# Machine Learning Lecture 9: Introduction to Deep Learning

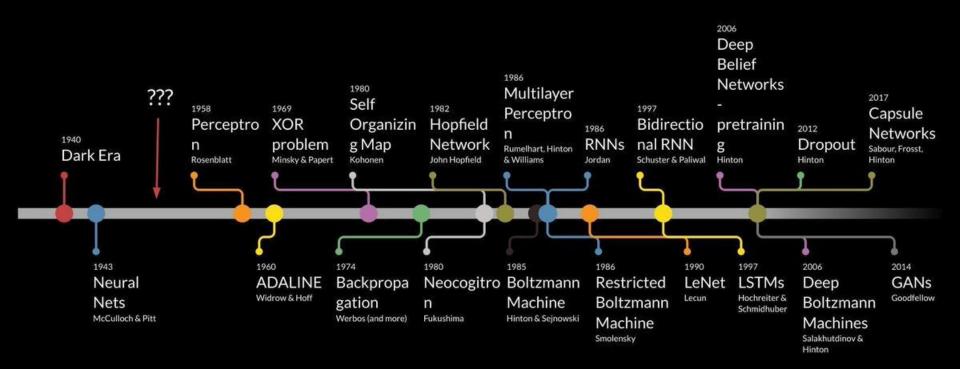
Harbour.Space University February 2020

Radoslav Neychev

#### Outline

- Neural Networks in different areas. Historical overview.
- 2. Backpropagation.
- 3. Playground.
- 4. More on backpropagation.
- 5. Activation functions.
- 6. PyTorch practice

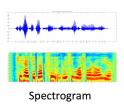
#### **Deep Learning Timeline**

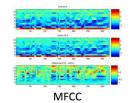


#### **Audio Features**

# Real world problems



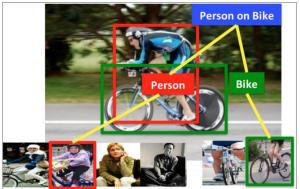




- Object detection
- Action classification
- Image captioning
- •



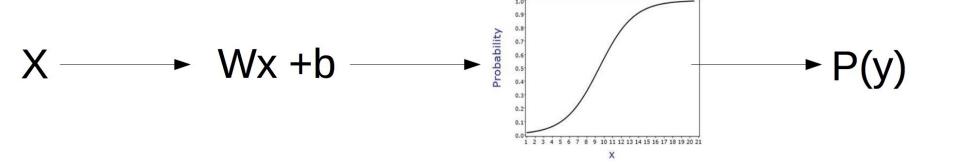






"man in black shirt is playing guitar."

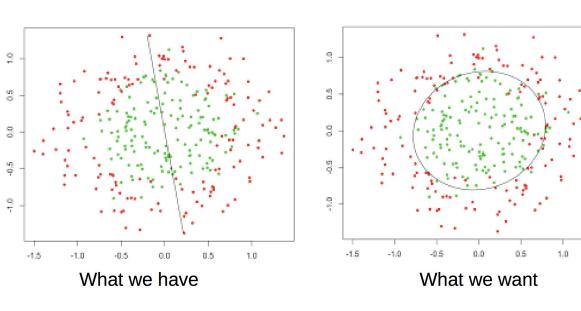
# Logistic regression



$$P(y|x) = \sigma(w \cdot x + b)$$

$$L = -\sum_{i} y_{i} \log P(y|x_{i}) + (1 - y_{i}) \log (1 - P(y|x_{i}))$$

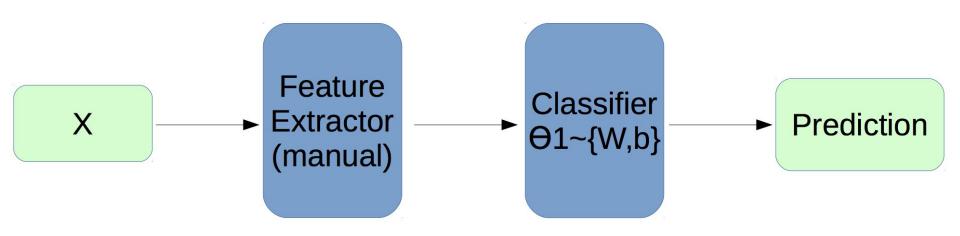
#### Problem: nonlinear dependencies



Logistic regression (generally, linear model) need feature engineering to show good results.

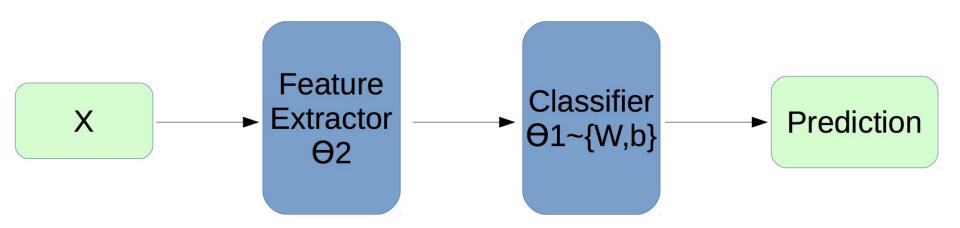
And feature engineering is an *art*.

#### Classic pipeline



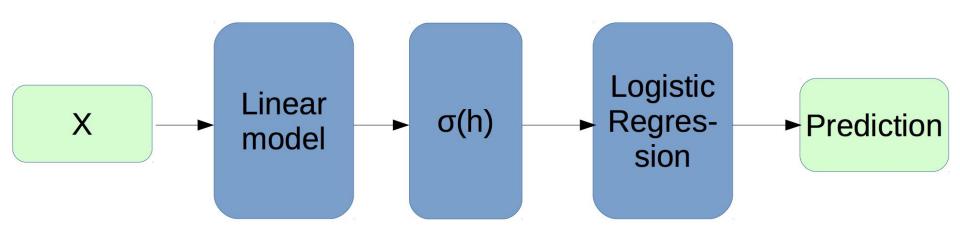
Handcrafted features, generated by experts.

#### NN pipeline



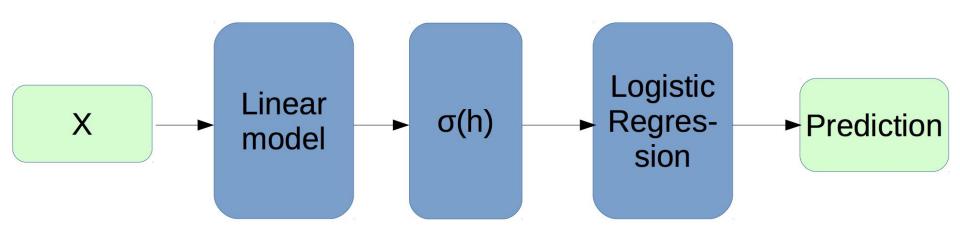
Automatically extracted features.

#### NN pipeline: example



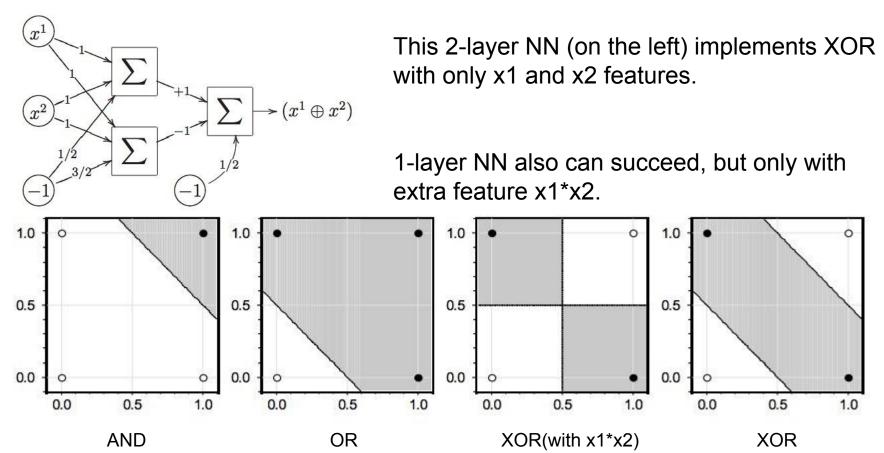
E.g. two logistic regressions one after another.

#### NN pipeline: example



Actually, it's a neural network.

#### XOR problem



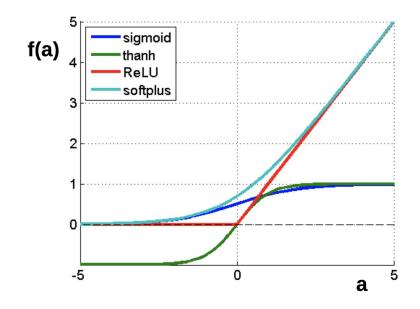
#### Activation functions: nonlinearities

$$f(a) = \frac{1}{1 + e^a}$$

$$f(a) = \tanh(a)$$

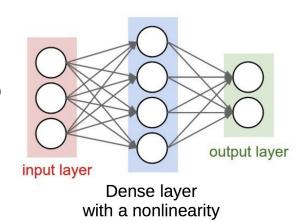
$$f(a) = \max(0, a)$$

$$f(a) = \log(1 + e^a)$$



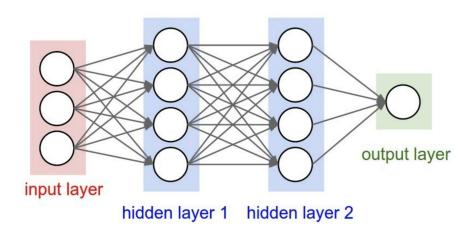
#### Some generally accepted terms

- Layer a building block for NNs :
  - o Dense layer: f(x) = Wx+b
  - Nonlinearity layer:  $f(x) = \sigma(x)$
  - Input layer, output layer
  - A few more we will cover later
- Activation function function applied to layer output
  - Sigmoid
  - o tanh
  - ReLU
  - Any other function to get nonlinear intermediate signal in NN
- Backpropagation a fancy word for "chain rule"

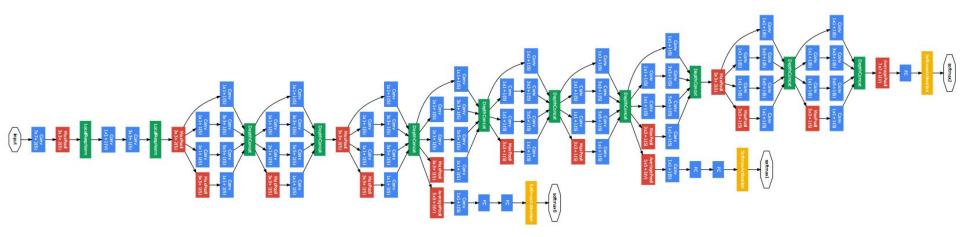


"Train it via backprop!"

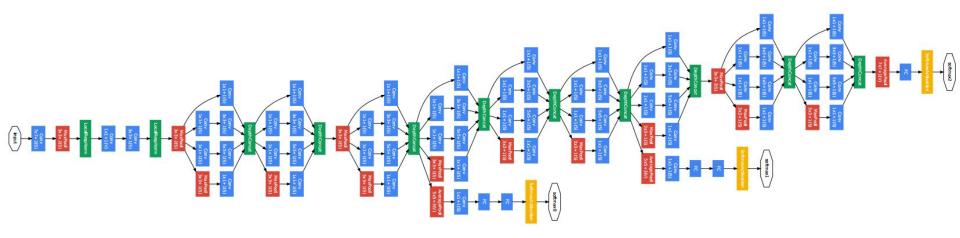
#### Actually, it can be deeper



#### Much deeper...



#### Much deeper...

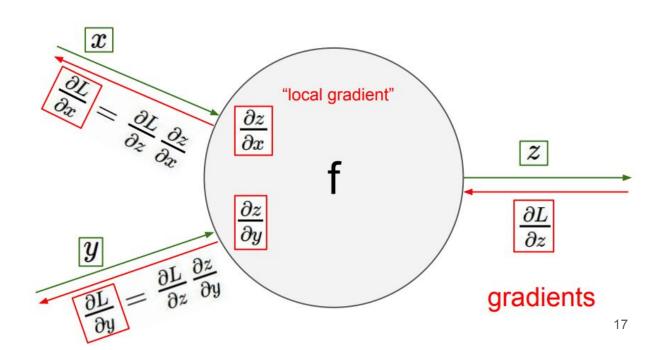


How to train it?

#### Backpropagation and chain rule

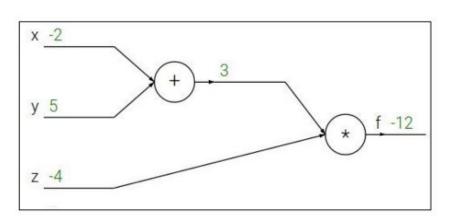
Chain rule is just simple math:  $\frac{\partial L}{\partial x} = \frac{\partial L}{\partial z} \frac{\partial z}{\partial x}$ 

Backprop is just way to use it in NN training.



source: <a href="http://cs231n.github.io">http://cs231n.github.io</a>

$$f(x, y, z) = (x + y)z$$
  
e.g. x = -2, y = 5, z = -4

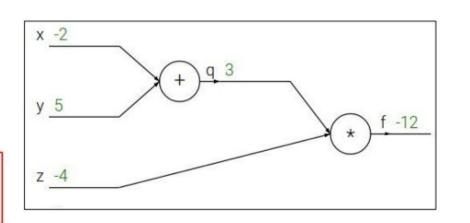


$$f(x, y, z) = (x + y)z$$
  
e.g. x = -2, y = 5, z = -4

$$q=x+y \qquad rac{\partial q}{\partial x}=1, rac{\partial q}{\partial y}=1$$

$$f=qz$$
  $rac{\partial f}{\partial q}=z, rac{\partial f}{\partial z}=q$ 

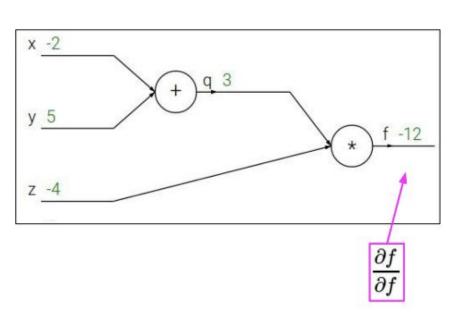
Want: 
$$\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$$



$$f(x, y, z) = (x + y)z$$
  
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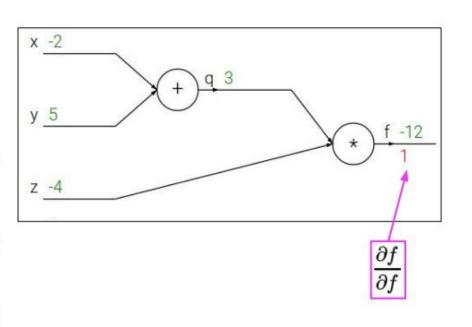
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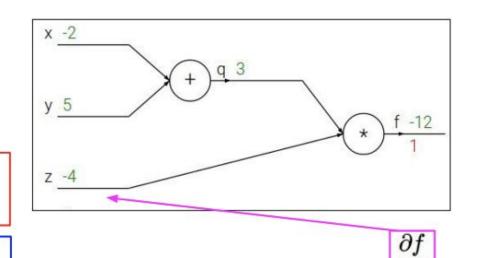
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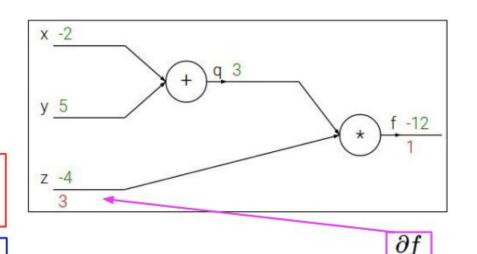
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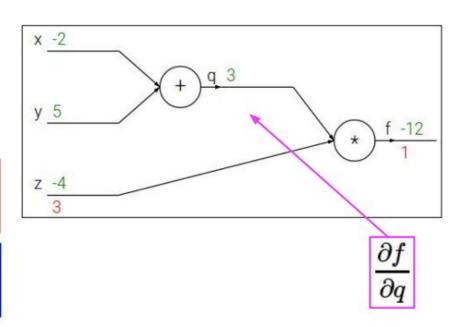
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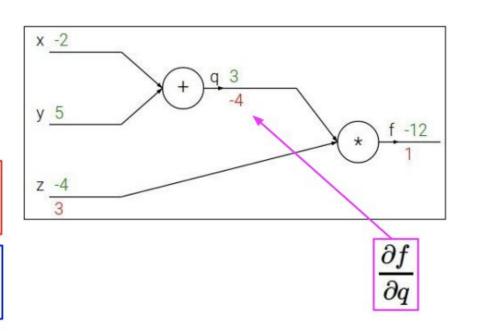
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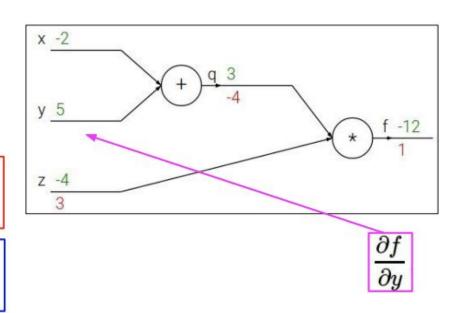
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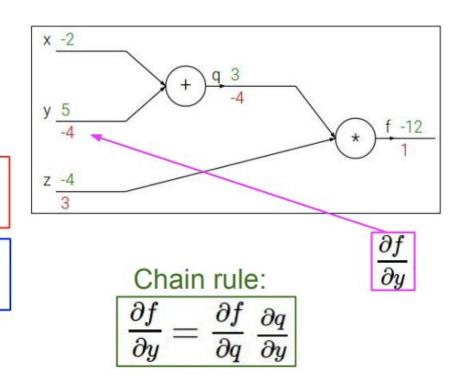
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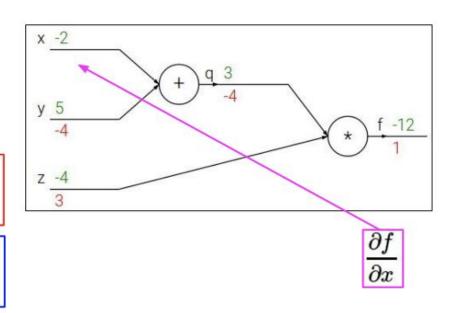
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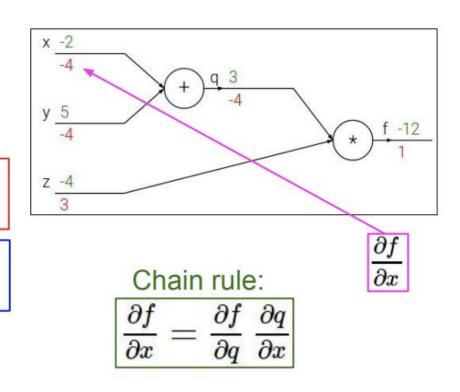
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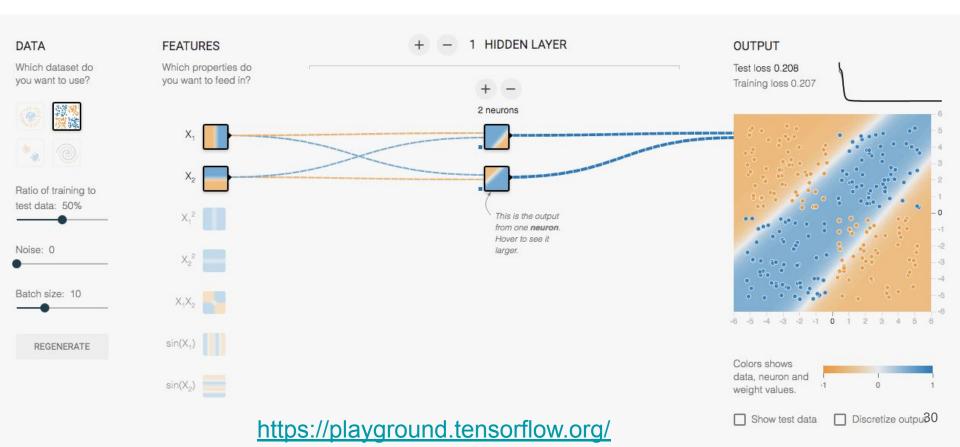
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#### Practice time: interactive playground

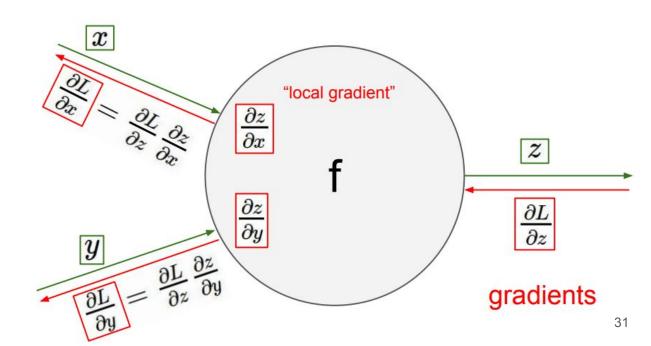


#### Backpropagation and chain rule

Chain rule is just simple math:

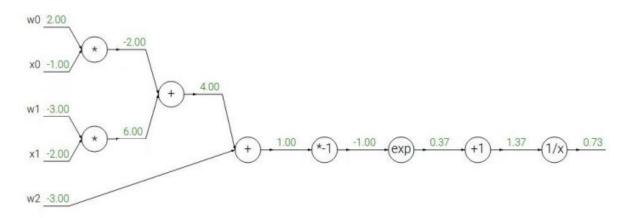
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Backprop is just way to use it in NN training.



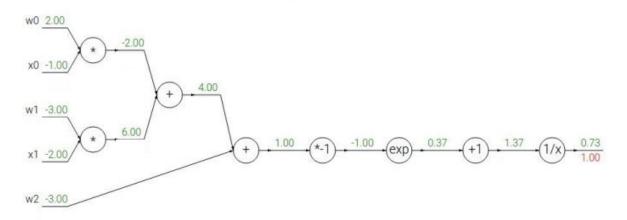
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Another example: 
$$f(w,x) = \frac{1}{1 + e^{-(w_0 x_0 + w_1 x_1 + w_2)}}$$



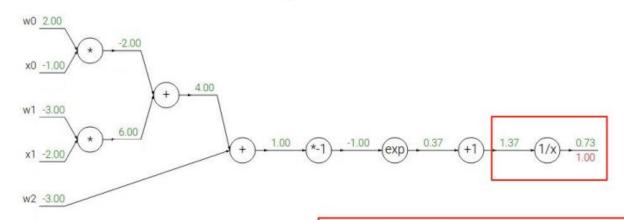
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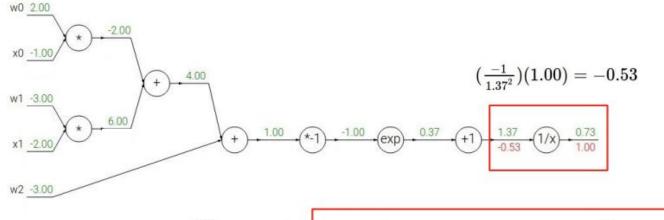


$$egin{array}{lll} f(x)=e^x & 
ightarrow & rac{df}{dx}=e^x & f(x)=rac{1}{x} & 
ightarrow & rac{df}{dx}=-1/x^2 \ f_a(x)=ax & 
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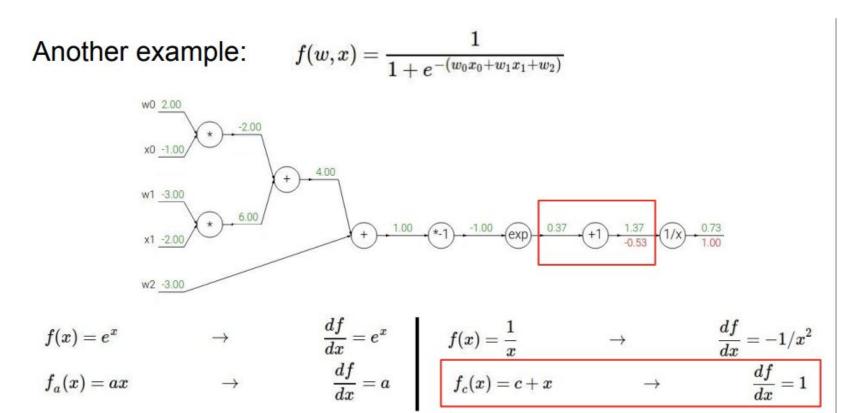
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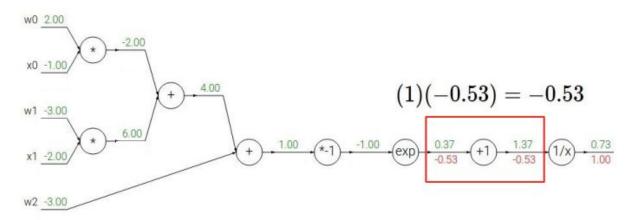


$$f(x)=e^x \hspace{1cm} o \hspace{1cm} rac{df}{dx}=e^x \hspace{1cm} f(x)=rac{1}{x} \hspace{1cm} o \hspace{1cm} rac{df}{dx}=-1 \ f_c(x)=c+x \hspace{1cm} o \hspace{1cm} o \hspace{1cm} rac{df}{dx}=-1 \ f_c(x)=c+x \hspace{1cm} o \hspace{$$



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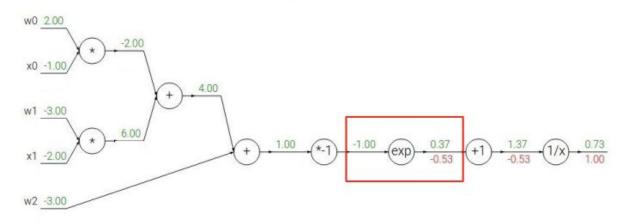
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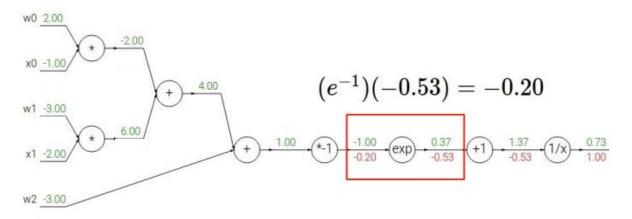
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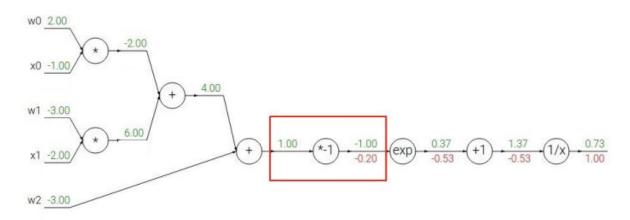
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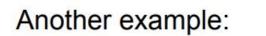
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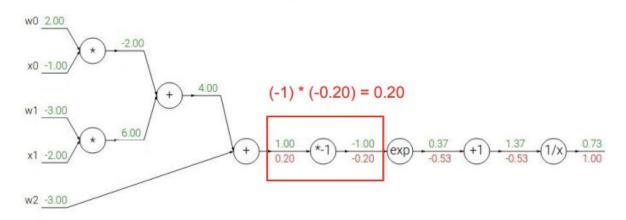


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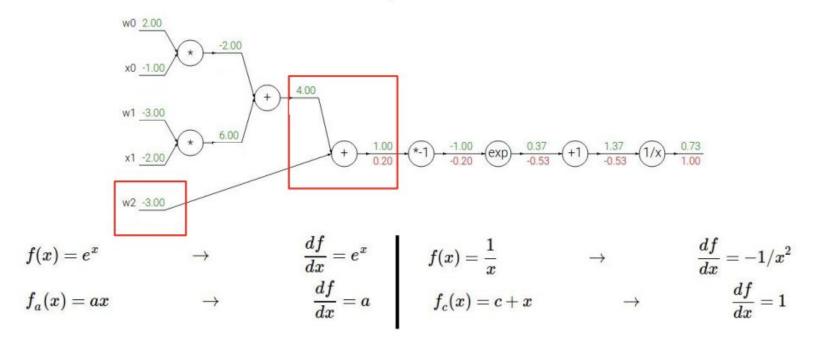
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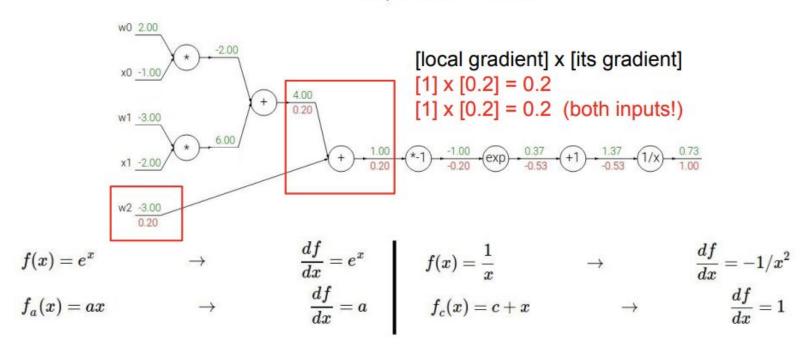
Another example: 
$$f(w, y)$$

$$f(w,x)=rac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$



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$$f(w,x)=rac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$

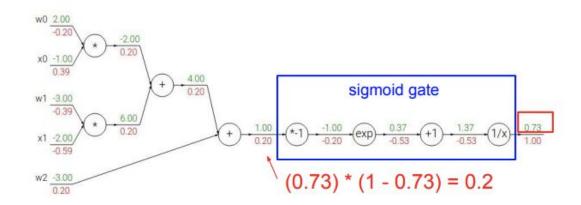


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Another example: 
$$f(w,x) = \frac{1}{1 + e^{-(w_0x_0 + w_1x_1 + w_2)}}$$
 [local gradient] x [its gradient] x [its

45

$$f(w,x)=rac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$
  $\sigma(x)=rac{1}{1+e^{-x}}$  sigmoid function  $rac{d\sigma(x)}{dx}=rac{e^{-x}}{(1+e^{-x})^2}=\left(rac{1+e^{-x}-1}{1+e^{-x}}
ight)\left(rac{1}{1+e^{-x}}
ight)=(1-\sigma(x))\,\sigma(x)$ 

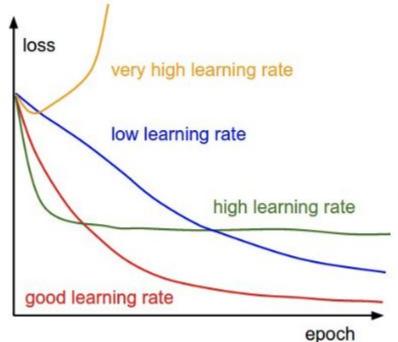


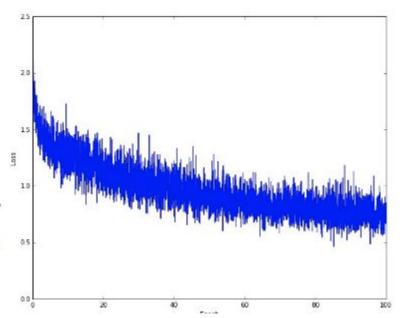
# Gradient optimization

Stochastic gradient descent (and variations)

is used to optimize NN parameters.

 $x_{t+1} = x_t - \text{learning rate} \cdot dx$ 





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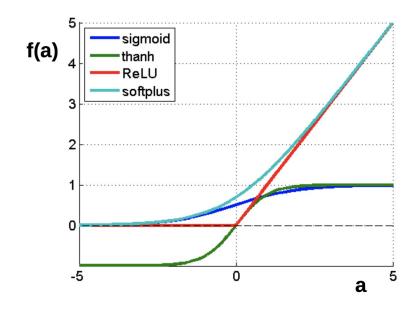
#### Once more: nonlinearities

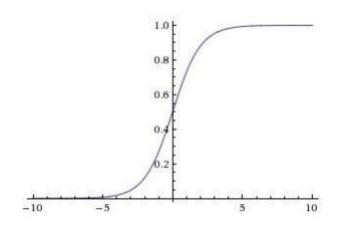
$$f(a) = \frac{1}{1 + e^a}$$

$$f(a) = \tanh(a)$$

$$f(a) = \max(0, a)$$

$$f(a) = \log(1 + e^a)$$





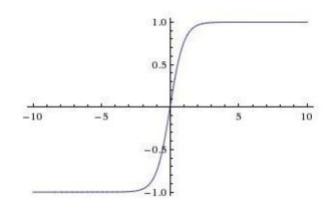
## **Sigmoid**

$$f(a) = \frac{1}{1 + e^a}$$

- Squashes numbers to range [0,1]
- Historically popular since they have nice interpretation as a saturating "firing rate" of a neuron

#### 3 problems:

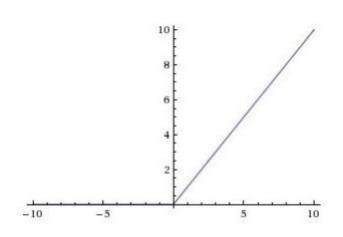
- Saturated neurons "kill" the gradients
- Sigmoid outputs are not zerocentered
- 3. exp() is a bit compute expensive



- Squashes numbers to range [-1,1]
- zero centered (nice)
- still kills gradients when saturated :(

#### tanh(x)

$$f(a) = \tanh(a)$$

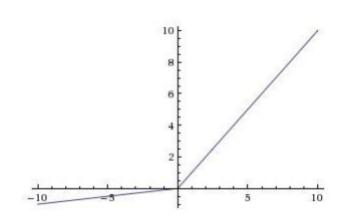


# ReLU (Rectified Linear Unit)

$$f(a) = \max(0, a)$$

- Does not saturate (in +region)
- Very computationally efficient
- Converges much faster than sigmoid/tanh in practice (e.g. 6x)
- Not zero-centered output
- An annoyance:

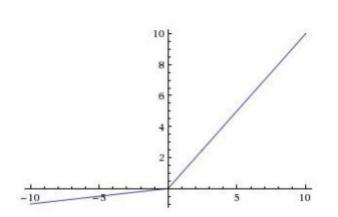
hint: what is the gradient when x < 0?



- Does not saturate
- Computationally efficient
- Converges much faster than sigmoid/tanh in practice! (e.g. 6x)
- will not "die".

## Leaky ReLU

$$f(x) = \max(0.01x, x)$$



#### Leaky ReLU

$$f(x) = \max(0.01x, x)$$

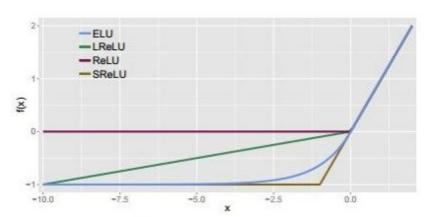
- Does not saturate
- Computationally efficient
- Converges much faster than sigmoid/tanh in practice! (e.g. 6x)
- will not "die".

#### Parametric Rectifier (PReLU)

$$f(x) = \max(\alpha x, x)$$

backprop into \alpha (parameter)

#### **Exponential Linear Units (ELU)**



$$f(x) = \begin{cases} x & \text{if } x > 0 \\ \alpha (\exp(x) - 1) & \text{if } x \le 0 \end{cases}$$

- All benefits of ReLU
- Does not die
- Closer to zero mean outputs
- Computation requires exp()

## Activation functions: sum up

- Use ReLU as baseline approach
- Be careful with the learning rates
- Try out Leaky ReLU or ELU
- Try out tanh but do not expect much from it
- Do not use Sigmoid

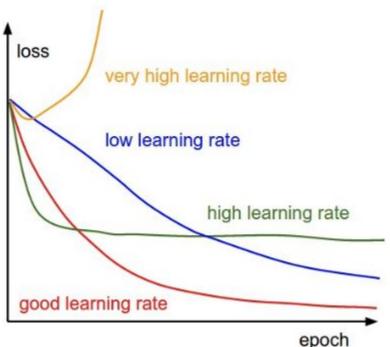
## That's all. Time to build some NN.



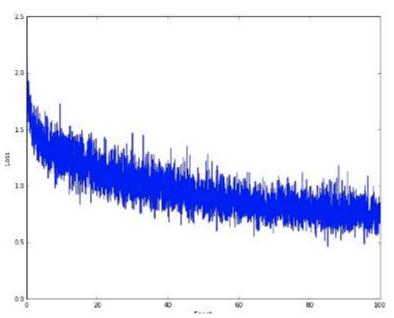
# Backup

# **Optimizers**

Stochastic gradient descent is used to optimize NN parameters.



 $x_{t+1} = x_t - \text{learning rate} \cdot dx$ 

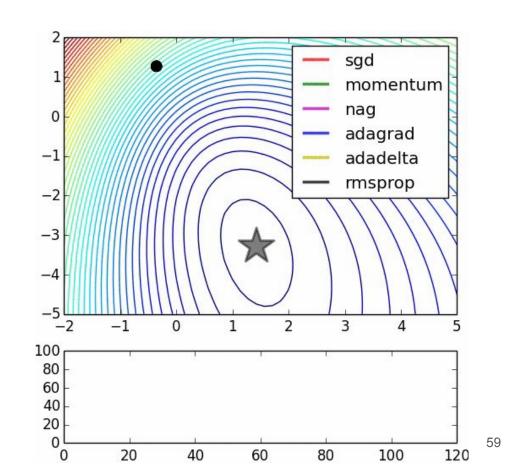


source: http://cs231n.github.io/neural-networks-3/

## **Optimizers**

#### There are much more optimizers:

- Momentum
- Adagrad
- Adadelta
- RMSprop
- Adam
- ...
- even other NNs

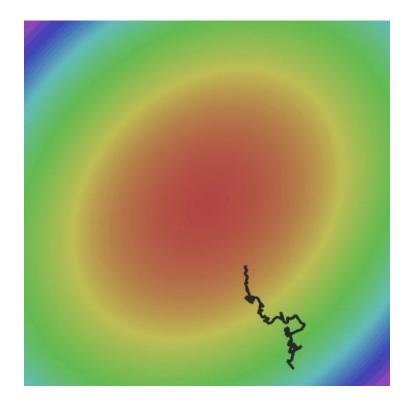


## Optimization: SGD

$$L(W) = \frac{1}{N} \sum_{i=1}^{N} L_i(x_i, y_i, W)$$

$$\nabla_W L(W) = \frac{1}{N} \sum_{i=1}^{N} \nabla_W L_i(x_i, y_i, W)$$

Averaging over minibatches ---> noisy gradient



## First idea: momentum

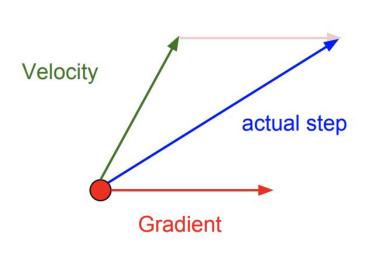
Simple SGD

$$x_{t+1} = x_t - \alpha \nabla f(x_t)$$

SGD with momentum

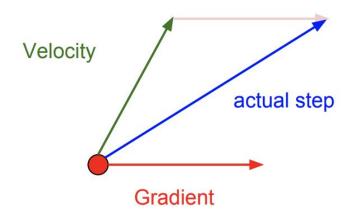
$$v_{t+1} = \rho v_t + \nabla f(x_t)$$
$$x_{t+1} = x_t - \alpha v_{t+1}$$

#### Momentum update:



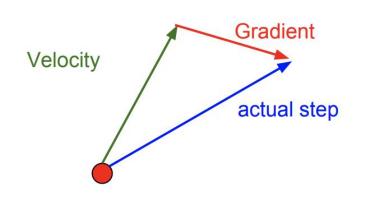
#### Nesterov momentum

#### Momentum update:



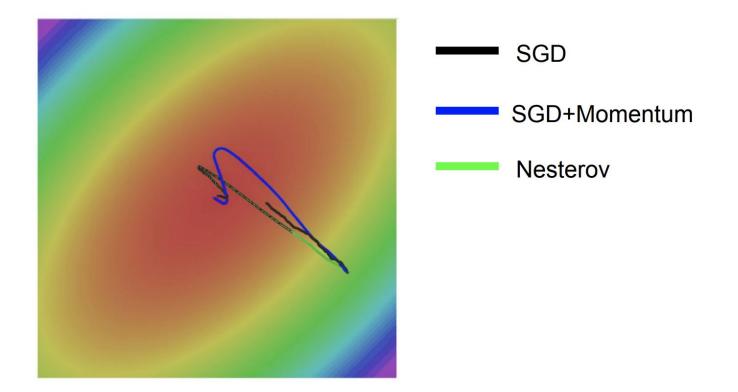
$$v_{t+1} = \rho v_t + \nabla f(x_t)$$
$$x_{t+1} = x_t - \alpha v_{t+1}$$

#### **Nesterov Momentum**



$$v_{t+1} = \rho v_t - \alpha \nabla f(x_t + \rho v_t)$$
$$x_{t+1} = x_t + v_{t+1}$$

## Comparing momentums



#### Second idea: different dimensions are different

Adagrad: SGD with cache

$$\operatorname{cache}_{t+1} = \operatorname{cache}_t + (\nabla f(x_t))^2$$
$$x_{t+1} = x_t - \alpha \frac{\nabla f(x_t)}{\operatorname{cache}_{t+1} + \varepsilon}$$

#### Second idea: different dimensions are different

Adagrad: SGD with cache

$$\operatorname{cache}_{t+1} = \operatorname{cache}_t + (\nabla f(x_t))^2$$
$$x_{t+1} = x_t - \alpha \frac{\nabla f(x_t)}{\operatorname{cache}_{t+1} + \varepsilon}$$

Problem: gradient fades with time

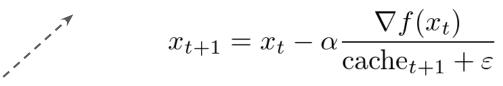
## Second idea: different dimensions are different

Adagrad: SGD with cache

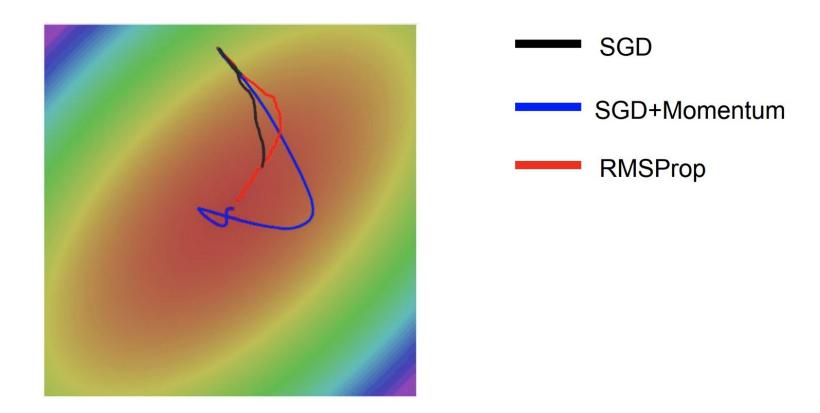
$$cache_{t+1} = cache_t + (\nabla f(x_t))^2$$
$$x_{t+1} = x_t - \alpha \frac{\nabla f(x_t)}{cache_{t+1} + \varepsilon}$$

RMSProp: SGD with cache with exp. Smoothing

$$cache_{t+1} = \beta cache_t + (1 - \beta)(\nabla f(x_t))^2$$



Slide 29 Lecture 6 of Geoff Hinton's Coursera class



## Adam

Let's combine the momentum idea and RMSProp normalization:

$$v_{t+1} = \gamma v_t + (1 - \gamma) \nabla f(x_t)$$

$$\operatorname{cache}_{t+1} = \beta \operatorname{cache}_t + (1 - \beta) (\nabla f(x_t))^2$$

$$x_{t+1} = x_t - \alpha \frac{v_{t+1}}{\operatorname{cache}_{t+1} + \varepsilon}$$

Let's combine the momentum idea and RMSProp normalization:

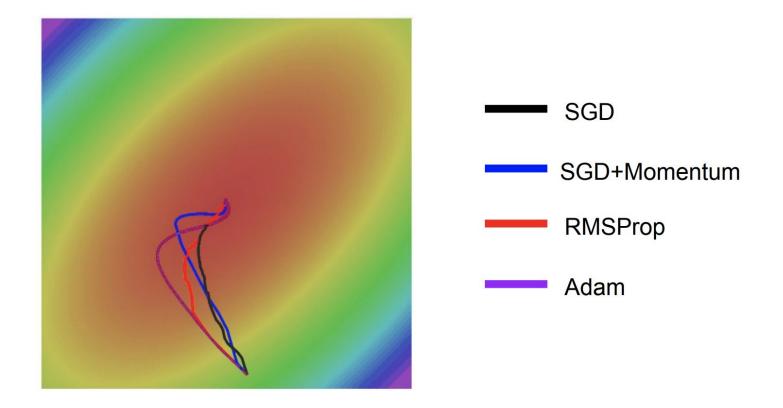
$$v_{t+1} = \gamma v_t + (1 - \gamma) \nabla f(x_t)$$

$$\operatorname{cache}_{t+1} = \beta \operatorname{cache}_t + (1 - \beta) (\nabla f(x_t))^2$$

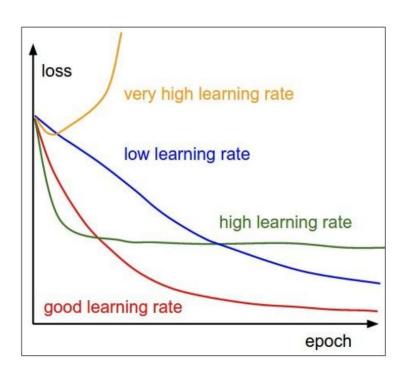
$$x_{t+1} = x_t - \alpha \frac{v_{t+1}}{\operatorname{cache}_{t+1} + \varepsilon}$$

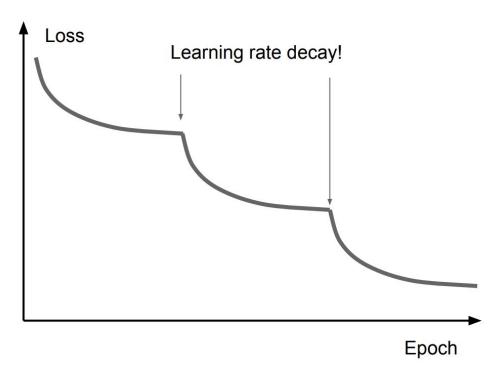
Actually, that's not quite Adam.

# Comparing optimizers



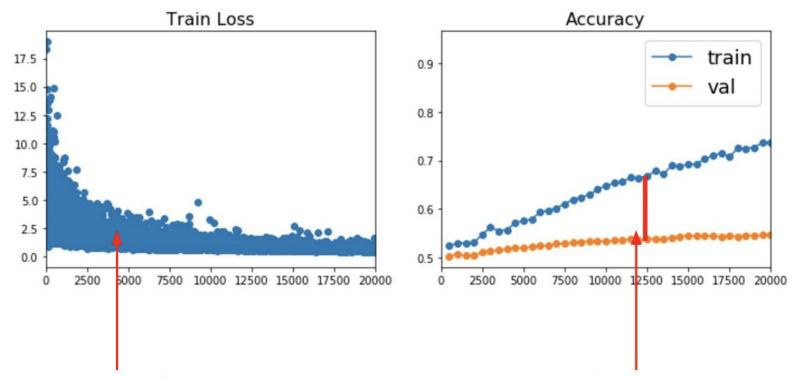
## Once more: learning rate





## Sum up: optimization

- Adam is great basic choice
- Even for Adam/RMSProp learning rate matters
- Use learning rate decay
- Monitor your model quality



Better optimization algorithms help reduce training loss

But we really care about error on new data - how to reduce the gap?