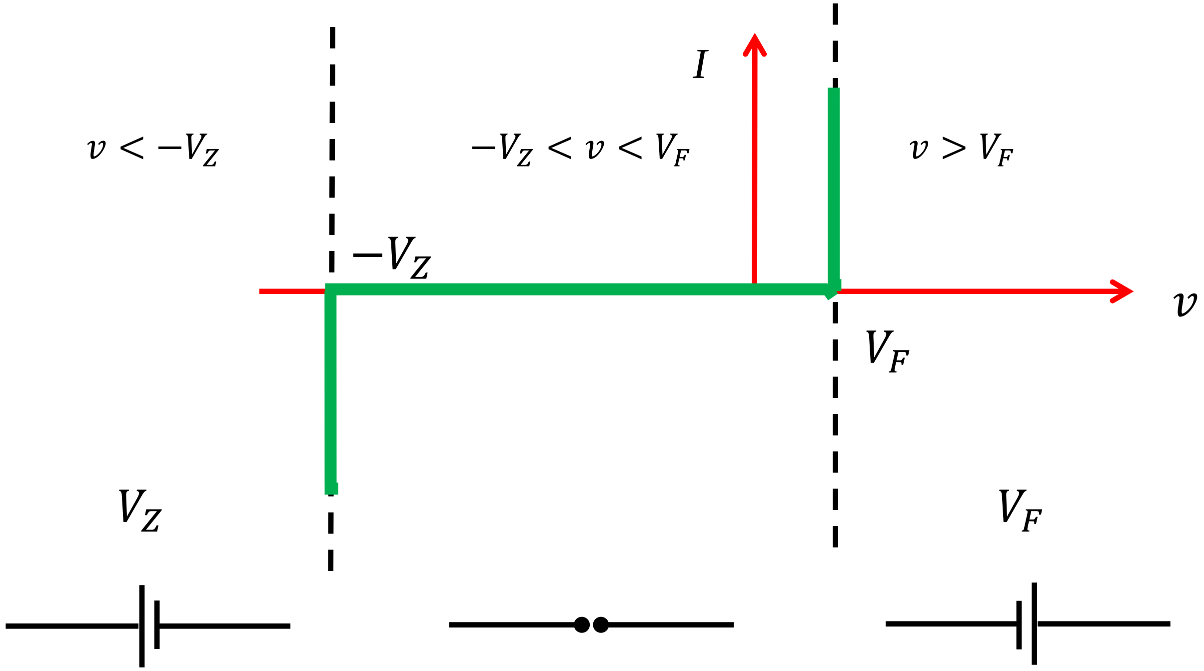
Breakdown and Zener Diode

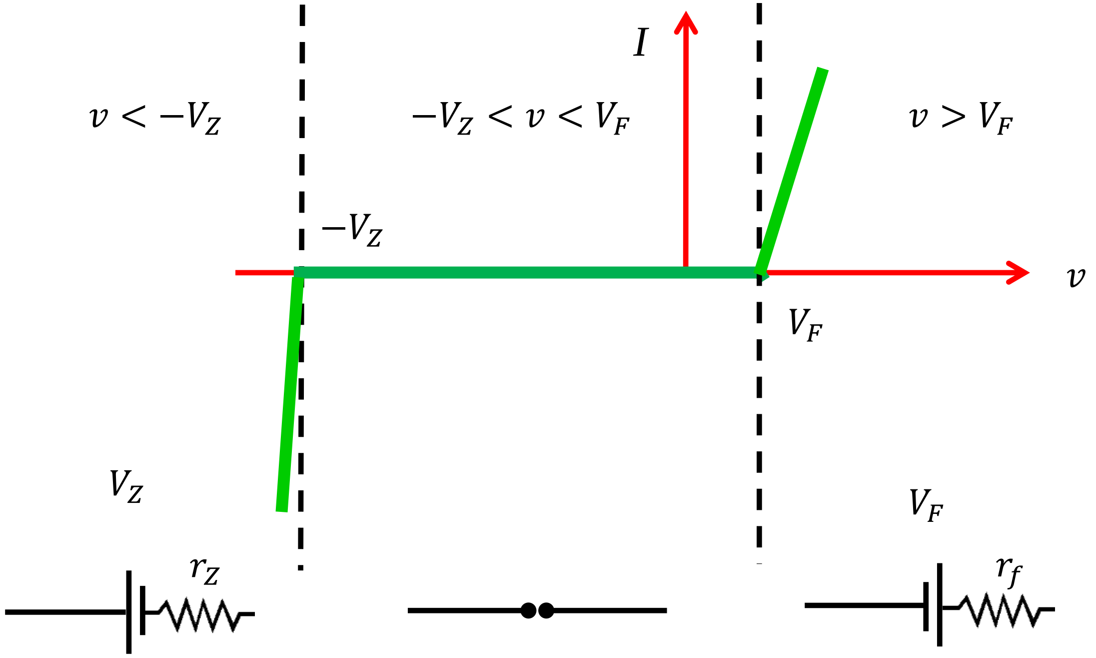




Zener Diode: Approximation I-V Model

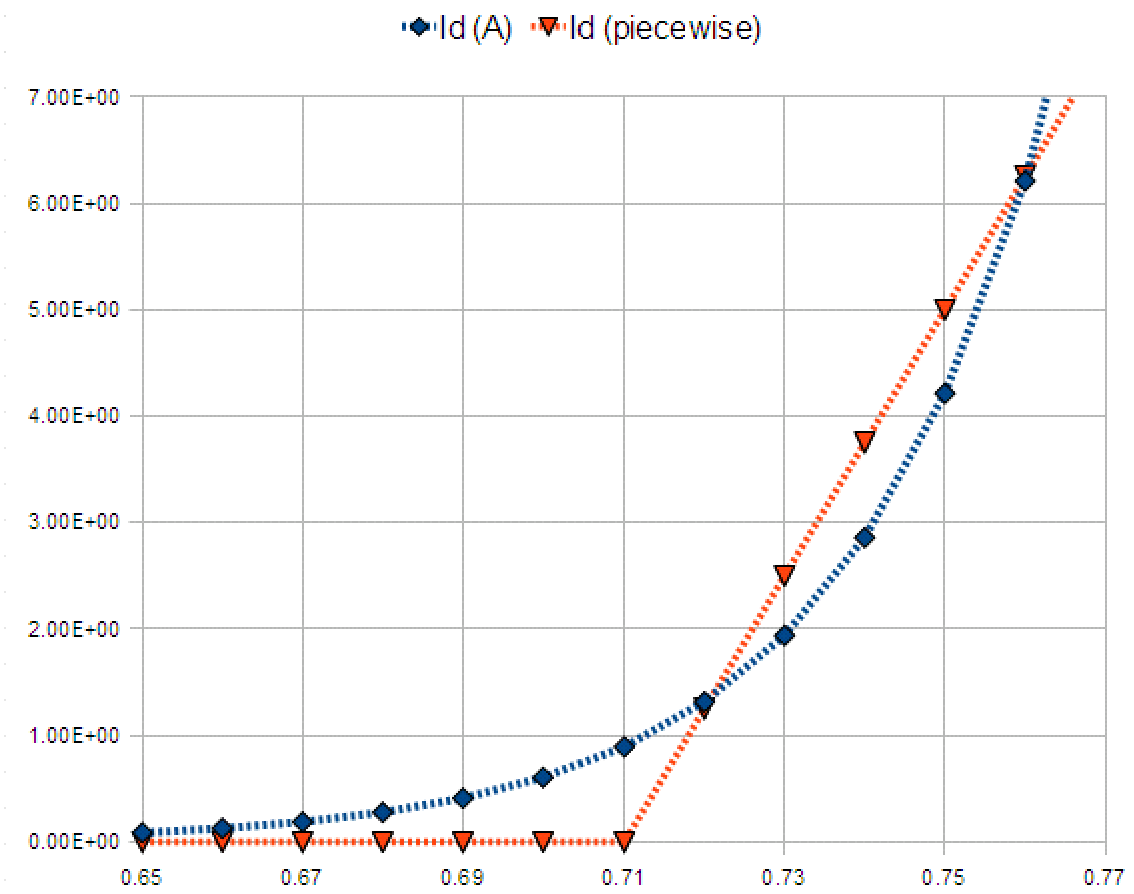




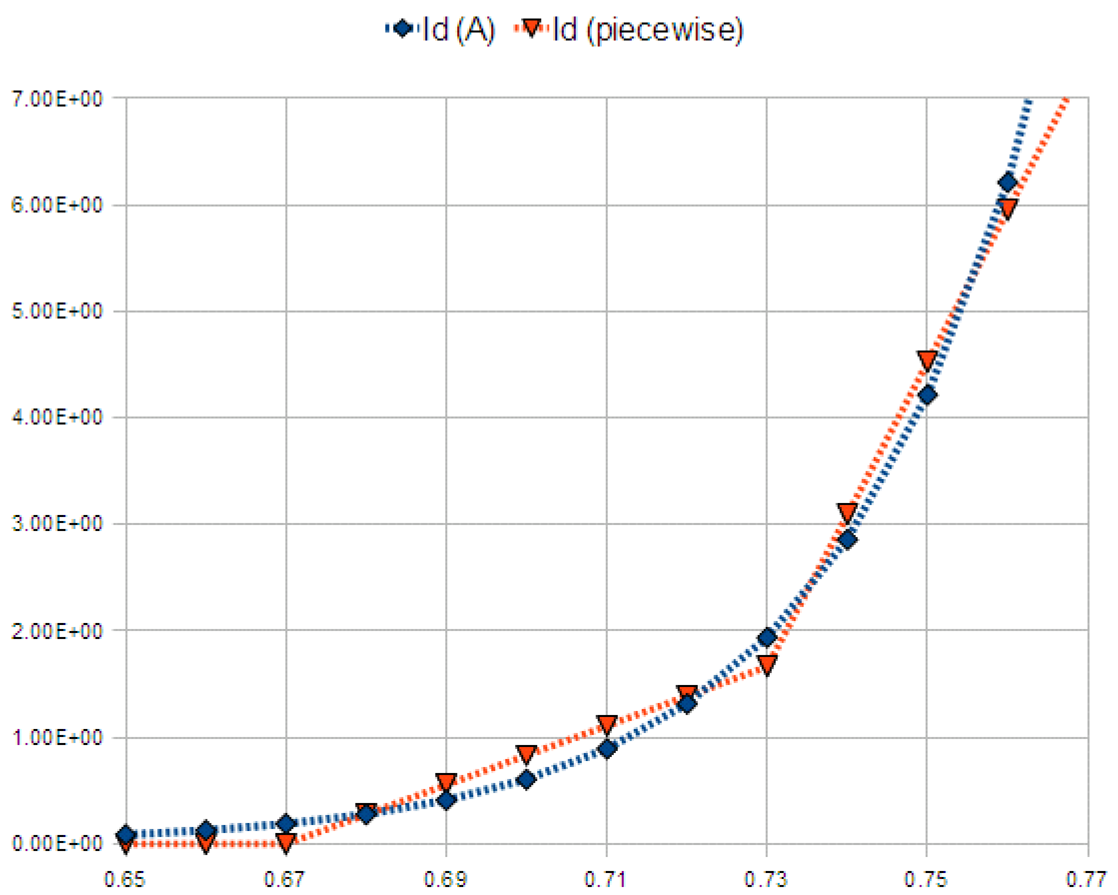


Method of assumed states: 3 possibilities

Diode: Piecewise Linear Approximation

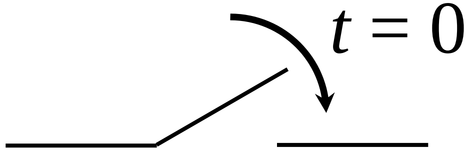


Method of assumed states: 2 possibilities



Method of assumed states: 3 possibilities

Switch vs Unidirectional Device



short circuit $t > 0$

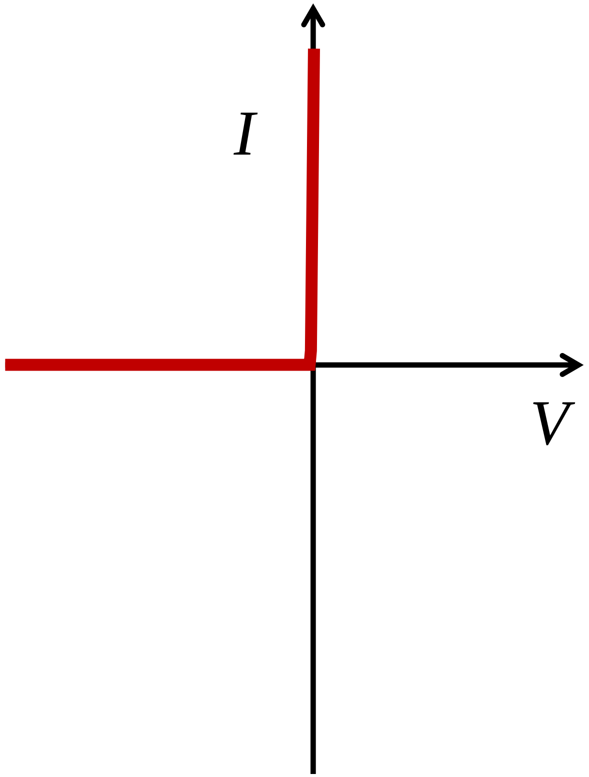
$t < 0$

open circuit

short circuit V > 0

V < 0

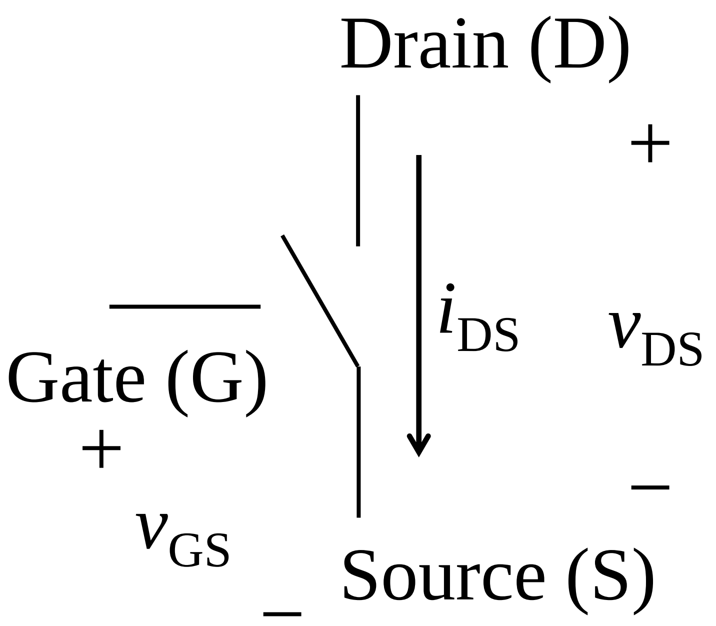
open circuit

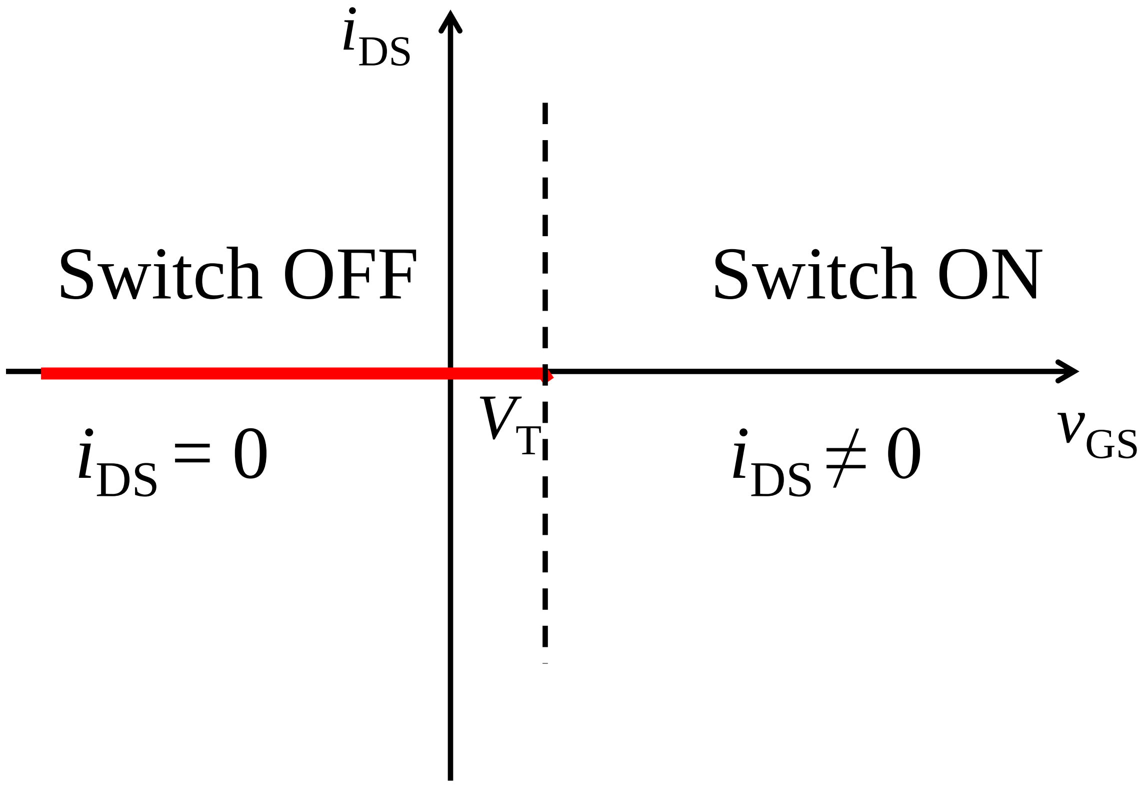


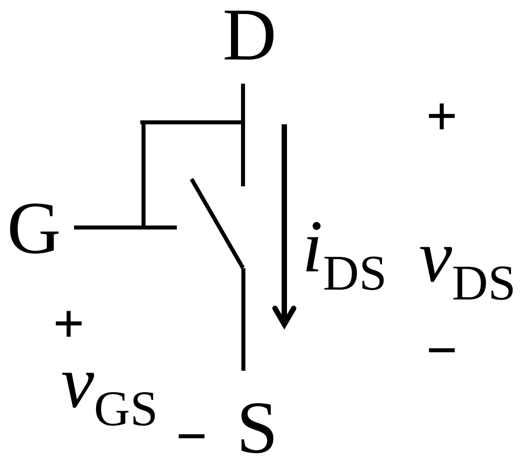
Switch: Time-Dependent

Diode: Voltage-Dependent

Electrical Switch: Three Terminal Device to Diode

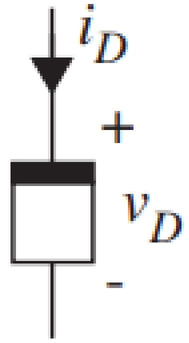


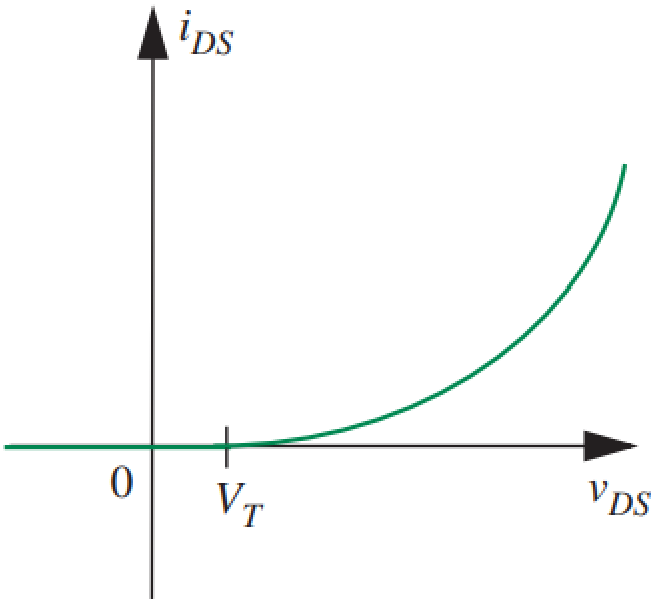




- For $v_{GS} = v_{DS}$, the device is two-terminal and it behaves like an unidirectional device
- Diode-connected transistor (MOSFET)

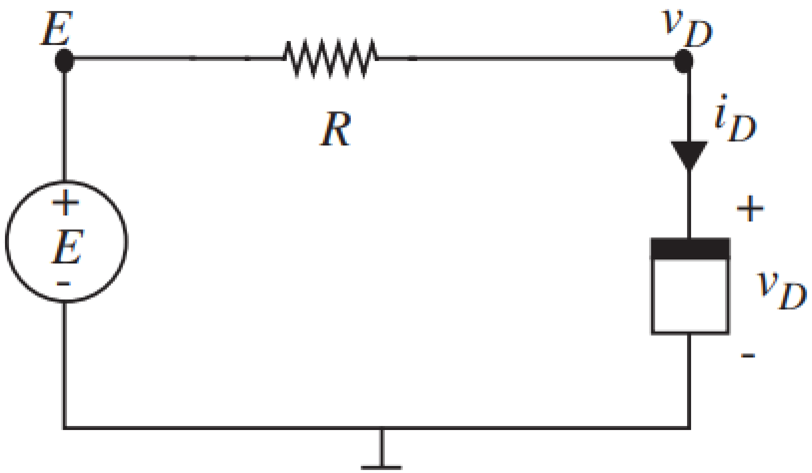
Unidirectional Device: Square Law





$$i_{DS} = \begin{cases} \dfrac{K(v_{DS} - V_T)^2}{2} & \text{for } v_{DS} \geq V_T \\ 0 & \text{for } v_{DS} < V_T \end{cases}$$

Square Law Device: Analytical Solution



$$i_D = \begin{cases} Kv_D^2 & \text{for } v_D > 0 \\ 0 & \text{for } v_D \le 0. \end{cases}$$

$V_T = 0$

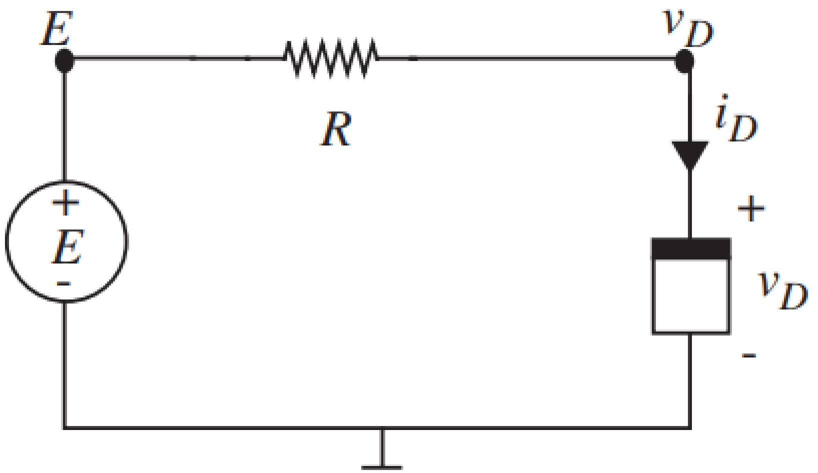
$$\frac{v_D - E}{R} + i_D = 0 \qquad i_D = Kv_D^2.$$

$$RKv_D^2 + v_D - E = 0.$$

$$\frac{v_D - E}{R} + Kv_D^2 = 0.$$

$$v_D = \frac{-1 + \sqrt{1 + 4RKE}}{2RK} \qquad i_D = K\left[\frac{-1 + \sqrt{1 + 4RKE}}{2RK}\right]^2$$

Square Law Device: Graphical Load Line Analysis

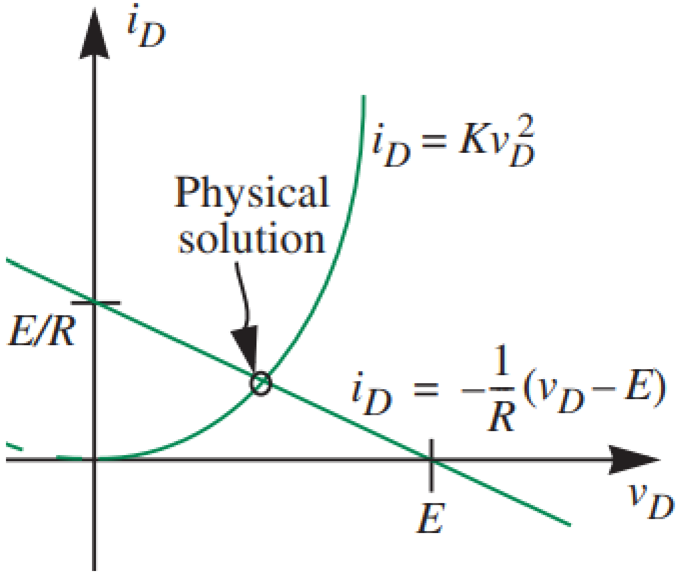


$$i_D = \begin{cases} Kv_D^2 & \text{for } v_D > 0 \\ 0 & \text{for } v_D \leq 0. \end{cases}$$

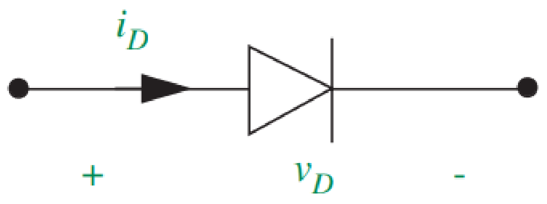
$$V_T = 0$$

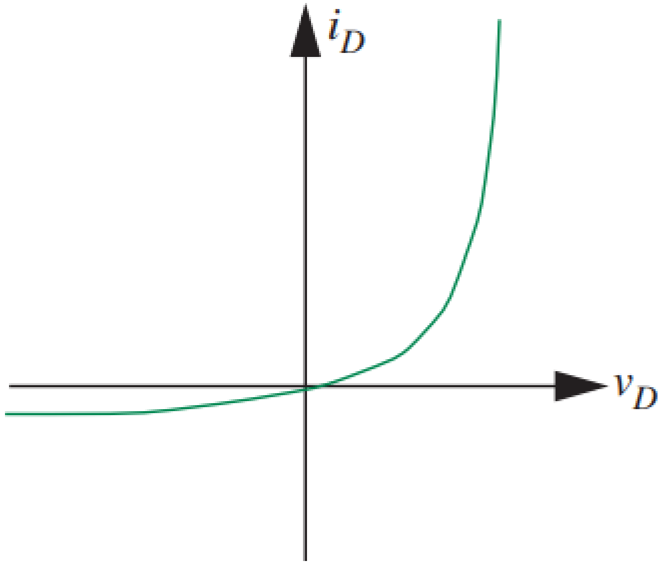
$$\frac{v_D - E}{R} + i_D = 0 \qquad i_D = Kv_D^2.$$

$$i_D = -\frac{v_D - E}{R}$$



Unidirectional Device: Exponential Law





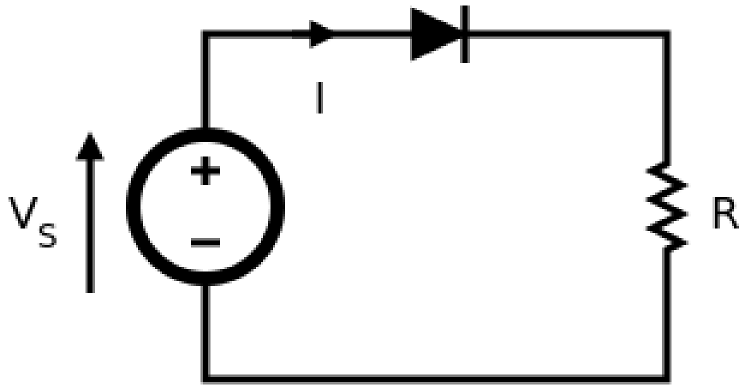
$$i_D = I_S\left(\exp\left(\frac{v_D}{nV_T}\right) - 1\right),\ n = 1$$

$$i_D = I_s(e^{v_D/V_{TH}} - 1).$$

$$V_T = V_{TH} = \frac{kT}{q} \approx 26\text{ mV}$$

at T = 300K

Exponential Law Device: Analytical Solution



$$I = \frac{V_S - V_D}{R}$$

$$I = I_S\left(\exp\left(\frac{V_D}{nV_T}\right) - 1\right)$$

$$w = \frac{I_S R}{nV_\mathrm{T}}\left(\frac{I}{I_S} + 1\right)$$

$$I/I_S = e^{V_D/nV_\mathrm{T}} - 1$$

$$we^w = \frac{I_S R}{nV_\mathrm{T}} e^{\frac{V_D}{nV_\mathrm{T}}} e^{\frac{I_S R}{nV_\mathrm{T}}\left(\frac{I}{I_S}+1\right)}$$

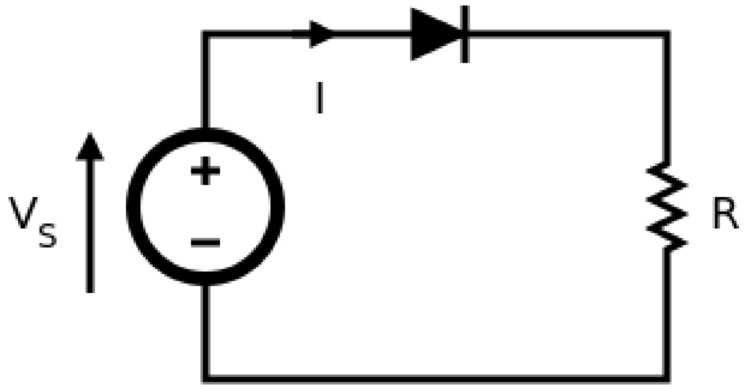
$$we^w = \frac{I_S R}{nV_\mathrm{T}} e^{\frac{V_S}{nV_\mathrm{T}}} e^{\frac{-IR}{nV_\mathrm{T}}} e^{\frac{IRI_S}{nV_\mathrm{T} I_S}} e^{\frac{I_S R}{nV_\mathrm{T}}}$$

$$we^w = \frac{I_S R}{nV_\mathrm{T}} e^{\frac{V_s + I_s R}{nV_\mathrm{T}}} = \text{c, Constant}$$

$$w = W\left(\frac{I_S R}{nV_\mathrm{T}} e^{\frac{V_s + I_s R}{nV_\mathrm{T}}}\right)$$

W(c) is the Lambert *W* function evaluated at the value c

Exponential Law Device: Iterative Solution



$$I = \frac{V_S - V_D}{R}$$

$$I = I_S\left(\exp\left(\frac{V_D}{nV_T}\right) - 1\right)$$

$$e^{\frac{V_D}{nV_\mathrm{T}}} = \frac{I}{I_S} + 1$$

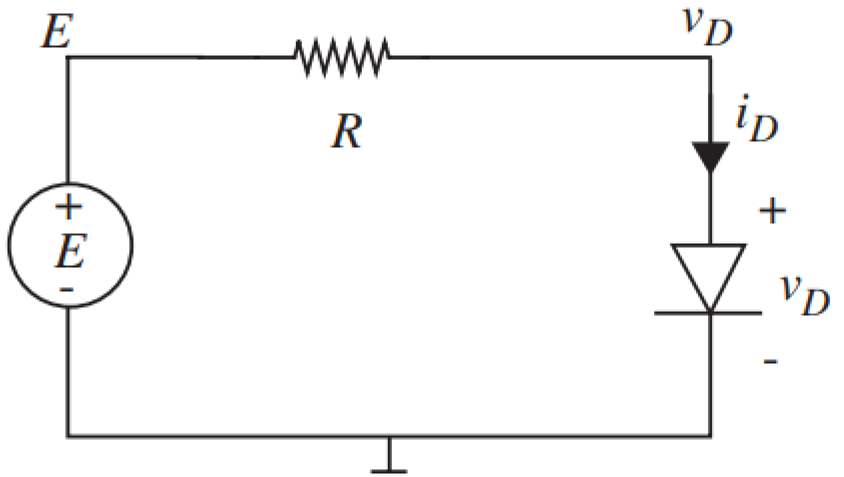
$$\frac{V_D}{nV_\mathrm{T}} = \ln\left(\frac{I}{I_S} + 1\right)$$

$$\frac{V_D}{nV_\mathrm{T}} = \ln\left(\frac{V_S - V_D}{RI_S} + 1\right)$$

$$V_D = nV_\mathrm{T}\ln\left(\frac{V_S - V_D}{RI_S} + 1\right)$$

- Start with an initial guess on the RHS
- Evaluate the LHS to improve upon
- Iterate until desired accuracy is achieved

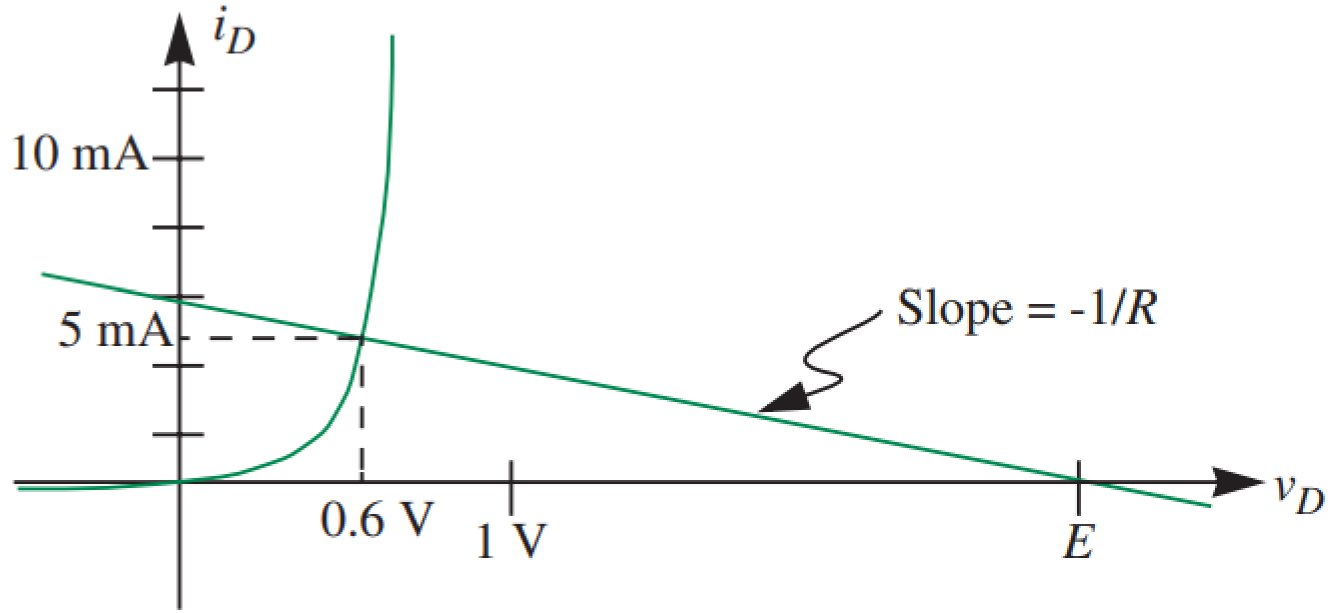
Square Law Device: Graphical Load Line Analysis



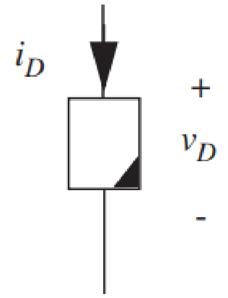
$$\frac{v_D - E}{R} + i_D = 0$$

$$i_D = -\frac{v_D - E}{R}$$

$$i_D = I_S\left(\exp\left(\frac{v_D}{nV_T}\right) - 1\right)$$

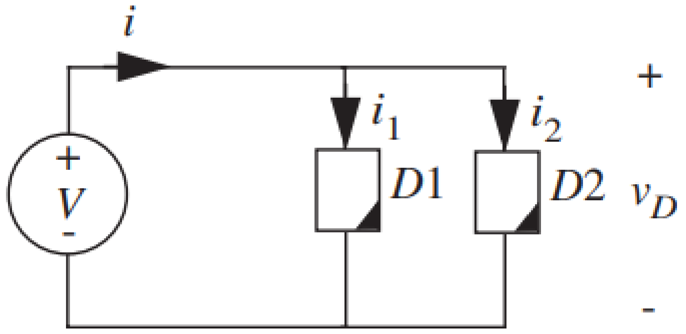


Square Law Device: Parallel Combination



$$i_D = \begin{cases} Kv_D^2 & \text{for } v_D > 0 \\ 0 & \text{for } v_D \le 0. \end{cases}$$

$V_T = 0$



$K = 0.1 \dfrac{\text{A}}{\text{V}^2}$

$v_D = 2\ \text{V}$

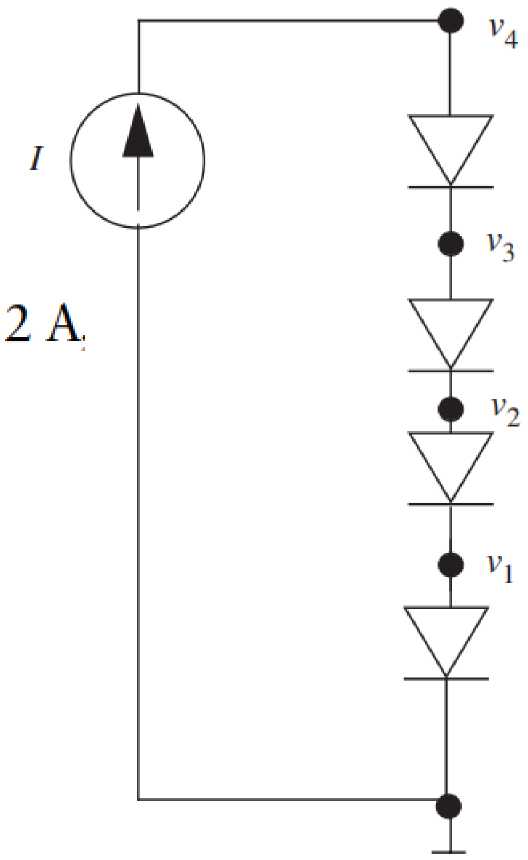
$V = 2\ \text{V}$

$i_D = 0.1 v_D^2 = 0.1 \times 2^2 = 0.4\ \text{A}$

$i_1 = i_2 = 0.4\ \text{A}$

$i = i_1 + i_2 = 0.8\ \text{A}$

Exponential Law Device: Series Combination



$I = 2$ A

$$i_D = I_s(e^{v_D/V_{TH}} - 1) \qquad I_s = 10^{-12} \text{ A}, V_{TH} = 0.025 \text{ V}.$$

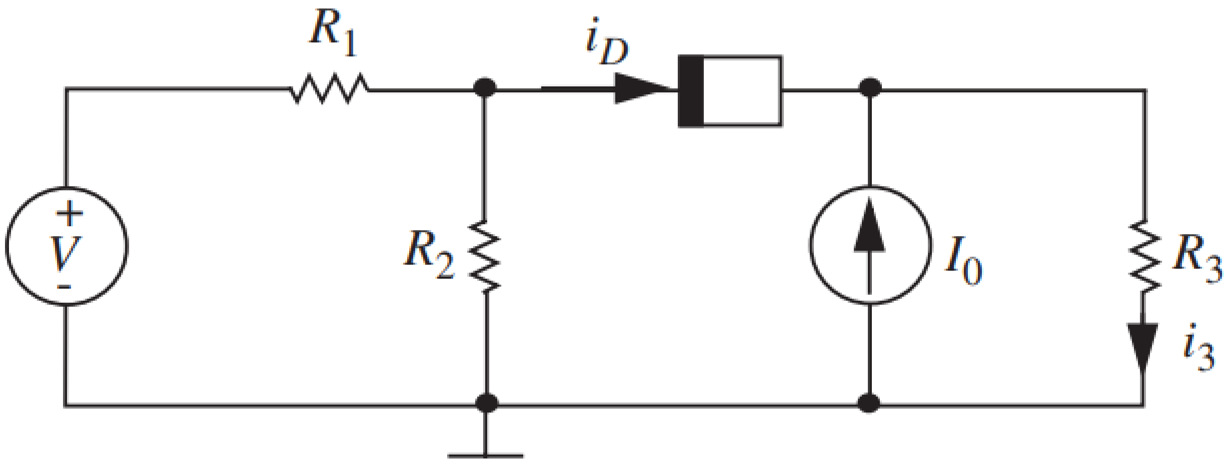
$$v_1 = 0.025 \ln(10^{12} I + 1) = 0.025 \ln(10^{12} \times 2 + 1) = 0.71 \text{ V}$$

$$v_1 = v_2 - v_1 \qquad v_2 = 2v_1 = 1.42 \text{ V}$$

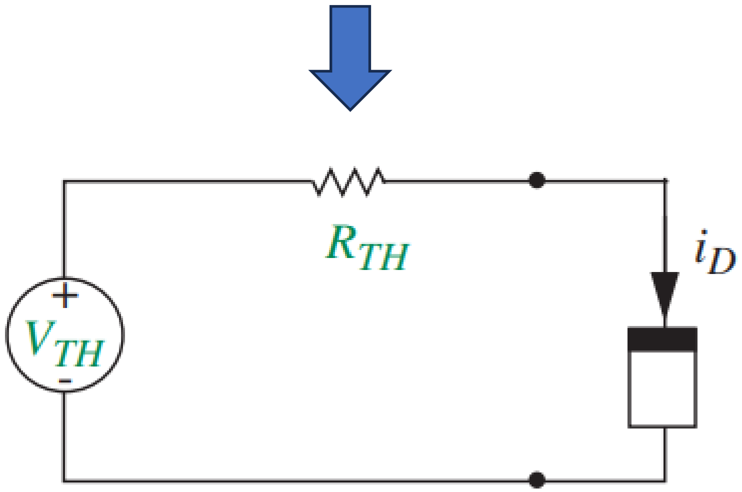
$$v_2 - v_1 = v_3 - v_2 \qquad v_3 = 3v_1 = 2.13 \text{ V}$$

$$v_3 - v_2 = v_4 - v_3 \qquad v_4 = 4v_1 = 2.84 \text{ V}.$$

Square Law Device: Circuit Analysis



$$i_D = \begin{cases} K v_D^2 & \text{for } v_D > 0 \\ 0 & \text{for } v_D \le 0. \end{cases}$$

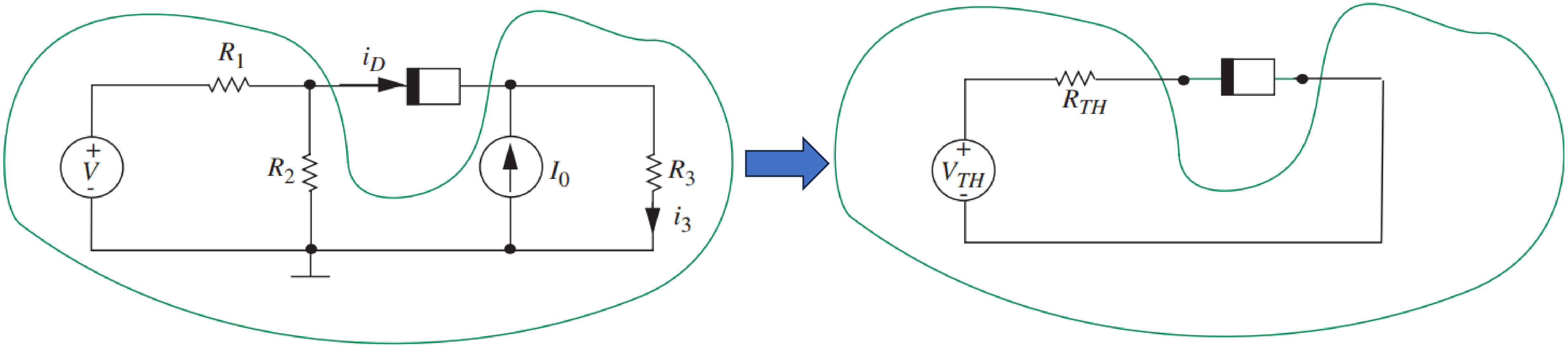


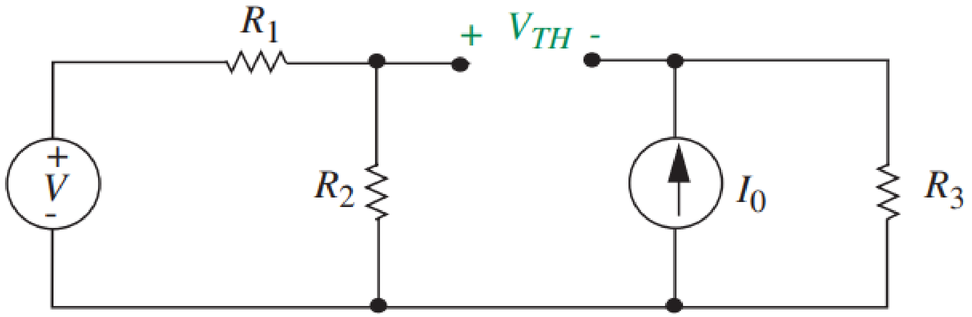
$$v_D = \frac{-1 + \sqrt{1 + 4RKE}}{2RK}$$

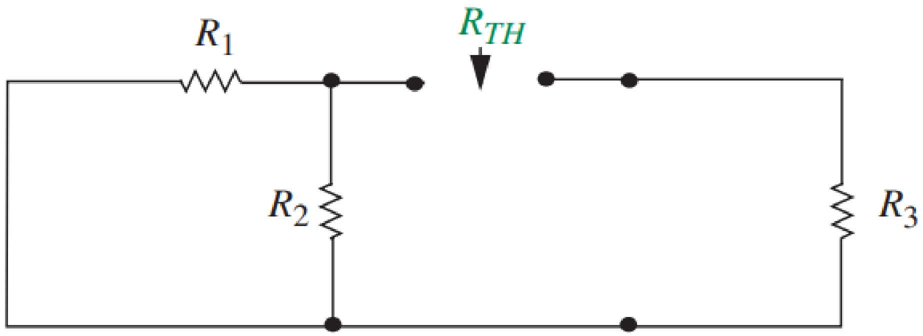
$E = V_{TH}$ $R = R_{TH}$

$$i_D = K\left[\frac{-1 + \sqrt{1 + 4RKE}}{2RK}\right]^2$$

Square Law Device: Circuit Analysis



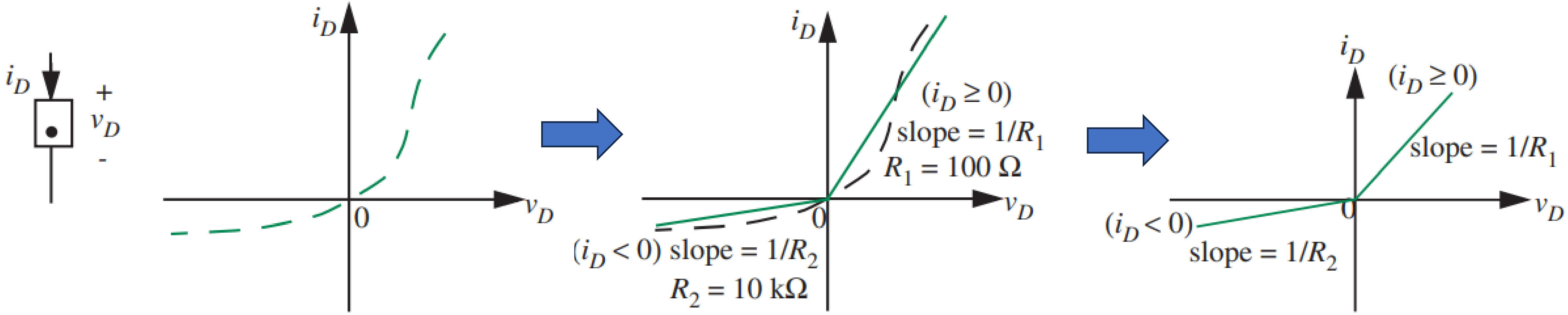


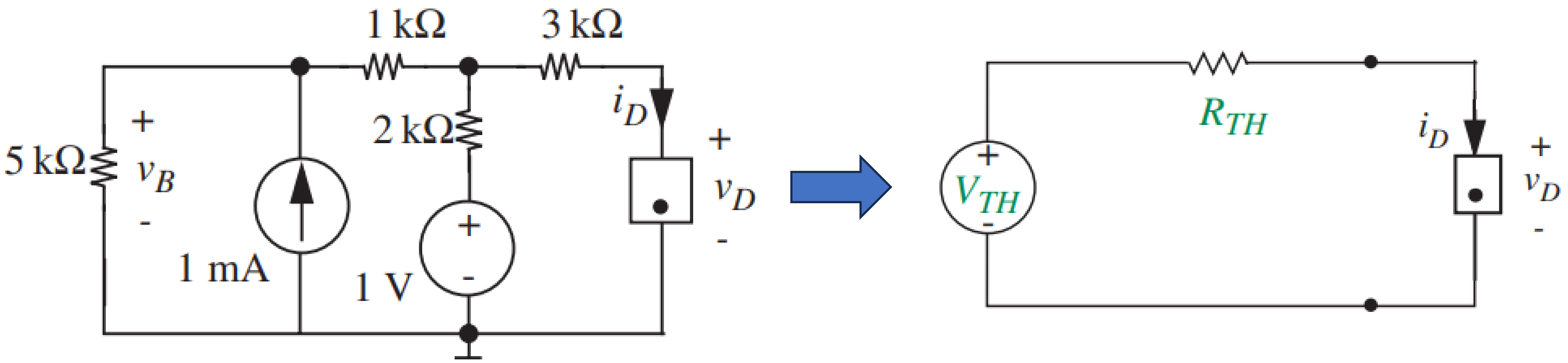


$$V_{TH} = V\frac{R_2}{R_1 + R_2} - I_0 R_3$$

$$R_{TH} = (R_1 || R_2) + R_3$$

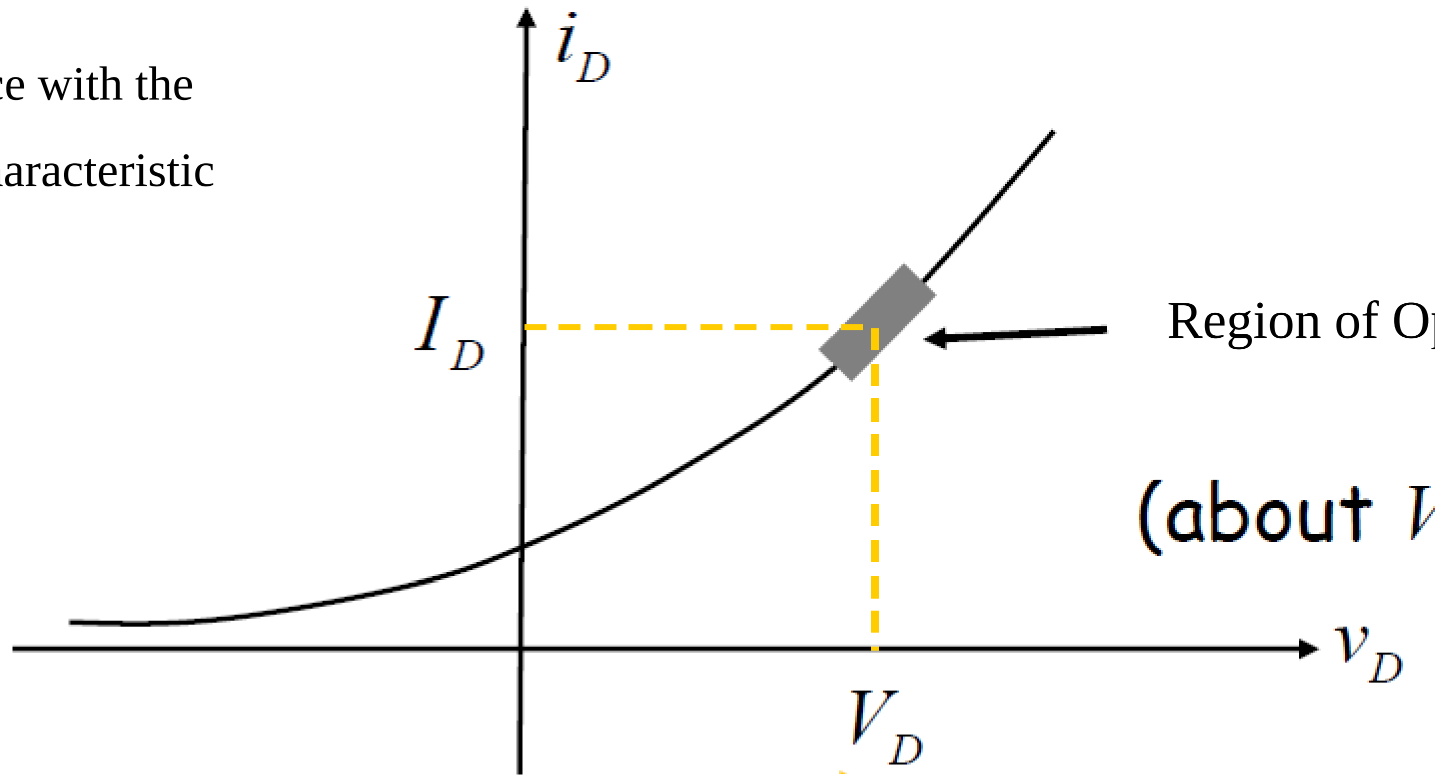
Piecewise Linear Approximation: Circuit Analysis





Small Signal Analysis

Consider a device with the following I-V characteristic

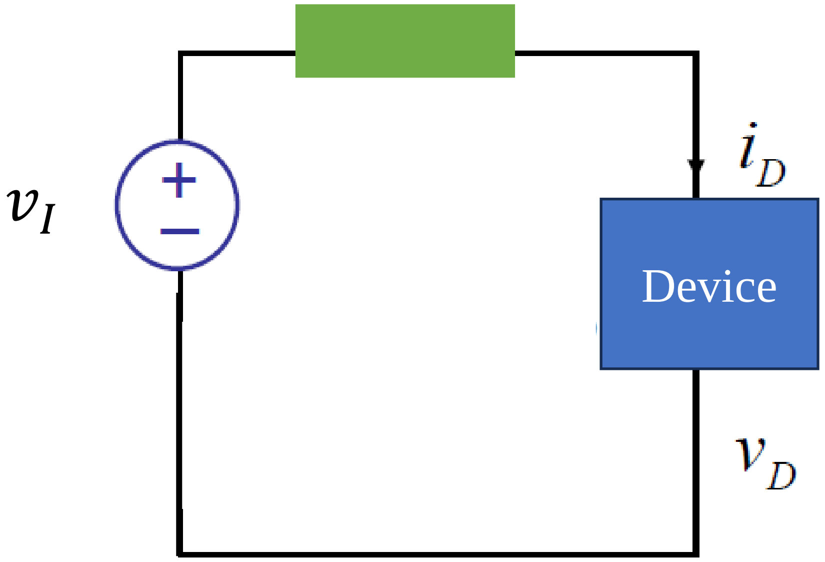


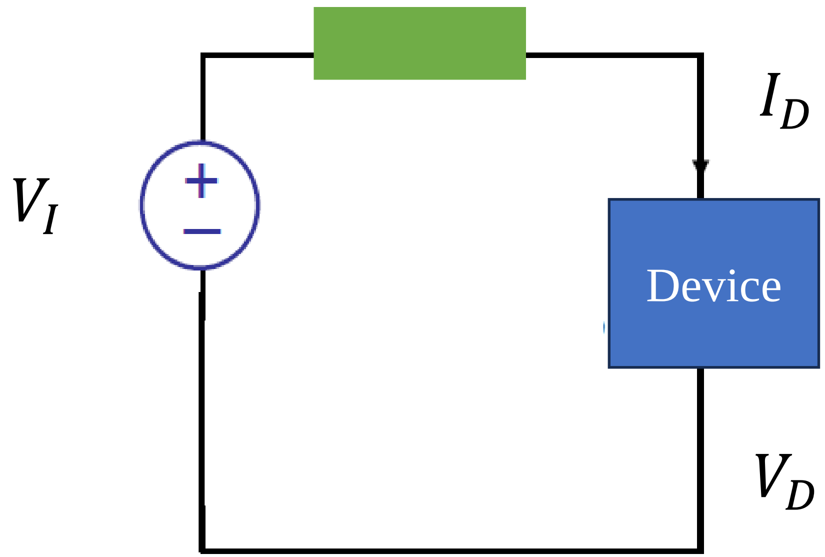
Region of Operation

(about V_D, I_D)

Bias
V_D, I_D

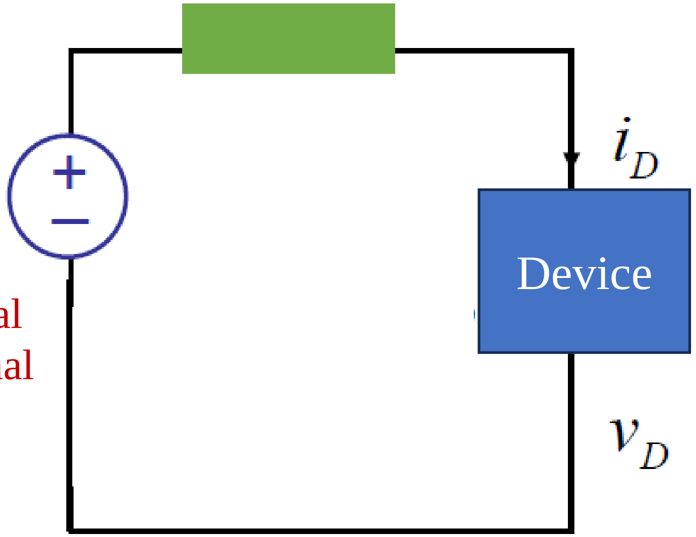
Small Signal Analysis





$$v_I = V_I + v_i$$

Bias Additional small signal



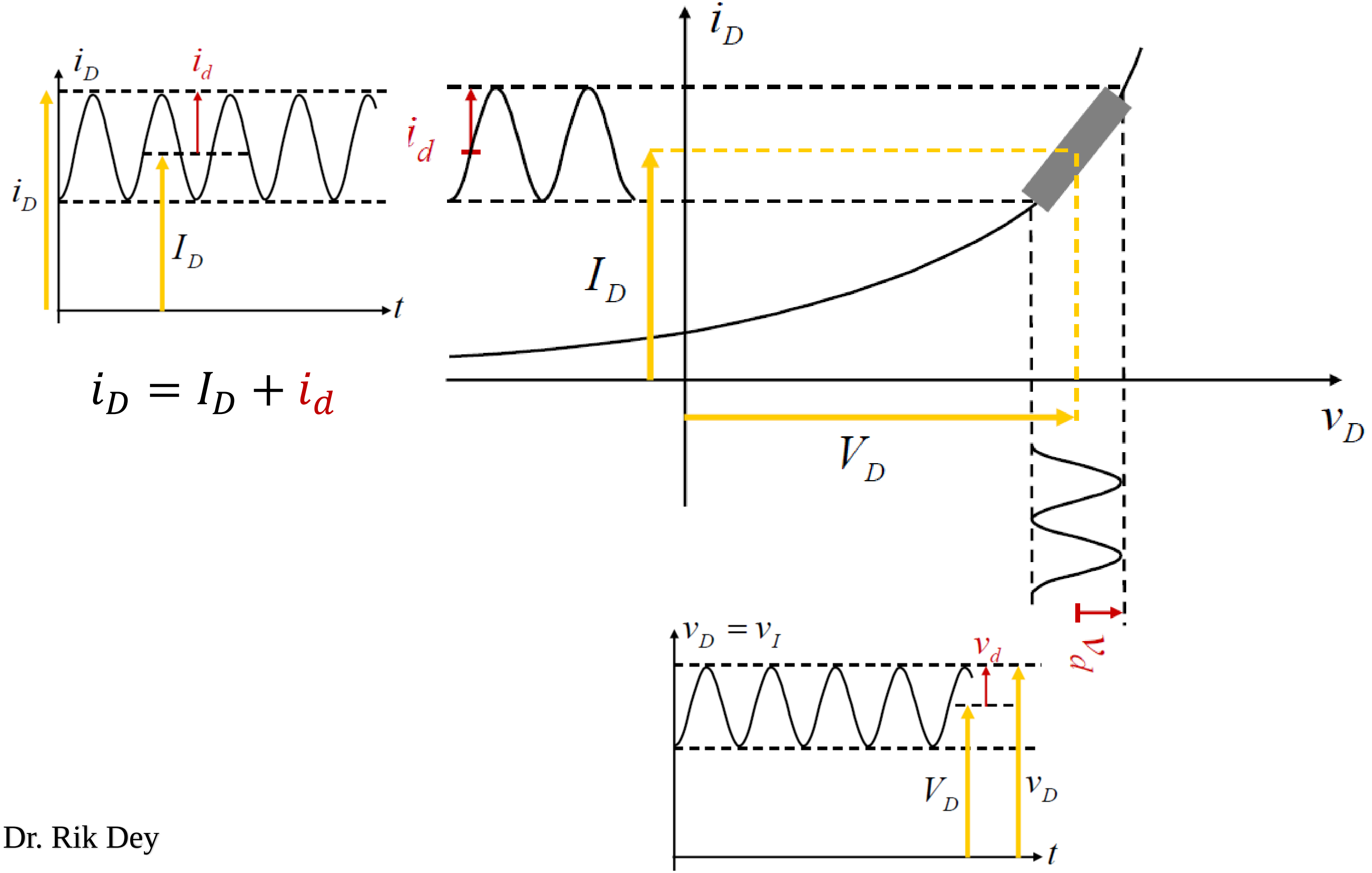
$$i_D = I_D + i_d$$

Relationship?

$$v_D = V_D + v_d$$

Superposition is not applicable

Small Signal Analysis Result



$$i_D = I_D + i_d$$

Linear

$$i_d = k\, v_d$$

k is the slope

$$k = \frac{di_D}{dv_D}$$

computed at Bias point

First order Taylor Series Approximation

Small Signal Analysis Method

- Operate at some bias point V_D, I_D
- Superimpose small signal v_d on top of V_D
- Response i_d to small signal v_d is approximately linear

$$v_D = V_D + v_d$$

signal Bias Additional small signal

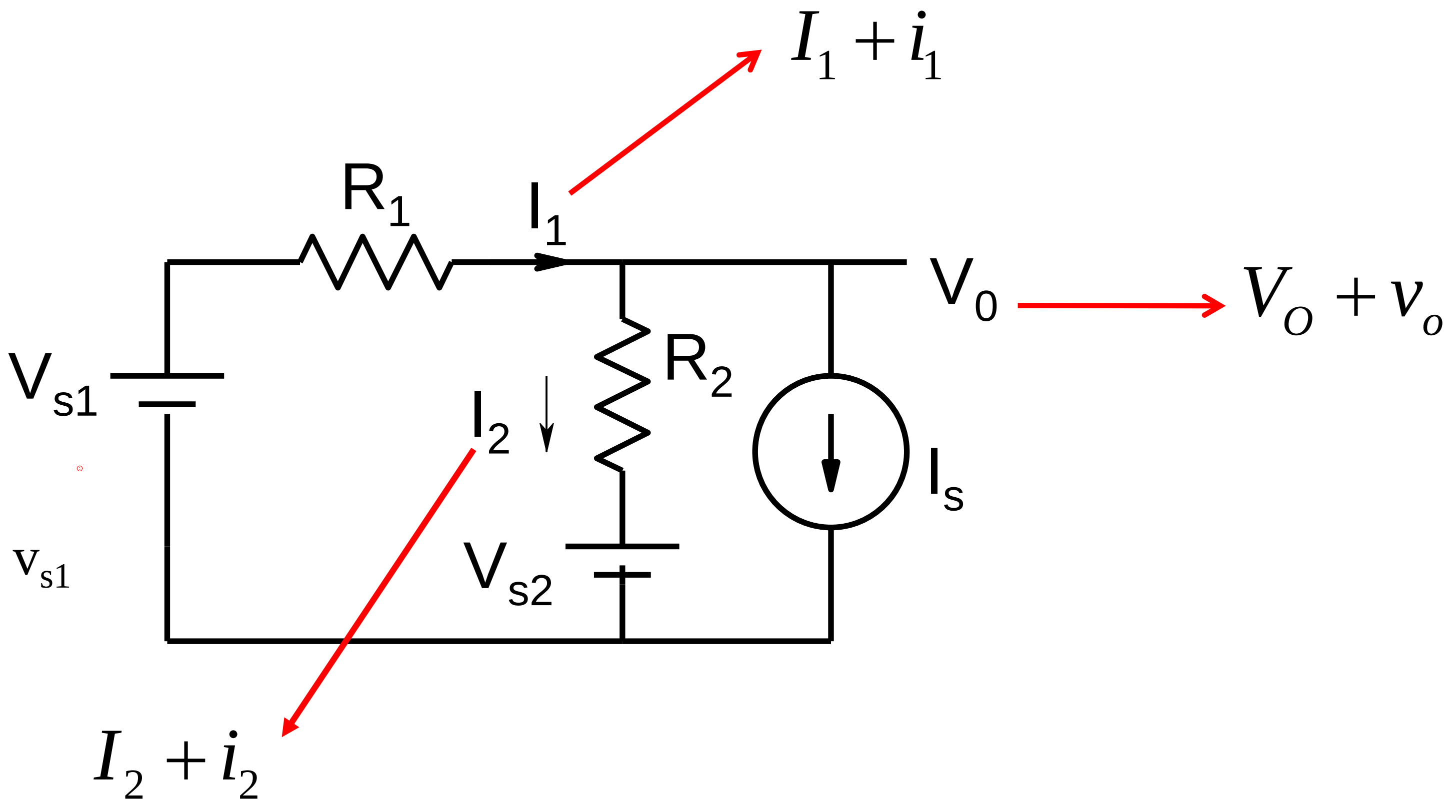
$$i_D = I_D + i_d$$

signal Bias Additional small signal

Linear

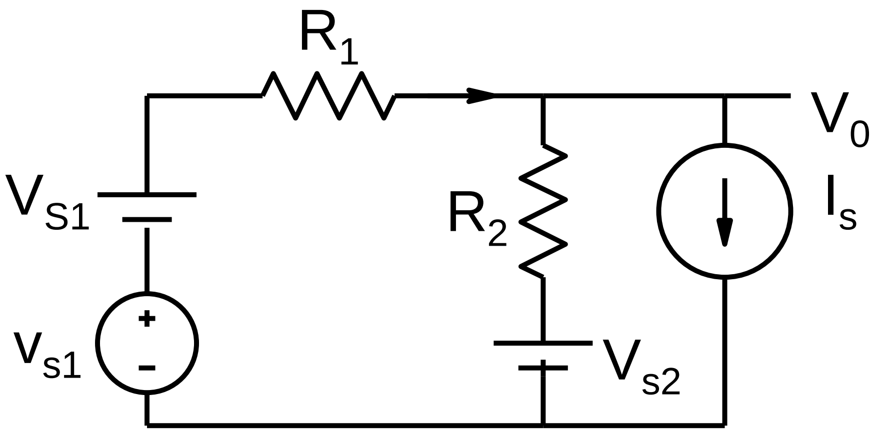
$$i_d = k\, v_d$$

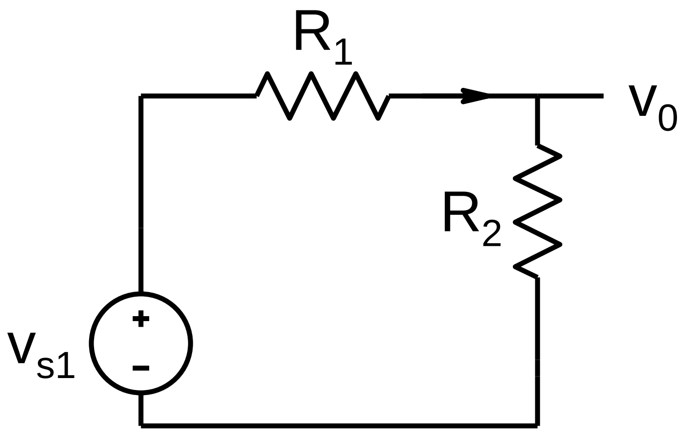
Incremental Model: General



Incremental Model: General

- Same model for R, C, L
- Voltage source with constant voltage -> short circuit
- Current source with constant current -> open circuit





$$v_o = v_{s1} \times \frac{R_2}{R_1 + R_2}$$