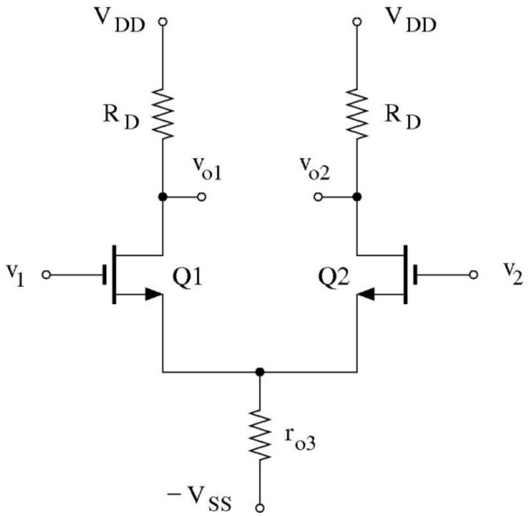
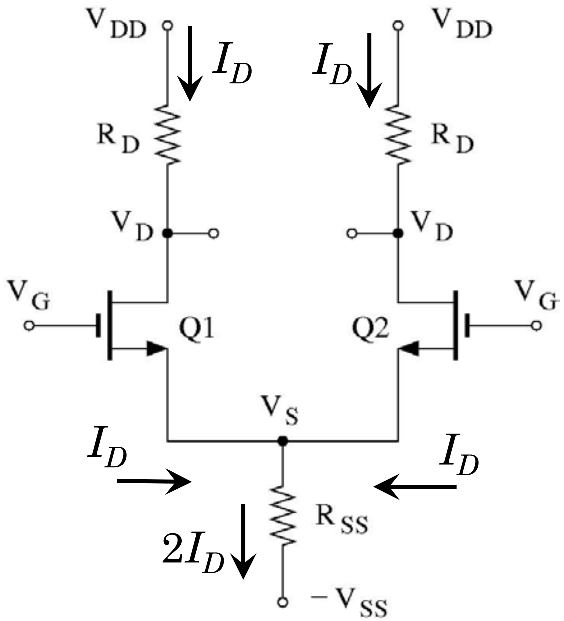
# Differential Amplifiers: Implementation on ICs

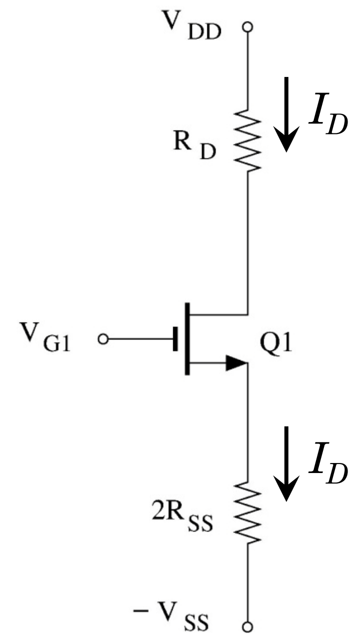
**Replacing $R_{SS}$ and $R_D$ with current-sources and active loads**



# Resistor $R_{ss}$ provides source degeneration for a stable bias



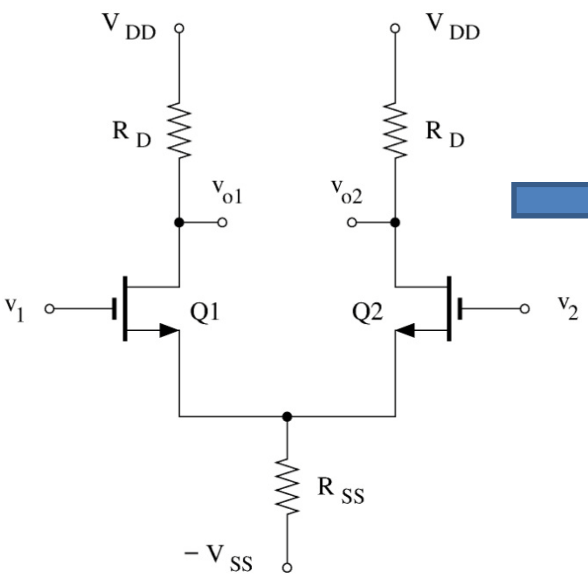

**Bias (Common Mode circuit )**

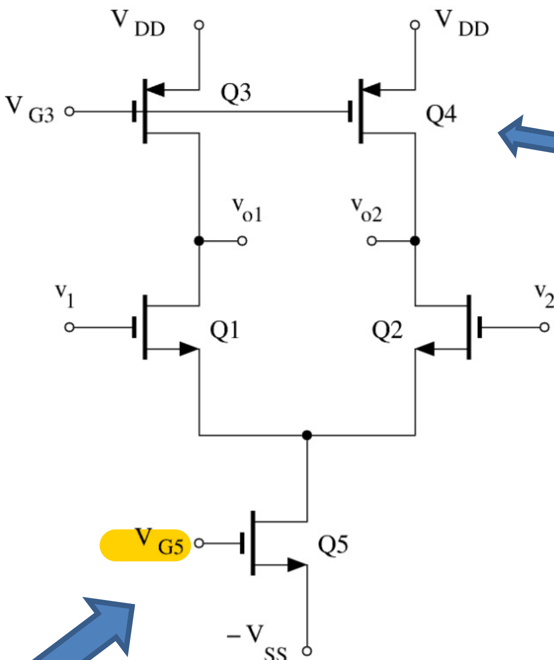



- In discrete circuits, bias is similar to that of a CS amplifier (source degeneration with a source resistor).
- **However** $R_{SS}$ does not affect the differential gain and , in fact, should be large to improve CMRR (no need for a by-pass capacitor!)

# Differential amplifier with current source active load

Q1 and Q2 are identical & $V_{G2} = V_{G1}$



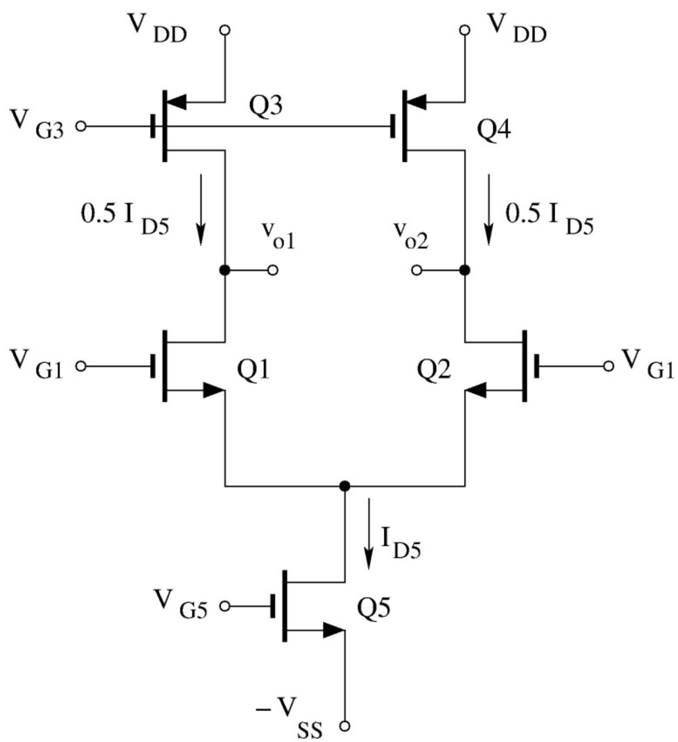

**Q3 and Q4 are identical**

- Q3/Q4 act active load/ current source (similar to a CS amplifier).

**Q5 is necessary**

- For signals, Q5 provides $R_{SS} = r_{o5}$ necessary for reducing common-mode gain (a large $R_{SS} = r_{o5}$ can be obtained without significant voltage drop across Q5).
- Parameters of Q5 (i.e., $W/L$, $V_G$) should be chosen such that $I_{D3} = I_{D4} = 0.5\ I_{D5}$ .
- Q5 eases the necessary precision in biasing Q1 and Q2 gates.

# Differential amplifier with current source active load – Bias




- Q1 and Q2 are identical & $V_{G2} = V_{G1}$
- Q3 and Q4 are identical
- Parameters of Q5 (i.e., $W/L$, $V_G$) are chosen such that $I_{D3} = I_{D4} = 0.5\ I_{D5}$

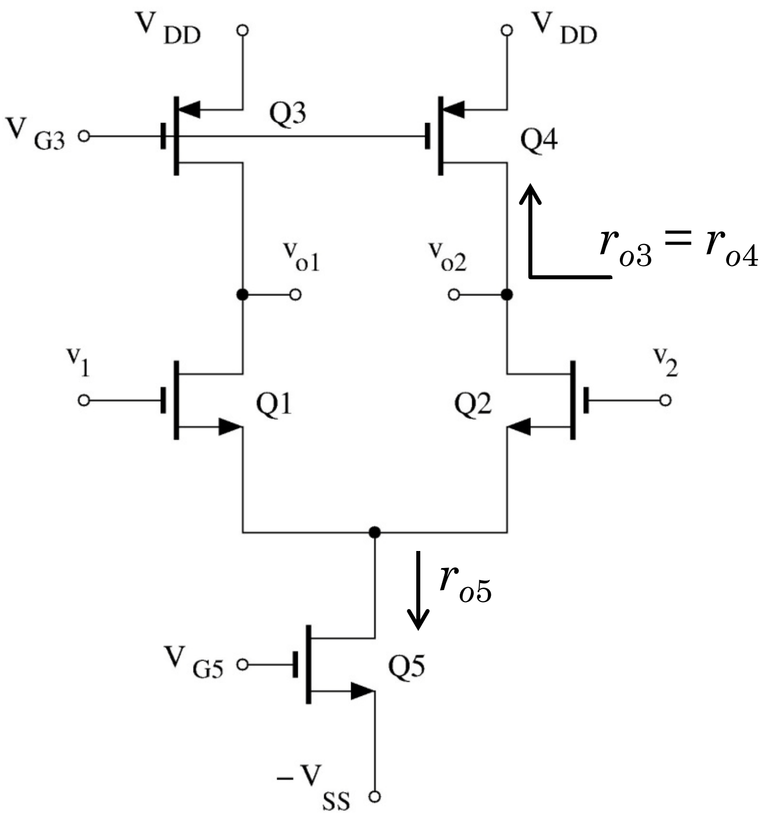
$$V_{GS1} = V_{GS2} \Rightarrow V_{OV1} = V_{OV2}$$
$$I_{D1} = I_{D2} = I_{D3} = I_{D4} = 0.5 I_{D5}$$

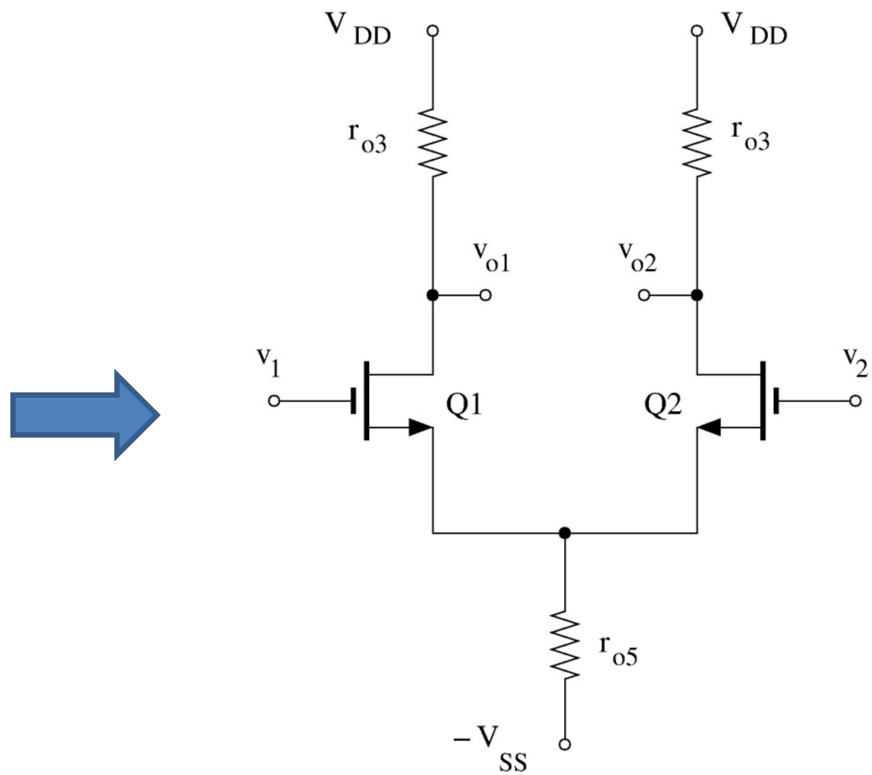
**Ignoring channel-width modulation:***

1. $I_{D1} = I_{D3} = 0.5\ I_{D5}$ sets $V_{OV1}$ and $V_{GS1}$
2. $V_{S1} = V_{GS1} - V_{G1}$
3. $V_{D5} = V_{S1}$
4. $V_{DS5} = V_{S1} + V_{SS}$
5. We need to include channel-width modulation to find $V_{DS1}$ and $V_{DS3}$
6. Precise biasing of Q1 and Q2 are not necessary to get correct $I_{D1}$ (it only affects $V_{DS1}$ and $V_{DS3}$ )*

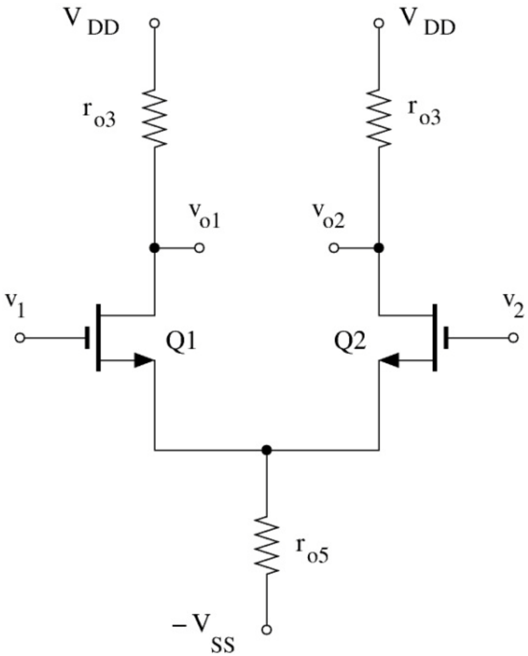
* Similar results are obtained if we do not ignore channel-width modulation: $V_S = V_{D5}$ will adjust to get the correct $V_{GS1}$ and $V_{OV1}$ (See problem set)

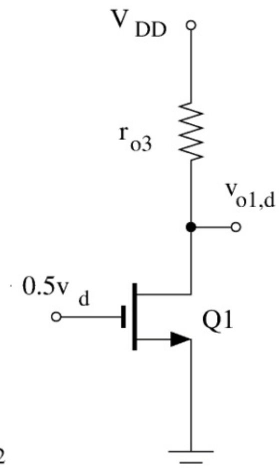
# Differential amplifier with current source active load – Signal analysis

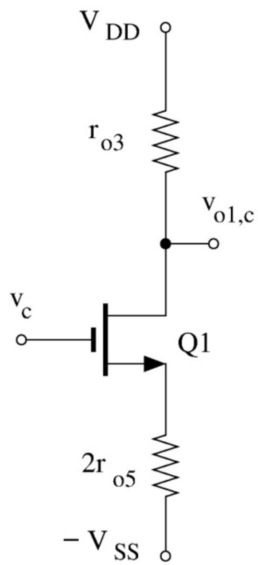


# Differential amplifier with current source active load – Signal analysis




**Differential Mode**

$$v_{o1,d} = -g_{m1}(r_{o1} \parallel r_{o3})\ (-0.5 v_d) = 0.5 g_{m1}(r_{o1} \parallel r_{o3}) v_d$$

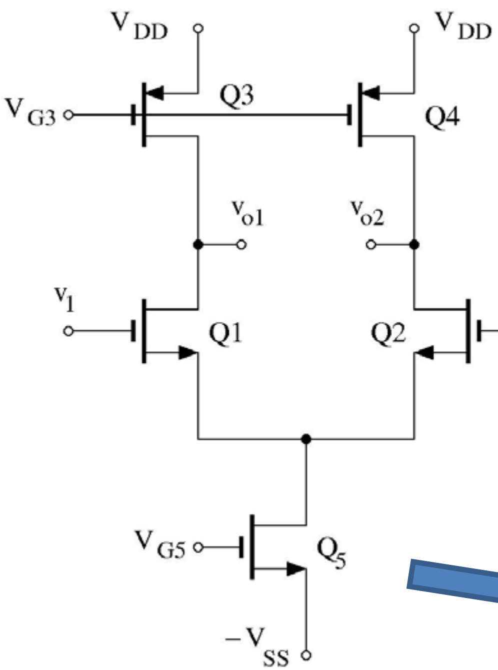
$$v_{o2,d} = -v_{o1} = -0.5 g_{m1}(r_{o1} \parallel r_{o3}) v_d$$

**Common Mode**

$$\frac{v_{o1,c}}{v_c} = -\frac{g_{m1} r_{o3}}{1 + 2 g_{m1} r_{o5} + r_{o3} / r_{o1}}$$

$$v_{o1,c} = v_{o2,c}$$

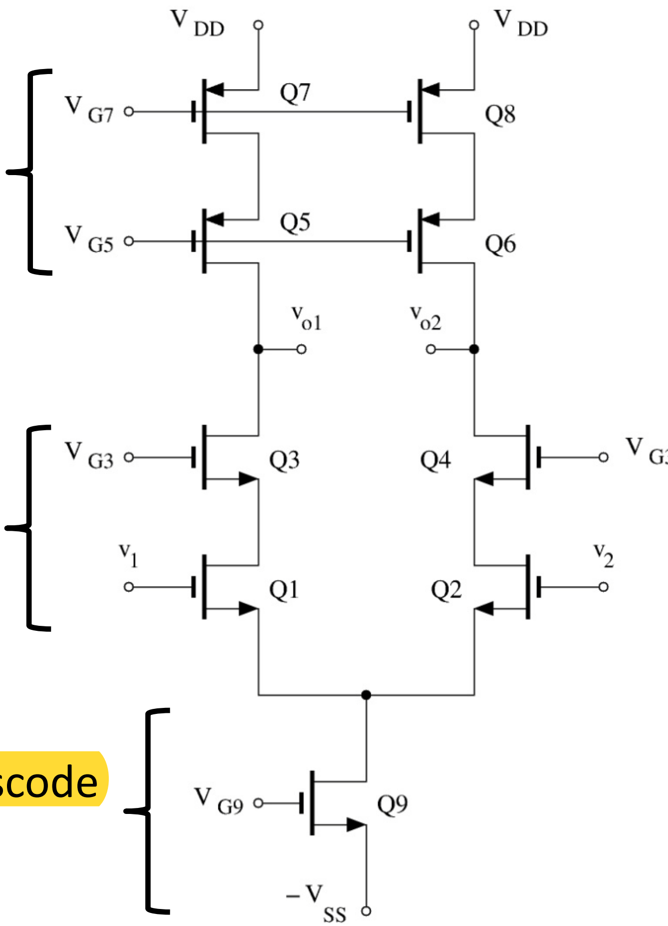
# Cascode differential amplifier




Cascode active load

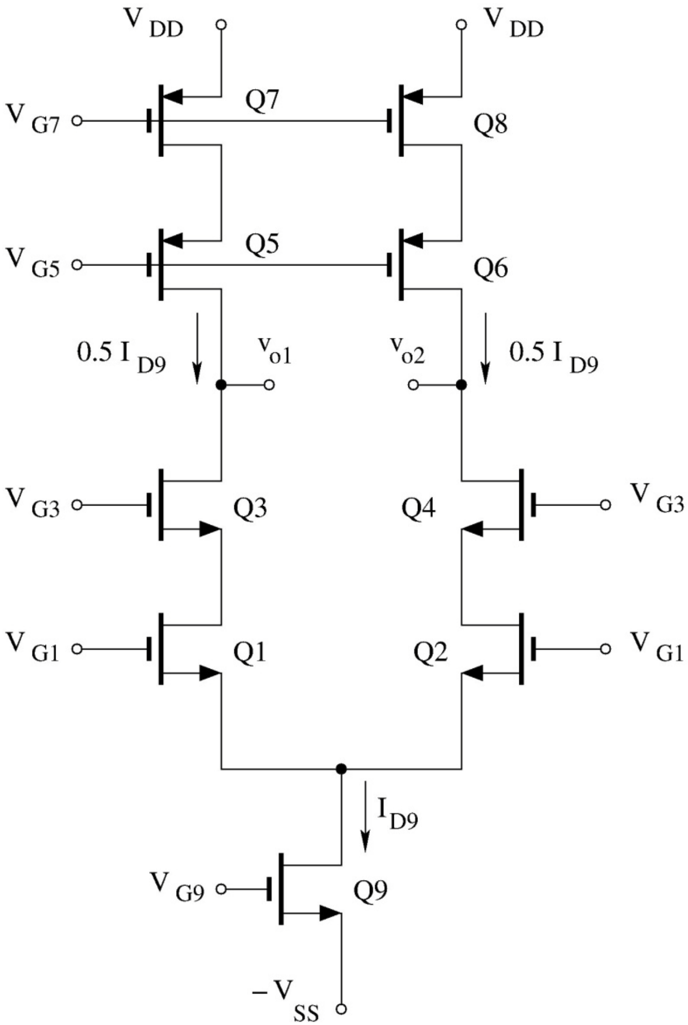
Cascode amplifier

No reason to put a cascode current source here.

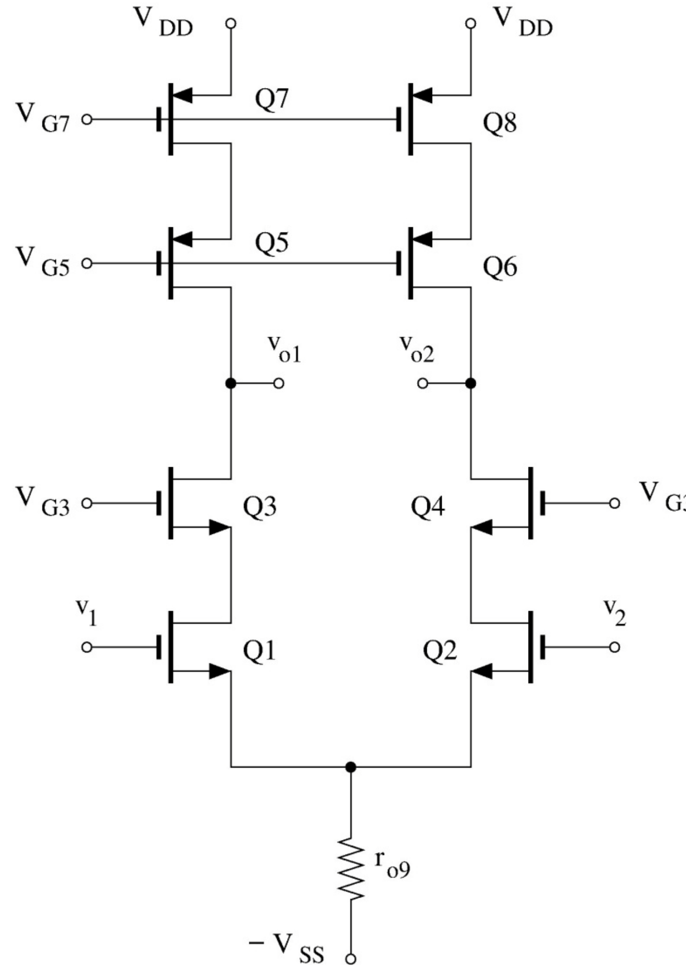



Bias analysis is similar to the case of differential amplifier with current-source active load.

# Cascode differential amplifier – Signal analysis

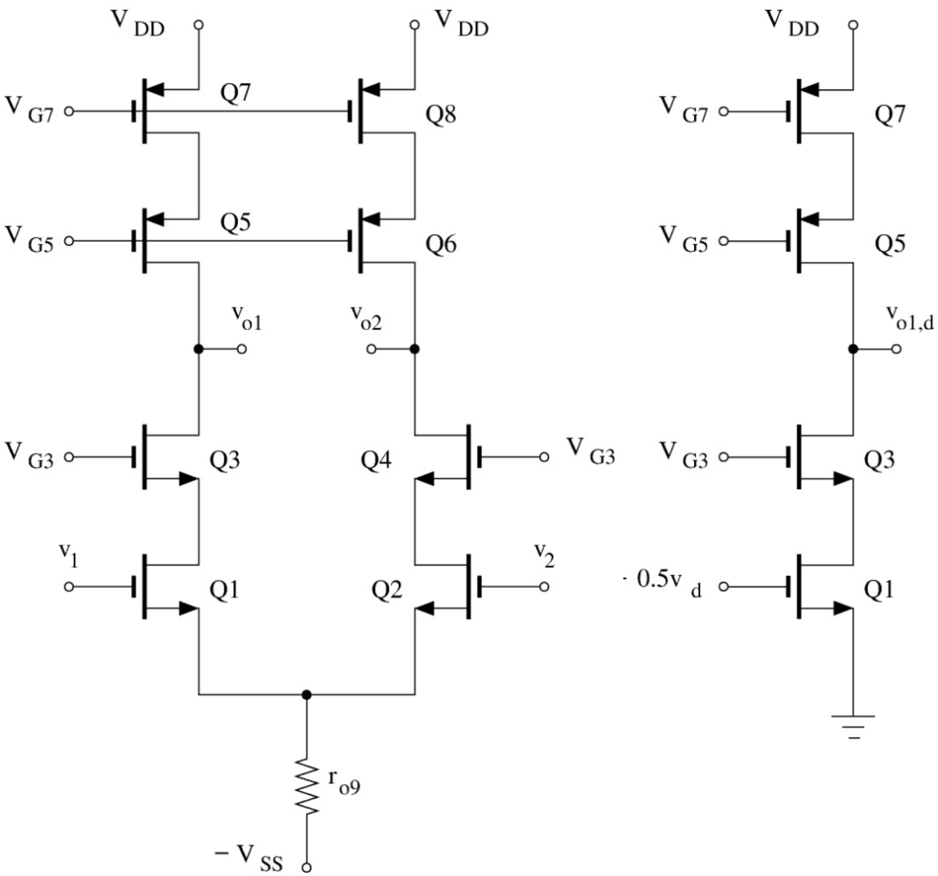



Small signal



# Cascode differential amplifier – Signal analysis

Differential Mode




From Lecture Set 6:

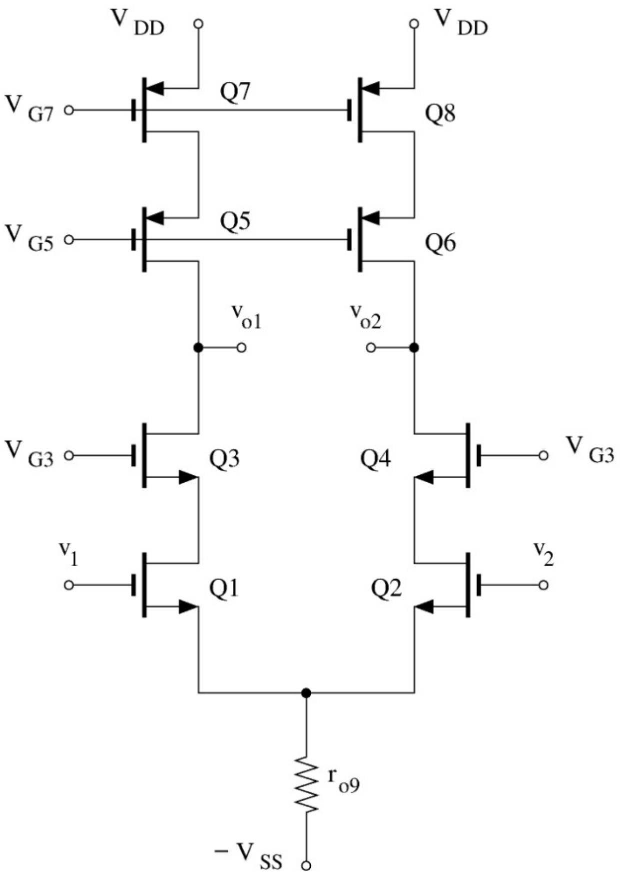
$$v_{o1,d} \approx -\frac{g_{m1}g_{m3}g_{m5}r_{o1}r_{o3}r_{o5}r_{o7}}{g_{m3}r_{o1}r_{o3} + g_{m5}r_{o5}r_{o7}} \times (-0.5v_d)$$

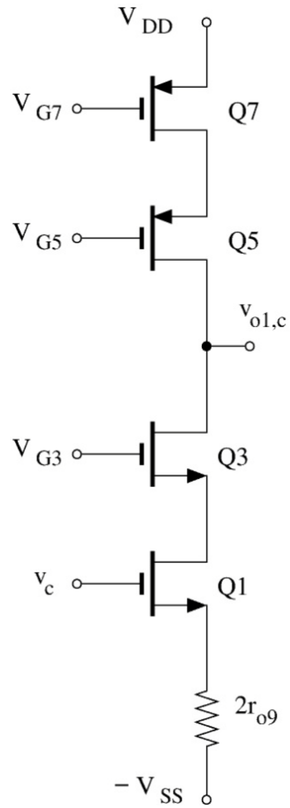
$$= -g_{m1}\left(g_{m3}r_{o3}r_{o1} \parallel g_{m5}r_{o5}r_{o7}\right)(-0.5v_d)$$

$$v_{o2,d} = -v_{o1,d}$$

# Cascode differential amplifier – Signal analysis

Common Mode



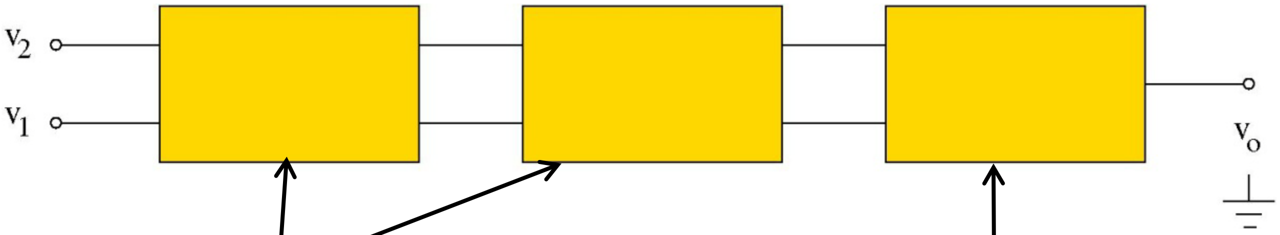

For $g_m \, r_o$ >> 1*

$$\boxed{v_{o2,c} = v_{o1,c} \approx -\frac{g_{m5} r_{o5} r_{o7}}{2 r_{o9}} \times v_c}$$

* Derive the expression for $v_{o1,c}$

# Differential Amplifiers – Output Configurations

Typical implementation of differential amplifier circuits

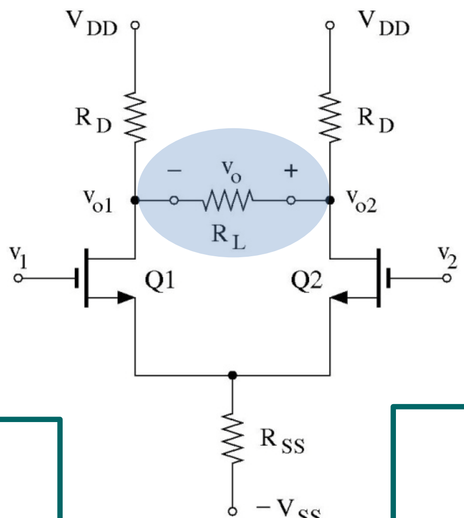



Two outputs

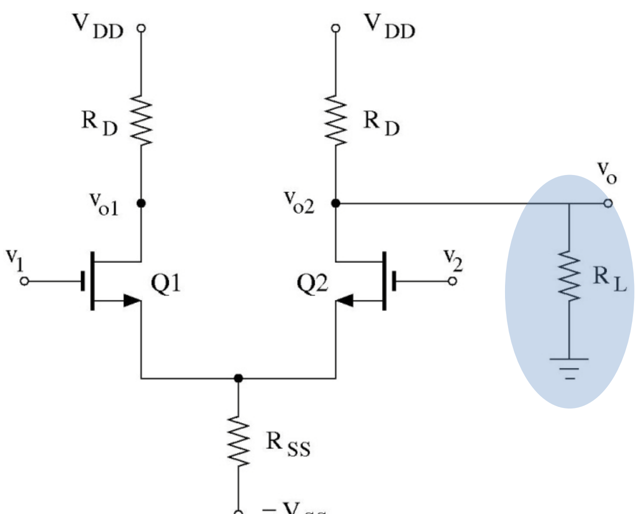
Single ended output

# Output Configurations of Differential Amplifiers

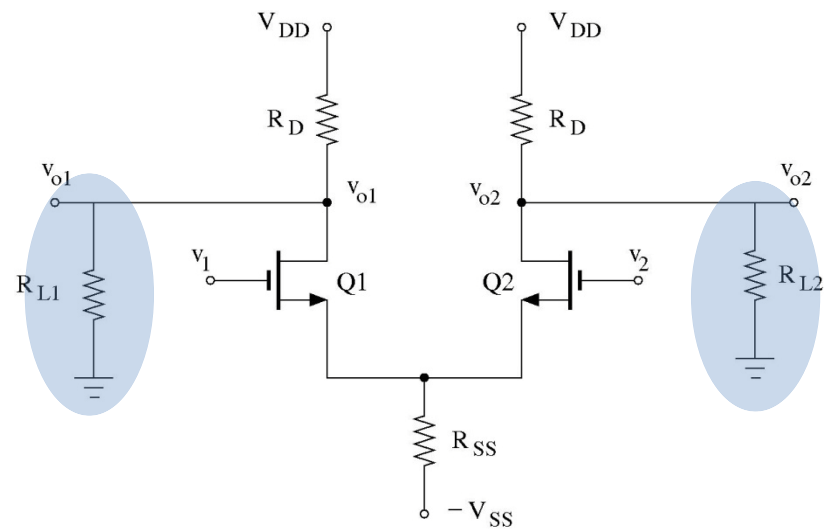
## Differential Output




## Single-ended Output

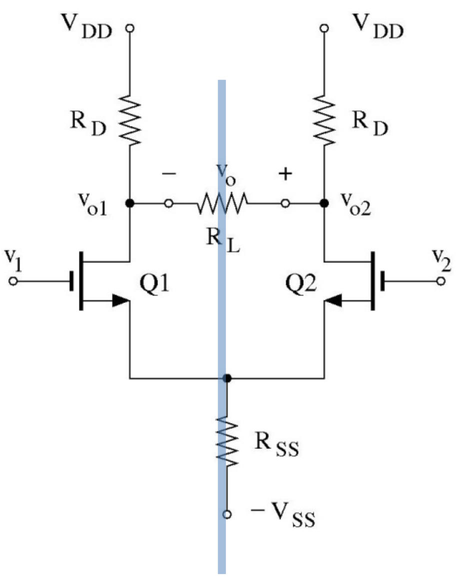


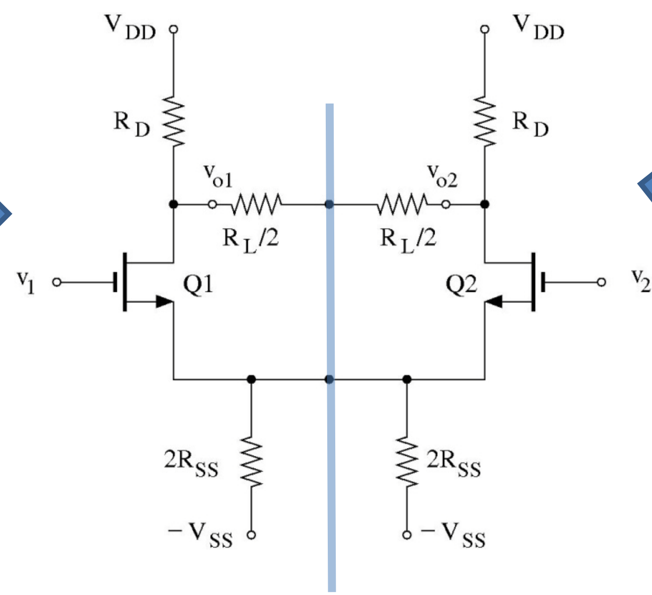

## Two Separate Outputs



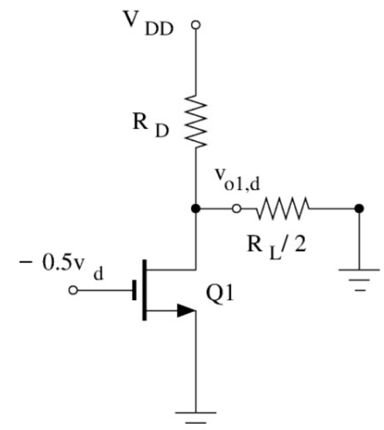
# Differential Amplifiers with Differential Output

### Differential Output

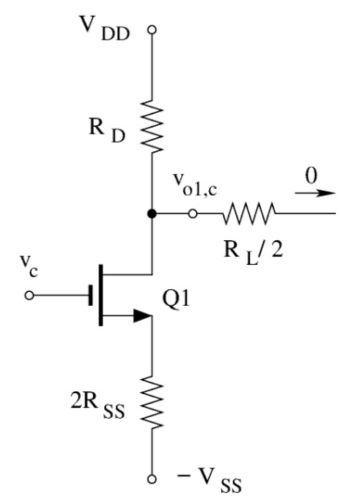



### Differential Mode



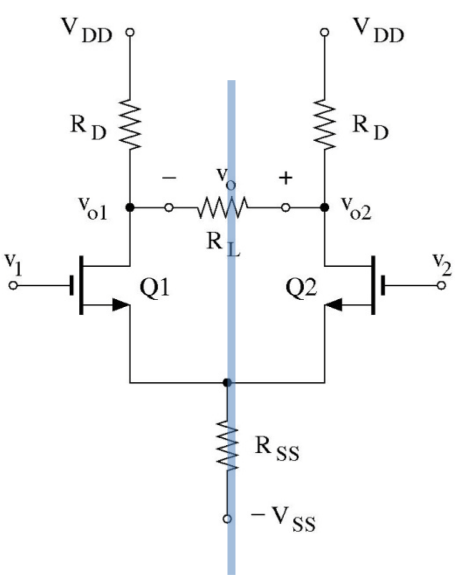

### Common Mode




Not used often because the load floats (i.e., not attached to the ground

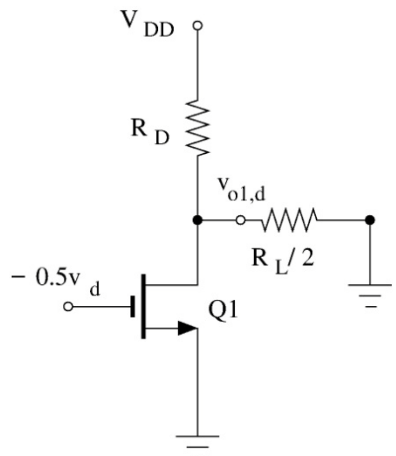
# Differential Amplifiers with Differential Output

## Differential Output




$$v_o = A_c \cdot v_c + A_d \cdot v_d$$

## Differential Mode



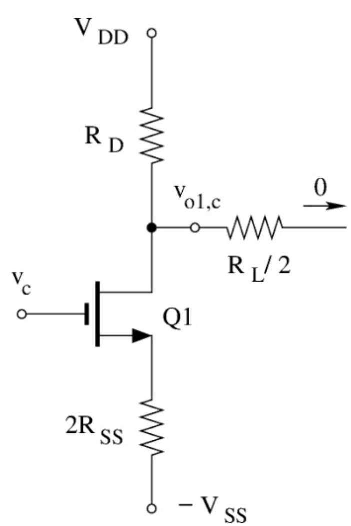

$$v_{o1,d} = -g_m (r_o \| R_D \| R_L/2)(-0.5 v_d)$$

$$v_{o2,d} = -v_{o1,d}$$

$$v_{od} = v_{o2,d} - v_{o1,d} = -2 v_{o1,d} = -g_m (r_o \| R_D \| R_L/2) v_d$$

$$A_d = \frac{v_{od}}{v_d} = -g_m (r_o \| R_D \| R_L/2)$$

## Common Mode




$$\frac{v_{o1,c}}{v_c} = -\frac{g_m R_D}{1 + 2 g_m R_{SS} + R_D / r_o}$$

$$v_{o2,c} = v_{o1,c}$$

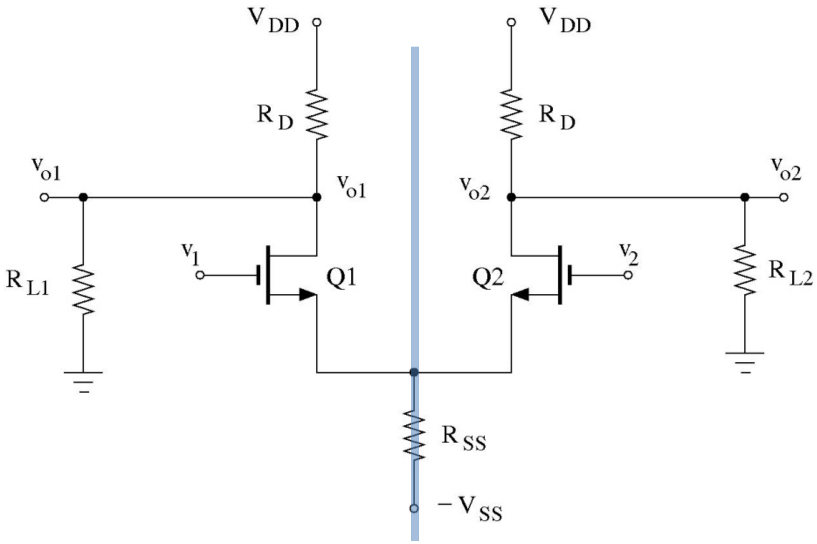
$$v_{oc} = v_{o2,c} - v_{o1,c} = 0$$

$$A_c = \frac{v_{oc}}{v_c} = 0$$

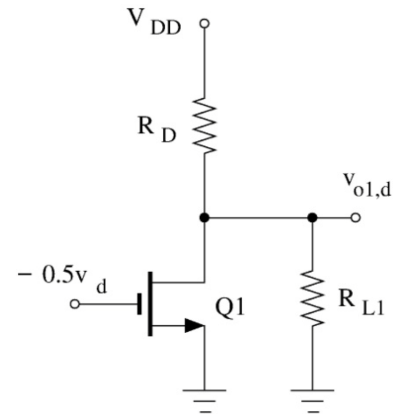
$$\text{CMRR} = \frac{|A_d|}{|A_c|} = \infty$$

# Differential Amplifiers with Two Outputs

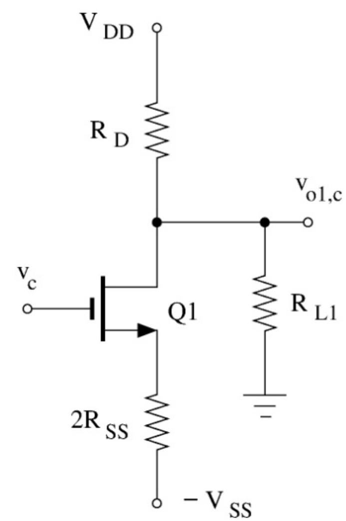
**Two Separate Outputs ($R_{L1} \approx R_{L2} = R_L$)**
(i.e., input to another difference amplifier)




Differential Mode



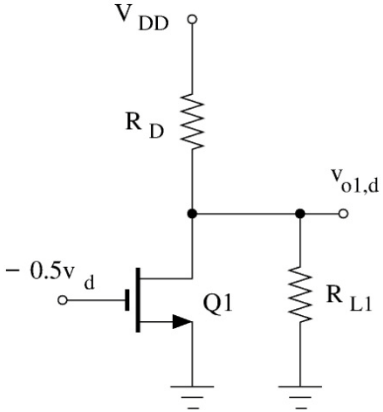

Common Mode




**Note:** To use half circuit, ($R_{L1} \approx R_{L2}$) <u>or</u> $R_L$ should be large enough so that symmetry is preserved (i.e. $R_{L1,2} >> R_o$)

# Differential Amplifiers with Two Outputs

Differential Mode




$$\frac{v_{o1,d}}{-0.5v_d} = -g_m(r_o \| R_D \| R_{L1})$$

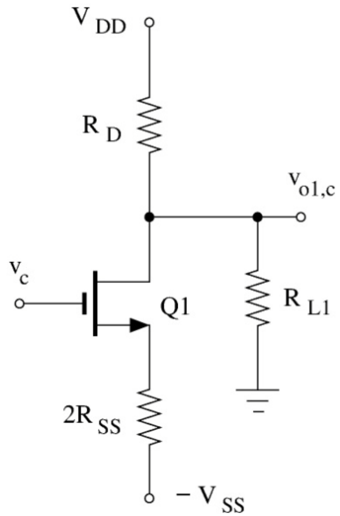
$$\frac{v_{o2,d}}{+0.5v_d} = -g_m(r_o \| R_D \| R_{L2})$$

**Note:** Each output has its own differential- and common-mode gains:

$$A_{1d} = \frac{v_{o1,d}}{v_d}, \quad A_{1c} = \frac{v_{o1,c}}{v_c}$$

$$v_{o1} = A_{1c} \cdot v_c + A_{1d} \cdot v_d$$

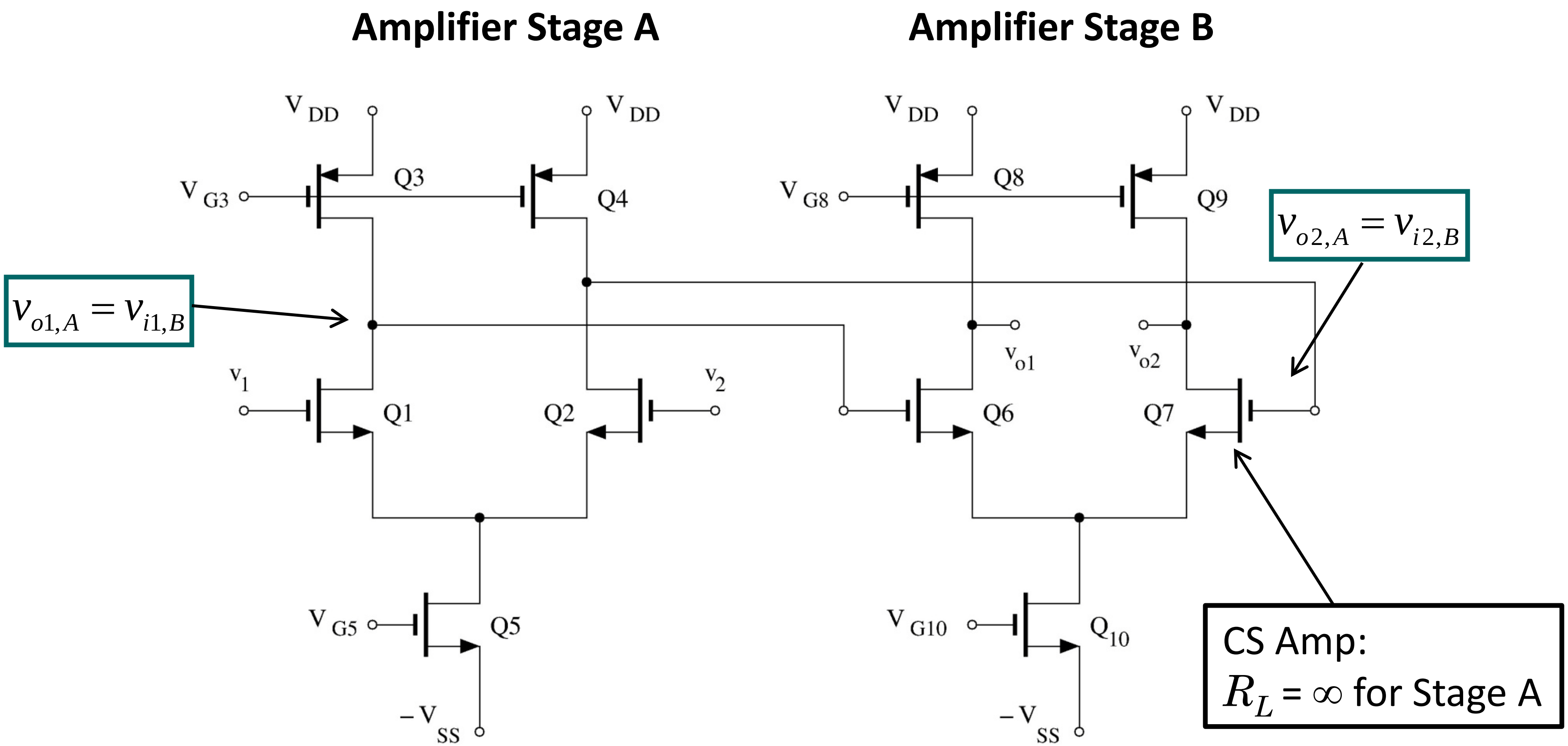
Common Mode




$$\frac{v_{o1,c}}{v_c} = -\frac{g_m(R_D \| R_{L1})}{1+2g_m R_{SS} + (R_D \| R_{L1})/r_o}$$

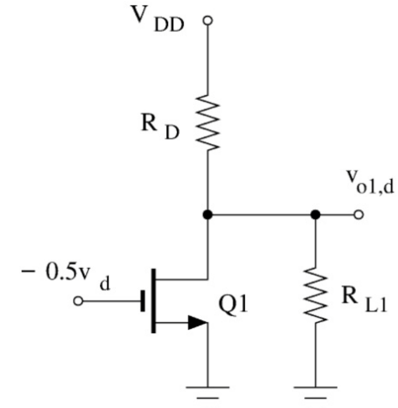
$$\frac{v_{o2,c}}{v_c} = -\frac{g_m(R_D \| R_{L2})}{1+2g_m R_{SS} + (R_D \| R_{L2})/r_o}$$

# Typical implementation of differential amplifiers with two outputs

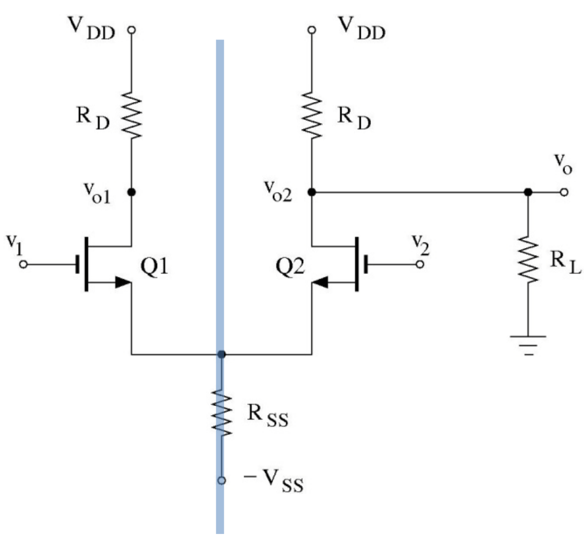


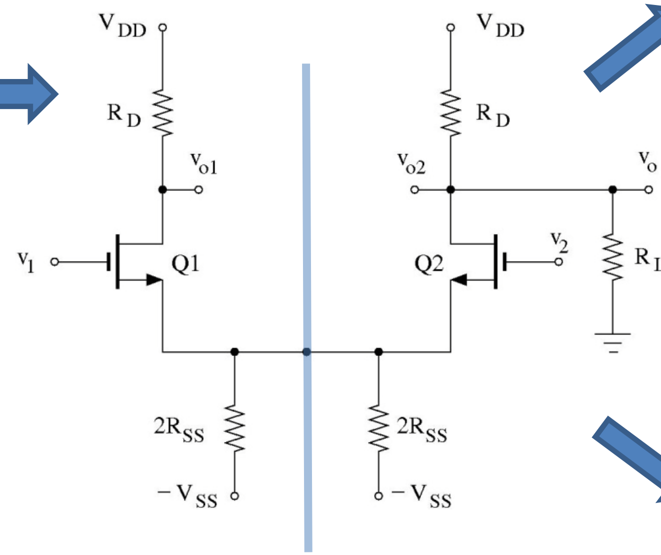
# Differential Amplifiers with Single-ended Output

Differential Mode

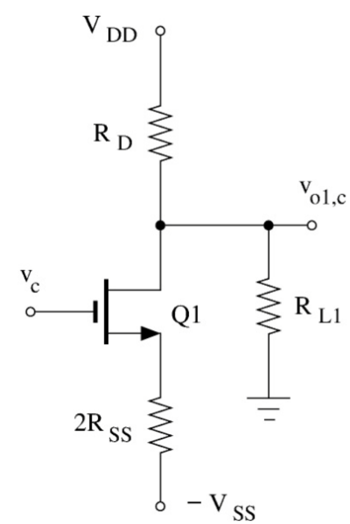



**Single-ended Output**




Common Mode

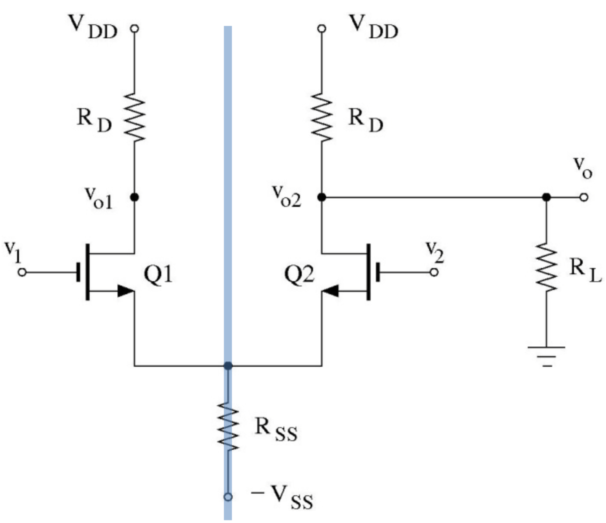



To use half circuit, $R_L$ should be large enough such that symmetry is preserved (i.e. $\boldsymbol{R_L >> R_o = R_D || r_o}$)

# Differential Amplifiers with Single-ended Output

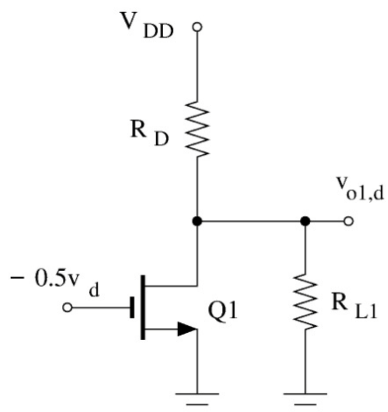
$$v_o = A_c \cdot v_c + A_d \cdot v_d$$

## Single-ended Output




To use half circuit, $R_L$ Should be large so that symmetry is preserved (i.e. $\boldsymbol{R_L >> R_o = R_D || r_o}$)

## Differential Mode

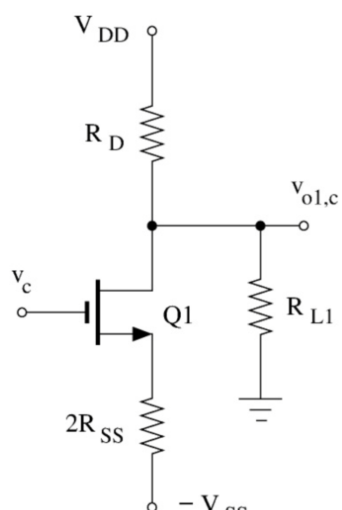



$$\frac{v_{o2}}{0.5 v_d} = -g_m (r_o \| R_D \| R_L)$$

$$v_{od} = v_{o2} = -0.5 g_m (r_o \| R_D \| R_L)\, v_d$$

$$A_d = \frac{v_{od}}{v_d} = -0.5 g_m (r_o \| R_D \| R_L)$$

## Common Mode



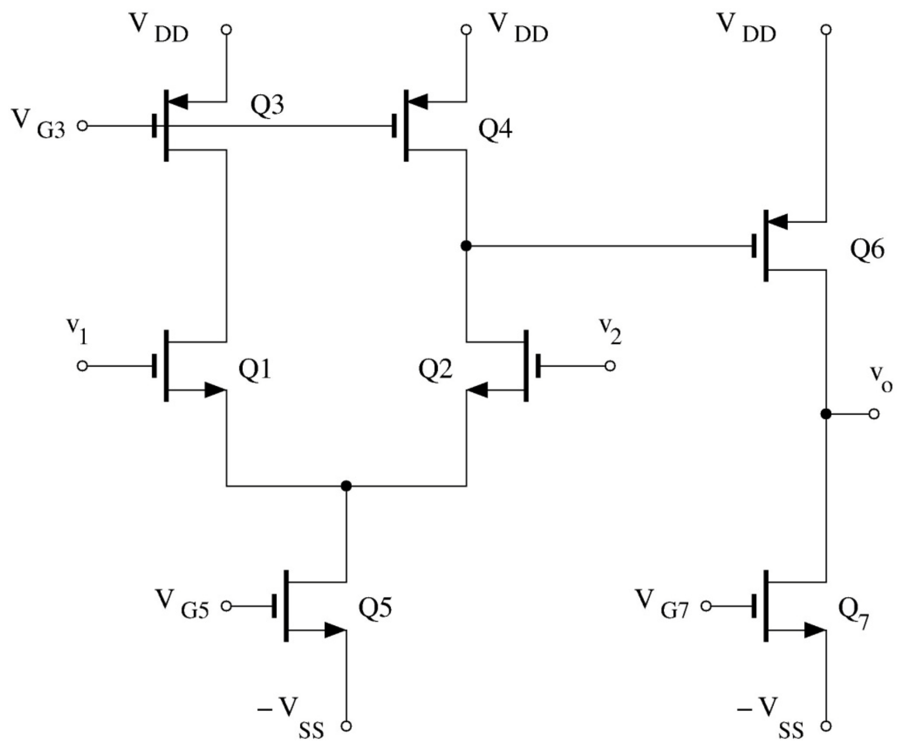

$$A_c = \frac{v_{oc}}{v_c} = -\frac{g_m (R_D \| R_L)}{1 + 2 g_m R_{SS} + (R_D \| R_L)/r_o}$$

Note: $A_c \neq 0$ which means CMMR is NOT infinite.

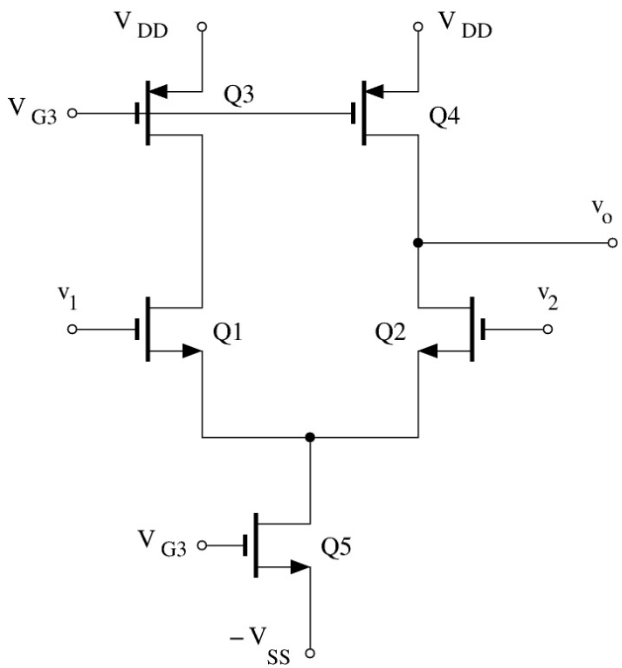
# An implementation of differential amplifiers with an output (coupled to a CS amplifier)

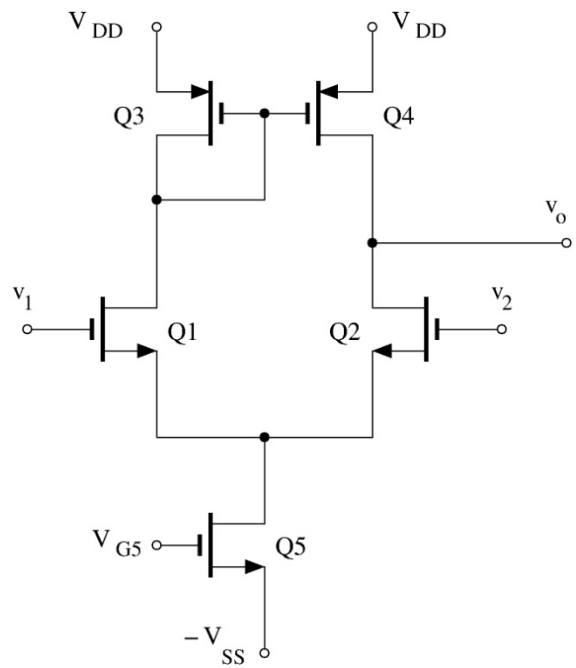
**Differential Amplifier with a single output**

**CS stage**



# Active load for a single-ended output



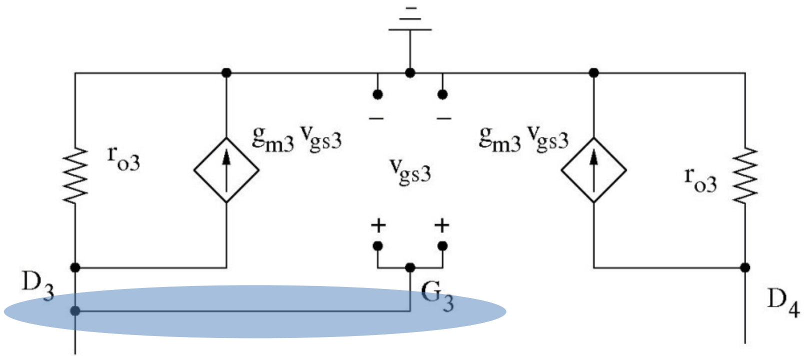




Works fine but require biasing of Q3 and Q4 (i.e., $V_{G3}$)

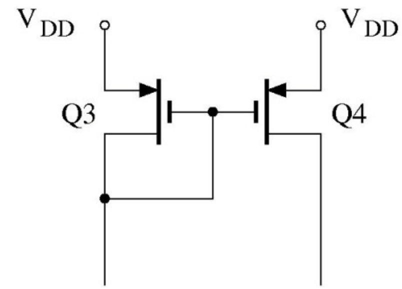
"Popular" active load for single-ended output

- Q3/Q4 are NOT current sources and do not require biasing (i.e., $V_{G3}$)
- Gets a similar gain and CMRR
- But, circuit is NOT symmetric (half-circuit does not work!)

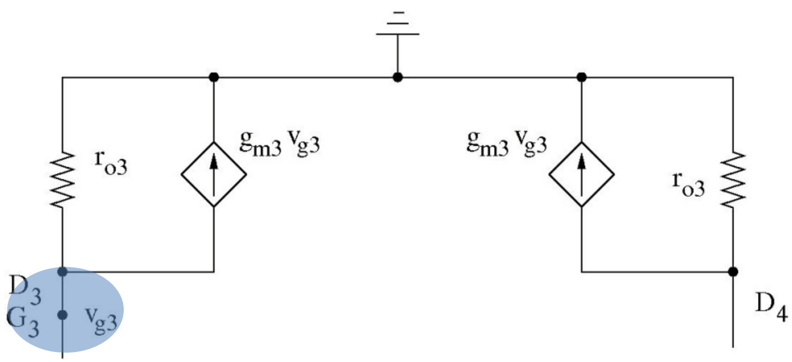
# Active load for a single-ended output: Small signal equivalent



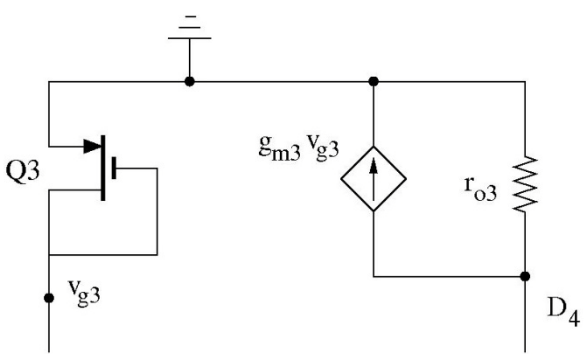

Small Signal

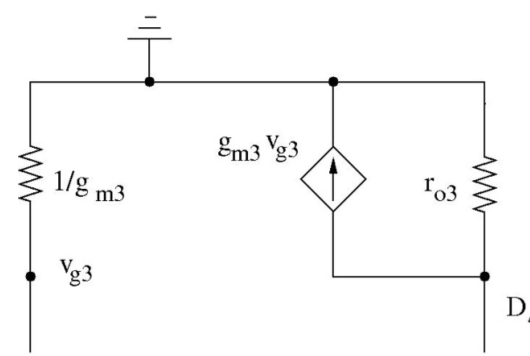



Note $r_{o4} = r_{o3}$ and $g_{m4} = g_{m3}$

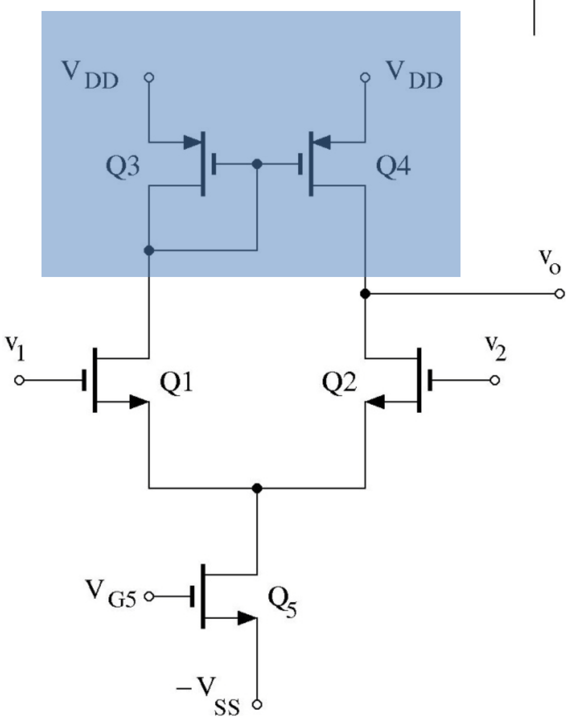


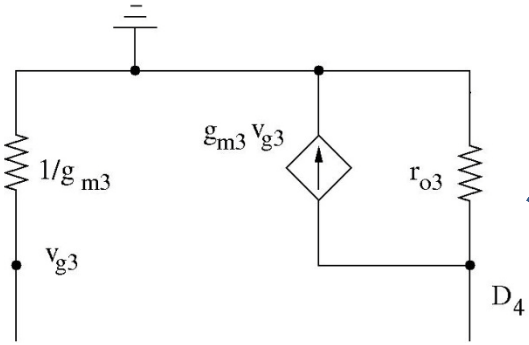

Diode-connected transistor



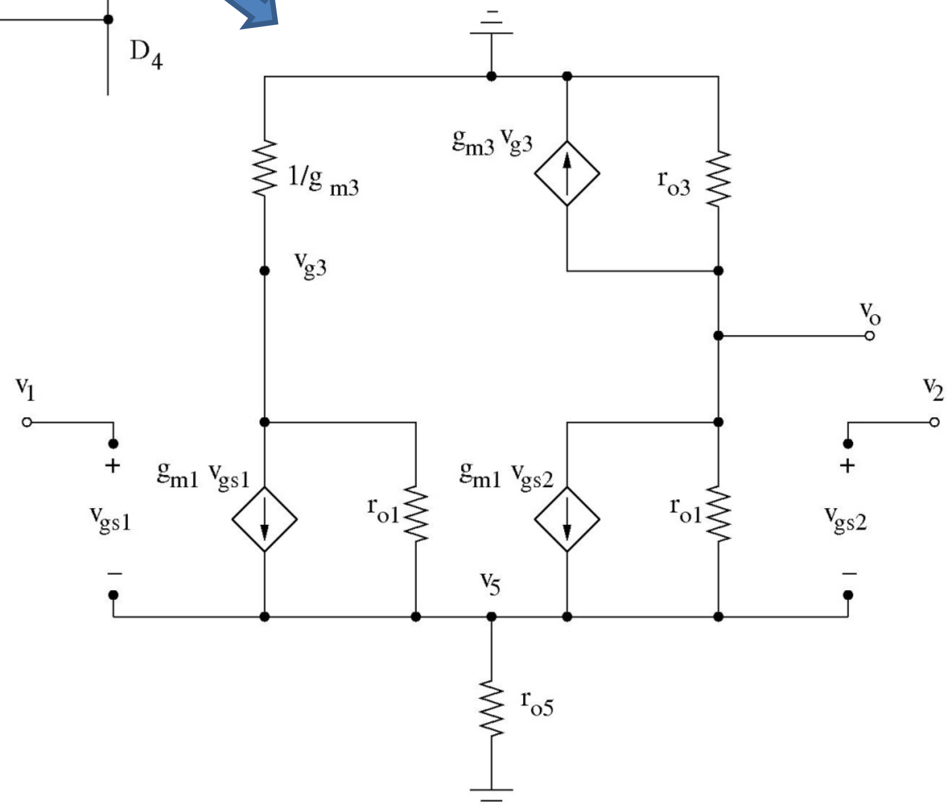



# Small-signal analysis of single-ended output



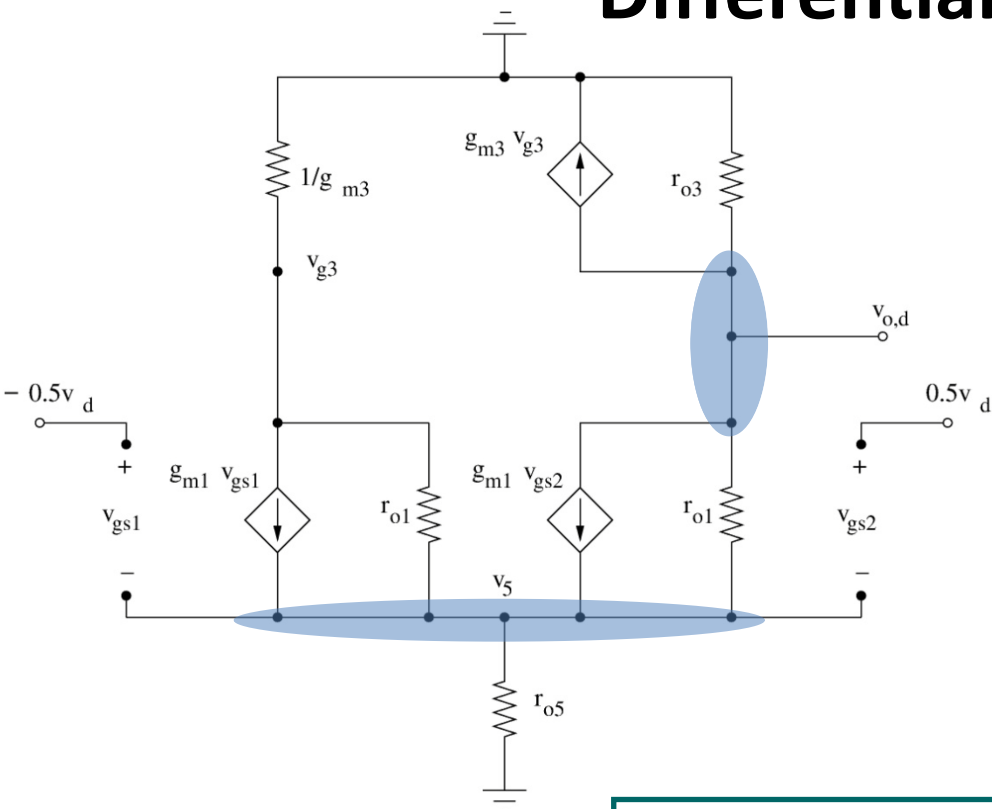

Small Signal




Note $r_{o4} = r_{o3}$ and $g_{m4} = g_{m3}$
$r_{o2} = r_{o1}$ and $g_{m2} = g_{m1}$

**Circuit is NOT symmetric**
**CANNOT use "half-circuit"**

# Small-signal analysis of single-ended output – Differential Gain (1)




$$r_{o4} = r_{o3} \text{ and } g_{m4} = g_{m3}$$
$$r_{o2} = r_{o1} \text{ and } g_{m2} = g_{m1}$$
$$v_{gs1} = -0.5v_d - v_5$$
$$v_{gs2} = +0.5v_d - v_5$$

Node $v_{g3}$

$$g_{m3}v_{g3} + g_{m1}(-0.5v_d - v_5) + \frac{v_{g3} - v_5}{r_{o1}} = 0$$

Node $v_o$

$$g_{m3}v_{g3} + \frac{v_o}{r_{o3}} + g_{m1}(+0.5v_d - v_5) + \frac{v_o - v_5}{r_{o1}} = 0$$

Node $v_5$

$$\frac{v_5}{r_{o5}} + \frac{v_5 - v_{g3}}{r_{o1}} + \frac{v_5 - v_o}{r_{o1}} - g_{m1}(-0.5v_d - v_5) - g_{m1}(+0.5v_d - v_5) = 0$$

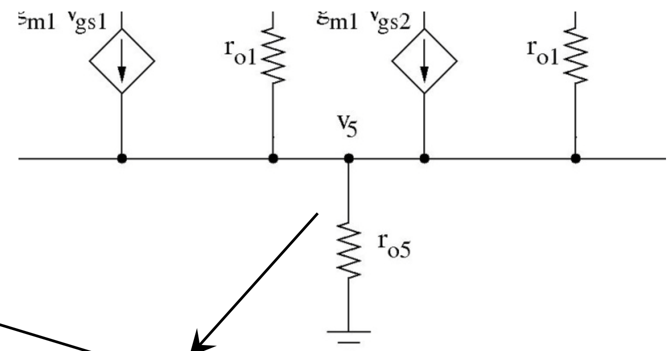
# Small-signal analysis of single-ended output – Differential Gain (2)

Rearranging terms:

$$v_{g3}\left(g_{m3}+\frac{1}{r_{o1}}\right)+v_5\left(-g_{m1}-\frac{1}{r_{o1}}\right)=+0.5g_{m1}v_d$$

$$v_{g3}\left(g_{m3}\right)+v_5\left(-g_{m1}-\frac{1}{r_{o1}}\right)+v_o\left(\frac{1}{r_{o3}}+\frac{1}{r_{o1}}\right)=-0.5g_{m1}v_d$$

$$v_{g3}\left(-\frac{1}{r_{o1}}\right)+v_5\left(+2g_{m1}+\frac{2}{r_{o1}}+\frac{1}{r_{o5}}\right)+v_o\left(-\frac{1}{r_{o1}}\right)=0$$




Dropping $1/r_o$ terms compared with $g_m$

$$v_{g3}\left(g_{m3}\right)+v_5\left(-g_{m1}\right)=+0.5g_{m1}v_d$$

$$v_{g3}\left(g_{m3}\right)+v_5\left(-g_{m1}\right)+v_o\left(\frac{1}{r_{o3}}+\frac{1}{r_{o1}}\right)=-0.5g_{m1}v_d$$

$$v_{g3}\left(-\frac{1}{r_{o1}}\right)+v_5\left(+2g_{m1}\right)+v_o\left(-\frac{1}{r_{o1}}\right)=0$$

Dropping $v_5/r_{o5}$ term implies that very little current flows into $r_{o5}$ (can remove $r_{o5}$ from the circuit as done in the textbook)

# Small-signal analysis of single-ended output – Differential Gain (3)

$$v_{g3}(g_{m3}) + v_5(-g_{m1}) = +0.5 g_{m1} v_d$$

$$v_{g3}(g_{m3}) + v_5(-g_{m1}) + v_o\left(\frac{1}{r_{o3}} + \frac{1}{r_{o1}}\right) = -0.5 g_{m1} v_d$$

$$v_{g3}\left(-\frac{1}{r_{o1}}\right) + v_5(+2g_{m1}) + v_o\left(-\frac{1}{r_{o1}}\right) = 0$$

Subtracting second equation from the first*:

$$\frac{v_o}{r_{o1} \| r_{o3}} = -g_{m1} v_d \quad \Rightarrow \quad v_o = -g_{m1}(r_{o1} \| r_{o3}) v_d \quad \Rightarrow \quad A_d = -g_{m1}(r_{o1} \| r_{o3})$$

Adding all three equations give:

Note: $v_{g3} << v_o$

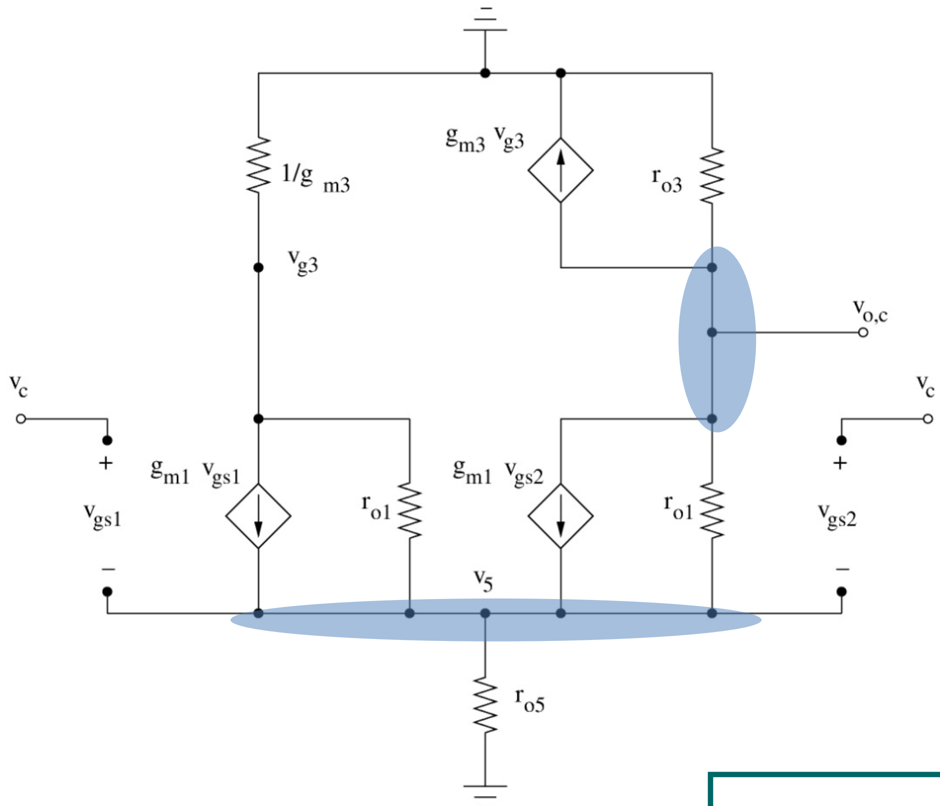
$$2 g_{m3} v_{g3} + \frac{v_o}{r_{o3}} = 0 \quad \Rightarrow \quad v_{g3} = -\frac{v_o}{2 g_{m3} r_{o3}}$$

$$v_{g3} = +\frac{g_{m1}(r_{o1} \| r_{o3})}{2 g_{m3} r_{o3}} v_d \approx \frac{g_{m1}}{4 g_{m3} r_o} v_d$$

Textbook Eq. 7.1.40 is incorrect

* This is sloppy math as if subtract 2nd equation from first before dropping $r_o$ terms, a $v_{g3}$ term appears in the above equation. Fortunately, as $v_{g3} << v_o$, ignoring $v_{g3}$ term is justified

# Small-signal analysis of single-ended output – Common-mode Gain (1)




$$
\begin{aligned}
r_{o4} &= r_{o3} \text{ and } g_{m4} = g_{m3} \\
r_{o2} &= r_{o1} \text{ and } g_{m2} = g_{m1} \\
v_{gs1} &= -0.5v_d - v_5 \\
v_{gs2} &= +0.5v_d - v_5
\end{aligned}
$$

Node $v_{g3}$ $$g_{m3}v_{g3} + g_{m1}(v_c - v_5) + \frac{v_{g3} - v_5}{r_{o1}} = 0$$

Node $v_o$ $$g_{m3}v_{g3} + \frac{v_o}{r_{o3}} + g_{m1}(v_c - v_5) + \frac{v_o - v_5}{r_{o1}} = 0$$

Node $v_5$ $$\frac{v_5}{r_{o5}} + \frac{v_5 - v_{g3}}{r_{o1}} + \frac{v_5 - v_o}{r_{o1}} - g_{m1}(v_c - v_5) - g_{m1}(v_c - v_5) = 0$$

# Small-signal analysis of single-ended output – Common-mode Gain (2)

$$g_{m3}v_{g3} + g_{m1}(v_c - v_5) + \frac{v_{g3} - v_5}{r_{o1}} = 0$$

$$g_{m3}v_{g3} + \frac{v_o}{r_{o3}} + g_{m1}(v_c - v_5) + \frac{v_o - v_5}{r_{o1}} = 0$$

$$\frac{v_5}{r_{o5}} + \frac{v_5 - v_{g3}}{r_{o1}} + \frac{v_5 - v_o}{r_{o1}} - g_{m1}(v_c - v_5) - g_{m1}(v_c - v_5) = 0$$

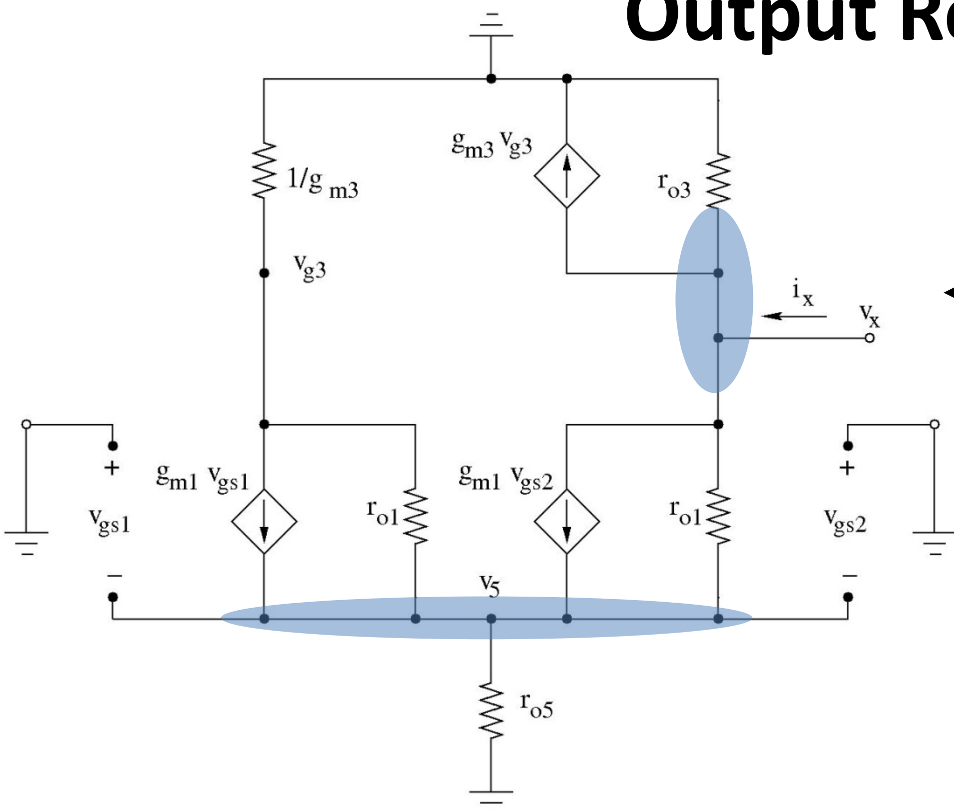
Subtracting second equation from the first and dropping $1/r_o$ terms compared with $g_m$

$$\frac{v_o}{r_{o1} \parallel r_{o3}} = 0 \;\Rightarrow\; A_c = 0 \;\Rightarrow\; \text{CMRR} = \infty$$

Solving equations without dropping $1/r_o$ terms compared with $g_m$

$$v_o = \frac{1}{2g_{m3}r_{o5}} v_c \;\Rightarrow\; A_c = \frac{1}{2g_{m3}r_{o5}} \;\Rightarrow\; \text{CMRR} = 2g_{m3}r_{o5}g_{m1}(r_{o1} \parallel r_{o3})$$

# Small-signal analysis of single-ended output – Output Resistance




Attach a source $v_x$ to the output and calculate $i_x$)

Node $v_{g3}$ $\quad g_{m3}v_{g3} + g_{m1}(-v_5) + \dfrac{v_{g3} - v_5}{r_{o1}} = 0$

Node $v_x$ $\quad g_{m3}v_{g3} + \dfrac{v_x}{r_{o3}} + g_{m1}(-v_5) + \dfrac{v_x - v_5}{r_{o1}} = i_x$

Node $v_5$ $\quad \dfrac{v_5}{r_{o5}} + \dfrac{v_5 - v_{g3}}{r_{o1}} + \dfrac{v_5 - v_x}{r_{o1}} - g_{m1}(-v_5) - g_{m1}(-v_5) = 0$

Subtracting second equation from the first and dropping $1/r_o$ terms compared with $g_m$

$$\frac{v_x}{r_{o1} \parallel r_{o3}} = i_x$$

$$R_o = \frac{v_x}{i_x} = r_{o1} \parallel r_{o3}$$