Practice 6

Elena Tuzhilina

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Question

A lottery ticket costs 2 dollars and the probability of winning is 0.1. If you win a lottery you receive 10 dollars, if you lose you receive 0 dollars.

1. Let X be a random variable that represents your money gain after playing one round of lottery, i.e. $X = 8$ if you win and $X = -2$ if you lose. Find a and b such that $X = a \cdot Y + b$ where $Y \sim Bernoulli(p)$, i.e. Y is a Bernoulli random variable. What is the value of p?

2. Use the properties of expectation and variance to find $E(X)$ and $Var(X)$.

3. If you play this lottery many-many times, do you think your average money gain will be positive or negative?

4. You decided to test your luck and bought 5 lottery tickets. Let Z denote the number of winning tickets. What is the expectation and variance of Z? Hint: use the link between Binomial and Bernoulli random variables.

5. What is the probability that at least one of these five tickets will win?

6. Let W be the *average* money gain for your five tickets. Find the expectation and variance of W. Hint: use $X_1, \dots X_5$ to represent the money gain of each ticket and find the formula that expresses W in terms of $X_1, \dots, X_5$

7. Find the chances that your average money gain is not negative, i.e. $P(W \geq 0)$? Is it higher than 50%? Hint: first find the formula that expresses W in terms of Z.

8. Now you decided to buy 100 tickets. Let W be the *average* money gain for your 100 tickets. What is the expectation and variance of W?

9. What is the approximate distribution of W?

10. Use the answer in 9 to find the chances that your new average money gain is not negative, i.e. $P(W \geq 0)$? Is it higher than 50%?

11. Use the 68–95–99.7 rule to find the interval $[c, d]$ that contains 95% of W values, i.e. such that $P(c \leq W \leq d) = 0.95$.

12. Use the 68–95–99.7 rule to find 2.5-th percentile for W. In other words we need to find the value t such that 2.5% of W values are less than t, i.e. $P(W \leq t) = 0.025$.

13. Use standardization and the distribution table to find the 2.5-th percentile for W.