1 Basic concepts and resistor circuits

1.1 Basics

We start our study of electronics with definitions and the basic laws that apply to *all* circuits. This is followed by an introduction to our first circuit element – the resistor.

In electronics, we are interested in keeping track of two basic quantities: the *currents* and *voltages* in a circuit. If you can make these quantities behave like you want, you have succeeded.

Current measures the flow of charge past a point in the circuit. The units of current are thus coulombs per second or *amperes*, abbreviated as A. In this text we will use the symbol I or i for current.

As charges move in circuits, they undergo collisions with atoms and lose some of their energy. It thus takes some work to move charges around a circuit. The work per unit charge required to move some charge between two points is called the *voltage* between those points. (In physics, this work per unit charge is equivalent to the difference in electrostatic potential between the two points, so the term *potential difference* is sometimes used for voltage.) The units of voltage are thus joules per coulomb or *volts*, abbreviated V. In this text we will use the symbol V or v for voltage.

In a circuit, there are sources and sinks of energy. Some sources of energy (or voltage) include batteries (which convert chemical energy to electrical energy), generators (mechanical to electrical energy), solar cells (radiant to electrical energy), and power supplies and signal generators (electrical to electrical energy). All other electrical components are sinks of energy.

Let's see how this works. The simplest circuit will involve one voltage source and one sink, with connecting wires as shown in Fig. 1.1. By convention, we denote the two sides of the voltage source as $+$ and $-$. A positive charge moving from the $-$ side to the $+$ side of the source gains energy. Thus we say that the voltage across the source is positive. When the circuit is complete, current flows out of the $+$ side of the source, as shown. The voltage across the component is negative when we

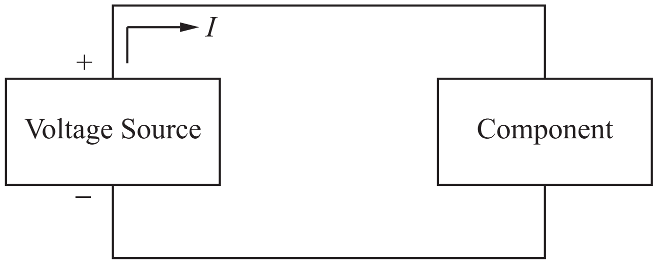

Figure 1.1 A simple generic circuit.

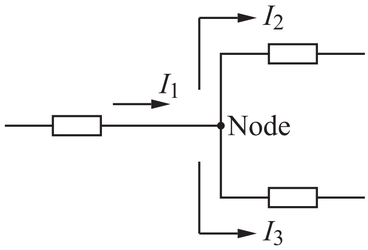

Figure 1.2 Example of Kirchoff's Current Law.

cross it in the direction of the current. We say there is a *voltage drop* across the component. Note that while we can speak of the current at any point in the circuit, the voltage is always between two points. It makes no sense to speak of the voltage at a point (remember, the voltage is a potential *difference*).

We can now write down some general rules about voltage and current.

1. The sum of the currents into a node (i.e. any point on the circuit) equals the sum of the currents flowing out of the node. This is Kirchoff's Current Law (KCL) and expresses conservation of charge. For example, in Fig. 1.2, $I_1 = I_2 + I_3$. If we use the sign convention that currents into a node are positive and currents out of a node are negative, then we can express this law in the compact form

$$\sum_k^{\text{node}} I_k = 0 \tag{1.1}$$

   where the sum is over all currents into or out of the node.
2. The sum of the voltages around any closed circuit is zero. This is Kirchoff's Voltage Law (KVL) and expresses conservation of energy. In equation form,

$$\sum_k^{\text{loop}} V_k = 0. \tag{1.2}$$

   Here we must use the convention that the voltage across a source is positive when we move across the source in the direction of the current and the voltage

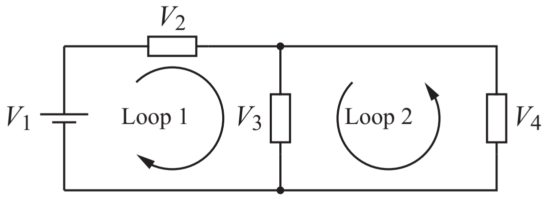

Figure 1.3 Example of Kirchoff's Voltage Law.

across a sink is negative when we move across the component in the direction of the current. If we traverse a source or sink in the direction opposite to the direction of the current, the signs are reversed. Figure 1.3 gives an example. Here we introduce the circuit symbol for an ideal battery, labeled with voltage V_1. The top of this symbol represents the positive side of the battery. The current (not shown) flows up out of the battery, through the component labeled V_2 and down through the components labeled V_3 and V_4. Looping around the left side of the circuit in the direction shown gives $V_1 - V_2 - V_3 = 0$ or $V_1 = V_2 + V_3$. Here we take V_2 and V_3 to be positive numbers and include the sign explicitly. Going around the right portion of the circuit as shown gives $-V_3 + V_4 = 0$ or $V_3 = V_4$. This last equality expresses the important result that components connected in parallel have the same voltage across them.

3. The power P provided or consumed by a circuit device is given by

$$P = VI \tag{1.3}$$

where V is the voltage across the device and I is the current through the device. This follows from the definitions:

$$VI = \left(\frac{\text{work}}{\text{charge}}\right)\left(\frac{\text{charge}}{\text{time}}\right) = \frac{\text{work}}{\text{time}} = \text{power.} \tag{1.4}$$

The units of power are thus joules per second or *watts*, abbreviated W. This law is of considerable practical importance since a key part of designing a circuit is to employ components with the proper power rating. A component with an insufficient power rating will quickly overheat and fail when the circuit is operated.

Finally, a word about prefixes and nomenclature. Some common prefixes and their meanings are shown in Table 1.1. As an example, recall that the unit *volts* is abbreviated as V, and *amperes* or *amps* is abbreviated as A. Thus 10^6 volts $= 1$ MV and 10^{-3} amps $= 1$ mA. Notice that case matters: 1 MA $\neq$ 1 mA.

Table 1.1 Some common prefixes used in electronics

Multiple	Prefix	Symbol
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f

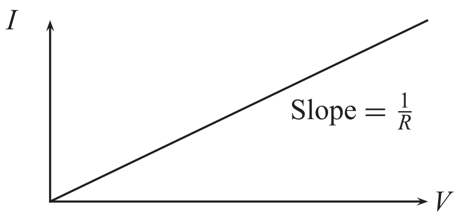

Figure 1.4 I–V curve for a resistor.

1.2 Resistors

A common way to represent the behavior of a circuit device is the I–V characteristic. This is a plot of the current I through the device as a function of applied voltage V across the device. Our first device, the resistor, has the simple linear I–V characteristic shown in Fig. 1.4. This linear relationship is expressed by Ohm's Law:

$$V = IR. \tag{1.5}$$

The constant of proportionality, R, is called the *resistance* of the device and is equal to one over the slope of the I–V characteristic. The units of resistance are *ohms*, abbreviated as Ω. Any device with a linear I–V characteristic is called a resistor.

The resistance of the device depends only on its physical properties – its size and composition. More specifically:

$$R = \rho \frac{L}{A} \tag{1.6}$$

Table 1.2 The resistivity of some common electronic materials

Material	ρ (10^{-8} Ωm)
Silver	1.6
Copper	1.7
Nichrome	100
Carbon	3500

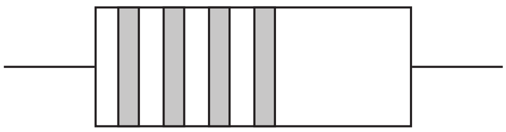

Figure 1.5 Value and tolerance bands on a resistor.

where ρ is the resistivity, L is the length, and A is the cross-sectional area of the material. The resistivity of some representative materials is given in Table 1.2.

The interconnecting wires or circuit board paths are typically made of copper or some other low resistivity material, so for most cases their resistance can be ignored. If we want resistance in a circuit we will use a discrete device made of some high resistivity material (e.g., carbon). Such resistors are widely used and can be obtained in a variety of values and power ratings. The low power rating resistors typically used in circuits are marked with color coded bands that give the resistance and the tolerance (i.e., the uncertainty in the resistance value) as shown schematically in Fig. 1.5.

As shown in the figure, the bands are usually grouped toward one end of the resistor. The band closest to the end is read as the first digit of the value. The next band is the second digit, the next band is the multiplier, and the last band is the tolerance value. The values associated with the various colors are shown in Table 1.3. For example, a resistor code having colors red, violet, orange, and gold corresponds to a value of $27 \times 10^3\ \Omega \pm 5\%$.

Resistors also come in variable forms. If the variable device has two leads, it is called a *rheostat*. The more common and versatile type with three leads is called a *potentiometer* or a "pot." Schematic symbols for resistors are shown in Fig. 1.6.

One must also select the proper power rating for a resistor. The power rating of common carbon resistors is indicated by the size of the device. Typical values are $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{2}$, 1, and 2 watts.

Table 1.3 Standard color scheme for resistors

Color	Digit	Multiplier	Tolerance (%)
none			20
silver		0.01	10
gold		0.1	5
black	0	1	
brown	1	10	
red	2	100	2
orange	3	10^3	
yellow	4	10^4	
green	5	10^5	
blue	6	10^6	
violet	7	10^7	
gray	8		
white	9		

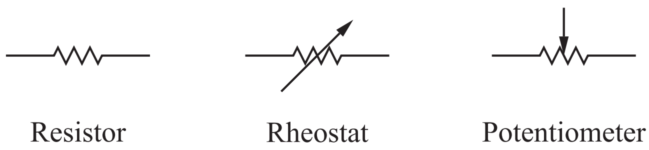

Figure 1.6 Schematic symbols for a fixed resistor and two types of variable resistors.

As noted in Eq. (1.3), the power consumed by a device is given by $P = VI$, but for resistors we also have the relation $V = IR$. Combining these we obtain two power relations specific to resistors:

$$P = I^2R \tag{1.7}$$

and

$$P = V^2/R. \tag{1.8}$$

1.2.1 Equivalent circuit laws for resistors

It is common practice in electronics to replace a portion of a circuit with its functional equivalent. This often simplifies the circuit analysis for the remaining portion of the circuit. The following are some equivalent circuit laws for resistors.

1.2.1.1 Resistors in series

Components connected in series are connected in a head-to-tail fashion, thus forming a line or series of components. When forming equivalent circuits, any

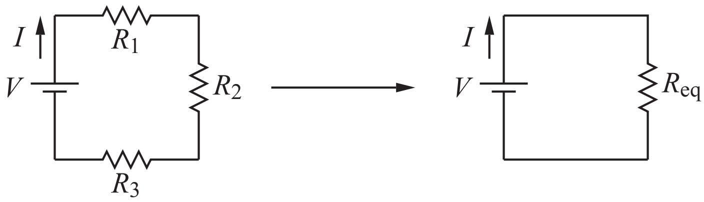

Figure 1.7 Equivalent circuit for resistors in series.

number of resistors in series may be replaced by a single equivalent resistor given by:

$$R_{eq} = \sum_i R_i \tag{1.9}$$

where the sum is over all the resistors in series. To see this, consider the circuit shown in Fig. 1.7. We would like to replace the circuit on the left by the equivalent circuit on the right. The circuit on the right will be equivalent if the current supplied by the battery is the same.

By KCL, the current in each resistor is the same. Applying KVL around the circuit loop and Ohm's Law for the drop across the resistors, we obtain

$$\begin{aligned} V &= IR_1 + IR_2 + IR_3 \\ &= I(R_1 + R_2 + R_3) \\ &= IR_{eq} \end{aligned} \tag{1.10}$$

where

$$R_{eq} = R_1 + R_2 + R_3. \tag{1.11}$$

This derivation can be extended to any number of resistors in series, hence Eq. (1.9).

1.2.1.2 Resistors in parallel

Components connected in parallel are connected in a head-to-head and tail-to-tail fashion. The components are often drawn in parallel lines, hence the name. When forming equivalent circuits, any number of resistors in parallel may be replaced by a single equivalent resistor given by:

$$\frac{1}{R_{eq}} = \sum_i \frac{1}{R_i} \tag{1.12}$$

where the sum is over all the resistors in parallel. To see this, consider the circuit shown in Fig. 1.8. Again, we would like to replace the circuit on the left by the equivalent circuit on the right.

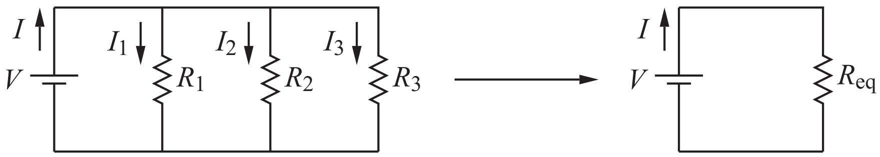

Figure 1.8 Equivalent circuit for resistors in parallel.

First, note that KCL requires

$$I = I_1 + I_2 + I_3. \tag{1.13}$$

Since the resistors are connected in parallel, the voltage across each one is the same, and, by KVL is equal to the battery voltage: $V = I_1R_1$, $V = I_2R_2$, $V = I_3R_3$. Solving these for the three currents and substituting in Eq. (1.13) gives

$$I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) = \frac{V}{R_{eq}} \tag{1.14}$$

where

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}. \tag{1.15}$$

Again, this derivation can be extended to any number of resistors in parallel, hence Eq. (1.12).

A frequent task is to analyze two resistors in parallel. Of course, for this special case of Eq. (1.12) we get $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$. It is often more illuminating to write this as an equation for R_{eq} rather than $\frac{1}{R_{eq}}$. After some algebra, we get

$$R_{eq} = \frac{R_1R_2}{R_1 + R_2}. \tag{1.16}$$

This special case is worth memorizing.

Example For the circuit shown in Fig. 1.9, how much current flows through the 20 kΩ resistor? What must its power rating be?

Solution As we will see, there is more than one way to solve this problem. Here we use a method that relies on basic electronics reasoning and our resistor equivalent circuit laws. We want the current through the 20 kΩ resistor. If we knew the voltage across this resistor (call this voltage V_{20k}), we could then get the current from Ohm's Law. In order to get the voltage across the 20 kΩ resistor, we need the voltage across the 10 kΩ resistor since, by KVL, $V_{20k} = 130 - V_{10k}$. In order to get the voltage across the 10 kΩ resistor, we need to know the current through

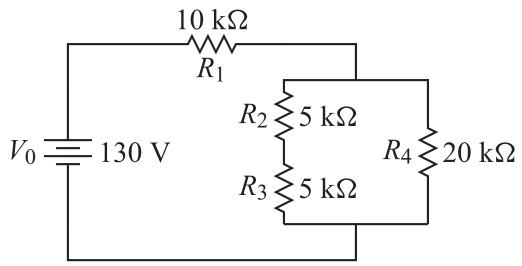

Figure 1.9 Example resistor circuit.

it, which is the same as the current supplied by the battery. Thus, if we can get the current supplied by the battery we can solve the problem. To get the battery current, we combine all our resistors into one equivalent resistor. The implementation of this strategy goes as follows.

1. Combine the two 5 kΩ series resistors into a 10 kΩ resistor.
2. This 10 kΩ resistor is then in parallel with the 20 kΩ resistor. Combining these we get (using Eq. (1.16))

$$R_{\text{eq}} = \frac{R_1 R_2}{R_1 + R_2} = \frac{(10\ \text{k}\Omega)(20\ \text{k}\Omega)}{10\ \text{k}\Omega + 20\ \text{k}\Omega} = 6.67\ \text{k}\Omega. \tag{1.17}$$

3. This 6.67 kΩ resistor is then in series with a 10 kΩ resistor, giving a total equivalent circuit resistance $R_{\text{eq}} = 16.67$ kΩ.
4. The current supplied by the battery is then

$$I = \frac{V_0}{R_{\text{eq}}} = \frac{130\ \text{V}}{16.67 \times 10^3\ \Omega} = 7.8 \times 10^{-3}\ \text{A} = 7.8\ \text{mA}. \tag{1.18}$$

5. KVL then gives $130\ \text{V} - (7.8\ \text{mA})(10\ \text{k}\Omega) - \text{V}_{20\text{k}} = 0$. Solving this gives $V_{20\text{k}} = 52$ V.
6. Ohm's Law then gives $I_{20\text{k}} = \frac{52\ \text{V}}{20\ \text{k}\Omega} = 2.6$ mA, which is the solution to the first part of our problem. As a check, it is comforting to note that this current is less than the total battery current, as it must be. The remainder goes through the two 5 kΩ resistors.
7. The power consumed by the 20 kΩ resistor is $P = I^2 R = (2.6 \times 10^{-3}\ \text{A})^2 (2 \times 10^4\ \Omega) = 0.135$ W. This is too much for a $\frac{1}{8}$ W resistor, so we must use at least a $\frac{1}{4}$ W resistor.

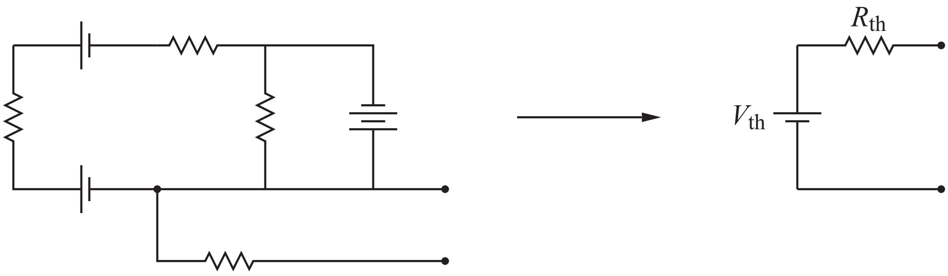

Figure 1.10 Representation of Thevenin's theorem.

1.2.1.3 Thevenin's theorem and Norton's theorem

The third of our equivalent circuit laws, Thevenin's theorem, is a more general result that actually includes the first two laws as special cases. The theorem states that any two-terminal network of sources and resistors can be replaced by a series combination of a single resistor R_{th} and voltage source V_{th}. This is represented by the example in Fig. 1.10. The sources can include both voltage and current sources (the current source is described below). A more general version of the theorem replaces the word *resistor* with *impedance*, a concept we will develop in Chapter 2.

The point of Thevenin's theorem is that when we connect a component to the terminals, it is much easier to analyze the circuit on the right than the circuit on the left. But there is no free lunch – we must first determine the values of V_{th} and R_{th}.

V_{th} is the voltage across the circuit terminals when nothing is connected to the terminals. This is clear from the equivalent circuit: if nothing is connected to the terminals, then no current flows in the circuit and there is no voltage drop across R_{th}. The voltage across the terminals is thus the same as V_{th}. In practice, the voltage across the terminals must be calculated by analyzing the original circuit.

There are two methods for calculating R_{th}; you can use whichever is easiest. In the first method, you start by short circuiting all the voltage sources and open circuiting all the current sources in the original circuit. This means that you replace the voltage sources by a wire and disconnect the current sources. Now only resistors are left in the circuit. These are then combined into one resistor using the resistor equivalent circuit laws. This one resistor then gives the value of R_{th}. In the second method, we calculate the current that would flow in the circuit if we shorted (placed a wire across) the terminals. Call this the short circuit current I_{sc}. Then from the Thevenin equivalent circuit it is clear that $R_{th} = \frac{V_{th}}{I_{sc}}$.

There is also a similar result known as Norton's theorem. This theorem states that any two-terminal network of sources and resistors can be replaced by a parallel

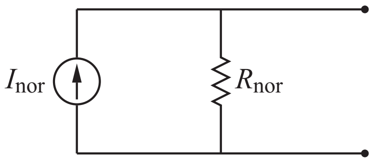

Figure 1.11 Equivalent circuit of Norton's theorem.

combination of a single resistor R_{nor} and current source I_{nor}. This equivalent circuit is shown in Fig. 1.11.

The current source is usually less familiar that the voltage source, but the two can be viewed as complements of one another. An ideal voltage source will maintain a constant voltage across it and will provide whatever current is required by the rest of the circuit. Similarly, an ideal current source will maintain a constant current through it while the voltage across it will be set by the rest of the circuit.

Returning now to the equivalent circuit, let's determine R_{nor} and I_{nor}. If we short the terminals, it is clear from the Norton equivalent circuit that all of I_{nor} will pass through the shorting wire. Thus $I_{nor} = I_{sc}$. We have seen previously that the voltage across the terminals when nothing is connected is equal to V_{th}. From the Norton equivalent circuit we then see that $V_{th} = I_{nor}R_{nor}$, so

$$R_{nor} = \frac{V_{th}}{I_{nor}} = \frac{V_{th}}{I_{sc}} = R_{th}. \tag{1.19}$$

1.2.2 Applications for resistors

Resistors are probably the most common circuit element and can be used in a variety of simple circuits. Here are a few examples.

1. Current limiting. Many electronic devices come with operating specifications. For example, the ubiquitous LED (light emitting diode) typically operates with a voltage drop of 1.7 V and a current of 20 mA. Suppose you have a 9 V battery and wish to light the LED. How can you operate the 1.7 V LED with a 9 V battery? By the discriminating use of a resistor! Consider the circuit in Fig. 1.12. KVL gives $V_0 - IR - V_{LED} = 0$, where V_{LED} is the voltage across the LED. We know that $V_0 = 9$ V, $V_{LED} = 1.7$ V, and we want $I = 20$ mA for proper operation. Solving for R gives

$$R = \frac{V_0 - V_{LED}}{I} = \frac{9 - 1.7\ \text{V}}{20 \times 10^{-3}\ \text{A}} = 365\ \Omega. \tag{1.20}$$

   This is an example of using a resistor as a current limiter. Without it, the LED would burn out immediately.

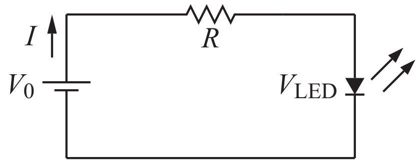

Figure 1.12 Application of a resistor as a current limiter.

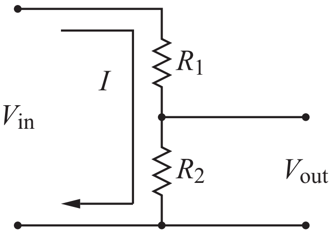

Figure 1.13 The ubiquitous voltage divider.

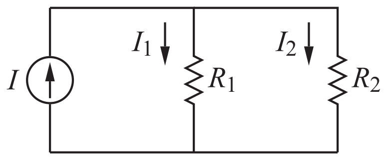

Figure 1.14 The current divider.

2. Voltage divider. Another very common resistor circuit is shown in Fig. 1.13. Some voltage V_{in} is applied to the input and the circuit provides a lower voltage at the output. The analysis is simple. KVL gives $V_{in} = I(R_1 + R_2)$ and Ohm's Law gives $V_{out} = IR_2$. Solving for I from the first equation and substituting in the second gives

$$V_{out} = IR_2 = \left(\frac{V_{in}}{R_1 + R_2}\right) R_2 = V_{in}\left(\frac{R_2}{R_1 + R_2}\right) \tag{1.21}$$

where this last form emphasizes that $V_{out} < V_{in}$ since $\frac{R_2}{R_1+R_2} < 1$. This equation is used so frequently it is worth memorizing.

3. The current divider circuit is shown in Fig. 1.14. A current source is applied to two resistors in parallel and we would like to obtain an expression that tells us how the current is divided between the two. By KCL, $I = I_1 + I_2$. Since the two resistors are in parallel, the voltage across them must be the same. Hence, $I_1R_1 = I_2R_2$. Solving this latter equation for I_2 and plugging into the first gives

$$I = I_1 + I_1\left(\frac{R_1}{R_2}\right) = I_1\left(\frac{R_1 + R_2}{R_2}\right) \tag{1.22}$$

or

$$I_1 = \left(\frac{R_2}{R_1 + R_2}\right) I. \tag{1.23}$$

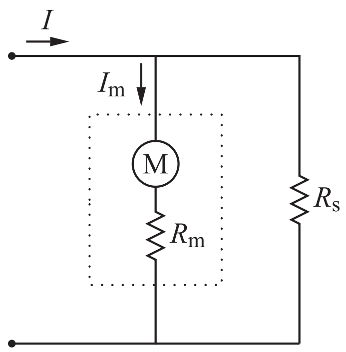

Figure 1.15 Using a resistor to extend the range of a current meter.

4. Multi-range analog voltmeter/ammeter. In electronics, one frequently has the need to measure voltage and current. The instrument of choice for many experimentalists is the multimeter, which can measure voltage, current, and resistance. The analog version of the multimeter uses a simple meter as a display. If you tear one of these multimeters apart, you find that the meter is a current measuring device that gives a full scale deflection of the needle for a given, small current, typically 50 μA. This is fine if you want to measure currents from zero to 50 μA, but what if you have a larger current to measure, or want to measure a voltage instead?

Both of these can be accomplished by judicious use of resistors. The circuit in Fig. 1.15 shows a meter in parallel with a so-called shunt resistor R_s. The physical meter (within the dotted lines) is represented by an ideal current measuring meter in series with a resistor R_m. When a current I is applied to the terminals, part goes through the meter and part through the shunt. The circuit is simply a current divider, so we have (cf. Eq. (1.22))

$$I = I_m\left(1 + \frac{R_m}{R_s}\right). \tag{1.24}$$

A full scale deflection of the meter will always occur when $I_m = 50\ \mu A$, and R_m is also set at the construction of the meter, but by adjusting the shunt resistance R_s we can make this full scale deflection occur for any input current I we choose.

Another simple addition will allow us to use our meter to measure voltage. Placing a resistor R_s in series with the meter gives the configuration in Fig. 1.16. It is convenient here to define the voltage $V_m = I_m R_m$ that will produce a full scale deflection when applied across the physical meter. This circuit is then seen to be a voltage divider. Inverting Eq. (1.21) then gives

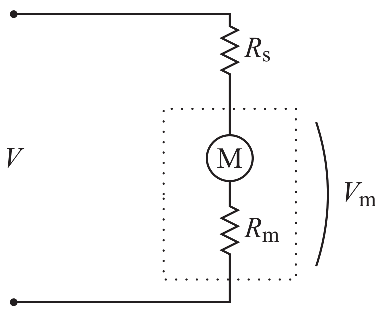

Figure 1.16 Using a resistor to measure voltage with a current meter.

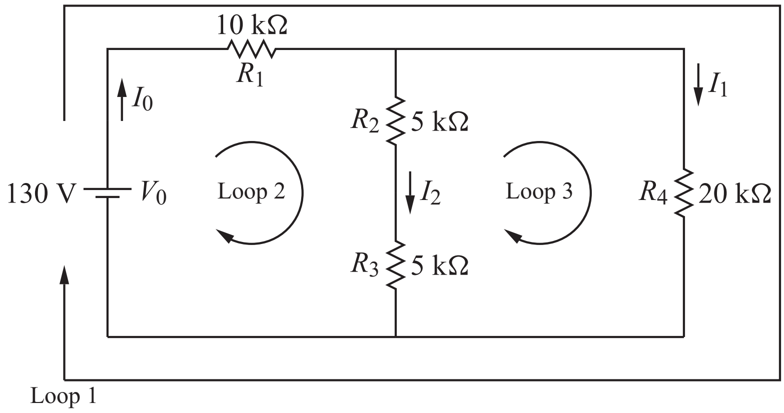

Figure 1.17 The standard method of solving circuit problems.

$$V = V_m \left(1 + \frac{R_s}{R_m}\right) \tag{1.25}$$

so by varying R_s we can make the full scale deflection of the meter correspond to any input voltage.

1.2.3 Techniques for solving circuit problems

We list here three methods for solving circuit problems, and illustrate the use of these techniques on the same problem that we solved previously using equivalent circuit laws for resistors. Our goal is to solve for the current through resistor R_4 in Fig. 1.17.

The standard method This method involves assigning currents to each branch of the circuit and then applying KVL and KCL. In Fig. 1.17 we have assigned currents I_0, I_1, and I_2. In this case, the application of KCL gives a single equation

$$I_0 = I_1 + I_2 \tag{1.26}$$

but in circuits with more than three branches KCL gives additional relations. Next we use KVL around the loops indicated in the figure. For Loop 1 we get

$$V_0 - I_0 R_1 - I_1 R_4 = 0 \tag{1.27}$$

while Loop 2 gives

$$V_0 - I_0 R_1 - I_2(R_2 + R_3) = 0 \tag{1.28}$$

and finally

$$-I_1 R_4 + I_2(R_2 + R_3) = 0 \tag{1.29}$$

for Loop 3. We thus have four equations relating the three unknown currents I_0, I_1, and I_2 and need to solve for I_1. In practice we need only three independent equations to solve for the currents, but we have given all four here to illustrate the method. Solving Eq. (1.26) for I_2 (one of the currents we are not interested in) and plugging into Eq. (1.29) gives

$$-I_1 R_4 + (I_0 - I_1)(R_2 + R_3) = 0 \tag{1.30}$$

and solving Eq. (1.27) for I_0 gives

$$I_0 = \frac{V_0 - I_1 R_4}{R_1}. \tag{1.31}$$

Plugging Eq. (1.31) into Eq. (1.30) and solving for I_1 gives, after some algebra,

$$I_1 = \frac{V_0(R_2 + R_3)}{R_1 R_4 + (R_1 + R_4)(R_2 + R_3)}. \tag{1.32}$$

Plugging the circuit values into this equation gives $I_1 = 2.6$ mA, our former answer.

The mesh loop method Our second method for solving circuit problems is the mesh loop method. In this method, currents are assigned to the circuit loops rather than the actual physical branches of the circuit. This is shown in Fig. 1.18 where we assign current I_1 to the outer loop and I_2 to the inner loop.

We then move around these loops, applying KVL, but including contributions from both loop currents. The outer loop then gives

$$V_0 - (I_1 + I_2)R_1 - I_1 R_4 = 0 \tag{1.33}$$

while the inner loop gives

$$V_0 - (I_1 + I_2)R_1 - I_2(R_2 + R_3) = 0. \tag{1.34}$$

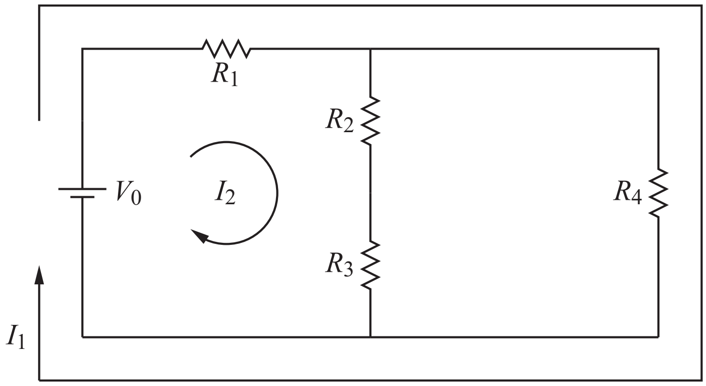

Figure 1.18 The mesh loop method of solving circuit problems.

Note that the resulting set of equations is simpler in this method: two equations in two unknowns I_1 and I_2. For this reason the mesh loop method is usually preferable for more complicated circuits. Furthermore, our equations can be rearranged into the conventional form of a system of linear algebraic equations. Thus Eq. (1.33) becomes

$$(R_1 + R_4)I_1 + R_1 I_2 = V_0 \tag{1.35}$$

while Eq. (1.34) gives

$$R_1 I_1 + (R_1 + R_2 + R_3)I_2 = V_0. \tag{1.36}$$

Students of linear algebra may wish to solve these using Cramer's Method of Determinants or with the built-in capabilities of many hand-held calculators (see Appendix B). The usual brute force method also works: solving Eq. (1.36) for I_2, plugging this into Eq. (1.35), and solving for I_1 produces (again, after some algebra),

$$I_1 = \frac{V_0(R_2 + R_3)}{(R_1 + R_4)(R_2 + R_3) + R_1 R_4}, \tag{1.37}$$

the same expression obtained with the standard method.

Thevenin's theorem Finally, we solve this problem by using Thevenin's theorem. We form the required two terminal network by removing R_4 and taking the two terminals at the points where R_4 was attached. This is shown in Fig. 1.19.

The remaining circuit should look familiar – if we combine R_2 and R_3 it is the previously considered voltage divider. Thus

$$V_{\mathrm{th}} = V_0 \left(\frac{R_2 + R_3}{R_1 + R_2 + R_3} \right) = 65\ \mathrm{V}. \tag{1.38}$$

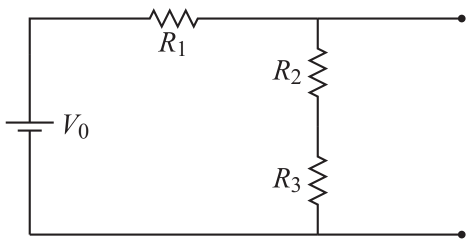

Figure 1.19 First step in solving the problem using Thevenin's theorem.

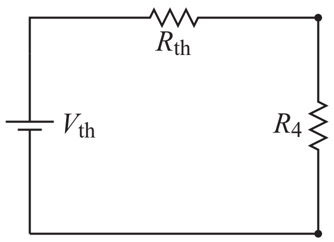

Figure 1.20 Last step: re-attach R_4 to the Thevenin equivalent circuit.

Shorting out the battery leaves R_1 in parallel with $R_2 + R_3$ so

$$R_{th} = \frac{R_1(R_2 + R_3)}{R_1 + R_2 + R_3} = 5\ \text{k}\Omega. \tag{1.39}$$

Reattaching R_4 then gives the simple circuit of Fig. 1.20 with the current through R_4 given by

$$I_1 = \frac{V_{th}}{R_{th} + R_4} = \frac{65\ \text{V}}{25\ \text{k}\Omega} = 2.6\ \text{mA} \tag{1.40}$$

as before.

1.2.4 Input resistance

A common measurement in the electronics lab is the voltage across a component. An important fact to keep in mind when making such measurements is that *the measuring instrument becomes part of the circuit*. The act of measuring thus inevitably changes the thing we are trying to measure because we are adding circuitry to the original circuit. To help us cope with this problem, test instrument manufacturers specify a quantity called the *input resistance* R_{in} (or, as we will see later, the *input impedance*). The effect of attaching the instrument is the same as attaching a resistor with value R_{in}. To see how this helps, suppose we are measuring the voltage across some resistor R_0 in a complicated circuit, as depicted in Fig. 1.21.

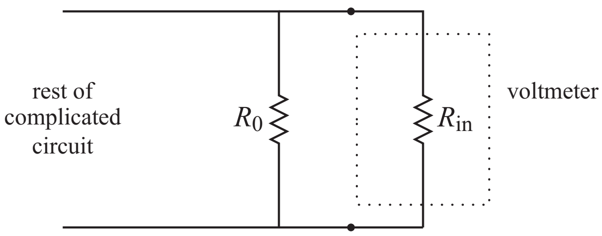

Figure 1.21 Measuring the voltage across resistor R_0 with a voltmeter.

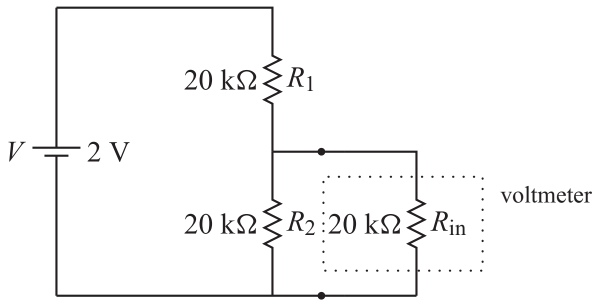

Figure 1.22 Measuring the output of a voltage divider with a voltmeter.

If we know the input resistance of our measuring device we see that the effect of making the measurement is to replace the original resistor R_0 with the parallel combination of R_0 with R_{in}

$$R_0 \to R_0 \parallel R_{in} = \frac{R_0 R_{in}}{R_0 + R_{in}} \tag{1.41}$$

where $R_0 \parallel R_{in}$ is shorthand for the parallel combination. From this, one can see that the circuit-altering effect of attaching the measuring instrument is mitigated by making the input resistance as high as possible, because

$$\frac{R_0 R_{in}}{R_0 + R_{in}} \to R_0 \tag{1.42}$$

as $R_{in} \to \infty$.

As an example of what happens when R_{in} is not large, consider the circuit in Fig. 1.22. Ignoring the meter for a moment we see that the original circuit is a voltage divider, and application of Eq. (1.21) gives $V_{out} = 1$ V. But the effect of the meter's input resistance is to change R_2 to $R_2 \parallel R_{in} = 10$ kΩ. Using this in Eq. (1.21) gives $V_{out} = \frac{2}{3}$ V, and this is what the meter will indicate. So, unless we are aware of the effect of input resistance, we run the danger of making a false measurement. On the other hand, if we are aware of this effect, we can analyze the effect and determine the true value of our voltage when the meter is unattached.

Figure 1.23 Impedance specification for a typical analog meter.

How does one determine the value of the input resistance for a given instrument? Here are some common ways.

1. Look in the instrument manual under *input resistance* or *input impedance*. The value should be in ohms.
2. For analog voltmeters, look for a specification with units of ohms per volt (Ω/V). This is usually printed on the face of the meter itself, as shown in Fig. 1.23. To get R_{in}, multiply this number by the full scale voltage selected. For example, suppose your meter is specified as 20000 Ω/V and you have selected the 2.5 V full scale setting. The input resistance is then $20000 \times 2.5 = 50\ \mathrm{k}\Omega$.
3. You may have to analyze the instrument circuitry itself. The relevant question is: when a voltage is applied to the input of the instrument, how much current flows into the instrument? Then, by Ohm's Law, the input resistance is just the ratio of this voltage and current.

1.3 AC signals

So far our examples have used constant voltage sources such as batteries. Constant voltages and currents are described as DC quantities in electronics. On the other

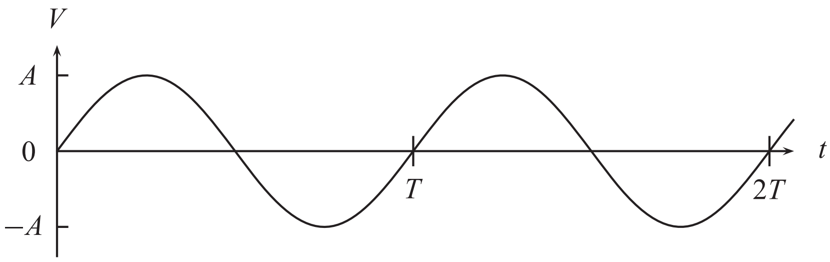

Figure 1.24 A sine wave.

hand, voltages and currents that vary in time are called AC quantities. For future reference, we list here some of the most common AC signals.

1. Sinusoidal signals. This is probably the most fundamental signal in electronics since, as we will see later, any signal can be constructed from sinusoidal signals. A typical sinusoidal voltage is shown in Fig. 1.24.

   Sinusoidal voltages can be written

$$V = A\sin(2\pi ft + \phi) = A\sin(\omega t + \phi) \tag{1.43}$$

   where A is the amplitude, f is the frequency in cycles/second or hertz (abbreviated Hz), ϕ is the phase, and ω is the angular frequency (in radians/second). The repetition time t_{rep} is also called the period T of the signal, and this is related to the frequency of the signal by $T = \frac{1}{f}$.

   There are several ways to specify the amplitude of a sinusoidal signal that are in common use. These include the following.

   (a) The peak amplitude A or A_{p}.

   (b) The peak-to-peak amplitude $A_{\text{pp}} = 2A$.

   (c) The rms amplitude $A_{\text{rms}} = A/\sqrt{2}$. This is useful for power calculations involving sinusoidal waves. For example, suppose we want the power dissipated in a resistor given the sinusoidally varying voltage across it. We cannot simply use Eq. (1.8) since our voltage is varying in time (what V would we use?). Instead, we calculate the time average of the power over one period:

$$P = \frac{1}{T}\int_0^T \frac{V^2}{R}\mathrm{d}t = \frac{1}{TR}\int_0^T A^2\sin^2(\omega t + \phi)\mathrm{d}t = \frac{A^2}{2R} = \frac{A_{\text{rms}}^2}{R}. \tag{1.44}$$

This last form shows that we *can* use Eq. (1.8) to calculate the power as long as we use the rms amplitude of the sinusoidal signal in the formula. The same argument applies to Eq. (1.7) for sinusoidal currents.

(d) Decibels (abbreviated dB) are used to compare the amplitude of two signals, say A_1 and A_2:

$$\mathrm{dB} = 20\log_{10}\frac{A_2}{A_1} = 10\log_{10}\left(\frac{A_2}{A_1}\right)^2 = 10\log_{10}\frac{P_2}{P_1} \tag{1.45}$$

where this last expression uses the power level of the two signals. So, for example, if $A_2 = 2A_1$, then we get $20\log 2 \approx 6$, so we say A_2 is 6 dB higher than A_1. Various related schemes specify the decibel level relative to a fixed standard. So dBV is the dB relative to a 1 $\mathrm{V_{rms}}$ signal and dBm is the dB relative to a 0.78 $\mathrm{V_{rms}}$ signal. For the curious, this latter voltage standard is 1 mW into a 600 Ω resistor.

Some other typical waveforms of electronics are shown in Figs. 1.25 through 1.30.

2. Square wave. Specified by an amplitude and a frequency (or period).

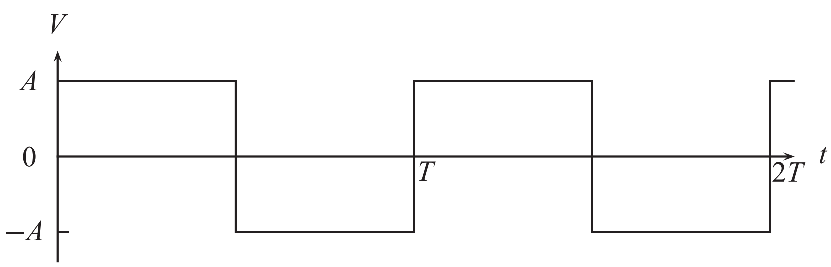

Figure 1.25 The square wave.

3. Sawtooth wave. Specified by an amplitude and a frequency (or period).

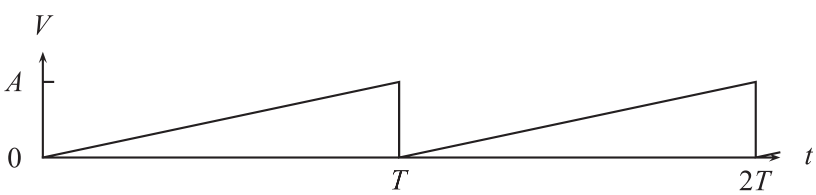

Figure 1.26 The sawtooth wave.

4. Triangle wave. Specified by an amplitude and a frequency (or period).

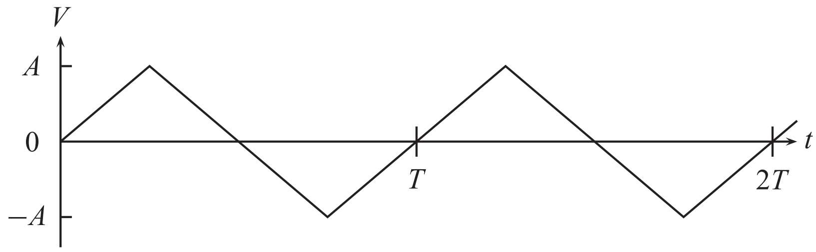

Figure 1.27 The triangle wave.

5. Ramp. Specified by an amplitude and a ramp time.

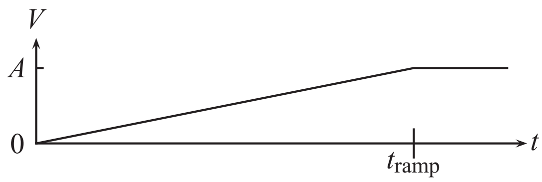

Figure 1.28 A ramp signal.

6. Pulse train. Specified by an amplitude, a pulse width τ, and a repetition time t_{rep}. The *duty cycle* of a pulse train is defined as τ/t_{rep}.

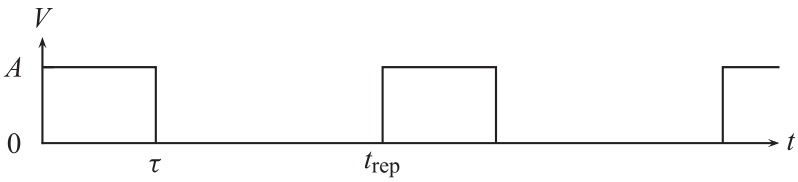

Figure 1.29 A pulse train.

7. Noise. These are random signals of thermal origin or simply unwanted signals coupled into the circuit. Noise is usually described by its frequency content, but that is a more advanced topic.

Figure 1.30 Noise.

EXERCISES

1. What is the resistance of a nichrome wire 1 mm in diameter and 1 m in length?
2. What is the maximum allowable current through a 10 kΩ, 10 W resistor? Through a 10 kΩ, 1/4 W resistor?
3. (a) What power rating is needed for a 100 Ω resistor if 100 V is to be applied to it? (b) For a 100 kΩ resistor?
4. Compute the current through R_3 of Fig. 1.31.

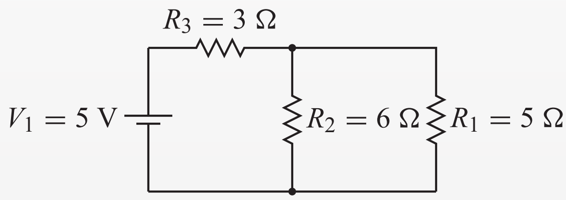

Figure 1.31 Circuit for Problems 4 and 5.

5. Compute the current through R_1 and R_2 of Fig. 1.31.
6. The output of the voltage divider of Fig. 1.32 is to be measured with voltmeters with input resistances of 100 Ω, 1 kΩ, 50 kΩ, and 1 MΩ. What voltage will each indicate?

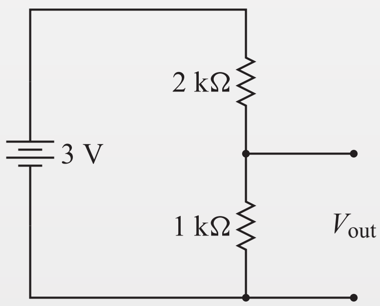

Figure 1.32 Circuit for Problem 6.

7. A real battery can be modeled as an ideal voltage source in series with a resistor (the *internal resistance*). A voltmeter with input resistance of 1000 Ω measures the voltage of a worn-out 1.5 V flashlight battery as 0.9 V. What is the internal resistance of the battery?
8. If the flashlight battery of the preceding problem had been measured with a voltmeter with input resistance of 10 MΩ, what would the reading be?
9. What is the resistance across the terminals of Fig. 1.33?
10. Suppose that a 25 V battery is connected to the terminals of Fig. 1.33. Find the current in the 10 Ω resistor.
11. Compute the current through R_2 and R_3 of Fig. 1.34.

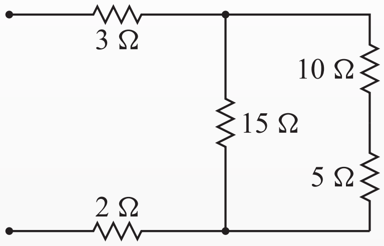

Figure 1.33 Circuit for Problems 9 and 10.

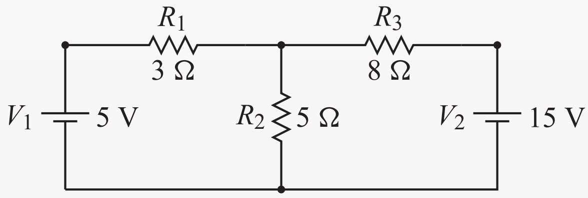

Figure 1.34 Circuit for Problem 11.

12. Find the Thevenin voltage and Thevenin resistance of the circuit shown in Fig. 1.35.

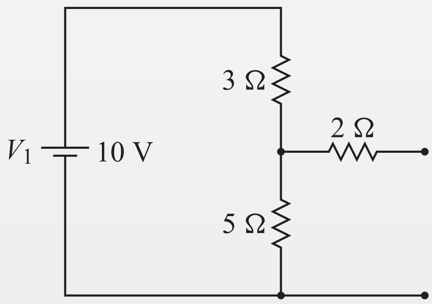

Figure 1.35 Circuit for Problem 12.

13. Find the Thevenin voltage and Thevenin resistance of the circuit shown in Fig. 1.36 with R_5 removed. The two terminals for this problem are the points where R_5 was connected.

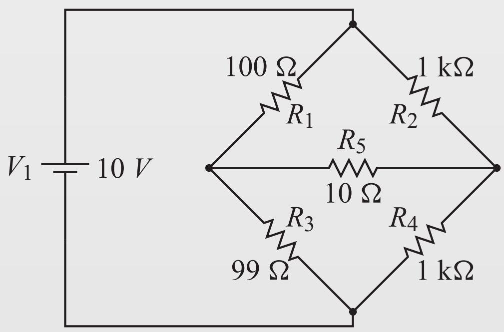

Figure 1.36 Circuit for Problems 13 and 14.

14. Using the result of the previous problem, find the current through R_5 of Fig. 1.36.

15. In the circuit of Fig. 1.37, compute the current in the 3 Ω resistor and find the value of V_2.

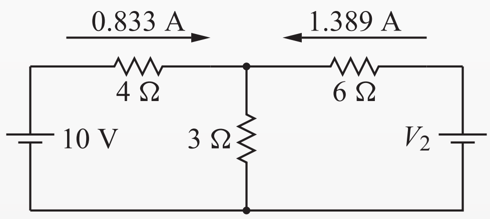

Figure 1.37 Circuit for Problem 15.

16. In the circuit of Fig. 1.38, find the value of V_3 such that the current in the 10 Ω resistor is zero.

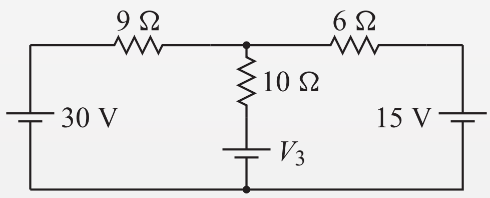

Figure 1.38 Circuit for Problem 16.

17. Compute all the currents labeled in the circuit of Fig. 1.39 assuming the following values: $V_1 = 5\,\text{V}$, $V_2 = 10\,\text{V}$, $V_3 = 15\,\text{V}$, $R_1 = 2\,\Omega$, $R_2 = 4\,\Omega$, $R_3 = 6\,\Omega$, $R_4 = 7\,\Omega$, $R_5 = 5\,\Omega$, $R_6 = 3\,\Omega$. Suggestion: use the mesh loop method.

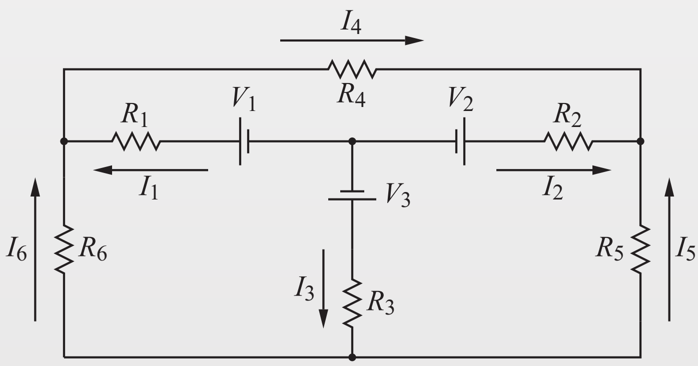

Figure 1.39 Circuit for Problem 17.

18. (a) Compute the current through the 10 Ω resistor in the circuit of Fig. 1.40. Do not use Thevenin's or Norton's theorems for this computation. (b) Now

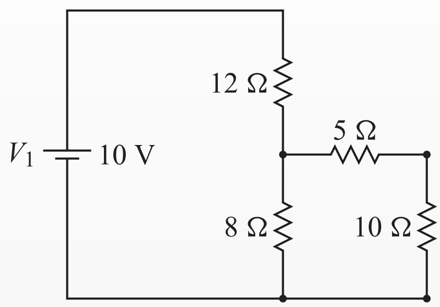

Figure 1.40 Circuit for Problem 18.

find the Thevenin voltage, the Thevenin resistance, and the Norton current when the 10 Ω resistor is removed. The two terminals for this problem are the points where the 10 Ω resistor was connected. (c) Show that, if the 10 Ω resistor is connected to the Thevenin equivalent circuit, the current through the 10 Ω resistor matches the value found in part (a). Do the same for the Norton equivalent circuit.

FURTHER READING

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