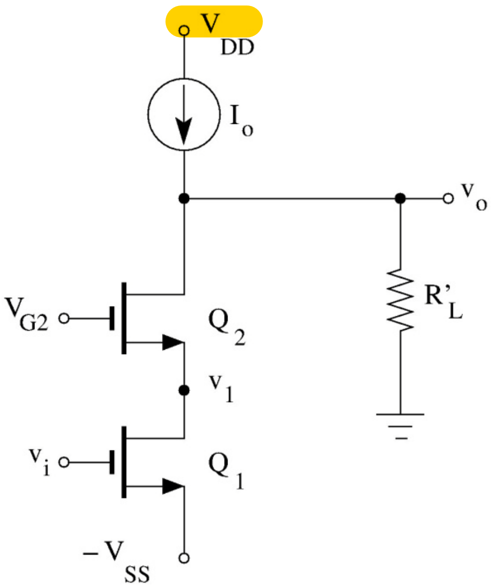
# 6. Cascode Amplifiers and Cascode Current Mirrors

Sedra & Smith Sec. 7 (MOS portion)

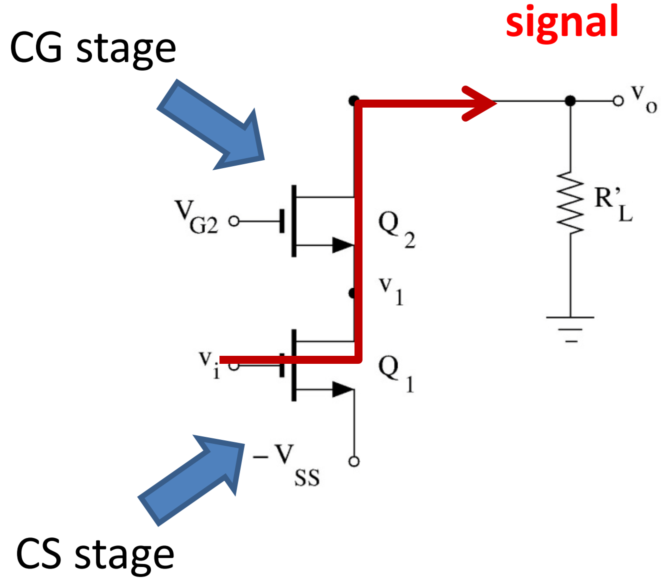
(S&S 5th Ed: Sec. 6 MOS portion & ignore frequency response)

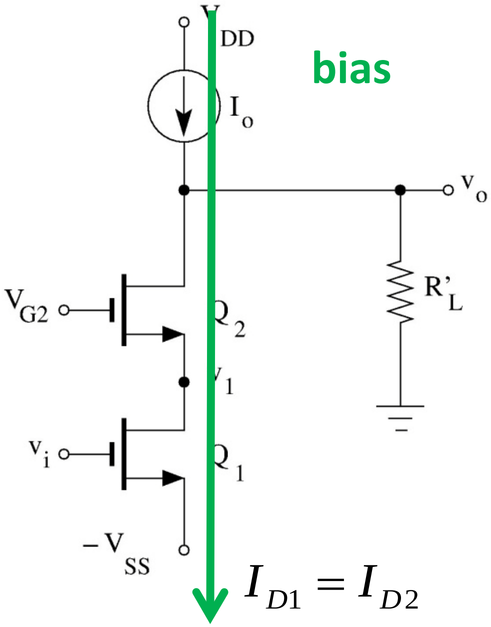
# Cascode amplifier
# is a popular building block of ICs

**Cascode Configuration**




**Signal circuit**: Current source becomes an open circuit

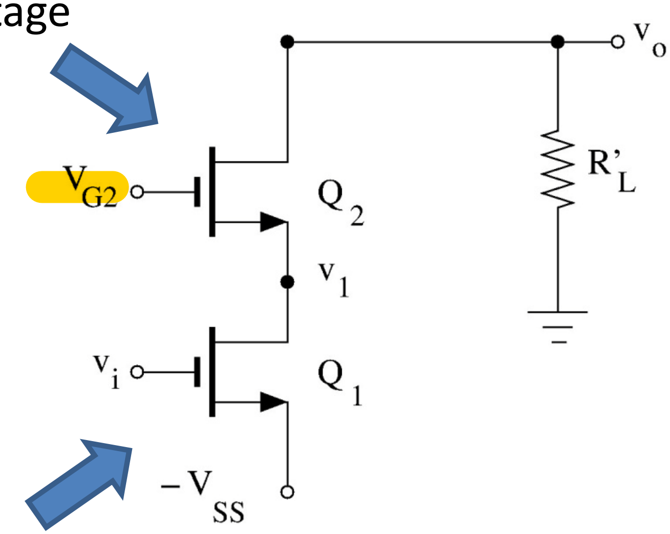



**Cascode amplifier is a two-stage, CS-CG configuration**

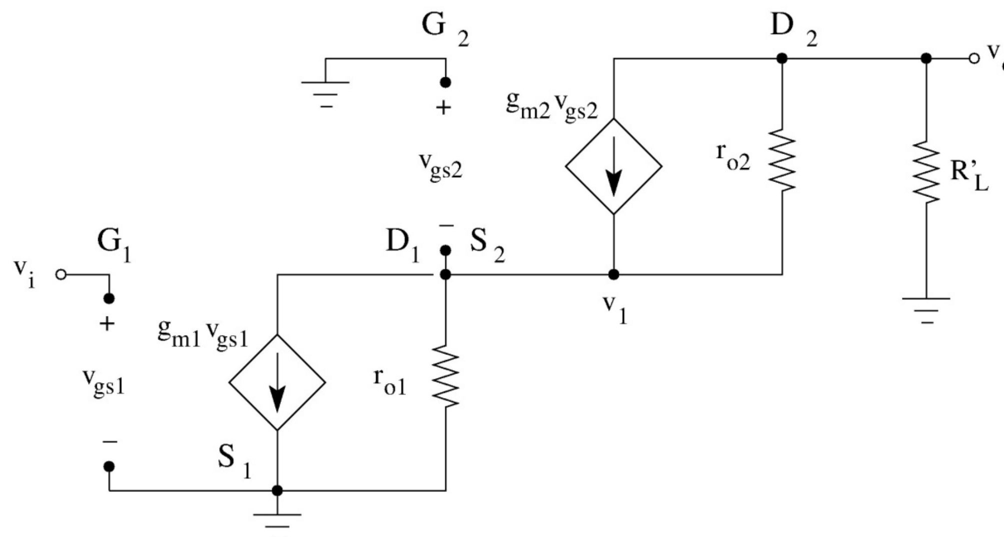
# Small Signal Model of a Cascode Amplifier

CG stage




CS stage

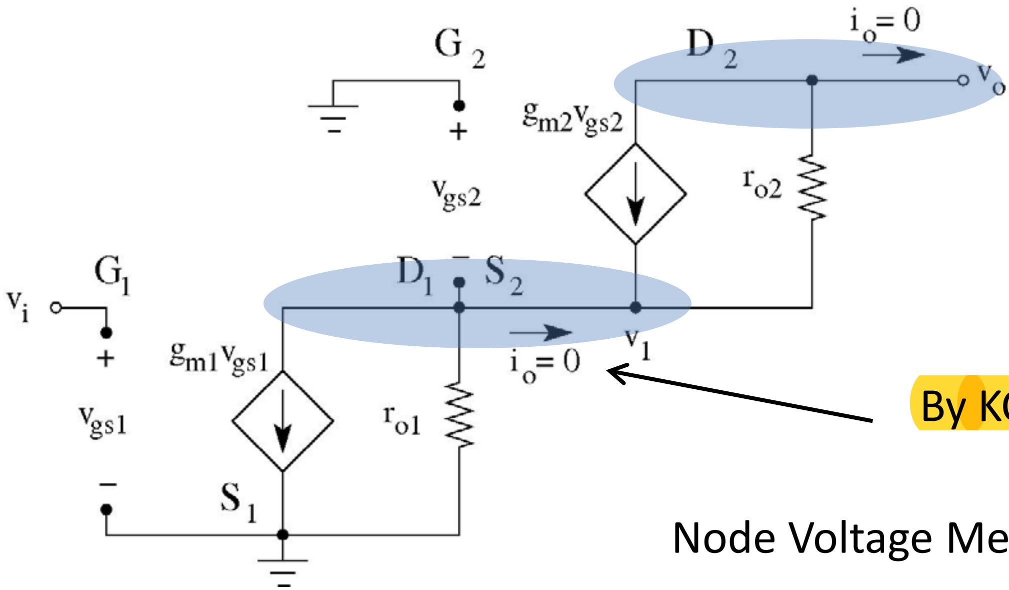
**Small Signal Model**




- Lengthy analysis to find $A_v$ (and a complicated equation). Simpler to compute open-loop gain ($A_{vo}$) and $R_o$.
- Text book introduces $G_m$ method to find $A_{vo}$ (See S&S Sec. 1)
- Here will find $A_{vo}$ directly from the small signal model.
- **However, the solution of and insight into Cascode amplifiers are best obtained using fundamental MOS configurations!**

Note that $A_{vo}$ and $R_o$ calculated here are meant to find $A_v$ and guide the choice of the active load. $A_{vo}$ and $R_o$ should be re-calculated for a practical circuit (see slides 14 & 15)

# Open-Loop gain of a Cascode amplifier (using small signal model)




Open Loop Gain ($R'_L \to \infty$, $i_o = 0$):

By KCL around Q2

Node Voltage Method:

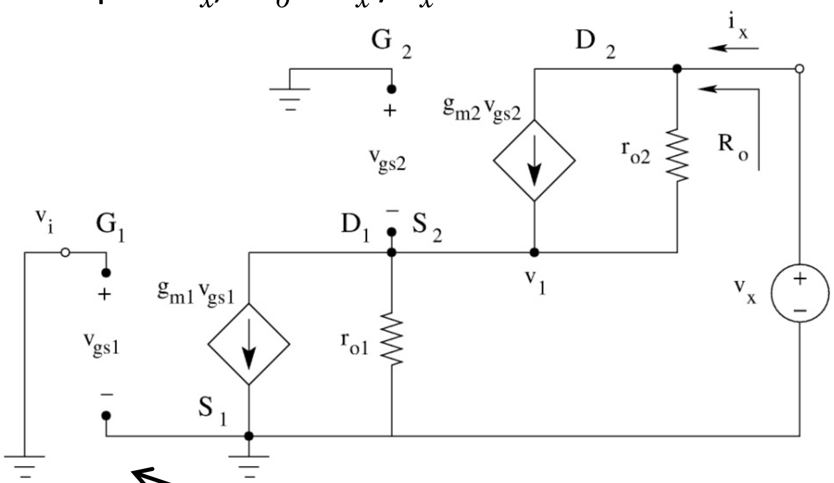
Node $v_o$: $$\frac{v_o - v_1}{r_{o2}} - g_{m2}v_1 = 0 \quad \Rightarrow \quad v_o = (1 + g_{m2}r_{o2})\, v_1$$

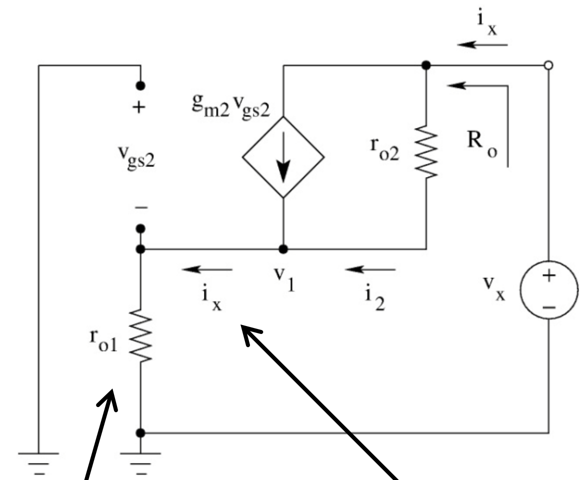
Node $v_1$: $$\frac{v_1}{r_{o1}} + g_{m1}v_i + 0 = 0 \quad \Rightarrow \quad v_1 = -g_{m1}r_{o1}\, v_i$$

$$\boxed{A_{vo} = \frac{v_o}{v_i} = -g_{m1}r_{o1} \times (1 + g_{m2}r_{o2}) \approx -g_{m1}r_{o1}g_{m2}r_{o2}}$$

# Output Resistance of a Cascode amplifier (using small signal model)

Set $v_i = 0$, attach a voltage source $v_x$, compute $i_x$, $R_o = v_x / i_x$




$v_i = v_{gs1} = 0 \;\rightarrow g_{m1}\; v_{gs1}$ current source becomes open circuit

By KCL around Q2

$$\text{KVL}: \quad v_{gs2} = -i_x r_{o1}$$

$$\text{KCL}: \quad i_2 = i_x - g_{m2} v_{gs2} = i_x + i_x g_{m2} r_{o1} = i_x (1 + g_{m2} r_{o1})$$

$$\text{KVL}: \quad v_x = i_2 r_{o2} + i_x r_{o1} = i_x (1 + g_{m2} r_{o1}) r_{o2} + i_x r_{o1}$$

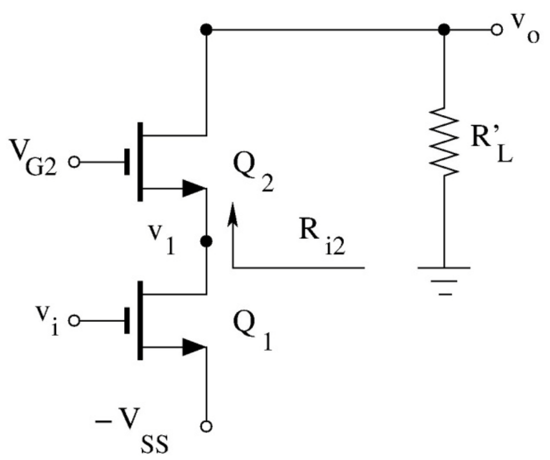
$$v_x = i_x [(1 + g_{m2} r_{o1}) r_{o2} + r_{o1}]$$

$$\boxed{R_o = \frac{v_x}{i_x} = r_{o1} + r_{o2} + g_{m2}\, r_{o1}\, r_{o2}}$$

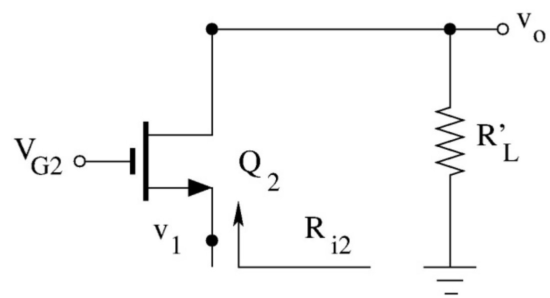
Note: $A_v = A_{vo} \times \dfrac{R_L' + R_o}{R_L'}$

# Gain of a Cascode Amplifier (using MOS Fundamental Configurations)

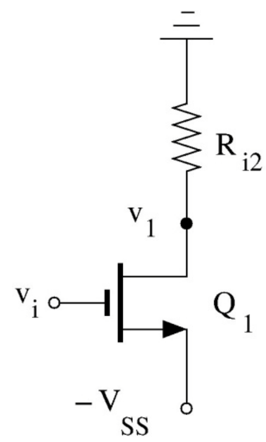
Cascode (signal circuit)




CG stage




CS stage




$$\boxed{A_{v2} = v_o / v_1 \approx g_{m2} (r_{o2} \,||\, R'_L)}$$

$$\boxed{A_{v1} = v_1 / v_i = -g_{m1} (r_{o1} \,||\, R_{i2})}$$

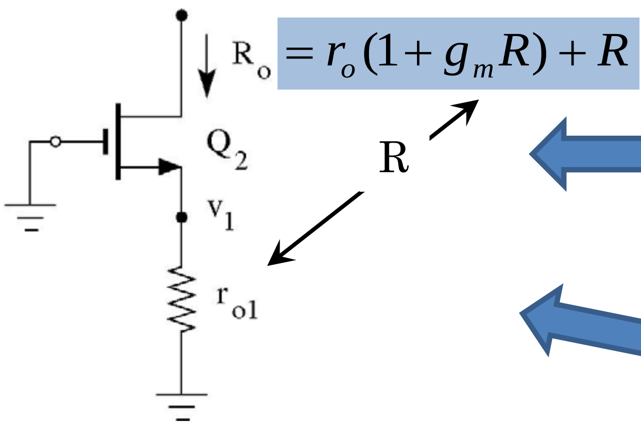
CG stages "reduces" the load seen by the CS stage by $g_{m2} r_{o2}$ →

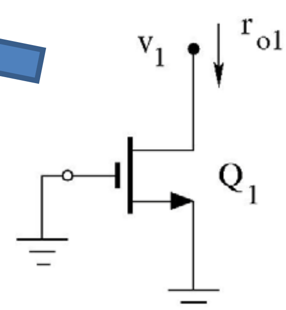
$$\boxed{R_{L1} = R_{i2} = \frac{r_{o2} + R'_L}{1 + g_{m2} r_{o2}}}$$

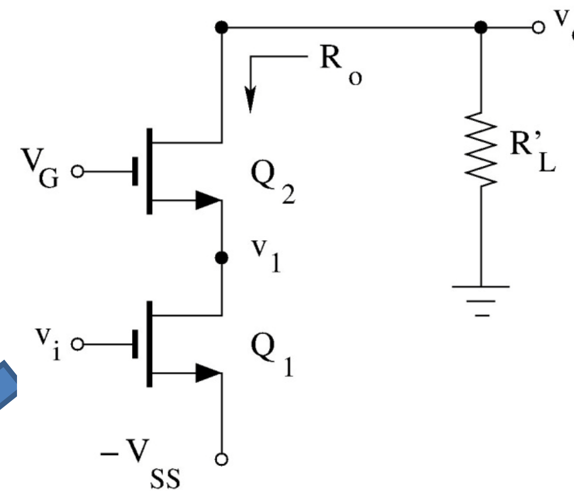
$$\boxed{A_v = v_o / v_i = A_{v1} A_{v2} = -g_{m1} g_{m2} (r_{o1} \,||\, R_{i2})\,(r_{o2} \,||\, R'_L)}$$

**Note:** Open Loop Gain: $(R'_L \to \infty)$ $\quad R_{L1} = R_{i2} = \dfrac{r_{o2} + R'_L}{1 + g_{m2} r_{o2}} = \infty \quad \to \quad A_{vo} = -g_{m1} r_{o1} g_{m2} r_{o2}$

# Output Resistance of a Cascode amplifier (from Elementary R forms)




$$R_o = r_{o2}(1 + g_{m2} r_{o1}) + r_{o1}$$

$$\boxed{R_o = r_{o1} + r_{o2} + g_{m2}\, r_{o1}\, r_{o2}}$$

$$R_o = r_{o2}(1 + g_{m2} r_{o1}) + r_{o1}$$

$$R_o \approx g_{m2} r_{o2} r_{o1} + r_{o1} = r_{o1}(1 + g_{m2} r_{o2})$$

$$\boxed{R_o \approx g_{m2}\, r_{o1}\, r_{o2}}$$

# Cascode Amplifier needs a large load

$$A_{v2} = g_{m2}(r_{o2} \parallel R'_L)$$

$$R_{L1} = R_{i2} = \frac{r_{o2} + R'_L}{1 + g_{m2} r_{o2}}$$

$$A_{v1} = -g_{m1}(r_{o1} \parallel R_{i2})$$

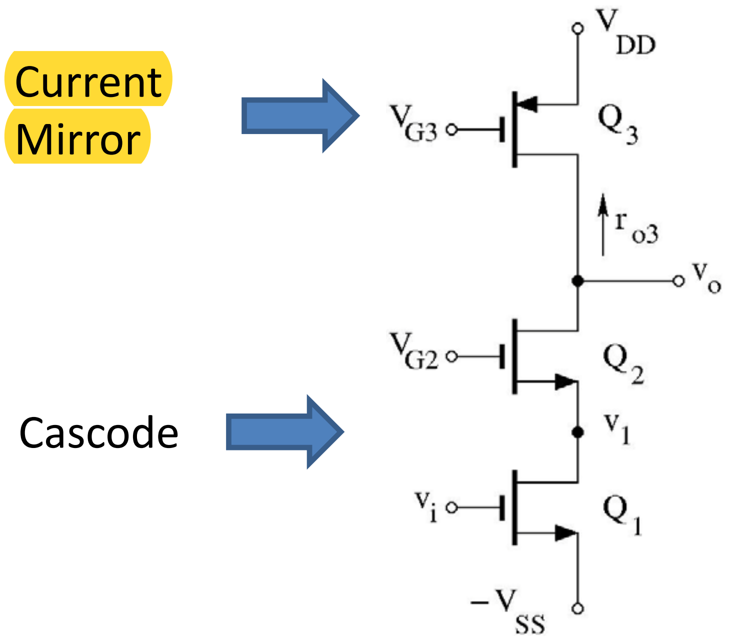
$$R_o \approx g_{m2}\, r_{o1}\, r_{o2}$$

**For simplicity assume $r_{o1} = r_{o2} = r_o$ and $g_{m1} = g_{m2} = g_m$**

| $R'_L$ | $A_{v2}$(CG) | $R_{i2} = R_{L1}$ | $A_{v1}$ (CS) | $A_v = A_{v1} A_{v2}$ | |
|---|---|---|---|---|---|
| $\infty$ | $g_m r_o$ | $\infty$ | $-g_m r_o$ | $-(g_m r_o)^2$ | **Max. Gain** |
| $(g_m r_o)\, r_o = R_o$ | $g_m r_o$ | $r_o$ | $-0.5 g_m r_o$ | $-0.5(g_m r_o)^2$ | **Practical Gain** |
| $r_o$ | $0.5\, g_m r_o$ | $2/g_m$ | $-2$ | $-g_m r_o$ | **Same gain as a single CS Amp.** |

- For comparison, a two-stage CS-amplifier (CS-CS) has a gain of $0.5\,(g_m r_o)^2$ for $R'_L = r_o$ and a gain of $(g_m r_o)^2$ for $R'_L = g_m r_o^2$.
  - Cascode amplifier needs a large load ($R'_L = g_m r_o^2$ ).

# Cascode amplifier needs a large load to get a high gain

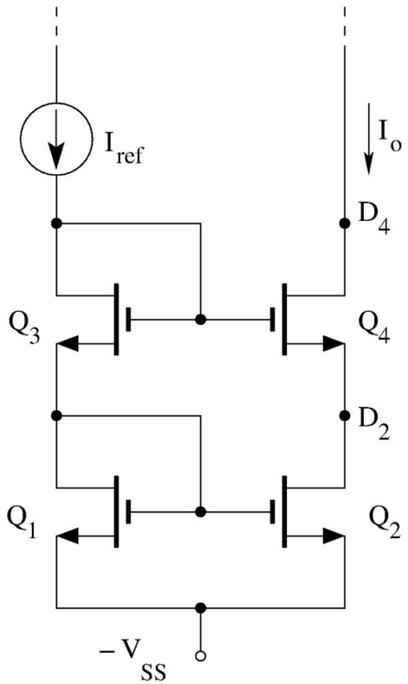



$R'_L = r_{o3}$

$A_v \approx -g_m r_o$

- Gain did not increase compared to a CS amplifier.
- This is still a useful circuit because of its high gain-bandwidth (we see this later).
- To get a high gain, $A_v = -0.5(g_m r_o)^2$ , we need to increase the small-signal resistance of the current mirror to $\approx (g_m r_o)\ r_o$
  - Cascode current mirror

# Cascode Current mirror



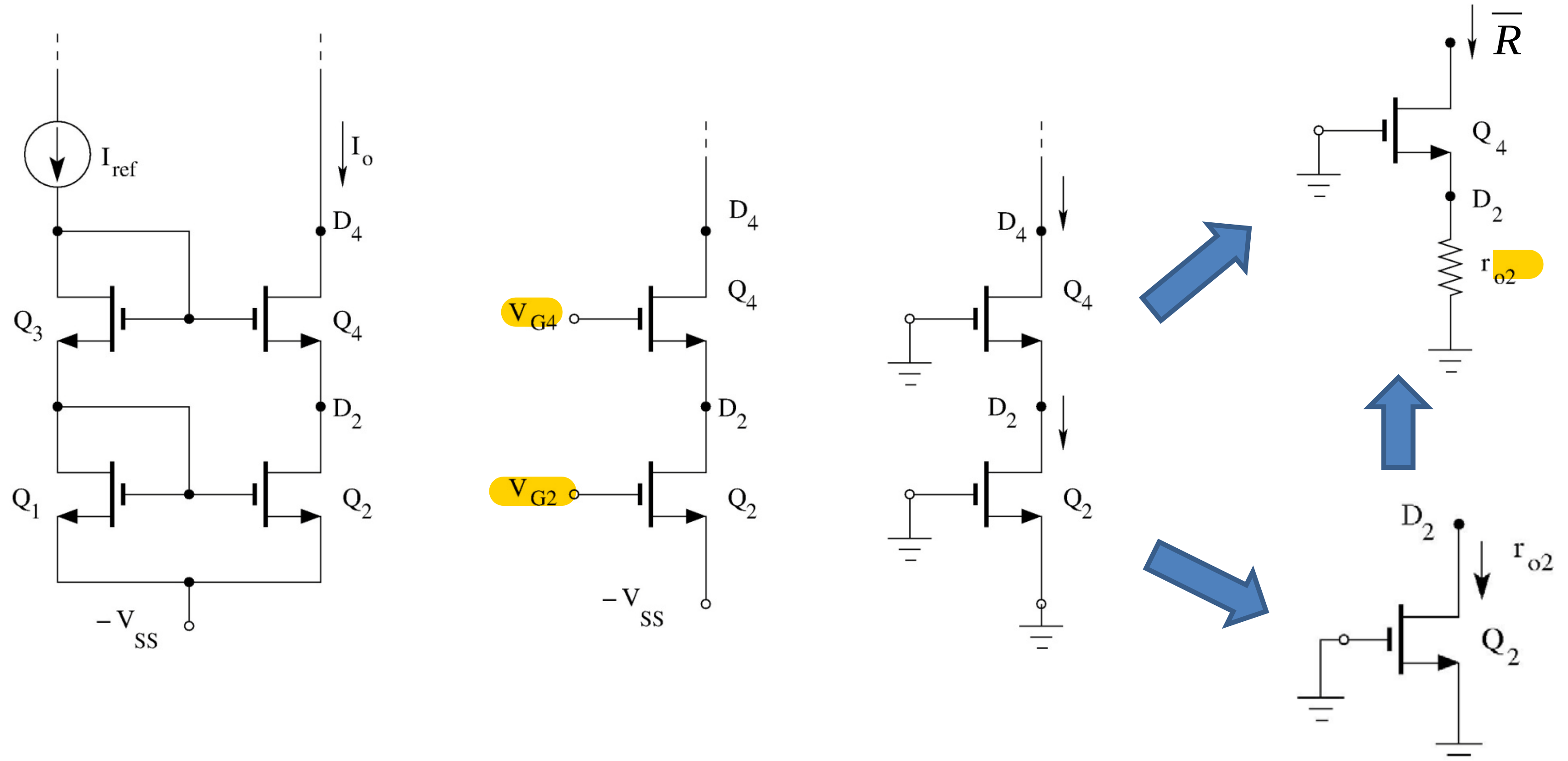

- Identical MOS: Same $\mu C_{ox}$ and $V_t$ , & $\dfrac{(W/L)_4}{(W/L)_3} = \dfrac{(W/L)_2}{(W/L)_1}$
  - $v_{GS1} = v_{GS2}$ & $v_{GS3} = v_{GS4}$
- Usually: $(W/L)_1 = (W/L)_3$ and $(W/L)_2 = (W/L)_4$
  - $v_{GS1} = v_{GS2} = v_{GS3} = v_{GS4} = v_{GS}$

- Q1 and Q3 are always in saturation
- Q2 and Q4 both have to be in saturation for current mirror to work
  - $V_{DS2} > V_{GS} - V_t$
  - $V_{DS4} > V_{GS} - V_t$
- Straight forward to show $I_o = \dfrac{(W/L)_2}{(W/L)_1} I_{ref}$

**Exercise:** Show that a single current mirror (no cascoding) works only if $V_{D2} > V_{OV} - V_{SS}$ and a cascode current mirror requires $V_{D4} > 2V_{OV} - V_{SS}$

# Small signal resistance of a cascode current mirror is quite large

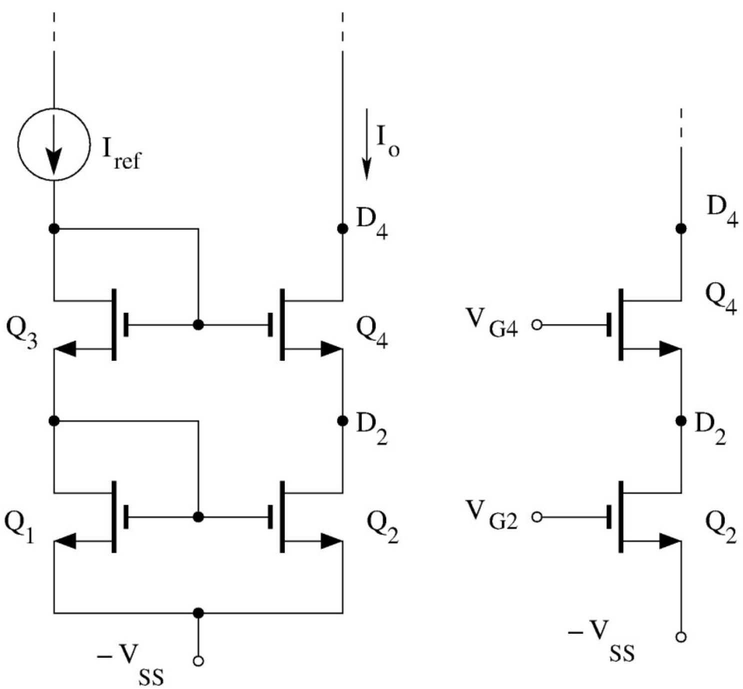
$$\overline{R} = r_{o4}(1 + g_{m4} r_{o2}) + r_{o2}$$



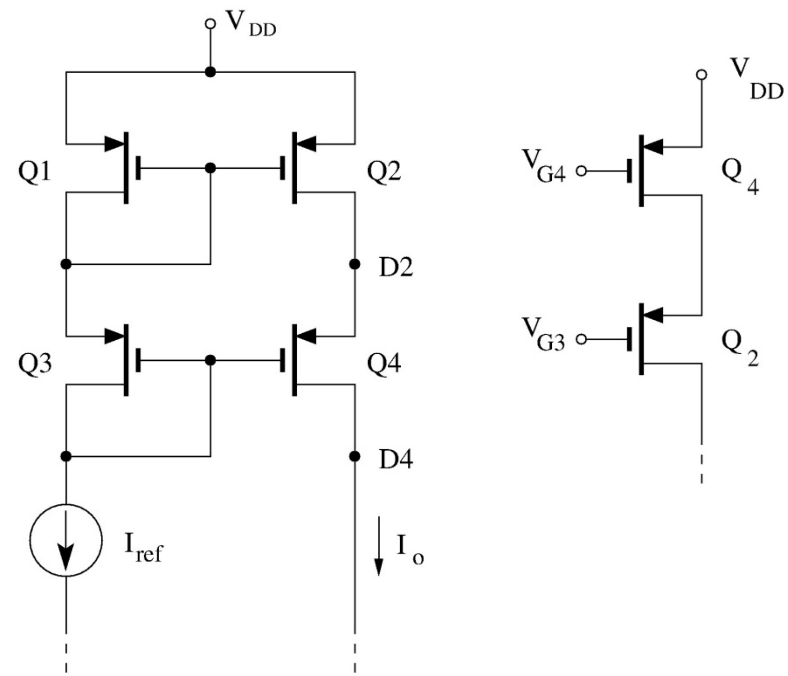

Transistor numbering is different in different circuits
Be careful in applying formulas!
It is best to use elementary R forms to find $\overline{R}$ instead of formula above.

# PMOS cascode current mirror is similar to NMOS version

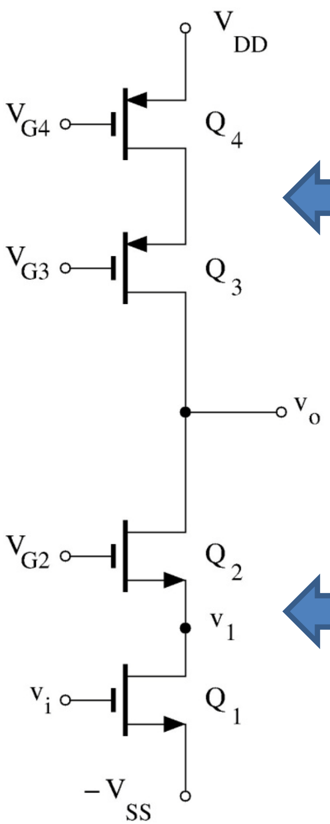
**NMOS Cascode current mirror**




**PMOS Cascode current mirror**

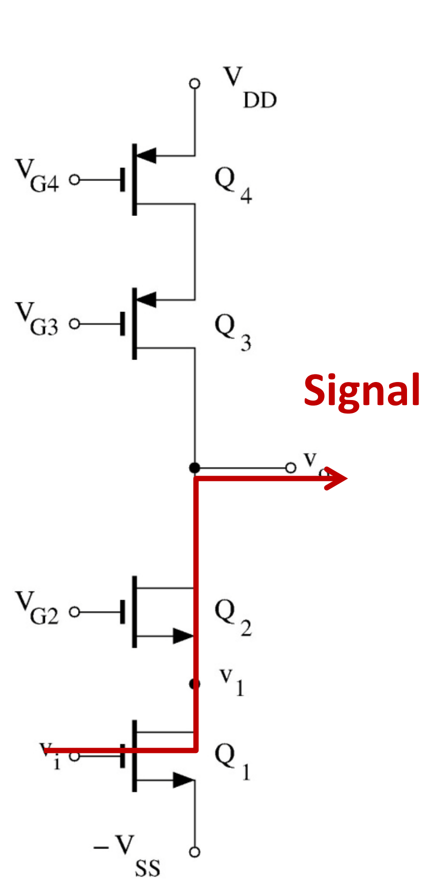


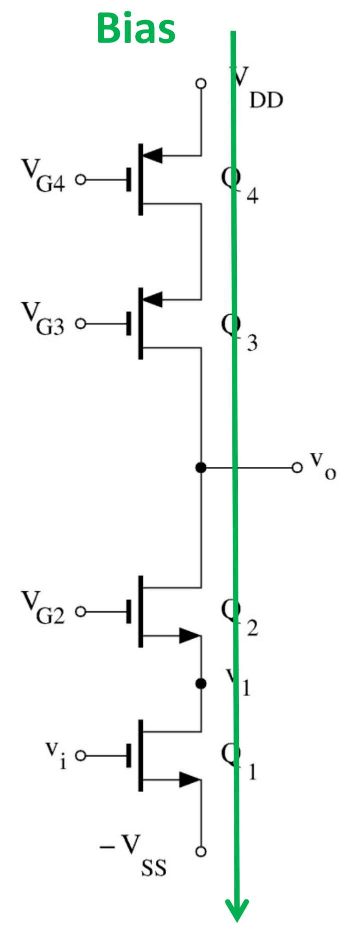
# Cascode amplifier with a cascode current mirror/active load




Cascode current mirror

Cascode amplifier




$$I_{D1} = I_{D2} = I_{D3} = I_{D4}$$

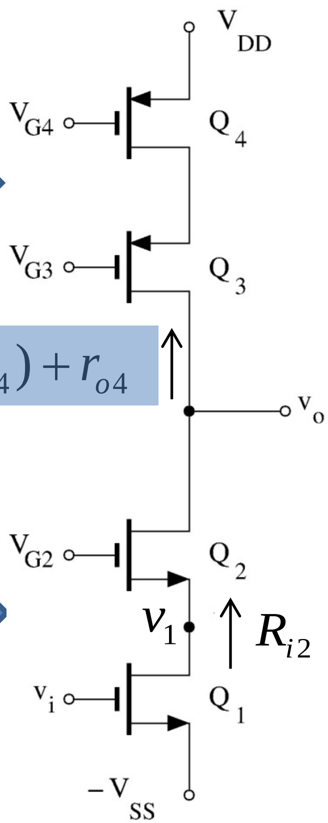
Exercise: Draw the circuit for a PMOS cascode with a cascode current mirror (cascode current mirror would be made of NMOS).

# Gain of a Cascode amplifier with a cascode current mirror/active load

Cascode current mirror

$r_{o3}(1+g_{m3}r_{o4})+r_{o4}$

Cascode amplifier




**Q2 (CG Amp)**

$$A_{v2} = v_o / v_1 \approx g_{m2}(r_{o2} \,||\, R'_L) \approx g_{m2} r_{o2}$$

$$R_{L1} = R_{i2} = \frac{r_{o2} + r_{o3}(1+g_{m3}r_{o4}) + r_{o4}}{1+g_{m2}r_{o2}} \approx \frac{g_{m3}r_{o3}r_{o4}}{g_{m2}r_{o2}}$$

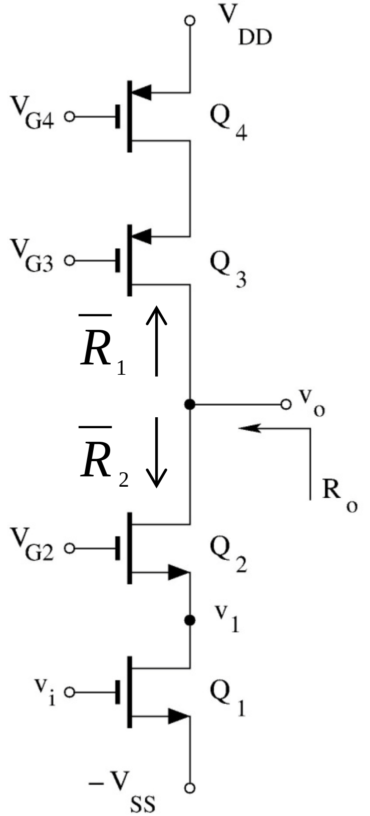
**Q1 (CS Amp)**

$$A_{v1} = v_1 / v_i = -g_{m1}\left(r_{o1} \,||\, \frac{g_{m3}r_{o3}r_{o4}}{g_{m2}r_{o2}}\right)$$

$$A_{vo} = A_{v1}A_{v2} = -\frac{g_{m1}g_{m2}g_{m3}r_{o1}r_{o2}r_{o3}r_{o4}}{g_{m2}r_{o1}r_{o2} + g_{m3}r_{o3}r_{o4}}$$

| Value for the same $g_m$ and $r_o$ |
|---|
| $A_{v2} \approx g_m r_o$ |
| $R_{L1} = R_{i2} \approx r_o$ |
| $A_{v1} = -0.5 g_m r_o$ |
| $A_{vo} = -0.5\,(g_m r_o)^2$ |

# Output Resistance of a Cascode amplifier with a cascode current mirror/active load




$$\overline{R}_1 = r_{o3}(1 + g_{m3} r_{o4}) + r_{o4}$$

$$\overline{R}_2 = r_{o2}(1 + g_{m2} r_{o1}) + r_{o1}$$

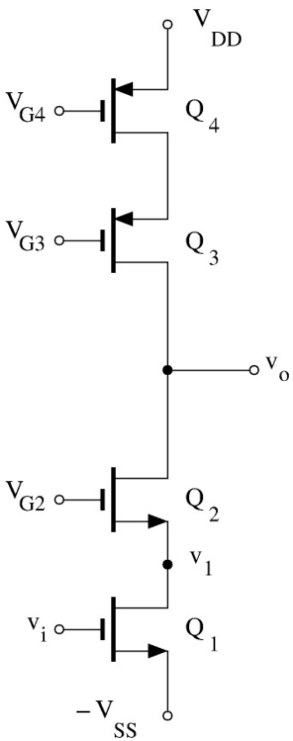
$$R_o = \overline{R}_1 \,\|\, \overline{R}_2$$

| Value for the same $g_m$ and $r_o$ |
| --- |
| $\overline{R}_1 \approx g_m r_o^2$ |
| $\overline{R}_2 \approx g_m r_o^2$ |
| $R_o = 0.5 g_m r_o^2$ |

# Why Cascode Amplifiers are popular?

**Cascode Amp.**

$$A_{vo} = -0.5\,(g_m r_o)^2$$
$$R_o = 0.5 g_m r_o^2$$




**Drawbacks:**

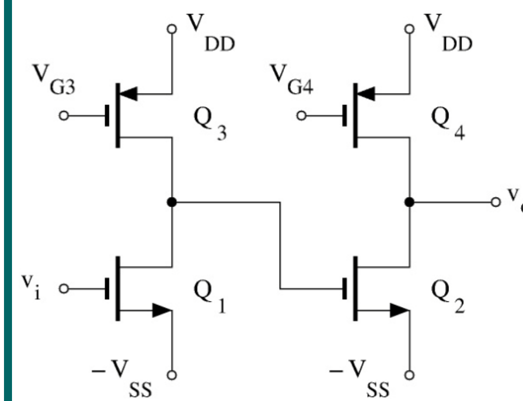
- While $A_{vo}$ are similar, Cascode has a very $R_o$ (MΩ level).
  - o should be followed with a CS or CD stage (infinite load for cascode)
  - o BJT cascodes are not useful.
- Low voltage headroom ($V_{DD}$ across 4 MOS)
  - o Folded cascodes solve this.

**Benefits:**

- **Much better high-frequency response (high gain-bandwidth).***
- Simpler biasing.

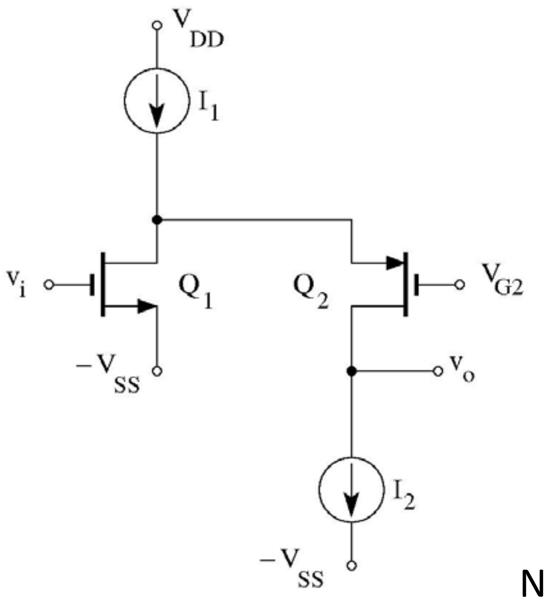
**2-stage CS Amp.**

$$A_{vo} = +0.25(g_m r_o)^2$$
$$R_o = r_o$$

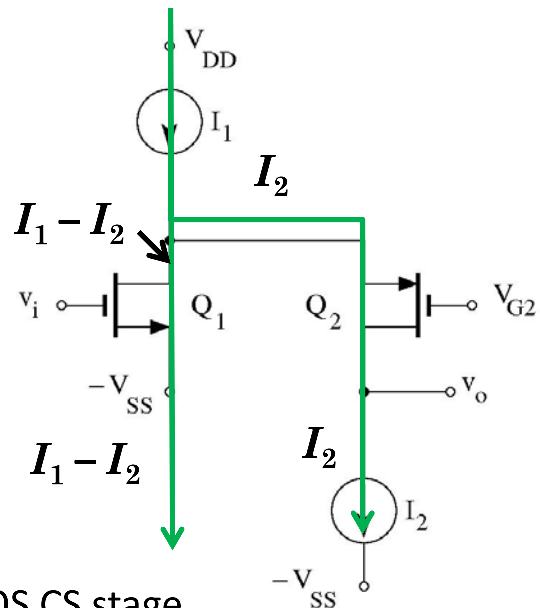



* We will see this later in our discussion of frequency response.

# Folded Cascode increases voltage overhead*



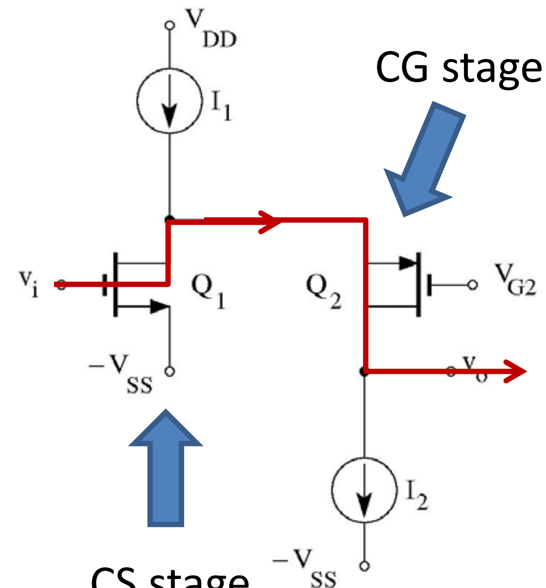

Bias




NMOS CS stage
Biased with $I_1 - I_2$

PMOS CG stage
Biased with $I_2$

Signal




* Folded cascode only helps the voltage overhead issue for difference amplifiers (see S&S pages 999-1000)