7. Differential Amplifiers

Sedra & Smith Sec. 2.1.3 and Sec. 8 (MOS Portion)

(S&S 5^{th} Ed: Sec. 2.1.3 and Sec. 7 MOS Portion & ignore frequency-response)

Common-Mode and Differential-Mode Signals & Gain

Differential and Common-Mode Signals/Gain

Consider a <u>linear</u> circuit with TWO inputs

By superposition:

$$\boxed{v_o = A_1 \cdot v_1 + A_2 \cdot v_2}$$

Define: $\boxed{v_d = v_2 - v_1}$ Difference (or differential) Mode

$\boxed{v_c = \dfrac{v_1 + v_2}{2}}$ Common Mode

$$\boxed{\begin{aligned} v_1 &= v_c - \frac{v_d}{2} \\ v_2 &= v_c + \frac{v_d}{2} \end{aligned}}$$

Substituting for $v_1 = v_c - \dfrac{v_d}{2}$ and $v_2 = v_c + \dfrac{v_d}{2}$ in the expression for υ_o:

$$v_o = A_1 \cdot \left(v_c - \frac{v_d}{2} \right) + A_2 \cdot \left(v_c + \frac{v_d}{2} \right) = \left(A_1 + A_2 \right) \cdot v_c + \left(\frac{A_2 - A_1}{2} \right) \cdot v_d$$

$$\boxed{v_o = A_c \cdot v_c + A_d \cdot v_d}$$

Differential and common-mode signal/gain is an alternative way of finding the system response

$$v_o = A_1 \cdot v_1 + A_2 \cdot v_2$$

$$v_d = v_2 - v_1 \qquad A_c = A_1 + A_2$$

$$v_c = \frac{v_1 + v_2}{2} \qquad A_d = \frac{A_2 - A_1}{2}$$

$$v_o = A_c \cdot v_c + A_d \cdot v_d$$

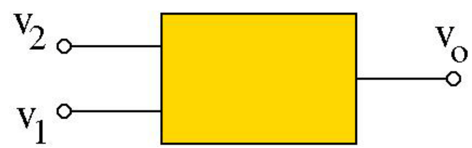
$$v_1 = v_c - \frac{v_d}{2} \qquad A_1 = \frac{A_c}{2} - A_d$$

$$v_2 = v_c + \frac{v_d}{2} \qquad A_2 = \frac{A_c}{2} + A_d$$

Differential Gain:	A_d
Common Mode Gain:	A_c
Common Mode Rejection Ratio (CMRR)*:	$\lvert A_d \rvert / \lvert A_c \rvert$

* CMRR is usually given in dB: CMRR(dB) = 20 log ($\lvert A_d \rvert / \lvert A_c \rvert$)

To find v_o, we can calculate/measure either A_1 A_2 pair or A_c A_d pair



Superposition (finding A_1 and A_2):

1. Set $v_2 = 0$, compute A_1 from $v_o = A_1 v_1$
2. Set $v_1 = 0$, compute A_2 from $v_o = A_2 v_2$
3. For any v_1 and v_2: $v_o = A_1 v_1 + A_2 v_2$

Difference Method (finding A_d and A_c):

1. Set $v_c = 0$ (or set $v_1 = -0.5\, v_d$ & $v_2 = +0.5\, v_d$) compute A_d from $v_o = A_d v_d$
2. Set $v_d = 0$ (or set $v_1 = +v_c$ & $v_2 = +v_c$) compute A_c from $v_o = A_c v_c$
3. For any v_1 and v_2: $v_o = A_d v_d + A_c v_c$

   $v_d = v_2 - v_1 \quad v_c = 0.5(v_1 + v_2)$

- **Both methods give the same answer for v_o (or A_v).**
- **The choice of the method is driven by application:**
  - **Easier solution**
  - **More relevant parameters**

Caution

- In Chapter 2.1.3, Sedra & Smith defines $\boldsymbol{v_\mathrm{d}} = \boldsymbol{v_2} - \boldsymbol{v_1}$

$$v_1 = v_c - \frac{v_d}{2} \qquad v_2 = v_c + \frac{v_d}{2}$$

- But in Chapter 8, Sedra & Smith uses $\boldsymbol{v_\mathrm{d}} = \boldsymbol{v_1} - \boldsymbol{v_2}$

$$v_1 = v_c + \frac{v_d}{2} \qquad v_2 = v_c - \frac{v_d}{2}$$

While keeping $\boldsymbol{v_o} = \boldsymbol{v_{o2}} - \boldsymbol{v_{o1}}$ as before (this is inconsistent)

- Here we use $\boldsymbol{v_\mathrm{d}} = \boldsymbol{v_2} - \boldsymbol{v_1}$ and $\boldsymbol{v_o} = \boldsymbol{v_{o2}} - \boldsymbol{v_{o1}}$ <u>throughout</u>

$$v_1 = v_c - \frac{v_d}{2} \qquad v_2 = v_c + \frac{v_d}{2}$$

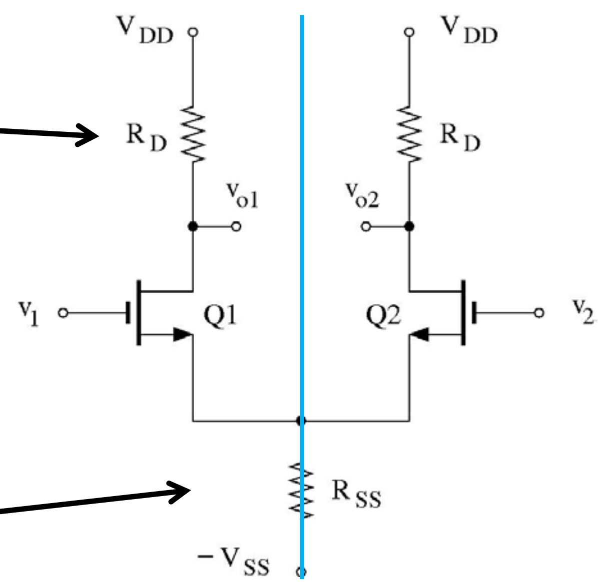
- Therefore, $\boldsymbol{A_d}$ **(lecture slides) =** $\boldsymbol{-A_d}$ **(Sedra & Smith)** for difference Amplifiers.
- <u>**Use Lecture Slides Notation!**</u>

Differential Amplifiers: Fundamental Properties

Differential Amplifier

- Identical transistors.
- Circuit elements are symmetric about the mid-plane.
- Identical bias voltages at Q1 & Q2 gates ($V_{G1} = V_{G2}$).
- Signal voltages & currents are different because $v_1 \neq v_2$.

Load R_D: resistor, current-mirror, active load, …



R_{SS}: Bias resistor, current source (current-mirror)

Q1 & Q2 are in CS-like configuration (input at the gate, output at the drain) but with sources connected to each other.

- For now, we keep track of "two" output, v_{o1} and v_{o2}, because there are several ways to configure "one" output from this circuit.

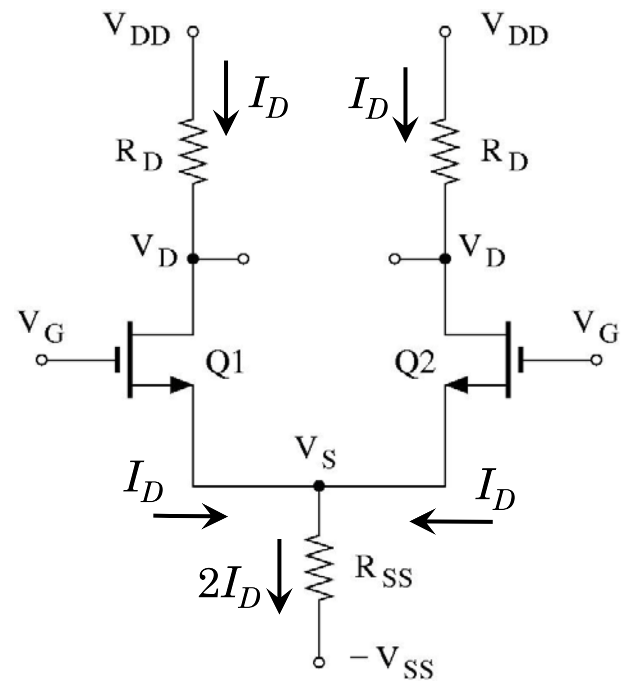
Differential Amplifier – Bias

Since $V_{G1} = V_{G2} = V_G$

and $V_{S1} = V_{S2} = V_S$

$$\boxed{\begin{aligned} V_{GS1} &= V_{GS2} = V_{GS} \\ V_{OV1} &= V_{OV2} = V_{OV} \\ I_{D1} &= I_{D2} = I_D \\ V_{DS1} &= V_{DS2} = V_{DS} \end{aligned}}$$

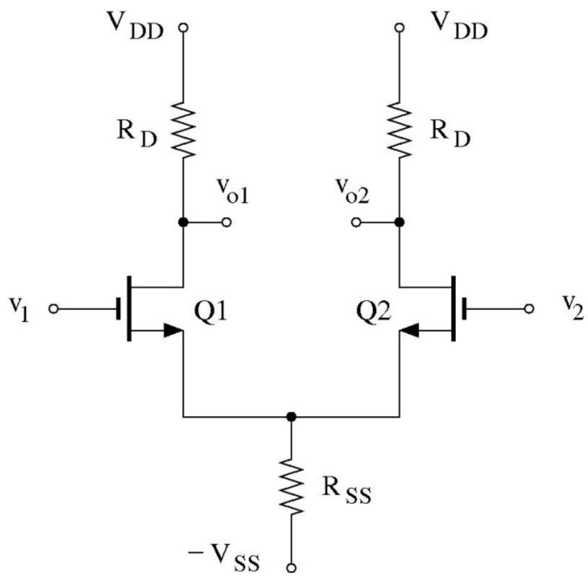
Also: $$\boxed{\begin{aligned} g_{m1} &= g_{m2} = g_m \\ r_{o1} &= r_{o2} = r_o \end{aligned}}$$



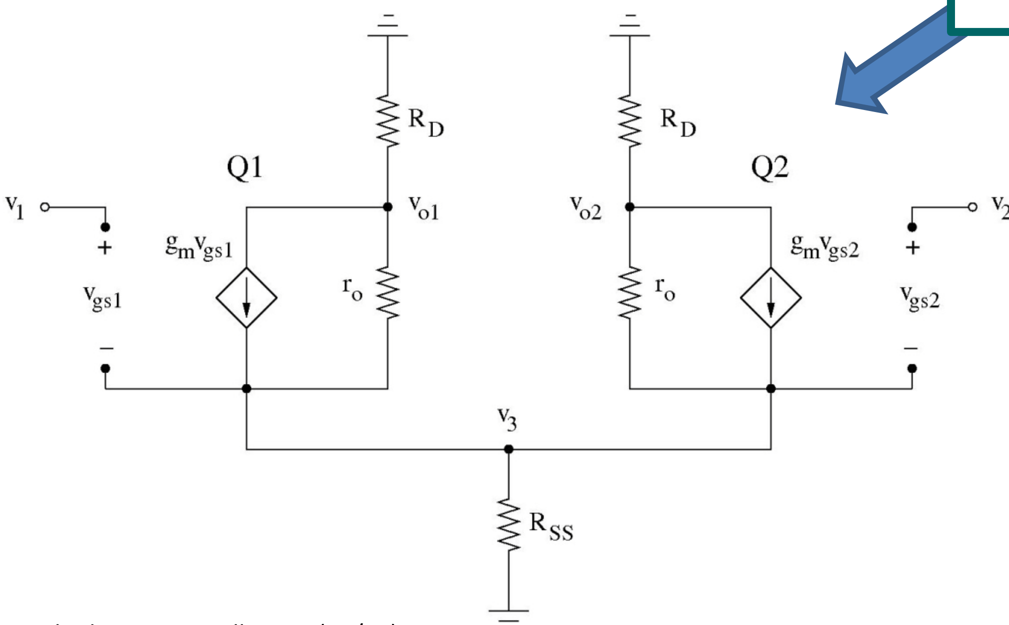
This is correct even if channel-width modulation is included because

$$I_{D1} R_D + V_{DS1} = I_{D2} R_D + V_{DS2}$$

Differential Amplifier – Gain

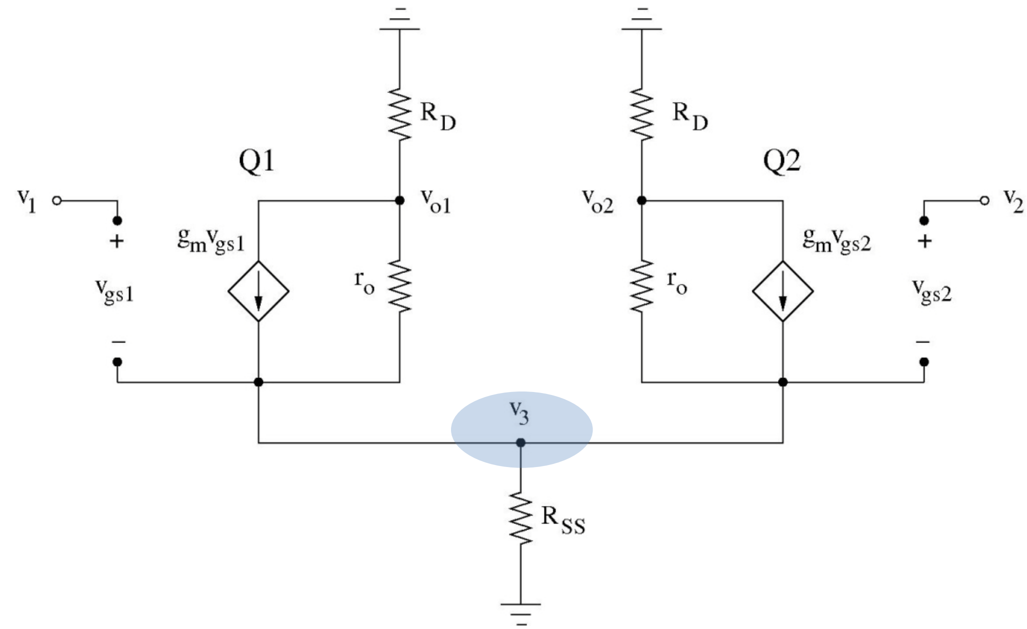


- Signal voltages & currents are different because $v_1 \neq v_2$
- We cannot use fundamental amplifier configuration for arbitrary values of v_1 and v_2.
- We have to replace each NMOS with its small-signal model.



Differential Amplifier – Gain

$$v_{gs1} = v_1 - v_3$$
$$v_{gs2} = v_2 - v_3$$



Node Voltage Method:

Node v_{o1}: $\dfrac{v_{o1}}{R_D} + \dfrac{v_{o1} - v_3}{r_o} + g_m(v_1 - v_3) = 0$

Node v_{o2}: $\dfrac{v_{o2}}{R_D} + \dfrac{v_{o2} - v_3}{r_o} + g_m(v_2 - v_3) = 0$

Node v_3: $\dfrac{v_3}{R_{SS}} + \dfrac{v_3 - v_{o2}}{r_o} + \dfrac{v_3 - v_{o1}}{r_o} - g_m(v_1 - v_3) - g_m(v_2 - v_3) = 0$

Above three equations should be solved to find v_{o1} , v_{o2} and v_3 (lengthy calculations)

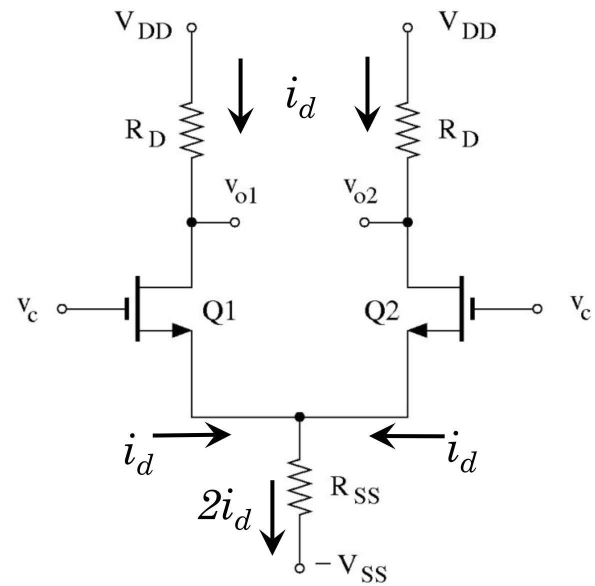
- **Because the circuit is symmetric, differential/common-mode method is the preferred method to solve this circuit (and we can use fundamental configuration formulas).**

Differential Amplifier – Common Mode (1)

Common Mode: Set $v_d = 0$ (or set $v_1 = +v_c$ and $v_2 = +v_c$)

Because of summery of
the circuit and input signals*:

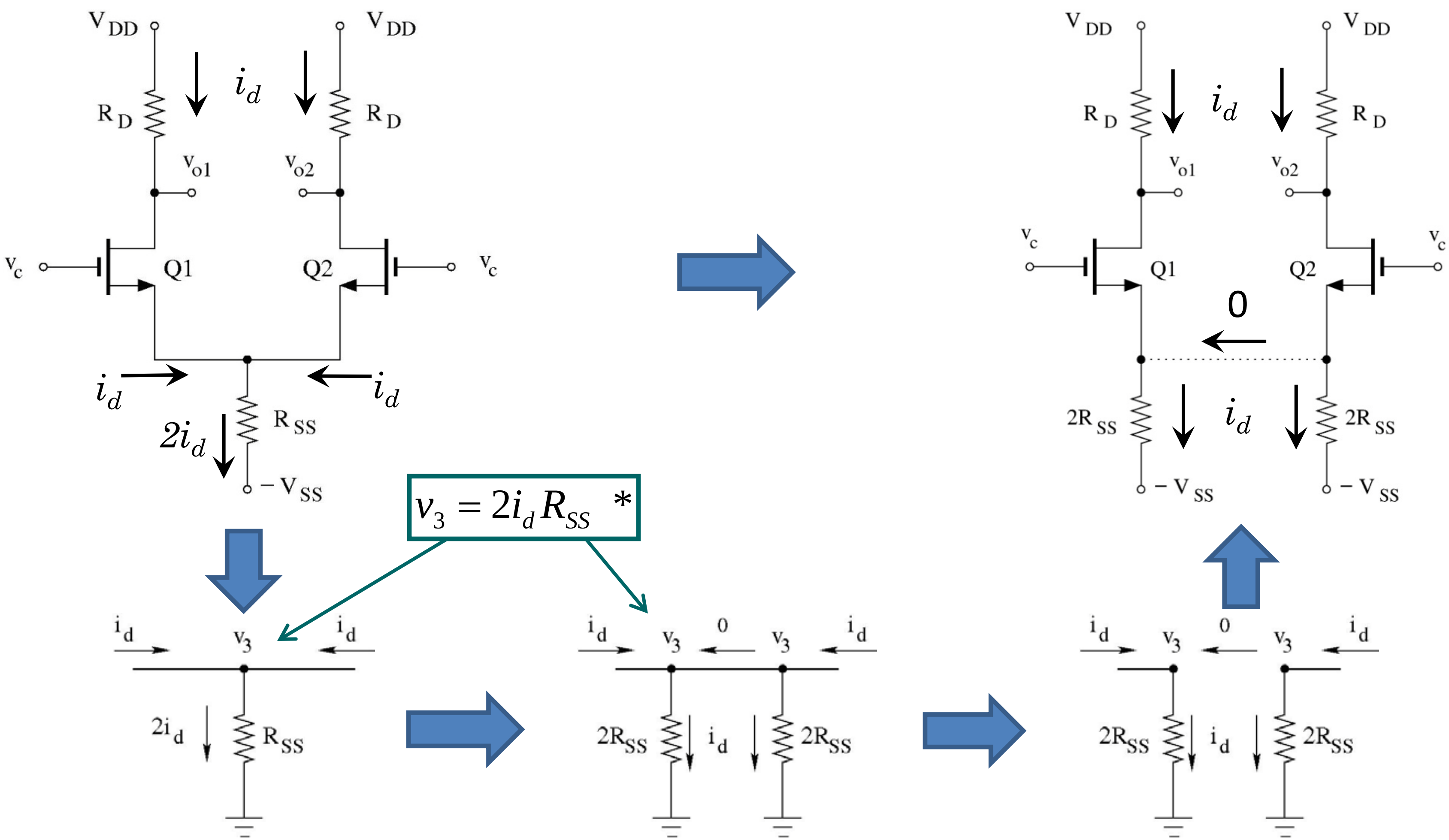
$$v_{o1} = v_{o2} \quad \text{and} \quad i_{d1} = i_{d2} = i_d$$



We can solve for v_{o1} by node voltage method but there is a simpler and more elegant way.

* If you do not see this, set $v_1 = v_2 = v_c$ in node equations of the previous slide, subtract the first two equations to get $v_{o1} = v_{o2}$. Ohm's law on R_D then gives $i_{d1} = i_{d2} = i_d$

Differential Amplifier – Common Mode (2)

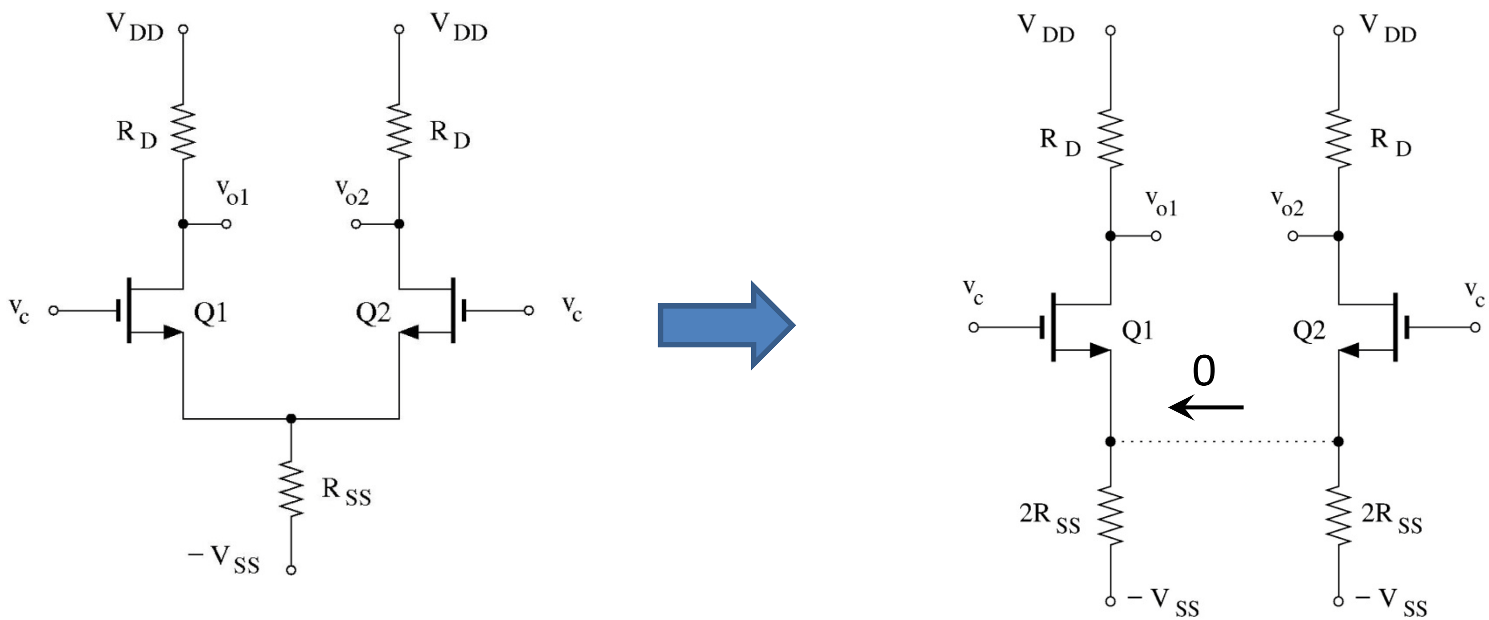


➢ **Because of the symmetry, the common-mode circuit breaks into two identical “half-circuits”.**

* V_{ss} is grounded for signal

Differential Amplifier – Common Mode (3)

➢ **The common-mode circuit breaks into two identical half-circuits.**



CS Amplifiers with Rs

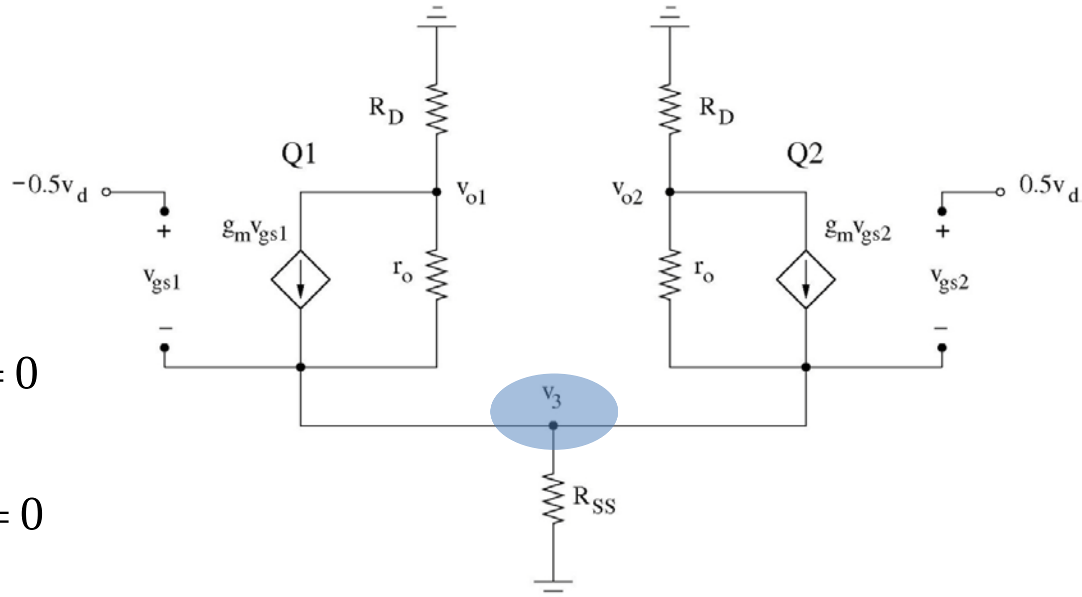
$$\frac{v_{o1}}{v_c} = \frac{v_{o2}}{v_c} = -\frac{g_m R_D}{1 + 2 g_m R_{SS} + R_D / r_o}$$

Differential Amplifier – Differential Mode (1)

Differential Mode: Set $\upsilon_c = 0$ (or set $\upsilon_1 = -\,\upsilon_d/2$ and $\upsilon_2 = +\,\upsilon_d/2$)

$$v_{gs1} = -0.5 v_d - v_3$$

$$v_{gs2} = +0.5 v_d - v_3$$



Node Voltage Method:

Node υ_{o1}: $$\frac{v_{o1}}{R_D} + \frac{v_{o1} - v_3}{r_o} + g_m(-0.5 v_d - v_3) = 0$$

Node υ_{o2}: $$\frac{v_{o2}}{R_D} + \frac{v_{o2} - v_3}{r_o} + g_m(+0.5 v_d - v_3) = 0$$

Node υ_3: $$\frac{v_3}{R_{SS}} + \frac{v_3 - v_{o2}}{r_o} + \frac{v_3 - v_{o1}}{r_o} - g_m(-0.5 v_d - v_3) - g_m(+0.5 v_d - v_3) = 0$$

Node υ_{o1} + Node υ_{o2} : $$\left(\frac{1}{R_D} + \frac{1}{r_o}\right)(v_{o1} + v_{o2}) - \left(\frac{2}{r_o} + 2 g_m\right) v_3 = 0$$

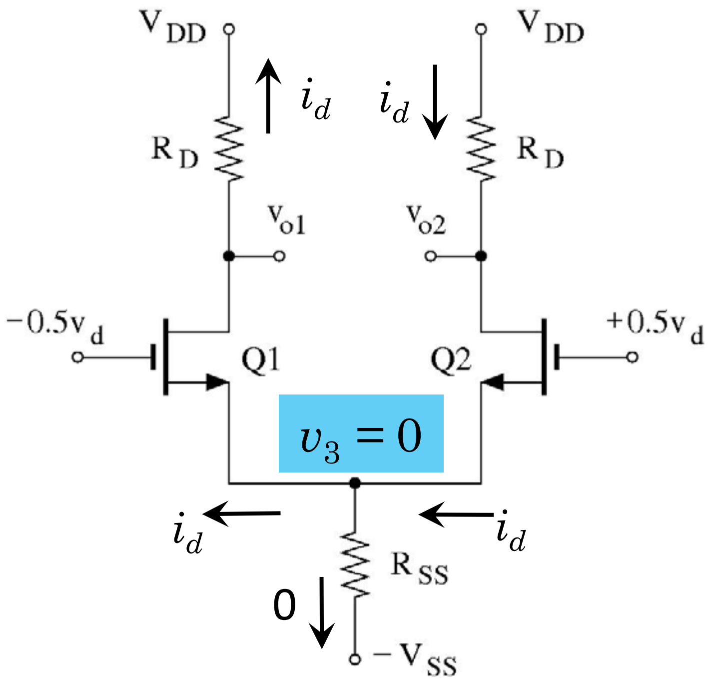
Node υ_3: $$-\frac{1}{r_o}(v_{o1} + v_{o2}) + \left(\frac{1}{R_{SS}} + \frac{2}{r_o} - 2 g_m\right) v_3 = 0$$

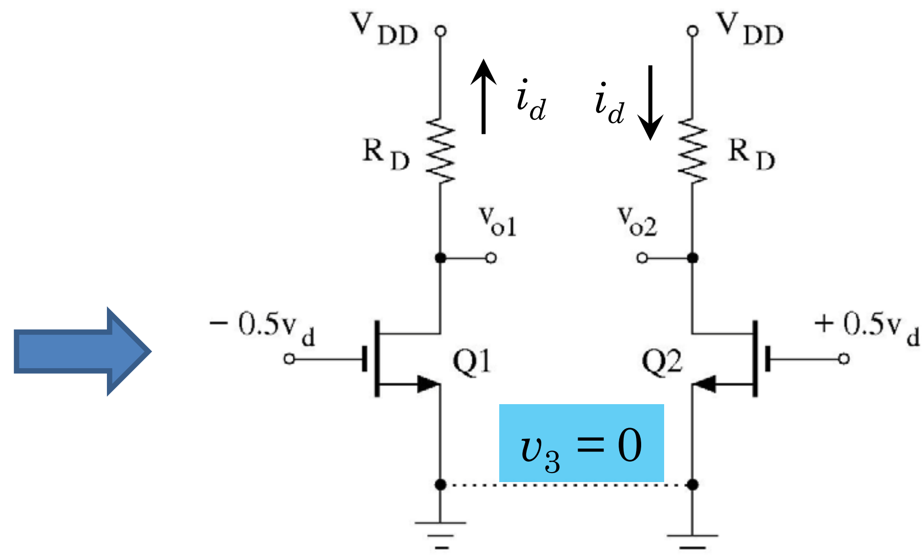
Only possible solution:

$$\boxed{\begin{array}{l} v_{o1} + v_{o2} = 0 \quad \Rightarrow \quad v_{o1} = -v_{o2} \\ v_3 = 0 \end{array}}$$

Differential Amplifier – Differential Mode (2)

$$\boxed{v_3 = 0 \quad \text{and} \quad v_{o1} = -v_{o2} \quad \Rightarrow \quad i_{d1} = -i_{d2}}$$





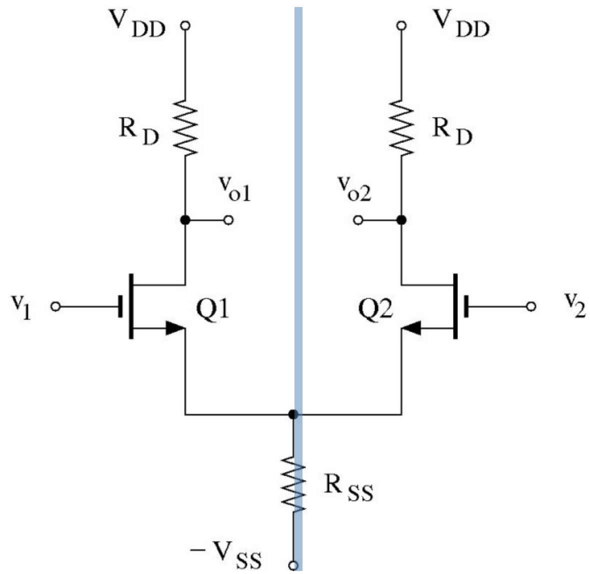
CS Amplifier

$$\frac{v_{o1}}{-0.5v_d} = -g_m(r_o \| R_D) \ , \quad \frac{v_{o2}}{+0.5v_d} = -g_m(r_o \| R_D)$$

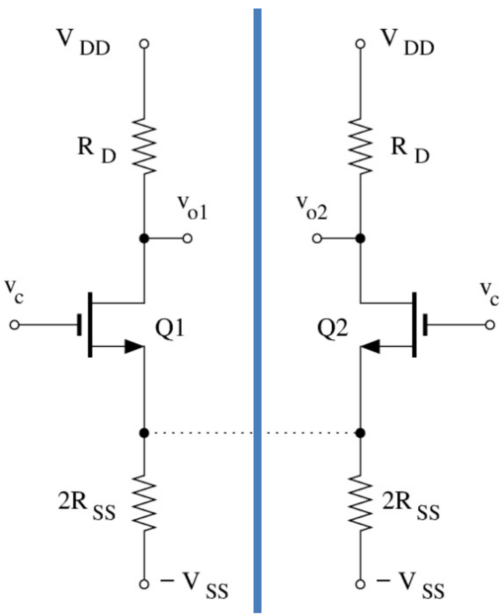
- **Because of the symmetry, the differential-mode circuit also breaks into two identical half-circuits.**

Concept of "Half Circuit"

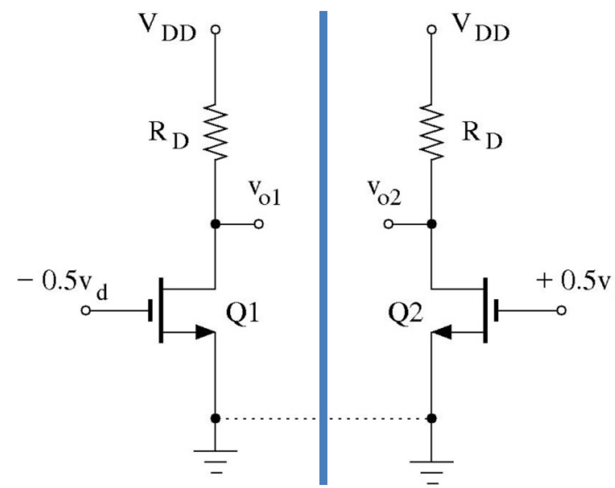
➢ **For a symmetric circuit, differential- and common-mode analysis can be performed using "half-circuits."**



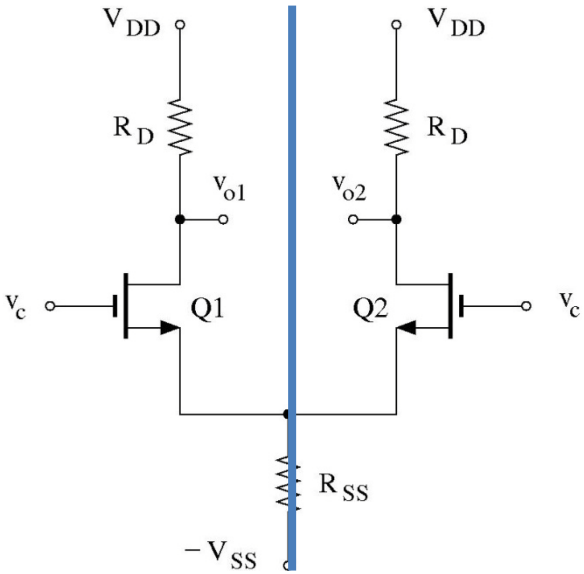
Common Mode



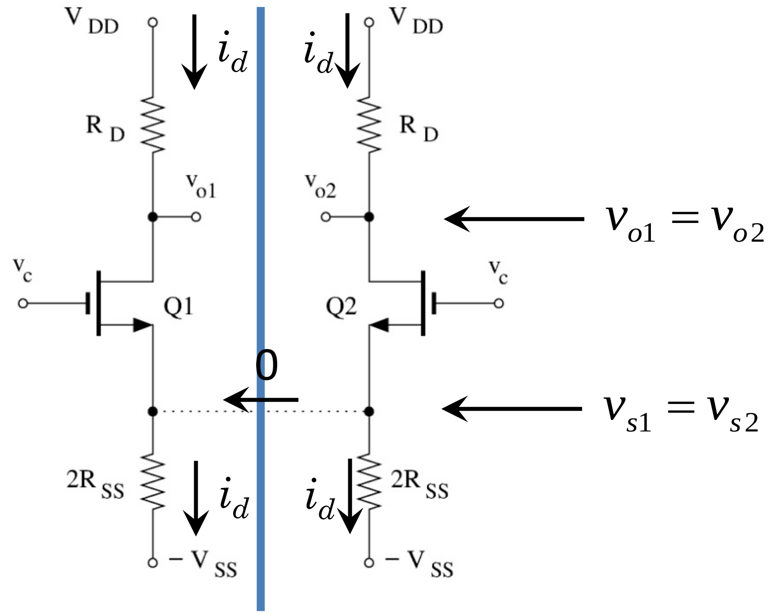
Differential Mode



Common-Mode "Half Circuit"



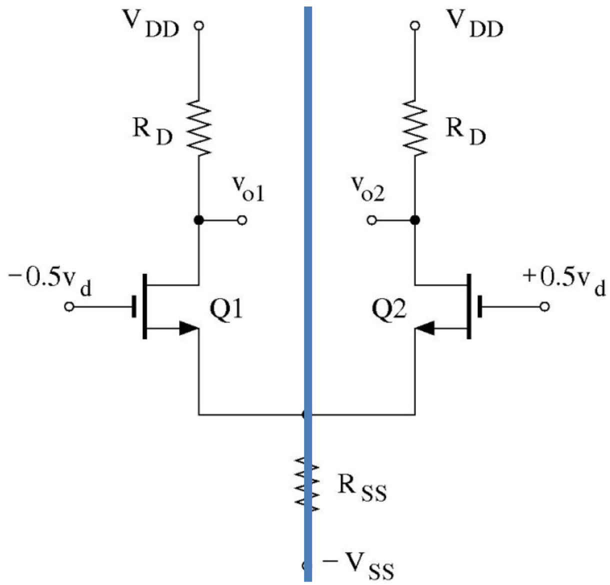
Common Mode circuit



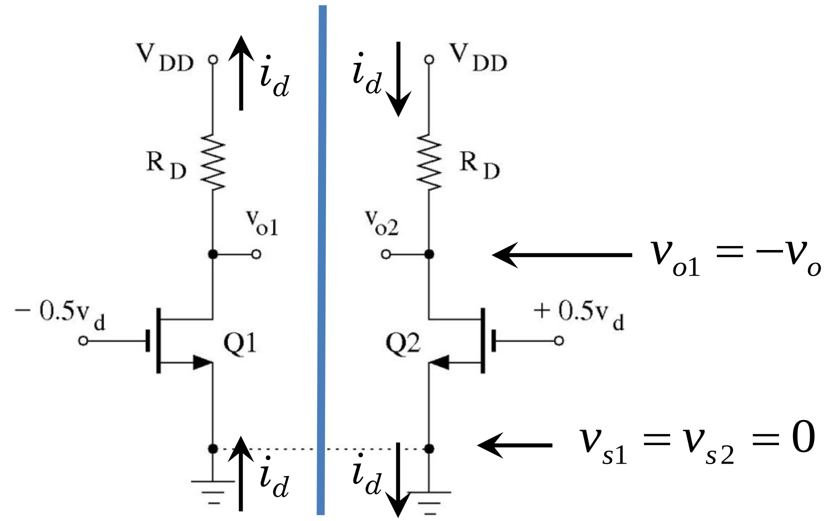
Common Mode Half-circuit

1. Currents about symmetry line are equal.
2. Voltages about the symmetry line are equal (e.g., $v_{o1} = v_{o2}$)
3. No current crosses the symmetry line.

Differential-Mode "Half Circuit"



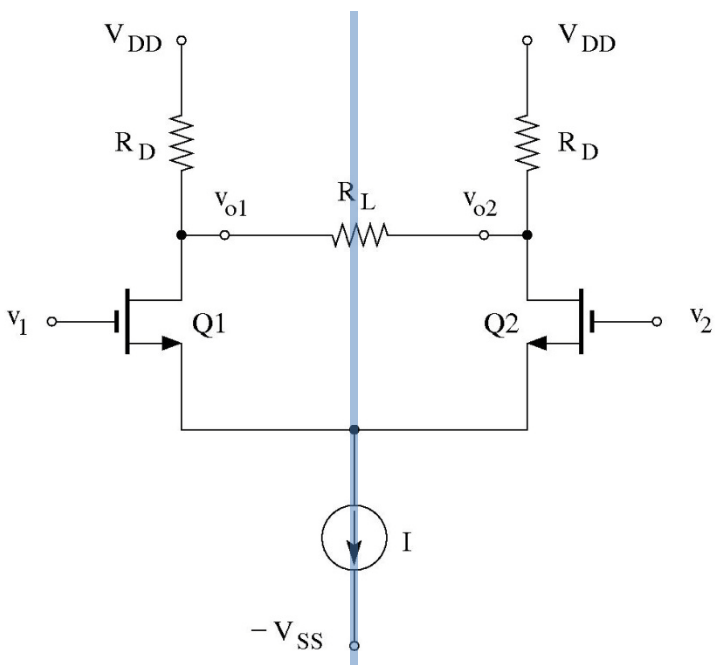
Differential Mode circuit

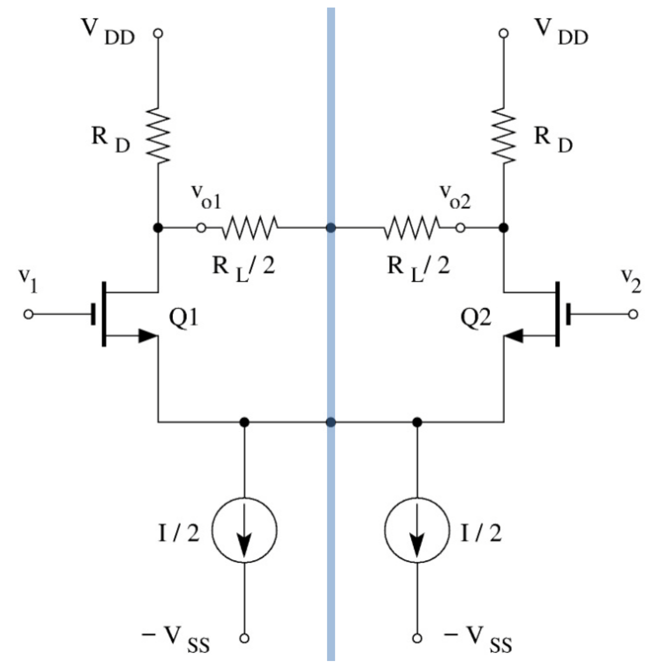


Differential Mode Half-circuit

1. Currents about the symmetry line are equal in value and opposite in sign.
2. Voltages about the symmetry line are equal in value and opposite in sign.
3. Voltage at the summery line is zero

Constructing "Half Circuits"





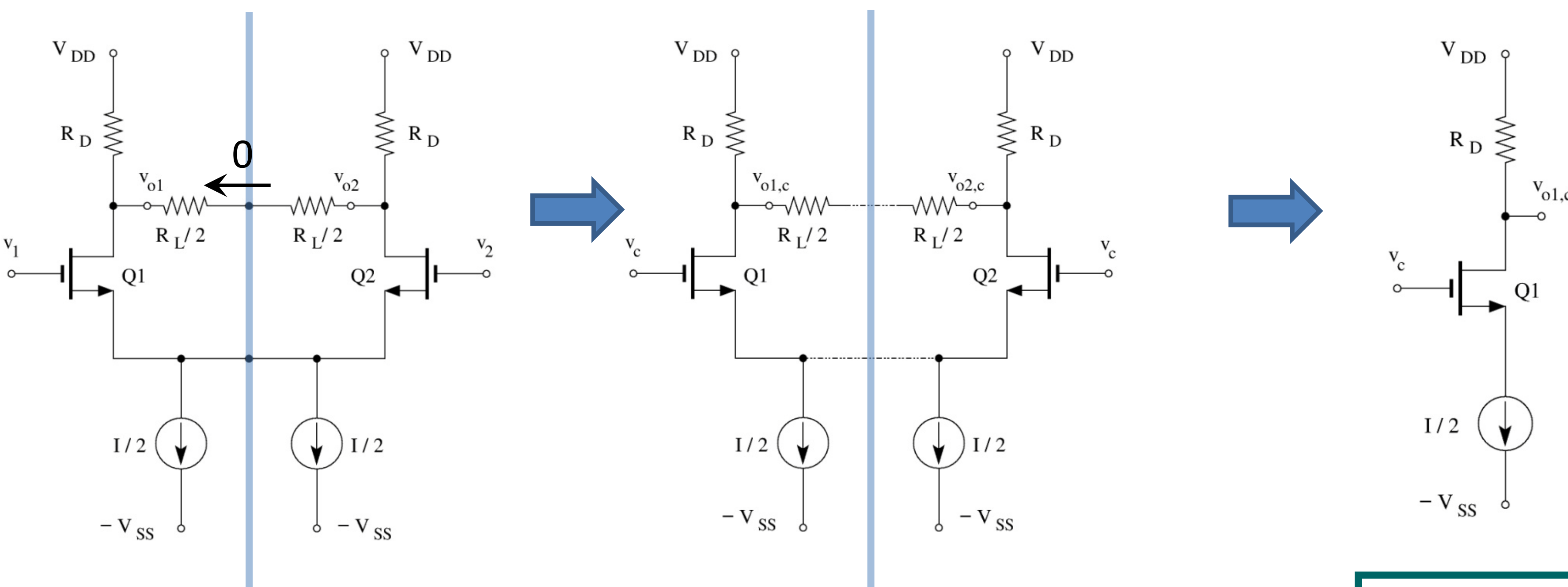
Step 1:

Divide **ALL elements** that <u>cross</u> the symmetry line (e.g., R_L) and/or <u>are located on</u> the symmetry line (current source) such that we have a symmetric circuit (only wires should cross the symmetry line, nothing should be located on the symmetry line!)

Constructing "Half Circuit"– Common Mode

Step 2: Common Mode Half-circuit

1. Currents about symmetry line are equal (e.g., $i_{d1} = i_{d2}$).
2. Voltages about the symmetry line are equal (e.g., $v_{o1} = v_{o2}$).
3. **No current crosses the symmetry line.**

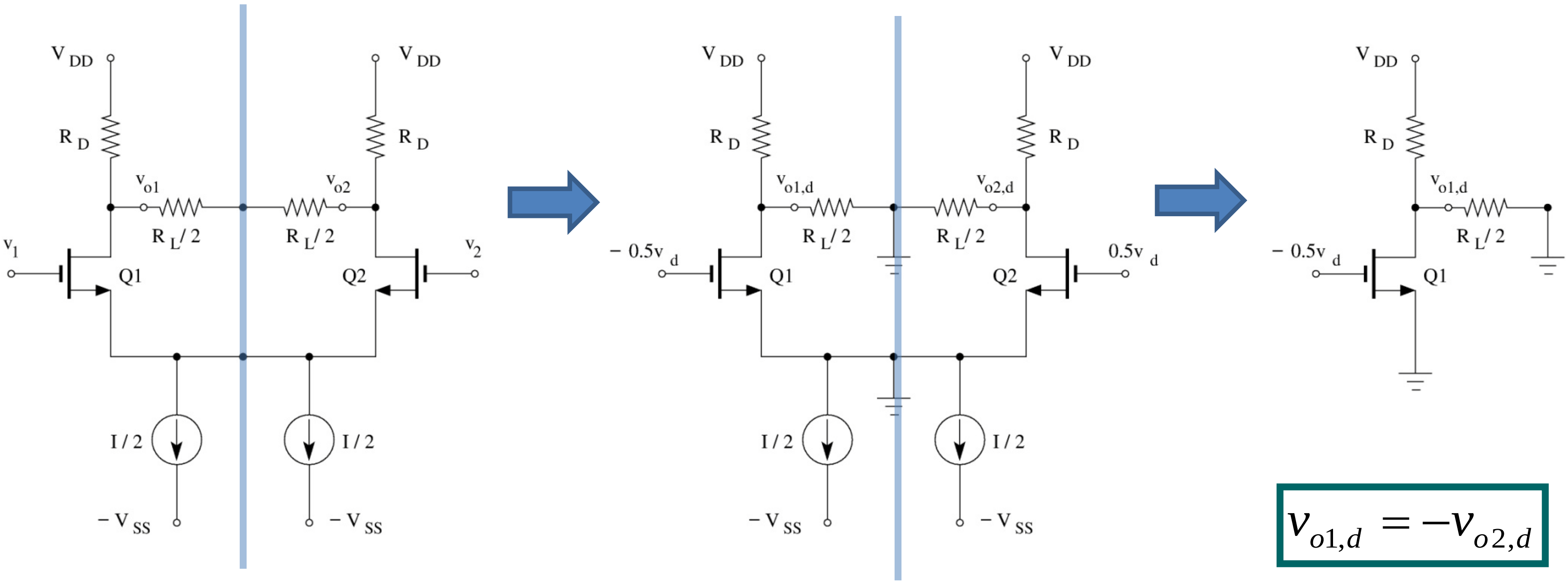


$$v_{o1,c} = v_{o2,c}$$

Constructing "Half Circuit"– Differential Mode

Step 3: Differential Mode Half-Circuit

1. Currents about symmetry line are equal but opposite sign (e.g., $i_{d1} = -i_{d2}$)
2. Voltages about the symmetry line are equal but opposite sign (e.g., $v_{o1} = -v_{o2}$)
3. **Voltage on the symmetry line is zero.**



$$v_{o1,d} = -v_{o2,d}$$

"Half-Circuit" works only if the circuit is symmetric!

- Half circuits for common-mode and differential mode are different.
- Bias circuit is similar to Half circuit for common mode.
- Not all difference amplifiers are symmetric. Look at the load carefully!

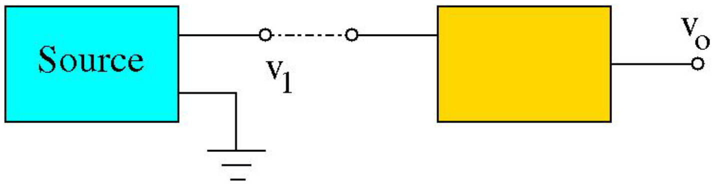
- We can still use half circuit concept if the deviation from prefect symmetry is small (i.e., if one transistor has R_D and the other $R_D + \Delta R_D$ with $\Delta R_D << R_D$).
  - However, we need to solve BOTH half-circuits (see slide 30)

Why are Differential Amplifiers popular?

- **They are much less sensitive to noise (CMRR >>1).**
- Biasing: Relatively easy direct coupling of stages:
  - Biasing resistor (R_{SS}) does not affect the differential gain (and does not need a by-pass capacitor).
  - No need for precise biasing of the gate in ICs
  - DC amplifiers (no coupling/bypass capacitors).
- …

Why is a large CMRR useful?

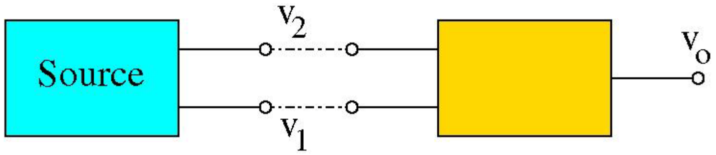
- A major goal in circuit design is to minimize the noise level (or improve signal-to-noise ratio). Noise comes from many sources (thermal, EM, …)
- A regular amplifier "amplifies" both signal and noise.



$$\boxed{v_1 = v_{sig} + v_{noise}}$$

$$\boxed{v_o = A \cdot v_1 = A \cdot v_{sig} + A \cdot v_{noise}}$$

- However, if the signal is applied between two inputs and <u>we use a difference amplifier with a large CMRR</u>, the signal is amplified a lot more than the noise which improves the signal to noise ratio.*



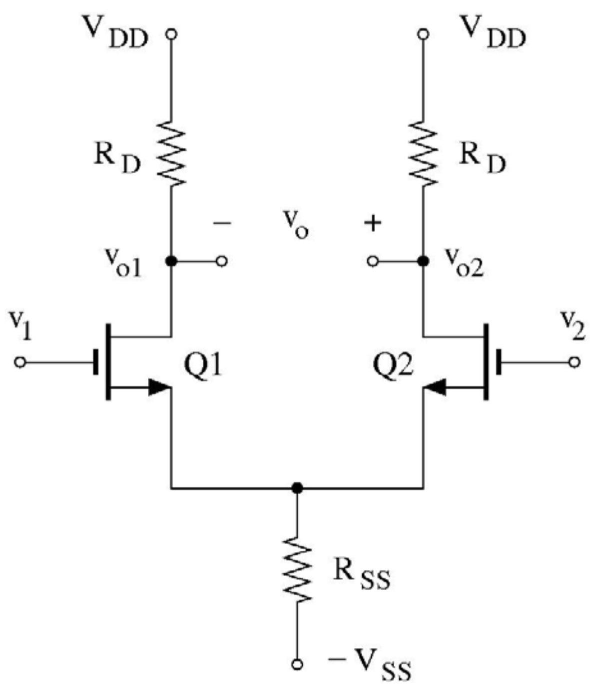
$$\boxed{\begin{array}{l} v_1 = -0.5 v_{sig} + v_{noise} \quad \& \quad v_2 = +0.5 v_{sig} + v_{noise} \\ v_d = v_2 - v_1 = v_{sig} \quad \& \quad v_c = v_{noise} \end{array}}$$

$$\boxed{v_o = A_d \cdot v_d + A_c \cdot v_c = A_d \cdot v_{sig} + \frac{A_d}{CMRR} \cdot v_{noise}}$$

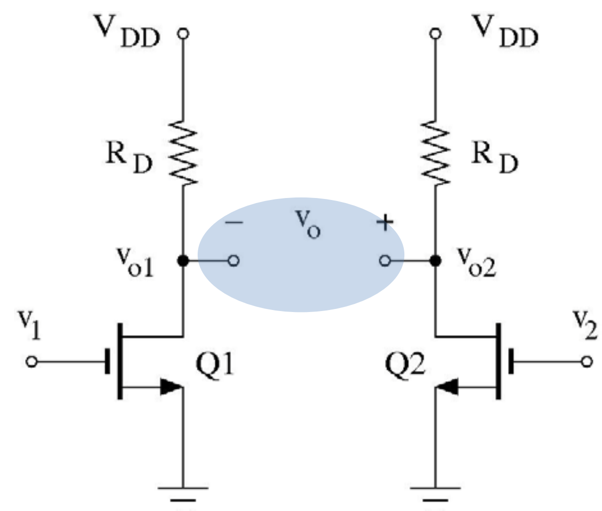
* Assuming that noise levels are similar to both inputs.

Comparing a differential amplifier two identical CS amplifiers (perfectly matched)

Differential Amplifier

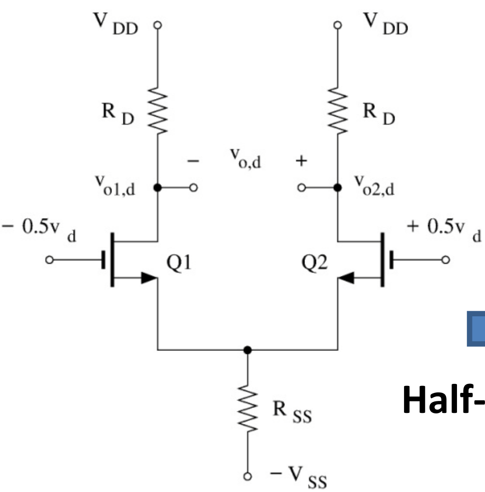


Two CS Amplifiers

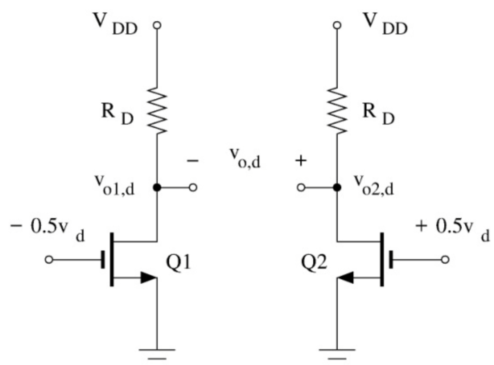


Comparison of a differential amplifier with two identical CS amplifiers – Differential Mode

Differential amplifier

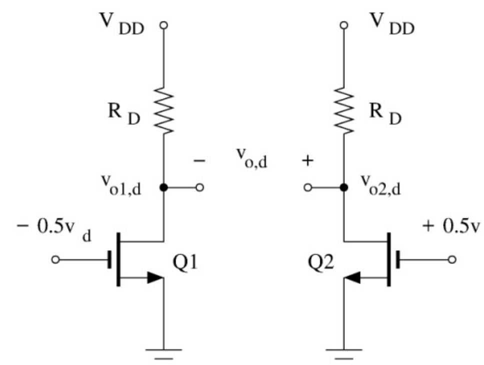


Half-Circuits



Identical

Two CS amplifiers



$$v_{o1,d} = -g_m (r_o \| R_D)\,(-0.5 v_d)$$

$$v_{o2,d} = -g_m (r_o \| R_D)\,(+0.5 v_d)$$

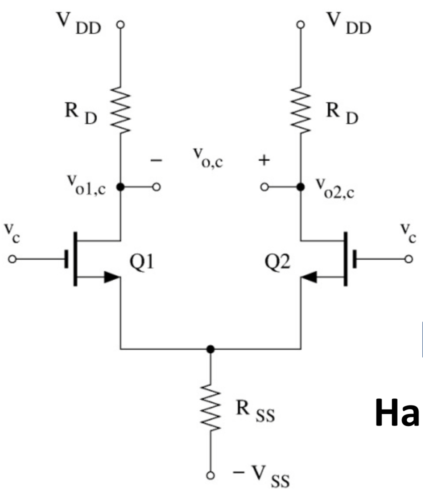
$$v_{od} = v_{o2,d} - v_{o1,d} = -g_m (r_o \| R_D) v_d$$

$$A_d = v_{od} / v_d = -g_m (r_o \| R_D)$$

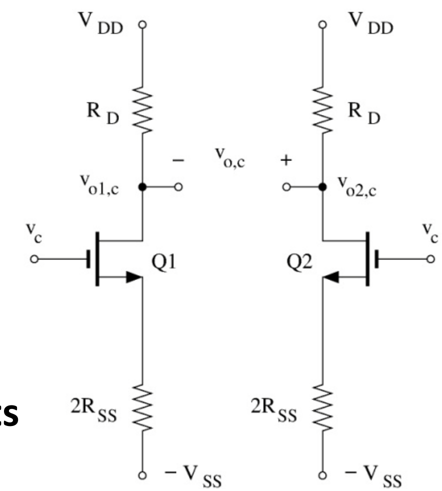
- $v_{o1,d}$, $v_{o2,d}$, v_{od}, **and differential gain, A_d, are identical.**

Comparison of a differential amplifier with two identical CS amplifiers – Common Mode

Differential amplifier

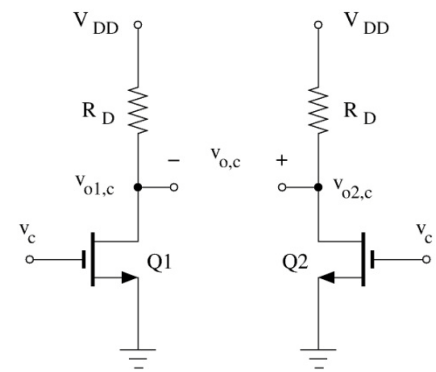


Half-Circuits



NOT Identical

Two CS amplifiers



$$v_{o1,c} = v_{o2,c} = -\frac{g_m R_D}{1 + 2 g_m R_{SS} + R_D / r_o} v_c$$

$$v_{oc} = v_{o2,c} - v_{o1,c} = 0$$

$$A_c = v_{oc} / v_c = 0$$

$$v_{o1,c} = v_{o2,c} = -g_m (r_o \| R_D) v_c$$

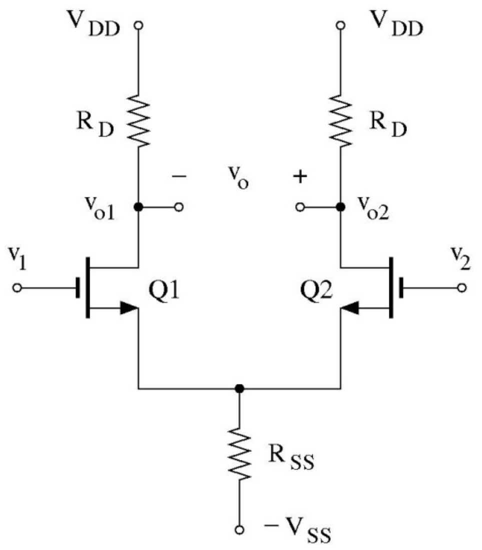
$$v_{oc} = v_{o2,c} - v_{o1,c} = 0$$

$$A_c = v_{oc} / v_c = 0$$

- **$v_{o1,c}$ & $v_{o2,c}$ are different! But v_{oc} = 0 and CMMR = ∞.**

Comparison of a differential amplifier with two identical CS amplifiers – Summary

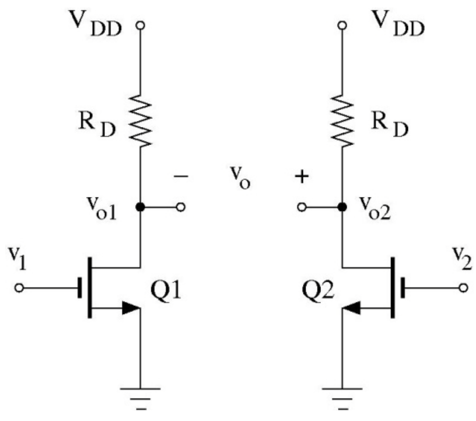
Differential Amplifier



$$A_d = \frac{v_{od}}{v_d} = -g_m (r_o \| R_D) \ , \ A_c = \frac{v_{oc}}{v_c} = 0$$

$$\text{CMRR} = \infty$$

Two CS Amplifiers

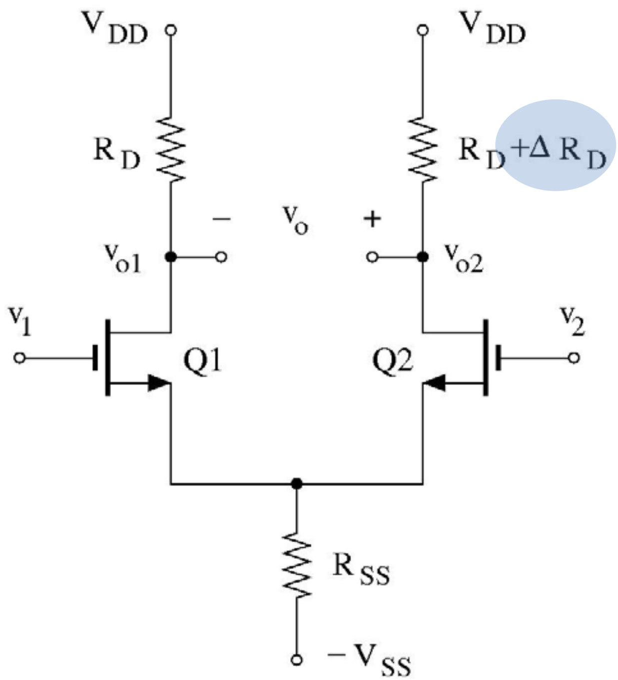


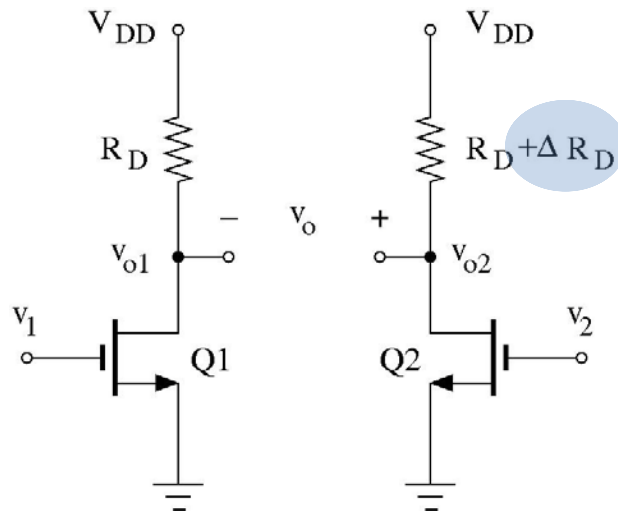
$$A_d = \frac{v_{od}}{v_d} = -g_m (r_o \| R_D) \ , \ A_c = \frac{v_{oc}}{v_c} = 0$$

$$\text{CMRR} = \infty$$

- **For perfectly matched circuits, there is no difference between a differential amplifier and two identical CS amplifiers.**
  - **But one can never make perfectly matched circuits!**

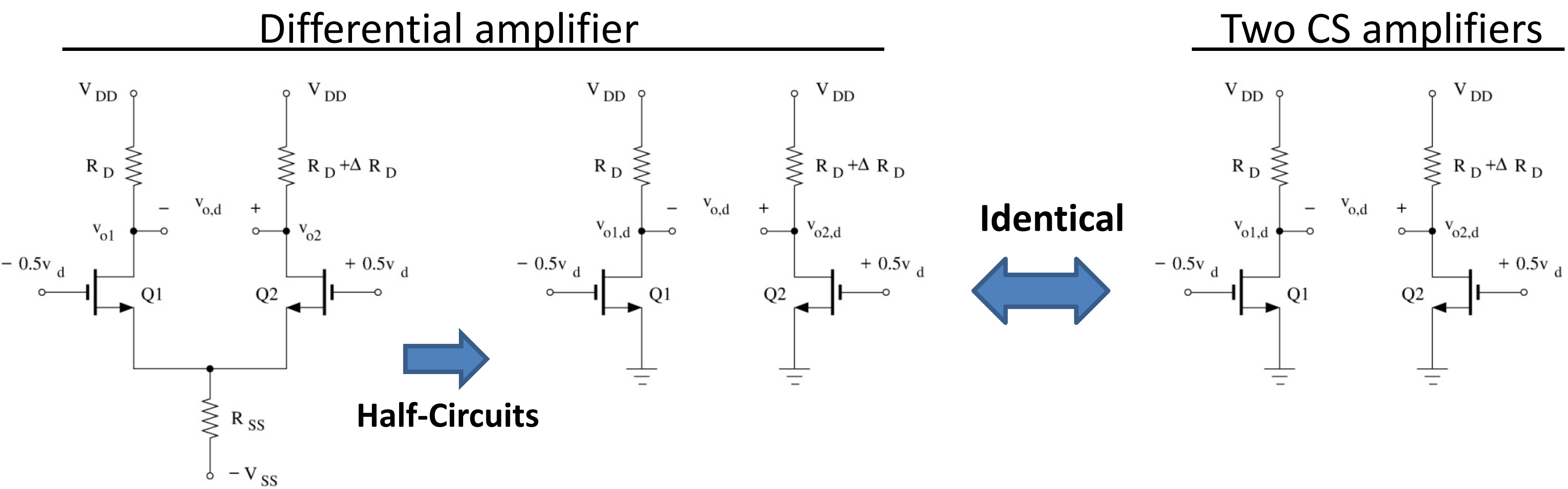
Consider a “slight” mis-match in the load resistors





➢ **We will ignore r_o in the this analysis (to make equations simpler)**

“Slightly” mis-matched loads – Differential Mode

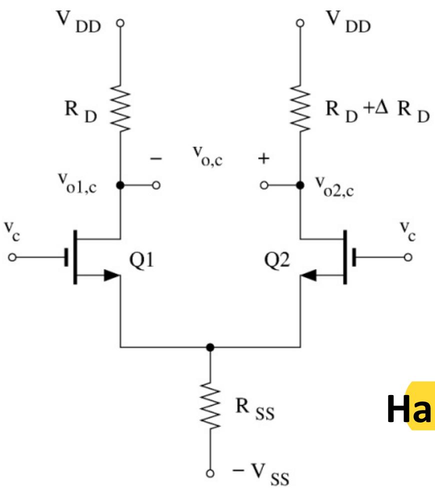


$$v_{o1,d} = -g_m (R_D)(-0.5 v_d)$$
$$v_{o2,d} = -g_m (R_D + \Delta R_D)(+0.5 v_d)$$
$$v_{od} = v_{o2,d} - v_{o1,d} = -g_m (R_D + 0.5\Delta R_D) v_d$$
$$A_d = v_{od} / v_d = -g_m (R_D + 0.5\Delta R_D)$$

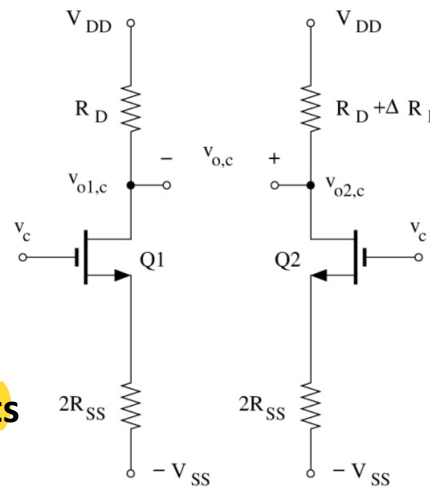
- **v_{o1}, v_{o2}, v_{od}, and differential gain, A_d, are identical.**

"Slightly" mis-matched loads – Common Mode

Differential amplifier

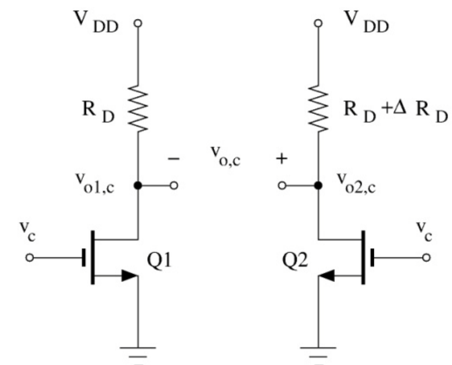


Half-Circuits



NOT Identical

Two CS amplifiers



$$v_{o1,c} = -\frac{g_m R_D}{1+2g_m R_{SS}} v_c \quad , \quad v_{o2,c} = -\frac{g_m (R_D + \Delta R_D)}{1+2g_m R_{SS}} v_c$$

$$v_{oc} = v_{o2,c} - v_{o1,c} = -\frac{g_m \Delta R_D}{1+2g_m R_{SS}} v_c$$

$$A_c = \frac{v_{oc}}{v_c} = -\frac{g_m \Delta R_D}{1+2g_m R_{SS}}$$

$$v_{o1,c} = -g_m R_D v_c$$

$$v_{o2,c} = -g_m (R_D + \Delta R_D) v_c$$

$$v_{oc} = v_{o2,c} - v_{o1,c} = +g_m \Delta R_D v_c$$

$$A_c = \frac{v_{oc}}{v_c} = +g_m \Delta R_D$$

- **v_{o1} and v_{o2} are different. In addition, $v_{oc} \neq 0$ and CMMR $\neq \infty$.**

A differential amplifier increases CMRR substantially for a slight mis-match ($\Delta R_D \neq 0$)

Two CS Amplifiers

$$A_d = -g_m(R_D + 0.5\Delta R_D)$$

$$A_c = +g_m \Delta R_D$$

$$\text{CMRR} \approx \frac{1}{\Delta R_D / R_D}$$

Differential Amplifier

$$A_d = -g_m(R_D + 0.5\Delta R_D)$$

$$A_c = -\frac{g_m \Delta R_D}{1 + 2 g_m R_{SS}}$$

$$\text{CMRR} \approx \frac{1 + 2 g_m R_{SS}}{\Delta R_D / R_D}$$

- Differential amplifier reduces A_c and increases CMRR substantially (by a factor of: $1 + 2\, g_m R_{SS}$).
  - The common-mode half-circuits for a differential amplifier are CS amplifiers with R_S (thus common mode gain is much smaller than two CS amplifiers).
  - We should use a large R_{SS} in a differential amplifier!

* Exercise: Compare a differential amplifier and two CS amplifiers with a mis-match in g_m