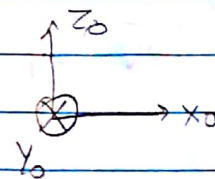
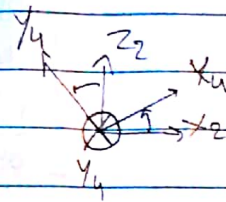
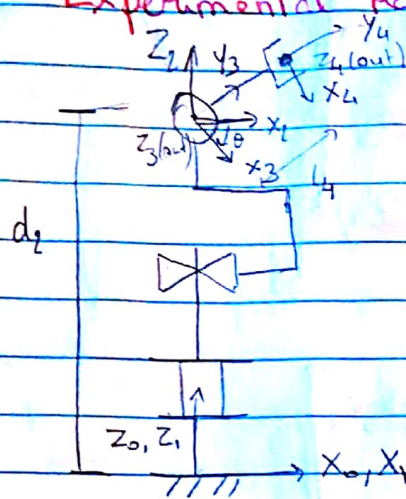


## Experimental Robotics

1/a)



$${}^0P_4$$

$${}^4P_u = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad {}^3P_4 = \begin{bmatrix} 0 \\ L_4 \\ 0 \end{bmatrix} \quad {}^2P_4 = \begin{bmatrix} -L_4 \sin \theta_3 \\ 0 \\ L_4 \cos \theta_3 \end{bmatrix} \quad {}^1P_4 = \begin{bmatrix} -L_4 \sin \theta_3 \\ 0 \\ L_4 \cos \theta_3 + d_2 \end{bmatrix}$$

Finally  ${}^0P_4 = \begin{bmatrix} -L_4 \sin \theta_3 \cos \theta_1 \\ -L_4 \sin \theta_3 \sin \theta_1 \\ L_4 \cos \theta_3 + d_2 \end{bmatrix}$

b)  ${}^2R_4 = \begin{bmatrix} \cos \theta_3 & -\sin \theta_3 & 0 \\ 0 & 0 & -1 \\ \sin \theta_3 & \cos \theta_3 & 0 \end{bmatrix} \quad {}^0R_2 = \begin{bmatrix} \cos \theta_1 & -\sin \theta_1 & 0 \\ \sin \theta_1 & \cos \theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$${}^0R_4 = {}^0R_2 {}^2R_4 = \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_3 & -s_3 & 0 \\ 0 & 0 & -1 \\ s_3 & c_3 & 0 \end{bmatrix}$$

$${}^0R_4 = \begin{bmatrix} c_1 c_3 & -c_1 s_3 & s_1 \\ s_1 c_3 & -s_1 s_3 & -c_1 \\ -s_3 & c_3 & 0 \end{bmatrix}$$



$${}^0P_4 = \begin{bmatrix} -L_4 \sin \theta_3 \cos \theta_4 & -L_4 \sin \theta_3 \sin \theta_4 & L_4 \cos \theta_3 + d_2 \\ L_4 \sin \theta_3 \cos \theta_4 & L_4 \sin \theta_3 \sin \theta_4 & L_4 \sin \theta_3 \\ 0 & 0 & 0 \end{bmatrix}$$

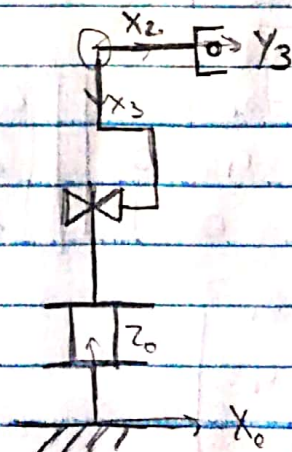
$$c) {}^0J_v = \begin{bmatrix} \frac{\partial x}{\partial \theta_1} & \frac{\partial x}{\partial \theta_2} & \frac{\partial x}{\partial \theta_3} \\ \frac{\partial y}{\partial \theta_1} & \frac{\partial y}{\partial \theta_2} & \frac{\partial y}{\partial \theta_3} \\ \frac{\partial z}{\partial \theta_1} & \frac{\partial z}{\partial \theta_2} & \frac{\partial z}{\partial \theta_3} \end{bmatrix}$$

$${}^0J_v = \begin{bmatrix} L_4 \sin \theta_3 \sin \theta_4 & 0 & -L_4 \cos \theta_3 \\ -L_4 \sin \theta_3 \cos \theta_4 & 0 & -L_4 \sin \theta_3 \\ 0 & 1 & -L_4 \sin \theta_3 \end{bmatrix}$$

$$d) {}^0J_w = \begin{bmatrix} 0 & 0 & \sin \theta_1 \\ 0 & 0 & -\cos \theta_1 \\ 1 & 0 & 0 \end{bmatrix}$$

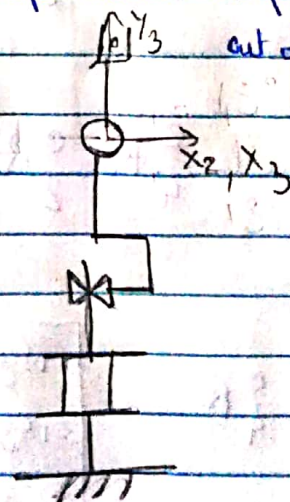
$$e) \theta_3 = \pm 90^\circ$$

In this configuration, we lose the translation with respect to  $\vec{x}_0$



$$\text{or } \theta_3 = 0 \text{ or } 180^\circ$$

We lose also the translation, but it can be expressed as the impossibility to go out of the plane  $(\vec{x}_0, \vec{z}_0)$



f)  $\theta_3 = 90^\circ$

$${}^0J = \begin{bmatrix} L_4 s_1 & 0 & 0 \\ L_4 c_1 & 0 & 0 \\ 0 & 1 & -L_4 \end{bmatrix}$$

$\theta_1 = 0^\circ$

$${}^0J_v = \begin{bmatrix} 0 & 0 & 0 \\ -L_4 & 0 & 0 \\ 0 & 1 & -L_4 \end{bmatrix}$$

$\theta_1 = 90^\circ$

$${}^0J_v = \begin{bmatrix} L_4 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & -L_4 \end{bmatrix}$$

- Column 2 and column 3 are not linearly independent.  
This confirms that this is a singularity.

We cannot translate horizontally ( $\vec{x}_0$ ) which is usually possible with the translate along  $\vec{z}_1$  and the rotation around  $\vec{z}_3$ .

$\theta_3 = 0$

$${}^0J_v = \begin{bmatrix} 0 & 0 & -L_4 c_1 \\ 0 & 0 & -L_4 s_1 \\ 0 & 1 & 0 \end{bmatrix}$$

$${}^0J_v = \begin{bmatrix} 0 & 0 & -L_4 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$${}^0J_v = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -L_4 \\ 0 & 1 & 0 \end{bmatrix}$$

- Column 1 is composed of 0s only.

We cannot rotate around  $\vec{z}_1$  without changing the end-effector position.

The end-effector can't be located out of the plane ( $\vec{x}_0, \vec{z}_0$ )



$$g) \quad M = m_4 J_{v_4}^T J_{v_4}$$

$$M = m_4 \begin{bmatrix} L_4 \delta_3 \delta_1 & -L_4 \delta_3 c_1 & 0 \\ 0 & 0 & 1 \\ -L_4 c_3 c_1 & -L_4 c_3 \delta_1 & -L_4 \delta_3 \end{bmatrix} \begin{bmatrix} L_4 \delta_3 \delta_1 & 0 & -L_4 c_3 \delta_1 \\ -L_4 \delta_3 c_1 & 0 & -L_4 c_3 c_1 \\ 0 & 1 & -L_4 \delta_3 \end{bmatrix}$$

$${}^0 P_4 = \begin{bmatrix} -L_4 \delta_3 c_1 \\ L_4 \delta_3 \delta_1 \\ L_4 c_3 + d_2 \end{bmatrix}$$

$$M = \begin{bmatrix} L_4^2 \delta_3^2 & 0 & 0 \\ 0 & 1 & -L_4 \delta_3 \\ 0 & -L_4 \delta_3 & L_4^2 \end{bmatrix}$$

$$m_{11} = 0$$

$$m_{22} = 0$$

$$m_{33} = -1.5 \sin(\theta_3)$$

$$m_{11} = 0$$

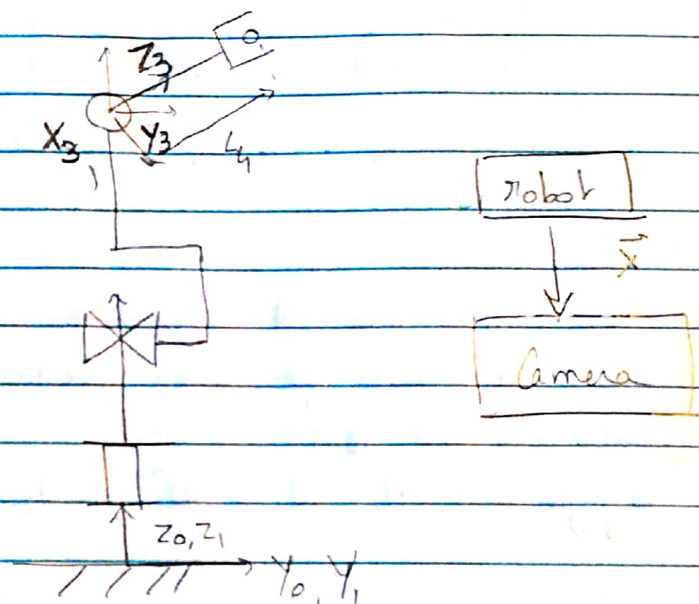
$$m_{22} = 0$$

$$m_{33} = 0$$

$$a) \quad G = -J_{v_4}^T m_4 \vec{g} \quad \vec{g} = \begin{bmatrix} 0 \\ 0 \\ -9.81 \end{bmatrix}$$

$$G = -g \times 1 \times \begin{bmatrix} 0 \\ 1 \\ -L_4 \sin \theta_3 \end{bmatrix}$$

2) a)



$$b) \quad {}^3P_4 = \begin{bmatrix} 0 \\ 0 \\ 1.5 \end{bmatrix} \neq {}^3P_4 = \begin{bmatrix} 0 \\ L_4 \\ 0 \end{bmatrix}$$

Frame {3} is not defined the same way as in question!!

180  
-90 90



c) i) Configurations:  $\theta_1 = 0^\circ$ ,  $d_2 = 0.5\text{m}$ ,  $\theta_3 = -90^\circ$

$${}^0P_4 = \begin{bmatrix} 0 & 1.5 & 0.5 \end{bmatrix}$$

$${}^0P_4 = \begin{bmatrix} -1.5 \sin(-90^\circ) \cos(0) & -1.5 \sin(-90^\circ) \sin(0) & 1.5 \cos(90) + 0.5 \end{bmatrix}$$

$${}^0P_4 = \begin{bmatrix} 1.5 & 0 & 0.5 \end{bmatrix}$$

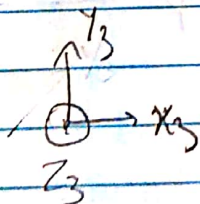
ii) Configurations:  $\theta_1 = 90^\circ$ ,  $d_2 = 0.5\text{m}$ ,  $\theta_3 = -90^\circ$

$${}^0P_4 = \begin{bmatrix} -1.5 & 0 & 0.5 \end{bmatrix}$$

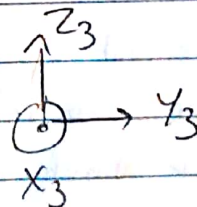
$$\begin{aligned} {}^0P_4 &= \begin{bmatrix} -1.5 \sin(-90) \cos(90) & -1.5 \sin(-90) \sin(90) & 1.5 \cos(90) + 0.5 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 1.5 & 0.5 \end{bmatrix} \end{aligned}$$

Overall the values are similar but the axes are different. This can be explained by the fact that the frames are not defined the same way.

1) c)



2) a)



The component of  $x_3$  in 1.c. is equivalent to the component of  $y_3$  in 2.a.  
 $d_2 \vec{z}_1$  on the contrary is defined the same way in the two questions.

d) Configuration (i)  $\theta_1 = 0^\circ$ ,  $d_2 = 0.5 \text{ m}$ ,  $\theta_3 = -90^\circ$

$${}^0J_v = \begin{bmatrix} -1.5 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 1.5 \end{bmatrix} \quad \text{in simulation}$$

$${}^0J_v = \begin{bmatrix} 0 & 0 & 0 \\ 1.5 & 0 & 0 \\ 0 & 1 & 1.5 \end{bmatrix} \quad \text{from l.c.}$$

As the frames are not defined the same way, we do not have the same jacobian matrices. Axis are switched, but values are the same.

Configuration (ii)  $\theta_1 = 90^\circ$ ,  $d_2 = 0.5$ ,  $\theta_3 = -90^\circ$

$${}^0J_v = \begin{bmatrix} 0 & 0 & 0 \\ -1.5 & 0 & 0 \\ 0 & 1 & 1.5 \end{bmatrix} \quad \text{in simulation}$$

$${}^0J_v = \begin{bmatrix} -1.5 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 1.5 \end{bmatrix}$$



$$e) \left. \begin{aligned} m_{11} &= L_4 \dot{\alpha}_3^2 \\ m_{22} &= 1 \\ m_{33} &= L_4^2 \end{aligned} \right\} \text{according to 1}$$

We can see that the mass matrix follows the same equations with  $m_{11}$  sinusoidal and  $m_{22}$  &  $m_{33}$  constant

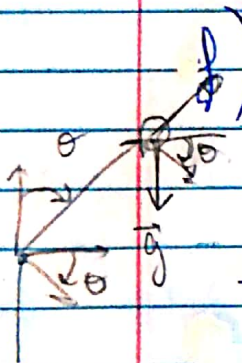
In frame 1, inertia is due to the eccentricity of  $m_4$  which explain  $L_4 \dot{\alpha}_3^2$

2] In configuration 2,

$$m_{11} = 0$$

$$m_{22} = 1$$

$$m_{33} = L_4^2 \quad \text{it does not depend on } d_2$$



$$1) \left. \begin{aligned} G &= - J_{c4}^T m_4 g \\ G &= - 9.81 \begin{bmatrix} 0 \\ 1 \\ L_4 \alpha_3 \end{bmatrix} \end{aligned} \right\}$$

Although  $J_{c4}$  differs from 1 c but the last row is the same

This is why we have the same results.

This is what we observe in the plot as well

The gravity vector is logical - as we do the projection of the mass along the z axis.

$$2) \quad G = - 9.81 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

This is exactly what we find on the plots