Formulaire vectoriel

Analyse vectorielle

$$\varepsilon_{ijk}\varepsilon_{mnk} = \delta_{im}\delta_{jn} - \delta_{in}\delta_{jm}$$
$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$$
$$(\mathbf{A} \times \mathbf{B}) \cdot (\mathbf{C} \times \mathbf{D}) = (\mathbf{A} \cdot \mathbf{C})(\mathbf{B} \cdot \mathbf{D}) - (\mathbf{A} \cdot \mathbf{D})(\mathbf{B} \cdot \mathbf{C})$$
$$\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$$

Coordonnées cartésiennes

$$\nabla = \hat{\mathbf{x}}\frac{\partial}{\partial x} + \hat{\mathbf{y}}\frac{\partial}{\partial y} + \hat{\mathbf{z}}\frac{\partial}{\partial z}$$
$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$
$$\nabla \times \mathbf{A} = \Big(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\Big)\hat{\mathbf{x}} + \Big(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\Big)\hat{\mathbf{y}} + \Big(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\Big)\hat{\mathbf{z}}$$

Coordonnées cylindriques

$$\nabla = \hat{\mathbf{r}}\frac{\partial}{\partial r} + \hat{\boldsymbol{\varphi}}\frac{1}{r}\frac{\partial}{\partial \varphi} + \hat{\mathbf{z}}\frac{\partial}{\partial z}$$
$$\nabla \cdot \mathbf{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\varphi}{\partial \varphi} + \frac{\partial A_z}{\partial z}$$
$$\nabla \times \mathbf{A} = \Big[\frac{1}{r}\frac{\partial A_z}{\partial \varphi} - \frac{\partial A_\varphi}{\partial z}\Big]\hat{\mathbf{r}} + \Big[\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\Big]\hat{\boldsymbol{\varphi}} + \frac{1}{r}\Big[\frac{\partial}{\partial r}(rA_\varphi) - \frac{\partial A_r}{\partial \varphi}\Big]\hat{\mathbf{z}}$$
$$\nabla^2 \Phi = \frac{1}{r}\frac{\partial}{\partial r}\Big(r\frac{\partial \Phi}{\partial r}\Big) + \frac{1}{r^2}\frac{\partial^2 \Phi}{\partial \varphi^2} + \frac{\partial^2 \Phi}{\partial z^2}$$

Coordonnées sphériques

$$\nabla = \hat{\mathbf{r}}\frac{\partial}{\partial r} + \hat{\boldsymbol{\theta}}\frac{1}{r}\frac{\partial}{\partial \theta} + \hat{\boldsymbol{\varphi}}\frac{1}{r\sin\theta}\frac{\partial}{\partial \varphi}$$
$$\nabla \cdot \mathbf{A} = \frac{1}{r^2}\frac{\partial}{\partial r}\big(r^2 A_r\big) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta A_\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\varphi}{\partial \varphi}$$
$$\nabla \times \mathbf{A} = \frac{1}{r\sin\theta}\Big[\frac{\partial}{\partial \theta}(\sin\theta A_\varphi) - \frac{\partial A_\theta}{\partial \varphi}\Big]\hat{\mathbf{r}} + \Big[\frac{1}{r\sin\theta}\frac{\partial A_r}{\partial \varphi} - \frac{1}{r}\frac{\partial}{\partial r}(rA_\varphi)\Big]\hat{\boldsymbol{\theta}} + \frac{1}{r}\Big[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial \theta}\Big]\hat{\boldsymbol{\varphi}}$$
$$\nabla^2 \Phi = \frac{1}{r^2}\frac{\partial}{\partial r}\Big(r^2\frac{\partial \Phi}{\partial r}\Big) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\Big(\sin\theta\frac{\partial \Phi}{\partial \theta}\Big) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 \Phi}{\partial \varphi^2}$$