Chapter 3: Quantum Computing

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3.0 Summary

Quantum Computation (QC) is a type of computation where unitary and measurement operations are executed on linear superpositions of basis states. This paper provides a brief introduction to QC. We begin with a discussion of basic models for QC such as quantum TMs, quantum gates and circuits and related complexity results. We then discuss a number of topics in quantum information theory, including bounds for quantum communication and I/O complexity, methods for quantum data compression. and quantum er-

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ror correction (that is, techniques for decreasing decoherence errors in QC), Furthermore, we enumerate a number of methodologies and technologies for doing QC. Finally, we discuss resource bounds for QC including bonds for processing time, energy and volume, particularly emphasizing challenges in determining volume bounds for observation apperatus.

3.1 Introduction

3.1.1 Reversible Computations

Reversible Computations are computations where each state transformation is a reversible function, so that any computation can be reversed without loss of information. Landauer [1] showed that irreversible computations must generate heat in the computing process, and that reversible computations have the property that if executed slowly enough, they (in the limit) can consume no energy in an adiabatic computation. Bennett [2] (also see Bennett, Landauer [3], Landauer [4], Toffoli [5]) showed that any computing machine (e.g., an abstract machine such as a Turing Machine) can be transformed to do reversible computations. Bennett's reversibility construction required extra space to store information to insure reversibility; Li, Vitanyi [6] give trade-offs between time and space in the resulting reversible ma-

chine. An innovative technique due to Bennett [2,7] can be used to make reversible functions bijective, as required for quantum computations. Given a bijective function f, suppose we can reversibly compute in time $T(x)$ a bijective function f and it's inverse f^{-1} using auxiliary registers for storage of the input. He proves in time $O(T(n))$ we can also reversibly compute the bijective mapping: $(x, 0, 0) \rightarrow (f(x), 0, 0)$ without use of auxiliary registers for storage of the input.

3.1.2 An Introduction to Quantum Computation

Computations and methods not making use of quantum mechanics will be termed *classical*. In contrast, *Quantum Computation (QC)* applies quantum mechanics to do computation. A single molecule (or collection of particles and/or atoms) may have a number n of degrees of freedom known as *qubits*. Associated with each fixed setting X of the n qubits to Boolean values is a *basis state* denoted $|a>$.

Quantum mechanics allows for a linear superposition (also termed an *entangled quantum state*) of these basis states to exist simultaneously. Each basis state $|a>$ of the superposition is assigned a given complex amplitude α; this is denoted $\alpha|a>$. *Unitary transformations* are reversible operations on the superpositions which can be represented by unitary matrices A (e.g.,

permutation matrices, rotation matrices, and the matrices of Fourier transforms) where $AA^T = I$. The sum of the squares of the magnitudes of the amplitudes of all basis states is 1. This sum remains invariant due to the application of a unitary transformations. The Hilbert space H_n is the set of all possible such linear superpositions.

QC is a method of computation where various operations can be executed on these superpositions:

- *unitary operations*, and
- *observation operations*, which allow for the (strong) measurement of each qubit, providing a mapping from the current superposition to a superposition where the measured qubit is assigned a Boolean value with probability given by the square of the amplitude of the qubit in its original superposition.

Elementary unitary operations that suffice for any quantum computation over qubits (see [8] and [9]) include a conditional form of the conditional XOR operation $\oplus$, the Boolean operation NOT, and a constant Boolean operation yielding 0. The time bound for a quantum computations is defined to be the number of such elementary unitary operations.

3.1.3 Surveys of QC

The following are reviews and surveys have been made of QC: Bennett [10], Barenco [11], Benio [12], Brassard [13,14], Haroche, Raimond [15], Brassard [16], Preskill [17], Scarani [18], Steane [19], Vedral, Plenio [20]. Also, Taubes [21] and Gershenfeld, Chuang [22] give popular press descriptions of QC. The following are texts quantum computing:

- Overviews: [23, 24, 25, 26].
- Quantum information processing: [27, 28, 29, 30, 31, 32, 33, 34].
- Quantum cryptography: [35] ,36, 37].
- Quantum coding theory:[38, 39].
- Quantum algorithms: [40].
- Experimental implementation of quantum computation: [41, 42, 43, 44, 45].

3.1.4 Initial Work in QC

Feynman [46,47] and Benioff [48] were the first to suggest the use of quantum mechanical principles for doing computation. Deutsch and Jozsa [49] give the first example of a quantum algorithm that gave a rapid solution of an

example problem, where the problem (for a given a black box function) is not quickly solvable by any deterministic conventional computing machine. But their problem could be quickly solved using randomization. Bernstein and Vazirani [50] then provided the first example of a fast quantum algorithm for a problem that could not be quickly solved by conventional computing machines even using randomization. (Also see Costantini, Smeraldi [51] for a generalization of Deutsch's example and see Collins et al [52] for a simplified Deutsch-Jozsa algorithm, and see Jozsa [53,54,55] for further work in quantum computation and complexity.)

3.1.5 Organization of this Paper

In this Section 3.1 we have introduced QC. In Section 3.2 we introduce formal quantum computing models and in Section 3.3 we discuss quantum complexity classes. Next we overview key topics concerning quantum information processing: in Section 3.4 we discuss bounds for quantum communication, then next in Section 3.5 we discuss methods for quantum errorless compression, in Section 3.6 we discuss methods for quantum error coding, and in Section 3.7 we describe methods for quantum cryptography. In Section 3.8 we discuss further algorithmic applications of QC. In Section 3.9 we enumerate various technologies for doing QC. In Section 3.10 we review of

the resource bounds of quantum computing as compared with the resources required by classical methods for computation. In Section 3.11 we conclude the paper. In Appendix 3.12 we discuss the challenge of providing volume bounds for observation apparatus when doing QC.

3.2 Quantum Computing Models

• **Quantum TMs and other Automata.** Deutsch [56] gave the first formal description of a quantum computer, known as a *quantum TM*. The tape contents of the TM are qubits. *Quantum configurations* of the QTM are superpositions of (classical) TM configurations. A transition of the QTM is a unitary mapping on quantum configurations of the QTM. Thus, a computation of the QTM is a unitary mapping from the initial quantum configuration to the final quantum configuration. Various papers generalize machines and automata to the quantum case. Moore, Crutchfield [57] propose quantum finite-state and push-down automata, and regular and context-free grammars, and they generalize several formal language and automata theorems, e.g. pumping lemmas, closure properties, rational and algebraic generating functions, and Greibach normal form. Kondacs and Watrous [58] partially characterize the power of quantum finite state automata. Dunlavey [59] gives a space-efficient simulation of a deterministic finite state machine

(FSM) on a quantum computer (using Grover's search algorithm discussed below). Watrous [60] investigates quantum cellular automata and Dürr et al [61,62] give decision procedures for unitary linear (one dimensional) quantum cellular automata.

• **Quantum Gates.** A set of Boolean gates are *universal* if any Boolean operation on arbitrarily many bits can be expressed as compositions of these gates. Toffoli [5] defined an extended XOR 3-bit gate (which is an XOR gate condition on one of the inputs and is known as the *Toffoli gate*) and showed that this gate, in combination with certain 1-bit gates, is universal. A set of quantum qubit gates are *universal* for Boolean computations for QC if any unitary operation on arbitrarily many qubits can be expressed as compositions of these gates. Deutsch defined the extended quantum XOR 3-qubit gate (known as the Deutsch-Toffoli gate) and proved this gate, in combination with certain one qubit gates, is universal. Barenco [63], Sleator et al [64], Barenco et al [65], and DiVincenzo [66] proved the 2-qubit XOR gates with certain 1-qubit gates can implement the Deutsch-Toffoli gate, so are universal for QC (also see Smolin and DiVincenzo [67], DiVincenzo et al [68, 69], Poyatos et al [70], Mozyrsky et al [71,72,73]). Lloyd [74] then proved that almost any 2-qubit quantum logic gate (with certain 1-qubit gates) is universal for QC. Monroe et al [75], DiVincenz et al [76] gave experimental

demonstrations of quantum gates. [77] defined a quantum computing model known as a *quantum gate array* which allows execution of a (possibly cyclic) sequence of quantum gates, where each input is a qubit, and each gate computes a unitary transformation.

• **Quantum Circuits.** Yao [78] restricted the concept to (acyclic) *quantum circuits* which are a generalization of Boolean logic circuits for quantum gates. It suffices that a quantum circuit use only these universal gates. Yao [78] proved that QTM computations are equivalent to uniform quantum circuit families. Bernstein and Vazirani [50] showed that quantum gates of only logarithmic accuracy suffice for polynomial time quantum circuits. Aharonov et al [79] discusses a generalization of quantum circuits to allow mixed states, where measurements can be done in the middle of the computation, and showed that such quantum circuits are equivalent in computational power to standard quantum circuits. This generalized an earlier result of Bernstein and Vazirani [50] that showed that all observation operations can be pushed to the end of the computation, by repeated use of a quantum XOR gate construction. Aharonov et al [80] considered a adiabatic model of quantum computation and showed it is equivalent to standard quantum computation.

• **Computer Simulations of QC.** Obenland, Despain [81, 82, 83] have

given efficient computer simulations of QC, including errors and decoherence, and Cerf, S. E. Koonin [84] have given Monte Carlo simulations of QC.

3.3 Complexity Bounds for QC

3.3.1 Quantum Complexity Classes and Structural Complexity

Berthiaume, Brassard [85] survey open QC structural complexity problems (also see Berthiaume [86]). QC can clearly execute deterministic and randomized computations with no slow down. P (NP, QP, respectively) are the class of problems solved by deterministic (nondeterministic, quantum, respectively) polynomial time computations. Thus QP is the quantum analog of the time efficient class P. It is not known if QP contains NP, that is if QC can solve NP search problems in polynomial time. It is also not known whether QP is a superset of P, nor if there are any problems QC can solve in polynomial time that are not in P (but this is true given quantum oracles; see Berthiaume, Brassard [87,88], Machta [89], van Dam [90, 91] for complexity bounds for computing with quantum oracles).

3.3.2 Bounded Precision QC

Let BQP be the class of polynomial time quantum computations that are computed within bounded error. Most of the algorithms we will mention (such as Shor's) are in the class BQP. [50] showed that BQP computations can be done using unitary operations with a fixed irrational rotation. Adleman et al [92] improved this to show that BQP can be computed using only unitary operations with rational rotations, and that BQP is in the class PSPACE of polynomial space computations of (classical) TMs. Practical implementations of QC most likely will need to be done via unitary transitions within some modest amplitude precision. Bernstein, Vazirani [50] proved that BQP computations running in time T can be done with unitary operations specified by only $O(\log T)$ bits of precision.

3.3.3 Quantum Parallel Complexity Classes

Let NC (QNC, respectively) be the class of (quantum, respectively) circuits with polynomial size and polylogorithmic depth. Thus QNC is the quantum analog of the processor efficient parallel class NC. Moore, Nilsson [93] define QNC and show various problems are in QNC, for example they show that the quantum Fourier transform can be parallelized to linear depth and

polynomial size.

3.4 Bounds on Measurement, Sensing, and Communication

3.4.1 Lower Bounds on Quantum Communication.

Cleve et al [94] prove linear lower bounds for the quantum communication complexity of the inner product function, and give a reduction from the quantum information theory problem to the problem of quantum computation of the inner product. Knill, Laflamme [95] characterize the communication complexity of one qubit.

3.4.2 Interaction-Free Quantum Measurement

A method for *(nearly) interaction-free measurement (IFM)* specifies the design of a quantum optical sensing system that is able to determine with arbitrarily high likelihood if an obstructing body has been inserted into the system, without moving or modifying its optical components; moreover, In the case that the obstructing body is present, IFM uses at most an arbitrarily small multiplicative factor of the input intensity to do the sensing. Kwiat et al [96] (also see [97]) have given a method for IFM which does repeated rounds of measurement to affect small phase changes that eventually determine (via the quantum Zeno effect) whether an obstructing body has

been inserted. Kwiat et al [97] assert their method can be applied to sensing tasks such as photography, but the use of their method for IMF has major practical limitations, since if the obstructing body has not been inserted, then the amount of sensing can be quite large.

3.4.3 Interaction-Free Quantum Sensing

Reif [98] defines *(nearly) interaction-free sensing (IFS)* similarly to IFM, except an upper bound is imposed on both the intensity to do the sensing (which again is an arbitrarily small multiplicative factor of the input intensity) whether or not the obstructing body is present. A quantum optical method for IFS (but not IFM) may be used to do I/O with bandwidth reduced by an arbitrarily small multiplicative factor of the bandwidth required for classical (e.g., conventional optical or electronic) I/O methods Reif [98] proves there is no method for IFS with unitary transformations, and so concludes I/O bandwidth can not be significantly reduced by such quantum methods for sensing. (Also see Holevo [99], Fuchs and Caves [100] for proof that quantum methods can not increase the bandwidth for transmission of classical information.)

3.5 Quantum Compression

Summary. Although as noted above, quantum methods can not increase the bandwidth for transmission of classical information, still in certain cases entangled states can be compressed to fewer qubits. This quantum compression could have important applications in practice, where the number of usable qubits is very limited. Schumacher [101] considered compression and decompression of a noiseless source of n quantum bits (qubits), each sampled independently from a given mixed state quantum ensemble. For such a quantum source, the compression factor obtainable by classical information theory is limited by the Shannon entropy, which in general (except in the case where the quantum ensemble has only orthogonal states) is less than the quantum compression factor given by the von Neumann entropy. In particular, Schumacher [101] proved a *quantum noiseless coding theorem* that states that the source's von Neumann entropy is the number of qubits per source state which is necessary and sufficient to asymptotically (in the limit of large code-block size) encode the output of the source with arbitrarily high fidelity. The quantum noiseless coding of Schumacher has asymptotically optimal fidelity and size; the resulting compressed number of qubits can be far fewer than in the classical case.

Shannon Entropy and the Limitations of Classical Methods for Noiseless Compression. Suppose n characters from a finite alphabet Σ are each sampled independently over some probability distribution p. In classical information theory, the Shannon entropy of each character is $H_S(p) = -\sum_{a\in\Sigma}^{1} p(a)\log p(a)$. A string of these n bits may be losslessly compressed to a bit string of mean length $H_S(p)n$.

The von Neumann entropy and Quantum Noiseless Compression. Following Schumacher [101], we assume there is a finite quantum state ensemble (S,p) which is a *mixed state* consisting of a finite number of qubit states $S = \{|a_0>,\ldots,|a_{|S|-1}>\}$, where each $|a_i> \in S$ has probability p_i. The compressor is assumed to act on blocks of n qubits (so is a block compressor), and is assumed to know this underlying ensemble (S,p). The *density matrix* of (S,p) is an $|S|\times|S|$ matrix $\rho = \sum_{i=0}^{|S|-1} p_i|a_i><a_i|$. The *von Neumann entropy* (see [102,101]) corresponding to (S,p) is $H_{VN}(\rho) = -Tr(\rho log\rho)$. In general, the Shannon entropy $H_S(p)$ is greater than or equal to the von Neumann entropy. These entropies are equal only when the states in S are mutually orthogonal.

An Example: Consider a slightly more complex example of a source consisting of a sequence of n photons polarized randomly, with equal probability of phase 0 or phase angle θ. (e.g., As a very simple example of

a source with low von Neumann entropy, consider N photons polarized randomly, equiprobably at 0 or 1.) In this case, the states are $S = \{|a_0>, |a_1>\}$, where the first state $|a_0> = |0>$, corresponds to phase 0, and the other state $|a_1> = \cos\theta|0> + \sin\theta|1>$ corresponds to phase angle θ, and the probabilities are both $p(0) = p(1) = \frac{1}{2}$. The density matrix is $\rho = \frac{1}{2}|a_0><a_0| + \frac{1}{2}|a_1><a_1| = \frac{1}{2}(|0><0| + (\cos\theta|0> + \sin\theta|1>)(\cos\theta<0| + \sin\theta<1|)) = \frac{1}{2}((1+\cos^2\theta)|0><0| + \cos\theta\sin\theta|0><1| + \cos\theta\sin\theta|1><0| + \sin^2\theta|1><1|)$ which has 2×2 matrix form $\frac{1}{2}\begin{bmatrix} 1+\cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$ over the basis vector $\begin{bmatrix} |0> \\ |1> \end{bmatrix}$. Then we can find an appropriate β which gives a change of basis with new basis states $|0'> = |0>$ and $|1'> = \cos\beta|0> + \sin\beta|1>$, providing a diagonal density matrix $\rho' = \frac{1}{2}\begin{bmatrix} (1+\cos^2\theta) + \cos\theta\sin\theta\tan\beta & 0 \\ 0 & \cos\theta\sin\theta + \sin^2\theta\tan\beta \end{bmatrix}$ over the basis vector $\begin{bmatrix} |0'> \\ |1'> \end{bmatrix}$. Although this source has high Shannon entropy $H_S(p)$, it will have low von Neumann entropy $H_{VN}(\rho)$ in the case of a small magnitude phase angle θ. However, note that the entropies are the same in the special case where $\theta = \pi/2$, so the states $|a_0> = |0>$, $|a_1> = |1>$ are orthogonal and the density matrix is simply the diagonal matrix $\rho = \frac{1}{2}(|0><0| + |1><1|)$ which has a diagonal density matrix $\rho =$

$\begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$ over the basis vector $\begin{bmatrix} |0> \\ |1> \end{bmatrix}$.

For technical reasons, the unitary compression and decompression mappings need to preserve the number of bits (some of which are ignored). An *n-to-n' quantum compressor* is a unitary transformation that maps n-qubit strings to n-qubit strings; the first n' qubits that are output by the compressor are taken as the compressed version of its input, and the remaining $n-n'$ qubits are discarded. An *n'-to-n decompressor* is a unitary transformation that maps n-qubit strings to n-qubit strings; the first n' qubits input to the decompressor are the compressed version of the uncompressed n qubits, and the remaining $n-n'$ qubits are all 0. The *source* to the compression scheme is assumed to be a sequence of n qubits sampled independently from (S,p). The *observed output* is the result of first compressing the input qubits, then decompressing them, and finally measurement of the result (over a basis containing the n inputs) The *compression rate* is n/n' and the *compression factor* is n'/n. The *fidelity* of the compression scheme is the probability the observed output is equal to the original input (that is the probability that the original qubits are correctly recovered, from the compressed qubits). The goal here is a quantum compression with both a high fidelity and a high compression rate.

Example (Continued): Consider again the example of a source consisting of a sequence of n photons polarized randomly, with equal probability of phase 0 or phase angle θ. If θ has small magnitude, then a quantum encoder can compress these photons into an entangled state using just a few photons. Furthermore, a quantum decoder can the recover n photons with the original distribution (with arbitrarily high fidelity for large n) from these compressed photons.

Schumacher Quantum Noiseless Compression. Schumacher [101] gave a *quantum noiseless coding theorem* which provided asymptotically optimal noiseless compression of a sequence of qubits independently sampled from a finite quantum state ensemble (S, p). The quantum noiseless coding theorem states that for any $\epsilon, \delta > 0$ and sufficiently large n, (i) there is an n-to-n' quantum compression scheme with fidelity at least $1 - \epsilon$ and compression to length $n' \leq n(H_{VN}(\rho) + \delta)$, and (ii) any n-to-n'' quantum compression scheme which gives compression to length $n'' \leq n(H_{VN}(\rho) - \delta)$, has fidelity $< 1 - \epsilon$. That is, in the limit of large code-block size, the source's von Neumann entropy $H_{VN}(\rho)$ is asymptotically the number of qubits per source state which is necessary and sufficient to encode the output of the source with arbitrarily high fidelity. Given a known finite quantum state ensemble (S, p), Schumacher's compression scheme assumes a known basis

for which the density matrix ρ is diagonal, with non-increasing values along the diagonal.

The proof of the Schumacher quantum noiseless coding theorem and its refinements by Jozsa and Schumacher [103] and H.Szeto [104] make use of the existence of a *typical subspace* Λ (see [103]) within a Hilbert space of n qubits over a source of von Neumann entropy $H_{VN}(\rho)$. The typical subspace Λ has dimension $\leq 2^{nH_{VN}(\rho)}$ and with high probability, a sample of n qubits has an almost unit projection onto Λ. The Schumacher compressor simply transposes (via a permutation mapping) the subspace Λ into the Hilbert space of a smaller block of $nH_{VN}(\rho)$ qubits. These proofs are not completely constructive.

Bennett [105] gave a constructive presentation of the Schumacher compression. He observes that the Schumacher compression can be done by a unitary mapping to a basis for which the density matrix ρ is diagonal (in certain simple cases the density matrix ρ is already diagonal, e.g., when the input is a set of n identical qubits) followed by certain combinatorial computation which we will call the *Schumacher compression function SCHUMACHER*. The Schumacher compression function SCHUMACHER simply orders the basis states first by the number of ones (from smallest to largest) that are in the binary expansion of the bits and then refines this order by

a lexical sort of the the binary expansion of the bits. That is, all strings with i ones are mapped before all strings with $i + 1$ ones, and those strings with the same number of ones are lexically ordered. Note that for any given value X of the qubits, this transformation SCHUMACHER(X) is simply a deterministic mapping from an n bit sequence to a n' bit sequence defined by a combinatorial computation. In particular, given an n bit binary string X, the transformation SCHUMACHER(X) is the number of n bit strings so ordered before X. It is easy to show that SCHUMACHER(X) is a permutation. Since it is a permutation, it is a bijective function which is uniquely reversible, and also is a unitary transformation. To insure that the overall transformation (for all the states) is a quantum computation, it is essential that the transformation SCHUMACHER(X) be done using only reversible, quantum-coherent elementary operations. (Bennett et al [106] gave a polynomial time quantum algorithm for the related problem of extraction of only classical information from a quantum noiseless coding.) Cleve, DiVincenzo [107] then developed the first polynomial time algorithm for Schumacher noiseless compression of n qubits. In particular, they explicitly computed the bijective function SCHUMACHER(X) and it's reverse using $O(n^3)$ reversible, elementary unitary operations. Up to then, this was the fastest previous algorithm for the Schumacher encoding and decoding functions.

Recently Reif and Chakraborty [108] gave a time efficient algorithm for asymptotically optimal noiseless quantum compression and decompression, costing only $O(n(\log^4 n)\log\log n)$ elementary quantum operations. This modified Schumacher encoding requires the evaluation of various combinatorial sums, for which Reif and Chakraborty provides efficient recursive, reversible quantum algorithms. The coding of [108] employed a modified Schumacher encoding that was still asymptotically optimal in fidelity and size.

The Schumacher quantum noiseless coding theorem assumes the compressor knows the source. Jozsa, et al [109] recently gave a generalization of the Schumacher compression to the case where the compressor does not know the source, thus providing the first asymptotically optimal universal algorithm for quantum compression. Also, Braustein, et al [110] have recently given a fast algorithm for an quantum analog of Huffman coding, but do not provide a proof that this coding gives asymptotically optimal noiseless quantum compression (that is, reaches the von Neumann entropy), as provided by Schumacher compression. ([104] assumes the compressor knows the source, but can be (extended to a asymptotically optimal universal algorithm for quantum compression where the compressor does not know the source, using the techniques of Jozsa, et al [109].)

3.6 Quantum Error Correcting Codes

3.6.1 Quantum Coding Theory.

The qubit can be defined in quantum information theory as the amount of information that can be carried in a quantum system with two basis states, e.g. the internal degree of freedom of a polarized photon. The qubit is thus fundamental unit of quantum channel capacity. Nielsen [111] (Svozil [112,113], Holevo [114], Knill, Laflamme [115,116], Ohya [117], develop a theory of quantum error-correcting codes and quantum information theory), e.g., they give the definition of *quantum mutual entropy* for an entangled state. Buhrman et al [[118], Adami, Cerf [119] contrast quantum information theory with classical information theory. Quantum channel capacity has been investigated for noisy channels (DiVincenzo, et al [120], Holevo [121], Barnum et al [122]), very noisy channels (Shor, Smolin [123]), and quantum erasure channels (Bennett et al [124]). Fuchs [125] showed that nonorthogonal quantum states maximize classical information capacity. (Also, Helstrom [126,127] defines a quantum theory of information detection, and Fuchs [128] defines a related quantum theory of information distinguishability.)

3.6.2 Decoherence Errors in QC.

Quantum decoherence is the gradual introduction of errors of amplitude in the quantum superposition of basis states. All known experimental implementations of QC suffer from the gradual decoherence of entangled states. The rate of decoherence per step of QC depends on the specific technology implementing QC. A significant property of Shor's algorithm is that the precision of the amplitudes in the superpositions need be only a polynomial number of bits. Although the addition of decoherence errors in the amplitudes may at first not have a major effect on the QC, the affect of the errors may accumulate over time and completely destroy the computation. Researchers have dealt with decoherence errors by extending classical error correction techniques to quantum analogs. Generally, there is assumed a decoherence error model where the errors introduced are assumed to be uniform random with bounded magnitude, independently for each qubit.

3.6.3 Quantum Codes.

Shor [129] and Steane [130] gave the first techniques for reducing quantum decoherence, by the addition of extra qubits which are then projected via observation operations to eliminate errors in the superposition. Calderbank,

Shor [131] and Steane [132] then proved that QC can be done with bounded decoherence error, assuming the error correction mechanism is without error itself. Bennett et al [133], Laflamme [134] gave the first optimal 5-qubit codes, leading to asymptotically optimal (for large code blocks) quantum error correction codes. Shor [135] and Kitaev [136,137] extended these techniques to do fault tolerant quantum computation on quantum networks, in the presence of bounded decoherence error, even if the error correction mechanism also suffers from error decoherence errors. A final innovation (Gottesman et al [138], Aharonov, Ben-O [139], Knill et al [140,141]) was concatenated versions of the above quantum codes that allow for arbitrarily long QC in the presence of arbitrary (i.e., not necessarily random) decoherence error below a fixed constant threshold. Current bounds on this threshold are very small, and it seems likely (although it is not yet known) they can be increased to above the decoherance error bounds of experimental techniques for QC.

Also see the texts [38,39] on quantum coding theory.

3.7 Quantum Cryptography

Here we overview Quantum cryptography; also see the following texts: [35,36,37]

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3.7.1 Quantum Keys

Bennett et al [142] and Bennett and Brassard [143] gave the first methods for quantum cryptography using qubits as keys, which were proved to be secure (see however the remarks at the end of this subsection) against certain types of attacks. Surveys of quantum cryptography are given in Bennett, Brassard, Ekert [144], Brassard [145], Bennett, Brassard [146], Brassard [147] , and Gisin 148] . Ozhigov [149] gives a protocol for security of information in quantum databases. Hruby [149] discusses further methods for quantum cryptography. Bennett et al [150] , Hughes et al [151] describes experiments of quantum cryptography, including optical fibers.

Bennett et al [152] gave a protocol for quantum oblivious transfer. Mayers [153] gives quantum oblivious transfer and key distribution protocols and Mayers [154] extends the protocols to noisy channels. Lo, Chau [155] give a quantum key distribution protocol which is unconditionally secure over arbitrarily long distance.

Brassard, Crpeau [156] gave quantum bit commitment and quantum coin tossing protocols. Brassard et al [157] gives quantum bit commitment scheme provably unbreakable by both parties. Yao [158] proved quantum protocols secure against coherent measurements. Brassard et al [159] shows

how to defeat classical bit commitments with a quantum computer. Chau, Lo [160] gives further methods for qubit commitment. Crpeau et al [161] gives protocols for quantum oblivious mutual identification.

Is quantum cryptography actually unbreakable? Unfortunately, some of the methods for quantum cryptography that are claimed to be unbreakable, can in fact be broken by sidestepping assumptions assumed in the proofs of their security. For example, to break the well known quantum cryptography method of Bennett and Brassard [143], Brandt [162] provided a method (experimentally demonstrated by Kim et al [163]) that exploited entanglement of momentum with the phase of photons, making observations of the momentum portions to infer transmitted phases. It is not clear what other prior results in quantum cryptography could be broken by similar techniques, which places the field of quantum cryptography in some doubt. (Also, Lo, Chau [164] have recently argued that quantum bit commitment and ideal quantum coin tossing are impossible in certain cases that are not covered in the above results.)

3.7.2 Distributed Quantum Networks

Future hardware will have to be fast, scalable, and highly parallelizable. A *quantum network* is a network of QCs executing over a spatially distributed

network, where quantum entanglement is distributed among distant nodes in the quantum network. Thus, using *distributed entanglement*, a quantum network distributes the parts of an entangled state to various processors, which can to act on the parts independently. Pellizzari [165] proposes quantum networks using optical fibers, and Cirac, Zoller et al [166], and Bose, Vedral [167] show state transfer distribution can be done among distant nodes. For example, [166] use a cavity QED device that traps atoms in multiple cavities and exchanges photons between the cavities to establish the distributed entanglement. Various basic difficulties were overcome:

– *How can one do state transfer distribution?* Bennett et al [168, 169] , Brassard [170] developed a technique known as *teleportation* to transmit arbitrary input states with perfect fidelity. It does this by separating the input state into classical and quantum components. The input can then be reconstructed from these components with perfect fidelity.

– *How can one cope with communication errors and attenuation in a quantum network?* Wootters, Zurek [171] proved that a single quantum cannot be cloned. (note: Buzek, Hillery [172] recently claimed a universal optimal cloning of qubits and quantum registers in a distributed quantum network, but this seem inconsistent with the no-coning theorem). That no-cloning theorem implies that once a signal becomes attenuated in a an optical fiber

communication channel, then it cannot in general be amplified. Hence it would at first appear that communication and quantum network links may be limited to distances of the order of the attenuation length in the fiber. However, the range of quantum communication could be extended using *quantum repeaters* that do quantum error correction, restoring the quantum signal without reading the quantum information. Ekert, Huelga et al [173] extend the techniques of distributed quantum computation to noisy channels, and showed that for quantum memories and quantum communication, a state can be transmitted over arbitrary distances with bounded error, provided a minimum gate accuracy can be achieved which is a constant factor of this error.

3.8 Other Algorithmic Applications of QC

The early literature in QC provided some examples of QC algorithms for problems constructed for the reasonable purpose of showing that QC can solve some problems more efficiently than conventional sequential computing models. Later, quantum algorithms were developed for variety of useful applications. Also see the texts on quantum algorithms: [40, 27, 28, 29, 30, 31, 32, 33, 34].

• **Quantum Fourier Transforms.** Drutsch, Jozsa [49] gave an $O(n)$ time

quantum algorithm for creating a uniform superposition of all possible values of n bits, which is a *quantum Fourier transform* over the finite field of size 2. Simon [174] used this quantum Fourier transform to gave an efficient time quantum algorithm for determining whether a function over a finite domain is invariant under some XOR-mask. This provided the one of the first examples of a quantum algorithm that efficiently solves an interesting problem that is costly for classical computation. Brassard, Hoyer [175] gave improvements to Simon's algorithm. There have been a number of efficient quantum algorithms for extensions of the quantum Fourier transform: to the approximate quantum Fourier transform (Coppersmith [176]), over various domains (Griffiths, Niu [177], Hoyer [178]), over symmetric groups (Beals [179]), over certain non-abelian groups (Pueschel, Roetteler, Bet [180]), Vedral, Barenco, Ekert [181] give efficient quantum networks for elementary arithmetic operations, using the quantum Fourier transform. Grigoriev [182] used the quantum Fourier transform to test shift-equivalence of polynomials.

• **Quantum Factoring.** The most notable algorithmic result in QC to date is the quantum algorithm of Shor [183].184] (also see a review of the algorithm is given by Ekert and Jozsa [185]) for discrete logarithm and integer factorization in polynomial time (with modest amplitude precision). Shor's algorithm uses an efficient reduction (due to Miller [186]) from integer factor-

ing to the problem of approximately computing the period (length of a orbit) within an integer ring. Shor approximates the period by repeated the use of a quantum Fourier transform over an integer ring and greatest common divisor computations. There has been considerable further work on Shor's quantum factoring algorithm: Zalka [187] improved the time complexity, Beckman et al [188] describe it's execution on quantum networks with small size and depth, Obenland, Despain [189], Plenio, Knight [190] consider the feasibility of executing Shor's quantum factoring algorithm on various quantum computer architectures (the latter provide somewhat pessimistic lower bounds for the factorization time of large numbers on a quantum computer in the presence of decoherance errors.) [191] describes a 7 qubit demonstration of Shor's factorization algorithm using nuclear magnetic resonance. Kitaev [192] gave an independent derivation of Shor's factoring result using a reduction to find an abelian stabilizer.

• **Quantum Search.** Another significant efficient QC algorithmic result is the algorithm of Grover [193], which searches within a data base of size N in time $\sqrt{N}$ (An interesting property of the Grover's algorithm for search is its similarity to the quantum Zeno affect technique for quantum measurement Kwiat et al [96,97]. In particular, the algorithm also uses $O(\sqrt{N})$ stages of unitary operations, each quite similar to a stage of the quantum Zeno

sensing method.) Grover refined his result to require only a single query [194], and to use almost any unitary transformation [195], Zalka [196] showed Grover's algorithm can not be further asymptotically sped up and so is optimal for data base search, and Pati [197] gave further improvements to the bounds. Biron et al [198], extended Grover's algorithm to arbitrary initial amplitude distribution. Cockshott [199] gave fast quantum algorithms for executing more general operations on relational databases, and Benjamin, Johnson [200] discuss the use of Grover's algorithm and related quantum algorithms for other data processing problems. Farhi et al [201] showed that Grover's algorithm could not be extend to quickly determine parity of N bits; in particular they showed that any quantum algorithm for parity takes at least $N/2$ steps. Meyer [202] and Terhal & Smolin [203] propose quantum search algorithms that do not to require entanglement. Brassard et al [204,205] combine the algorithmic techniques of Grover and Shor to give a fast quantum algorithm for approximately counting (i.e., finding the number of matches in a database).

While Grover's algorithm is clearly an improvement over linear sequential search in a data base, it appears less impressive in the case of an explicitly defined data base which needs to be stored in volume N. Parallel computation can do search in a data base of size N in time at most polylogarithmic

with N (that is, in time $O(\log^{O(1)} N)$) by relatively straightforward use of parallel search Moreover, Grover's algorithm may not have a clear advantage even in the case of an implicitly defined data base, which does not need to be stored, but instead can be constructed on the fly (e.g., that arising from NP search methods). In this case, Grover's search algorithm can be used to speed up combinatorial search within a domain of size N to a time bound of $O(\sqrt{N})$, (Hogg [206], Hogg, Yanik [207] investigate similar quantum search techniques for local and other combinatorial search problems), and in this case Grover's algorithm appears to require only volume logarithmic in the search space size N. In contrast, parallel computation takes volume linear in the combinatorial search space, but takes just time polylogorithmic in the search space.

• **Quantum Simulations in Physics.** The first application proposed for QC (Feynman [46]) was for simulating quantum physics. In principle, quantum computers provide universal quantum simulation of any quantum mechanical physical system (Lloyd [208], Zalka[209], Boghosian [210])). Proposed QC simulations of quantum mechanical systems include: many-body systems (Wiesner [211]), many-body Fermi systems (Abrams, Lloyd [212]), multiparticle (ballistic) evolution (Benioff [213]), quantum lattice-gas models (Boghosian, Taylor [214]), Meyer [215,216]), Ising spin glasses (Lidar,

Biham [217]), the thermal rate constant (Lidar, Wang [218], quantum chaos (Schack [219]).

• **Quantum Learning.** QC may have some interesting applications the learning theory and related problems. Bshouty, Jackson [220] describe learning Boolean formulas in disjunctive normal form (DNF) over the uniform distribution of inputs, using a quantum example oracle, and Ventura, Martinez [221] describe a QC learning algorithm for learning DNF using a classical example oracle. Also, Yu, Vlasov [222] describe image recognition using QC, Tucci [223] investigates quantum bayesian networks, and Ventura, Martinez [224] describe a quantum associative memory,

• **Quantum Robotics.** Benioff [225] considers a distributed QC system with mobile *quantum robots* that can carry out carrying out measurements and physical experiments on the environment, and as an example gives an algorithm for the problem of measuring the distance between a quantum robot and a particle on a 1D space lattice. Hogg [206] proposes the use of distributed QC to allow small-scale sensors and actuators to be controlled in a distributed manner. Further discussion of the applications of QC are given by Landauer [226,227].

• **Winding Up Quantum Clocks.** The precision of atomic clocks are limited by the spontaneous decay lifetimes of excited atomic states. An

interesting application of QC proposed by Huelga [228] (also see Bollinger et al [229]) is to extend these lifetimes by using quantum error correcting codes to inhibit the spontaneous decay. A similar idea can be used for improving the precision of frequency standards and interferometers.

• **Quantum Strategies.** Meyer [230, 231] has proposed a class of generalized games that allow for quantum strategies which he proves provide an improvement over conventional mixed (randomized) strategies for certain games.

3.9 Possible Technologies for Doing QC

Here we overview various experimental implementations of quantum computation; also see the texts: [41, 42, 43, 44, 45]. As noted previously, any QC can be realized by a *universal* set of gates consisting of the 2-qubit XOR operation along with some 1-qubit operations. There are two basic approaches known to do QC:

(A) Micromolecular QC. Here QC on n qubits is executed using n individual atoms, ions or photons, and each qubit is generally encoded using the quantized states of each individual atom, ion or photon. The readout (observation operation) is by measurement of the (eigen) state of each individual atom, ion or photon. In the following we enumerate a number of

proposed micromolecular QC methods:

• **Quantum Dots.** Burkard [232], Loss et al [233], Meekhof et al [234] describe the use of coupled quantum dots to do QC. ([80] proposes quantum computation using Cooper pairs.)

• **Ion Trap QC.** Cirac, Zoller [235,236], James [237] proposed using a linear array of cold trapped ions (the ions are trapped by electromagnetic fields) whose energy states are used to store the qubits (also, vibrational modes between consecutive ions also can be used to store states of qubits). The coupling of the qubits is by electrostatic repulsion between the ions. Unitary transitions on superpositions can be executed via an associated array of lasers, each of which pulses a distinct ion; these induce electric dipole moments that determine the transitions. A group at the National Institute of Standards at Boulder, CO (Meekhof et al [234], Wineland et al [238,239] , King et al [240], Turchette et al [241]) and a group at Los Alamos (Hughes [242], Hughes et al [243], James [244]) have experimentally demonstrated trapped ion QC. These and other researchers have addressed various key issues associated with quantum computation with trapped ions:

– Deterministic entanglement of two trapped ions (Turchette et al [241]),

– decoherence bounds (Hughes et al [245] and Plenio, Knight [246]),

– measurement and state preparation, i.e., initialization of the collective

motion of the trapped ions (Schneider et al [247] and King et al [240]),

− coherent quantum-state manipulation of trapped atomic ions (Wineland et al [238,239]),

− heating of the quantum ground state of trapped ions (James [248]) and quantum computation with "hot" trapped ions (Schneider et al [249]).

• **Cavity QED.** A group at Cal Tech (Turchette [250]) have experimentally demonstrated the use of trapped photons in a cavity QED system to execute 2-qubit XOR gates and thus in principle can do universal QC. The qubits are encoded by the circular polarization of photons. interacting. The XOR unitary transitions on superpositions can be executed by resonance between interacting photons in the cavity; The coupling of qubits is via resonance between interacting photons using a Cesium atom also in the cavity, and the coupling is tuned by the spacing of mirrors in the cavity.

• **Photonics.** Various groups Chuang et al [[251, 252] , Torma, Stenholm [253] have experimentally demonstrated QC using optical systems where qubits are encoded by photon phases and universal quantum gates are implemented by optical components consisting of beamsplitters and phase shifters as well as nonlinear media (also see the linear optics QC proposed by Adami, Cerf [254]).

• **Heteropolymer.** This is a polymer consisting of a linear array of atoms,

each of which can be either in a ground or excited energy state. Teich et al [255] first proposed classical (without quantium superpositions) molecular computations using heteropolymer. Later Lloyd [256] extended the use of heteropolymers to QC, using the energy states to store the state of the qubits. The coupling of qubits may be via electric dipole moments which causes energy shifts on adjacent atoms. Unitary transitions on superpositions can be executed via pulses of a laser at particular frequencies; these induce electric dipole moments that determine the transitions.

• **Nuclear Spin.** DiVincenzo [257] Wei et al [258, 259] proposed the use of nuclear spin to do QC; see the remarks following the discussion of Bulk QC.

• **Quantum Propagation Delays.** Castagnoli [260] proposed to do QC using retarded and advanced propagation of particles through various media.

Of these, Ion Trap QC, Cavity QED QC, and Photonics have been experimentally demonstrated up to a very small number of qubits (about 3 bits). The apparent intention of such micromolecular methods for QC is to have an apparatus for storing qubits and executing unitary operations (but not necessarily executing observation operations) which requires only volume linear in the number of qubits. One difficulty (addressed by Kak [261], Murao et al [262]) is *purification of the initial state*: if the state of a QC is initially in an entangled state, and each of the quantum gate trans-

formations introduces phase uncertainty during the QC, then effect of these perturbations may accumulate to make the output to the QC incorrect. A more basic difficulty for these micromolecular methods is that they all use experimental technology that is not well established as might be; in particular their approaches each involve containment of atomic size objects (such as individual atoms, ions or photons) and manipulations of their states. A further difficulty of the micromolecular methods for QC is that apparatus for the observation operation, for even if observation is approximated, seems to require volume growing exponential with the number of qubits, as described earlier in this paper.

(B) Bulk (or NMR) QC. Nuclear magnetic resonance (NMR) spectroscopy is an imaging technology using the spin of the nuclei of a large collection of atoms. Bulk QC is executed on a macroscopic volume containing, in solution a large number of identical molecules, each of which encodes all the qubits. The molecule can be chosen so that it has n distinct quantized spins modes (e.g., each of the n nuclei may have a distinct quantized spins). Each of the n qubits is encoded by one of these spin modes of the molecule. The coupling of qubits is via spin-spin coupling between pairs of distinct nuclei. Unitary operations such as XOR can be executed by radio frequency (RF) pulses at resonance frequencies determined in part

by this spin-spin coupling between pairs of nuclei (and also by the chemical structure of the molecule). Bulk QC was independently proposed by Cory, Fahmy, Havel [263] and Gershenfeld, Chuang [264, 22]. Also see Berman et al [265] and the proposal of Wei et al [259] for doing NMR QC on doped crystals rather than in solutions, and see Kane [266] for another solid state NMR architecture for quantum computing using silicon.

• **Bulk QC** was experimentally tested (Jones et al [267]) and applied to demonstrate the following tasks: quantum search (Jones [268]), approximate quantum counting (Jones, Mosca [269]) Deutsch's problem (Jones, Mosca [270]), Deutsch-Jozsa algorithm on 3 qubits (Linden, Barjat, Freeman [271]).

• **Advantages of Bulk QC:** (i) it can use well established NMR technology and in particular macroscopic devices, The main advantages are (ii) the long time duration until decoherence (due to a low coupling with the environment) and (iii) it currently scales to more qubits than other proposed technologies for QC.

• **Disadvantages of Bulk QC:** A possible disadvantage of Bulk QC is that it appears to allow only a *weak* measurement of the ensemble average which does not provide a quantum state reduction; that is the weak measurement does not alter (at least by much) the superposition of states. Another disadvantage of Bulk QC is that it may require, for a variety of

reasons, macroscopic volumes, and in particular volumes which grow exponential with the number of qubits. Macroscopic volumes may be required for measurement via conventional means. However, known quantum algorithms can still be executed even in this case (e.g., see Gershenfeld, Chuang [264, 22]). So the lack of strong measurement is not a major disadvantage.

Also, Bulk QC requires the initialization to close to a pure state. If Bulk QC is done at room temperature, the initialization methods of Cory, Fahmy, Havel [263] (using logical labeling) and Gershenfeld, Chuang [264, 22](using spatial averaging) yield a pseudo-pure state, where the number of molecules actually in the pure state drops exponentially as $1/c^n$ with the number n of qubits, for some constant c (as noted by Warren [272]). If we approximate the resulting measurement error by a normal distribution, the measurement error is (with high likelihood) at least a multiplicative factor of $1 - c'/\sqrt{N}$, for some constant c'. To overcome this measurement error, we need $1/c^n > c'/\sqrt{N}$, and so we require that the volume be at least $N > (c^n/c')^2$. Hence, for the output of the Bulk QC to be (weakly) measured, the volume (the number N molecules) of Bulk QC needs to grow exponentially with the number n of qubits. Recently, there have been various other proposed methods for initialization to a pure state:

• Barnes [273] proposes the use of very low temperatures,

• Gershenfeld, Chuang [264, 22] suggest the use of gradient fields.

• Knill et al [274] suggest a randomization technique they call temporal averaging.

• Recent work of Schulman, Vazirani [275] provides polynomial volume for initialization, with the assumption of an exponential decrease in spin-spin correlations with the distance between the nuclei located within a molecule (in particular, they assume that the statistical correlation between and two bits on a molecule falls of exponentially with the distance between these bits). Although their methods may provide a solution in practice, known inter-atomic interactions such as the spin-spin correlations are generally considered to be governed by potential force laws which decrease by inverse polynomial powers rather than by an exponential decrease.

It has not yet been experimentally established which of these pure state initialization methods scale to a large number of qubits without large volume.

(Note: Some physicists feel that it has not been clearly established whether: (a) NMR is actually a quantum phenomenon with quantum superposition of basis states, or (b) if NMR just mimics a quantum phenomenon and is actually just classical parallelism, where the quantum superposition of basis states is encoded using multiple molecules where each molecule is in

a distinct basis state. If the latter is true with each molecule is in a distinct basis state, then (see Williams and Clearwater [23]) the volume may grow grow exponentially with the number n of qubits, since each basis state may need to be stored by at least one molecule, and the number of basis states can be 2^n. Also, even if each molecule is in some partially mixed quantum state (see Zyczkowski et al [276]), the volume may still need to grow very large.)

In summary, some possible disadvantages of Bulk QC that may make it difficulty to scale are (i) the inability to do observation (strong measurement with quantum state reduction), (ii) the difficulty to do even a weak measurement without the use of exponential volume, (iii) difficulty (possibly now resolved) to obtain pure initial states without the use of exponential volume, (iv) the possibility that Bulk QC is not a quantum phenomena at all (an unresolved controversy within physics), and so may require use of exponential volume.

It is interesting to consider whether NNR can be scaled down from the macroscopic to molecular level. DiVincenzo [257], Wei et al [258, 259] propose doing QC using the nuclear spins of atoms or electrons in a single trapped molecule. The main advantages are (i) small volume and (ii) the long time duration until decoherence (an advantage shared with NMR). The

key difficulty of this approach is the measurement of the state of each spin, which does not appear to be feasible by the mechanical techniques for detection of magnetic resonance usual used in NMR, which can only do detection of the spin for large ensembles of atoms.

3.10 Resource Bounds

In this paper, we have discussed many applications of quantum computation which provide advantages over classical methods of computation. Certain of the applications of QC (e.g., quantum cryptography) require only a small or constant number of qubits, where as other applications (e.g., factoring and data base search) require a large number of qubits and moreover require an observation operation at least as the final step of the QC. For these advantages to be practical, we need to determine that there are no unfeasible large resources required by QC. Hence we complete the paper with a review of the resource bounds of quantum computing as compared with the resources required by classical methods for computation. In particular, we will conclude that for the advantages of QC (with a large number of qubits) to be practical for applications requiring a large number of qubits, there needs to be determined (theoretical and practically) bounds on the volume required of observation operations. This seems to us a major missing element in the

field of QC.

The energy consumption, processing rate, and volume, are all important resources to consider in computing devices. Conventional (classical) electronic supercomputers of the size of a work station operate in the range of 10^{-9} Joules per operation, at up to about 50 giga-ops per second, with memory of about 10 to 100 giga-bytes, and in a volume of about 10 cm^2. The volume scales as the number of bits of storage.

• **Energy Bounds for QC.** The conventional linear model of QC allows only unitary state transformations and so by definition is reversible (with the possible exception of the observation operation which does quantum state reduction). Benioff [48] noted that as a consequence of the reversibility of the unitary state transformations of QC, these transformations dissipate no energy. But this does not consider (i) the precision of the amplitudes to be preserved nor (ii) the expected time duration required to drive the operation to completion. Gea-Banacloche [277] and Ozawa [278] independently derived lower bounds on the energy needed to execute, within a given precision of the amplitudes and in a given time, an elementary qubit logical operation on a quantum computer. They derived energy lower bounds depending inversely on the time duration for the operation and on the precision of the amplitudes to be preserved. Hence, for polynomial time quantum

computations requiring polynomial relative precision, the lower bounds on energy are polynomial, though the constant factors could be a limitation for practical implementations. (Recall that Bernstein, Vazirani [50] proved that BQP computations can be done with unitary operations specified by only logarithmic bits of precision, which corresponds to relative precision ϵ where $\epsilon > 1/n^{O(1)}$.) These energy lower bound results were stated to be independent of the nature of the physical system encoding the qubits, and under what the authors claimed was normal circumstances in a wide variety of conditions for implementations of quantum computers. Nevertheless, the matter is still appears not to becompletely resolved, since there may be physical implementations of quantum computers that do not abide by their assumptions. Energy bounds for the quantum qubit logical operations require better understanding and study, particularly with respect to their dependance on the technology used.

• **Processing Rate of QC.** The rate of execution unitary operations in QC depend largely on the implementation technology (see Section 3.9); certain techniques can execute unitary operations in microseconds (e.g., Bulk NMR) and some might execute at microsecond or even picosecond rates (e.g., photonic techniques for NMR) The time duration to do observation can also be very short, but may be highly dependant on the size of the measuring

apparatus and on the required precision (see the below discussion on the observation operation and its volume).

• **Volume Bounds for QC.** We now consider (perhaps more closely than usual in the quantum literature) the volume bounds of QC. Potentially, the modest volume bounds of QC may be the one significant advantage over classical methods for computation. Due to the *quantum parallelism* (i.e., the superposition of the basis states allow each basis state to exist in parallel), the volume would at *appear* to be no more that the number of qubits. This may be true, but there are a number of substantial issues that need to be carefully considered. Recall the observation operation both provides a measurement of a qubit with a resulting state reduction. However, the QC literature has not yet carefully considered the volume bounds for the observation operation and as we shall see, it is not yet at all clear what the volume is required. In spite of major works on the mathematical and physical foundations of quantum observation, the precise nature of quantum state reduction via a strong quantum measurement remains somewhat of a mystery. Two distinct approaches to the mathematical and physical foundations of observation have been developed:

(a) The *Copenhagen Formulation,* where the observation is simply *an assumed basic operation* and is considered to be done by a macroscopic mea-

suring device, and

(b) The *Von Neumann Formulation* [102, Chapter 4: Macroscopic Measurement], which views the measuring apparatus as well as the quantum system measured as both part of a quantum system. Hence the evolution of the system (and resulting experimental predictions) can be distinct from that predicted by the Copenhagen formulation of observation (which does not take this into account since the measuring apparatus is assume in their formulation to be very large).

See Cerf and Adami [18] for a comparison the Copenhagen and Von Neumann formulations and see Hay and Peres [279] for an example of this difference. In summary, the Copenhagen and the von Neumann formulations for observation differ in the assumed context (macroscopic or microscopic measurement apparatus). (Note: Attempts to rectify the difference between the Copenhagen and the von Neumann formulation for observation are given in Hay and Peres [279] and in Zurek [280], but it appears not yet resolved.) The Copenhagen formulation for observation is generally used in the context of quantum physics experiments which use macroscopic measurement apparatus. However, the Copenhagen formulation does not seem to be applicable in the context of a microscopic measurement apparatus, which is so small that it is subject to quantum effects (and thus is within a unitary

quantum system). So the Copenhagen formulation for observation may not appropriate for molecular size QC. Although the von Neumann formulation of observation is not relevant to the vast majority of physics experiments (since their experiments generally use large measuring apparatus and small number of degrees of freedom (qubits)), nevertheless the von Neumann formulation for observation appears to be appropriate for molecular size QC. It is possible that the volume for quantum observation apparatus grows very quickly with the number of qubits in the von Neumann formulation. In particular, no one has proved a upper bound on the volume for quantum observation (as a function of the number of qubits) assuming the von Neumann formulation. See the Appendix for a further discussion of the problem of determining volume bounds for the observation operation in QC.

3.11 Conclusion and Acknowledgments

We have overviewed the field of quantum computing and surveyed its major algorithmic results and applications as well as physical implementations and their limitations. Some of issues such as energy costs as well as volume of observation apparatus for quantum computing are still unresolved, and the later are further addressed in the Appendix.

Acknowledgments. We would like to thank G. Brassard for his clear explanation of numerous results in the field of QC. Also, I would like to thank P. Shor and U. Vazirani for references and illuminating discussions on quantum computation, and in particular on the issue of volume bounds for quantum observation.

3.12 Appendix: Volume of Observation Apperatus for Quantum Computing

Here we discuss the challenge of providing volume bounds for observation apperatus when doing QC.

3.12.1 A Potentially Fallacious Proof of Small Volume

First we note that one might be tempted to give a constructive proof, that observation can be done on n qubits in small volume, along the following lines:

(i) *Basis Step.* We begin with a simple, well established experimental method for observation of a single qubit in small quantum system with say n_0 qubits, for a constant n_0. There are many other examples of experimentally verified methods for observation, using macroscopic measurement apparatus. (For example, a number of proposed QC architectures (e.g., the Cirac

and Zoller [232,233] proposed ion trap QC and Kane's [262] silicon-based NMR QC) give specific descriptions of measuring apparatus that have been experimentally verified for observation of a single qubit within a quantum computing systems with a constant number of qubits. While their measuring apparatus is macroscopic, it still must have just some finite volume.

(ii) *Inductive Step.* However, then we just scale up by using the same experimental apparatus to do observation on each of n qubits (that is, repeating the observation for each of the other qubits). This seems to result in a small volume (perhaps even liner size) apparatus for observation.

The potential fallacy of this line of argument is that:

(a) In the basis step, the experiments of [232,262] did not provide bounds on the errors (or fidelity) of the measurement as a function of the volume of the measuring apparatus.

(b) The inductive step fails to take into account quantum effects involving both the measuring apparatus and the n qubits, as might be predicted by the von Neumann formulation of quantum measurement in the case where the measuring apparatus is so small that it is subject to quantum effects.

That is, there needs to be given, in addition to the experimental description (which is only established for n_0 qubits):

(iii) *A mathematical analysis of the quantum effects (in the context of a*

closed unitary system) involving the measuring apparatus as the number n of qubits grows large. In particular, there need to be determined bounds on the errors (or fidelity) of the measurement as a function of the size of the measuring apparatus.

Without this crucial final element, the proof is certainly not complete. Since the observation operation is not reversible, such a proof (in the context of a closed unitary system) seem unlikely to be obtainable.

3.12.2 Possible Experimental Demonstrations of Measurement:

Another approach would be to experimentally test a proposed small volume apparatus for observation on n qubits for moderate size n (say, in the rang of a few hundred, which is required for a nontrivial factoring computation). But the experimental evidence of the volume bounds for observation is unclear, since the QC experiments have not yet been scaled to large or even moderate numbers (say dozens) of qubits, and there are few if any physics experiments for this case. (Shnirman, Schoen [123] describe the use of a single-electron transistor to perform quantum measurements, D'Helon, Milburn [281] describe quantum measurements with quantum computers, and Ozawa [282] describes methods for nondestructive (known as *nondemolition*) quantum measurements of certain quantum computations.)

Hence, at this time that there appears to be *neither a mathematical proof nor an experimental demonstration* (for even a moderately large number of qubits n) *that observation can be done in small volume* (in a closed quantum system). Thus at this time, there is no evidence (either mathematical or experimental) that QC using measurement scales to large numbers of qubits with small volume.

We first consider a number of related questions concerning measurement and quantum state reduction:

3.12.3 Is a Quantum Observation Instantaneous?

It appears not. Brune et al [283] describe the progressive decoherence of the meter in a quantum measurement.

3.12.4 Is an Observation Always Reversible?

It appears the answer be both no (in a narrow mathematical sense of a state reduction), yes (for small closed state spaces), and no (in a practical sense for entanglements in a large state space):

- By the strict mathematical definition of the state reduction due to observation, in general an observation is not reversible. Under what conditions is a measurement reversible in the strict mathematical sense?

That is, when can we measure classical information from a quantum source (yielding a set of pure states with their probabilities with a reduction of quantum entropy), but later be able to reverse this process to regenerate the entangled source state ? Bennett et al [106] show that this is possible in the very special case where the source states can be partitioned into two or more mutually orthogonal subsets. (Other necessary and sufficient conditions for measurements to be reversible have been proved in Bennett, et al [106] and Chuang, Yamamoto [284] describe how to regenerate a qubit if it has observable error.)

- There is experimental evidence that the physical execution of some reductions via measurement are in fact reversible (at least in very small closed systems). Mabuchi, Zoller [285] have observed inversions of quantum jumps in very small quantum-optical systems under continuous observation, and Ueda [286] compares the notions of mathematical and physical reversibility.

- On the other hand, in the case of entanglements in a large state space, even if a measurement is in principle reversible in a closed system due the reversible nature of the diffusion process, the likelihood of such a reverse to the original state, within a moderate (say polynomial in

n) time duration, appears to drop exponentially with the number of qubits n. Gottfield [287] Diosi, Lukacs [288] (also see Pearle [289,290]) explain quantum state vector reduction via strong measurement as a physical process, e.g, state diffusion into the atoms of the measurement apparatus. This diffusion due to reduction may be modeled by a system similar to a rapidly mixing markov system in probability theory, which seems to provide a very low (dropping exponentially with n) likelihood for reversibility within a polynomial time duration. (Others have modeled measurement by a nonlinear interactions with the environment, which are irreversible.)

3.12.5 Avoiding Observation Operations ?

An alternative approach is to completely avoid observation operations on the basis that the observation operation is not actually essential to many quantum computations. (This seems somewhat surprising, given the extensive use of the observation operation in the QC literature for both algorithms and quantum error correction.) Bernstein and Vazirani [50] (by showing that any given observation operation can be delayed to future steps by use of the using XOR operation) proved that all observation operations can be delayed to the final step of a quantum computation. For a small $\epsilon > 0$, let some

particular qubit (of the linear superposition of basis states) be *ϵ-near classic* if had the qubit been observed, the measured value would be a fixed value (either be 0 or 1) with ϵ probability. Suppose the output of a QC consists of the observation of a subset S of the qubits; the resulting reduced superposition will be termed the *output superposition*. Bernstein and Vazirani [50] and Brassard et al [172,201] observe that any QC can be repeated to insure the output qubits are ϵ-near classic in the final output superposition after the repetitions. Note that if a QC with bounded amplitude precision is reduced by an observation, the output qubits yield the correct value with high likelihood. Hence we may consider simply not doing the observation reduction to a basis state in the final step; in place of this (reduced) output superposition we simply output the non-reduced quantum state superposition of the QC that exists just prior to the final observation step. This alternative approach can entirely eliminate the observation operation from many quantum computations, and so provides small volume, but has the drawback of providing a non-classic output consisting of a non-reduced quantum state superposition. The potential difficulty with this approach is as follows: if this (non-reduced quantum state superposition) output is then processed by a classical computing machine, it may propagate unwanted quantum effects to the classical computing machine.

3.12.6 Approximate Observation Operations?

An approach to this difficulty is to only do the observation operation approximately within accuracy ϵ; this may suffice for many QC applications. However, even if the observation operation is done ϵ-approximately by unitary operations, it appears to require a number of additional qubits n' growing exponentially with the n, the original number of qubits of the QC. In fact, we know of no upper bound on n' better than $2^n \log(1/\epsilon)$.

3.12.7 Why the Volume Required by Observation Apperatus May Not Be Small.

We next consider whether it is reasonable to expect that a mathematical proof (or such experimental demonstrations) of small volume quantum observation will ever be done. We provide an informal argument (it should be emphasized that the following is not a formal proof in any sense) that even an ϵ-approximate observation can not be done in polynomial time using small volume, where ϵ is the inverse of a polynomial. Since for n qubits, the size of the basis state space grows as 2^n in the general case, it seems reasonable to assume (e.g., where the physics of the strong measurement is modeled by a diffusion process [287,288,289] that is rapidly mixing) that the likelihood

of reversibility within polynomial time bounds drops exponentially with the number n of qubits. Thus, in the context of polynomial time computations, the ϵ-approximate observation is assumed irreversible with high likelihood.

The argument will hinge on the assumption, made by conventional formulations of quantum computation, that quantum computation (including both unitary qubit operations as well as the non-unitary observation (or projection) operation) can be exectuted for any given number n qubits, which makes an implicit assumption that both unitary and non-unitary operations can be executed at any scale.

Let us also assume that the number of qubits n is small (at most a few hundreds). For sake of contradiction, let us for the moment suppose that (i) quantum computing scales to at least moderate size (say a few tens of thousands of qubits), and (ii) an ϵ-approximate observation operation can be done on one of n qubits by a microscopic measuring device of size $n' = n^c$, for a constant c, and operating within time polynomial in n. Since n is small, the measuring device is surely of sufficiently small size so that it's physics is consistent with established quantum physics (for observe that if quantum computing is to scale to at least moderate size n', then surely quantum effects need to hold for molecules of size n'). This implies we need to view the apparatus for the observation as executing polynomial

time unitary quantum computation, which is reversible, so the reverse of the observation also executes in quantum polynomial time. Hence we have an apparent contradiction, since we have assumed the ϵ-approximate observation is not reversible in polynomial time. (Note. This argument does not require that the governing physical laws shift at some definite size from a quantum-mechanical paradigm to a classical paradigm; instead the argument requires that if the quantum-mechanical paradigm is valid at size n then it also is valid at some what larger size $n' = n^c$.)

Due to informal nature of this argument, it only provides partial evidence that (with the above assumption), QC with the observation operation does not scale to a large number of qubits within small volumes, and in particular that a polynomial time ϵ-approximate observation operation requires very large volume and can not be done at the micromolecular scale for moderate large n. We feel our above argument is far too informal to provide a resolution of the issue. It remains a major open problem in QC to *provide a formal proof that either (i) there is large volume required for observation or (ii) there is not.*

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Note: QCQC 98 is an acronym for: Proc. of 1st NASA Workshop on Quantum

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