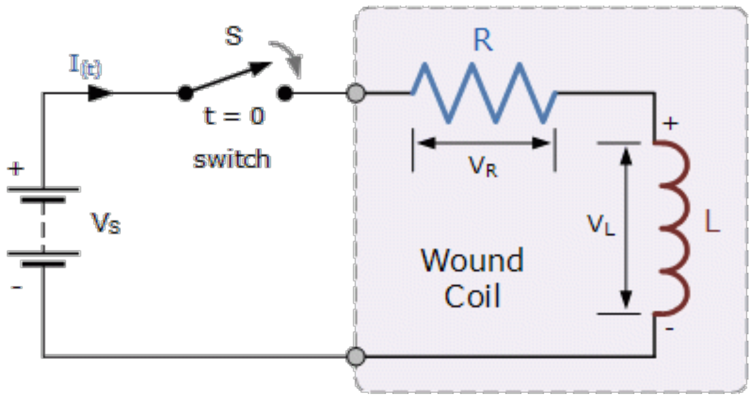


LR Series Circuit

All coils, inductors, chokes and transformers create a magnetic field around themselves consist of an Inductance in series with a Resistance forming an LR Series Circuit

The first tutorial in this section about Inductors, we looked briefly at the time constant of an inductor stating that the current flowing through an inductor could not change instantaneously, but would increase at a constant rate determined by the self-induced emf in the inductor.

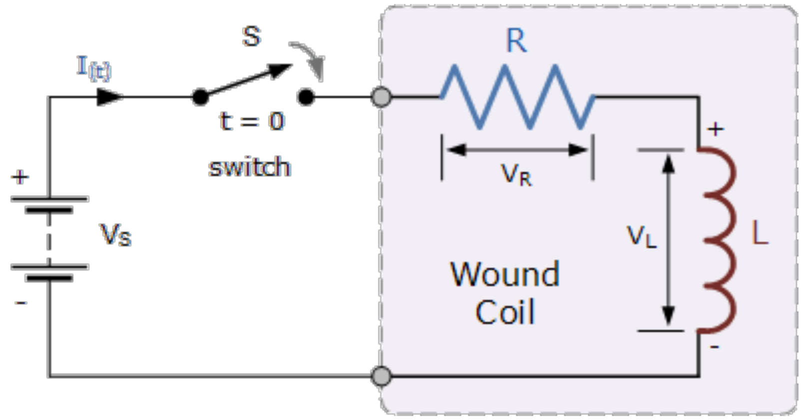
In other words, an inductor in an electrical circuit opposes the flow of current, (i) through it. While this is perfectly correct, we made the assumption in the tutorial that it was an ideal inductor which had no resistance or capacitance associated with its coil windings.

However, in the real world “ALL” coils whether they are chokes, solenoids, relays or any wound component will always have a certain amount of resistance no matter how small. This is because the actual coils turns of wire being used to make it uses copper wire which has a resistive value.

Then for real world purposes we can consider our simple coil as being an “Inductance”, **L** in series with a “Resistance”, **R**. In other words forming an **LR Series Circuit**.

A **LR Series Circuit** consists basically of an inductor of inductance, L connected in series with a resistor of resistance, R. The resistance “R” is the DC resistive value of the wire turns or loops that goes into making up the inductors coil. Consider the LR series circuit below.

The LR Series Circuit



The above *LR series circuit* is connected across a constant voltage source, (the battery) and a switch. Assume that the switch, S is open until it is closed at a time t = 0, and then remains permanently closed producing a “step response” type voltage input. The current, i begins to flow through the circuit but does not rise rapidly to its maximum value of Imax as determined by the ratio of V / R (Ohms Law).

This limiting factor is due to the presence of the self induced emf within the inductor as a result of the growth of magnetic flux, (Lenz’s Law). After a time the voltage source neutralizes the effect of the self induced emf, the current flow becomes constant and the induced current and field are reduced to zero.

We can use Kirchhoff’s Voltage Law, (KVL) to define the individual voltage drops that exist around the circuit and then hopefully use it to give us an expression for the flow of current.

Kirchhoff’s voltage law (KVL) gives us:

$$V_{(t)} - \left(V_R + V_L\right) = 0$$

The voltage drop across the resistor, R is I*R (Ohms Law).

$$V_R = I \times R$$

The voltage drop across the inductor, L is by now our familiar expression L(di/dt)

$$V_L = L\frac{di}{dt}$$

Then the final expression for the individual voltage drops around the LR series circuit can be given as:

$$V_{(t)} = I \times R + L\frac{di}{dt}$$

We can see that the voltage drop across the resistor depends upon the current, i, while the voltage drop across the inductor depends upon the rate of change of the current, di/dt. When the current is equal to zero, (i = 0) at time t = 0 the above expression, which is also a first order differential equation, can be rewritten to give the value of the current at any instant of time as:

Expression for the Current in an LR Series Circuit

$$I_{(t)} = \frac{V}{R}\left(1 - e^{-Rt/L}\right) \quad (A)$$

Where:

V is in Volts

R is in Ohms

L is in Henries

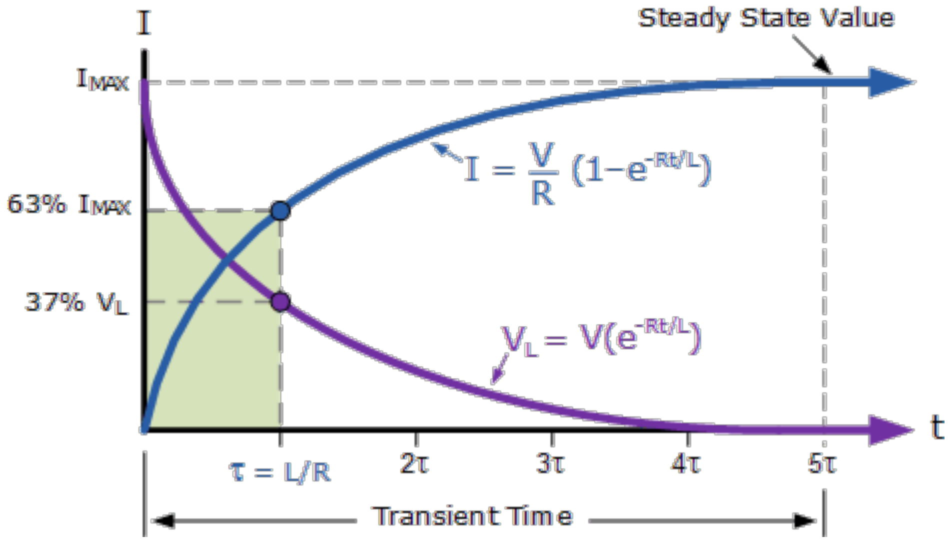
t is in Seconds

e is the base of the Natural Logarithm = 2.71828

The **Time Constant**, (τ) of the LR series circuit is given as L/R and in which V/R represents the final steady state current value after five time constant values. Once the current reaches this maximum steady state value at 5τ, the inductance of the coil has reduced to zero acting more like a short circuit and effectively removing it from the circuit.

Therefore the current flowing through the coil is limited only by the resistive element in Ohms of the coils windings. A graphical representation of the current growth representing the voltage/time characteristics of the circuit can be presented as.

Transient Curves for an LR Series Circuit



Since the voltage drop across the resistor, V_R is equal to I*R (Ohms Law), it will have the same exponential growth and shape as the current. However, the voltage drop across the inductor, V_L will have a value equal to: $Ve^{(-Rt/L)}$. Then the voltage across the inductor, V_L will have an initial value equal to the battery voltage at time t = 0 or when the switch is first closed and then decays exponentially to zero as represented in the above curves.

The time required for the current flowing in the LR series circuit to reach its maximum steady state value is equivalent to about **5 time constants** or 5τ. This time constant τ, is measured by τ = L/R, in seconds, where R is the value of the resistor in ohms and L is the value of the inductor in Henries. This then forms the basis of an RL charging circuit were 5τ can also be thought of as "5*(L/R)" or the *transient time* of the circuit.

The transient time of any inductive circuit is determined by the relationship between the inductance and the resistance. For example, for a fixed value resistance the larger the inductance the slower will be the transient time and therefore a longer time constant for the LR series circuit. Likewise, for a fixed value inductance the smaller the resistance value the longer the transient time.

However, for a fixed value inductance, by increasing the resistance value the transient time and therefore the time constant of the circuit becomes shorter. This is because as the resistance increases the circuit becomes more and more resistive as the value of the inductance becomes negligible compared to the resistance. If the value of the resistance is increased sufficiently large compared to the inductance the transient time would effectively be reduced to almost zero.

LR Series Circuit Example No1

A coil which has an inductance of 40mH and a resistance of 2Ω is connected together to form a LR series circuit. If they are connected to a 20V DC supply.

a). What will be the final steady state value of the current.

$$\text{Steady State Current, } I = \frac{V}{R} = \frac{20}{2} = 10A$$

b) What will be the time constant of the RL series circuit.

$$\text{Time Constant, } \tau = \frac{L}{R} = \frac{0.04}{2} = 0.02s \text{ or } 20mS$$

c) What will be the transient time of the RL series circuit.

$$\text{Transient Time, } 5\tau = 5 \times 0.02s = 100mS$$

d) What will be the value of the induced emf after 10ms.

$$\text{Induced emf, } V_L = Ve^{(-Rt/L)} = 20e^{(-2\times0.01/0.04)}$$

$$V_L = 20 \times 0.6065 = 12.13V$$

e) What will be the value of the circuit current one time constant after the switch is closed.

$$\text{Instantaneous Current, } I_{(t)} = \frac{V_S}{R}(1-e^{-Rt/L})$$

The Time Constant, τ of the circuit was calculated in question b) as being 20ms. Then the circuit current at this time is given as:

$$I_{(t)} = \frac{20}{2}(1 - e^{-2\times0.02/0.04})$$

$$I_{(t)} = 10(1\text{-}0.368) = 6.32A$$

You may have noticed that the answer for question (e) which gives a value of 6.32 Amps at one time constant, is equal to 63.2% of the final steady state current value of 10 Amps we calculated in question (a). This value of 63.2% or 0.632 x I_{MAX} also corresponds with the transient curves shown above.

Power in an LR Series Circuit

Then from above, the instantaneous rate at which the voltage source delivers power to the circuit is given as:

$$P = V\times I \quad \text{in Watts}$$

The instantaneous rate at which power is dissipated by the resistor in the form of heat is given as:

$$P = I^2\times R \quad \text{in Watts}$$

The rate at which energy is stored in the inductor in the form of magnetic potential energy is given as:

$$P = Vi = Li\frac{di}{dt} \quad \text{in Watts}$$

Then we can find the total power in a RL series circuit by multiplying by **i** and is therefore:

$$P = i^2R + Li\frac{di}{dt} \quad \text{(Watts)}$$

Where the first I^2R term represents the power dissipated by the resistor in heat, and the second term represents the power absorbed by the inductor, its magnetic energy.

106 Comments

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S

Shikha

À simple R-L circuit is excited with a constant voltage source the speed of the response depends on what?

Posted on January 10th 2021 | 10:38 am

Reply

mohamed

how to calculate steady state error for RL circuit

Posted on December 26th 2020 | 2:01 am

Reply

YACKZY

It's a great job the experiment is correct

Posted on November 18th 2020 | 11:27 am

Reply

John

How do i calculate the winding resistance when given only R, L and frequency.

Posted on November 15th 2020 | 11:12 pm

Reply

Wayne Storr

Please read the tutorial about Inductive Reactance

Posted on November 16th 2020 | 11:10 am

Reply

Newton

Curves for transients in L-R circuits in both induced voltage and resistor voltage

Posted on November 05th 2020 | 7:10 pm

Reply

Era

How can I find out the power dissipated by the resistor in equilibrium?

Posted on October 05th 2020 | 1:17 pm

Reply

Wayne Storr

$P = V*I = I^2R = V^2/R$

Posted on October 05th 2020 | 6:35 pm

Reply

Veronicah

I have been assisted greatly in forming derivatives of RL circuit.thanks to the administrators.

Posted on September 07th 2020 | 7:29 am

Reply

Arko Dey

Diploma in Electrical Engineering.
2015-2018.
B.tech in Electrical Engineering.
2019 -2022.

Arko Dey
West Bengal, Bankura, Indas, Shankrul.
Pin code –
Current location – Kolkata, Dum Dum (Gosh para).
Pin code –
Contact number –
Thank you.

Posted on August 25th 2020 | 6:21 am

Reply

Sooraj

Please explian what happens when the volatage drop across both resistance and indactance are same

Posted on August 02nd 2020 | 7:31 am

Reply

Jbatarseh@yahoo.com

when you wind a wire on an iron rod and connect both ends to a battery positive and negative terminal to magnetize the rod , why doesn't the battery shorted out , even though it is winded still the wire is directly connected from positive to negative ?

Thanks ;
Jacob

Posted on June 25th 2020 | 10:48 pm

Reply

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david

hello, I want to ask... induced emf has exponencial equation which has negative degree .. so.... it cant be more than 20 volt .... but in reality it can.... so is that equation true or I misunderstand something ?

Posted on June 23rd 2020 | 10:49 pm

Reply

More

JOHN MWIKA

Nice otes

Posted on June 22nd 2020 | 4:05 pm

Reply

EDWIN

Thanks a lot your explanation is the best

Posted on June 04th 2020 | 1:42 pm

Reply

NATHANIEL MOWL

I am really with some difficulties on the question below on my work doing;
Explain with diagrams the relationship between Current (I) and Voltage (E) in a circuit consisting of an inductor and resistor in series.

Posted on May 25th 2020 | 10:07 am

Reply

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mehrdad

homemade ducking systems

Posted on April 19th 2020 | 1:32 pm

Reply

Nguyễn Tấn Đạt

thank you very much!

Posted on April 18th 2020 | 6:38 am

Reply

Kashish

Very well explained

Posted on December 06th 2019 | 3:29 pm

Reply

Kelvin

Awesome

Posted on November 18th 2019 | 5:57 am

Reply

rahul shekhar

thanks for explaination

Posted on November 16th 2019 | 5:14 am

Reply

Mohit verma

Thanks

Posted on September 16th 2019 | 2:40 pm

Reply

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