Practice 4

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January 31, 2023

Question 1

You poll 1000 people and ask 2 questions:

(a) Are you physically active?
(b) Have you ever had a heart attack?

You get the following information - 10% of people get a heart attack - 60% of people who had a heart attack were not physically active - 70% of people who did not have a heart attack were physically active

1. Convert these statement to marginal and conditional probabilities.

$P(attack) = 0.1$

$P(not\ active|attack) = 0.6$

$P(active|no\ attack) = 0.7$

2. Compute the probability to have no heart attack.

$P(no\ attack) = 0.9$

3. Compute the probability to be active given a heart attack.

$P(active|attack) = 1 - 0.6 = 0.4$

4. Compute the probability to be not active given no heart attack.

$P(not\ active|no\ attack) = 1 - 0.7 = 0.3$

5. Create a tree diagram

First level: attack

- yes (0.1)
- no (0.9)

Second level: active|attack

- yes|yes (0.4)
- no|yes (0.6)
- yes|no (0.7)
- no|no (0.3)

6. Compute all joint probabilities (four probabilities in total).

$P(active \cap attack) = 0.1 \cdot 0.4 = 0.04$
$P(not\ active \cap attack) = 0.1 \cdot 0.6 = 0.06$
$P(active \cap no\ attack) = 0.9 \cdot 0.7 = 0.63$
$P(not\ active \cap no\ attack) = 0.9 \cdot 0.3 = 0.27$

7. Compute all marginal probabilities (four probabilities in total).

$P(attack) = 0.1$
$P(no\ attack) = 0.9$
$P(active) = 0.04 + 0.63 = 0.67$
$P(not\ active) = 0.06 + 0.27 = 0.33$

8. What is the probability to have a heart attack given that you are active?

$P(attack|active) = 0.04/0.67 = 0.0597$

9. Apply the Bayes' rule and make sure you get the same answer.

Question 2

1. Compute the expectation of a Bernoulli random variable with p = 0.1.

$E(X) = 0 \cdot 0.9 + 1 \cdot 0.1 = 0.1$

2. What is the general formula for arbitrary p?

$E(X) = 0 \cdot (1-p) + 1 \cdot p = p$

3. Compute the variance of a Bernoulli random with p = 0.1.

$Var(X) = (0-0.1)^2 \cdot 0.9 + (1-0.1)^2 \cdot 0.1 = 0.01 \cdot 0.9 + 0.81 \cdot 0.1 = 0.009 + 0.081 = 0.09$

4. What is the general formula for arbitrary p?

$Var(X) = (0-p)^2 \cdot (1-p) + (1-p)^2 \cdot p = p^2 \cdot (1-p) + (1-p)^2 \cdot p = p \cdot (1-p)(p+1-p) = p \cdot (1-p)$

Question 3

You have a bag with 7 items, the probability of an item to be defective is 0.2.

1. What type of random variable can be used to represent defectiveness of one item?

Bernoulli with p = 0.2. X = 1 if defective and X = 0 if not defective.

2. What type of random variable can be used to represent the number of defective items among these 7?

Binomial with p = 0.2 and n = 7.

3. Use the table (see Files/Tables on Quercus) to find the probability to have 3 out of 7 defective items?

The probability is 0.1147

4. Use the table to draw the distribution diagram the number of defective items.
5. How would you use this table if I ask you to find the probability to have 3 out of 7 defective items if the probability for a an item to be defective is 0.8?

It is the same as the probability to have 4 out of 7 not defective* items if the probability for an item to be *not defective* is 0.2. Using the table we get 0.0287.