Research on How the Professor's Questions Equilibrate Students' Seat Strategies

Team member:

Xu Yinqiu, student ID:15220162202423

Jiang Hao, student ID:15220162202141

Ke Xueying, student ID:15220162202161

1 Abstract:

In college classes, active interactions are always important. However, since professors and students are not familiar with each other and the professor will randomly choose someone to answer questions, all of them will be unable to get into their optimal state. Thus, we study how professors' questions equilibrate student' seat strategy.

Suppose there are some students who like answering questions while the others are not. When a student who likes answering questions is chosen by the professor, he will get a higher utility. When a student who doesn't like answering the questions is chosen, he will get lower utility than not being chosen. For professors, choosing someone who likes answering the question will make his utility higher. Based on these assumptions, we construct two 2 by 2 matrices and solve for the equilibria for each student and professor by simulation.

2 Notations

- N: number of students in the class
- c: the additional utility of a student who sits in the front
- k_c: the utility of a student who likes answering questions and was chosen
- k_n: the utility of a student who likes answering questions but was not chosen
- d_c: the utility of a student who dislikes answering questions but was chosen
- d_n: the utility of a student who dislikes answering questions and was not chosen.
- t_k: the utility of the professor when he chose a student who likes answering

questions

- t_d: the utility of the professor when he chose a student who dislikes answering questions
- p: the probability that the professor chose a student in the front
- α: the proportion of students who like answering questions sitting in the front
- β: the proportion of students who like answering questions sitting in the back

3 Definition of the Game

- **Players: one** professor, $\mathbf{N}$ students
- **Action Spaces:**

  Professor (Mixed Strategy): $A_P = \{choose\ a\ student\ in\ the\ front\ with\ probability\ p, choose\ a\ student\ in\ the\ back\ with\ probability\ 1-p\}$

  Students (Pure Strategy): $A_i = \{sit\ in\ the\ front, sit\ in\ the\ back\}$
- **Type Spaces:**

  Professor : $T_P = \{those\ who\ tend\ to\ choose\ in\ the\ front, those\ who\ tend\ to\ choose\ in\ the\ back\}$

  Student: $T_i = \{those\ who\ like\ answering\ questions, those\ who\ dislike\ answering\ questions\}$
- **Beliefs:**

  Professor: the professor has the belief that the α, β in this class is the same as the values in the former class, i.e. $p_P = 1$.

  Student: the student has the belief that the α, β, p in this class is the same as the values in the former class, i.e. $p_i = 1$.

  Remark: The beliefs are used between lectures in the game process.
- **Payoffs:**

  Professor: $E[u_P]$

  Students $E[u_i]$

4 Assumptions

- The seats in the front are identical, and the seats in the back are identical.
- The students who like answering questions share the same utility, the students who dislike answering questions also share the same utility.
- A professor can't change his type, i.e. we always have $p > \frac{1}{2}$ or $p < \frac{1}{2}$

5 The Process of the Game

Professor and students will get familiar with each other, and they will change their choice base on their knowledge about each other's preference.

Lecture 1: They know nothing about each other. Professor will randomly choose students.

After this lecture, students will think that the professor has a probability $\mathbf{p}$ choose the students sitting in the front. And professor will have an assumption that $\boldsymbol{\alpha}$ proportion of students sitting in the front like answering the question, and $\boldsymbol{\beta}$ proportion of students sitting in the back like answering questions.

Lecture 2: They will make choice based on their experience from lecture 1.

And they will get a new $\mathbf{p}$, a new $\boldsymbol{\alpha}$ and a new $\boldsymbol{\beta}$

Lectures …

Lecture N: Everyone reaches his optimal utility.

6 Payoff Matrices

The payoff matrices have two types, one of the students who like answering questions, one of the students who dislike answering questions.

Type I

<table>
<tr><td></td><td colspan="3">Student (like answering questions)</td></tr>
<tr><td rowspan="3">Professor</td><td></td><td>Front</td><td>Back</td></tr>
<tr><td>Front (p)</td><td>$(\mathrm{\alpha t_k} + (1-\alpha)t_d,$
$\mathrm{c} + \frac{2}{N}\mathrm{k_c} + \frac{N-2}{N}\mathrm{k_n})$</td><td>$(\mathrm{\alpha t_k} + (1-\alpha)t_d, \mathrm{k_n})$</td></tr>
<tr><td>Back
(1 − p)</td><td>$(\mathrm{\beta t_k} + (1-\beta)t_d, \mathrm{c} + \mathrm{k_n})$</td><td>$(\mathrm{\beta t_k} + (1-\beta)t_d,$
$\frac{2}{N}\mathrm{k_c} + \frac{N-2}{N}\mathrm{k_n})$</td></tr>
</table>

Type II

<table>
<tr><td></td><td colspan="3">Student (dislike answering questions)</td></tr>
<tr><td rowspan="3">Professor</td><td></td><td>Front</td><td>Back</td></tr>
<tr><td>Front (p)</td><td>$(\mathrm{\alpha t_k} + (1-\alpha)t_d,$
$\mathrm{c} + \frac{2}{N}d_{\mathrm{c}} + \frac{N-2}{N}d_{\mathrm{n}})$</td><td>$(\mathrm{\alpha t_k} + (1-\alpha)t_d, \mathrm{d_n})$</td></tr>
<tr><td>Back
(1 − p)</td><td>$(\mathrm{\beta t_k} + (1-\beta)t_d, \mathrm{c} + \mathrm{d_n})$</td><td>$(\mathrm{\beta t_k} + (1-\beta)t_d,$
$\frac{2}{N}d_{\mathrm{c}} + \frac{N-2}{N}d_{\mathrm{n}})$</td></tr>
</table>

7 Expected Utilities

- **Professor:**

$$\mathrm{E}[\mathrm{u_P}(p)] = \mathrm{p}[\mathrm{\alpha t_k} + (1-\alpha)t_d] + (1-p)[\mathrm{\beta t_k} + (1-\beta)t_d]$$

- **Students who like answering questions:**

$$\mathrm{E}[\mathrm{u_i}(F)] = \mathrm{p}\left[\mathrm{c} + \frac{2}{N}\mathrm{k_c} + \frac{N-2}{N}\mathrm{k_n}\right] + (1-p)(c + k_n)$$

$$\mathrm{E}[\mathrm{u_i}(B)] = \mathrm{pk_n} + (1-p)\left[\frac{2}{N}\mathrm{k_c} + \frac{N-2}{N}\mathrm{k_n}\right]$$

- **Students who dislike answering questions**

$$\mathrm{E}[\mathrm{u_i}(F)] = \mathrm{p}\left[\mathrm{c} + \frac{2}{N}\mathrm{d_c} + \frac{N-2}{N}\mathrm{d_n}\right] + (1-p)(c+d_n)$$

$$\mathrm{E}[\mathrm{u_i}(B)] = \mathrm{pd_n} + (1-p)\left[\frac{2}{N}\mathrm{d_c} + \frac{N-2}{N}\mathrm{d_n}\right]$$

8 Simulation

To find the Nash Equilibria and to verify our model, we simulated all possible contingencies; we shall then illustrate how we did this.

We first simulate a classroom with $\mathbf{8}$ seats, with $\mathbf{4}$ seats in the front and $\mathbf{4}$ seats in the back.

We then simulate $\mathbf{8}$ students, in which the number of students who like answering questions ($\mathrm{N_k}$) satisfies the condition $\mathrm{N_k} \in \{n|n \in \mathbb{N}, n \leq N\}$, of whom the number of those who like sitting in the front ($\mathrm{N_{kf}}$) satisfies the condition that $\mathrm{N_{kf}} \in \{n|n \in \mathbb{N}, n \leq N_k\}$. (If the number of students who like answering questions and want to sit in the front is greater than 4, we just take the extra ones in the back.).

For the students' utilities, we set $\mathrm{k_c} > k_n = 0, d_n > d_c = 0$. For the professor's utility, it's better for him to choose a student who likes answering question than one who dislikes that. So, we set $\mathrm{t_k} > t_d = 0$.

Then based on the utilities we just defined, once we have a seat distribution, we simulate the choosing action for 10,000 times. We can then calculate the utilities of everyone, then we just simply estimate the expected utilities with sample mean.

Finally, we find the NE by selecting the greatest value of all utilities.

9 Nash Equilibria

Professor chooses p to maximize his utility and two types of students who like answering questions and don't like it choose between sitting in the front (F) or in the back (B) respectively to maximize their utility.

Nash Equilibrium1:

$$((1,0), \mathrm{F(like)/B(dislike)})$$

The professor always chooses the students in the front and students who like answering questions always sit in the front.

Actually, in the equilibrium, both professor and students will choose a pure strategy.

The professor always chooses a student who will offer a good answer to his question which enables his lecture to go through smoothly.

Students corresponding to professor's choice always sit in the front to obtain additional knowledge maximizing their utilities.

A representative example of this equilibrium would be the condition of a major course. Outstanding students sit in the front and the professor picks students sitting in the front.

Nash Equilibrium2:

$$((0,1), \mathrm{B(like)/F(dislike)})$$

The professor always chooses the students in the back and the students who like answering questions always sit in the back.

Similarly, the professor tends to choose students who can offer a good answer. So if he likes to choose students sitting in the back, students who like answering questions will choose seats in the back of the classroom. Simultaneously, the professor will maximize his utility by picking students in the back.

A representative example of this equilibrium is politics lectures.

10 Modifications

In real cases, we can extend the distribution of the seats and the professor's preference or tendency; that is, we can have more than just **Front** and **Back**, but **Row 1**, **Row 2** …

For the students, we first assumed that the students of the same category are identical; however, in real cases, we can override this assumption and regard these students differently to come to a Nash Equilibrium for the cases that the number of students who like answering questions and sitting in the front exceeds the number of seats in the front.