Task 1

Introduction

In the present part of the report, we will investigate to what extent we will be able to classify respondents in their country, and then we will compare the performance of different classifiers.

Data

The data have been obtained from the 6^{th} Wave of the World Value Survey, which was carried out between 2010 and 2013. The data include the standardized scores of 3929 respondents of 3 countries on 32 variables, that have been summarized with 7 factors obtained using exploratory factor analysis with oblique rotation. The 7 factors related to the 32 variables are:

1. **Rights**, that it's related to homosexuality, prostitution, abortion, divorce, sex before marriage, suicide;
2. **Steal**, that it's related to claiming benefits, avoiding fare, stealing property, cheating taxes, accept a bribe;
3. **Crime**, that it's related to robberies, alcohol, police-military, racist behavior, drug sale;
4. **Religion**, that it's related to attend religious services, pray, the importance of God;
5. **Realize self**, that it's related to creative, rich, spoil oneself, be successful, exciting life;
6. **Do good**, that it's related to security, do good, behave properly, protect environment, tradition;
7. **Violence**, that it's related to beat wife, parents beating children, violenc.

Methodology

To investigate the possibility to classify the respondents in their country based on the 7 factors we have used the canonical discriminant analysis. We have applied the linear regression function with 7 predictors and 1 dependent variable, the Country. Then to the output, we have applied the Canonical Discriminant Analysis.

```
lm.out<-lm(cbind(F_rights, F_steal, F_crime,F_religion,F_realizeself,F_dogood,
                 F_violence)~as.factor(country), data=dwvs)
candisc.out<-candisc(lm.out)
print(candisc.out)
```

```
Canonical Discriminant Analysis for as.factor(country):

   CanRsq Eigenvalue Difference Percent Cumulative
1 0.80691    4.17882     3.5622  87.142     87.142
2 0.38142    0.61661     3.5622  12.858    100.000

Test of H0: The canonical correlations in the
current row and all that follow are zero
```

```
  LR test stat approx F numDF denDF   Pr(> F)
1      0.11944  1059.53    14  7834 < 2.2e-16 ***
2      0.61858   402.65     6  3918 < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

As we can see both the Square Canonical Correlation are significant, but the discriminating power to separate between the groups is higher for the first than for the second discriminant function: 0.81 and 0.38 respectively. The LR test indicates that the discriminant analysis is meaningful. The first test's null hypothesis is H_0 : $\lambda_1 = \lambda_2 = 0$ and this hypothesis as we can see from the *p-value* is rejected. The *null* hypothesis of the first test it's equivalent to the test for $H_0 : \mu Netherlands = \mu Nigeria = \mu Philippines$.

The second LR test indicates that $H_0 : \lambda_2 = 0$, and also this null hypothesis is rejected. So even if the second discriminant function has less discriminant power cannot be omitted and it's statistically meaningful.

On our analysis, we have also applied two different tests for centroids and to test the equal covariance.

To see if the three-country has different centroids and confirm the results of the canonical discriminant analysis we have applied on the linear regression the function *Manova*:

```
res_t1_2 <- summary(Manova(lm.out), test="Wilks")

summary.default(Manova(lm.out), test="Wilks")
```

```
          Length Class  Mode
SSP        1     -none- list
SSPE      49     -none- numeric
df         1     -none- numeric
error.df   1     -none- numeric
terms      1     -none- character
repeated   1     -none- logical
type       1     -none- character
test       1     -none- character
```

The *p-value* is small, and the test confirms that the analysis is meaningful and that at least there is a pair of centroids that differs significantly. The function *Manova* in r doing the *Wilks Lambda test* uses the Rao approximation. To test the assumption on equal population covariance we have applied to the linear regression the function *boxM*:

```
boxM(lm.out)
```

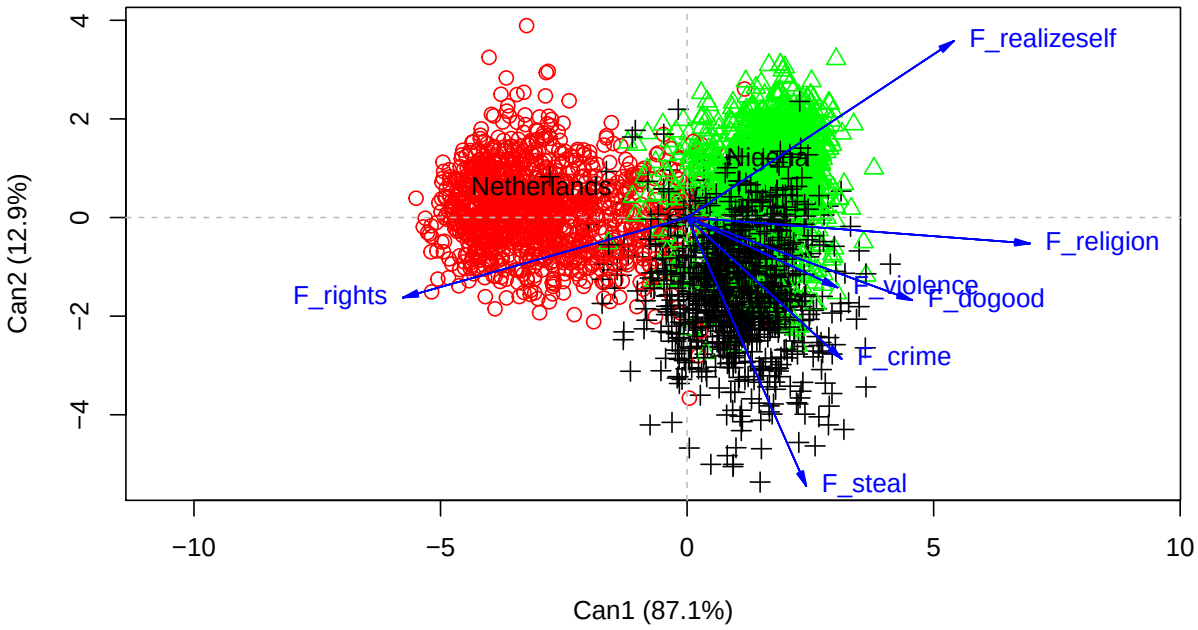
```
    Box's M-test for Homogeneity of Covariance Matrices

data:  Y
Chi-Sq (approx.) = 5479.2, df = 56, p-value < 2.2e-16
```

The test of Box indicates that H_0 of equal covariance matrices across groups is not supported by data.

Plot

To complete the Canonical Discriminant analysis, we have plotted the three countries and the 7 variables.

```
Vector scale factor set to 7.827
```



We can see that the group of individuals in *red* are Netherlands citizens, the group of individuals in *green* are Nigerians citizens and the group of individuals in *black* are Philippines citizens. In blue we can see the 7 explanatory variables. The plot shows a clear separation between Netherlands and the other two countries on the first discriminant function while the second discriminant function could help to separate Nigeria and the Philippines. The first discriminant function especially correlates with the factors: rights, religion, realize self, and do good; whereas the second discriminant function correlates with the factors: steal and realize self. The two-factor crime and violence have a lower correlation on the two factors and so has in this analysis lower importance on separate the 3 countries.

Compare the performance of different classifiers

We are going now to compare the performance of different classifiers to classify respondents in their country based on the 7 factors. To be able to do that we are going to compute the training error and the leave-one-out cross-validation error. The classification method that we are going to compare are: - Linear discriminant analysis; - Quadratic discriminant analysis; - K-nearest neighbors with k ranging from *1 to 100*; - High Dimensional Discriminant Analysis;

Linear discriminant analysis

This method aims to separate in the clearest possible way different groups using the linear combination of observed independent variables. The linear discriminant analysis method assumes that the covariance

structure of the independent variable is the same across groups. In our analysis, we know from the Box test previously computed that this assumption is not supported by the data. It will be an interesting test if in this case the Quadratic discriminant analysis, where the assumption on the equality of covariance matrix is relaxed, will perform better. In the linear discriminant analysis, we have applied the method of Fisher correcting for the different prior probability.

```
lda.out3<-lda(country ~ F_rights + F_steal + F_crime + F_religion + F_realizeself +
                F_dogood + F_violence, data=dwvs)
#print(lda.out3)
pred.train3 <- predict(lda.out3,dwvs, prior=c(1,1,1)/3)
```

```
tab3 <- table(dwvs$country,pred.train3$class)
#print(tab3)
kbl(tab3)
```

	Netherlands	Nigeria	Philippines
Netherlands	1145	39	76
Nigeria	10	1327	241
Philippines	17	220	851

```
#training hit rate
kbl(sum(diag(tab3))/sum(tab3))
```

x
0.8464086

```
#classify test observations using LDA
pred.loocv4<-lda(country~F_rights+F_steal+F_crime+F_religion+F_realizeself+F_dogood+F_violence,data=dwvs
tab4<-table(dwvs$country,pred.loocv4$class)
print(tab4)
```

```
             Netherlands Nigeria Philippines
  Netherlands        1145      39          76
  Nigeria              11    1326         241
  Philippines          17     224         847
```

```
#LOOCV hit rate
kbl(sum(diag(tab4))/sum(tab4))
```

x
0.845135

We can see that in that case, the difference between the performance for training error and LOOCV error is really small, so there is no evidence for overfitting.

Quadratic discriminant analysis

The second method that we have applied is Quadratic discriminant analysis. It should perform better considering the difference in the covariance matrix for the different groups. QDA even if has a lower bias

with a different covariance matrix, has a larger variance, and as in our case with a small dataset can be problematic.

Even in this case, we have applied the method of Fisher correcting for prior probabilities.

```
qda.out7<-qda(country ~ F_rights + F_steal + F_crime + F_religion + F_realizeself +
                F_dogood + F_violence, data = dwvs)

pred.train7<-predict(qda.out7,dwvs, prior=c(1,1,1)/3)

tab7<-table(dwvs$country,pred.train7$class)
#print(tab7)
#training hit rate
kbl(sum(diag(tab7))/sum(tab7))
```

x
0.8550688

```
#classify test observations
#use equal population sizes classes
pred.test8 <- qda(country ~ F_rights + F_steal + F_crime + F_religion + F_realizeself +
                    F_dogood + F_violence, data=dwvs,prior=c(1,1,1)/3,CV=TRUE)
tab8<-table(dwvs$country,pred.test8$class)
#LOOCV hit rate
kbl(sum(diag(tab8))/sum(tab8))
```

x
0.8525217

In that case there is also no evidence of overfitting. We can see from the results that the QDA performs better than the LDA but not with a significant improvement.

K-nearest Neighbors

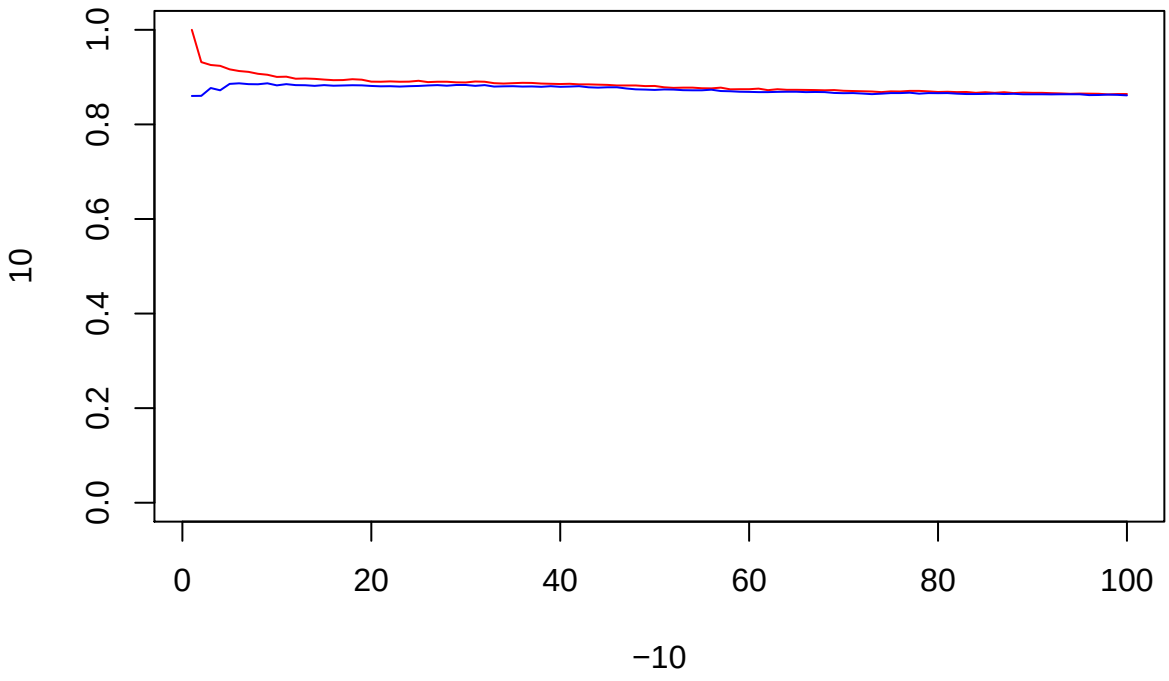
The third model that we had analyzed is the K-nearest Neighbors. We have computed the model using all the 3926 observations, and to choose which is the correct number k of parameters to use, we have compared the training error with the Leave one out cross-validation error.

```
#str(dwvs)
table(dwvs$country)
```

```
Netherlands     Nigeria Philippines
       1260        1578        1088
```

```
set.seed(9850)    # -> random number generator
gp<-runif(nrow(dwvs))
dwvs2<-dwvs[order(gp),]
#str(dwvs)
#str(dwvs2)
#head(dwvs)
#head(dwvs2)
```

```
hitratknn<-function(observed,predicted){
  tab<-table(observed,predicted)
  hitratknn<-sum(diag(tab))/sum(tab)
  return(hitratknn)
}

knnmax<-100
err<-matrix(rep(0,knnmax*2), nrow=knnmax)

for(j in 1:knnmax) {
  predknn.train<-knn(dwvs2[,2:8], dwvs2[,2:8], dwvs2$country, k=j)
  err[j,1]<-hitratknn(dwvs2$country,predknn.train)
}

for(j in 1:knnmax) {
  predknn.train<-knn.cv(dwvs2[,2:8], dwvs2$country, k=j)
  err [j,2]<-hitratknn(dwvs2$country,predknn.train)
}

plot(-10, 10,xlim=c(1,knnmax),ylim=c(0,1))
lines(c(1:knnmax),err[,1],col="red") # -> training error
lines(c(1:knnmax),err[,2],col="blue")
```



We can see that with K=1 the model is flexible and by definition, we have training hit rate (red line) of 0, but the LOOCV hit rate (blue line) is higher in this case, while with model less flexible, as with k=98, the

two errors are similar. Since both the errors increase if we increase the parameter K, probably the model that describes the dataset better is the model with K=30 or K=66.

Table 1:

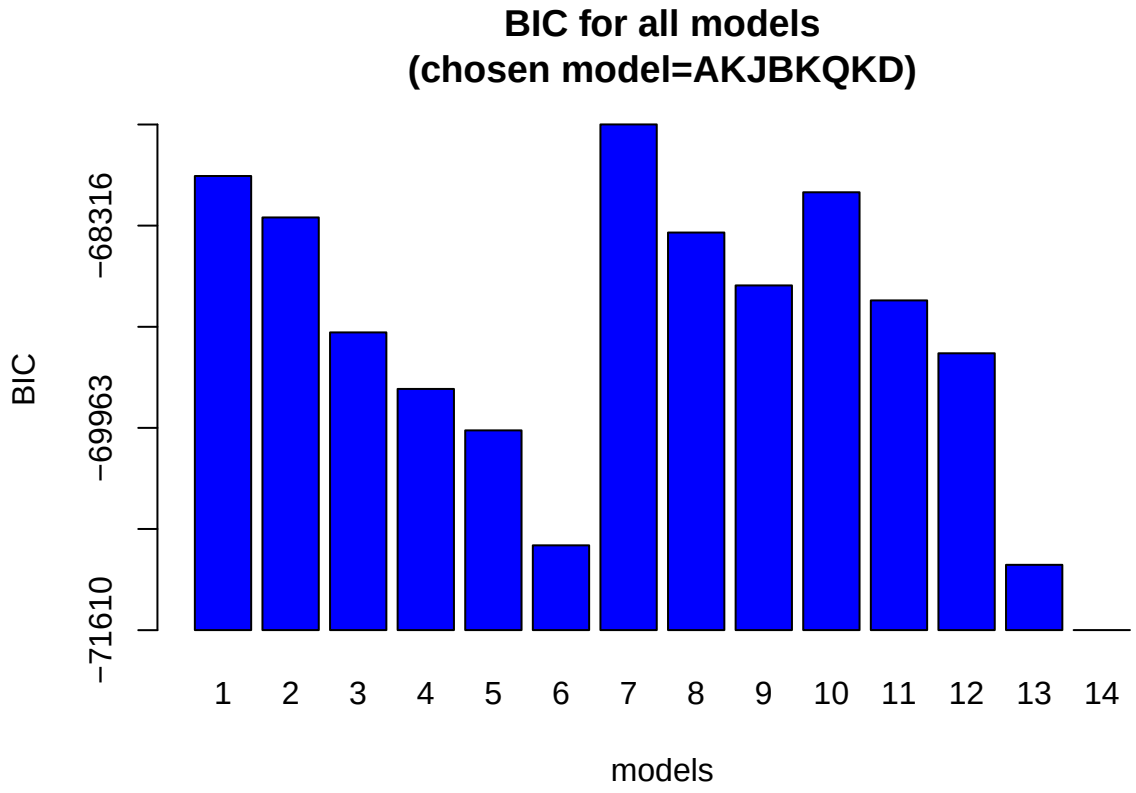
K	TRAINING HIT RATE	LOOCV HIT RATE
1	1	0.8601630
30	0.8886908	0.8833418
66	0.8728986	0.8683138
100	0.8642384	0.8614366

High Dimensional Discriminant Analysis

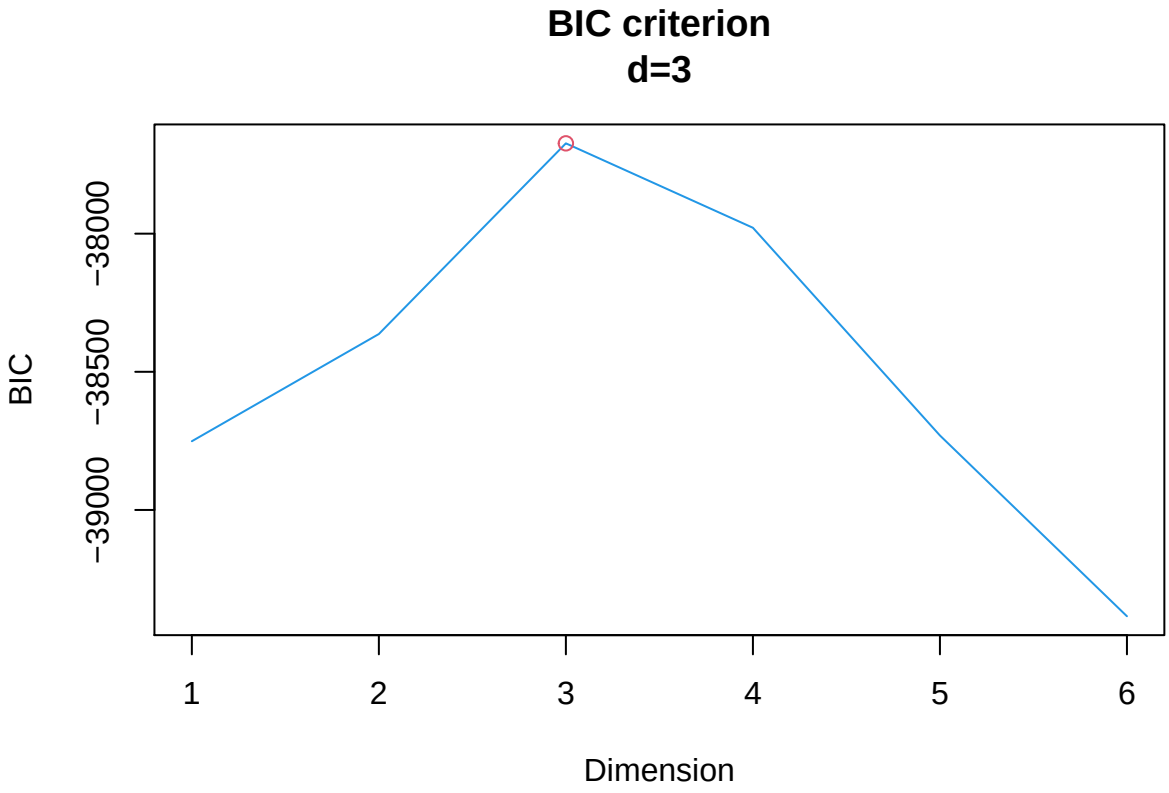
The fourth method that we have used to discriminate between different groups is the HDDA method. This method could be useful while the number of parameters is high compared to the number of data.

```
w <- dwvs[,-1]
cls <- dwvs[,1]
#HDDA on the learning dataset:
hdda.out11 <- hdda(w, cls, scaling=TRUE, model="all", d="BIC",graph=TRUE,show=TRUE)
```

```
 # :     Model     BIC
 1 :     AKJBKQKDK   -67912.32
 2 :     AKBKQKDK    -68249.76
 3 :     ABKQKDK     -69185.72
 4 :     AKJBQKDK    -69646.01
 5 :     AKBQKDK     -69983.46
 6 :     ABQKDK      -70919.41
 7 :     AKJBKQKD    -67492.38
 8 :     AKBKQKD     -68372.73
 9 :     ABKQKD      -68803.13
10 :     AKJBQKD     -68044.89
11 :     AKBQKD      -68925.24
12 :     ABQKD       -69355.63
13 :     AJBQD       -71078.02
14 :     ABQD        -71609.34

SELECTED: Model AKJBKQKD, BIC=-67492.38.
```

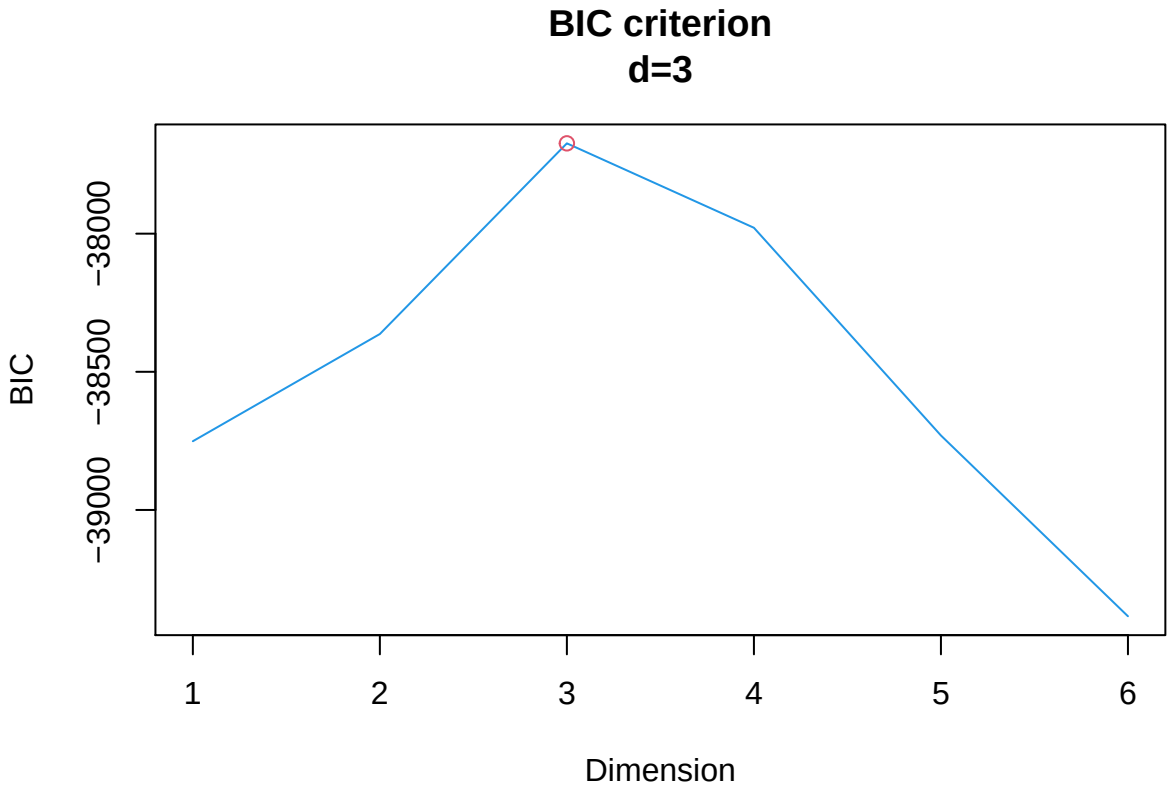



```
plot(hdda.out11)
```



The model used from the HDDA analysis applying the BIC criterion is model 7($A_{kj}K_jB_kQ_kd$). The decision following the *BIC* criteria is to choose the model with the lowest value, in that case, the reference system is negative, so the lowest value is for model 7.

```
plot(hdda.out11,method="BIC")
```



The dimension choose for the model is 3. The model chosen by the *BIC* criteria has the same number of principal components for all the three different classes.

```
pred.train11<-predict(hdda.out11,w,cls)
```

```
Correct classification rate: 0.8530311.
               Initial class
Predicted class Netherlands Nigeria Philippines
    Netherlands        1196      20          40
    Nigeria              26    1356         251
    Philippines          38     202         797
```

```
tab11<-table(dwvs$country,pred.train11$class)
#print(tab11)
#training hit rate
kbl(sum(diag(tab11))/sum(tab11))
```

x
0.8530311

```
pred.loocv12 <- hdda(w, cls,scaling=TRUE, d="BIC", LOO=TRUE)
tab12<-table(cls,pred.loocv12$class)
#print(tab12)
kbl(tab12)
```

	Netherlands	Nigeria	Philippines
Netherlands	1197	26	37
Nigeria	22	1378	178
Philippines	44	285	759

```
#LOOCV hit rate
kbl(sum(diag(tab12))/sum(tab12))
```

x
0.8492104

Also with the *HDDA* model, there is rather small evidence of overfitting, but the HDDA model does not perform better than the other models.

Error comparison for the different models

We have computed for all 4 models the hit rate, for the comparison in the table we will present the training and LOOCV errors computing $1 - *hit$ rate:

Table 2:

MODEL	TRAINING ERROR	LOOCV ERROR
LDA	0.1535914	0.154865
QDA	0.1449312	0.1474783
KNN (K=30)	0.1113092	0.1166582
HDDA	0.1469689	0.1507896

In all the present models there is little evidence of overfitting, the two errors computed are in all the cases similar. The K-nearest Neighbors is a good model to compare the others and we can see that even if it performs betters it has not a huge difference. The model that performs better between the other 3 is the Quadratic discriminant analysis, it is the most complex one with the highest number of parameters used.

Confronting the results, we can say that even if there is a difference between the models, no one of the computed ones has outstanding results.

Table of *QDA LOOCV* rate:

Table 3:

-	Netherlands	Nigeria	Philippines
Netherlands	1210	21	29
Nigeria	40	1315	223
Philippines	43	223	822

As we were expecting in the analysis of the canonical discriminant analysis, the model has a high ability to differentiate between Netherlands and the two other countries, when it has a high error rate discriminating between Nigeria and the Philippines.

Multinomial logistic regression model

```
m1<- multinom(country~F_rights+F_steal+F_crime+F_religion+F_realizeself+F_dogood +F_violence, family=mu
```

```
# weights:  27 (16 variable)
initial  value 4313.151845
iter  10 value 1376.724330
iter  20 value 1352.584896
final  value 1346.617148
converged
```

```
t1_15_result <- summary (m1)
t1_15_result
```

```
Call:
multinom(formula = country ~ F_rights + F_steal + F_crime + F_religion +
    F_realizeself + F_dogood + F_violence, data = dwvs, family = multinomial,
    maxit = 3926, hess = TRUE)

Coefficients:
            (Intercept)  F_rights   F_steal   F_crime F_religion F_realizeself
Nigeria       0.6472240 -2.122889 0.5537755 0.5976742   2.887712     2.8922393
Philippines   0.9959826 -1.466848 1.5556448 1.0662700   1.932117     0.9680894
               F_dogood F_violence
Nigeria     -0.01110756  1.2844737
Philippines  1.16765772  0.7560728

Std. Errors:
            (Intercept)  F_rights   F_steal   F_crime F_religion F_realizeself
Nigeria       0.1612100 0.1642741 0.1735127 0.1334749  0.2143403     0.1695936
Philippines   0.1510987 0.1475679 0.1685630 0.1296852  0.1866007     0.1556415
             F_dogood F_violence
Nigeria     0.1218099  0.1534901
Philippines 0.1197516  0.1470007

Residual Deviance: 2693.234
AIC: 2725.234
```

We can see that there are 2 different regression model estimates: the first one compares the probability of Nigeria to the probability of the Netherlands, the second model compares the probability of the Philippines to the probability of the Netherlands. All the parameters are significant except *F_dogood* for Nigeria, which's not significantly different from 0. The sign of the parameters is the same for all the parameters in both the regressions, except for *F_dogood* where the Nigeria coefficient is not significant. This could be explained because have we analyzed previously Netherlands strongly differs from the other two countries, while Nigeria and the Philippines have not a clear separation.

```
#### compute hitrate training data####
train.pred<-predict(m1,newdata=dwvs)
tab<-table(dwvs$country, train.pred)
kbl(sum(diag(tab))/sum(tab))
```

x
0.8571065

Error rate: 0.1428935

```
########compute LOOCV
nobs<- 3926
hit<-rep(0,nobs)
for (i in 1:nobs){
  train<-c(1:nobs)
  mod<- multinom(country ~ F_rights + F_steal + F_crime + F_religion +
                   F_realizeself + F_dogood + F_violence, data=dwvs,
                 subset=train[-i], print= FALSE,maxit=3926)
  pred<- predict(mod, newdata=dwvs[i,])
  hit[i]<-ifelse(pred==dwvs$country[i],1,0)
}
```

```
#hitrate
mean(hit)  #### LOOCV
```

Error rate: 0.1444218

The Multinomial logistic regression model has slightly better error values than the other models, except for KNN.