$$\boxed{H(s)}$$

$$s=\sigma+j\omega$$

$$H(s)=\frac{U_o(s)}{U_{in}(s)}=\frac{a_m\ s^m+\ldots+a_0}{b_n\ s^n+\ldots+b_0}=\frac{a_m}{b_n}\ \frac{\Pi_i(s-z_i)}{\Pi_j(s-p_j)}$$

a_i, b_j

$\mathrm{z}_\mathrm{i}, p_j$

j=0, 1, 2,...,n.

$i=0,1,2,\ldots\ldots,m$,

$$H(s)=|H(\omega)|\exp\left[j\varphi(\omega)\right]$$

$$|H(\omega)|$$

$$\varphi(\omega)$$

$$\boxed{H_{(k-2)}(s)}\ \boxed{H_{(k-1)}(s)}\ \boxed{H_k(s)}\ \boxed{H_{(k+1)}(s)}\ \boxed{H_{(k+2)}(s)}$$

$$H\ (s)=\frac{N(s)}{D(s)}=\prod_k H_k(s)=\prod_k\frac{N_k(s)}{D_k(s)}$$

$$H_k(s)=\frac{a_2\ s^2+a_1\ s+a_0}{s^2+b_1\ s+b_0}=\frac{N_k\ (s)}{s^2+b_1\ s+b_0}$$

$$H_k(s)=\frac{N_k\ (s)}{s^2+\dfrac{\omega_0}{Q}\ s+\omega_0^2}=\frac{N_k\ (s)}{(s-p_1)(s-p_2)}$$

$$Q > 1/2$$

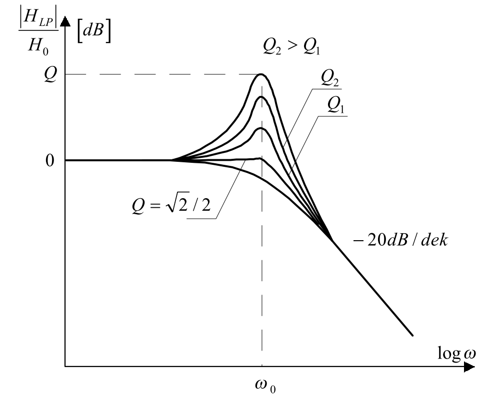
$$p_{1,2} = -\frac{\omega_0}{2Q} \pm j\frac{\omega_0}{2Q}\sqrt{4Q^2 - 1} = \sigma_p \pm j\omega_p \qquad \sigma_p = -\frac{\omega_0}{2Q} \qquad \omega_p = \omega_0\sqrt{1 - \frac{1}{4Q^2}}$$

$$\omega_0 = \sqrt{\sigma_p^2 + \omega_p^2} \qquad Q = \frac{\sqrt{\sigma_p^2 + \omega_p^2}}{2\left|\sigma_p\right|} = \frac{\omega_0}{2\left|\sigma_p\right|} \qquad \xi = \frac{1}{2\,Q}$$

$$s^2 + b_1\,s + b_0 = s^2 + \frac{\omega_0}{Q} + \omega_0 \qquad Q = \sqrt{b_0}\,/\,b_1 \text{ oraz } \omega_0 = \sqrt{b_0}$$

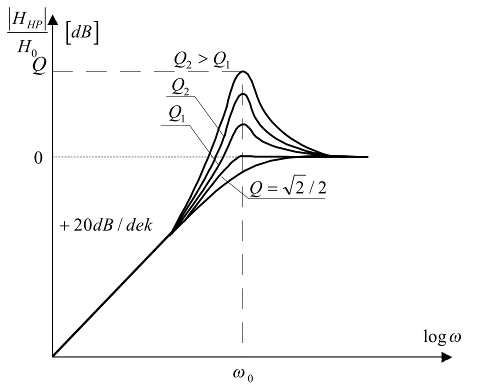
Filtro pasa baja

$$H_{LP} = H_0\,\frac{\omega_0^2}{s^2 + \frac{\omega_0}{Q}\,s + \omega_0^2}$$



Filtro pasa alta

$$H_{HP}(s) = H_0\,\frac{s}{s^2 + \frac{\omega_0}{}\,s + \omega_0^2}$$

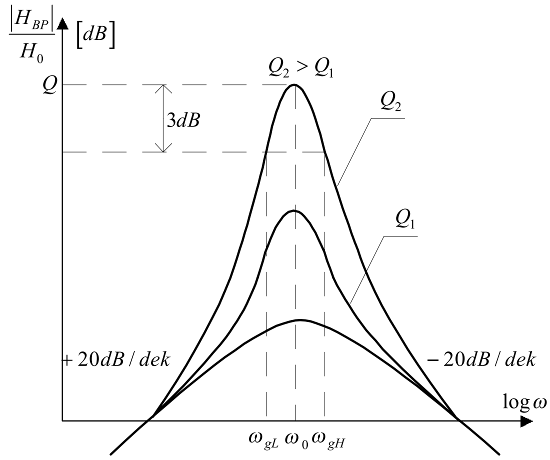


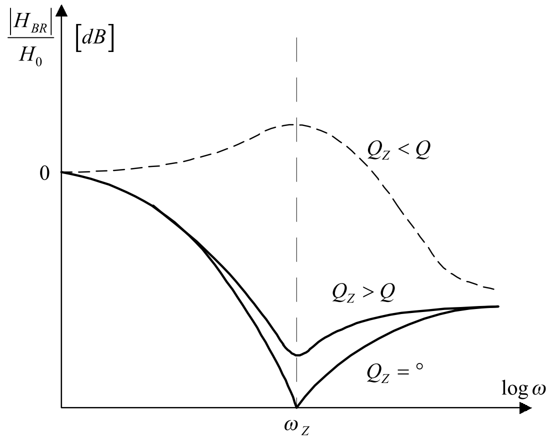
Filtro pasa banda

$$H_{BP}(s) = H_0\,\frac{\omega_0\,s}{s^2 + \frac{\omega_0}{Q}\,s + \omega_0^2}$$

Filtro supresor de banda

$$H_{BP}(s) = H_0\,\frac{s^2 + \frac{\omega_z}{Q_z}\,s + \omega_z^2}{s^2 + \frac{\omega_0}{Q}\,s + \omega_0^2}$$





$$H_{AP}(s) = H_0 \frac{s^2 - \frac{\omega_0}{Q_z} s + \omega_0^2}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2} \qquad \Phi(\omega) = -2\, arc\, tg \frac{\frac{\omega}{2\, Q\, \omega_0}}{1 - \frac{\omega^2}{\omega_0^2}}$$

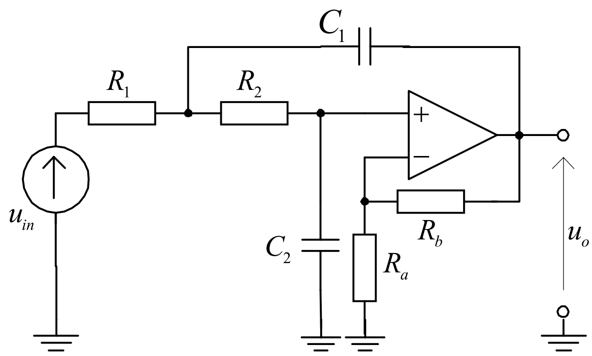
$$S_{x_i}^{Q} = \frac{x_i}{Q} \frac{\partial Q}{\partial x_i}; \quad S_{x_i}^{\omega_0} = \frac{x_i}{\omega_0} \frac{\partial \omega_0}{\partial x_i}$$

$$i = 1, 2, \ldots n$$

$$S_{x_i}^{z} = \frac{x_i}{z} \frac{\partial z}{\partial x_i}; \quad S_{x_i}^{p} = \frac{x_i}{p} \frac{\partial p}{\partial x_i}$$

$$i = 1, 2, \ldots\ldots, n$$

Sallen Key



$$k_u = 1 + R_b / R_a$$

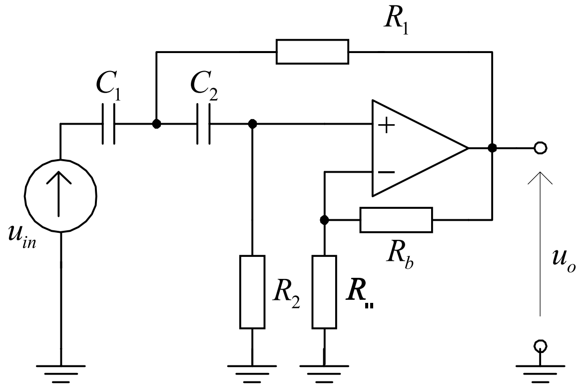
$$H_{LP}(s) = \frac{U_o(s)}{U_{in}(s)} = \frac{\frac{k_u}{R_1\, R_2\, C_1\, C_2}}{s^2 + \left(\frac{1}{R_1\, C_1} + \frac{1}{R_2\, C_1} + \frac{1 - k_u}{R_2\, C_2} \right) s + \frac{1}{R_1\, R_2\, C_1\, C_2}}$$

$$\omega_0 = \frac{1}{\sqrt{R_1\, R_2\, C_1\, C_2}} \qquad Q = \frac{\frac{1}{\sqrt{R\, R_2\, C\, C_2}}}{\left(\frac{1}{R\, C} + \frac{1}{R_2\, C} + \frac{1 - k_u}{R_2\, C_2} \right)} \qquad H_0 = k_u$$

$$R_1 = R_2 = R$$

$$C_1 = \frac{2\, Q}{\omega_0} \qquad C_2 = \frac{1}{2\, Q\, \omega_0}$$

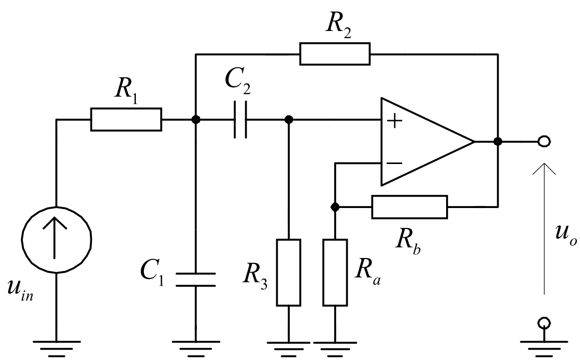
$$S_R^Q = 0; \quad S_C^Q = \pm 1/2; \quad S_R^{\omega_0} = S_C^{\omega_0} = -1/2$$



$$H_{HP}(s) = \frac{U_o(s)}{U_{in}(s)} = \frac{k_u\, s^2}{s^2 + \left(\frac{1}{R_2\, C_1} + \frac{1}{R_2\, C_2} + \frac{1-k_u}{R_1\, C_1}\right) s + \frac{1}{R_1\, R_2\, C_1\, C_2}}$$

$$k_u = 1; \quad C_1 = C_2$$

$$Q = \frac{1}{2}\sqrt{\frac{R_1}{R_2}} \qquad \omega_0 = \frac{1}{C\sqrt{R_1 R_2}}$$



$$\omega_0 = \sqrt{\frac{R_1 + R_2}{R_1\, R_2\, R_3\, C_1\, C_2}}$$

$$Q = \frac{\sqrt{\frac{R_1 + R_2}{R_1\, R_2\, R_3\, C_1\, C_2}}}{\left(\frac{1}{R_1 C_1} \quad \frac{1}{R_3 C_1} + \frac{1}{R_3 C_2} + \frac{1-k_u}{R_2 C_1}\right)}$$

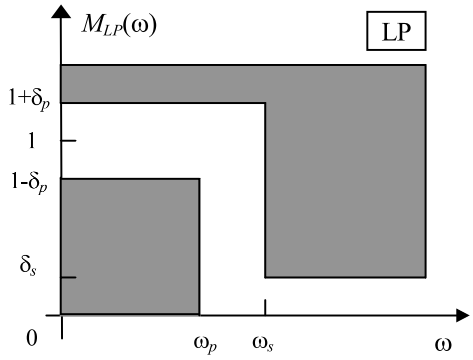
$$H_{BP}(s) = \frac{U_o(s)}{U_{in}(s)} = \frac{\frac{k_u}{R\,C}\, s}{s^2 + \left(\frac{1}{R_1 C_1} + \frac{1}{R_3 C_1} + \frac{1}{R_3 C_2} + \frac{1-k_u}{R_2 C_1}\right) s + \frac{R_1 + R_2}{R_1\, R_2\, R_3\, C_1\, C_2}}$$

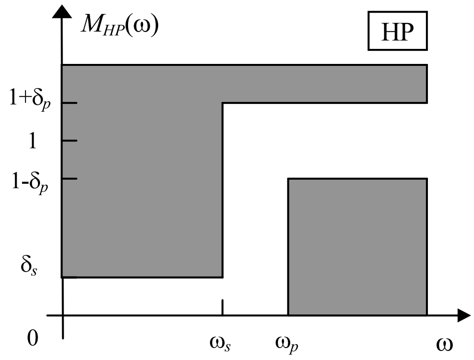
LP – *Low Pass*

$$\begin{cases} 1-\delta_{pass} \le \left|H_{LP}(j\omega)\right| \le 1+\delta_{pass}, & \text{dla } \left|\omega\right| \le \omega_{pass} \\ 0 \le \left|H_{LP}(j\omega)\right| \le 1+\delta_{pass}, & \text{dla } \omega_{pass} < \left|\omega\right| < \omega_{stop} \\ 0 \le \left|H_{LP}(j\omega)\right| \le \delta_{stop}, & \text{dla } \omega_{stop} \le \left|\omega\right| \end{cases}$$

HP – *High Pass*

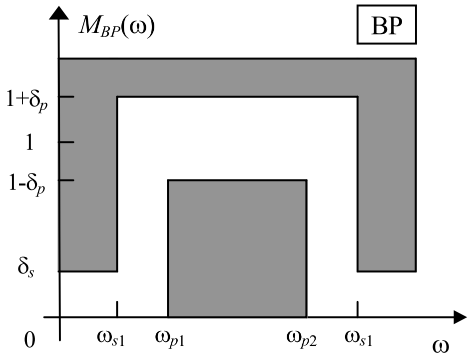
$$\begin{cases} 0 \le \left|H_{HP}(j\omega)\right| \le \delta_{stop}, & \text{dla } \left|\omega\right| \le \omega_{stop} \\ 0 \le \left|H_{HP}(j\omega)\right| \le 1+\delta_{pass}, & \text{dla } \omega_{stop} < \left|\omega\right| < \omega_{pass} \\ 1-\delta_{pass} \le \left|H_{HP}(j\omega)\right| \le 1+\delta_{pass}, & \text{dla } \omega_{pass} \le \left|\omega\right| \end{cases}$$





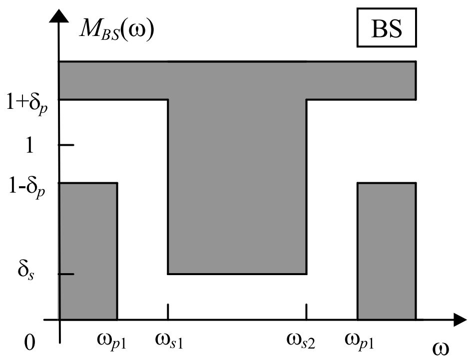
BP – *Band Pass*

$$\begin{cases} 1-\delta_{pass} \le \left|H_{BP}(j\omega)\right| \le 1+\delta_{pass}, & \text{dla } \omega_{pass1} \le \left|\omega\right| \le \omega_{pass2} \\ 0 \le \left|H_{BP}(j\omega)\right| \le \delta_{stop}, & \text{dla } \left|\omega\right| \le \omega_{stop1} \text{ lub } \omega_{stop2} \le \left|\omega\right| \\ 0 \le \left|H_{BP}(j\omega)\right| \le 1+\delta_{pass}, & \text{dla } \omega_{stop1} < \left|\omega\right| < \omega_{pass1} \text{ lub } \omega_{pass2} < \left|\omega\right| < \omega_{stop2} \end{cases}$$



BS – *Band Stop*

$$\begin{cases} 1-\delta_{pass} \le \left|H_{BS}(j\omega)\right| \le 1+\delta_{pass}, & \text{dla } \left|\omega\right| \le \omega_{pass1} \text{ lub } \omega_{pass2} \le \left|\omega\right| \\ 0 \le \left|H_{BS}(j\omega)\right| \le \delta_{stop}, & \text{dla } \omega_{stop1} \le \left|\omega\right| \le \omega_{stop2} \\ 0 \le \left|H_{BS}(j\omega)\right| \le 1+\delta_{pass}, & \text{dla } \omega_{pass1} < \left|\omega\right| < \omega_{stop1} \text{ lub } \omega_{stop2} < \left|\omega\right| < \omega_{pass2} \end{cases}$$



$$H_{B,N}(s) = \prod_{k=1}^{N}(-p_k) \Big/ \prod_{k=1}^{N}(s-p_k)$$

ω_{pass}, ω_{stop}, δ_{pass}, δ_{stop}

$$\left|H_{B,N}\left(j(\omega/\omega_{3dB})\right)\right| = \frac{1}{\sqrt{1+(\omega/\omega_{3\mathrm{dB}})^{2N}}}$$

$$\begin{cases} \dfrac{1}{\sqrt{1+(\omega_{pass}/\omega_{3\mathrm{dB}})^{2N}}} = 1-\delta_{pass} \\ \dfrac{1}{\sqrt{1+(\omega_{stop}/\omega_{3\mathrm{dB}})^{2N}}} = \delta_{stop} \end{cases}$$

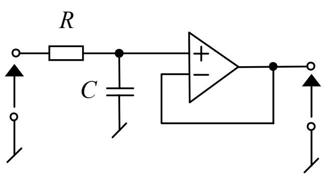
$$\begin{cases} -10\log_{10}\left(1+(\omega_{pass}/\omega_{3\mathrm{dB}})^{2N}\right) = 20\log_{10}(1-\delta_{pass}) = -A_{pass} \\ -10\log_{10}\left(1+(\omega_{stop}/\omega_{3\mathrm{dB}})^{2N}\right) = 20\log_{10}(\delta_{stop}) = -A_{stop} \end{cases}$$

$$\begin{cases} 1+(\omega_{pass}/\omega_0)^{2N} = 10^{A_{pass}/10} \\ 1+(\omega_{stop}/\omega_0)^{2N} = 10^{A_{stop}/10} \end{cases}$$

$$2N\log_{10}\left(\frac{\omega_{stop}}{\omega_{pass}}\right) = \log_{10}\left(\frac{10^{A_{stop}/10}-1}{10^{A_{pass}/10}-1}\right)$$

$$N = \lceil M \rceil, \quad M = \log_{10}\left(\frac{10^{A_{stop}/10}-1}{10^{A_{pass}/10}-1}\right) \Big/ 2\log_{10}(\Omega), \quad \Omega = \frac{\omega_{stop}}{\omega_{pass}}$$

$$\omega_{3\mathrm{dB}} = \frac{\omega_{stop}}{\left(10^{A_{stop}/10}-1\right)^{1/2N}}$$

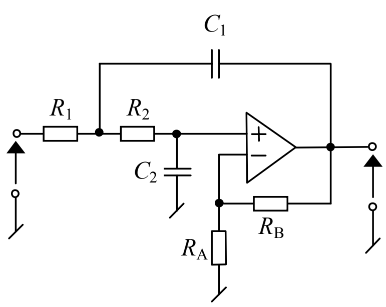


$$H(s) = \frac{1/RC}{s+1/RC}$$

$$p_k = \omega_{3\mathrm{dB}} e^{j\phi_k} = \omega_{3\mathrm{dB}} \exp\left[j\left(\frac{\pi}{2}+\frac{\pi}{2N}+(k-1)\frac{\pi}{N}\right)\right], \quad k = 1, 2, 3, \ldots, N$$

$$p_0 \in \mathbb{R} \Rightarrow \frac{1}{s-p_0}$$

$$p_0 \in \mathbb{C} \Rightarrow \frac{1}{(s-p_0)(s-p_0^*)} \Rightarrow \frac{1}{s^2+2\Re(p)s+|p|^2}$$



$$H(s) = \frac{\prod_{k=1}^{N}(-p_k)}{\prod_{i=1}^{P}\left(s^2+2\Re(p_i)s+|p_i|^2\right)\cdot\prod_{j=1}^{R}(s-p_j)}$$

$$H(s) = \frac{K/R_1R_2C_1C_2}{s^2 + \left(\dfrac{1}{R_1C_1}+\dfrac{1}{R_2C_1}+\dfrac{1-K}{R_2C_2}\right)s + \dfrac{1}{R_1R_2C_1C_2}}$$

$$N = 2P + R$$

(HP):

$$(v_{p1},\ \delta_{p1}) \to (\omega_{p1},\ \delta_{p1})$$

$$(v_{s1}, \delta_{s1}) \to (\omega_{s1},\ \delta_{s1})$$

$$\omega_0 = v_{pass}$$

$$v = -\frac{\omega_0}{\omega}$$

$$s = \frac{\omega_0}{s'} \qquad \{\omega_{pass} = 1,\ \omega_{stop} = v_{pass}/v_{stop},\ \delta_{pass},\ \delta_{stop}\}$$

$$H_{HP}(s') = \frac{b_M}{a_N} \cdot \frac{\prod_{m=1}^{M}(-z_m)}{\prod_{n=1}^{N}(-p_n)} \cdot \frac{(s')^{N-M}\prod_{m=1}^{M}(s' - \omega_0 z_m^{-1})}{\prod_{n=1}^{N}(s' - \omega_0 p_n^{-1})}$$

$$\delta_{pass} = \min\left(\delta_{p1}, \delta_{p2}\right), \quad \delta_{stop} = \min\left(\delta_{s1}, \delta_{s2}\right)$$

BP

$$s = \frac{(s')^2 + \omega_0^2}{\Delta\omega \cdot s'}, \quad \omega_0 = \sqrt{v_{p1}v_{p2}}, \quad \Delta\omega = v_{p2} - v_{p1}$$

$$\omega = \frac{v^2 - \omega_0^2}{\Delta\omega \cdot v}$$

$$\omega_{s1} = \frac{v_{s1}^2 - v_{p1}v_{p2}}{(v_{p2} - v_{p1}) \cdot v_{s1}}, \quad \omega_{s2} = \frac{v_{s2}^2 - v_{p1}v_{p2}}{(v_{p2} - v_{p1}) \cdot v_{s2}}$$

$$\omega_{stop} = \min\{|\omega_{s1}|, |\omega_{s2}|\}$$

BS

$$s = \frac{\Delta\omega \cdot s'}{(s')^2 + \omega_0^2}, \quad \omega_0 = \sqrt{v_{p1}v_{p2}}, \quad \Delta\omega = v_{p2} - v_{p1}$$

$$\omega = \frac{\Delta\omega \cdot v}{\omega_0^2 - v^2}$$

$$\omega_{s1} = \frac{(v_{p2} - v_{p1}) \cdot v_{s1}}{v_{s1}^2 - v_{p1}v_{p2}}, \quad \omega_{s2} = \frac{(v_{p2} - v_{p1}) \cdot v_{s2}}{v_{s2}^2 - v_{p1}v_{p2}}$$

$$\omega_s = \min\{|\omega_{s1}|, |\omega_{s2}|\}$$

$$H_{BP}(s') = \frac{b_M}{a_N} \cdot \frac{(\Delta\omega)^{N-M}(s')^{N-M}\prod_{m=1}^{M}\left[(s')^2 - (z_m\Delta\omega)s' + \omega_0^2\right]}{\prod_{n=1}^{N}\left[(s')^2 - (p_n\Delta\omega)s' + \omega_0^2\right]}$$

$$H_{BS}(s') = \frac{b_M}{a_N} \cdot \frac{\prod_{m=1}^{M}(-z_m)}{\prod_{n=1}^{N}(-p_n)} \cdot \frac{\left[(s')^2 + \omega_0^2\right]^{N-M}\prod_{m=1}^{M}\left[(s')^2 - (z_m^{-1}\Delta\omega)s' + \omega_0^2\right]}{\prod_{n=1}^{N}\left[(s')^2 - (p_n^{-1}\Delta\omega)s' + \omega_0^2\right]}$$