# Introduction to Reinforcement Learning

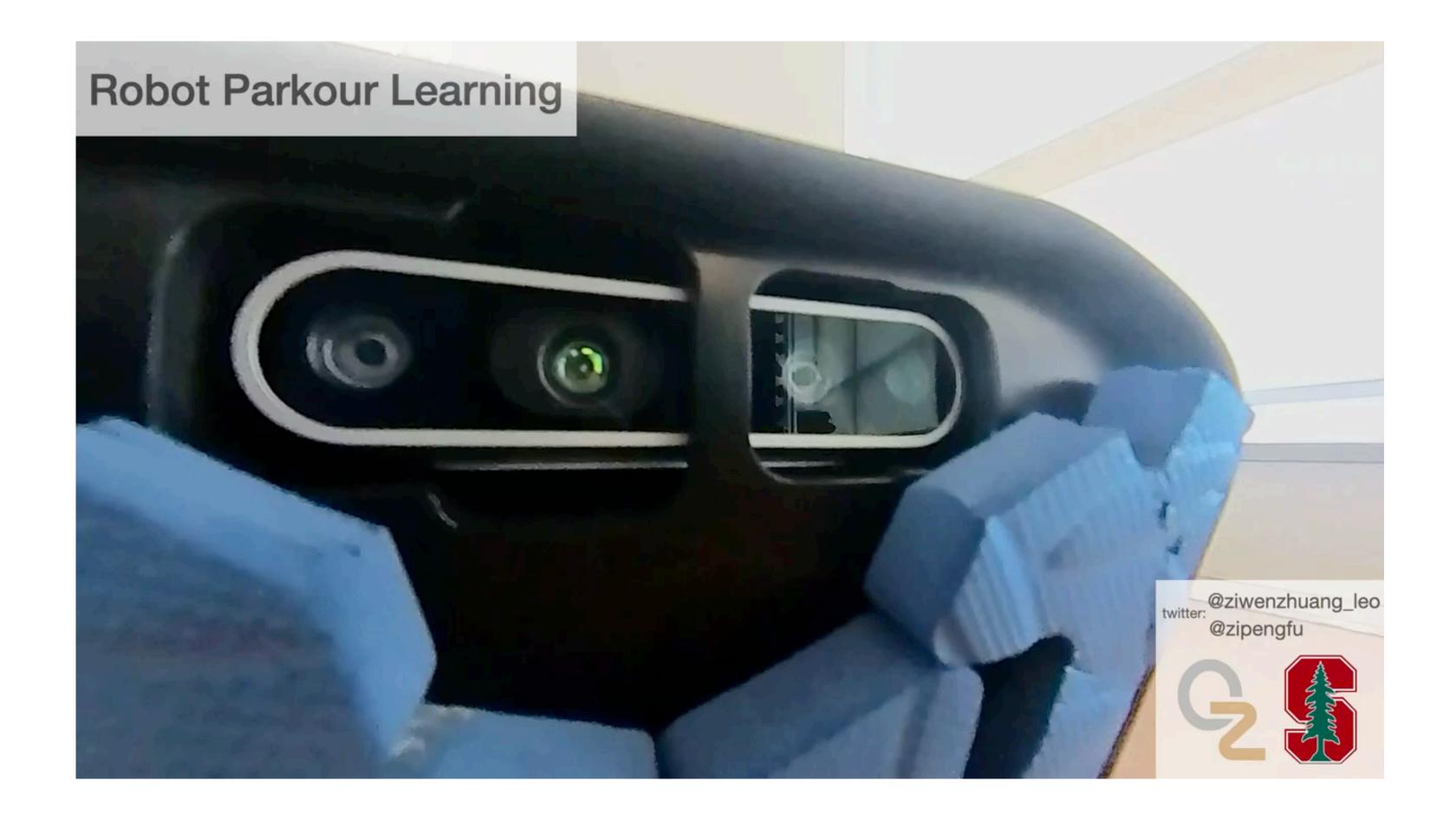
Elena Congeduti, 09-12-2024



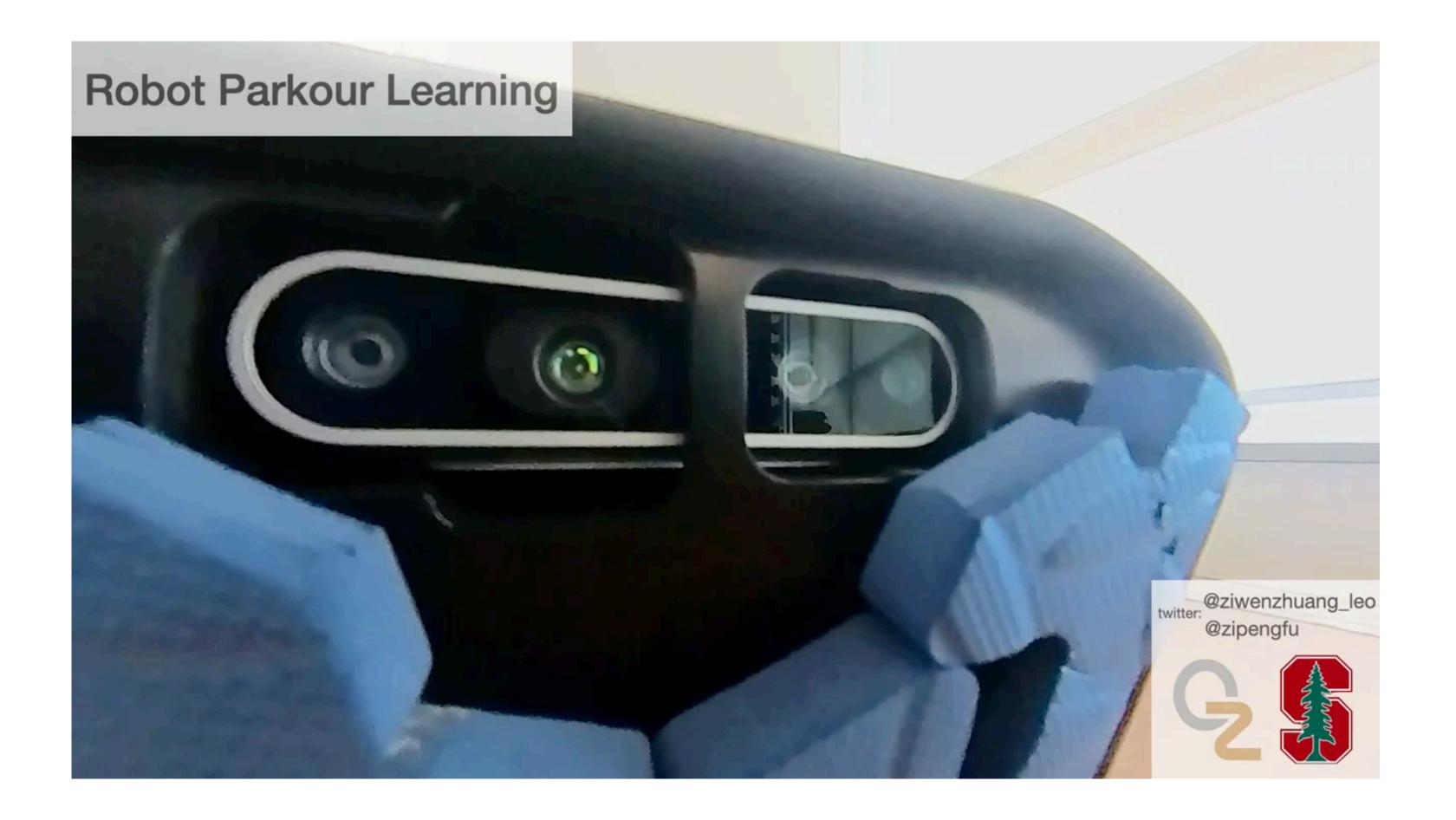
## Lecture's Agenda

- Sequential decision making
- Markov decision processes (MDPs)
- Value iteration
- Tabular Q-learning

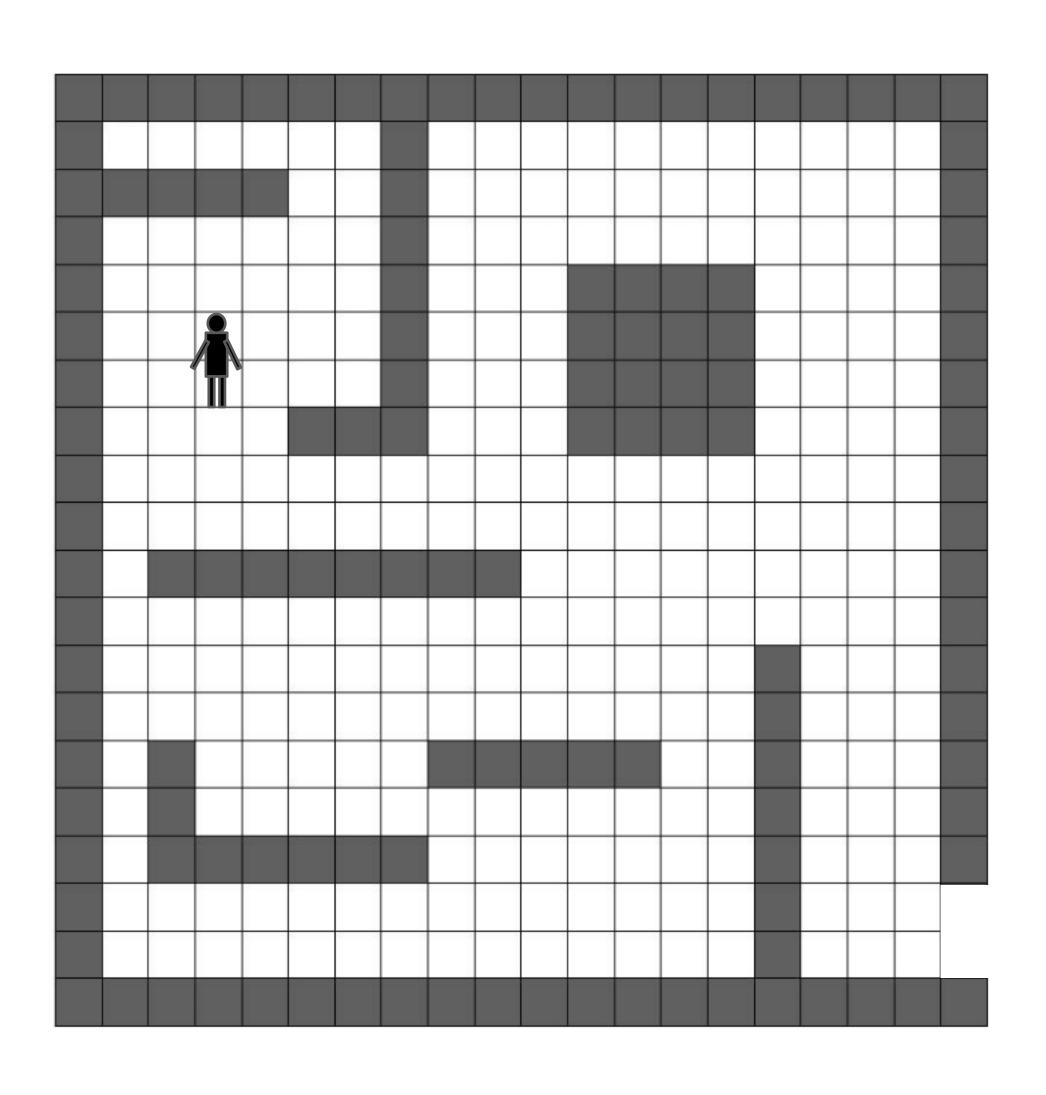
# What is Reinforcement Learning?

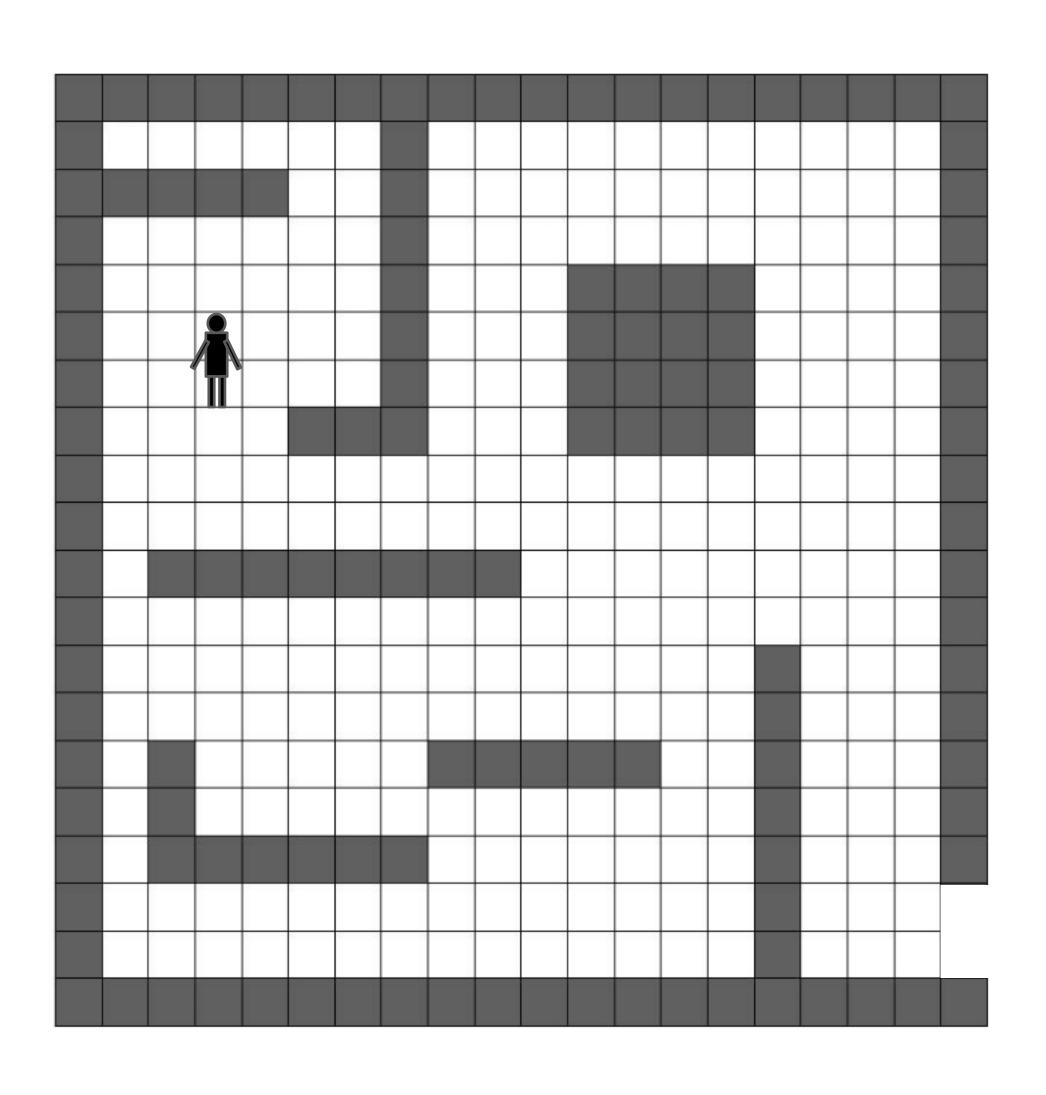


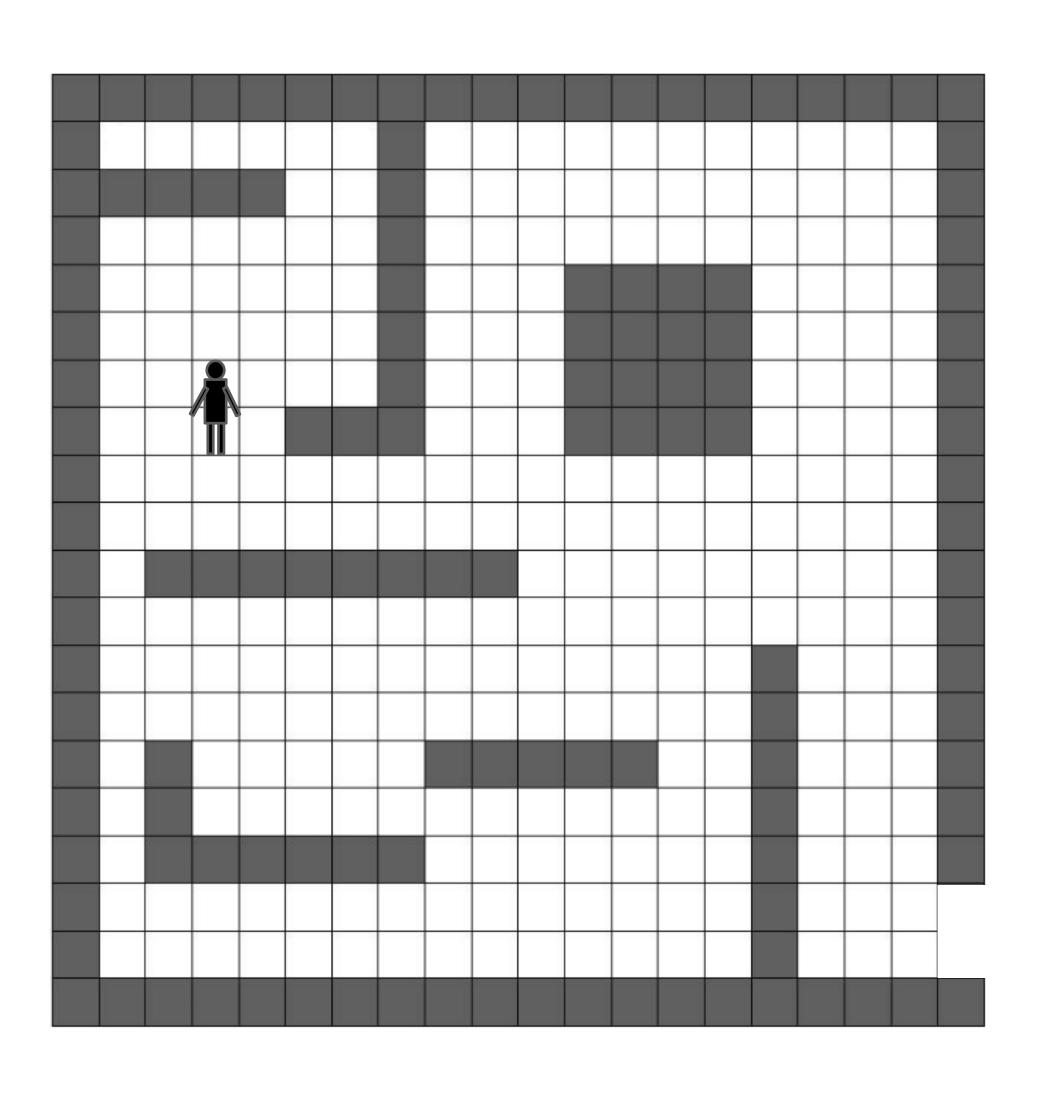
## What is Reinforcement Learning?

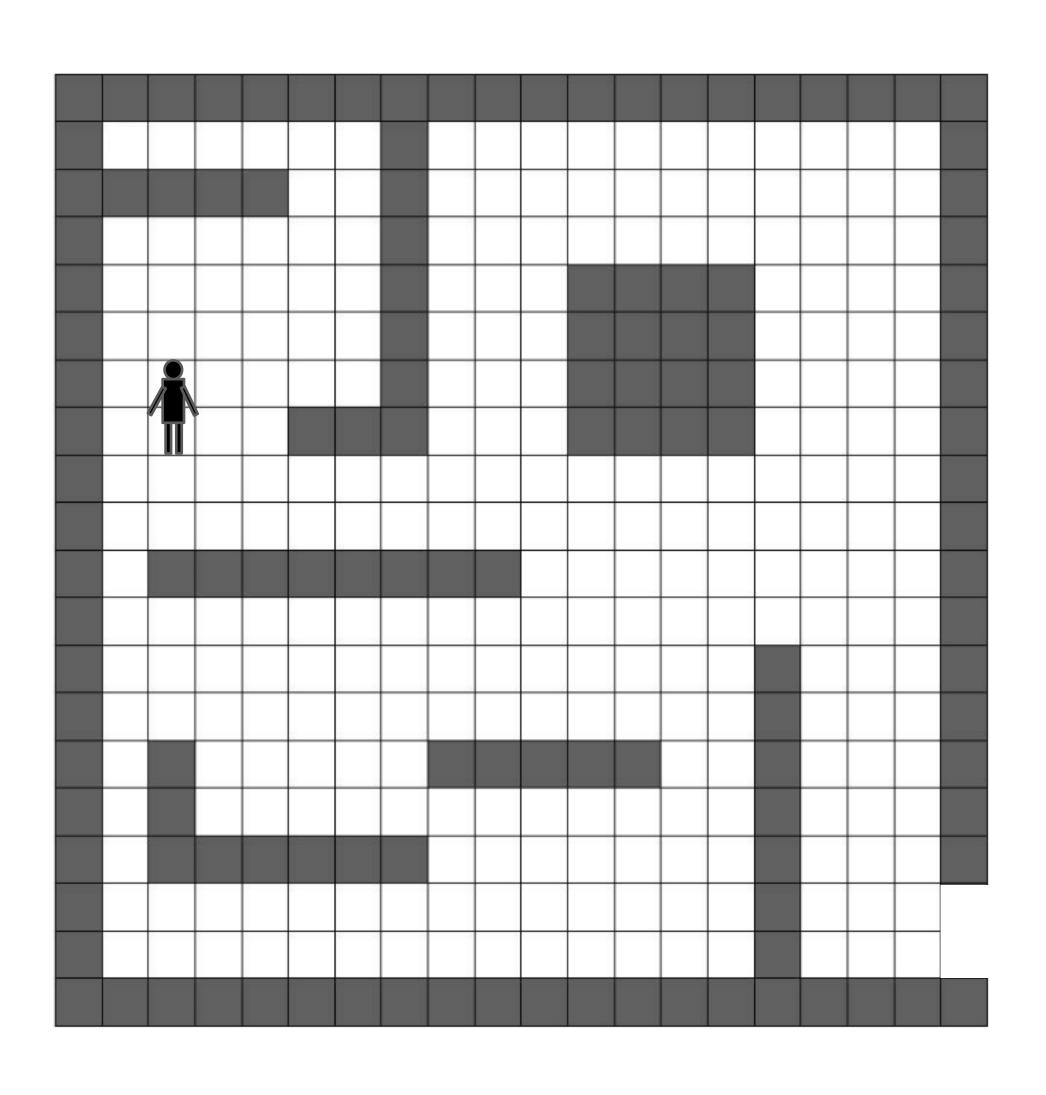


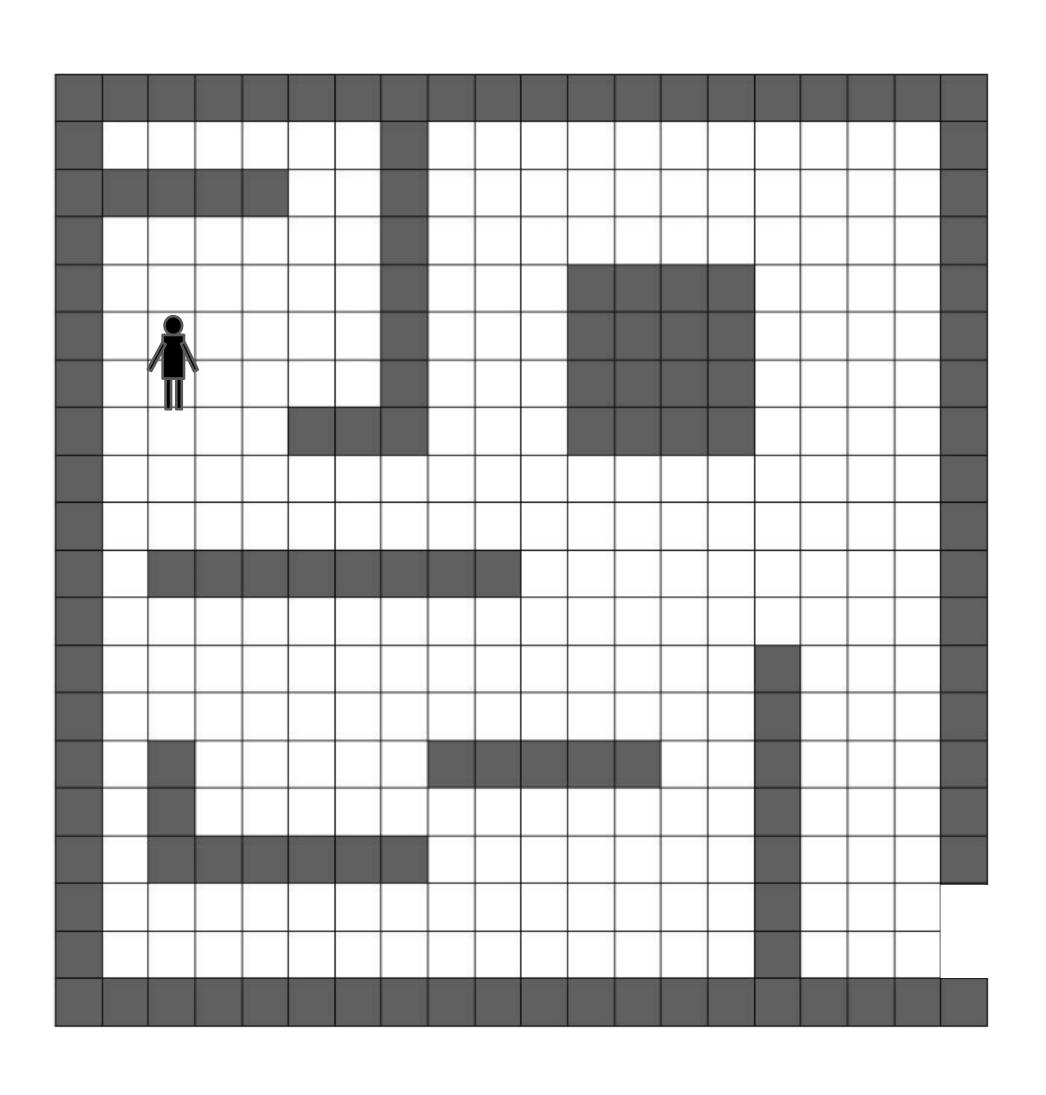
Intelligent agent learns from experience how to make decisions that maximize its return in the face of uncertainty

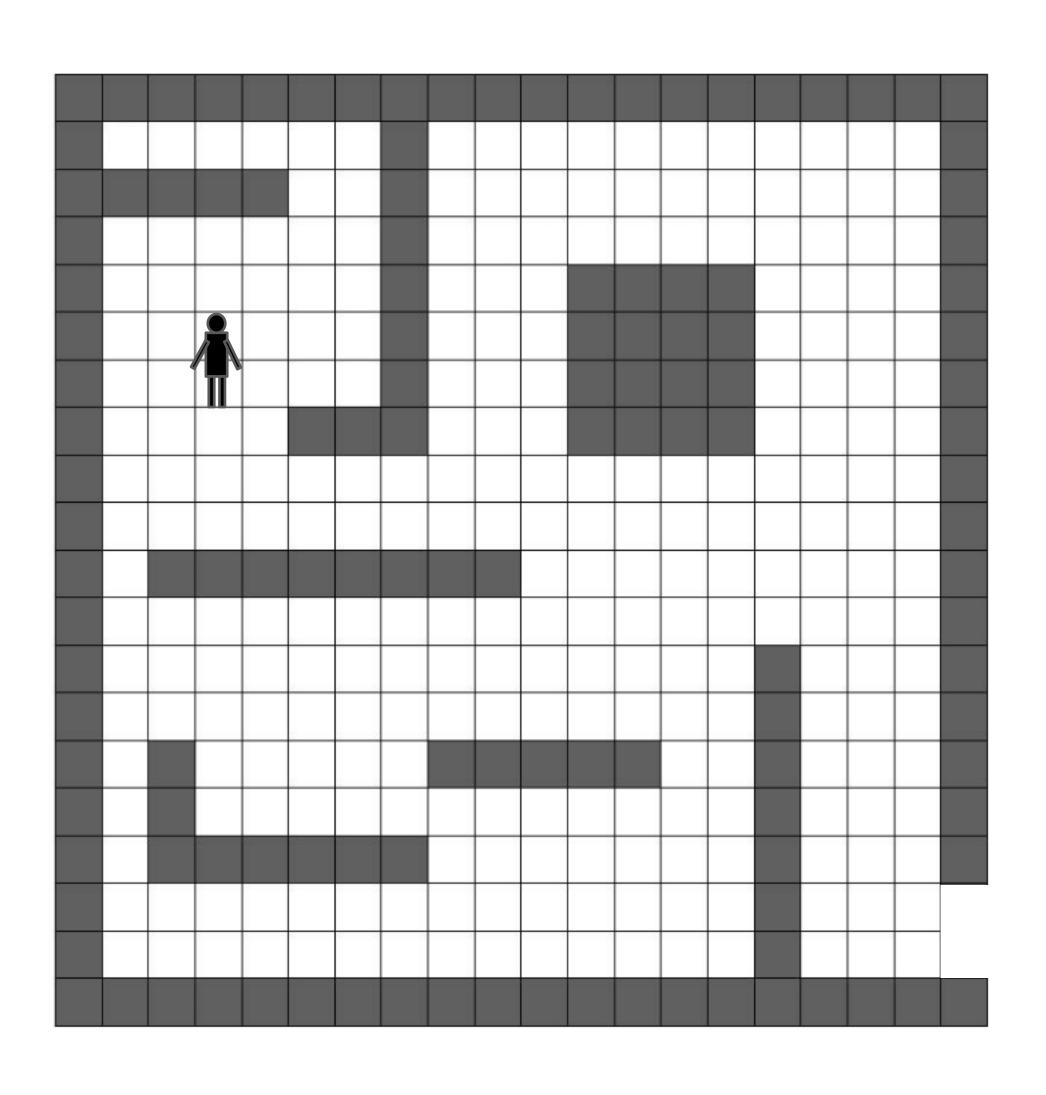


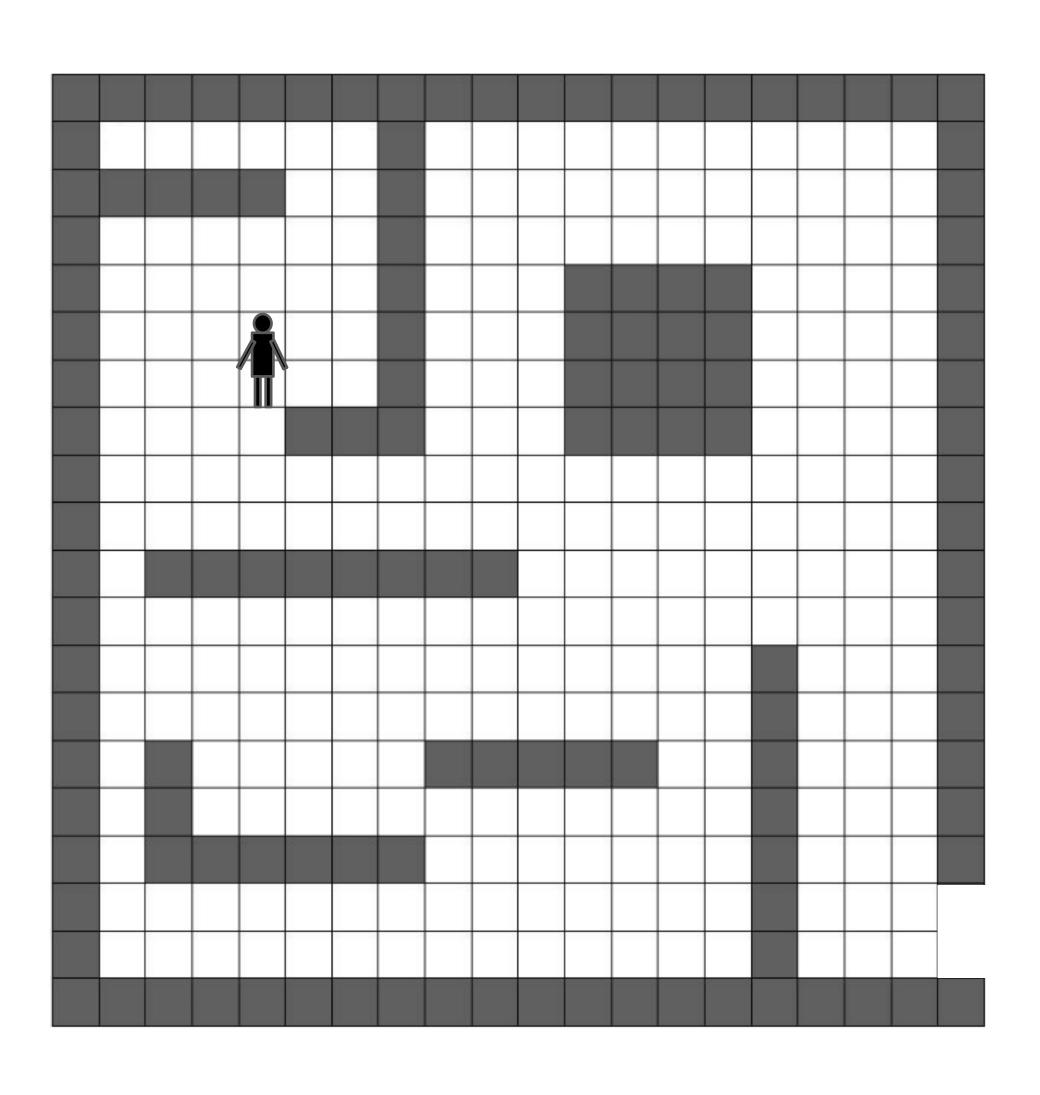


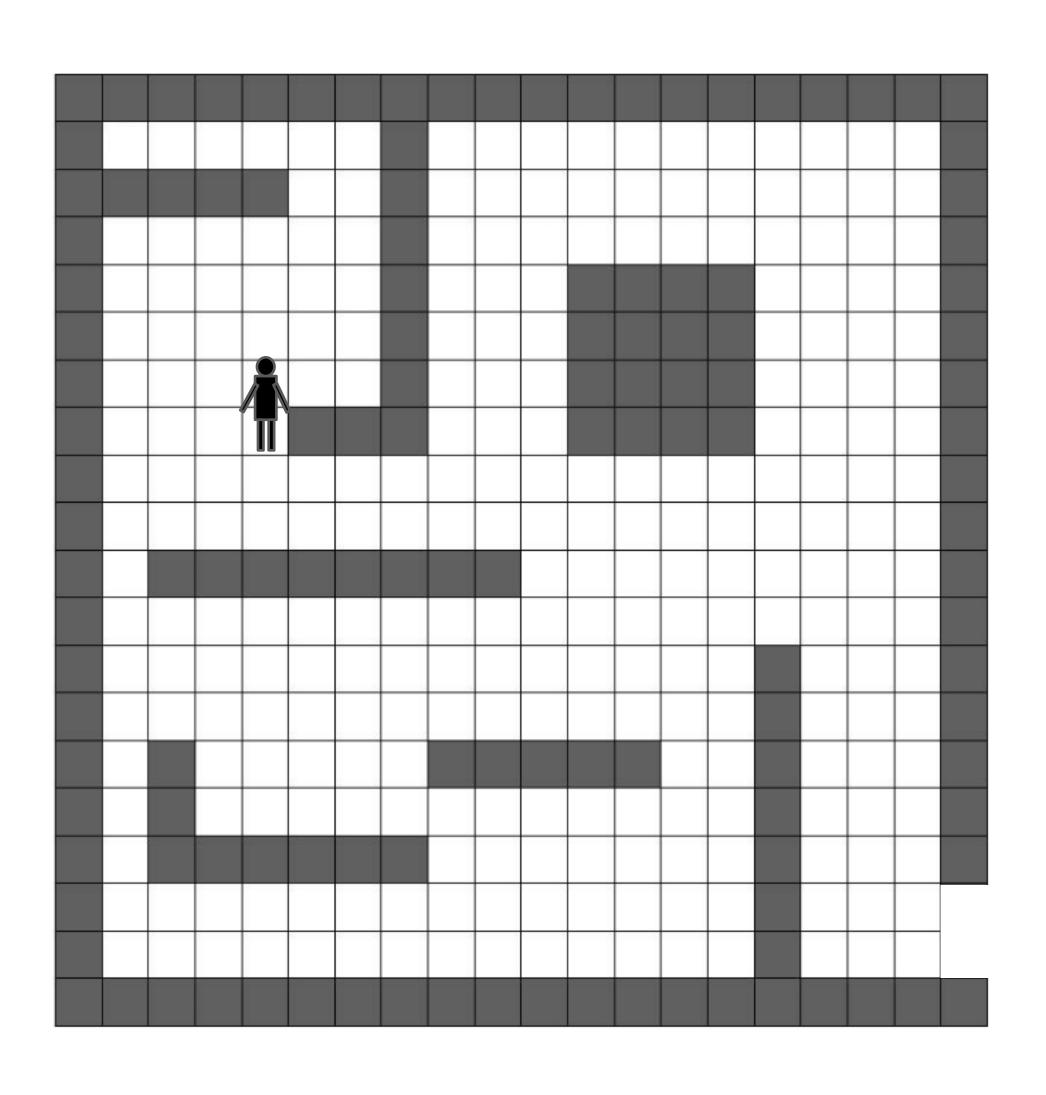


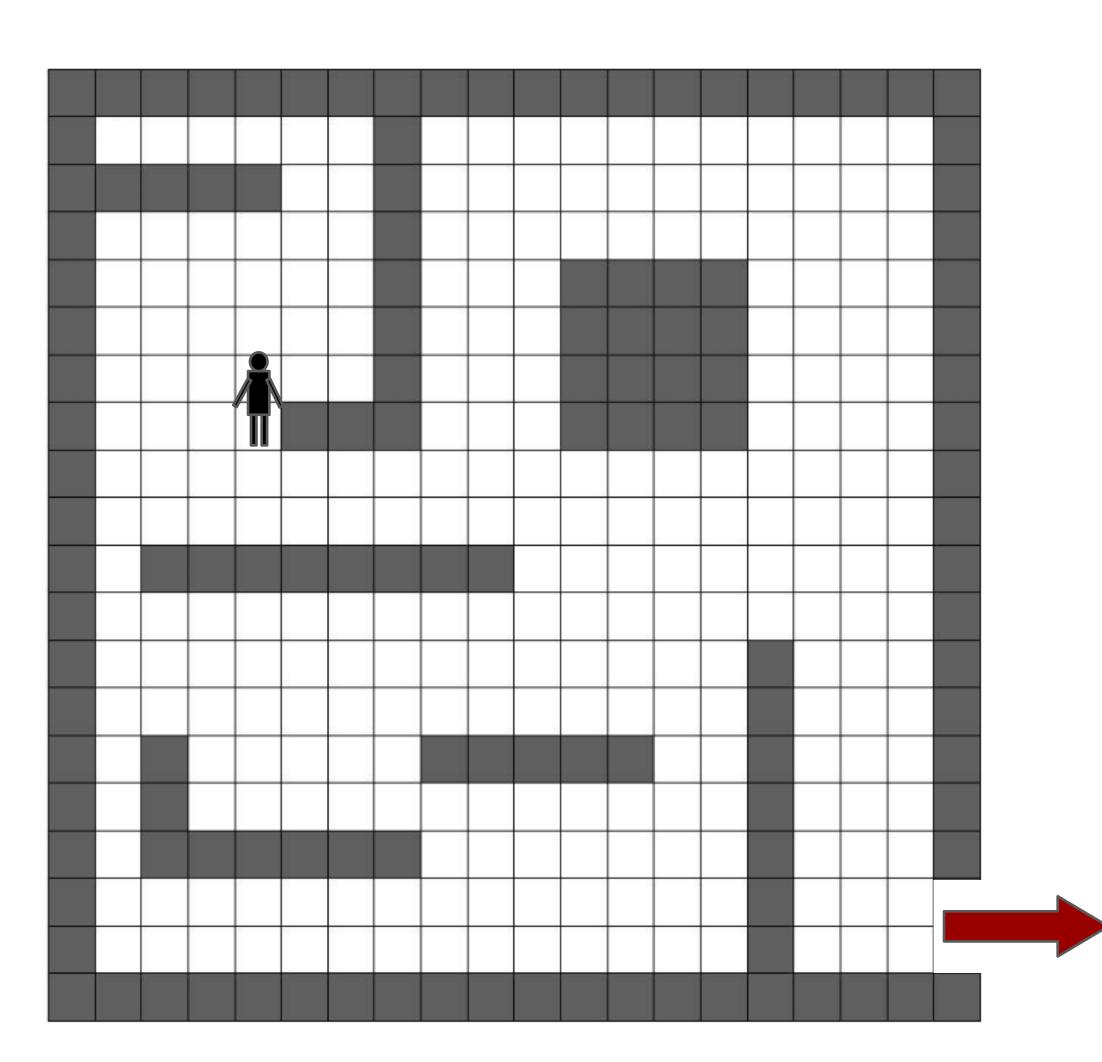




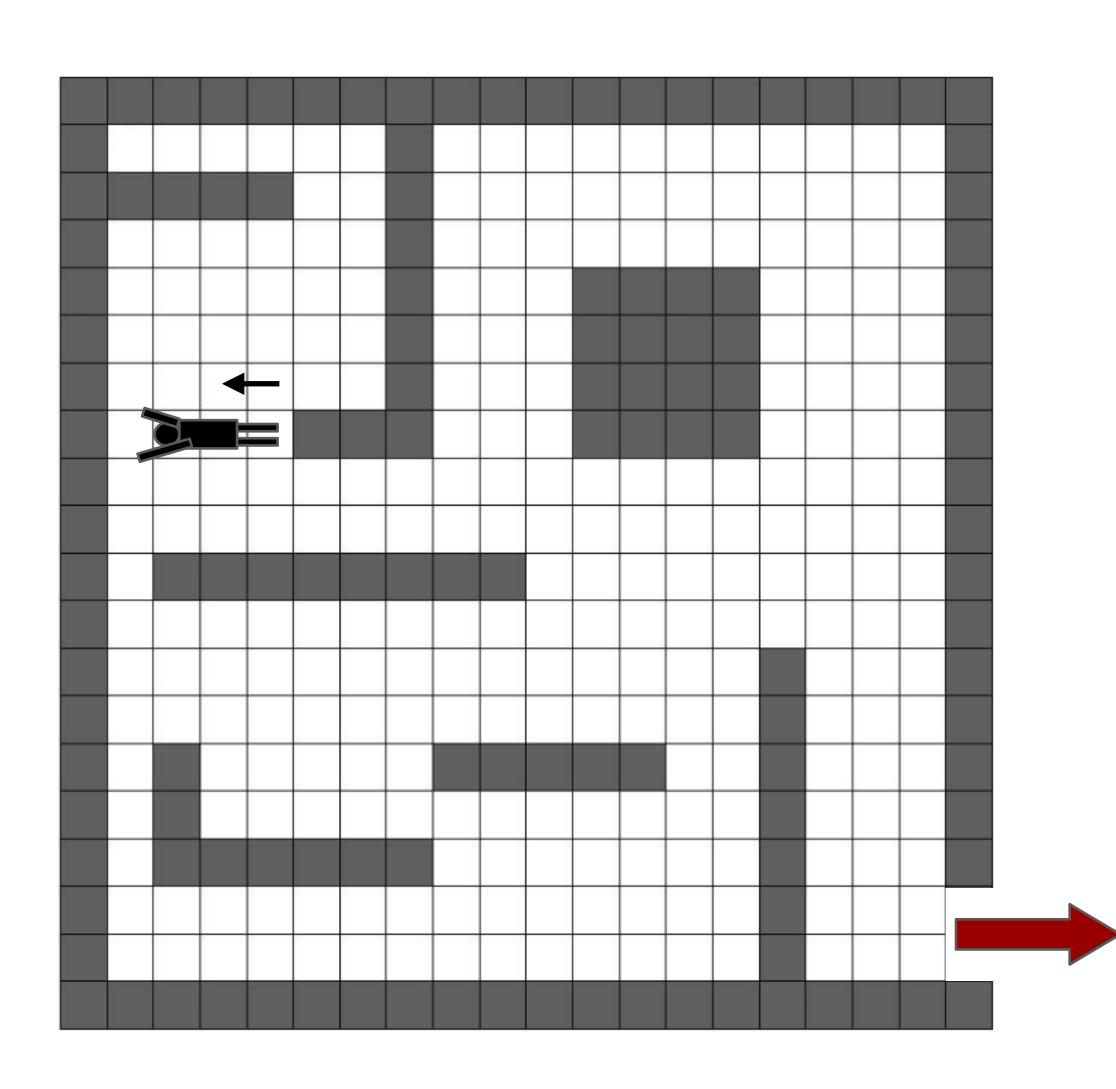








- Robot moving in a maze
- Goal: find the exit

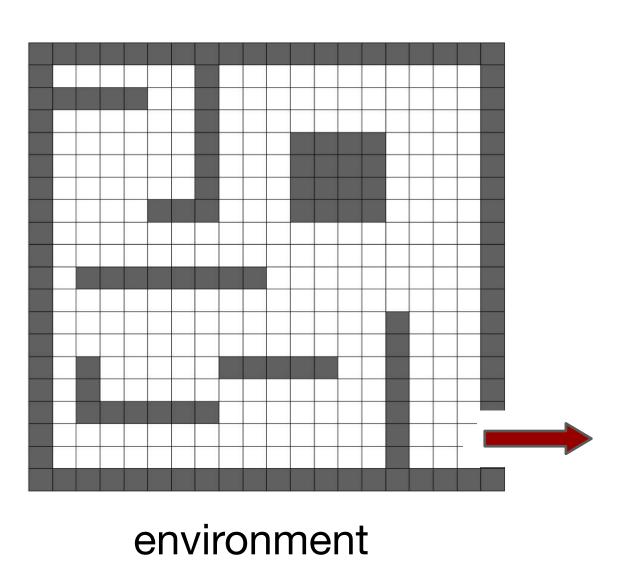


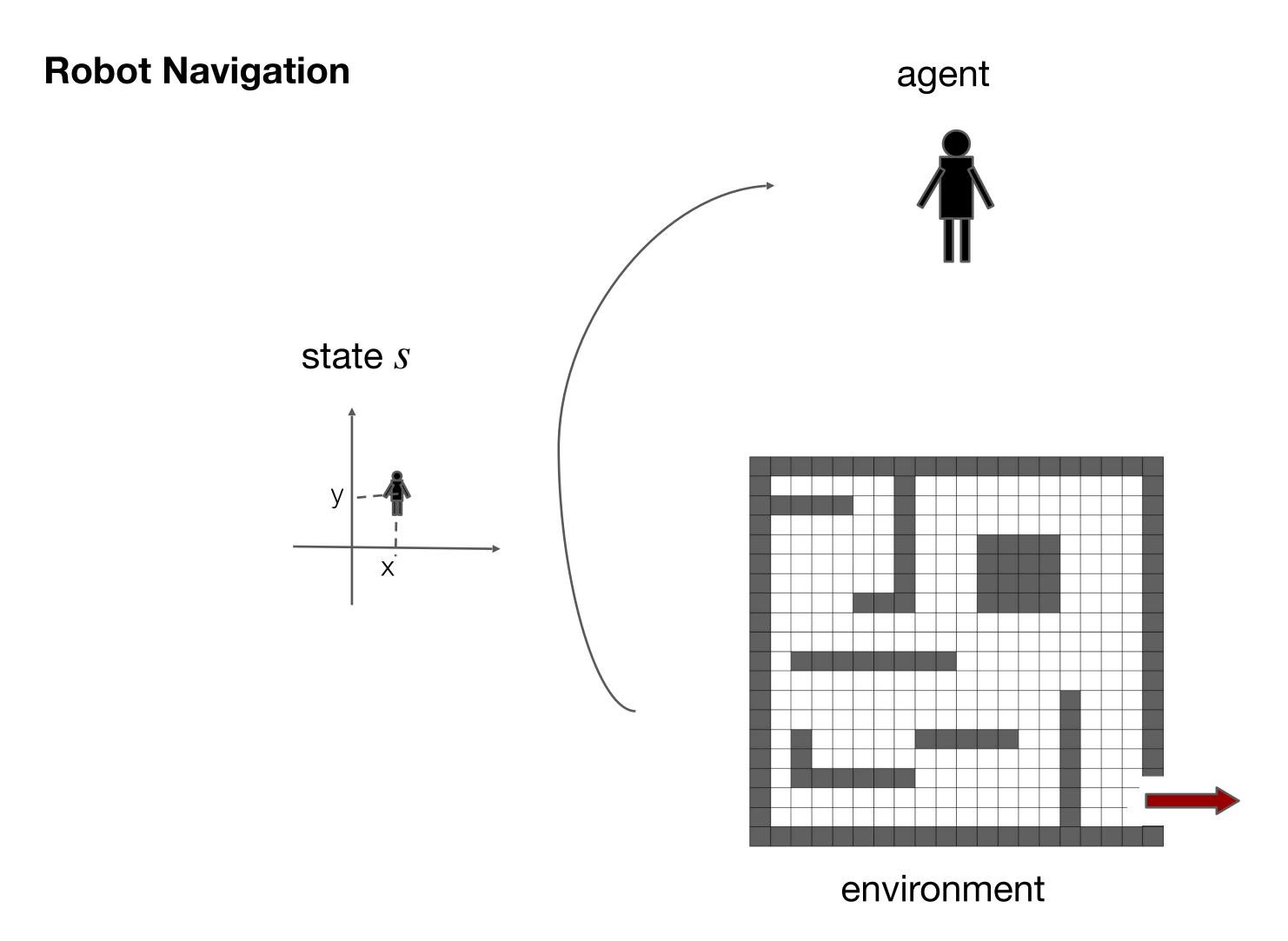
- Robot moving in a maze
- Goal: find the exit
- Chance of failing

**Robot Navigation** 

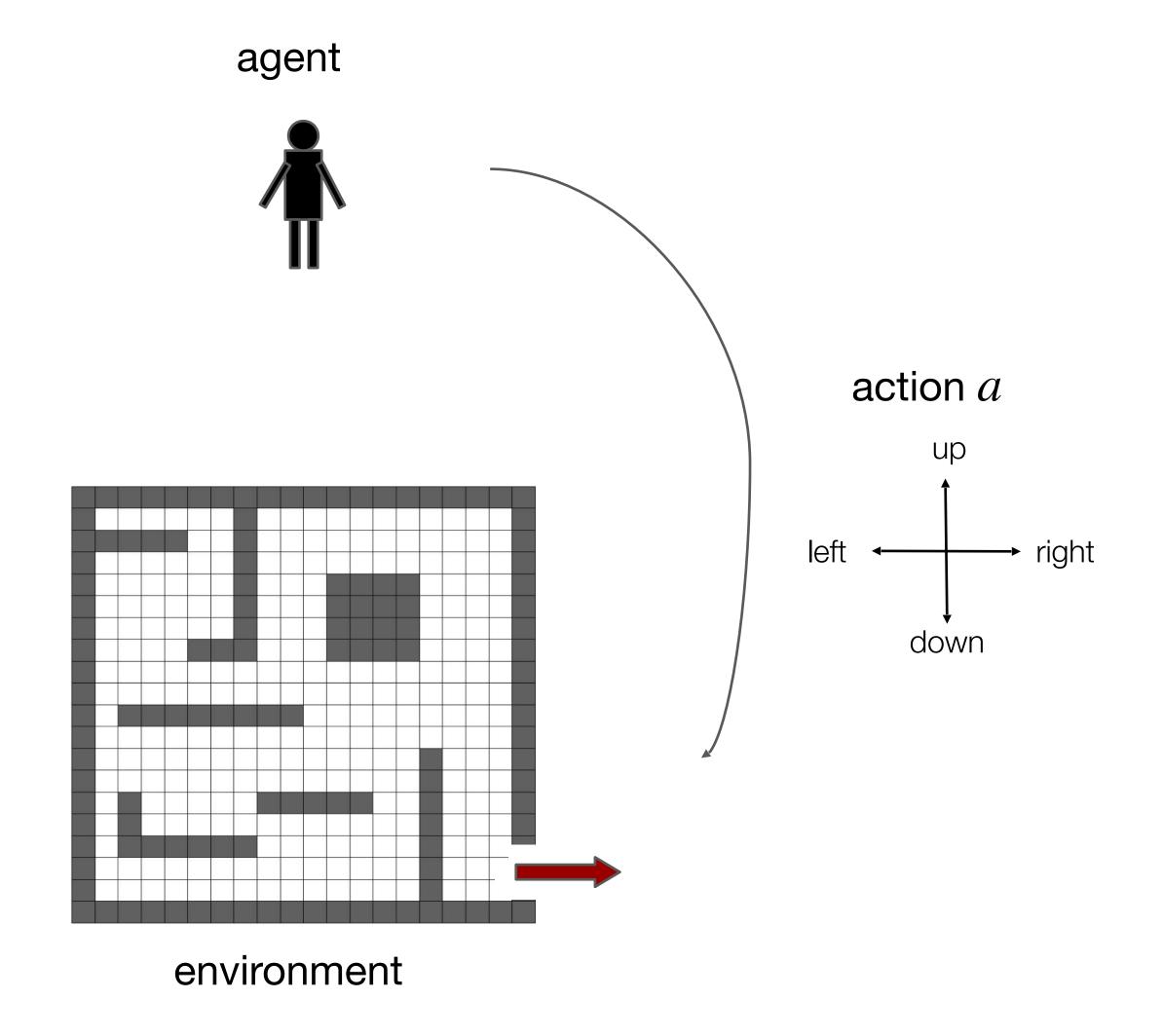
agent



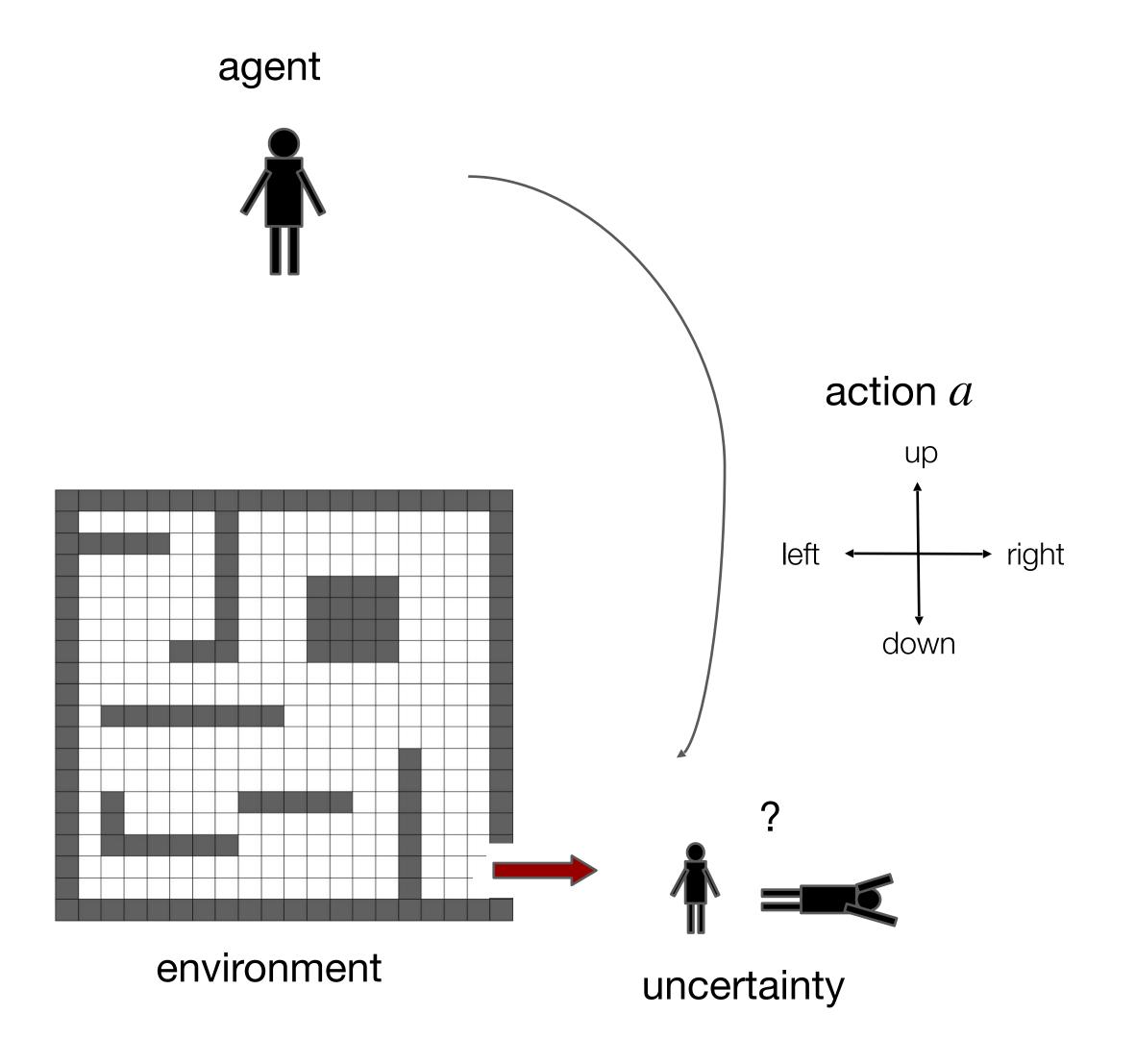


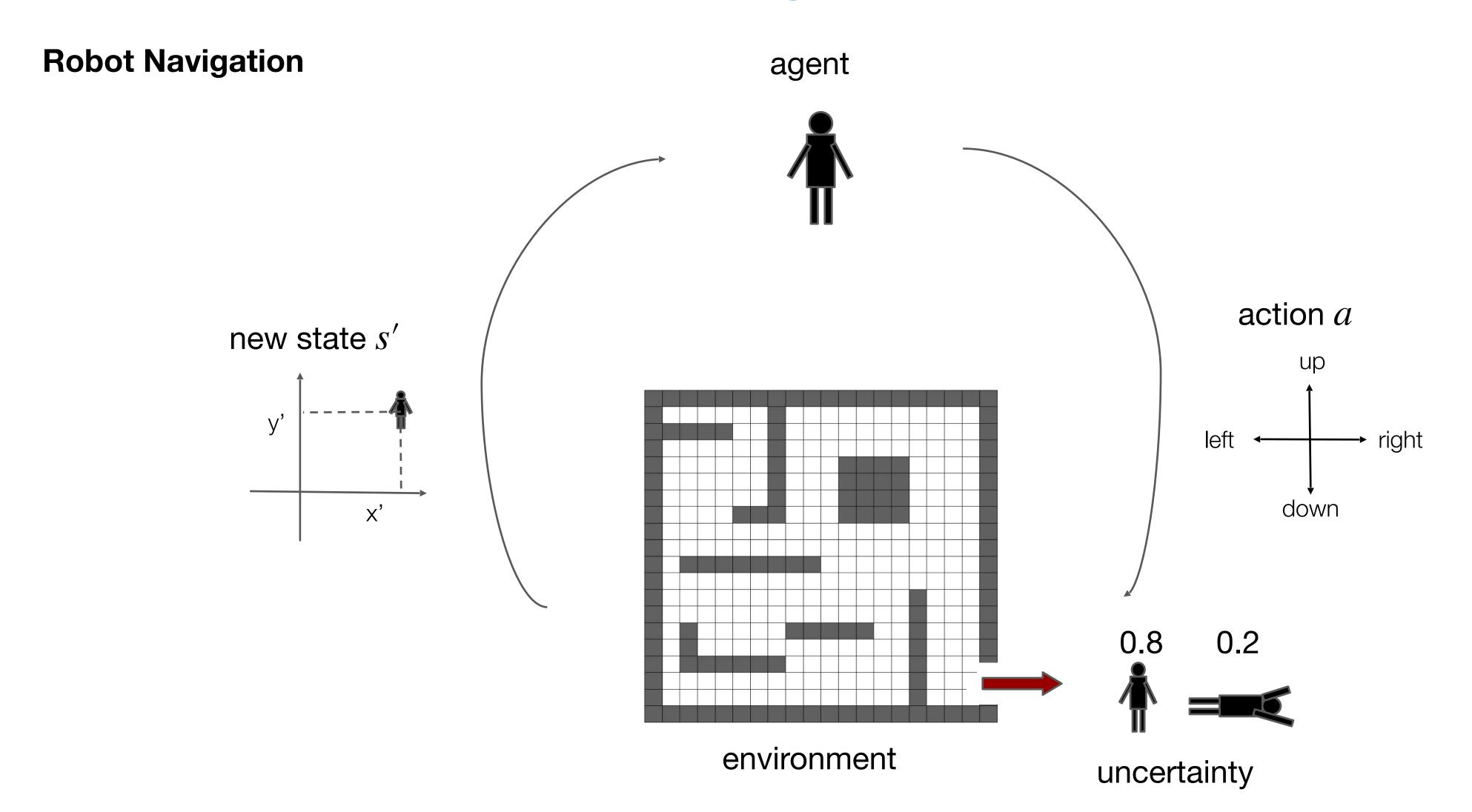


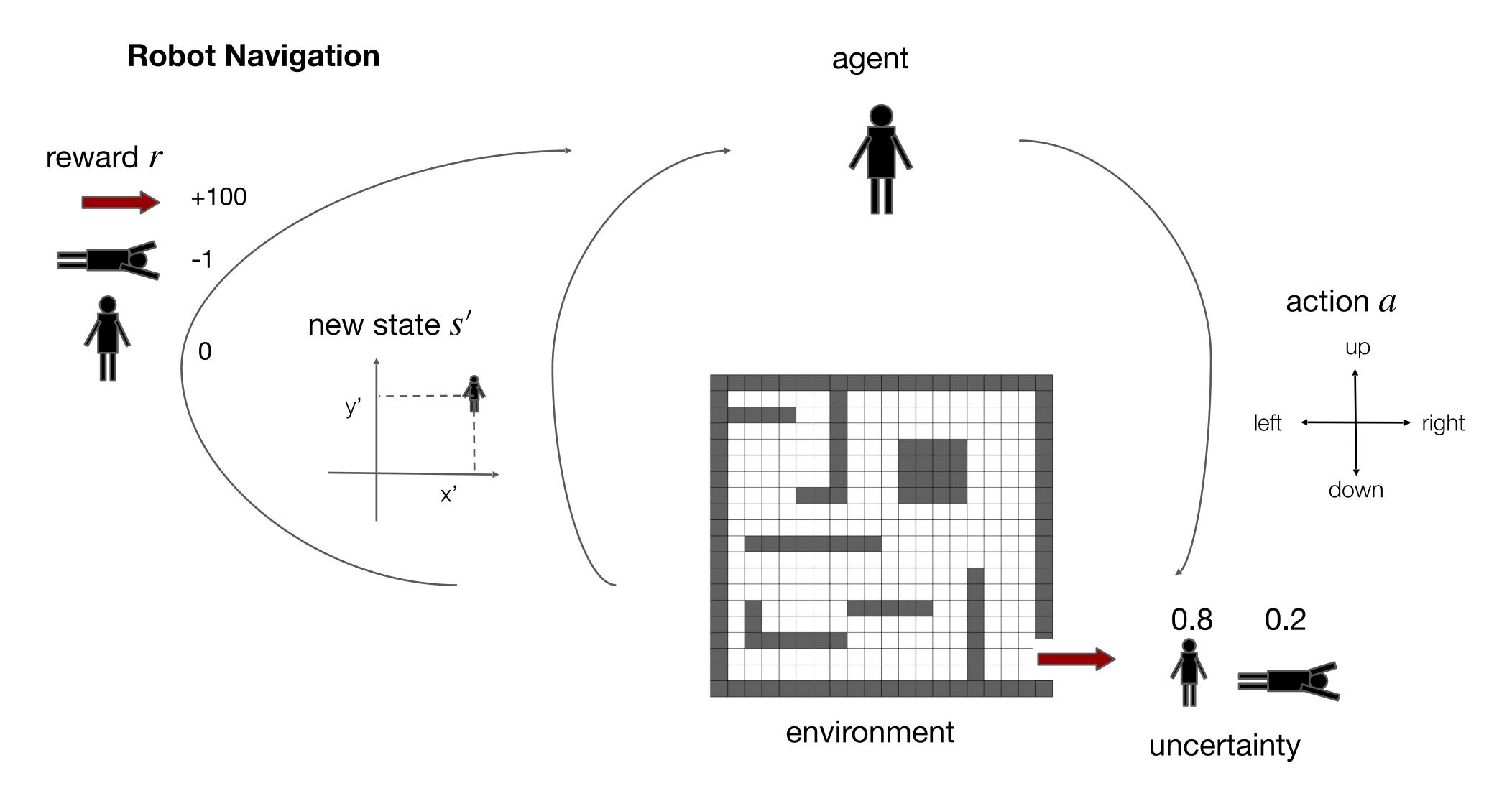
**Robot Navigation** 

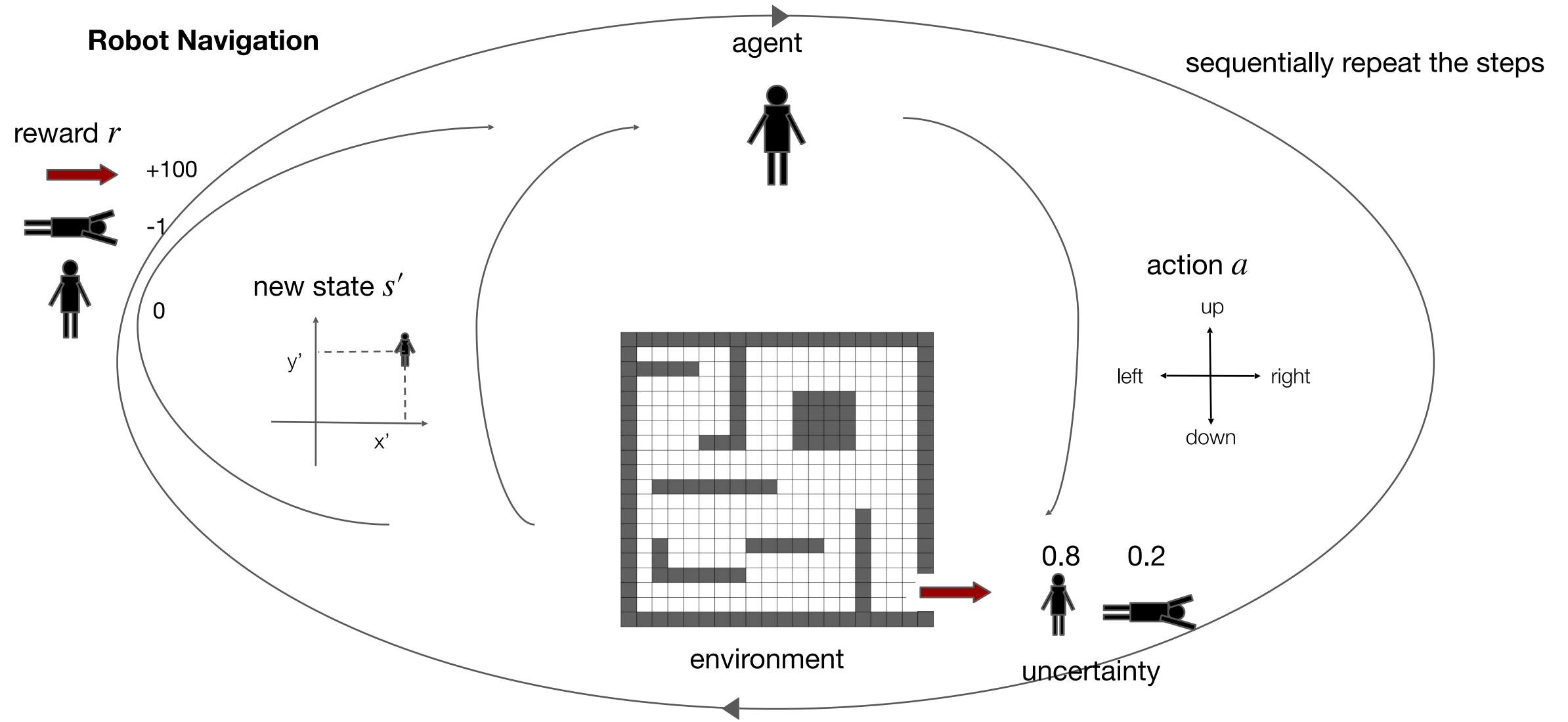


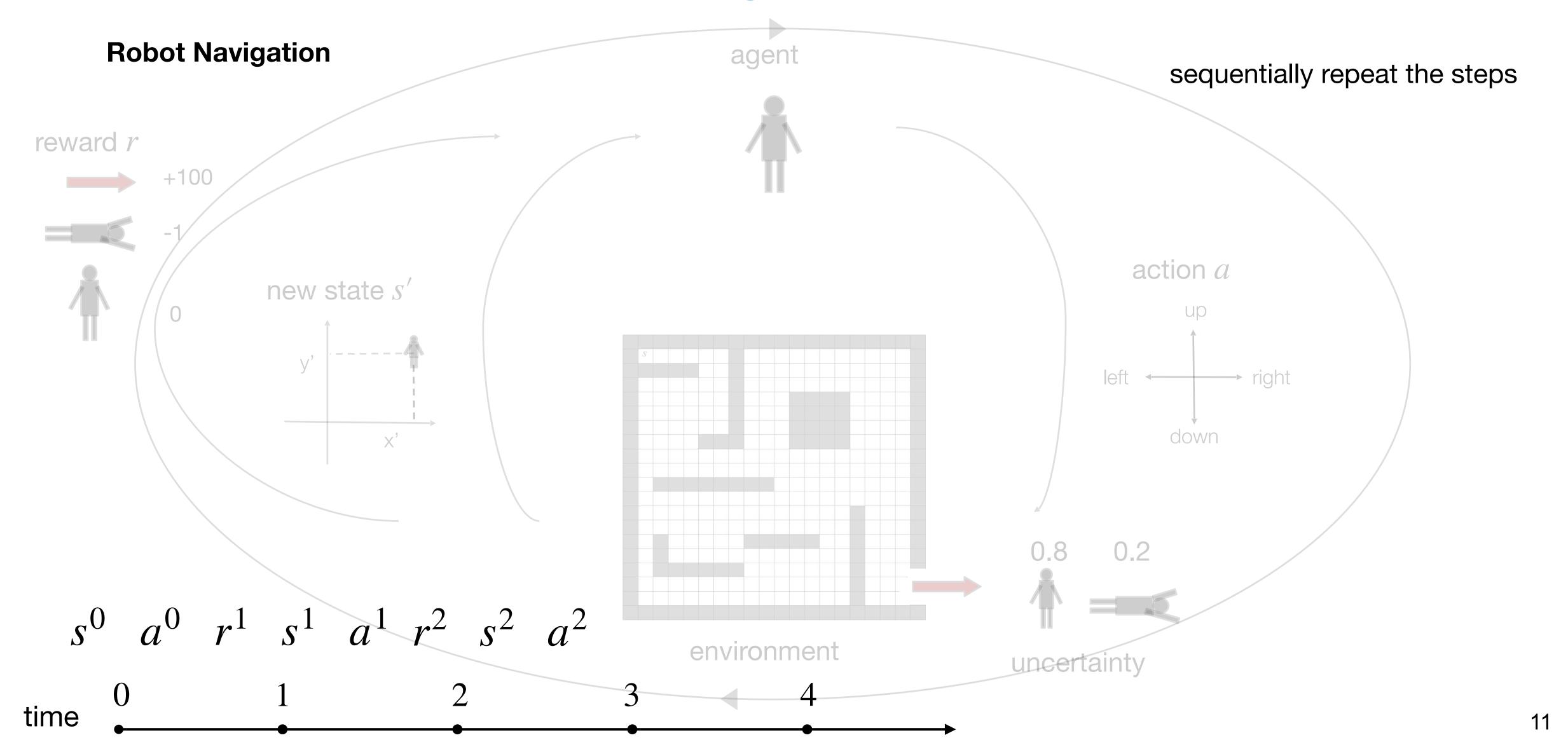
**Robot Navigation** 

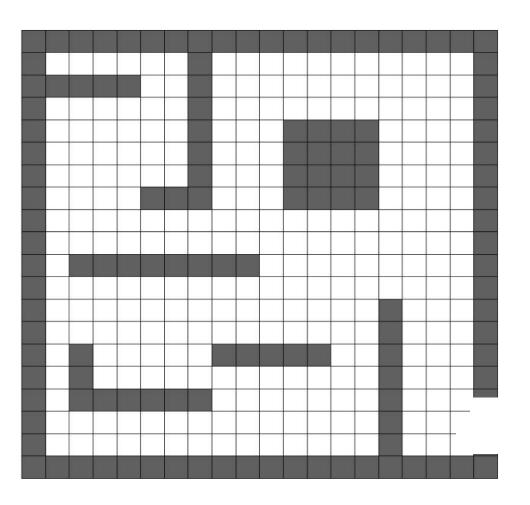




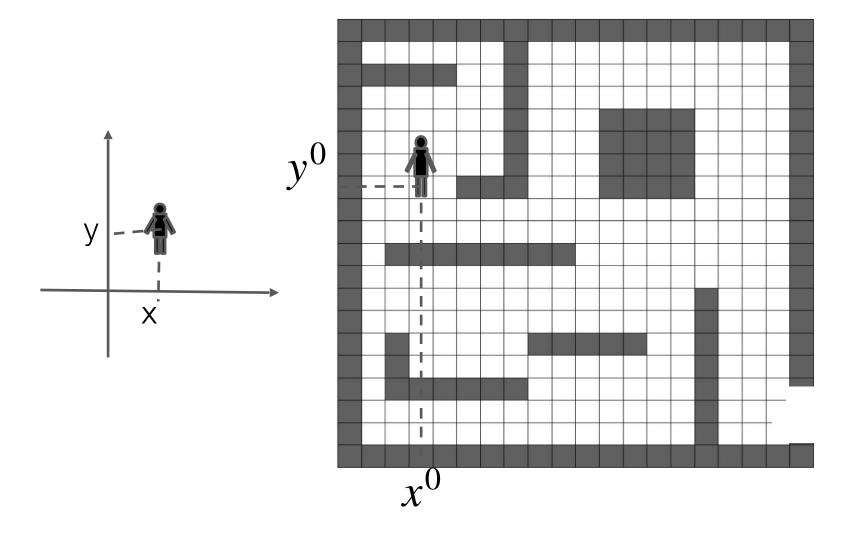








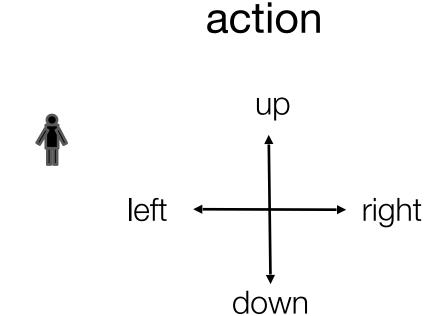
- State space S of the environment
- Initial state  $s^0 \in S$



$$S = \{ \text{feasible } (x, y) \}$$

$$s^0 = (x^0, y^0)$$
 initial robot coordinates

- State space S of the environment
- Initial state  $s^0 \in S$
- ullet Action space A of possible agent actions

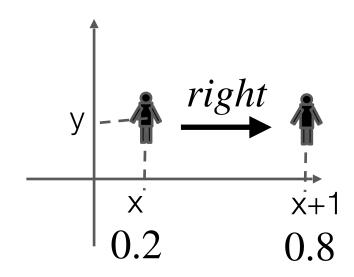


$$A = \{right, down, left, up\}$$

Markov decision process (MDP)  $M = (S, A, s^0, P, R)$ 

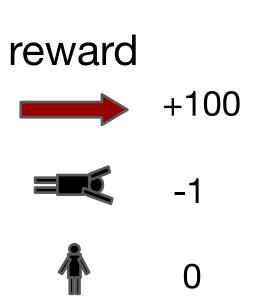
- State space S of the environment
- Initial state  $s^0 \in S$
- ullet Action space A of possible agent actions
- Transition probabilities P(s'|s,a) from state s to s' after choosing a

#### **Transitions**



$$P(s'|a = right, s = (x, y)) = \begin{cases} 0.8 & if \ s' = (x + 1, y) \\ 0.2 & if \ s' = (x, y) \\ 0 & otherwise \end{cases}$$

- State space S of the environment
- Initial state  $s^0 \in S$
- ullet Action space A of possible agent actions
- Transition probabilities P(s'|s,a) from state s to s' after choosing a
- Reward R(s', a, s) or R(s, a) for resulting in state s' from state s after choosing action a



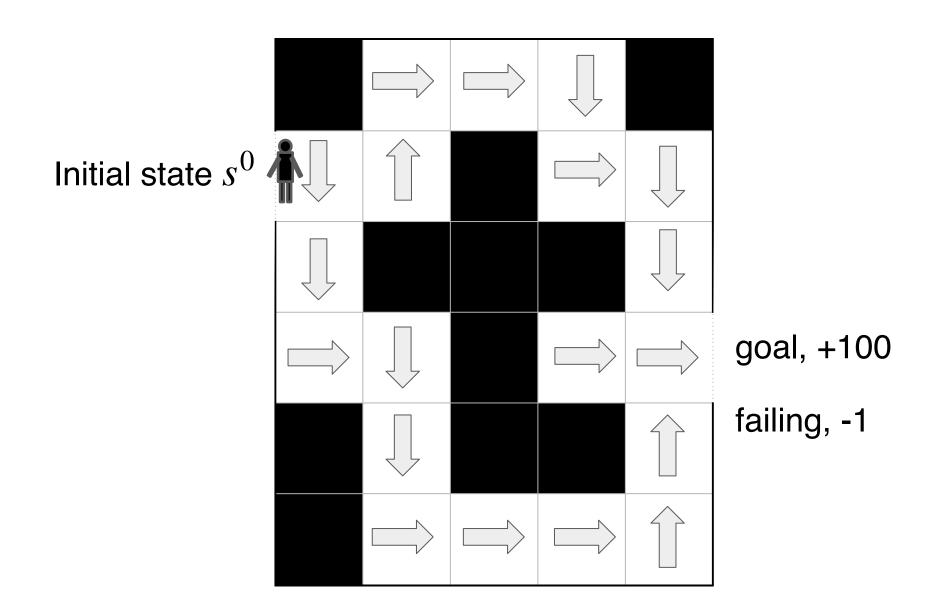
$$R(s', a, s) = \begin{cases} 100 & if \ s' = s_{goal} \\ -1 & if \ s = s' \\ 0 & otherwise \end{cases}$$

## Policy

- A policy  $\pi$  encodes the agent's behaviour
- A map from states to actions,  $\pi(s) = a$  is the agent's action when the environment state is s

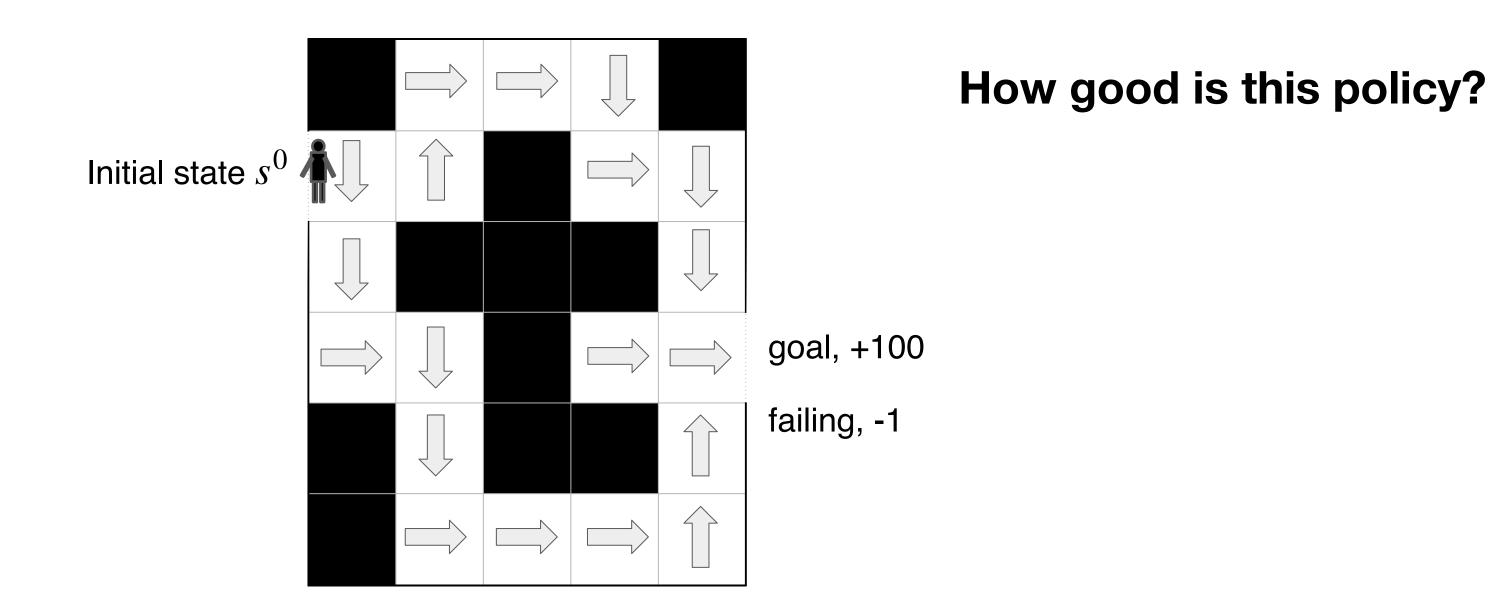
## Policy

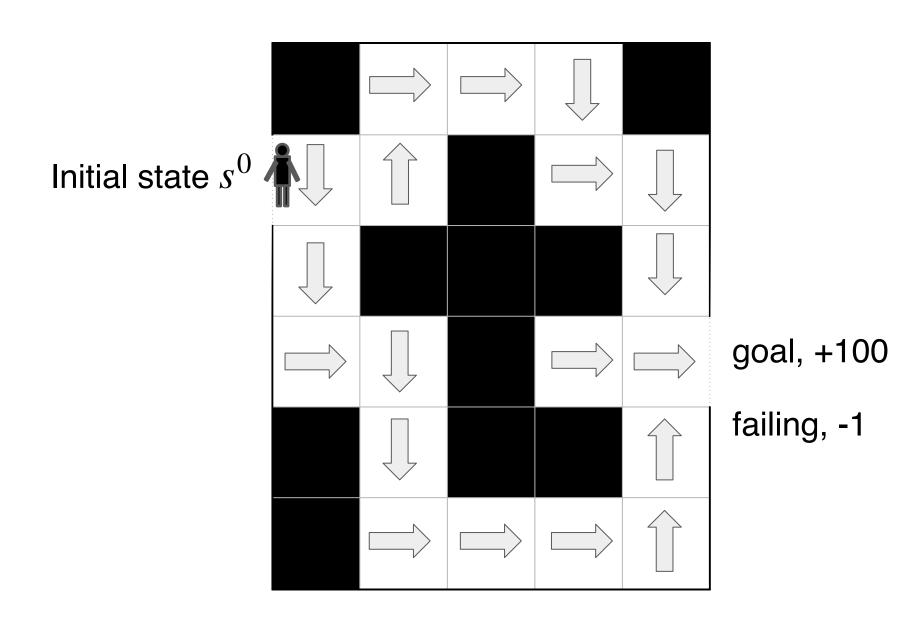
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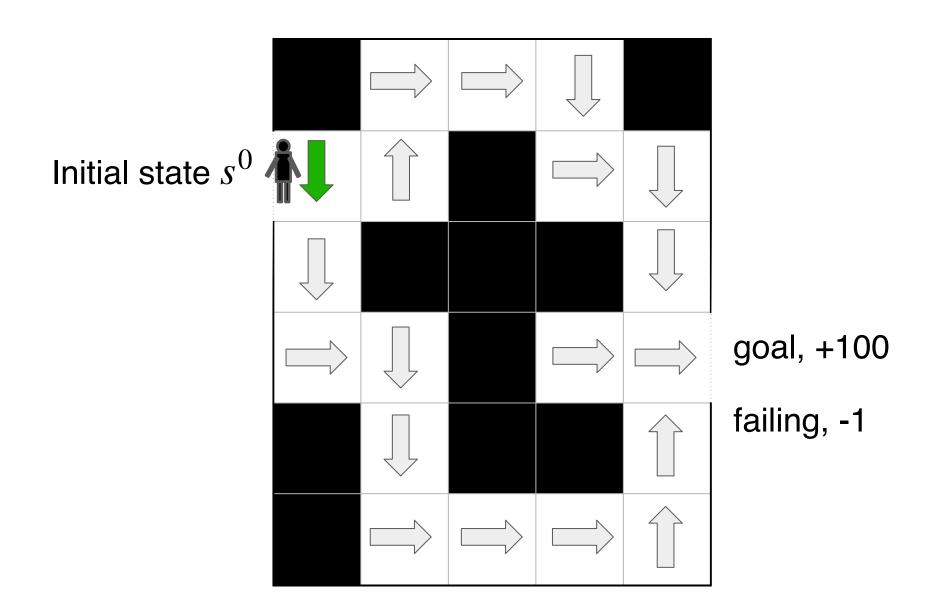
## Policy

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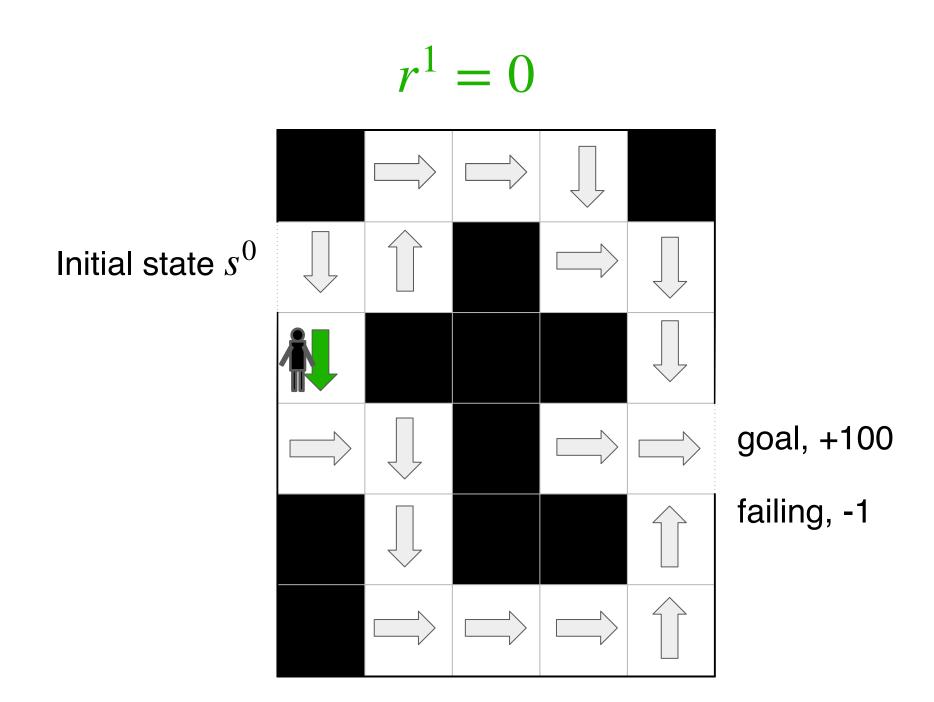




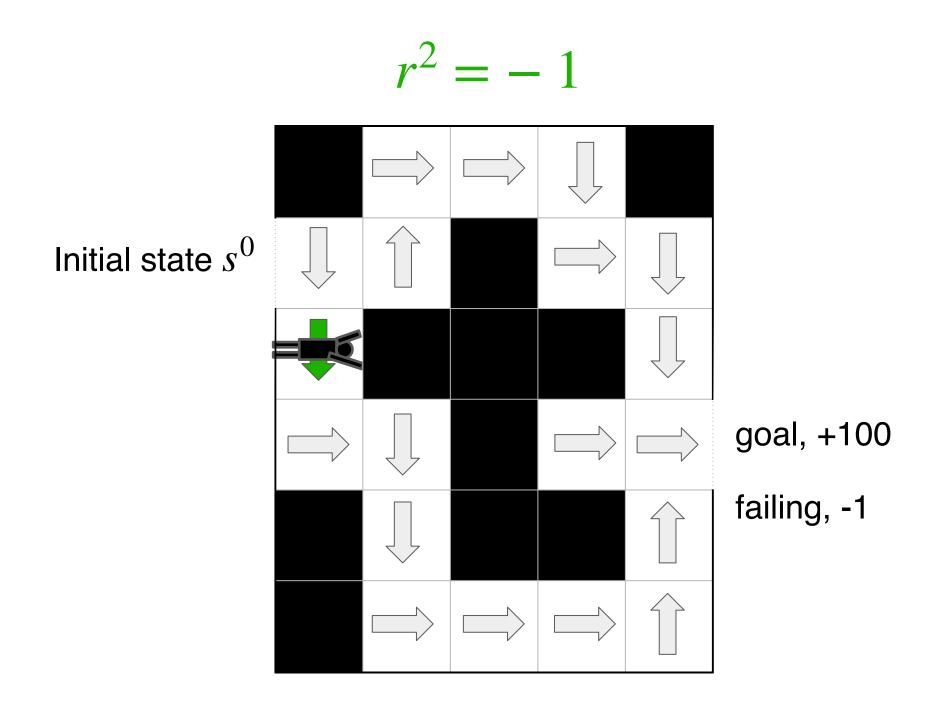
$$r^1$$
,  $r^2$ ,  $r^3$  ...



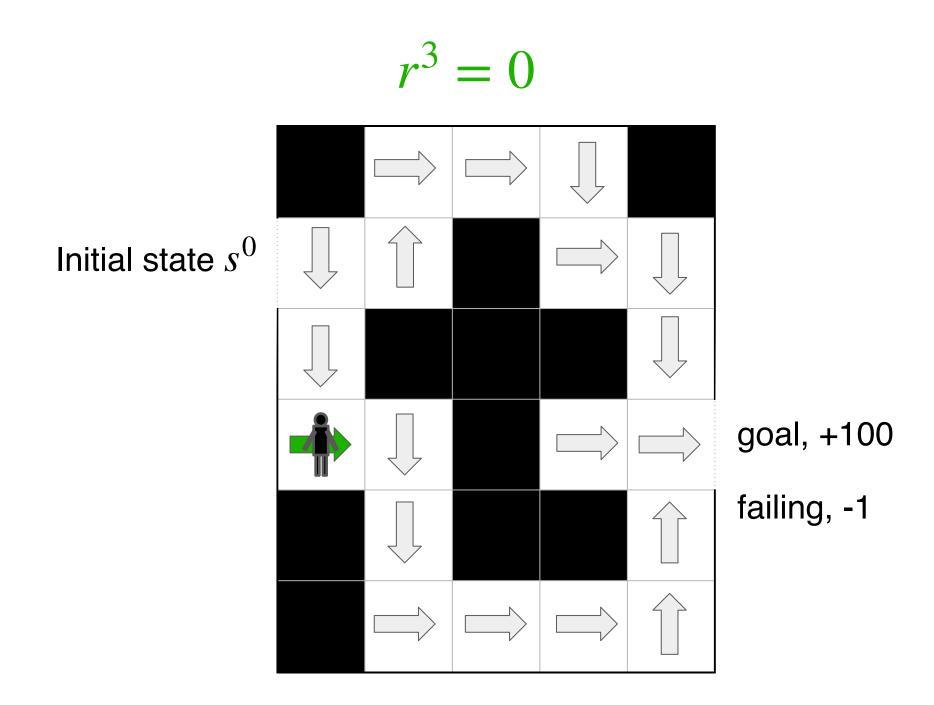
$$0, r^2, r^3 \dots$$



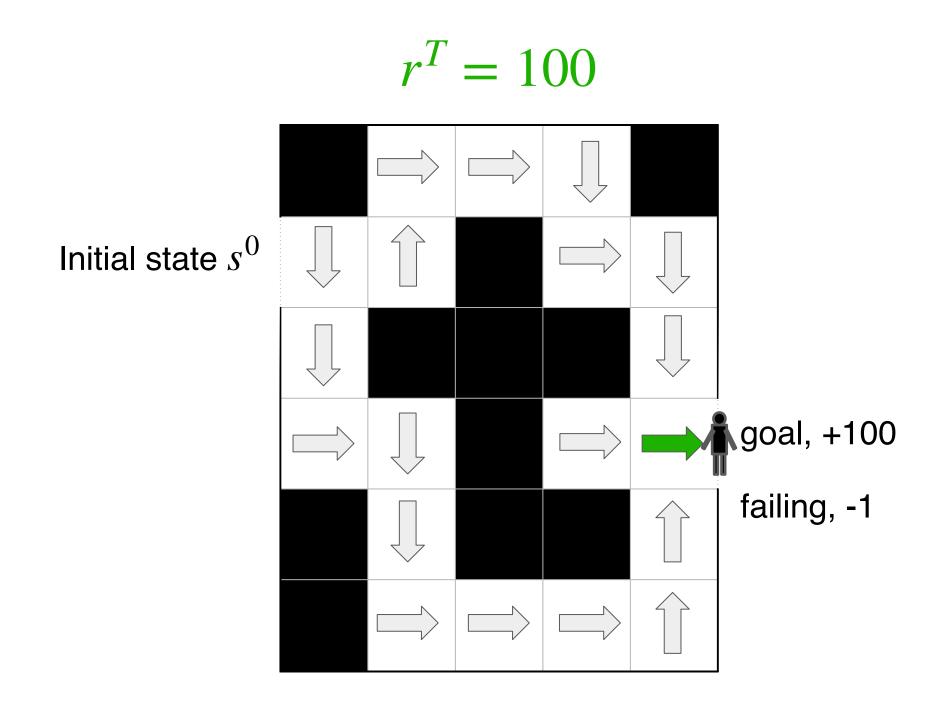
$$0, -1, r^3 \dots$$



$$0, -1, 0 \dots$$



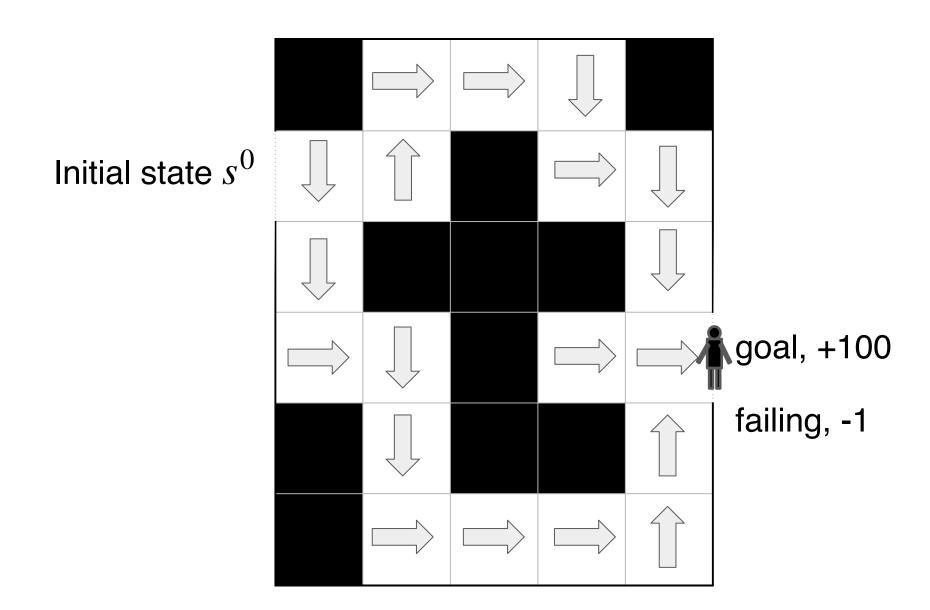
$$0, -1, 0 \dots 100$$



 $V^{\pi}(s)$  is the expected **sum of discounted** future rewards for employing a policy  $\pi$  starting from an initial state s

$$\gamma^{1}0 + \gamma^{2}(-1) + \gamma^{3}0 + ... \gamma^{T}(100)$$

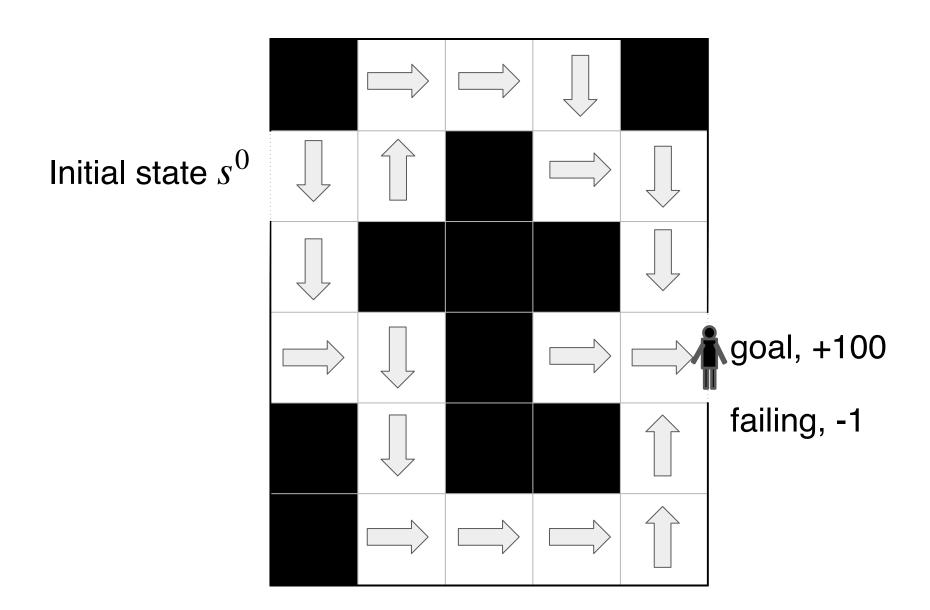
 $0 < \gamma < 1$  discount factor



 $V^{\pi}(s)$  is the expected **sum of discounted** future rewards for employing a policy  $\pi$  starting from an initial state s

$$\gamma^{1}0 + \gamma^{2}(-1) + \gamma^{3}0 + \dots \gamma^{T}(100) = \sum_{t} \gamma^{t} r^{t}$$

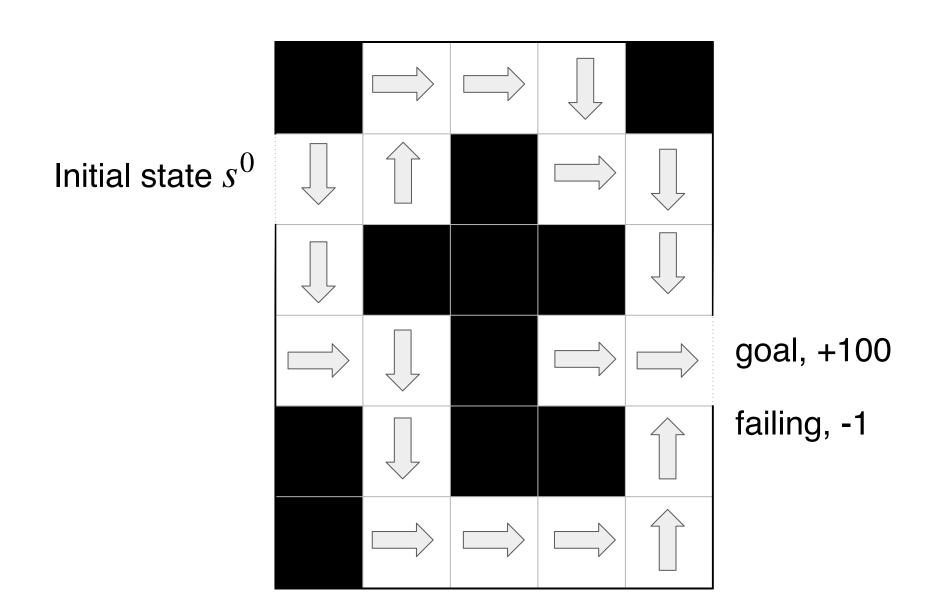
 $0 < \gamma < 1$  discount factor



 $V^{\pi}(s)$  is the **expected** sum of discounted future rewards for employing a policy  $\pi$  starting from an initial state s

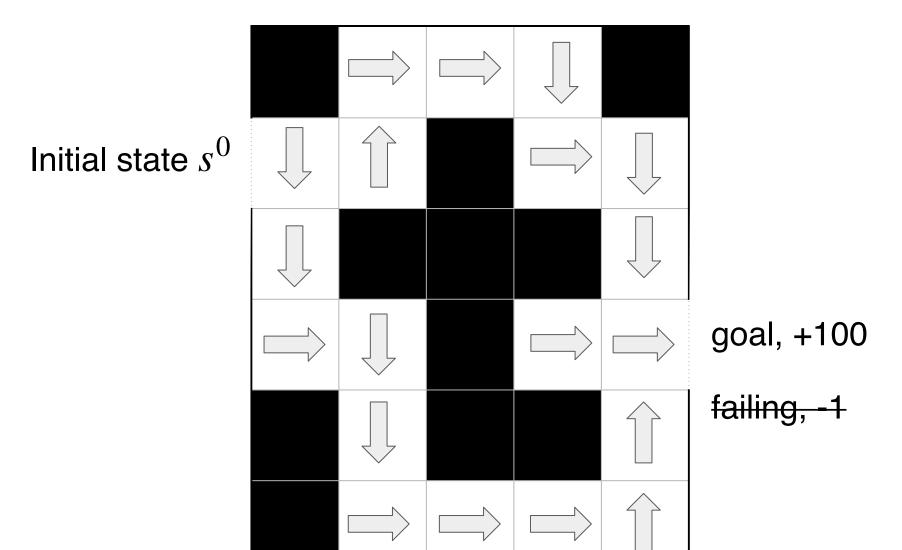
$$V^{\pi}(s) = \mathbb{E}\left[\sum_{t} \gamma^{t} r^{t} | s^{0} = s, \pi\right]$$

Average over all possible futures



 $V^\pi(s)$  is the expected sum of discounted future rewards for employing a policy  $\pi$  starting from an initial state s

$$V^{\pi}(s) = \mathbb{E}\left[\sum_{t} \gamma^{t} r^{t} | s^{0} = s, \pi\right]$$

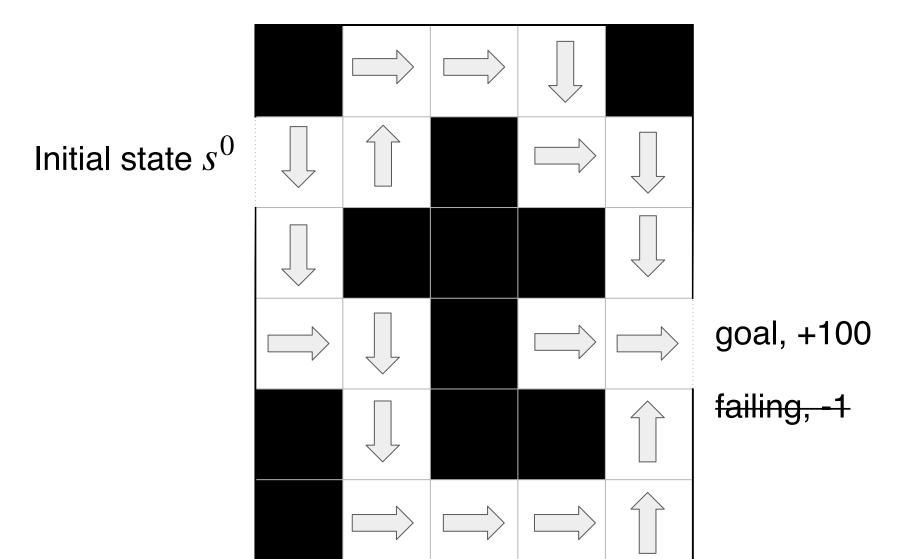


#### How good is this policy?

$$V^{\pi}(s^0) = ?$$

 $V^\pi(s)$  is the expected sum of discounted future rewards for employing a policy  $\pi$  starting from an initial state s

$$V^{\pi}(s) = \mathbb{E}\left[\sum_{t} \gamma^{t} r^{t} \mid s^{0} = s, \pi\right]$$

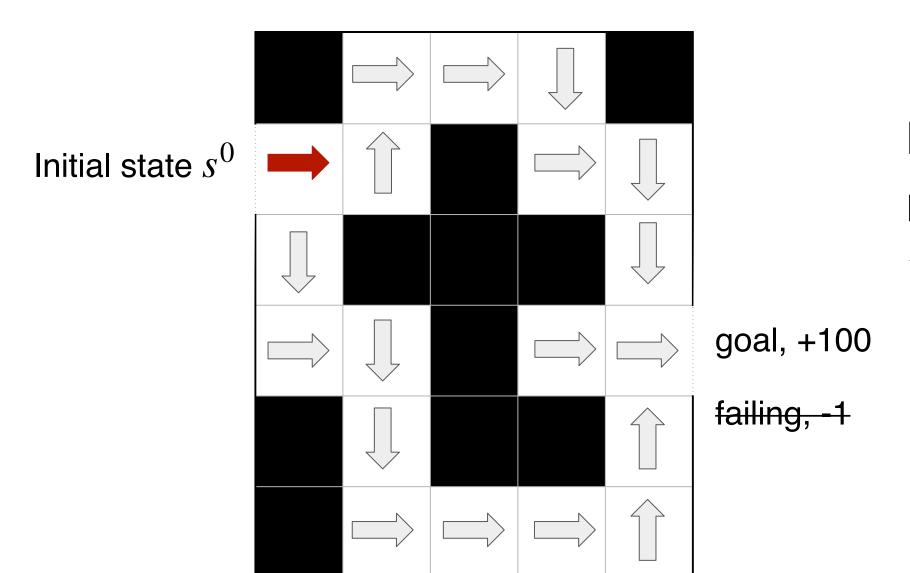


#### How good is this policy?

$$V^{\pi}(s^0) = 0 + 0 + \dots + 0 + \gamma^{11}100 \approx 31$$

 $V^\pi(s)$  is the expected sum of discounted future rewards for employing a policy  $\pi$  starting from an initial state s

$$V^{\pi}(s) = \mathbb{E}\left[\sum_{t} \gamma^{t} r^{t} \mid s^{0} = s, \pi\right]$$

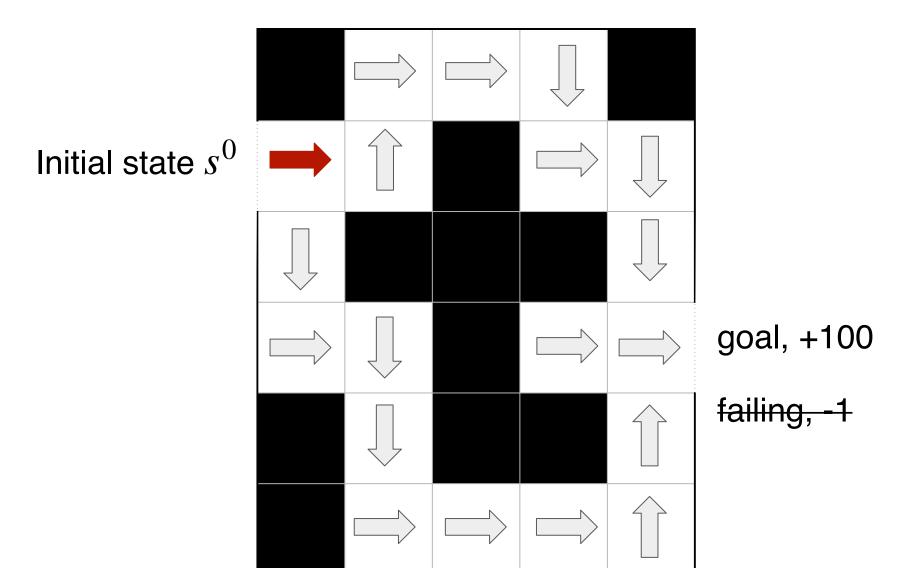


#### How good is this policy?

$$V^{\pi}(s^0) = \gamma^{11}100 \approx 31$$
  $V^{\pi}(s^0) = ?$ 

 $V^{\pi}(s)$  is the expected sum of discounted future rewards for employing a policy  $\pi$  starting from an initial state s

$$V^{\pi}(s) = \mathbb{E}\left[\sum_{t} \gamma^{t} r^{t} \mid s^{0} = s, \pi\right]$$

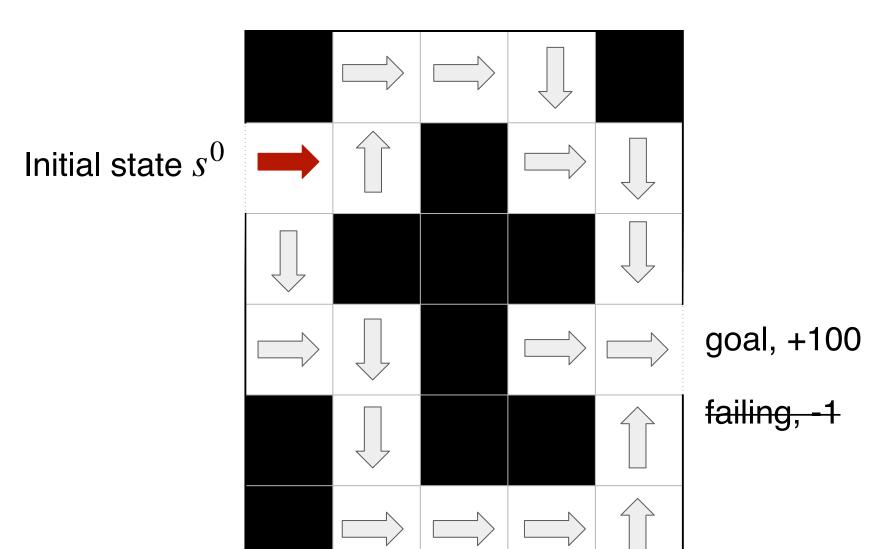


#### How good is this policy?

$$V^{\pi}(s^0) = \gamma^{11}100 \approx 31$$
  $V^{\pi}(s^0) = \gamma^9100 \approx 39$ 

 $V^\pi(s)$  is the expected sum of discounted future rewards for employing a policy  $\pi$  starting from an initial state s

$$V^{\pi}(s) = \mathbb{E}\left[\sum_{t} \gamma^{t} r^{t} \mid s^{0} = s, \pi\right]$$



#### How good is this policy?

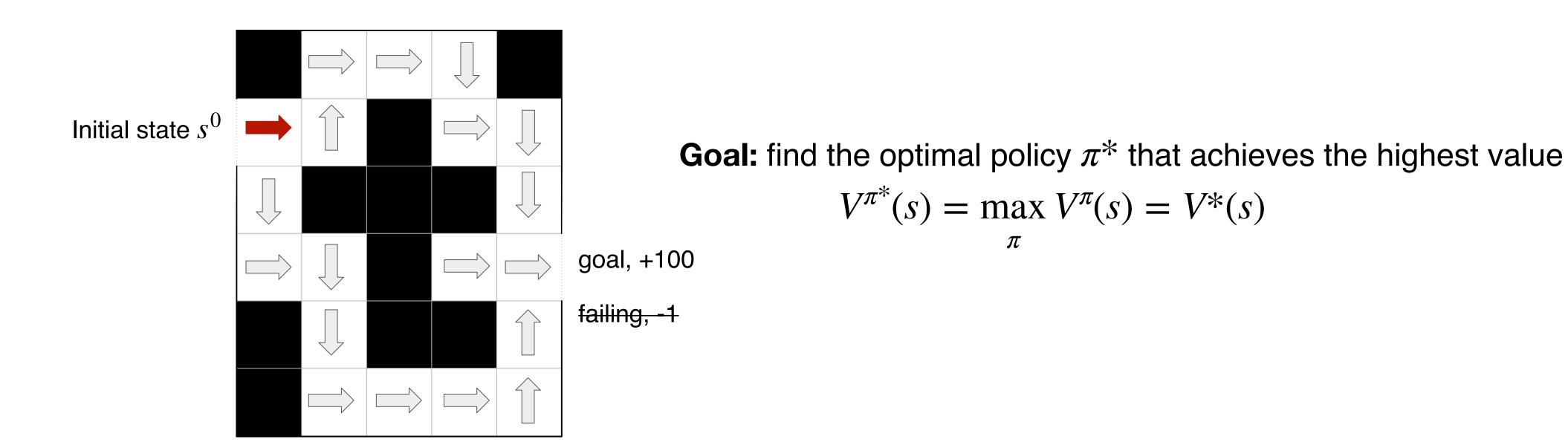
Deterministic transitions - no chance of failing,  $\gamma = 0.9$ 

$$V^{\pi}(s^0) = \gamma^{11}100 \approx 31 < V^{\pi}(s^0) = \gamma^9100 \approx 39$$

Value measures the quality of a policy: best policy gives the highest value

 $V^\pi(s)$  is the expected sum of discounted future rewards for employing a policy  $\pi$  starting from an initial state s

$$V^{\pi}(s) = \mathbb{E}\left[\sum_{t} \gamma^{t} r^{t} | s^{0} = s, \pi\right]$$



Value measures the quality of a policy: best policy gives the highest value

## Quiz Show

- 1. Go to brightspace → 9. Reinforcement Learning → Quiz Show
- 2. Read the description and play the quiz show
- 3. Do not compute anything just follow your intuition
- 4. 3 mins then discussion



## Quiz Show: discussion

#### How do you formulate this game as a Markov decision process?

- 1. Who is the agent?
- 2. What are the states and initial state?
- 3. What are the actions?
- 4. What is the reward?



### Quiz Show: discussion

#### How do you formulate this game as a Markov decision process?

- 1. Who is the agent?
- 1. You, the player
- 2. What are the states and initial state?

2. 
$$S = \{Q_1, Q_2, Q_3, Q_4, home\}$$
 and  $s^0 = Q_1$ 

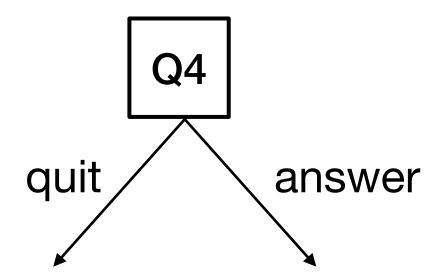
- 3. What are the actions?
- 3.  $A = \{quit, answer\}$
- 4. What is the reward?

4. Reward 
$$R(home, ...) = 0$$
\$,  $R(Q_2, quit) = 100$ \$,  $R(Q_3, quit) = 1100$ \$ ...,



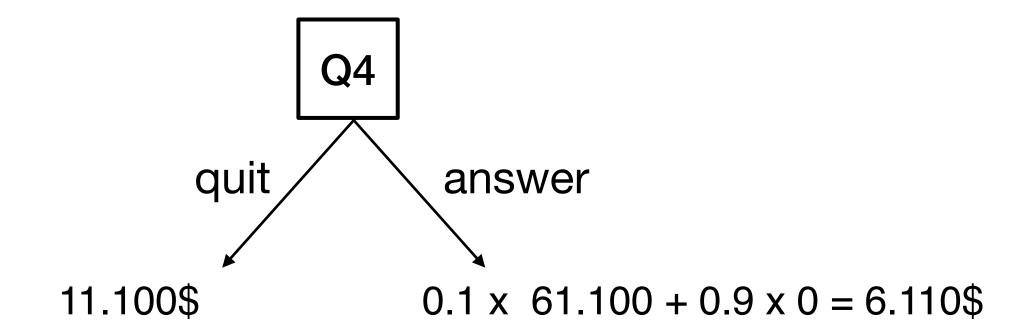
Optimal policy 
$$\begin{cases} \pi^*(Q_4) &= quit \\ \pi^*(Q) &= answer \quad \text{for } Q \neq Q_4 \end{cases}$$
 with optimal value  $V^*(Q_1) = 3746$ 

Assume we are at question 4, what is the optimal action?



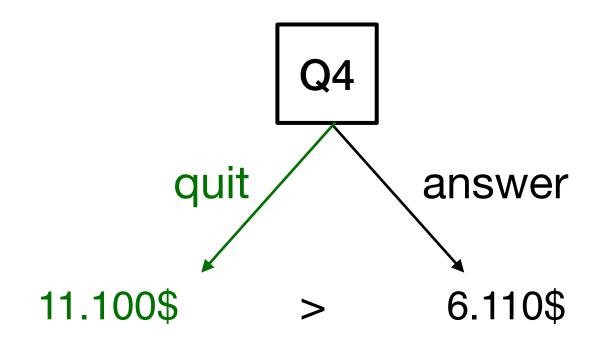


Assume we are at question 4, what is the optimal action?





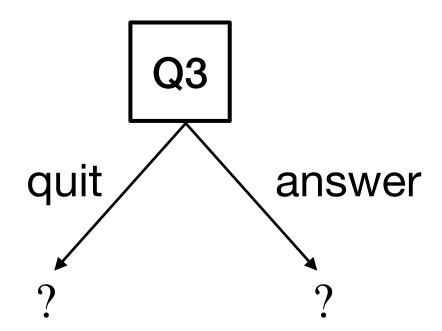
Assume we are at question 4, what is the optimal action?





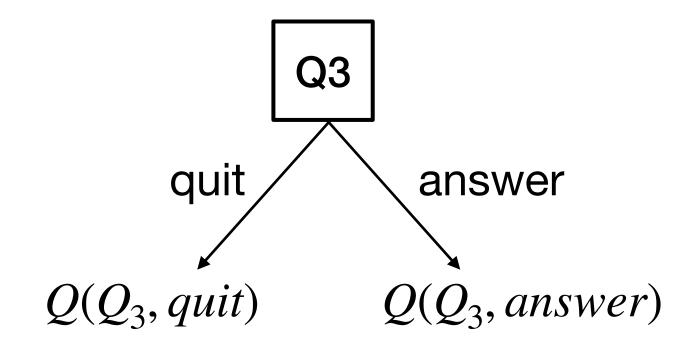
$$\pi^*(Q_4) = quit$$
  $V^*(Q_4) = 11.100$ \$

Assume we are at question 3, what is the optimal action?



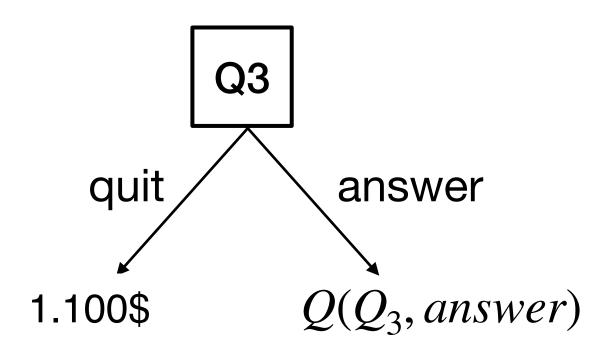


• Expected value for executing an action in a given state assuming that you will act optimally in the future

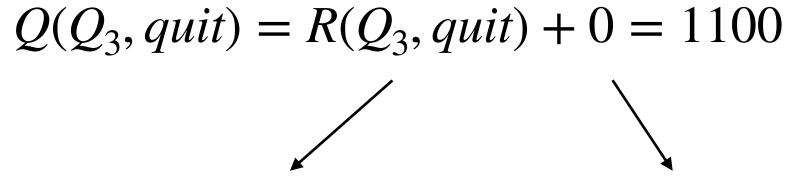




• Expected value for executing an action in a given state assuming that you will act optimally in the future



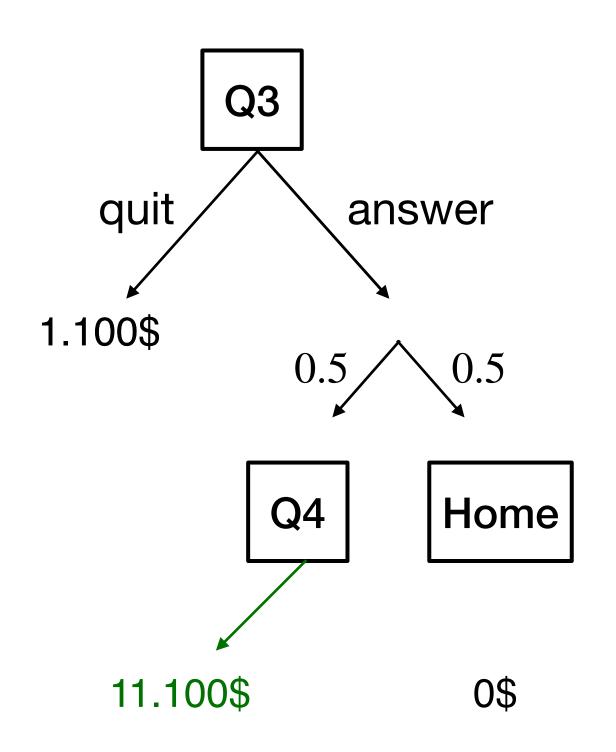




Immediate reward for quitting at question 3

There's no future state as your game is terminated

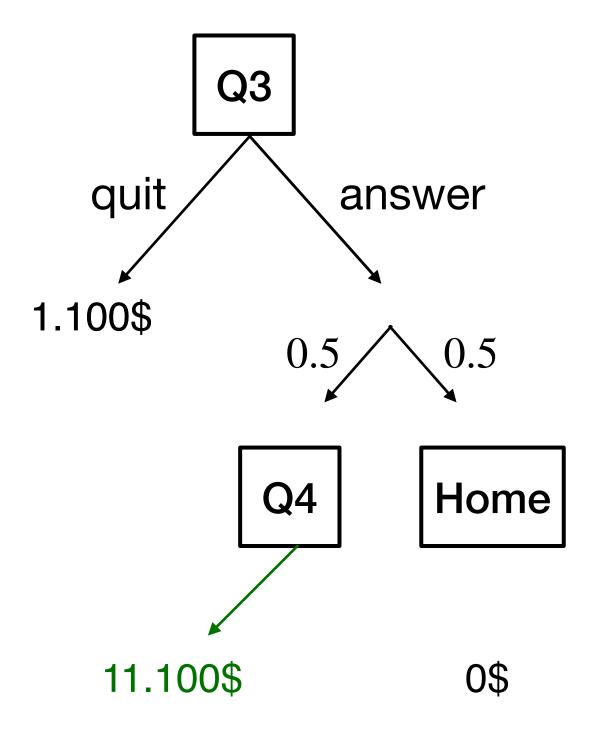
• Expected value for executing an action in a given state assuming that you will act optimally in the future





 $Q(Q_3, answer) =$ 

• Expected value for executing an action in a given state assuming that you will act optimally in the future





$$Q(Q_3, answer) = 0 + (0.5)$$

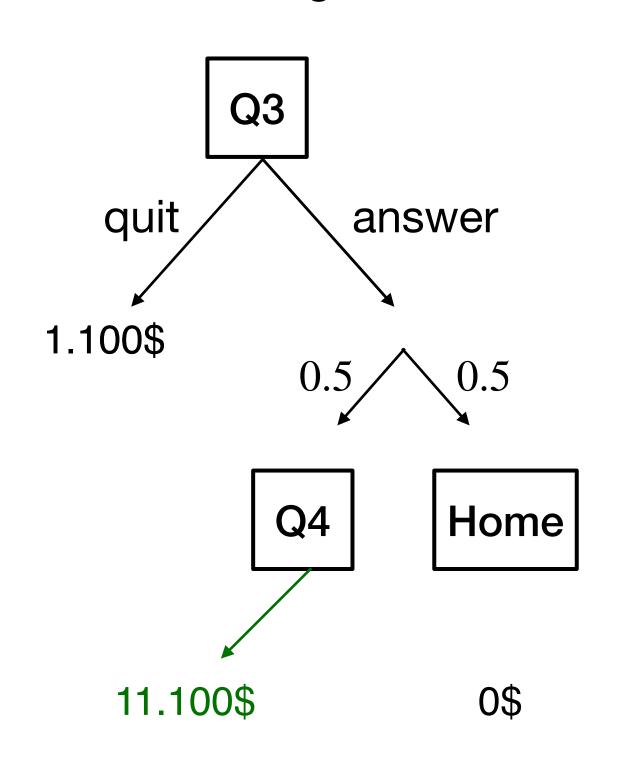
 $\times 11.100 +$ 

0.5

) = 5.550

Immediate reward for answering at question 3 Expected value for acting optimally in the future

• Expected value for executing an action in a given state assuming that you will act optimally in the future



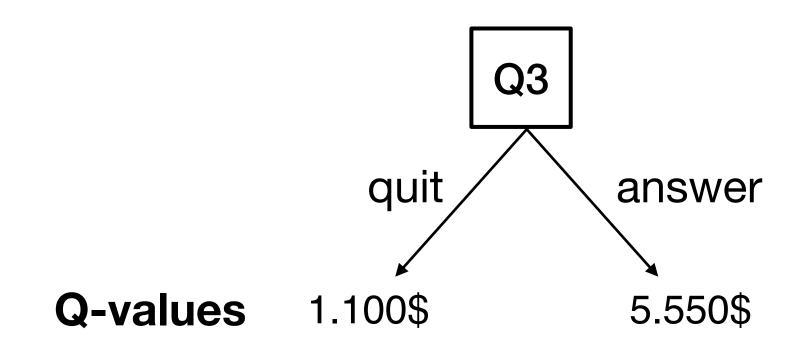


$$Q(Q_3, answer) = 0 + (0.5 \times 11.100 + 0.5 \times 0) = 5.550$$
  
 $R(Q_3, answer) + (P(Q_4 | answer, Q_3) \times V^*(Q_4) + P(home | answer, Q_3) \times V^*(home))$ 

Immediate reward for answering at question 3

Expected value for acting optimally in the future

• Expected value for executing an action in a given state assuming that you will act optimally in the future





$$Q(Q_3, answer) = 0 + (0.5 \times 11.100 + 0.5 \times 0) = 5.550$$

$$R(Q_3, answer) + (P(Q_4 \mid answer, Q_3) \times V^*(Q_4) + P(home \mid answer, Q_3) \times V^*(home))$$

Immediate reward for answering at question 3

Expected value for acting optimally in the future

• Expected value for executing an action in a given state assuming that you will act optimally in the future

$$Q(s,a) = R(s,a) + \gamma \sum_{s'} P(s'|s,a)V^*(s')$$

Immediate reward Discounted expected value for acting optimally in the future

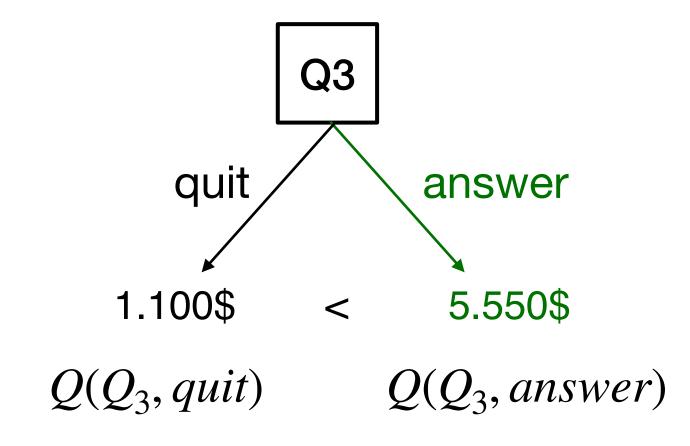


 $Q(Q_3, answer) = R(Q_3, answer) + P(Q_4 | answer, Q_3)V*(Q_4) + P(home | answer, Q_3)V*(home)$ 

• Expected value for executing an action in a given state assuming that you will act optimally in the future

$$Q(s, a) = R(s, a) + \gamma \sum_{s'} P(s' | s, a) V^*(s')$$

Select the optimal action as the one that maximizes the Q-value





$$\pi^*(Q_3) = \operatorname{argmax}_a Q(Q_3, a) = \operatorname{answer} V^*(Q_3) = \max_a Q(s, a) = 5500$$

• Compute approximation of  $V^*(s)$  for every state s

Compute Q-values for every state s and action a

$$Q(s, a) = R(s, a) + \gamma \sum_{s'} P(s'|s, a)V^*(s')$$

• Build the optimal policy as  $\pi^*(s) = argmax_a Q(s, a)$  that maximizes the Q-value

#### **Algorithm**

```
Initialize random values {\cal V}^0
                                   \Delta \longleftarrow 0
Repeat:
    For s \in S:
        For a \in A
            Q^{k}(s,a) \longleftarrow R(s,a) + \gamma \sum_{s'} P(s'|s,a) V^{k-1}(s')
        V^k(s) = \max Q^k(s, a)
        \Delta \longleftarrow \max(\Delta, |V^k(s) - V^{k-1}(s)|)
Until \Delta < \varepsilon
Return V^k \approx V^* \in \mathbb{R}^{|S|}
```

#### **Algorithm**

```
Initialize random values V^{
m 0}
Repeat:
    For s \in S:
        For a \in A
           Q^{k}(s,a) \longleftarrow R(s,a) + \gamma \sum_{s'} P(s'|s,a) V^{k-1}(s')
        V^k(s) = \max_{a} Q^k(s, a)
        \Delta \longleftarrow \max(\Delta, |V^k(s) - V^{k-1}(s)|)
Until \Delta < \varepsilon
Return V^k \approx V^* \in \mathbb{R}^{|S|}
```

Vector of values  $V \in \mathbb{R}^{|S|}$ , one value for each state

#### **Algorithm**

Initialize random values 
$$V^0$$
  $\Delta \longleftarrow 0$  Repeat: For  $s \in S$ : For  $a \in A$  
$$Q^k(s,a) \longleftarrow R(s,a) + \gamma \sum_{s'} P(s'|s,a) V^{k-1}(s')$$
 
$$V^k(s) = \max_a Q^k(s,a)$$
  $\Delta \longleftarrow \max(\Delta, |V^k(s) - V^{k-1}(s)|)$ 

Until  $\Delta < \varepsilon$ 

Return 
$$V^k \approx V^* \in \mathbb{R}^{|S|}$$

Updates the Q-values for each action a and state s

#### **Algorithm**

Until  $\Delta < \varepsilon$ 

Return  $V^k \approx V^* \in \mathbb{R}^{|S|}$ 

Initialize random values 
$$V^0$$
 
$$\Delta \longleftarrow 0$$
 Repeat: For  $s \in S$ : For  $a \in A$  
$$Q^k(s,a) \longleftarrow R(s,a) + \gamma \sum_{s'} P(s'|s,a) V^{k-1}(s')$$
 
$$V^k(s) = \max_a Q^k(s,a)$$
 
$$\Delta \longleftarrow \max(\Delta, |V^k(s) - V^{k-1}(s)|)$$

Retain the maximum Q-value

#### **Algorithm**

Initialize random values 
$$V^0$$
  $\Delta \longleftarrow 0$  Repeat: For  $s \in S$ : For  $a \in A$   $Q^k(s,a) \longleftarrow R(s,a) + \gamma \sum_{s'} P(s'|s,a) V^{k-1}(s')$   $V^k(s) = \max_a Q^k(s,a)$   $\Delta \longleftarrow \max(\Delta, |V^k(s) - V^{k-1}(s)|)$ 

Until  $\Delta < \varepsilon$ 

Return  $V^k \approx V^* \in \mathbb{R}^{|S|}$ 

Repeat until the updates are small enough

#### **Algorithm**

Initialize random values 
$$V^0$$
 
$$\Delta \longleftarrow 0$$
 Repeat: For  $s \in S$ : For  $a \in A$  
$$Q^k(s,a) \longleftarrow R(s,a) + \gamma \sum_{s'} P(s'|s,a) V^{k-1}(s')$$
 
$$V^k(s) = \max_a Q^k(s,a)$$
 
$$\Delta \longleftarrow \max(\Delta, |V^k(s) - V^{k-1}(s)|)$$

Until  $\Delta < \varepsilon$ 

Return 
$$V^k \approx V^* \in \mathbb{R}^{|S|}$$

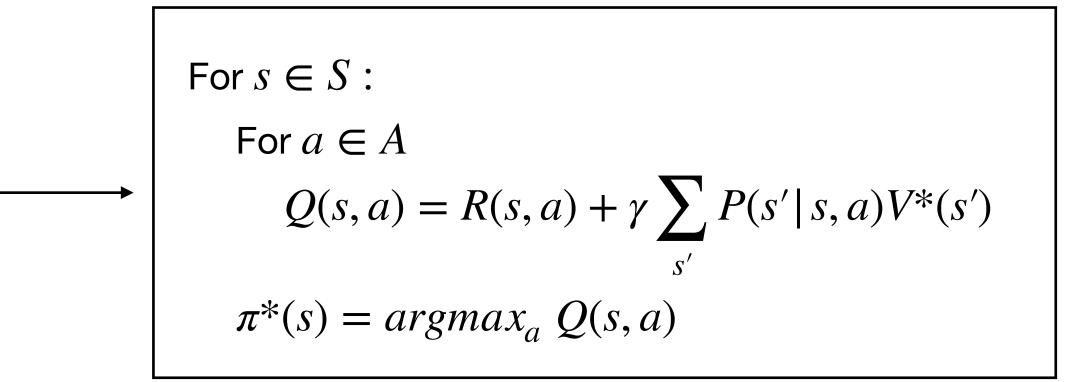
Return the optimal value  $V^*$ 

#### Algorithm

Until  $\Delta < \varepsilon$ 

Return  $V^k \approx V^* \in \mathbb{R}^{|S|}$ 

```
Initialize random values V^0 \Delta \longleftarrow 0 Repeat: For s \in S: For a \in A Q^k(s,a) \longleftarrow R(s,a) + \gamma \sum_{s'} P(s'|s,a) V^{k-1}(s') V^k(s) = \max_a Q^k(s,a) \Delta \longleftarrow \max(\Delta, |V^k(s) - V^{k-1}(s)|)
```



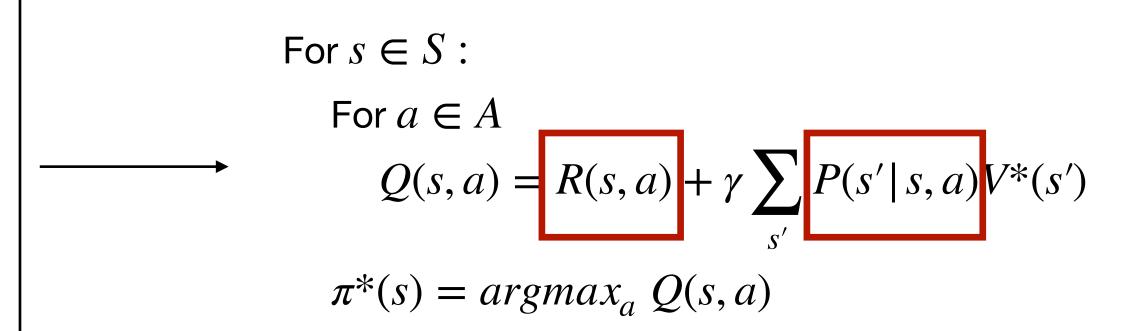
From the optimal value  $V^*$  to the optimal policy  $\pi^*$ 

#### Algorithm

Initialize random values  $V^0$   $\Delta \longleftarrow 0$  Repeat: For  $s \in S$ : For  $a \in A$   $Q^k(s,a) \longleftarrow R(s,a) + \gamma \sum_{s'} P(s'|s,a) V^{k-1}(s')$   $V^k(s) = \max_a Q^k(s,a)$   $\Delta \longleftarrow \max(\Delta, |V^k(s) - V^{k-1}(s)|)$ 

Until  $\Delta < \varepsilon$ 

Return 
$$V^k \approx V^* \in \mathbb{R}^{|S|}$$



#### Assumption that transitions and rewards are known









# Planning vs Reinforcement Learning

#### Planning

- → The transitions and reward model are known
- → Quiz show, robot navigation
- → Use value iteration to find optimal policy to deploy afterwards



#### Reinforcement learning

- → No prior knowledge of transition or reward model
- → Robot parkour, Atari Breakout
- → Find an optimal policy while interacting with an unfamiliar environment



# Supervised Learning vs Reinforcement Learning

#### Supervised Learning

- → Examples of correct or incorrect behaviour
- → Independent sample of experience
- → Learn a model and use it afterwards

#### Reinforcement learning

- → Only reward for a sequence of actions tried
- → Agent has partial control over the training data
- → Agent learns on-line: it must maximize performance during learning



Estimate Q-value while interacting with the environment: MDP is unknown but can be sampled!

Table of q-values Q with shape  $|S| \times |A|$  (one Q(s, a) for every s, a)

Estimate Q-value while interacting with the environment: MDP is unknown but can be sampled!

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After observing a transition s, a, s', r update the table Q

$$Q(s,a) \longleftarrow (1-\alpha)Q(s,a) + \alpha(r + \gamma \max_{a'} Q(s',a'))$$

Estimate Q-value while interacting with the environment: MDP is unknown but can be sampled!

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$$\downarrow \qquad \qquad \downarrow \qquad \qquad \downarrow$$
old value update

Estimate Q-value while interacting with the environment: MDP is unknown but can be sampled!

Table of q-values Q with shape  $|S| \times |A|$  (one Q(s, a) for every s, a)

After observing a transition s, a, s', r update the table Q

Estimate Q-value while interacting with the environment: MDP is unknown but can be sampled!

Table of q-values Q with shape  $|S| \times |A|$  (one Q(s, a) for every s, a)

After observing a transition s, a, s', r update the table Q

$$Q(s,a) \longleftarrow (1-\alpha)Q(s,a) + \alpha(r+\gamma \max_{a'} Q(s',a')) = Q(s,a) + \alpha(r+\gamma \max_{a'} Q(s',a') - Q(s,a))$$

Temporal difference TD error

Temporal average between the old value and the update weighted by the learning rate lpha

#### **Algorithm**

```
Initialize constant Q-table Q(s, a) \leftarrow c
Repeat
   Initialize the state s^0
    Repeat:
       Learning agent chooses a^t = \max Q(s^t, a)
       Perform one env step s^{t+1}, r^{t+1} \leftarrow \text{env}(s^t, a^t)
       Update Q-table
      Q(s^t, a^t) \longleftarrow Q(s^t, a^t) + \alpha \left( r^{t+1} + \gamma \max_{a'} Q(s^{t+1}, a') - Q(s^t, a^t) \right)
     Until the episode is done
Until max number of steps
```

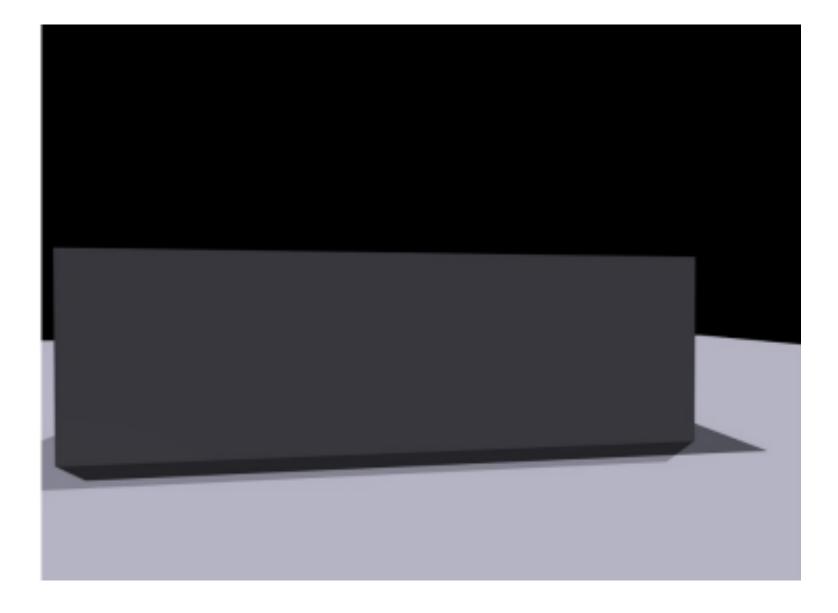
If all state-action pairs are visited often enough, tabular Q-learning converges to Q-value

# From Tabular to Deep RL

- The size of the Q table grows exponentially with the state and action dimensions
- Tabular RL can only work for small state-action spaces

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- How can the robot learn parkour?
  - No transitions and reward models available
  - High dimensional state space

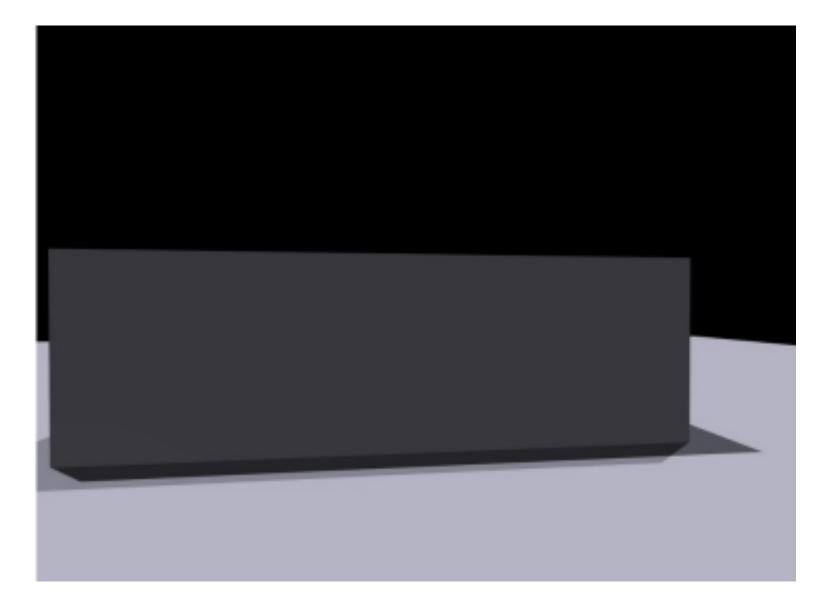


Robot Parkour vision of an obstacle

$$|S| = 48 \times 64 \times 255 \approx 10^6$$

# From Tabular to Deep RL

- The size of the Q table grows exponentially with the state and action dimensions
- Tabular RL can only work for small state-action spaces
- How can the robot learn parkour?
  - No transitions and reward models available
  - High dimensional state space
- Idea: use a neural network to approximate Q

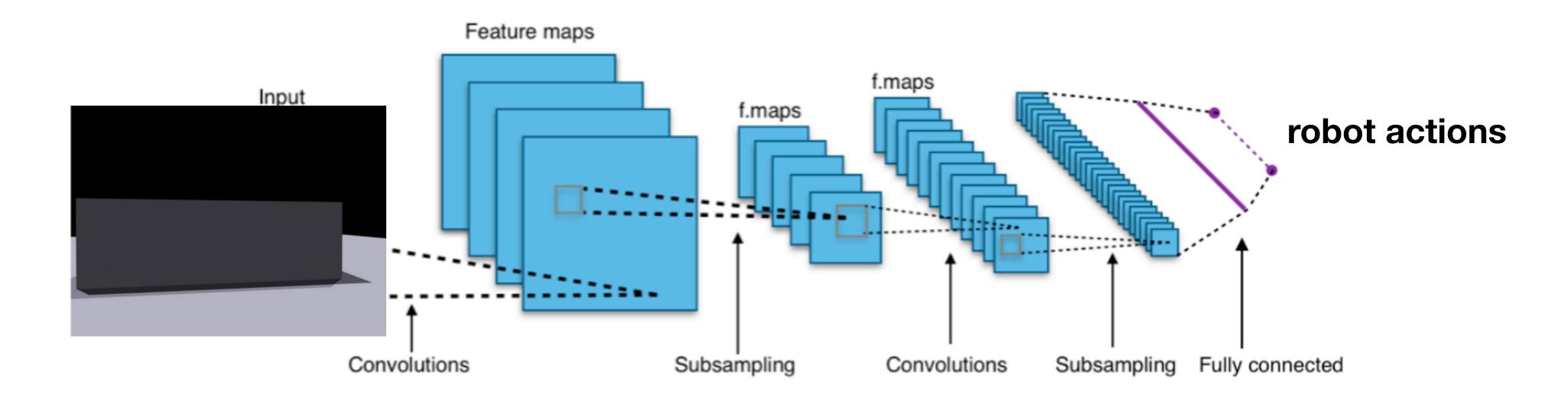


Robot Parkour vision of an obstacle

$$|S| = 48 \times 64 \times 255 \approx 10^6$$

# Deep Q-Network

Convolutional neural networks  $q(s;\theta) \in \mathbb{R}^{|A|}$ : output Q-value approximation per action



Gradient descent on mean-squared TD error

$$\theta \longleftarrow \theta - \alpha \nabla \left( r + \gamma \max_{a'} q(s'; \theta))_{a'} - q(s; \theta)_{a} \right)^{2}$$
TD-error

## Summary

#### **Topics**

- Sequential decision making: agents, environment, reward...
- MDPs
- Planning and Value Iteration
- Reinforcement learning and Tabular Q-learning

#### Reading material

- Reinforcement Learning: An Introduction Chapter 3
- Wendelin Böhmer Learning to interact, tabular Q-learning