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LArIAT INTERNAL DOCUMENT – NOT FOR PUBLIC DISTRIBUTION

Measurement of the Highland Formula in Liquid Argon LArIAT experiment

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This is where the abstract goes.

PACS numbers: 13.15.+g Neutrino interactions, 14.40.-n Mesons, 14.40.Be Light mesons (S=C=B=0), 13.75.-n Hadron-induced low- and intermediate-energy reactions and scattering (energy *le* 10 GeV)

MOTIVATIONS

What are we measuring and why?

EXPERIMENTAL SETUP

What experimental setup?

MEASUREMENT'S STATISTICAL FOUNDATION

The propagation of a particle in a medium depends upon the interaction of the particle with the medium itself. Multiple coulomb scattering (MCS) represents the multiple electromagnetic interactions between a charged particle and the atomic nuclei of the medium the particle traverses. The effect of MCS results in a deviation of the charged particle from its original trajectory. In the simple bi-dimensional representation shown in Figure 1, we define the incident momentum as p_{Inc}, the outgoing momentum as p_{out} and the scattering angle as θ_0. For a given incoming momentum, it is customary to model the distribution of the scattering angles for small angles ($< 10°$) [cite Leo] with a gaussian centered at zero and standard deviation σ_{MSC} given by the Highland-Lynch-Dahl formula (referred to as the Highland formula in what follows). The Highland formula reads

$$\sigma_{MCS} = \frac{13.6 \text{ MeV}}{p_{\text{inc}}\beta c} z \sqrt{\frac{l}{X_0}}[1 + 0.0038 \ln(\frac{l}{X_0})], \quad (1)$$

where c is the speed of light, β is the velocity of the particle in units of c, l the length of the material traversed and X_0 is the radiation length in the medium.

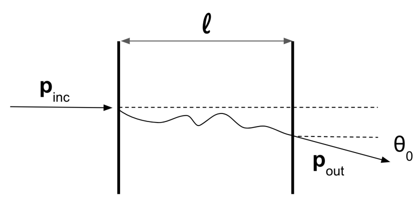

FIG. 1: 2D sketch of MCS.

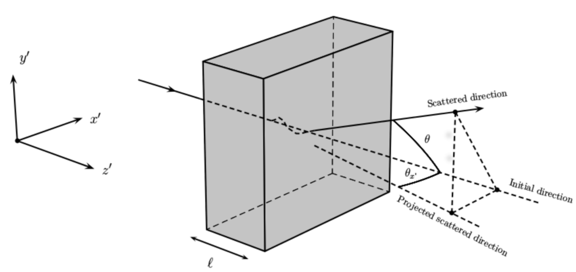

FIG. 2: 3D sketch of MCS.

In a more realistic tri-dimensional representation, show in Figure 2 we define θ_x and θ_y as the angle between the projections in the XZ and YZ planes. These angles are both distributed as gaussians centered at zero and standard deviation σ_{MSC}, in symbols:

$$\theta_x \sim \mathcal{N}(0, \sigma^2_{MSC}) \text{ and } \theta_y \sim \mathcal{N}(0, \sigma^2_{MSC}). \quad (2)$$

For small angles, we can approximate the 3D angle between the incoming and outgoing momenta as

$$\theta_{3D} = \sqrt{\theta_x^2 + \theta_y^2}. \quad (3)$$

Thus, we can assume that θ^2_{3D} is distributed as the sum of two independent gaussian distributions with the same mean $\mu_x = \mu_y = 0$ and same standard deviation $\sigma_x = \sigma_y = \sigma_{MCS}$; in symbols,

$$\theta^2_{3D} = \sum_{i=x,y} \theta_i^2 \sim \sum_{i=x,y} \Gamma(1/2, 2\sigma_i^2) = \Gamma(n/2, 2\sigma^2_{MCS}), \quad (4)$$

where n is the number of gaussian-distributed variables in the sum (in our case $n = 2$) and Γ is the gamma distribution. Substituting n, we simply find

$$\theta^2_{3D} \sim \Gamma(1, 2\sigma^2_{MCS}). \quad (5)$$

A common analytical parametrization of the gamma distribution in the k and α parameters is as follows:

$$\Gamma(k, \alpha) = \frac{1}{\Gamma(k)\alpha^k} x^{k-1} e^{-\frac{x}{\alpha}}, \quad (6)$$

where $\Gamma(k)$ is the nicely confusing gamma function, i.e. the compact version of the factorial, $\Gamma(k) = (k-1)!$ – don't you love statisticians?
In our case, the form of the gamma distribution is greatly simplified by the fact that $k = 1$. In fact, $\Gamma(1) = 1$, $x^{k-1} = x^0 = 1$ and the gamma function becomes:

$$\theta^2_{3D} \sim \Gamma(1, 2\sigma^2_{MCS}) = \frac{1}{2\sigma^2_{MCS}} e^{-\frac{\theta^2_{3D}}{2\sigma^2_{MCS}}}. \quad (7)$$

In order to calculate the Highland formula as a function of the momentum, we divide the LArIAT events in bins of incident momentum and we measure σ_{MCS} in each bin. For each bin, we plot the θ^2_{3D} distribution, we fit it with an exponential and we find the slope; then we calculate $\sigma_{MCS} \pm \delta\sigma_{MCS}$ from the estimated slope and the fit uncertainty.

The form of the function used to fit the θ^2_{3D} distributions is the following

$$\theta^2_{3D} \sim C e^{\alpha \theta^2_{3D}}, \quad (8)$$

where C is a normalization factor and $\alpha = -\frac{1}{2\sigma^2_{MCS}}$. Thus, accounting for the propagation of uncertainties, we find

$$\sigma_{MCS} \pm \delta\sigma_{MCS} = \sqrt{-\frac{1}{2\alpha}} \pm \sigma_{MCS}\frac{\delta\alpha}{2\alpha}, \quad (9)$$

where $\delta\alpha$ is the uncertainty of the fit parameter.

MEASUREMENT METHODOLOGY

How are we measuring it?

RESULTS

[1] Standard LArIAT detector reference:
R. Acciarri *et al.* (LArIAT Collaboration), hopefully we'll have one soon.