$$
\begin{aligned}
C_{\text{Int}}^{\pi\ \text{MC}}(E_i) &= \frac{N_{\text{Int}}^{\pi\ \text{MC}}(E_i)}{N_{\text{Int}}^{\text{TOT MC}}(E_i)} &(1)\\
&= \frac{N_{\text{Int}}^{\text{TOT MC}}(E_i) - B_{\text{Int}}^{\text{TOT MC}}(E_i)}{N_{\text{Int}}^{\text{TOT MC}}(E_i)} &(2)
\end{aligned}
$$

$$
\begin{aligned}
C_{\text{Inc}}^{\pi\ \text{MC}}(E_i) &= \frac{N_{\text{Inc}}^{\pi\ \text{MC}}(E_i)}{N_{\text{Inc}}^{\text{TOT MC}}(E_i)} &(3)\\
&= \frac{N_{\text{Inc}}^{\text{TOT MC}}(E_i) - B_{\text{Inc}}^{\text{TOT MC}}(E_i)}{N_{\text{Inc}}^{\text{TOT MC}}(E_i)} &(4)
\end{aligned}
$$

$$E_{\text{Front Face}}^{\text{kin}} = \sqrt{p_{Beam}^2 + m_{Beam}^2} - m_{Beam} - E_{Loss} \tag{5}$$

$$U_{i,j}^{\text{Int}}(E_i) \tag{6}$$

$$U_{i,j}^{\text{Inc}}(E_i) \tag{7}$$

$$E_j^{\text{kin}} = E_{\text{Front Face}}^{\text{kin}} - \sum_{j<i} E_{\text{dep i}} \tag{8}$$

$$E_i = \sqrt{p_{Beam}^2 + m_{Beam}^2} - m_{Beam} - E_{Loss} - E_{\text{dep FF-j}} \tag{9}$$

$$E_{\text{slice j}} = \sqrt{p_{Beam}^2 + m_{Beam}^2} - m_{Beam} - E_{Loss} - E_{\text{dep FF-j}} \tag{10}$$

$$\delta E_{\text{slice j}} = \sqrt{\delta p_{Beam}^2 + \delta E_{Loss}^2 + \delta E_{\text{dep FF-j}}^2} \tag{11}$$

	Low E Beam	High E Beam
G4Pions	70.9 %	82.3 %
G4Muons	14.6 %	13.5 %
G4Electrons	14.5 %	4.2 %

Table 1: Simulated beamline composition per magnet settings

$$E_{\text{dep FF-j}} = \sum_{j<s} E_{\text{dep s}} \Rightarrow \delta E_{\text{dep FF-j}} = (j-1)\delta E_{\text{dep s}} \tag{12}$$

$$\sigma^{\pi^-}_{TOT}(E_i) = \frac{1}{n\ \delta X} \frac{\epsilon^{\text{Inc}}(E_i)\ \ C^{\pi MC}_{\text{Int}}(E_i)\ \ N^{\text{TOT}}_{\text{Int}}(E_i)}{\epsilon^{\text{Int}}(E_i)\ \ C^{\pi MC}_{\text{Inc}}(E_i)\ \ N^{\text{TOT}}_{\text{Inc}}(E_i)}. \tag{13}$$

$$\sigma^{K^+}_{TOT}(E_i) = \frac{1}{n\ \delta X} \frac{\epsilon^{\text{Inc}}(E_i)\ \ C^{KMC}_{\text{Int}}(E_i)\ \ N^{\text{TOT}}_{\text{Int}}(E_i)}{\epsilon^{\text{Int}}(E_i)\ \ C^{KMC}_{\text{Inc}}(E_i)\ \ N^{\text{TOT}}_{\text{Inc}}(E_i)}. \tag{14}$$

$$P_{\text{Int}} = \frac{N_{\text{Int}}}{N_{\text{Inc}}} = 1 - e^{-\sigma_{\text{TOT}}\ \text{n}\ \delta X} \tag{15}$$

$$\sigma_{\text{TOT}}(E) \sim \frac{1}{\text{n}\ \delta X} \frac{N_{\text{Int}}}{N_{\text{Inc}}} \tag{16}$$

$$\sigma_{\text{TOT}}(E_i) = \frac{1}{\text{n}\ \delta X} \frac{N^{\pi^-}_{\text{Int}}(E_i)}{N^{\pi^-}_{\text{Inc}}(E_i)} \tag{17}$$

$$\mathcal{L}(\mu_0; \sigma_0^2; \Delta\theta_0, \Delta\theta_1) = \prod_{i=0}^{1} f_X(\Delta\theta_i, \mu_0, \sigma_0^2) \Rightarrow \tag{18}$$

$$\log \mathcal{L} = -\frac{1}{2}\log(2\pi) - \log\sigma_0 - \frac{1}{2}\frac{(\Delta\theta_0 - \mu_0)^2}{\sigma_0^2} + \text{same for } \Delta\theta_1 \tag{19}$$

	Run-II Neg Pol
1. Events Reconstructed in Beamline	158396
2. Events with Plausible Trajectory	147468
3. Beamline $\pi^-/\mu^-/e^-$ Candidate	138481
4. Events Surviving Pile Up Filter	108929
5. Events with WC2TPC Match	41757
6. Events Surviving Shower Filter	40841
7. Available Events For Cross Section	40841

Table 2: Number of data events for Run-II Negative and Positive polarity