

Practice 3

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Question 1

You toss four coins.

1. What is the sample space? How many outcomes do we have?

$$S = \{(H, H, H, H), (H, H, H, T), (H, H, T, H), (H, H, T, T), (H, T, H, H), (H, T, H, T), (H, T, T, H), (H, T, T, T), (T, H, H, H),$$

$$N = 16$$

2. What is the probability of each outcome?

$$P(O_i) = 1/16$$

3. You want to compute the probability of having four tails. What is the event?

$$E = \{(T, T, T, T)\}$$

4. What is the probability of having four tails? Can you apply the multiplication rule to compute this?

$$P(E) = \frac{\text{number of outcomes in } E}{N} = \frac{1}{16}$$

We apply the multiplication rule because each of the four tosses happens independently.

$$P(E) = P(1st\ toss\ is\ tail) \cdot P(2nd\ toss\ is\ tail) \cdot P(3rd\ toss\ is\ tail) \cdot P(4th\ toss\ is\ tail) = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$$

5. What is the probability to get heads in at least one toss?

The complement to “getting all tails” is “not getting all tails”. This is the same to “getting at least one heads”.

$$P(\bar{E}) = 1 - P(E) = 1 - \frac{1}{16} = \frac{15}{16}$$

6. Denote A the event of getting heads in tosses 1,2 and 3 (toss 4 can be anything). Denote B the event of getting heads in tosses 3 and 4 (tosses 1 and 2 can be anything). Find $P(A)$ and $P(B)$.

By the multiplication rule

$$P(A) = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

$$P(B) = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

7. What is $P(A \cap B)$? Do you think A and B are independent events?

If both A and B occur, then we will have heads in all rolls.

$$A \cap B = \{(H, H, H, H)\}$$

$$P(A \cap B) = \frac{1}{16}$$

Since

$$\frac{1}{16} = P(A \cap B) \neq P(A) \cdot P(B) = \frac{1}{32}$$

these events are not independent.

9. What is $P(A \cup B)$?

If one of A or B occur then we will have one of the five outcomes

$$A \cup B = \{(H, H, H, T), (H, H, H, H), (H, T, H, H), (T, H, H, H), (T, T, H, H)\}$$

Therefore

$$P(A \cup B) = \frac{5}{16}$$

We can also compute this using addition rule:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{1}{8} + \frac{1}{4} - \frac{1}{16} = \frac{5}{16}$$

10. Find $P(A|B)$ and $P(B|A)$.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1/16}{1/4} = \frac{1}{4}$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{1/16}{1/8} = \frac{1}{2}$$

11. What is the interpretation of these probabilities?

$P(A|B) = \frac{1}{4}$. Given that the last two tosses are heads there is 1/4 chance to get heads in the first three rolls.

Why? Given that the last two tosses are heads you can have one of (H,H,H,H), (H,T,H,H), (T,H,H,H), (T,T,H,H) combinations. Among these four combinations only one gives you the heads in the first three rolls, i.e. (H,H,H,H).

$P(B|A) = \frac{1}{2}$. Given that the first three tosses are heads there is 1/2 chance to get heads in the last two rolls.

Why? Given that the first three tosses are heads you can have one of (H,H,H,H), (H,H,H,T) combinations. Among these two combinations only one gives you the heads in the last two rolls, i.e. (H,H,H,H).

Question 2

You poll 1000 people and ask 2 questions:

- (a) Are you physically active?
- (b) Have you ever had a heart attack?

You get the following contingency table (measured in frequencies)

##	active	not-active
## heart attack	50	30
## no heart attack	550	370

1. What is the probability of being active and having a heart attack? (joint probability)

$$P(\text{active} \cap \text{attack}) = \frac{50}{1000} = 0.05$$

2. What is the probability of being active? (marginal probability)

$$P(\text{active}) = \frac{50+550}{1000} = 0.6$$

3. What is the probability of having a heart attack? (marginal probability)

$$P(\text{attack}) = \frac{50+30}{1000} = 0.08$$

4. What is the probability of having a heart attack given that you are physically active? (conditional probability)

$$P(\text{attack}|\text{active}) = \frac{P(\text{active} \cap \text{attack})}{P(\text{active})} = \frac{0.05}{0.6} = 0.083333$$

5. What is the probability of being physically active given that you had a heart attack? (conditional probability)

$$P(\text{active}|\text{attack}) = \frac{P(\text{active} \cap \text{attack})}{P(\text{attack})} = \frac{0.05}{0.08} = 0.625$$