ActiveFilters

November 20, 2018

1 Active Filters

All the calculus are made using the s domain, where $s = j\omega$ and corresponds to the cross section of the s plane at the imaginary axis.

```
In [2]: #  s domain generation
        #  1 Hz to 10 kHz
        f = logspace(0, 4, 5000);
        w = 2*pi*f;
        s = w*im;
```

1.1 First order Low Pass filter

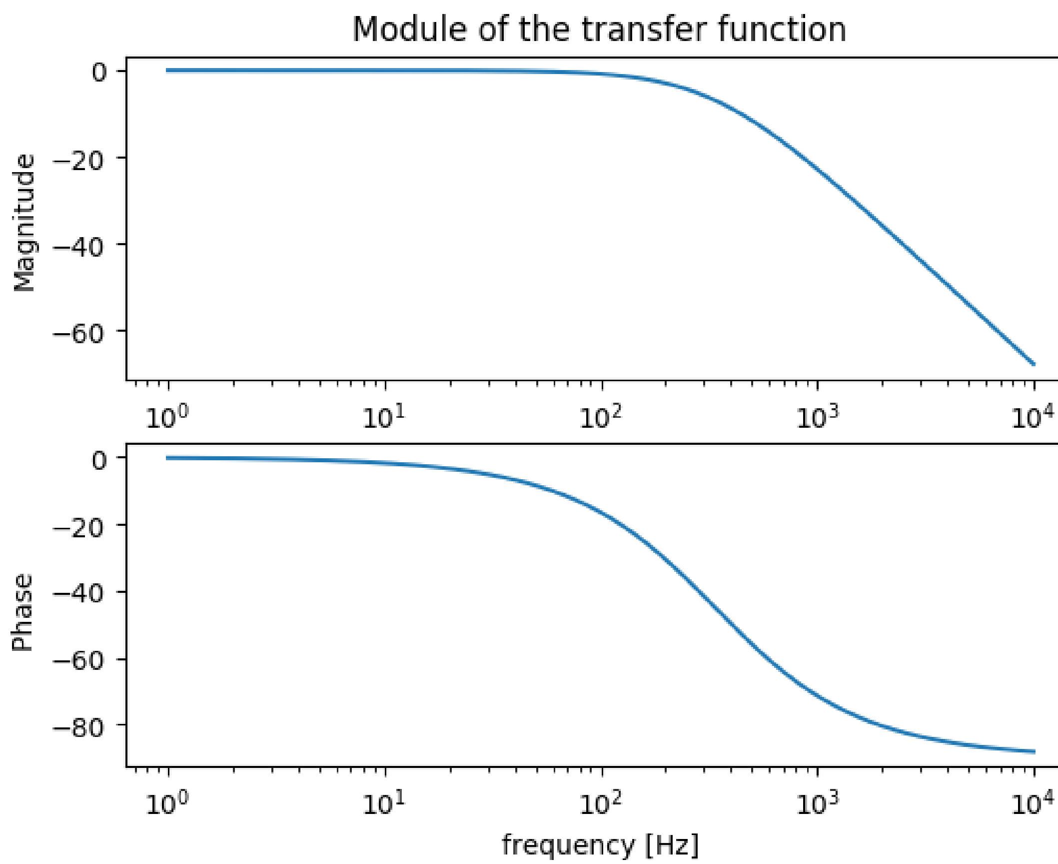
$$V_{out} = \frac{1/sC}{R + 1/sC} V_{in} \Rightarrow \frac{V_{out}}{V_{in}} = \frac{1}{sRC + 1} \Rightarrow \frac{V_{out}}{V_{in}} = \frac{1/RC}{s + 1/RC}$$

$$H(s) = \frac{s_0}{s + s_0}$$

where $s_0 = 1/RC$

```
In [4]: using PyPlot;
```

```
In [5]: # circuit parameters
        R = 470;
        C = 10.0^(-6)
        s0 = 1/(R*C);
        H = s0./(s+s0);
        mag = 20*log.(abs.(H))
        phase = atan(imag(H)./real(H))*180/pi;
        clf()
        subplot(2,1,1); semilogx(f,mag);
        title("Module of the transfer function");
        ylabel("Magnitude");
        subplot(2,1,2); semilogx(f,phase);
        ylabel("Phase ");
        xlabel("frequency [Hz]");
```



1.2 First order High Pass filter

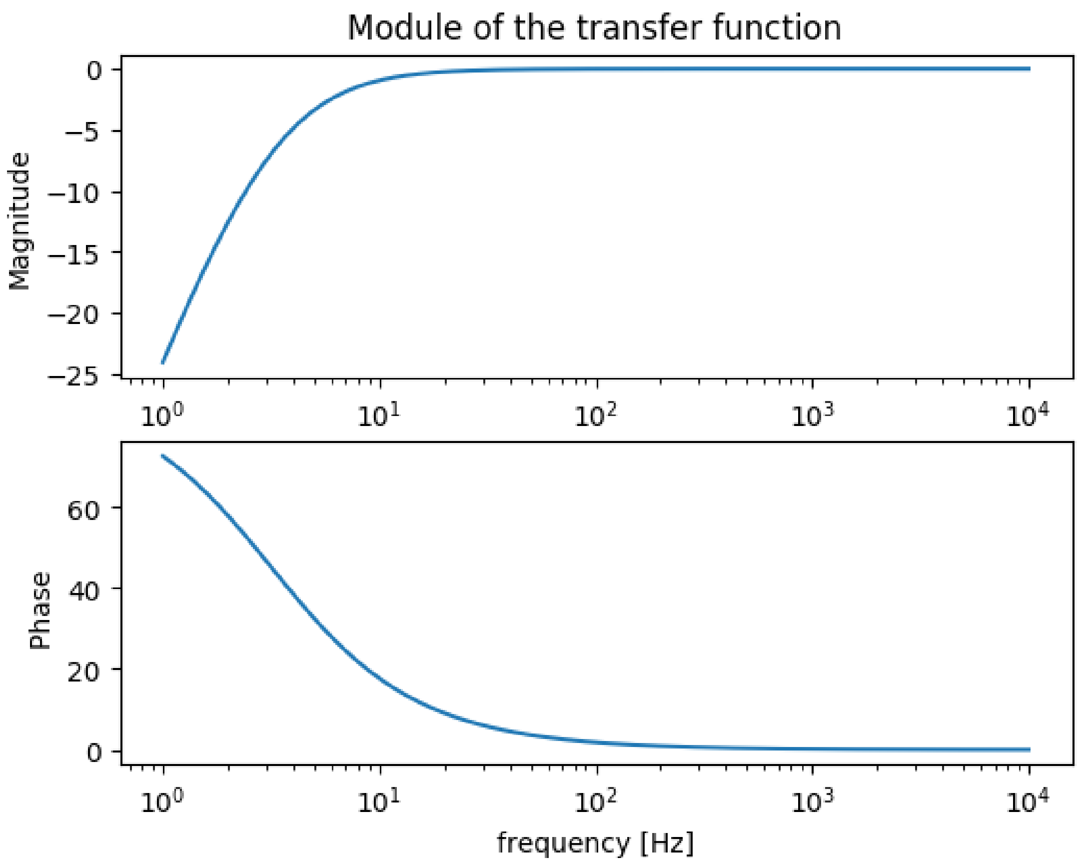
$$V_{out} = \frac{R}{R+1/sC}V_{in} \Rightarrow \frac{V_{out}}{V_{in}} = \frac{sRC}{sRC+1} \Rightarrow \frac{V_{out}}{V_{in}} = \frac{s}{s+1/RC}$$

$$H(s) = \frac{s}{s+s_0}$$

where $s_0 = 1/RC$

```
In [4]: # circuit parameters
        R = 50000;
        C = 10.0^(-6)
        s0 = 1/(R*C);
        H = s./(s+s0);
        mag = 20*log.(abs.(H))
        phase = atan(imag(H)./real(H))*180/pi;
        clf()
        subplot(2,1,1); semilogx(f,mag);
        title("Module of the transfer function");
        ylabel("Magnitude");
```

```
subplot(2,1,2); semilogx(f,phase)
ylabel("Phase ");
xlabel("frequency [Hz]");
```



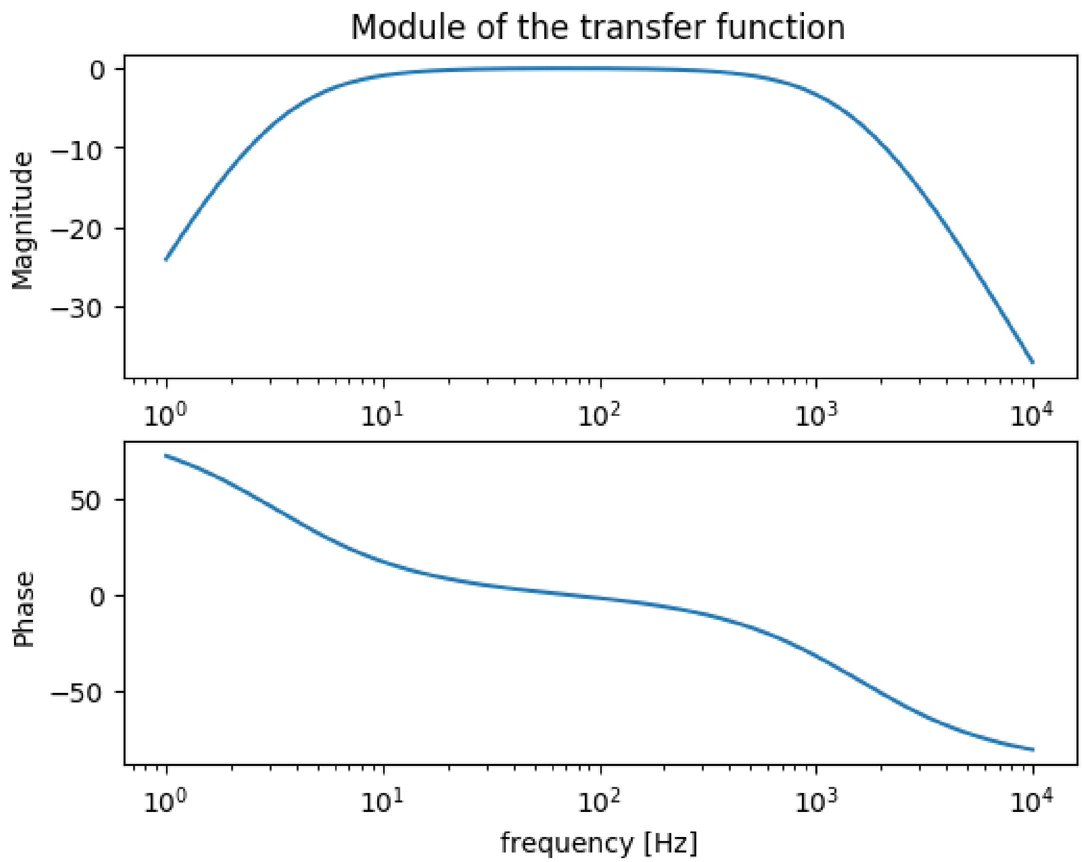
1.3 Cascade filter

$$H(s) = H_1(s) \cdot H_2(s)$$

$$H(s) = \frac{s}{s+s_0} \cdot \frac{s_0}{s+s_0} = \frac{s \cdot s_0}{s^2 + 2ss_0 + s_0^2}$$

```
In [5]: # circuit parameters
        Rl = 1000;
        Cl = 10.0^(-7);
        s0l = 1/(Rl*Cl);
        Hl = s0l./(s+s0l);
        Rh = 50000;
        Ch = 10.0^(-6)
        s0h = 1/(Rh*Ch);
        Hh = s./(s+s0h);
```

```
H = Hl.*Hh;             # product of the transfer functions
mag = 20*log.(abs.(H))
phase = atan(imag(H)./real(H))*180/pi;
clf();
subplot(2,1,1); semilogx(f,mag);
title("Module of the transfer function");
ylabel("Magnitude");
subplot(2,1,2); semilogx(f,phase)
ylabel("Phase");
xlabel("frequency [Hz]");
```



1.4 Sallen Key Low Pass filter

The circuit matrix can be presented as

$$\begin{bmatrix} 1/R_1 & 0 & -1/R_1 \\ 0 & s(C_1+C_2)+1/R_2 & -1/R_2 - sC_1 \\ -1/R_1 & -sC_1 - 1/R_2 & 1/R_1 + 1/R_2 + sC_1 \end{bmatrix} \cdot \begin{bmatrix} V_{in} \\ V_{out} \\ V_{mid} \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

The transfer function is given by the equation:

$$\frac{V_{out}}{V_{in}} = H(s) = \frac{\omega_0^2}{s^2 + 2\alpha s + \omega_0^2}$$

$$\omega_0 = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}$$

$$2\alpha = \frac{\omega_0}{Q} = \frac{1}{C_1}\frac{R_1 + R_2}{R_1 R_2}$$

Poles

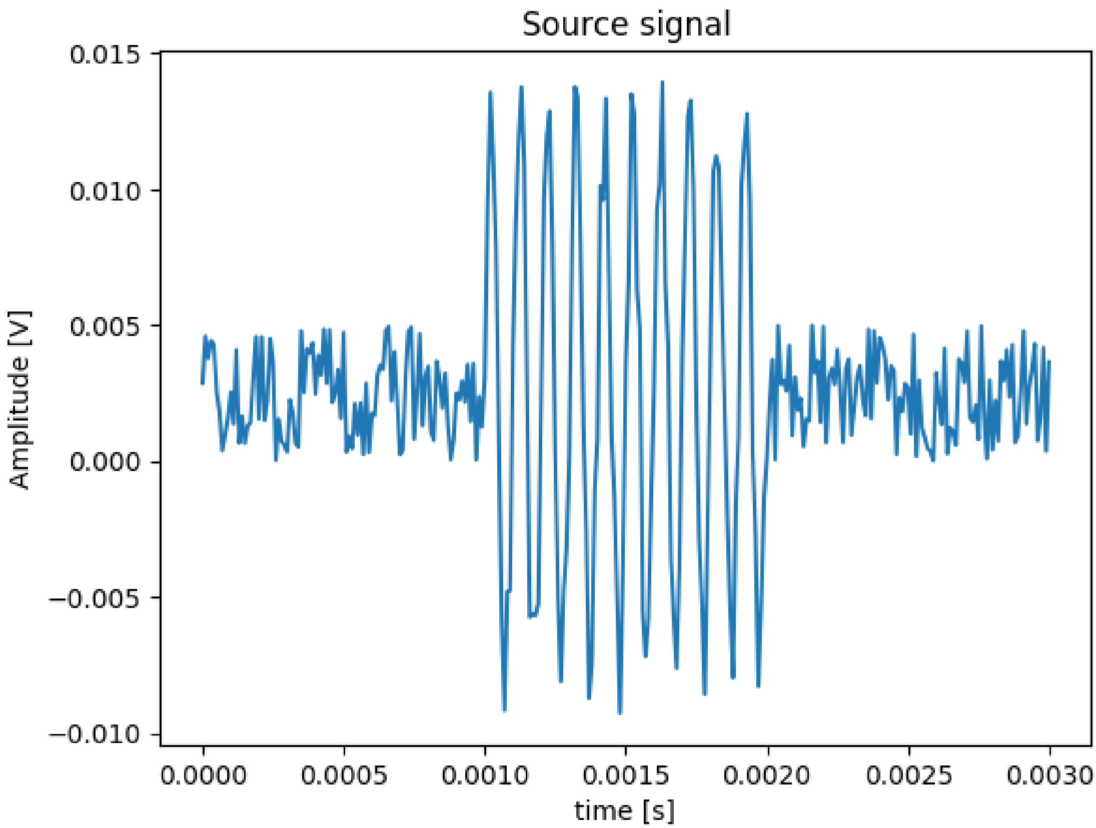
$$s = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

$$E_g = Asin(\omega t) + \alpha N(t)$$

$$\omega = 2\pi f$$

In [6]:
```
A = 0.01;                    # amplitude
f = 10^4;                    # frequency 10 kHz
alfa  = 0.0050;               # noise amplitude
t = 0:0.00001:0.001;          # 1 ms  time vector with a resolution of 10 us
E = A*sin.(2 *pi * f *t) + alfa*rand(length(t),1);   # sinusoidal signal definition

# add noise in front and at the back of the sine signal
Eg = [alfa*rand(100,1); E; alfa*rand(100,1)];
t = 0:0.00001:0.003;
Rg = 500;                # signal source thevenin impedance
plot(t,Eg)
title("Source signal")
xlabel("time [s]");
ylabel("Amplitude [V]");
```


Source signal
Amplitude [V]
time [s]
0.015
0.010
0.005
0.000
−0.005
−0.010
0.0000
0.0005
0.0010
0.0015
0.0020
0.0025
0.0030

```
In [ ]:
```