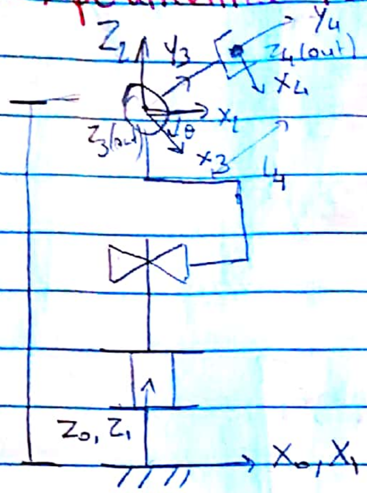
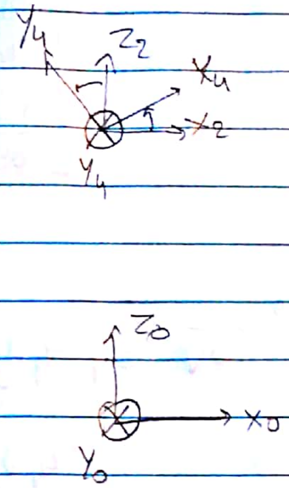
Experimental Robotics

1) a)





$\boxed{{}^0P_4}$

$${}^4P_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad {}^3P_4 = \begin{bmatrix} 0 \\ L_4 \\ 0 \end{bmatrix} \quad {}^2P_4 = \begin{bmatrix} -L_4 \sin\theta_3 \\ 0 \\ L_4 \cos\theta_3 \end{bmatrix} \quad {}^1P_4 = \begin{bmatrix} -L_4 \sin\theta_3 \\ 0 \\ L_4 \cos\theta_3 + d_2 \end{bmatrix}$$

Finally $${}^0P_4 = \begin{bmatrix} -L_4 \sin\theta_3 \cos\theta_1 \\ -L_4 \sin\theta_3 \sin\theta_1 \\ L_4 \cos\theta_3 + d_2 \end{bmatrix}$$

b) $${}^2_4R = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 \\ 0 & 0 & -1 \\ \sin\theta_3 & \cos\theta_3 & 0 \end{bmatrix} \qquad {}^0_2R = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$${}^0_4R = {}^0_2R\,{}^2_4R = \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_3 & -s_3 & 0 \\ 0 & 0 & -1 \\ s_3 & c_3 & 0 \end{bmatrix}$$

$${}^0_4R = \begin{bmatrix} c_1c_3 & -c_1s_3 & s_1 \\ s_1c_3 & -s_1s_3 & -c_1 \\ s_3 & c_3 & 0 \end{bmatrix}$$

$$^0P_4 = \begin{bmatrix} -L_4 s_3 c_1 \\ -L_4 s_3 s_1 \\ L_4 c_3 + d_2 \end{bmatrix}$$

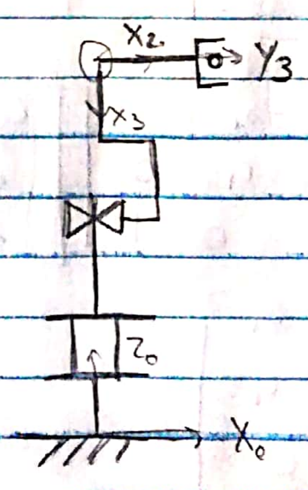
c) $$^0J_v = \begin{bmatrix} \frac{\partial x}{\partial \theta_1} & \frac{\partial x}{\partial d_2} & \frac{\partial x}{\partial \theta_3} \\ \frac{\partial y}{\partial \theta_1} & \frac{\partial y}{\partial d_2} & \frac{\partial y}{\partial \theta_3} \\ \frac{\partial z}{\partial \theta_1} & \frac{\partial z}{\partial d_2} & \frac{\partial z}{\partial \theta_3} \end{bmatrix}$$

$$^0J_v = \begin{bmatrix} L_4 s_3 s_1 & 0 & -L_4 c_3 c_1 \\ -L_4 s_3 c_1 & 0 & -L_4 c_3 s_1 \\ 0 & 1 & -L_4 s_3 \end{bmatrix}$$

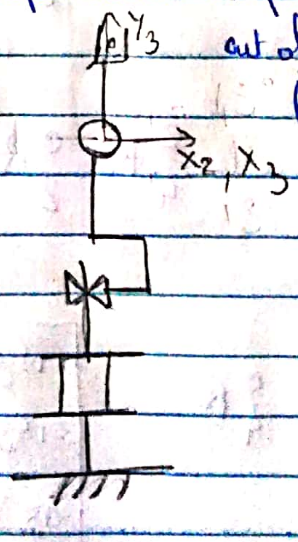
d) $$^0J_\omega = \begin{bmatrix} 0 & 0 & s_1 \\ 0 & 0 & -c_1 \\ 1 & 0 & 0 \end{bmatrix}$$

e) $\theta_3 = \pm 90$ or $\theta_3 = 0$ or 180

In this configuration, we lose the translation with respect to $\vec{x_0}$



We lose also the translation, but it can be expressed as the impossibility to go out of the plane $(\vec{X_0}, \vec{Z_0})$



f) $\theta_3 = 90°$

$$ {}^0J_v = \begin{bmatrix} L_4 s_1 & 0 & 0 \\ L_4 c_1 & 0 & 0 \\ 0 & 1 & -L_4 \end{bmatrix} $$

$\theta_1 = 0°$

$$ {}^0J_v = \begin{bmatrix} 0 & 0 & 0 \\ -L_4 & 0 & 0 \\ 0 & 1 & -L_4 \end{bmatrix} $$

$\theta_1 = 90$

$$ {}^0J_v = \begin{bmatrix} L_4 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & -L_4 \end{bmatrix} $$

- Column 2 and column 3 are not linearly independent.

This confirms that this is a singularity.

We cannot translate horizontally ($\vec{X_0}$) which is usually possible with the translate along $\vec{Z_1}$ and the rotation around $\vec{Z_3}$.

$\theta_3 = 0$

$$ {}^0J_v = \begin{bmatrix} 0 & 0 & -L_4 c_1 \\ 0 & 0 & -L_4 s_1 \\ 0 & 1 & 0 \end{bmatrix} $$

$$ {}^0J_v = \begin{bmatrix} 0 & 0 & -L_4 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} $$

$$ {}^0J_v = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -L_4 \\ 0 & 1 & 0 \end{bmatrix} $$

- Column 1 is composed of 0s only.

We cannot rotate around $\vec{Z_1}$ without changing the end-effector position.

The end-effector can't be located out of the plan ($\vec{X_0}$, $\vec{Z_0}$)

g) $M = m_4 J_{v_4}^T J_{v_4}$

$$M = m_4 \begin{bmatrix} L_4 s_3 s_1 & -L_4 s_3 c_1 & 0 \\ 0 & 0 & 1 \\ -L_4 c_3 c_1 & -L_4 c_3 s_1 & -L_4 s_3 \end{bmatrix} \begin{bmatrix} L_4 s_3 s_1 & 0 & -L_4 c_3 c_1 \\ -L_4 s_3 c_1 & 0 & -L_4 c_3 s_1 \\ 0 & 1 & -L_4 s_3 \end{bmatrix}$$

$${}^0P_4 = \begin{bmatrix} -L_4 s_3 c_1 \\ -L_4 s_3 s_1 \\ L_4 c_3 + d_2 \end{bmatrix}$$

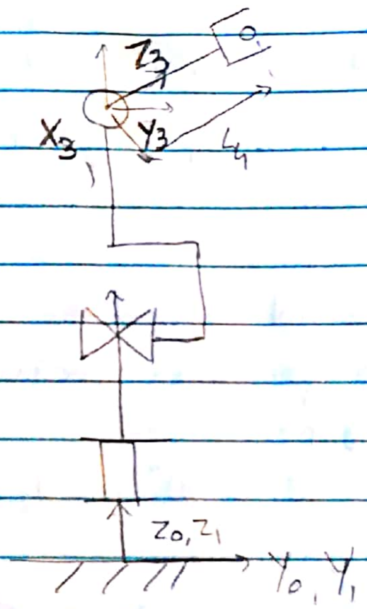
$$M = \begin{bmatrix} L_4^2 s_3^2 & 0 & 0 \\ 0 & 1 & -L_4 s_3 \\ 0 & -L_4 s_3 & L_4^2 \end{bmatrix}$$

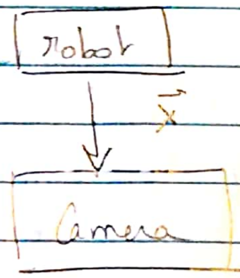
$m_{11} = 0$	$m_{11} = 0$
$m_{22} = 0$	$m_{22} = 0$
$m_{33} = -1.5 \sin(\theta_3)$	$m_{33} = 0$

a) $G = -J_{v_4}^T m_4 \vec{g}$ $\vec{g} = \begin{bmatrix} 0 \\ 0 \\ -9.81 \end{bmatrix}$

$G = -g \times 1 \times \begin{bmatrix} 0 \\ 1 \\ -L_4 s_3 \end{bmatrix}$

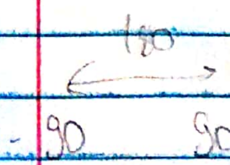
2] a)





b) ${}^3P_4 = \begin{bmatrix} 0 \\ 0 \\ 1.5 \end{bmatrix} \neq {}^3P_4 = \begin{bmatrix} 0 \\ L_4 \\ 0 \end{bmatrix}$

Frame{3} is not defined the same way as in question!



c) i. Configurations: $\theta_1 = 0°$, $d_2 = 0.5m$, $\theta_3 = -90°$

$${}^0P_4 = [0 \quad 1.5 \quad 0.5]$$

$${}^0P_4 = [1.5 \times \sin(-90°) \times \cos(0) \quad -1.5 \sin(-90°) \sin(0°) \quad 1.5\cos 90 + 0.5]$$

$${}^0P_4 = [1.5 \quad 0 \quad 0.5]$$

ii. Configurations: $\theta_1 = 90°$ $d_2 = 0.5m$ $\theta_3 = -90°$

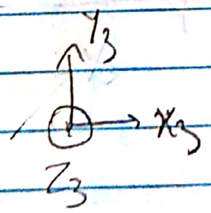
$${}^0P_4 = [-1.5 \quad 0 \quad 0.5]$$

$${}^0P_4 = [-1.5 \sin(-90)\cos(90) \quad -1.5 \sin(-90)\sin(90) \quad 1.5\cos(90) + 0.5]$$

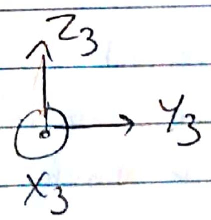
$$= [0 \quad 1.5 \quad 0.5]$$

Overall the values are similar but the axis are different. This can be explained by the fact that the frames are not defined the same way.

1) c)



2) a)



- The component of X_3 in 1.c. is equivalent to the component of Y_3 in 2.a.
  $d_2 \vec{z}_1$ on the contrary is defined the same way in the two questions.

d) Configuration (i) $\theta_1 = 0°$, $d_2 = 0.5\,m$, $\theta_3 = -90°$

$$^0J_v = \begin{bmatrix} -1.5 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 1.5 \end{bmatrix}$$ in simulation

$$^0J_v = \begin{bmatrix} 0 & 0 & 0 \\ 1.5 & 0 & 0 \\ 0 & 1 & 1.5 \end{bmatrix}$$ from 1.c.

As the frames are not defined the same way, we do not have the same jacobian matrices. Axis are switched, but values are the same.

Configuration (ii) $\theta_1 = 90°$, $d_2 = 0.5$, $\theta_3 = -90°$

$$^0J_v = \begin{bmatrix} 0 & 0 & 0 \\ -1.5 & 0 & 0 \\ 0 & 1 & 1.5 \end{bmatrix}$$ in simulation

$$^0J_v = \begin{bmatrix} -1.5 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 1.5 \end{bmatrix}$$

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e) 1] $m_{11} = L_4 s_3^2$

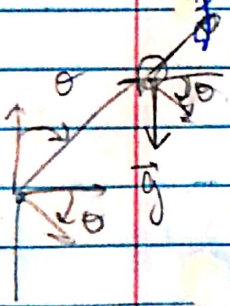
$m_{22} = 1$ } according to 1

$m_{33} = L_4^2$

We can see that the mass matrix follows the same equations
with m_{11} sinusoidal
and m_{22} & m_{33} constant
In frame 1, inertia is due to the excentricity of m_4 which explain $L_4 s_3^2$

2] In configuration 2,

$m_{11} = 0$
$m_{22} = 1$
$m_{33} = L_4^2$ it does not depend on d_2



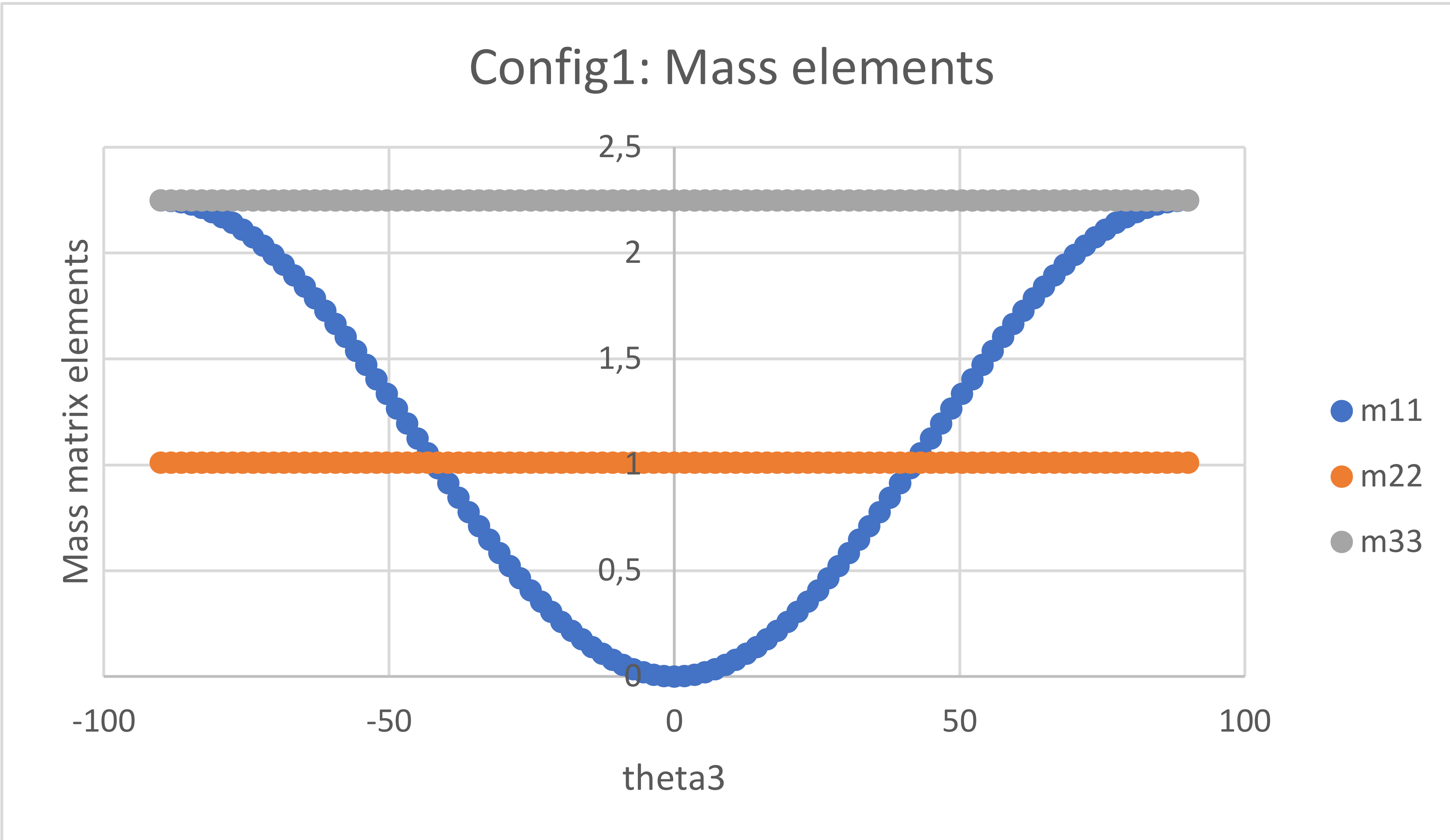
f) 1] $G = - J_{v_4}^T m_4 g$

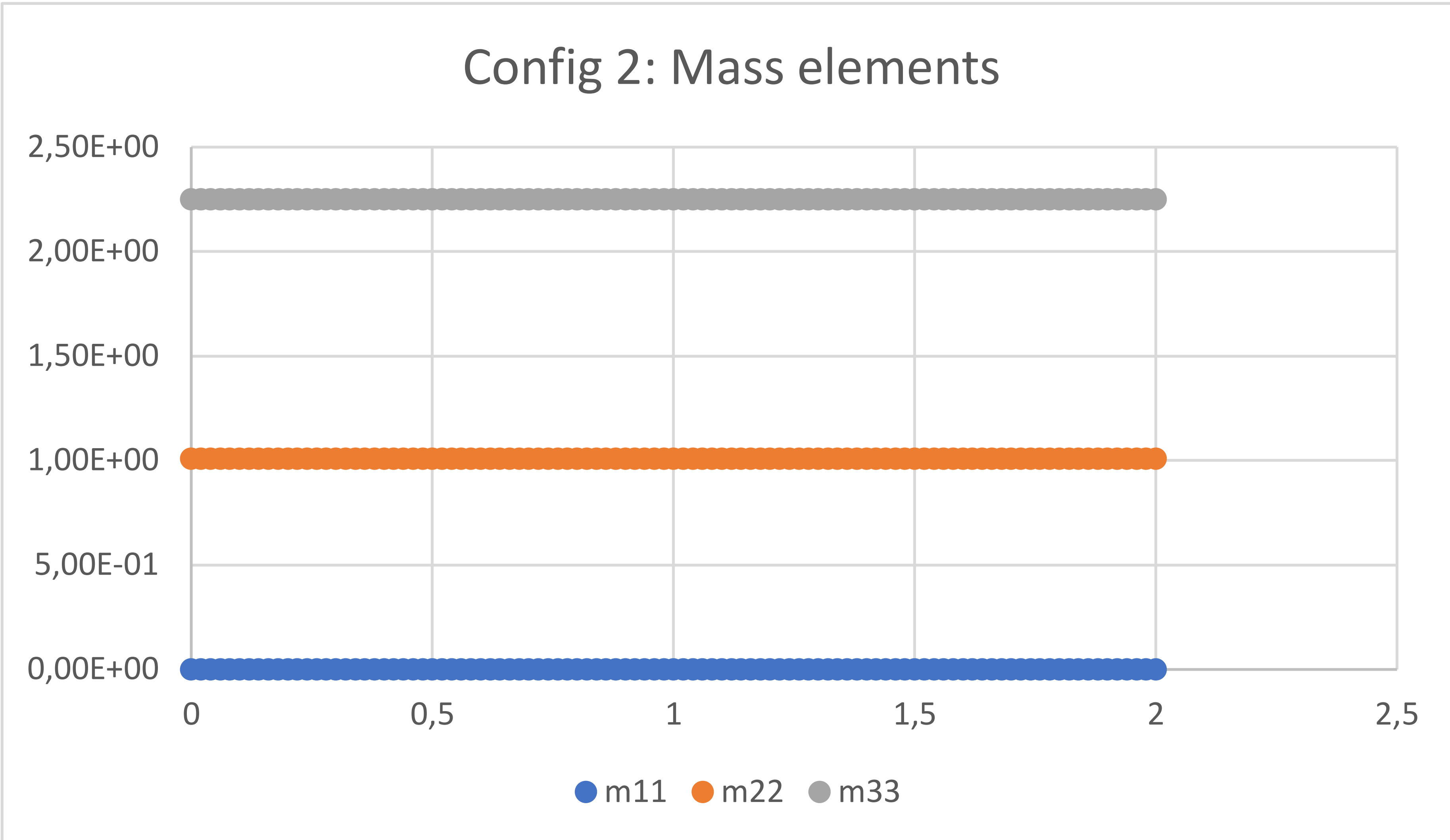
$$G = -9.81 \begin{bmatrix} 0 \\ 1 \\ L_4 s_3 \end{bmatrix}$$

Although J_{v_4} differs from 1c but the last row is the same. This is why we have the same results.

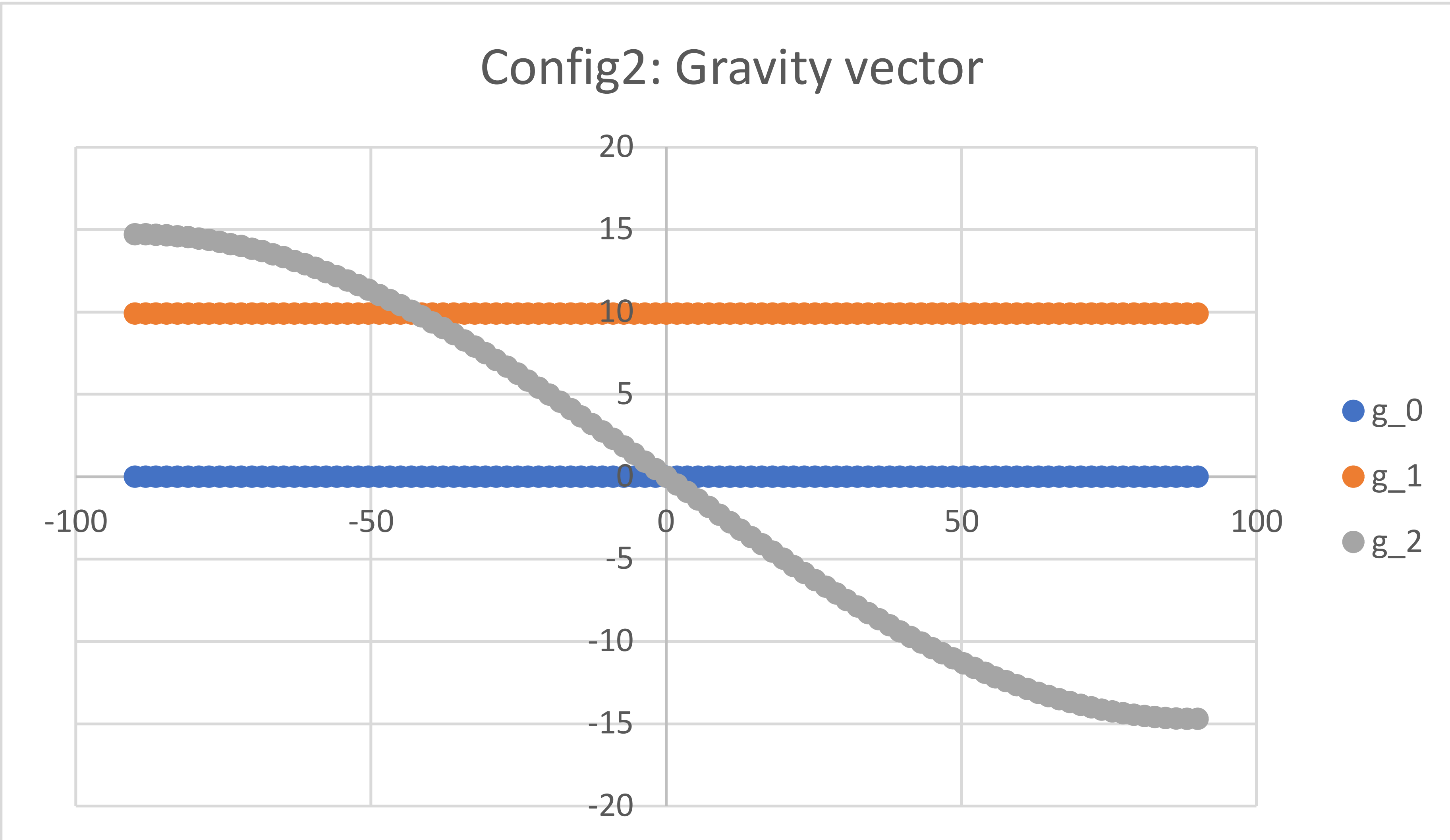
This is what we observe in the plot as well
The gravity vector is logical as we do the projection of the mass along the z axis.

2] $G = -9.81 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ This is exactly what we find on the plots.

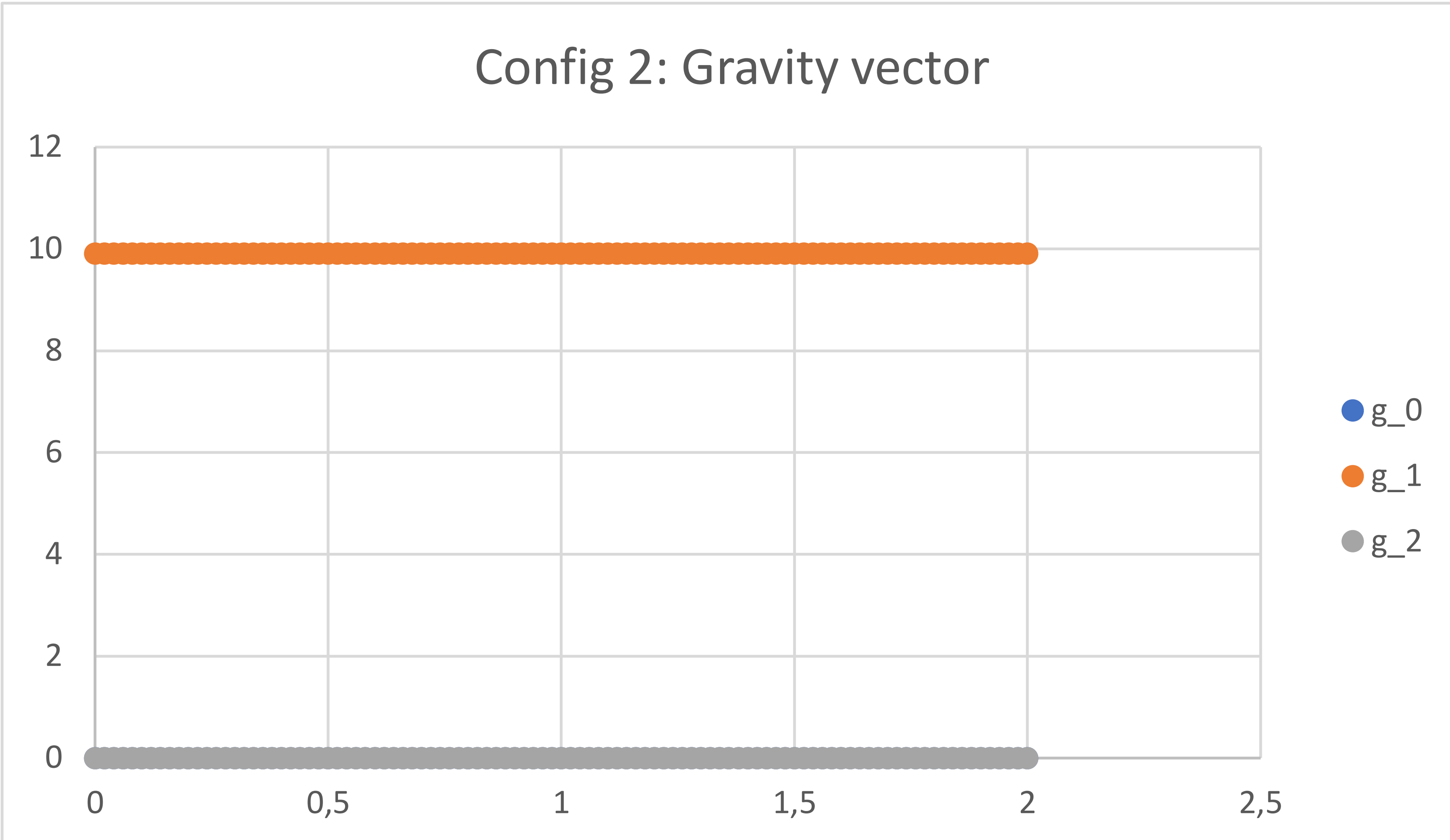

Config1: Mass elements
Mass matrix elements
theta3
2,5
2
1,5
1
0,5
0
-100
-50
0
50
100
m11
m22
m33



Config 2: Mass elements
2,50E+00
2,00E+00
1,50E+00
1,00E+00
5,00E-01
0,00E+00
0
0,5
1
1,5
2
2,5
m11
m22
m33



Config2: Gravity vector
20
15
10
5
0
-5
-10
-15
-20
-100
-50
0
50
100
g_0
g_1
g_2



Config 2: Gravity vector
12
10
8
6
4
2
0
0
0,5
1
1,5
2
2,5
g_0
g_1
g_2