

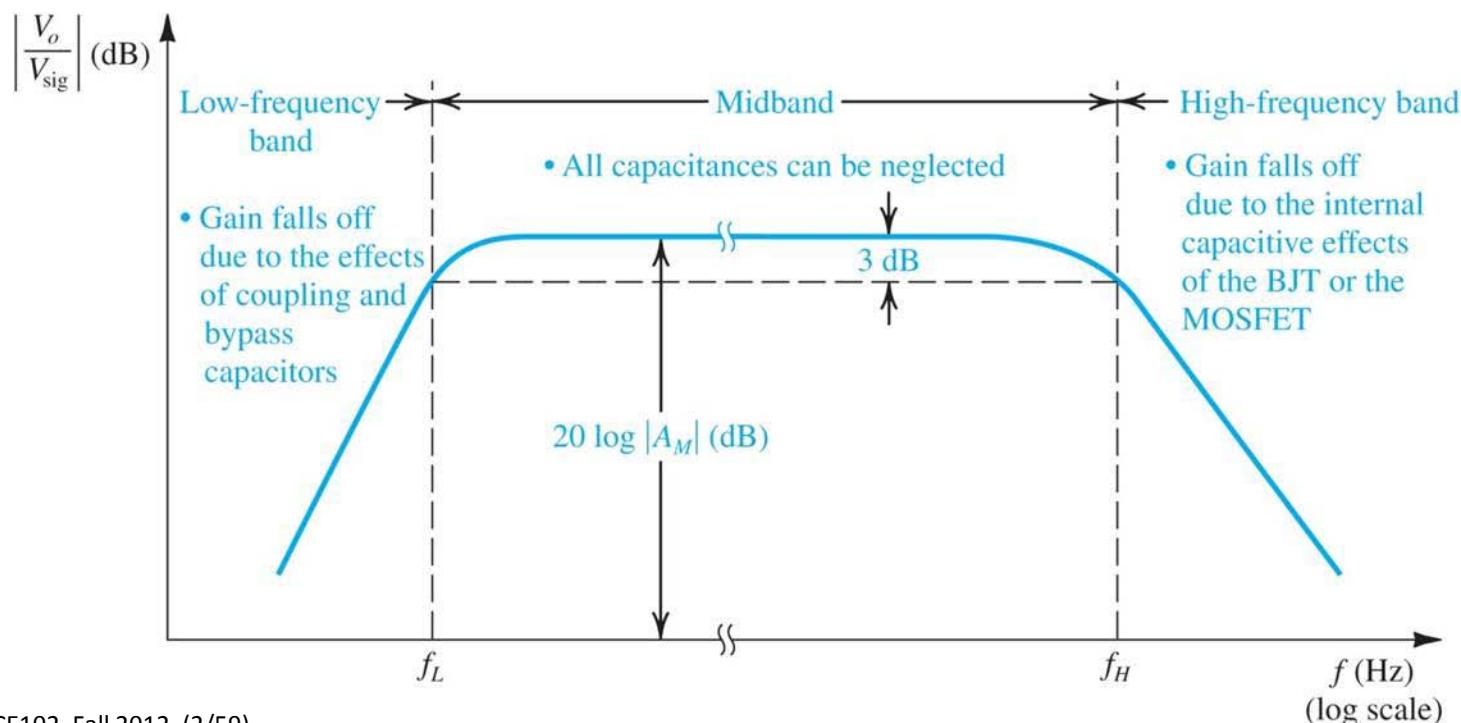
8. Frequency Response

Reading: Sedra & Smith: Sec. 1.6, Sec. 3.6 and Sec. 9
(MOS portions),

(S&S 5th Ed: Sec. 1.6, Sec. 3.7 (capacitive effects), Sec. 4.8, Sec. 4.9, ,Sec. 6. (Frequency response sections, i.e., 6.4, 6.6, ...), Sec. 7.6

Typical Frequency response of an Amplifier

- Up to now we have “ignored” capacitors in circuits & computed mid-band properties. We have to solve the circuit in the frequency domain in order to see the impact of capacitors (a typical response is shown below):
 - Lower cut-off frequency: f_L
 - Upper cut-off frequency: f_H
 - Band-width: $B = f_H - f_L$



Observation on the frequency response of an Amplifier

Observations:

- Analytical solution of amplifiers in frequency domain is complicated!
- Response (e.g., gain) of an ideal linear amplifier should be independent of frequency (otherwise signal “shape” would be distorted by the amplifier). Thus:
 - A practical amplifier acts as an ideal linear amplifier only for a range of frequencies, called the “mid-band”.
 - The lower and the upper cut-off frequencies (f_L and f_H) identify the frequency range over which the amplifier acts linearly.
 - Amplifier response at high frequencies (near the upper cut-off frequency , f_H) is important for stability considerations (gain and phase margins).
- Thus, we are mainly interested in **mid-band properties** (where capacitors can be ignored) and in **poles and zeros of the amplifier response** (due to capacitors).

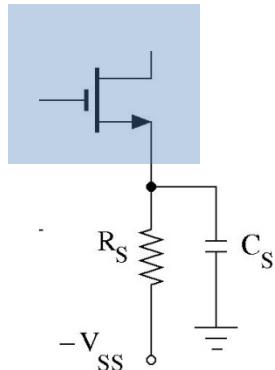
What do we mean by “capacitors can be ignored?”

- Capacitor impedance depends on the frequency: $|Z| = 1/(j\omega C)$.
 - At high frequency $|Z| \rightarrow 0$: capacitor acts as a short circuit.
 - At low frequency $|Z| \rightarrow \infty$: capacitor acts as an open circuit.
- For the above two limits, circuit becomes a “resistive” circuit and we do NOT need to solve the circuit in the frequency domain.
- Thus, ignoring capacitors means that we operate at either a high enough or at a low enough frequency such that capacitors become either open or short circuits, leading to a “resistive” circuit.
 - Note that the circuit is modified by the presence of the capacitors (e.g., elements may be shorted out).
- **But “high” and/or “low” frequency compare to what?**

Capacitor behavior depends on the frequency of interest.

Example:

$$|Z| = R \parallel (1/\omega C)$$



Capacitor approximates an open circuit at low frequencies:

$$R \ll (1/\omega C) \rightarrow |Z| = R \parallel (1/\omega C) \approx R$$

$$R \ll (1/\omega C) \rightarrow \omega \ll (1/RC)$$

Capacitor approximates a short circuit at high frequencies:

$$R \gg (1/\omega C) \rightarrow |Z| = R \parallel (1/\omega C) \approx (1/\omega C) \rightarrow 0$$

$$R \gg (1/\omega C) \rightarrow \omega \gg (1/RC)$$

We cannot ignore the capacitor when

$$R \sim (1/\omega C) \rightarrow \omega \sim (1/RC)$$

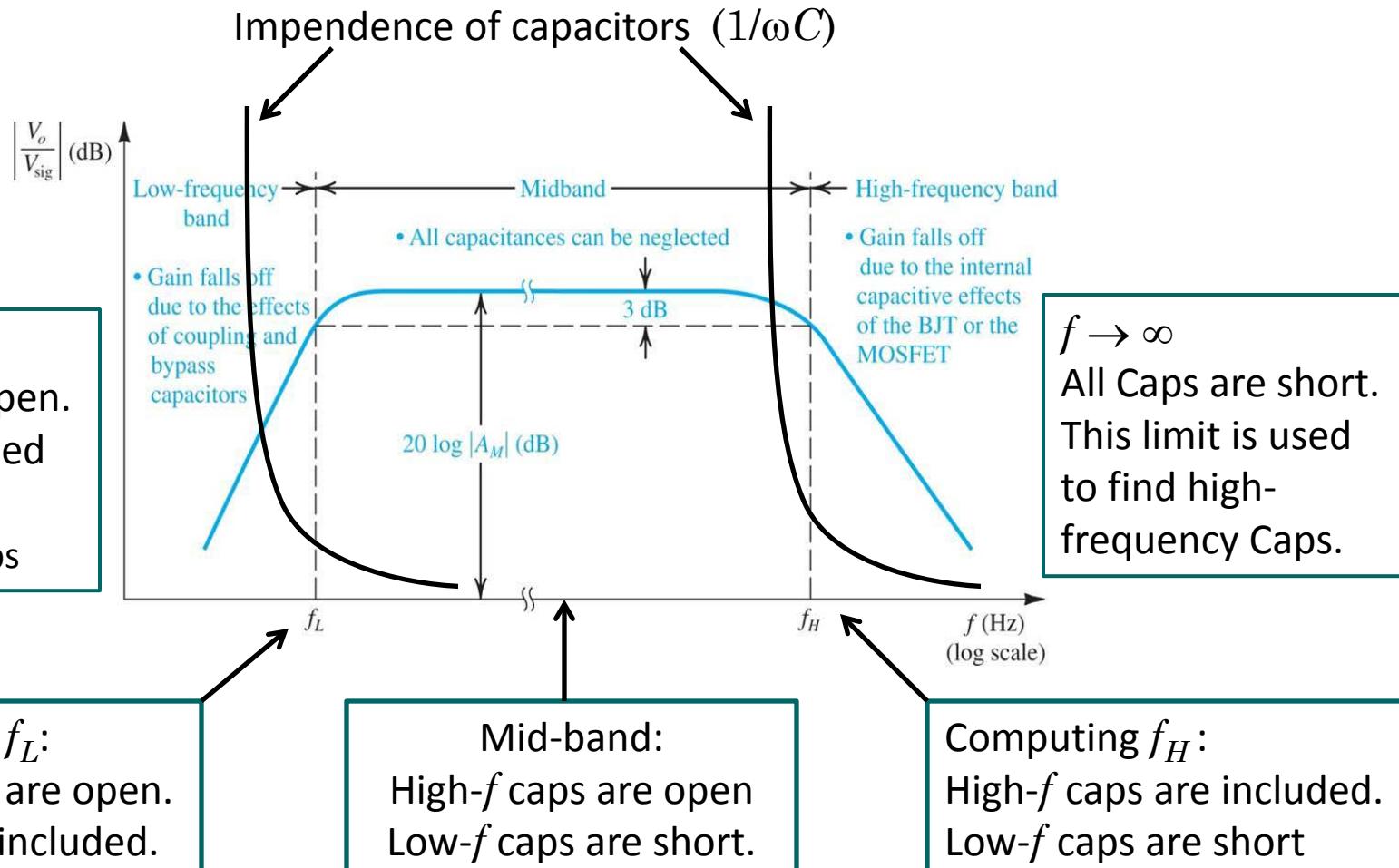
This defines the reference frequency for high- f and low- f

Note: The above circuit is like a low-pass filter with a cut-off frequency of $1/RC$

Finding Frequency response of amplifiers

- Capacitors typically divide into two groups: low- f capacitors (setting f_L) and high- f capacitors (setting f_H).
 - **We need to identify low- f and high- f caps.** We will use **absolute limits** of $f = 0$ (ALL capacitors open) and $f = \infty$ (ALL capacitors short) for this purpose.
- For bias ($f = 0$) all caps are open circuit!
- For “mid-band” properties ($f_L \ll f \ll f_H$)
 - Low- f capacitors will be short circuit (because $f_L \ll f$).
 - High- f capacitors will be open circuit (because $f \ll f_H$).
 - **The resulting “resistive” circuit** gives mid-band properties.
- We will use **time-constant method** to find f_L and f_H (separately)
 - To find f_L all high- f capacitors will be open circuit (because $f_L \ll f_H$)
 - To find f_H all low- f capacitors will be short circuit (because $f_H \gg f_L$)

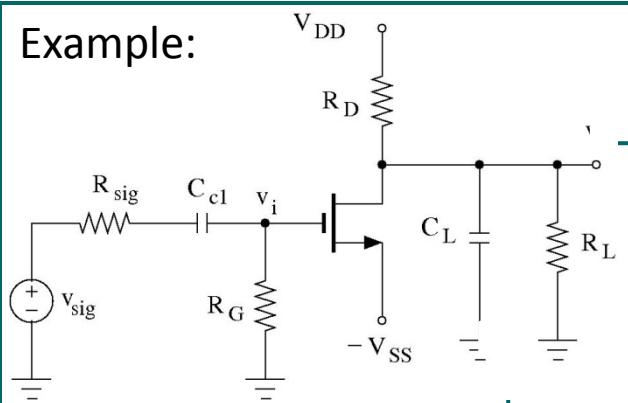
Impact of various capacitors depend on the frequency of interest



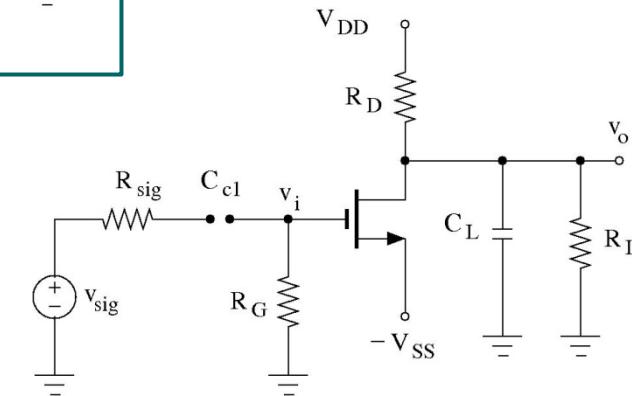
How to find which capacitors contribute to the lower cut-off frequency

- Consider each capacitor individually. Let $f = 0$ (capacitor is open circuit):
 - If v_o (or A_M) does not change, capacitor does NOT contribute to f_L (i.e., it is a high- f cap)
 - If v_o (or A_M) $\rightarrow 0$ or is reduced substantially, capacitor contributes to f_L (i.e., it is a low- f cap)

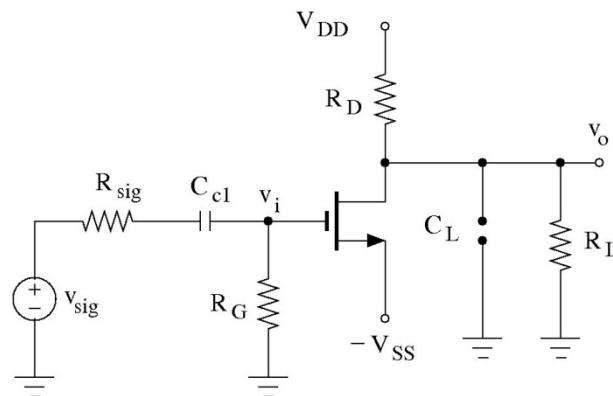
Example:



C_{c1} open:
 $v_i = 0 \rightarrow v_o = 0$
Contributes to f_L



C_L open:
No change in v_o
Does NOT contribute to f_L



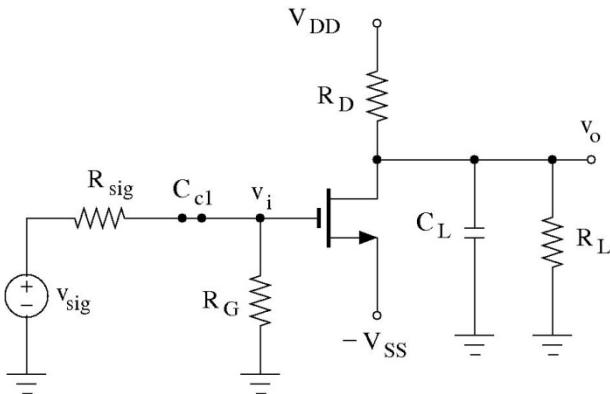
How to find which capacitors contribute to the upper cut-off frequency

- Consider each capacitor individually. Let $f \rightarrow \infty$ (capacitor is short circuit):
 - If v_o (or A_M) does not change, capacitor does NOT contribute to f_H (i.e., it is a low- f cap)
 - If v_o (or A_M) $\rightarrow 0$ or reduced substantially, capacitor contributes to f_H (i.e., it is a high- f cap)

C_{c1} short:

No change in v_o

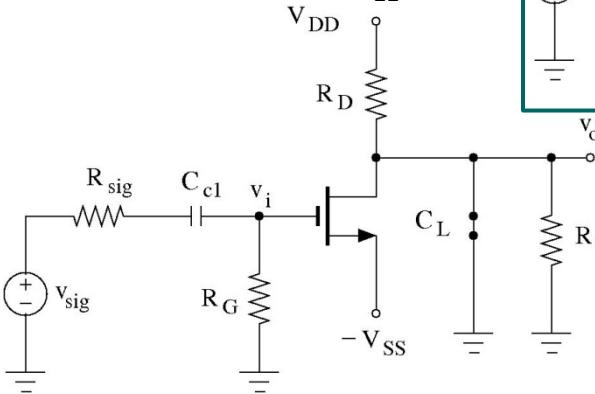
Does NOT contribute to f_H



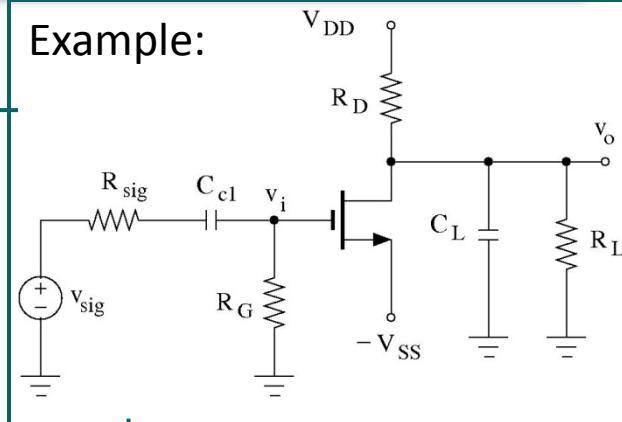
C_L short:

$$v_o = 0$$

Contributes to f_H



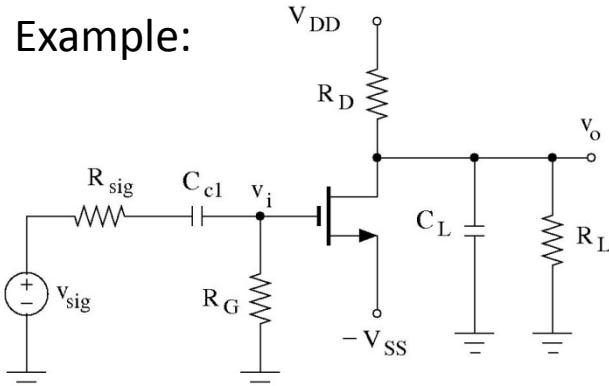
Example:



Constructing appropriate circuits

C_{c1} : Low- f capacitor
 C_L : High- f capacitor

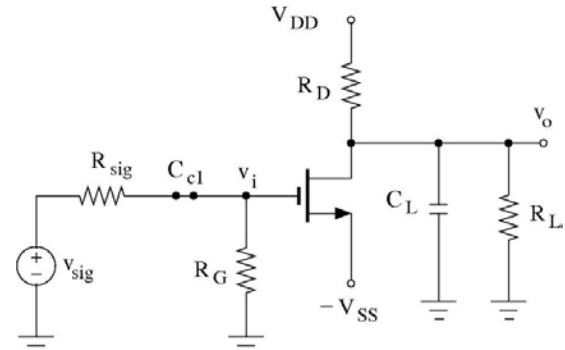
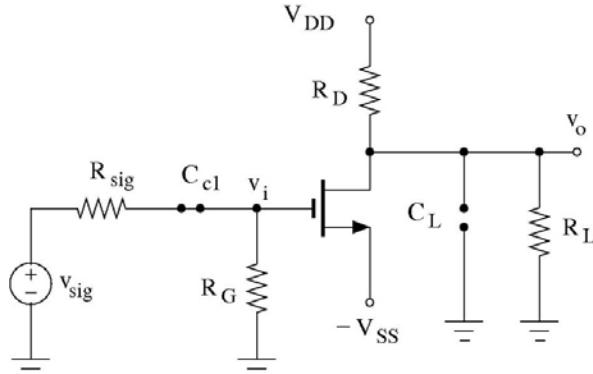
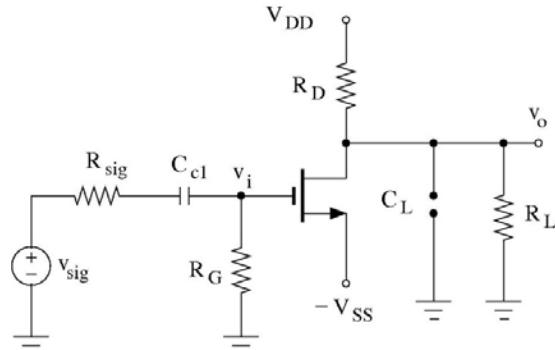
Example:



Computing f_L :
High- f caps are open.
Low- f caps included.

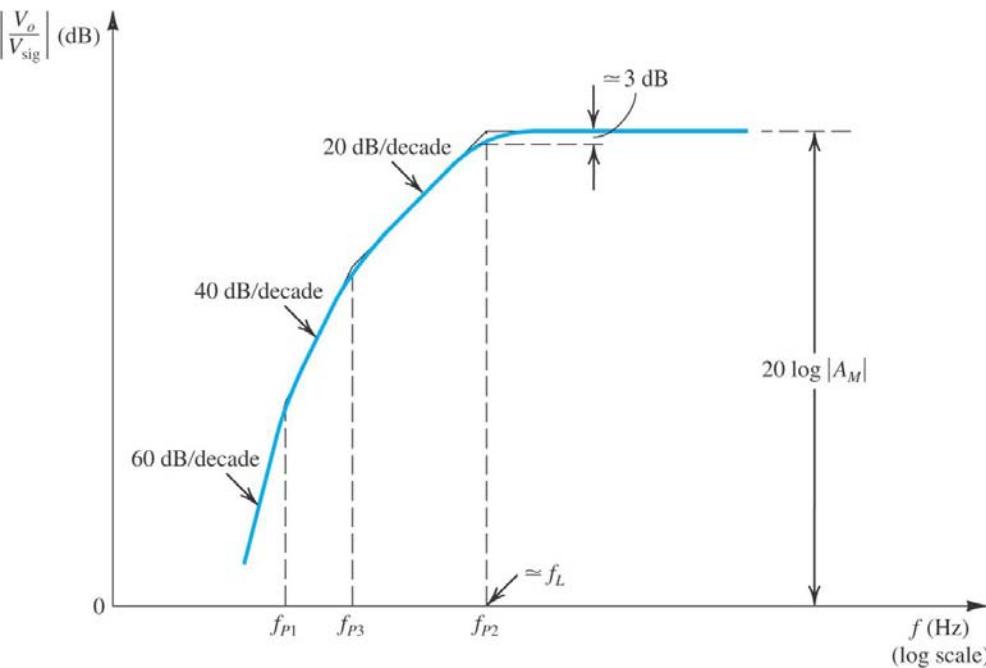
Mid-band:
High- f caps are open
Low- f caps are short.

Computing f_H :
High- f caps are included.
Low- f caps are short



Low-Frequency Response

Low-frequency response of an amplifier



Example: an amplifier with three poles

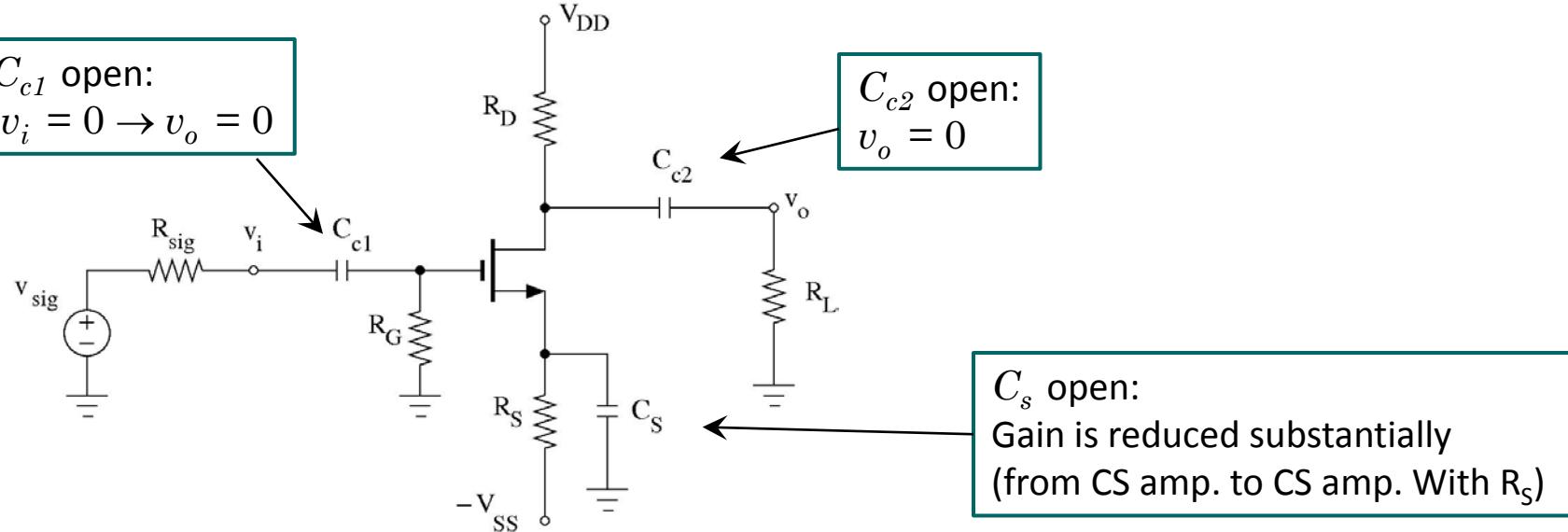
$$\frac{V_o}{V_{sig}} = A_M \times \frac{s}{s + \omega_{p1}} \times \frac{s}{s + \omega_{p2}} \times \frac{s}{s + \omega_{p3}}$$

(Set $s = j\omega$ to find Bode Plots)

- Each capacitor gives a pole.
- All poles contribute to f_L (exact value of f_L from computation or simulation)
- A good approximation for design & hand calculations:
$$f_L \approx f_{p1} + f_{p2} + f_{p3} + \dots$$
- If one pole is at least a factor of 4 higher than others (e.g., f_{p2} in the above figure), f_L is approximately equal to that pole (e.g., $f_L \approx f_{p2}$ in above within 20%)

Low-frequency response of a CS amplifier (from detailed frequency response analysis)

All capacitors contribute to f_L (as v_o is reduced when $f \rightarrow 0$ or caps open circuit)



Lengthy calculations: See S&S pp689-692 for detailed calculations (S&S assumes $r_o \rightarrow \infty$ and $R_s \rightarrow \infty$)

$$\frac{V_o}{V_{sig}} = A_M \times \frac{s}{s + \omega_{p1}} \times \frac{s}{s + \omega_{p2}} \times \frac{s}{s + \omega_{p3}}$$

$$A_M = -\frac{R_G}{R_G + R_{sig}} g_m (r_o \parallel R_D \parallel R_L)$$

$$\omega_{p1} = \frac{1}{C_{c1}(R_G + R_{sig})}, \quad \omega_{p3} = \frac{1}{C_{c2}(R_D \parallel r_o + R_L)}$$

$$\omega_{p2} \approx \frac{1}{C_s [R_s \parallel [(r_o + R_D \parallel R_L)/(1 + g_m r_o)]]},$$

Finding poles by inspection

1. Set $v_{sig} = 0^*$
2. Consider each capacitor separately, e.g., C_n (assume all others are short circuit!)
3. Find the total resistance seen between the terminals of the capacitor, e.g., R_n (treat ground as a regular “node”).
4. The pole associated with that capacitor is

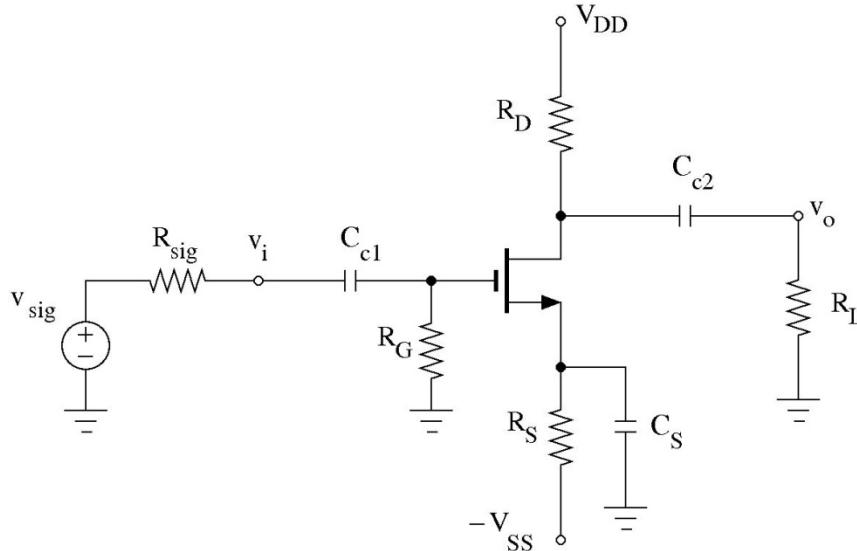
$$f_{pn} = \frac{1}{2\pi R_n C_n}$$

5. Lower-cut-off frequency can be found from

$$f_L \approx f_{p1} + f_{p2} + f_{p3} + \dots$$

* Although we are calculating frequency response in frequency domain, we will use time-domain notation instead of phasor form (i.e., v_{sig} instead of V_{sig}) to avoid confusion with the bias values.

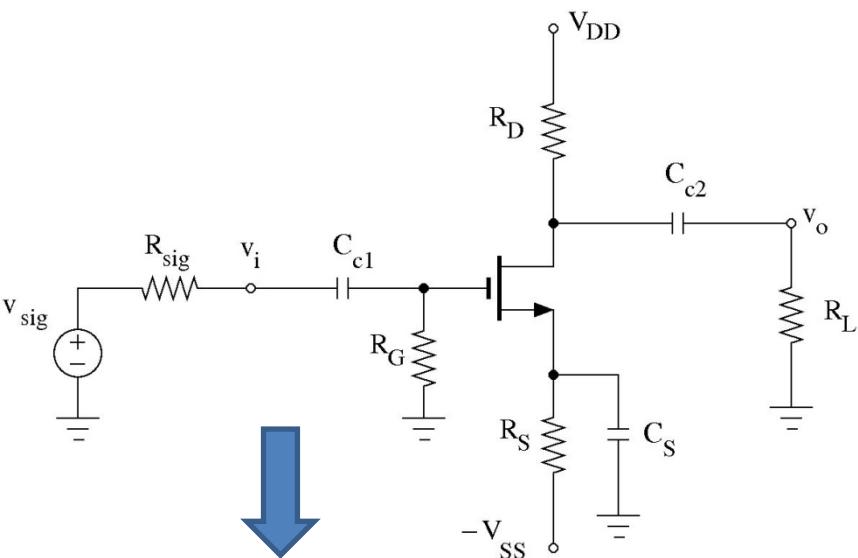
Example: Low-frequency response of a CS amplifier (from pole inspection)



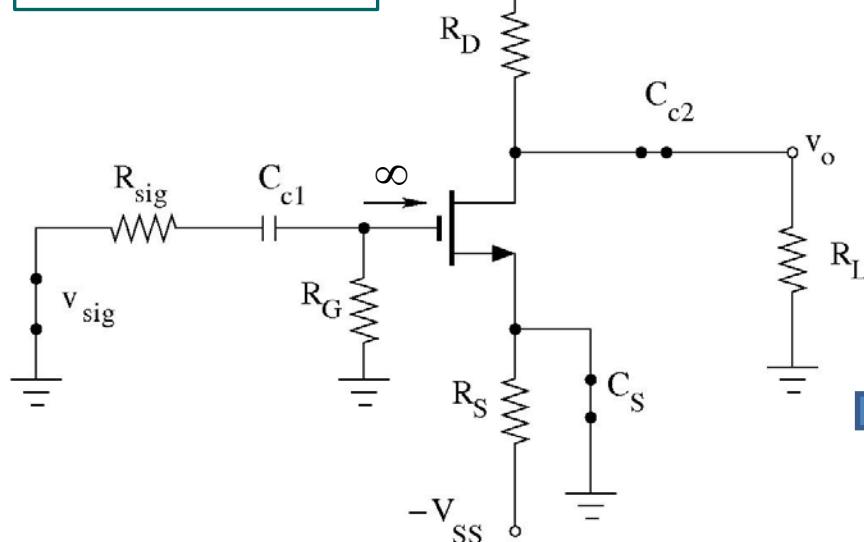
- Examination of circuit shows that ALL capacitors are low- f capacitors.
- In the following slides we will compute poles introduced by each capacitor. (Compare with the detailed calculations of slide 13!)

$$f_L \approx f_{p1} + f_{p2} + f_{p3}$$

Low-frequency response of a CS amplifier (f_{p1})

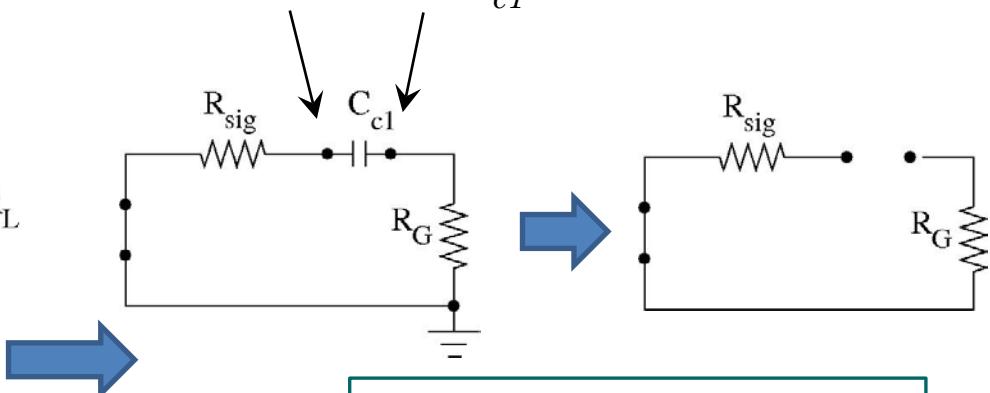


1. Consider C_{c1} :



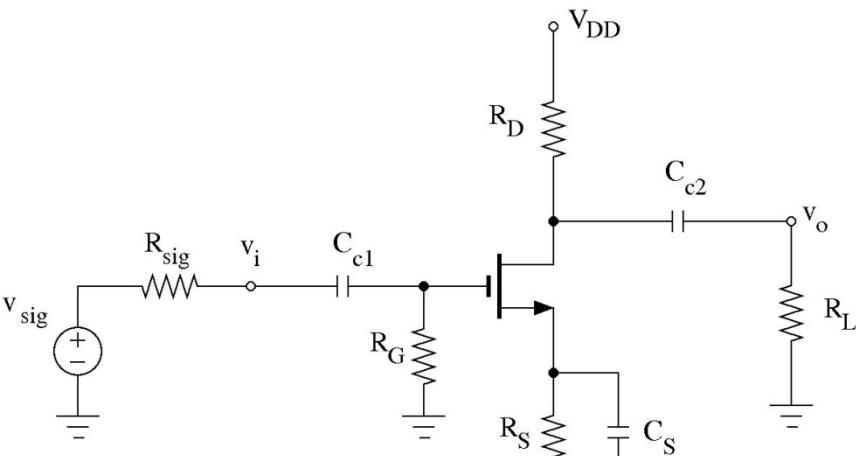
$$f_{p1} = \frac{1}{2\pi C_{c1} (R_G + R_{sig})}$$

Terminals of C_{c1}

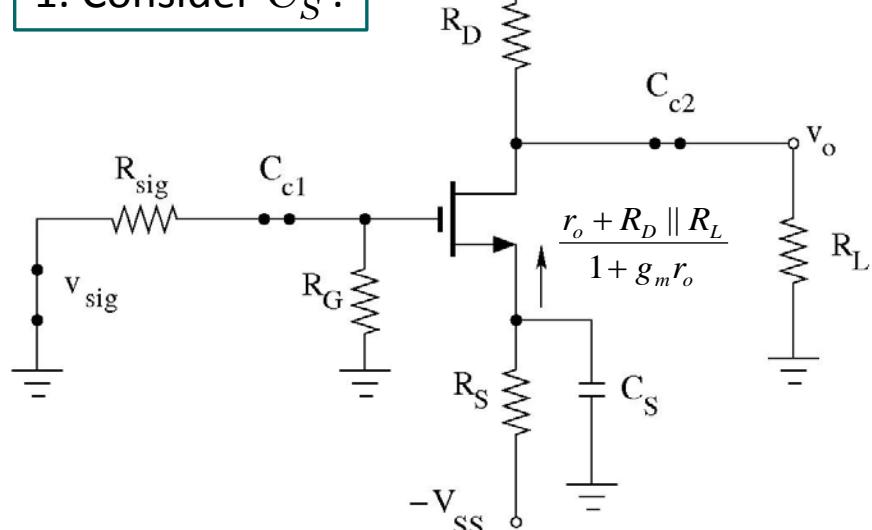


2. Find resistance between
Capacitor terminals

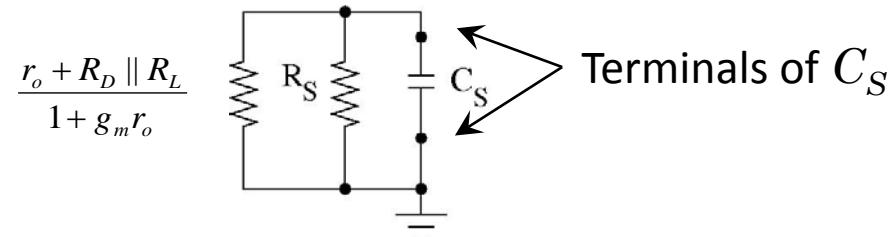
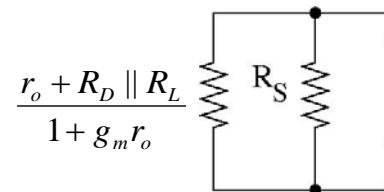
Low-frequency response of a CS amplifier (f_{p2})



1. Consider C_S :

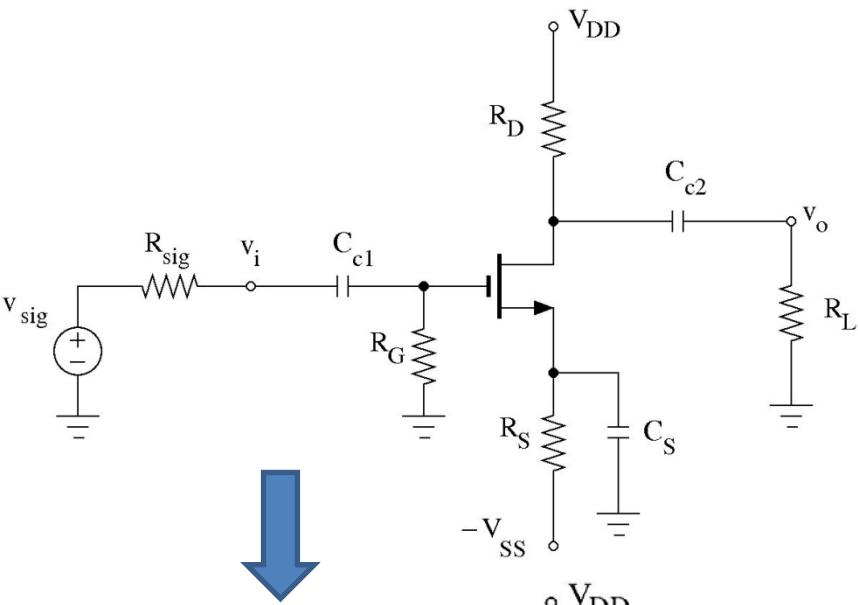


$$f_{p2} = \frac{1}{2\pi C_S [R_S \parallel ((r_o + R_D \parallel R_L) / (1 + g_m r_o))]}$$

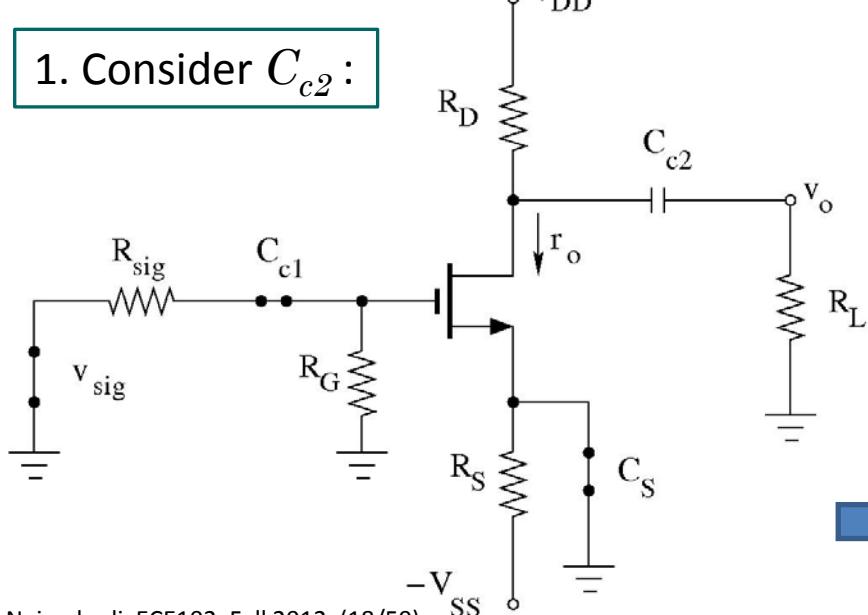


2. Find resistance between
Capacitor terminals

Low-frequency response of a CS amplifier (f_{p3})



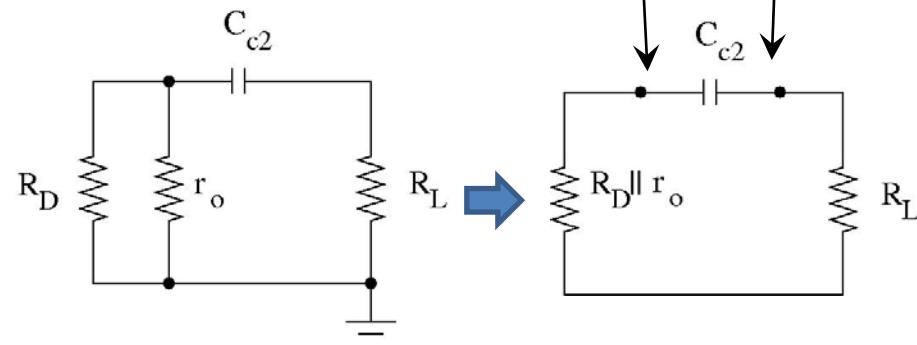
1. Consider C_{c2} :



$$f_{p3} = \frac{1}{2\pi C_{c2} (R_L + R_D \parallel r_o)}$$



Terminals of C_{c2}



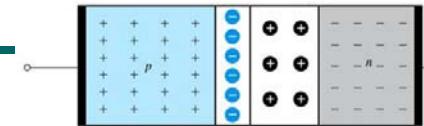
2. Find resistance between Capacitor terminals

High-Frequency Response

- **Amplifier gain falls off due to the internal capacitive effects of transistors as well as possible capacitors in the circuit.**

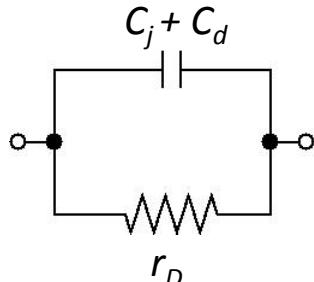
Capacitive Effects in pn Junction

- Charge stored in the pn junction, leading to a capacitance
- For majority carriers, stored charge is a function of applied voltage leading to a “small-signal” junction capacitance, C_j
- For minority carriers, stored charge depends on the time for these carriers to diffuse across the junction and recombine, leading to a diffusion capacitance, C_d
- Both C_j and C_d depend on bias current and/or voltage.
- Junction capacitances are small and are given in femto-Farad (fF)



$$1 \text{ fF} = 10^{-15} \text{ F}$$

High-f small signal model of diode



Forward Bias

$$C_j \approx 2 \cdot C_{j0}$$

$$C_d = \frac{\tau_T \cdot I_D}{V_T}$$

Reverse Bias

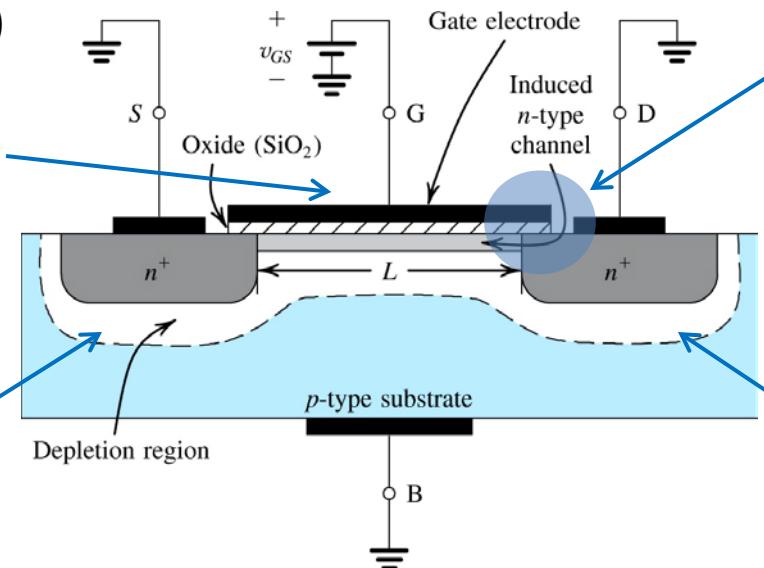
$$C_j = \frac{C_{j0}}{(1 + V_R/V_0)^m}$$

$$C_d = 0$$

*See S&S pp154-156 for detailed derivations

Capacitive Effects in MOS*

1. Capacitance between Gate and channel
(Parallel-plate capacitor)
appears as 2 capacitors:
between gate/source &
between gate/drain

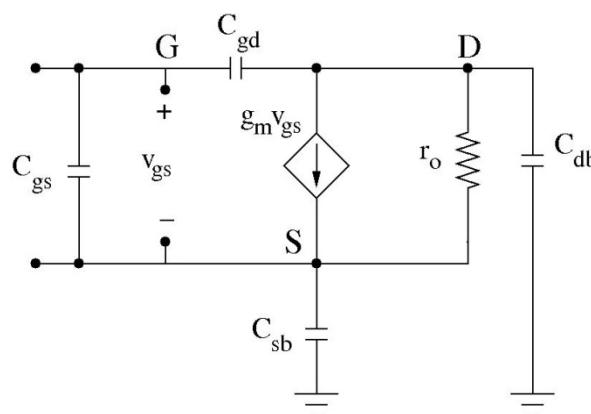


2. Capacitance between Gate & Source and Gate & Drain due to the overlap of gate electrode
(Parallel-plate capacitor)

3. Junction capacitance between Source and Body
(Reverse-bias junction)

4. Junction capacitance between Drain and Body
(Reverse-bias junction)

MOS High-frequency small signal model

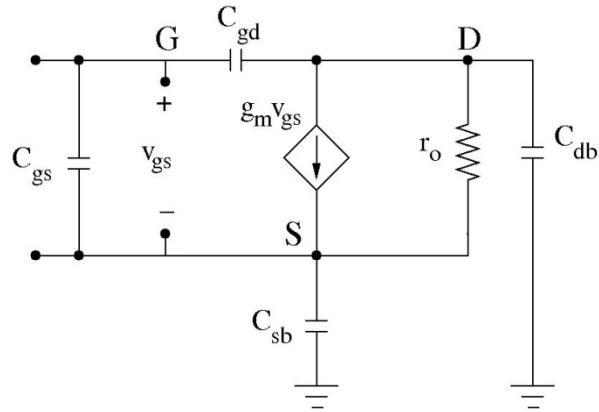


*See S&S pp154-156 for detailed derivations

MOS high-frequency small signal model

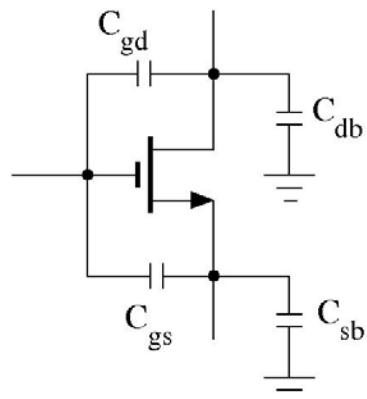
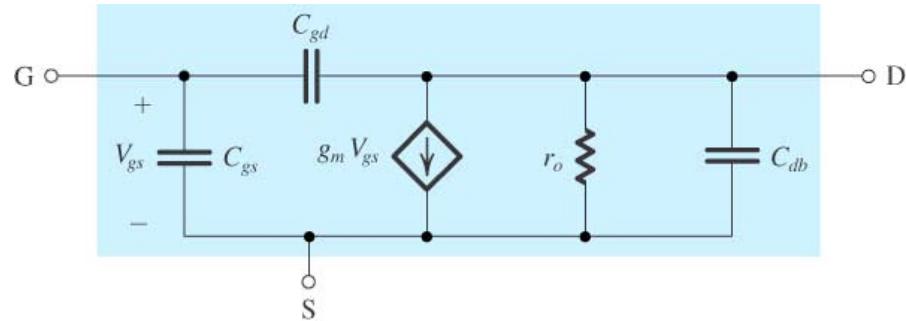
Accurate Model

(we use this model here)



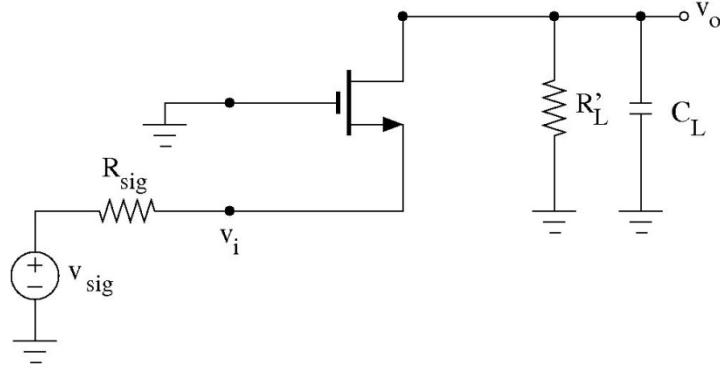
For source connected to body

(used by S&S)

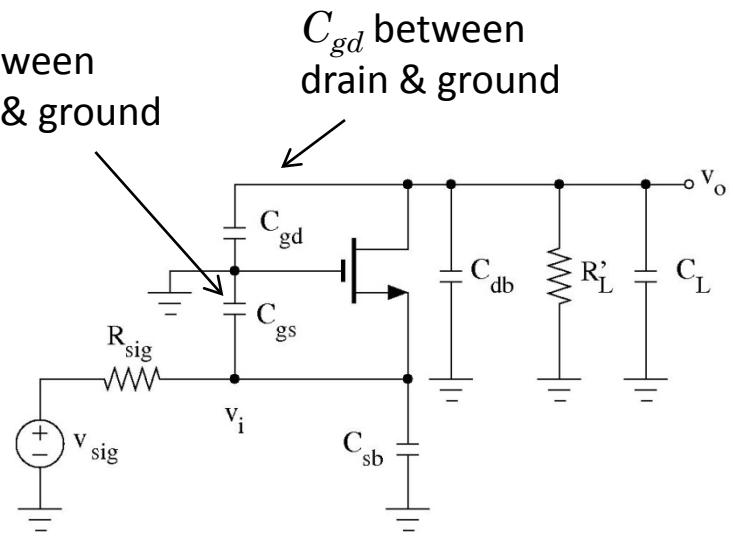


Generally, transistor internal capacitances are shown outside the transistor so that we can use results from the mid-band calculations.

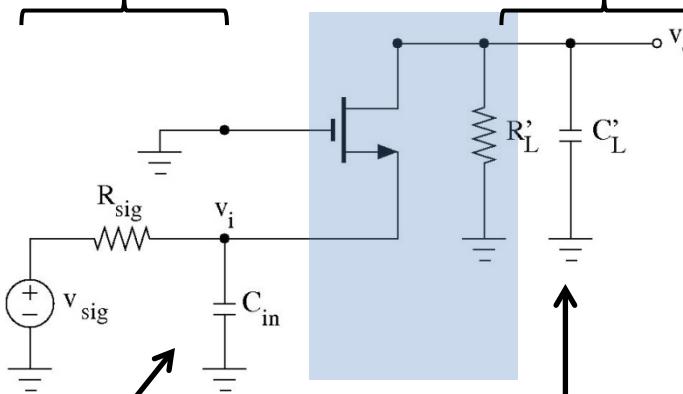
High-frequency response of a CG amplifier



C_{gs} between source & ground

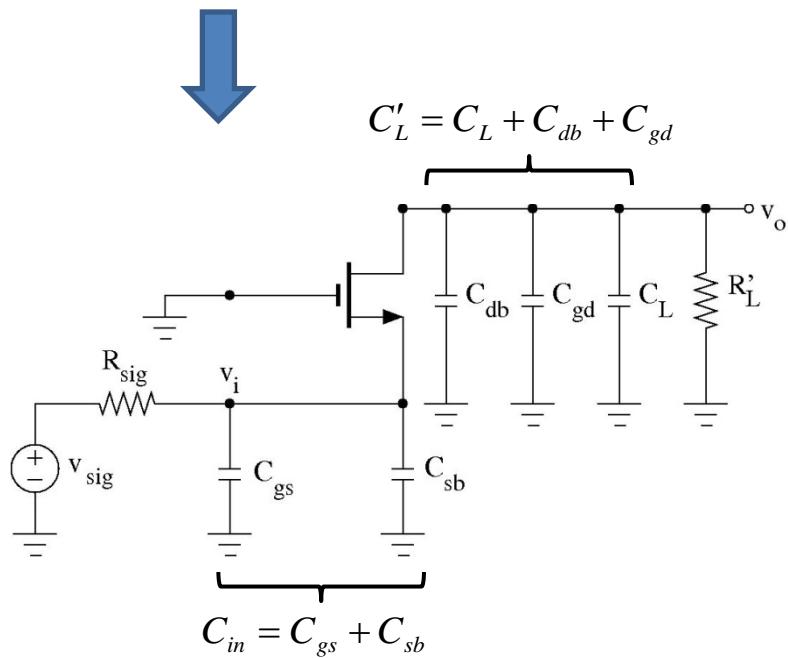


Low-pass filter Mid-band Amp Low-pass filter



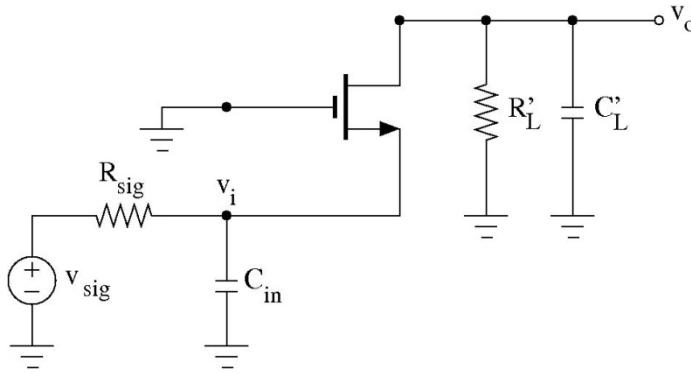
“Output Pole”

C_{gd} between drain & ground



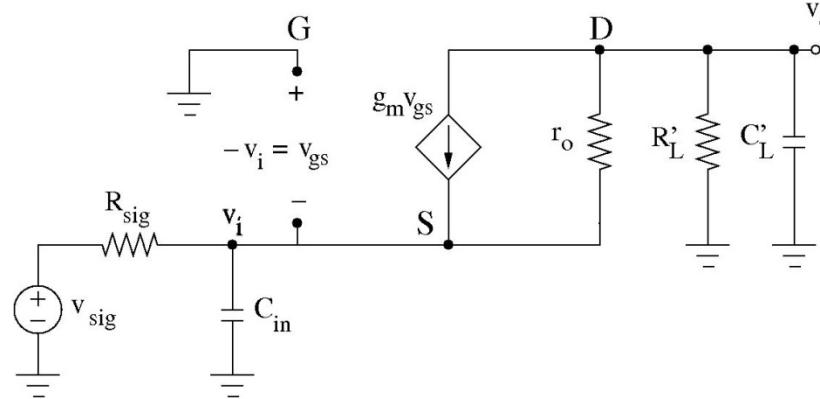
“Input Pole”

High- f response of a CG amplifier – Exact Solution (1)



$$C'_L = C_L + C_{db} + C_{gd}$$

$$C_{in} = C_{gs} + C_{sb}$$



$$\left\{ \begin{array}{l} \text{Node } v_i : \frac{v_i - v_{sig}}{R_{sig}} + \frac{v_i}{1/sC_{in}} - g_m(-v_i) + \frac{v_i - v_o}{r_o} = 0 \\ \text{Node } v_o : \frac{v_o}{R'_L} + \frac{v_o}{1/sC'_L} + g_m(-v_i) + \frac{v_o - v_i}{r_o} = 0 \end{array} \right.$$

Can be solved to find v_o/v_{sig}

High- f response of a CG amplifier – Exact Solution (2)

Compact solution can be found by ignoring r_o (i.e., $r_o \rightarrow \infty$)

$$\text{Node } v_i : v_i - v_{sig} + (sC_{in} + g_m)R_{sig}v_i = 0$$

$$\frac{v_i}{v_{sig}} = \frac{1}{1 + g_m R_{sig} + sC_{in} R_{sig}} = \frac{1}{1 + g_m R_{sig}} \times \frac{1}{1 + sC_{in} R_{sig} / (1 + g_m R_{sig})}$$

$$\frac{v_i}{v_{sig}} = \frac{1/g_m}{1/g_m + R_{sig}} \times \frac{1}{1 + sC_{in} (R_{sig} \parallel 1/g_m)}$$

Voltage divider
($R_i = 1/g_m$ and R_{sig})

“Input Pole”

$$\text{Node } v_o : \frac{v_o}{R'_L} + sC'_L v_o - g_m(v_i) = 0 \Rightarrow \frac{v_o}{v_i} = \frac{g_m R'_L}{1 + sC'_L R'_L}$$

Mid-band Gain

“Output Pole”

$$\frac{v_o}{v_{sig}} = \frac{R_i}{R_i + R_{sig}} \times (g_m R'_L) \times \frac{1}{1 + sC_{in} (R_{sig} \parallel 1/g_m)} \times \frac{1}{1 + sC'_L R'_L}$$

Open-Circuit Time-Constants Method

$$H(s) = \frac{1 + a_1 s + a_2 s^2 + \dots}{1 + b_1 s + b_2 s^2 + \dots} = \frac{1 + a_1 s + a_2 s^2 + \dots}{(1 + s/\omega_{p1})(1 + s/\omega_{p2})\dots}$$

$$b_1 = \frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}} + \dots$$

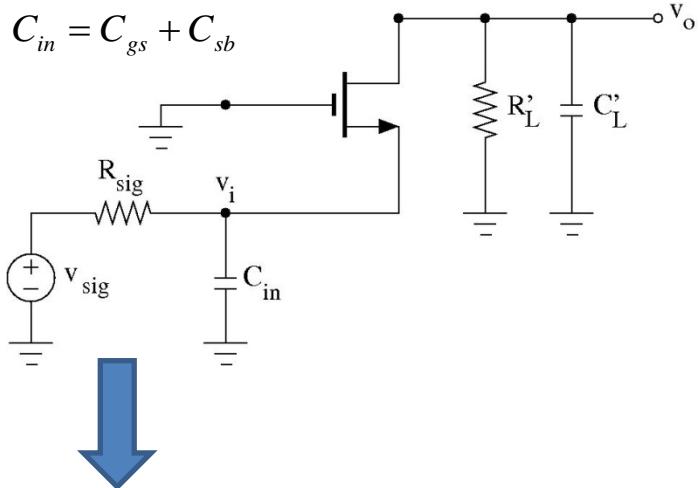
b_1 can be found by the **open-circuit time-constants method**.

1. Set $v_{sig} = 0$
2. Consider each capacitor separately, e.g., C_j (assume others are open circuit!)
3. Find the total resistance seen between the terminals of the capacitor, e.g., R_j (treat ground as a regular “node”).
4. $b_1 = \sum_{j=1}^n R_j C_j$

A good approximation to f_H is: $f_H = \frac{1}{2\pi b_1}$

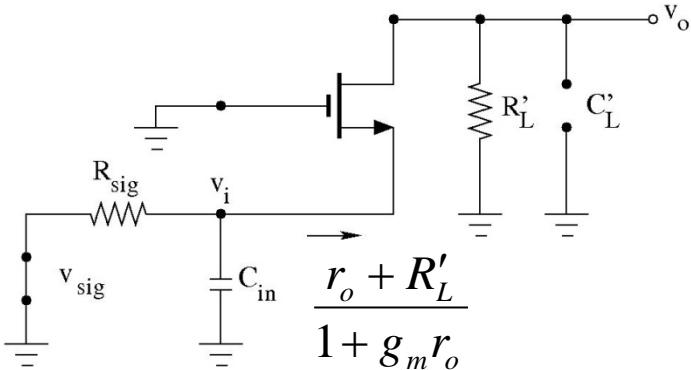
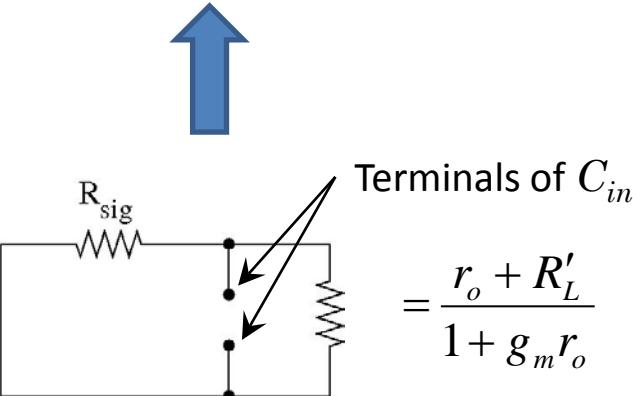
High- f response of a CG amplifier – time-constant method (input pole)

$$C'_L = C_L + C_{db} + C_{gd}$$



1. Consider C_{in} :

$$\tau_1 = C_{in} [R_{sig} \parallel (r_o + R'_L) / (1 + g_m r_o)]$$

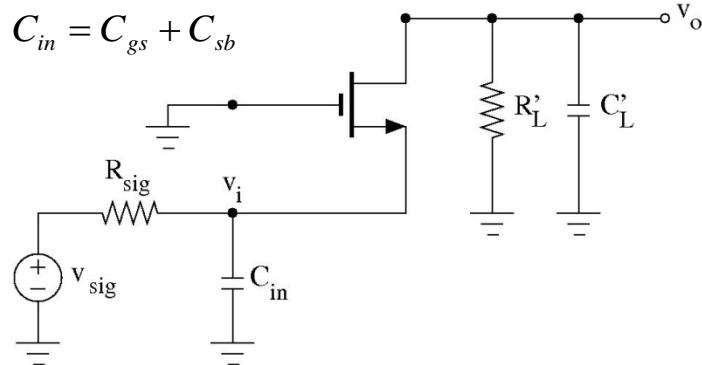


$$= \frac{r_o + R'_L}{1 + g_m r_o}$$

2. Find resistance between Capacitor terminals

High- f response of a CG amplifier – time-constant method (output pole)

$$C'_L = C_L + C_{db} + C_{gd}$$

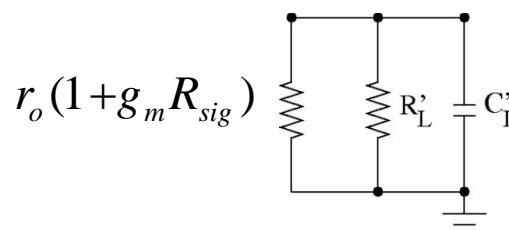
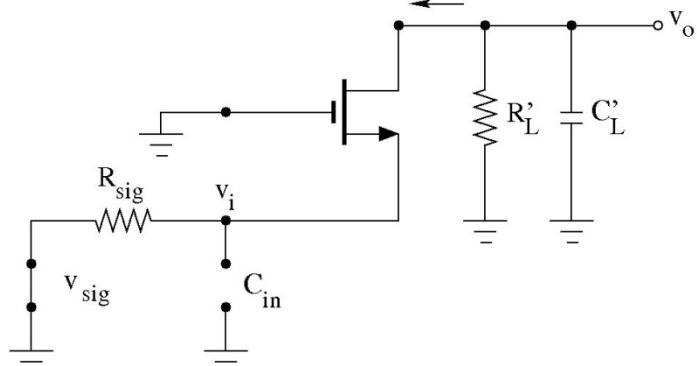


$$\tau_2 = C'_L [R'_L \parallel r_o (1 + g_m R_{sig})]$$

1. Consider C'_L :



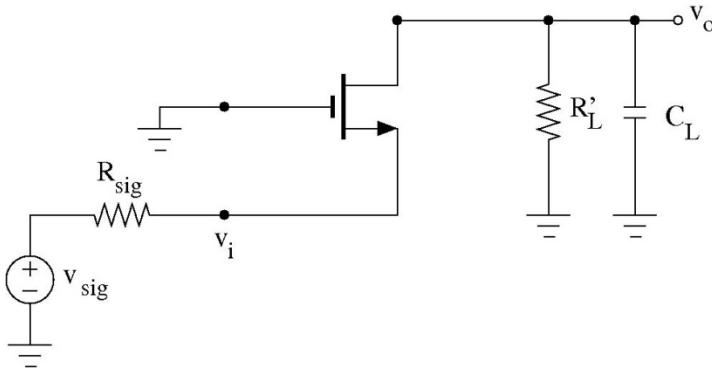
$$\approx r_o (1 + g_m R_{sig})$$



2. Find resistance between Capacitor terminals



High- f response of a CG amplifier – time-constant method



$$A_M = + \frac{R_i}{R_i + R_{sig}} g_m (r_o \parallel R'_L)$$

$$R_i = (r_o + R'_L) / g_m r_o$$

$$C'_L = C_L + C_{db} + C_{gd}$$

$$C_{in} = C_{gs} + C_{sb}$$

$$\tau_1 = C_{in} [R_{sig} \parallel (r_o + R'_L) / (1 + g_m r_o)]$$

$$\tau_2 = C'_L [R'_L \parallel r_o (1 + g_m R_{sig})]$$

$$f_H = \frac{1}{2\pi b_1} = \frac{1}{2\pi (\tau_1 + \tau_2)}$$

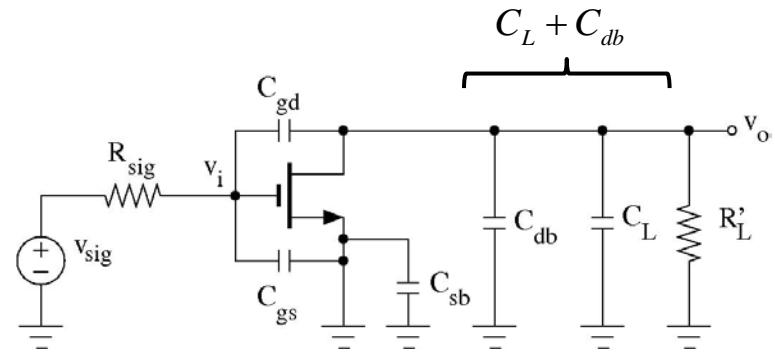
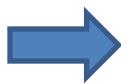
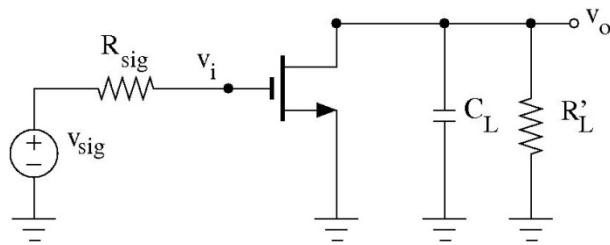
Comparison of time-constant method with the exact solution ($r_o \rightarrow \infty$)

$$\frac{v_o}{v_{sig}} = \frac{1/g_m}{1/g_m + R_{sig}} \times (g_m R'_L) \times \underbrace{\frac{1}{1 + sC_{in}(R_{sig} \parallel 1/g_m)} \times \frac{1}{1 + sC'_L R'_L}}$$

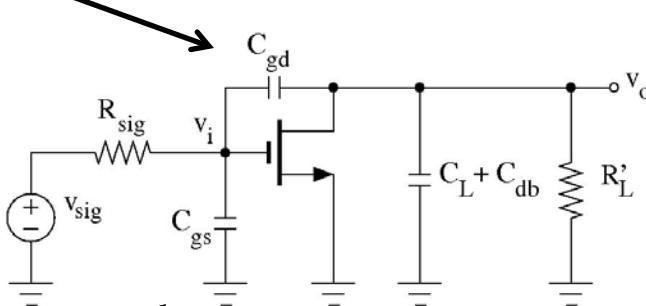
$$A_M$$

Input pole : Output pole :
 $\omega_{p1} = 1/\tau_1$ $\omega_{p2} = 1/\tau_2$

High-frequency response of a CS amplifier



C_{gd} is between output and input!

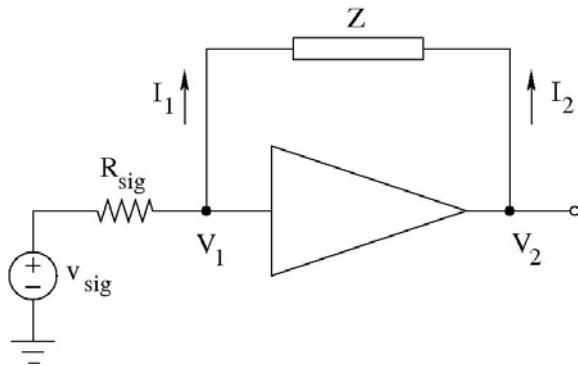


Two methods to find f_H

- 1) Miller's theorem
- 2) Direct calculation of resistance between terminals of C_{gd} (see Problem Set 8, Exercise 4)

Miller's Theorem

- Consider an amplifier with a gain A with an impedance Z attached between input and output
- V_1 and V_2 “feel” the presence of Z only through I_1 and I_2
- We can replace Z with any circuit as long as a current I_1 flows out of V_1 and a current I_2 flows out of V_2 .



$$V_2 = A \cdot V_1$$

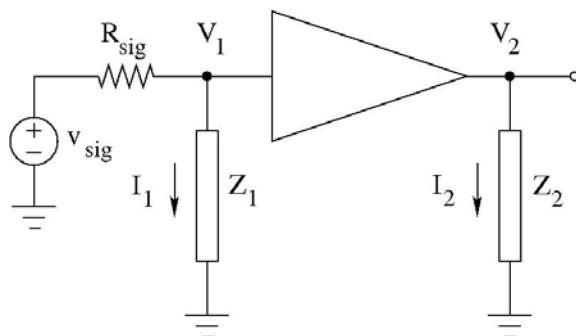
$$I_1 = \frac{V_1 - V_2}{Z} = \frac{(1-A) \cdot V_1}{Z}$$

$$I_2 = \frac{V_2 - V_1}{Z} = \frac{(A-1) V_1}{Z} = \frac{(A-1) V_2}{Z A}$$



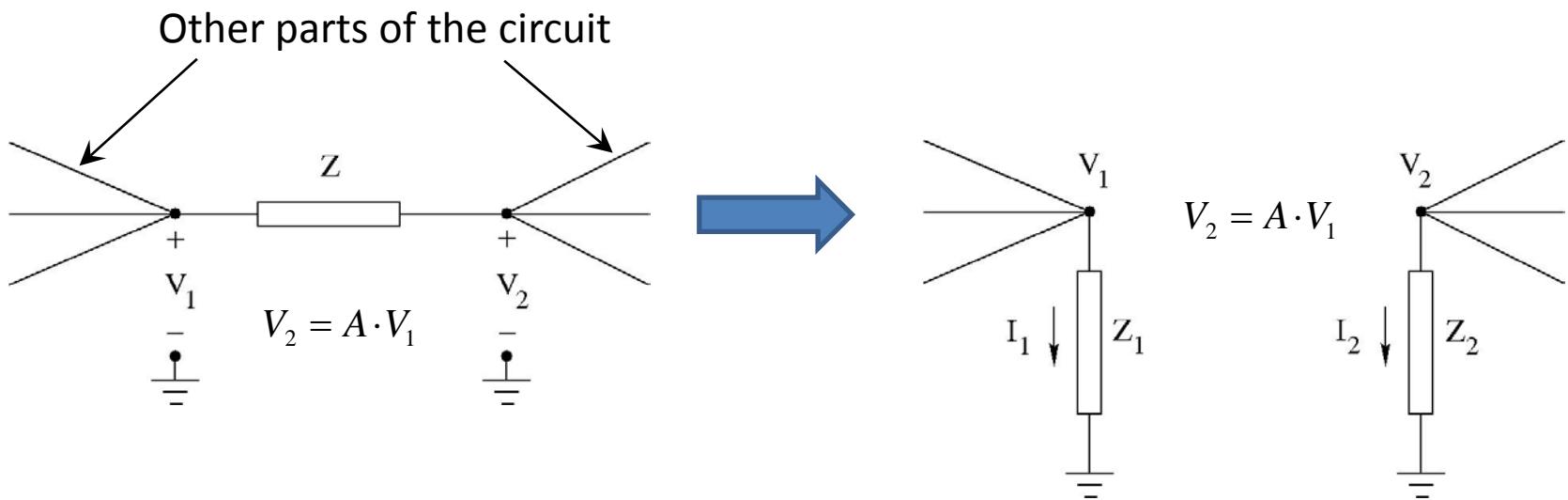
$$I_1 = \frac{V_1}{Z/(1-A)} = \frac{V_1}{Z_1}, \quad Z_1 = \frac{Z}{(1-A)}$$

$$I_2 = \frac{V_2}{ZA/(A-1)} = \frac{V_2}{Z_2}, \quad Z_2 = \frac{Z}{1-1/A}$$



Miller's Theorem – Statement

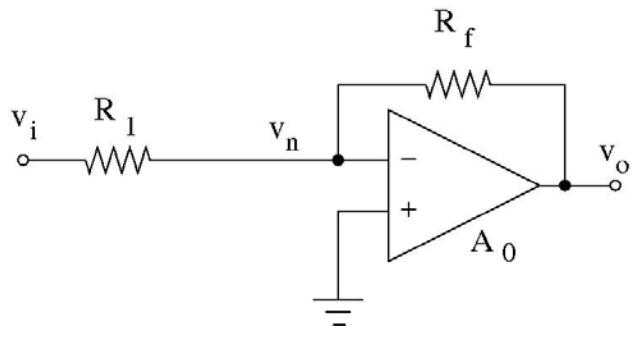
- If an impedance Z is attached between input and output an amplifier with a gain A , Z can be replaced with two impedances between input & ground and output & ground.



$$Z_1 = \frac{Z}{1 - A}$$

$$Z_2 = \frac{Z}{1 - \frac{1}{A}}$$

Example of Miller's Theorem: Inverting amplifier

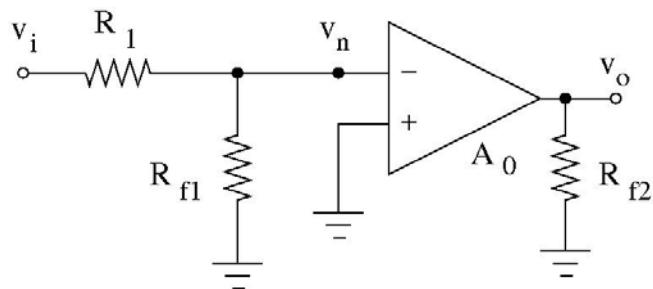


$$\text{OpAmp: } v_o = A_0 \cdot (v_p - v_n) = -A_0 \cdot v_n$$

Recall from ECE 100, if A_0 is large

$$\frac{v_o}{v_i} = -\frac{R_f}{R_1}$$

Solution using Miller's theorem:



$$Z_1 = \frac{Z}{1-A}$$

$$Z_2 = \frac{Z}{1-1/A}$$

$$R_{f1} = \frac{R_f}{1+A_0} \approx \frac{R_f}{A_0}$$

$$R_{f2} = \frac{R_f}{1+1/A_0} \approx R_f$$

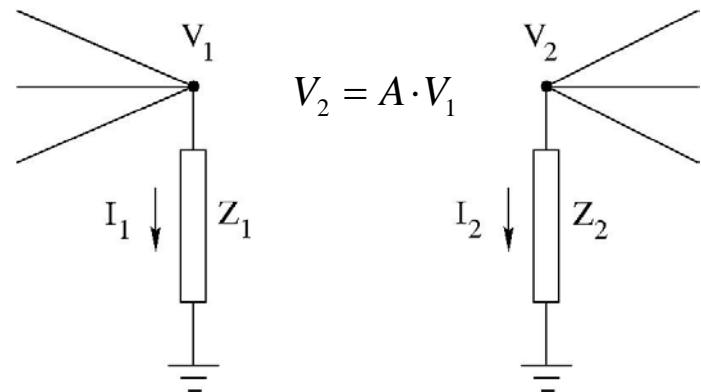
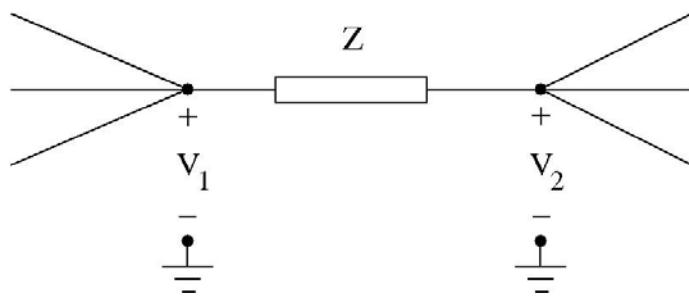
$$\frac{v_n}{v_i} = \frac{R_{f1}}{R_1 + R_{f1}}$$

$$\frac{v_o}{v_i} = -A_0 \frac{v_n}{v_i} = \frac{-A_0 R_{f1}}{R_1 + R_{f1}}$$

$$= \frac{-A_0 (R_f / A_0)}{R_1 + (R_f / A_0)} = \frac{-R_f}{R_1 + (R_f / A_0)}$$

$$\frac{v_o}{v_i} \approx \frac{-R_f}{R_1}$$

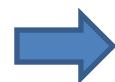
Applying Miller's Theorem to Capacitors



$$Z_1 = \frac{Z}{1-A}$$

$$Z_2 = \frac{Z}{1 - \frac{1}{A}}$$

Large capacitor at
the input for $A \gg 1$

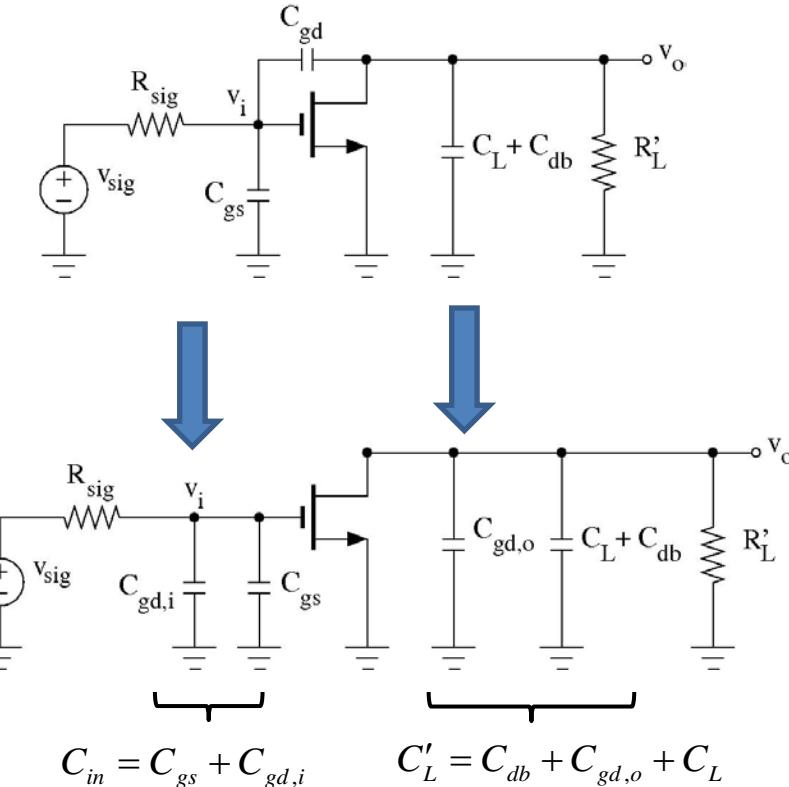


$$Z = \frac{1}{j\omega C}$$

$$Z_1 = \frac{Z}{1-A} \Rightarrow C_1 = (1-A)C$$

$$Z_2 = \frac{Z}{1-1/A} \Rightarrow C_2 = (1-1/A)C$$

High-frequency response of a CS amplifier – Using Miller's Theorem

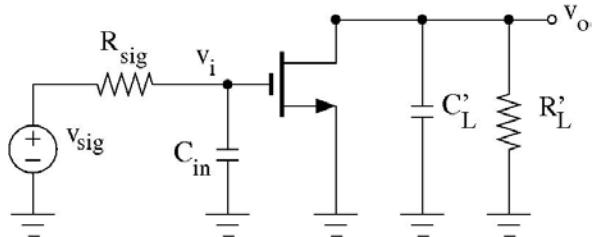


Use Miller's Theorem to replace capacitor between input & output (C_{gd}) with two capacitors at the input and output.

$$A = \frac{v_d}{v_g} = -g_m(r_o \parallel R'_L)$$

$$C_{gd,i} = C_{gd}(1 - A) = C_{gd}[1 + g_m(r_o \parallel R'_L)]$$

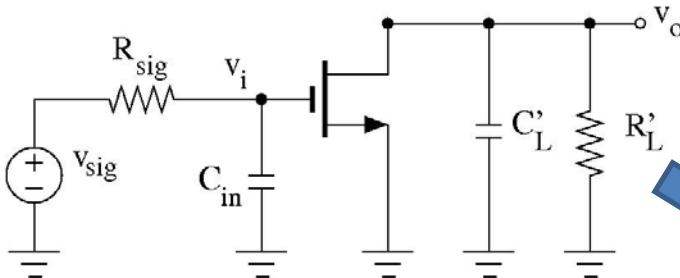
$$C_{gd,o} = C_{gd}(1 - 1/A) = C_{gd}[1 + 1/g_m(r_o \parallel R'_L)] \approx C_{gd} \quad *$$



Note: C_{gd} appears in the input ($C_{gd,i}$) as a “much larger” capacitor.

* Assuming $g_m R'_L \gg 1$

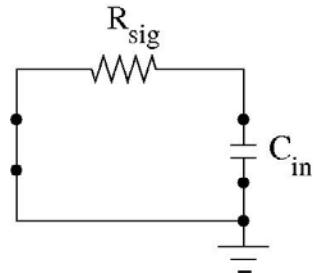
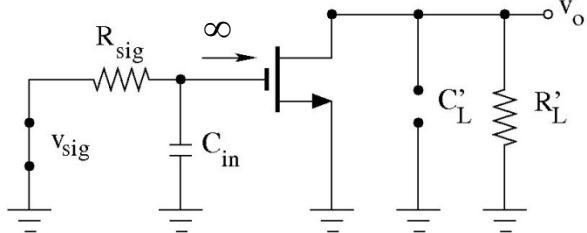
High- f response of a CS amplifier – Miller's Theorem and time-constant method



$$C_{in} = C_{gs} + C_{gd} [1 + g_m (r_o \parallel R'_L)]$$

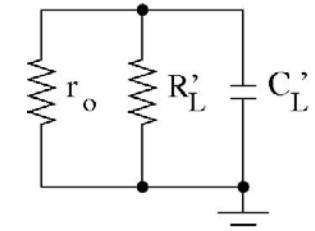
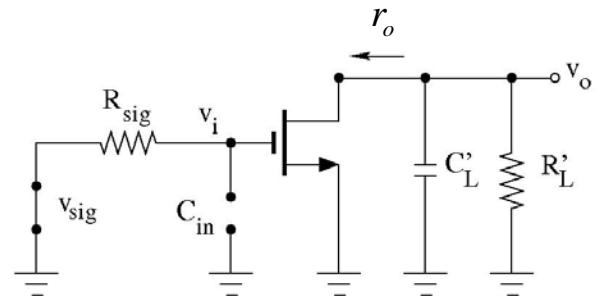
$$C'_L = C_{db} + C_{gd} + C_L$$

Input Pole (C_{in}):



$$\tau_1 = C_{in} R_{sig}$$

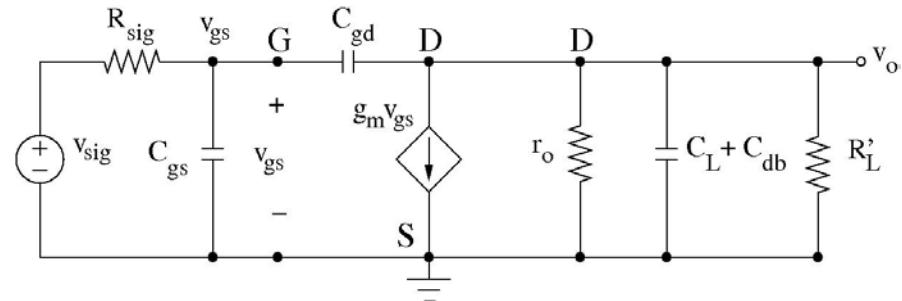
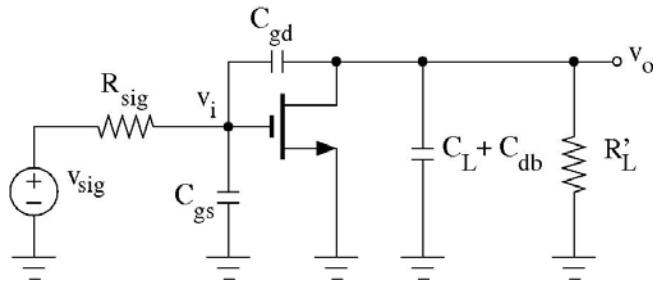
Output Pole (C'_L):



$$\tau_2 = C'_L (r_o \parallel R'_L)$$

$$\frac{1}{2\pi f_H} = b_1 = C_{in} R_{sig} + C'_L (r_o \parallel R'_L)$$

High- f response of a CS amplifier – Exact solution



Solving the circuit (node voltage method):

$$\left. \begin{aligned} \frac{v_o}{v_{sig}} &= \frac{-g_m(r_o \parallel R'_L) \times (1 - sC_{gd} / g_m)}{1 + b_1s + b_2s^2} \\ b_1 &= C_{in}R_{sig} + C'_L(r_o \parallel R'_L) \\ b_2 &= [(C_L + C_{db})(C_{gs} + C_{gd}) + C_{gs}C_{gd}] \times \\ &\quad R_{sig}(r_o \parallel R'_L) \\ C_{in} &= C_{gs} + C_{gd}[1 + g_m(r_o \parallel R'_L)] \\ C'_L &= C_{db} + C_{gd} + C_L \end{aligned} \right\}$$

Miller & time-constant method:

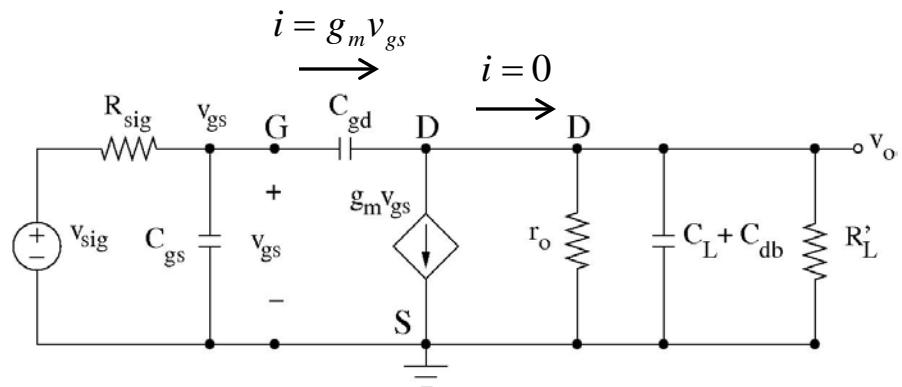
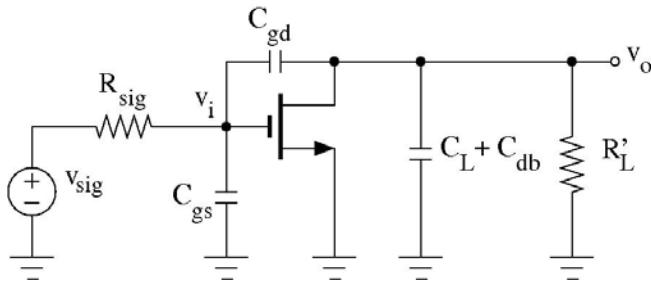
1. Same b_1 and same f_H as the exact solution

$$\frac{1}{2\pi f_H} = b_1 = C_{in}R_{sig} + C'_L(r_o \parallel R'_L)$$
2. Although, we get the same f_H , there is a substantial error in individual input and output poles.
3. Miller approximation did not find the zero!

Miller's Theorem vs Miller's Approximation

- For Miller Theorem to work, ratio of V_2/V_1 (amplifier gain) should be calculated in the presence of impedance Z .
- In our analysis, we used mid-band gain of the amplifier and ignored changes in the gain due to the feedback capacitor, C_{gd} . This is called “Miller’s Approximation.”
 - In the OpAmp example the gain of the chip, A_0 , remains constant when R_f is attached (as the output resistance of the chip is small).
- Because the amplifier gain in the presence of C_{gd} is smaller than the mid-band gain (we are on the high- f portion of the Bode gain plot), Miller’s approximation overestimates $C_{gd,i}$ and underestimates $C_{gd,o}$
 - There is a substantial error in individual input and output poles. However, b_1 and f_H are estimated well.
- More importantly, Miller’s Approximation “misses” the zero introduced by the feedback capacitor (This is important for stability of feedback amplifiers as it affects gain and phase margins).
 - Fortunately, we can calculate the zero of the transfer function easily (next slide).

Finding the “zero” of the CS amplifier

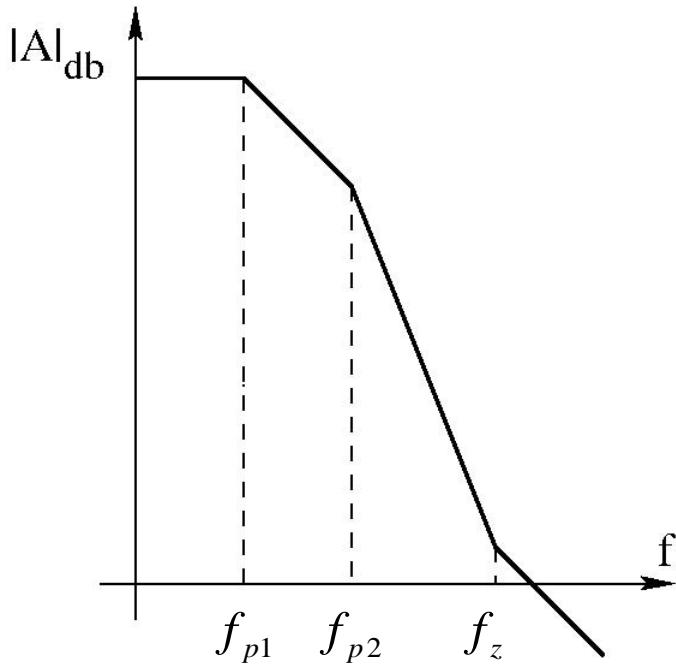


- 1) Definition of Zero: $v_o(s = s_z) = 0$
- 2) Because $v_o = 0$, zero current will flow in r_o , $C_L + C_{db}$ and R'_L
- 3) By KCL, a current of $g_m v_{gs}$ will flow in C_{gd} .
- 4) Ohm's law for C_{gd} gives:

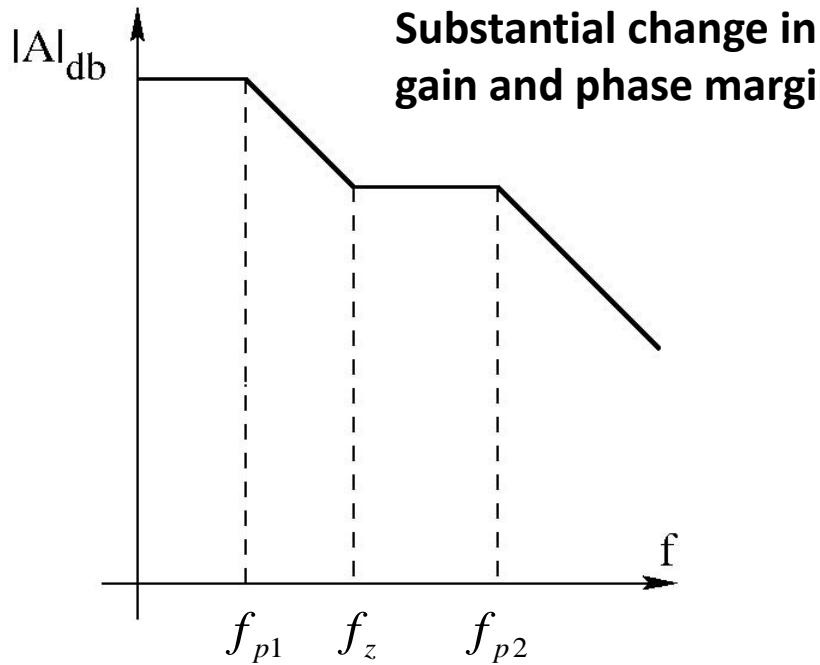
$$v_{gs} - 0 = Z i = \frac{g_m v_{gs}}{s_z C_{gd}}$$

$$s_z = \frac{g_m}{C_{gd}}, \quad f_z = \frac{g_m}{2\pi C_{gd}}$$

Zero of CS amplifier can play an important role in the stability of feedback amplifiers



Case of $f_z \gg f_{p2} > f_{p1}$



Case of $f_{p2} > f_z > f_{p1}$

Note: Since the input pole is at $1/(2\pi\tau_1) = 1/(2\pi C_{in}R_{sig})$
Small R_{sig} can push f_{p2} to very large values!

Comparison of CS and CG amplifiers

- Both CS and CG amplifiers have a high gain of $g_m(r_o \parallel R'_L)$
- CS amplifier has an infinite input resistance while CG amplifier suffers from a low input resistance.
- CG amplifier has a much better high- f response:
 - CS amplifier has a large capacitor at the input due to the Miller's effect: $C_{in} = C_{gs} + C_{gd}[1 + g_m(r_o \parallel R'_L)]$ compared to that of a CG amplifier $C_{in} = C_{gs} + C_{sb}$
 - In addition a CS amplifier has a zero.
- The Cascode amplifier combines the desirable properties of a high input resistance with a reasonable high- f response. (It has a much better high- f response than a two-stage CS amplifier)

Caution: Time-constant Method & Miller's Approximation

- The time constant method approximation to f_H (see S&S page 724).

$$b_1 = \sum_{j=1}^n R_j C_j \approx \frac{1}{\omega_H}$$

- Since, $b_1 = \frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}} + \dots \Rightarrow \frac{1}{\omega_H} \approx \frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}} + \dots$

This is the correct formula to find f_H

- However, S&S gives a different formula in page 722 (contradicting formulas of pp724). **Ignore this formula (S&S Eq. 9.68)**

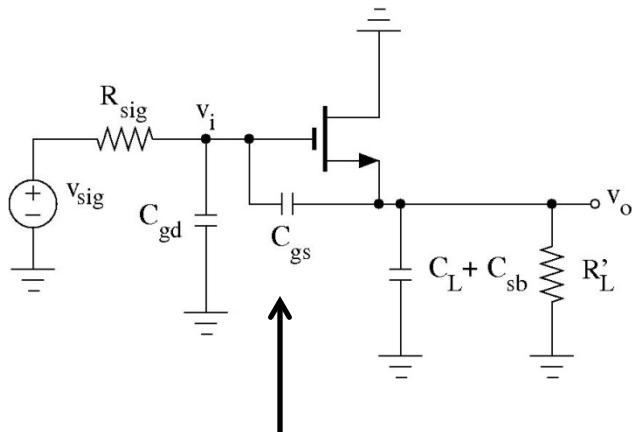
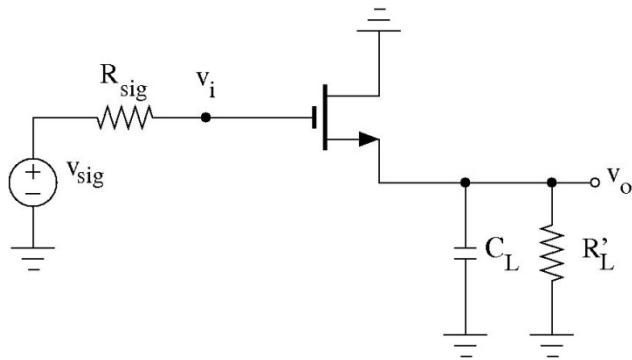
$$\frac{1}{\omega_H} = \sqrt{\frac{1}{\omega_{p1}^2} + \frac{1}{\omega_{p2}^2} + \frac{1}{\omega_{p3}^2} + \dots}$$

- Discussions (and some conclusions re Miller's theorem) in Examples 9.8 to 9.10 are incorrect. The discrepancy between f_H from Miller's approximation and exact solution is due to the use of Eq. 9.68 (Not Miller's fault!)

Caution: Miller's Approximation

- The main value of Miller's Theorem is to demonstrate that a large capacitance will appear at the input of a CS amplifier (Miller's capacitor).
- While Miller's Approximation gives a reasonable approximation to f_H , it fails to provide accurate values for each pole and misses the zero.
 - Miller's approximation should be used only as a first guess for analysis. Simulation should be used to accurately find the amplifier response.
 - Stability analysis (gain and phase margins) should utilize simulations unless a dominant pole far away from f_H is introduced.
- Miller's approximation breaks down when gain is close to 1 (See source follower, following slides).

High- f response of a Source Follower

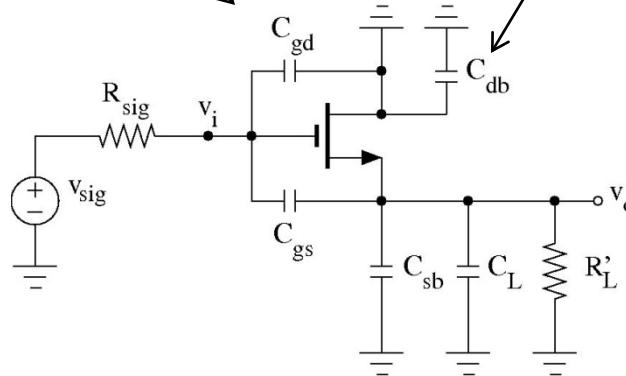


C_{gs} between output and input



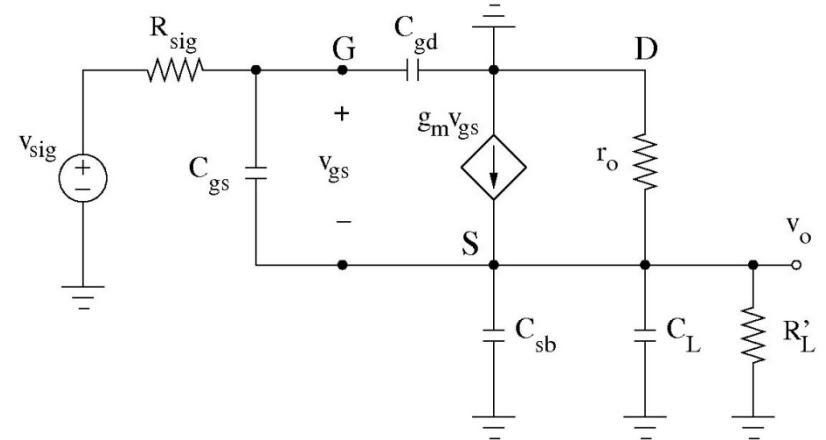
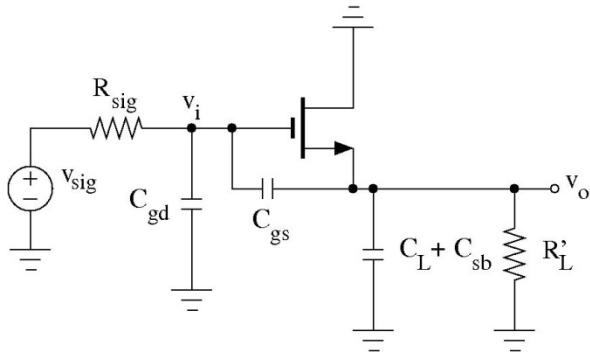
C_{gd} between gate and ground

C_{db} is shorted out.



- Because mid-band gain is close to 1 and positive, Miller approximation will not work well.
 - Need to apply Miller's theorem exactly
 - Lengthy calculations

High- f response of a source follower – Exact Solution



$$\text{Node } v_i : \frac{v_i - v_{sig}}{R_{sig}} + sC_{gd}v_i + sC_{gs}(v_i - v_o) = 0$$

$$\text{Node } v_o : \frac{v_o}{R'_L \parallel r_o} + s(C_L + C_{sb})v_o + g_m(v_i - v_o) + sC_{gs}(v_o - v_i) = 0$$

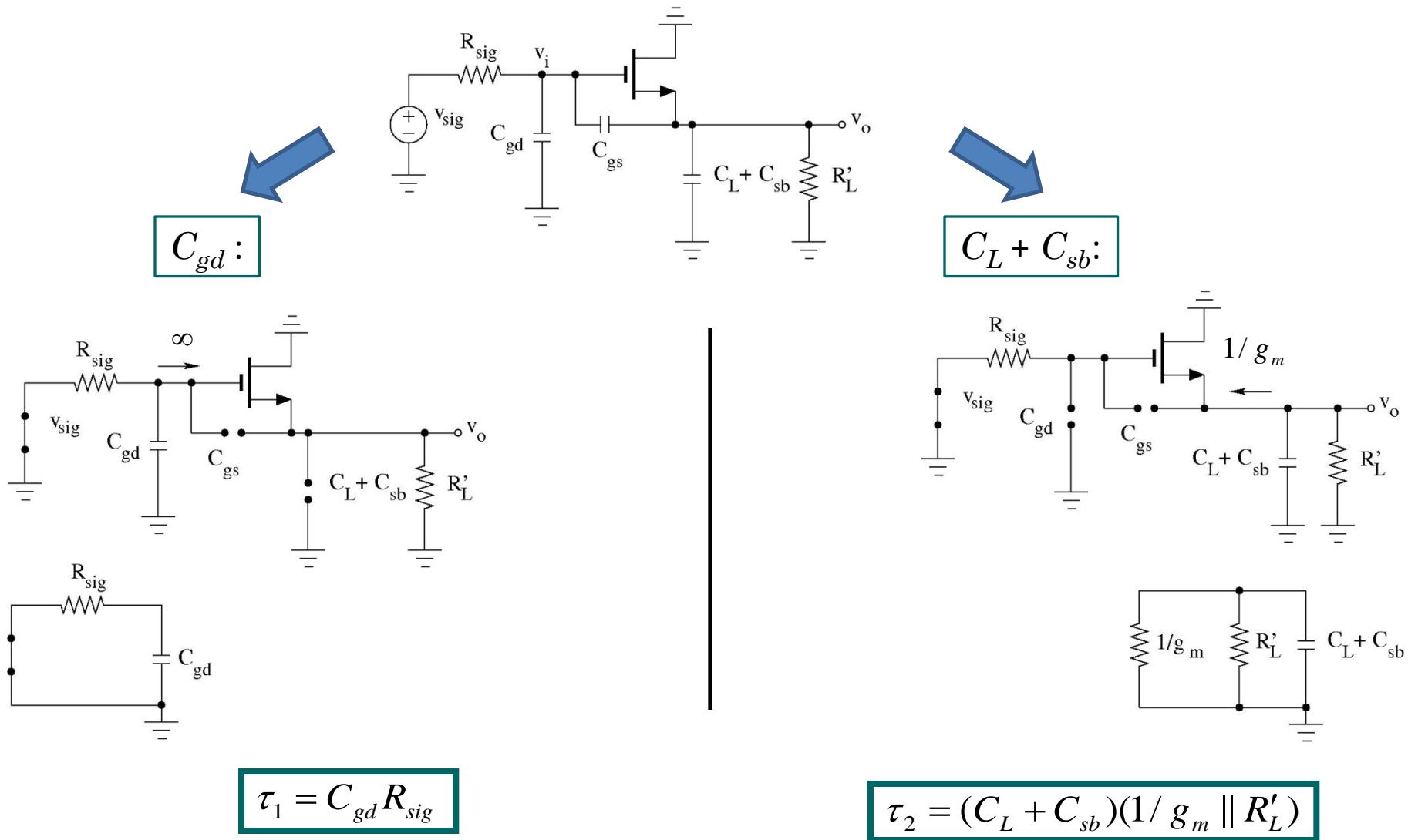
$$\frac{v_o}{v_{sig}} = \frac{g_m(r_o \parallel R'_L)}{1 + g_m(r_o \parallel R'_L)} \times \frac{(1 + sC_{gs}/g_m)}{1 + b_1s + b_2s^2}$$

Mid-band gain

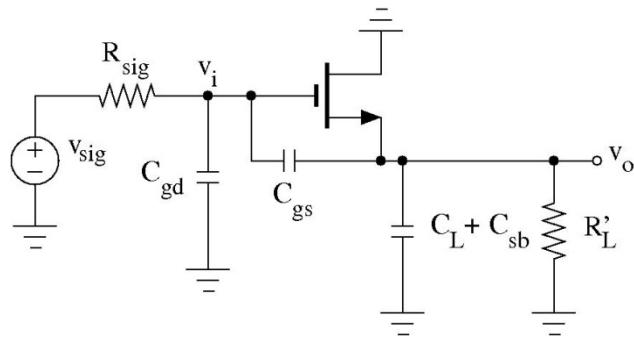
Zero: $s_z = -\frac{g_m}{C_{gs}}$, $f_z = \frac{g_m}{2\pi C_{gs}}$

Lengthy analysis is needed to find b_1 , b_2 , and two poles

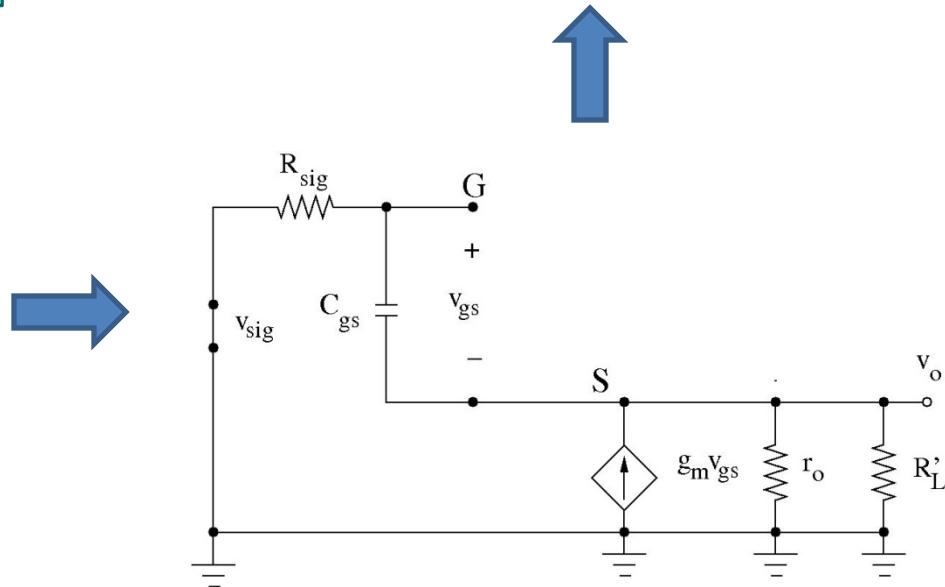
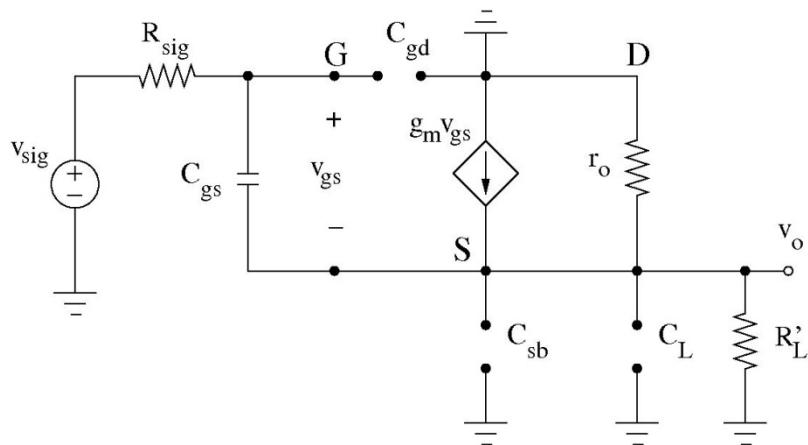
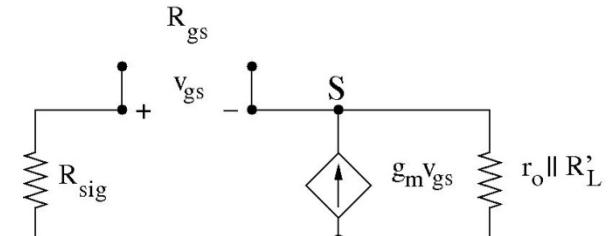
High- f response of a source follower – time-constant method (1)



High-frequency response of a follower – Time-constant method (2)

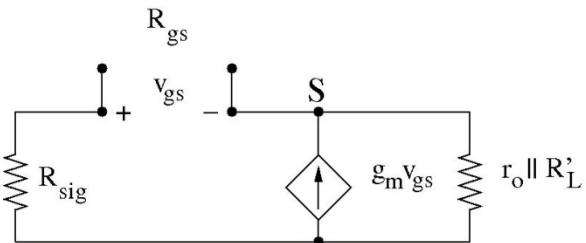


3. C_{gs} (cannot use elementary R forms)



High-frequency response of a follower – Time-constant method (3)

3. C_{gs} (cannot use elementary R forms), continued



$$V_x = v_{gs}$$

$$\text{KVL} \quad V_x = I_x R_{sig} + (R'_L \parallel r_o)(I_x - g_m v_{gs})$$

$$V_x = I_x R_{sig} + (R'_L \parallel r_o) I_x - g_m (R'_L \parallel r_o) V_x$$

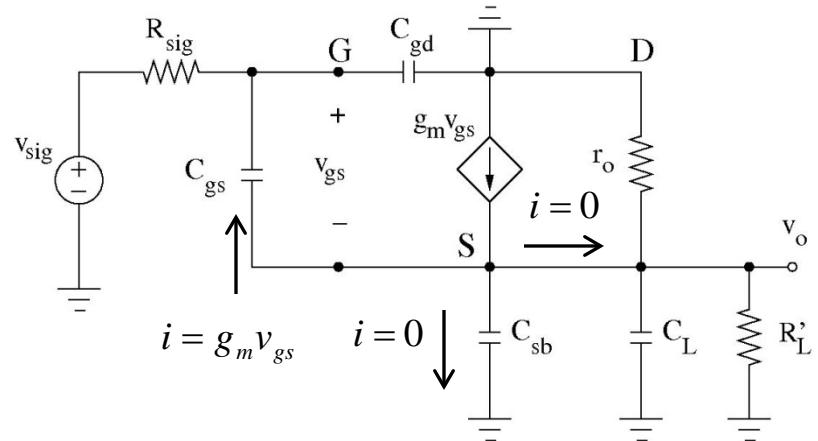
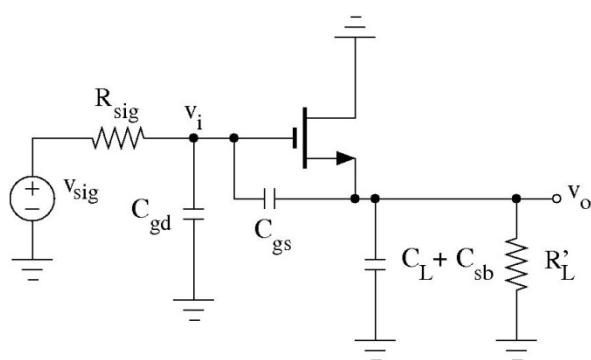
$$V_x [1 + g_m (R'_L \parallel r_o)] = I_x [R_{sig} + (R'_L \parallel r_o)]$$

$$R_{gs} = \frac{V_x}{I_x} = \frac{R_{sig} + (R'_L \parallel r_o)}{1 + g_m (R'_L \parallel r_o)}$$

$$\tau_3 = C_{gs} R_{gs}$$

$$\frac{1}{2\pi f_H} = b_1 = \tau_1 + \tau_2 + \tau_3 = C_{gd} R_{sig} + (C_L + C_{sb}) (1/g_m \parallel R'_L) + C_{gs} \times \frac{R_{sig} + (R'_L \parallel r_o)}{1 + g_m (R'_L \parallel r_o)}$$

Finding the “zero” of a source follower



- 1) Definition of Zero: $v_o(s = s_z) = 0$
- 2) Because $v_o = 0$, zero current will flow in r_o , C_L , C_{sb} and R'_L
- 3) By KCL, a current of $g_m v_{gs}$ will flow in C_{gs} .
- 4) Ohm's law for C_{gs} gives:

$$0 - v_{gs} = Z i = \frac{g_m v_{gs}}{s_z C_{gs}}$$

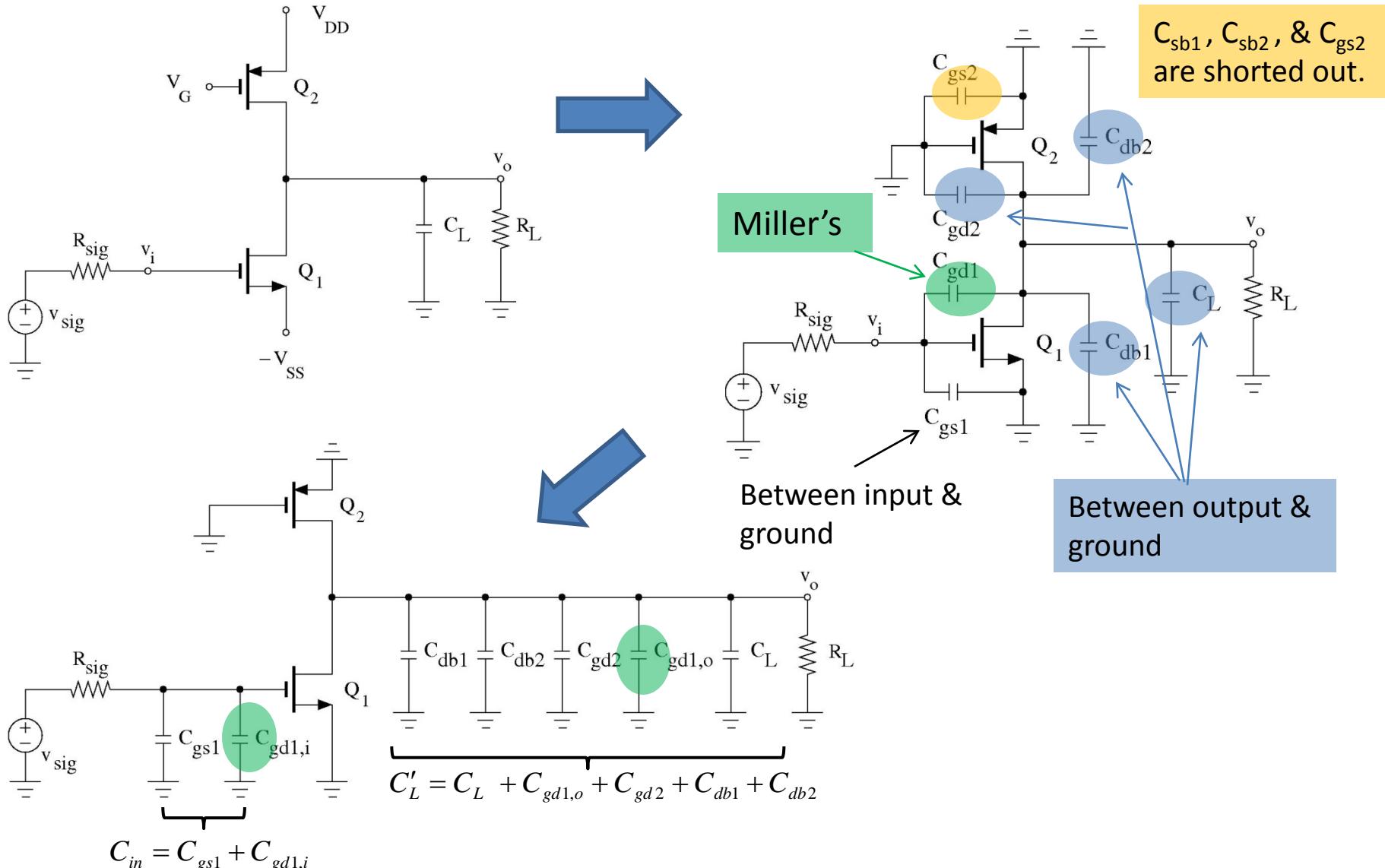
$$s_z = -\frac{g_m}{C_{gs}}, \quad f_z = \frac{g_m}{2\pi C_{gs}}$$

Examples of Computing high- f response of various amplifiers

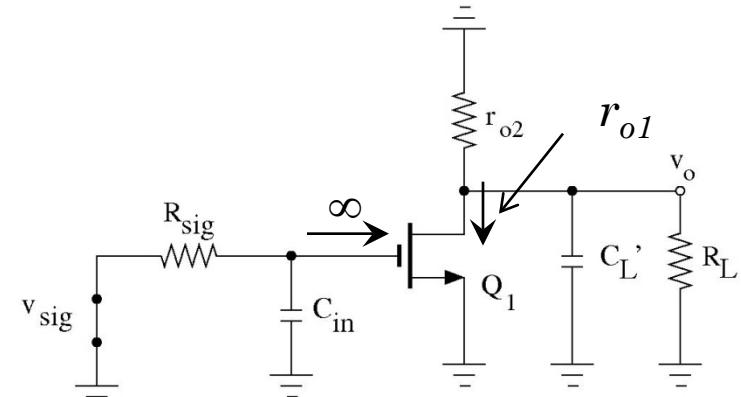
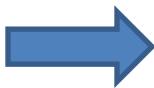
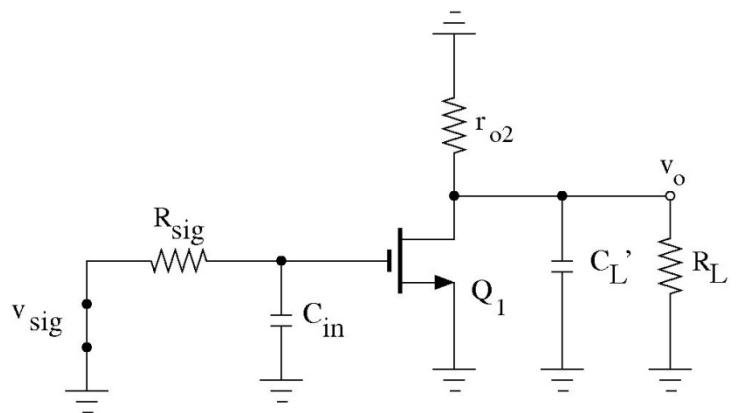
Procedure:

1. Include internal-capacitances of NMOS and simplify the circuit.
2. Use Miller's approximation for "Miller" capacitors in configurations with large (and negative) A.
3. Use time-constant method to find f_H
4. Do not forget about zeros in CS and CD configurations.

High-*f* response of a CS amplifier with current-source/active load



High- f response of a CS amplifier with current-source/active load



$$A = \frac{v_{d1}}{v_{g1}} = -g_m (r_{o1} \parallel r_{o2} \parallel R_L) \equiv -g_m R'_L$$

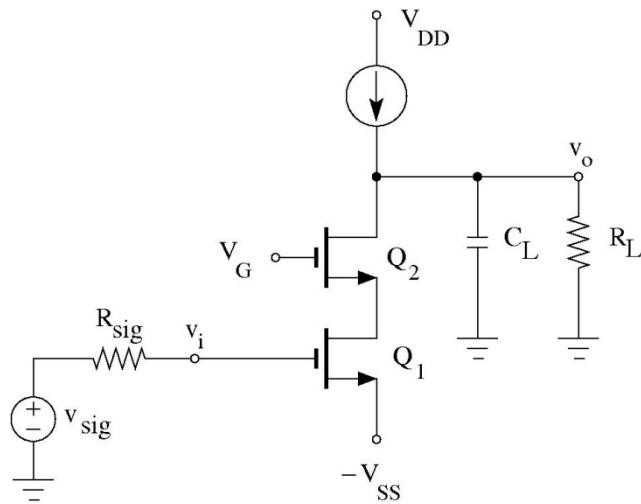
$$\begin{aligned} C_{in} : \quad R &= R_{sig} \parallel \infty = R_{sig} \\ \Rightarrow \quad \tau_1 &= C_{in} R_{sig} \end{aligned}$$

$$\begin{aligned} C'_L : \quad R &= r_{o1} \parallel r_{o2} \parallel R_L \\ \Rightarrow \quad \tau_2 &= C'_L (r_{o1} \parallel r_{o2} \parallel R_L) \end{aligned}$$

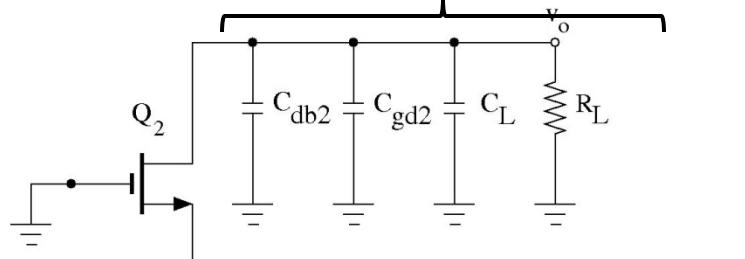
$$\frac{1}{2\pi f_H} = b_1 = C_{in} R_{sig} + C'_L (r_{o1} \parallel r_{o2} \parallel R_L)$$

$$s_z = \frac{g_m}{C_{gd}}, \quad f_z = \frac{g_m}{2\pi C_{gd}}$$

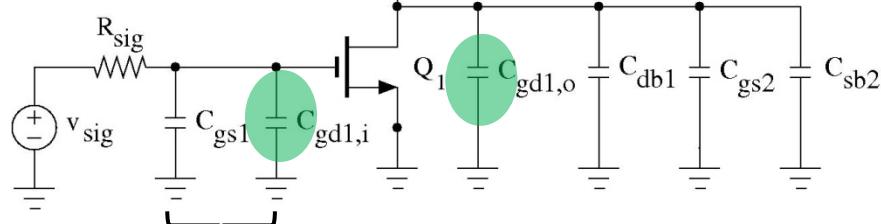
High- f response of Cascode amplifiers



$$C'_L = C_L + C_{gd2} + C_{db2}$$

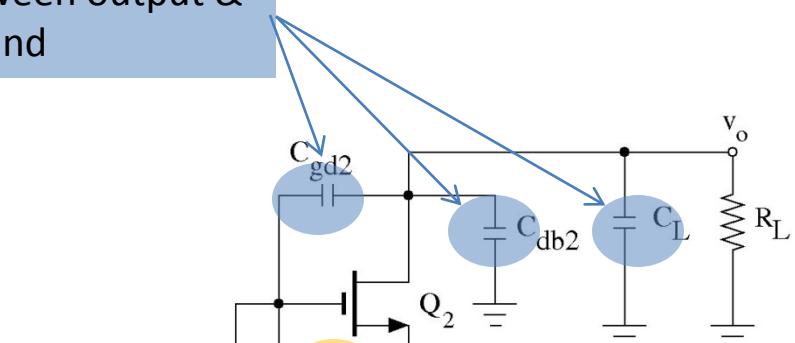


$$C_1 = C_{gd1,o} + C_{db1} + C_{gs2} + C_{sb2}$$

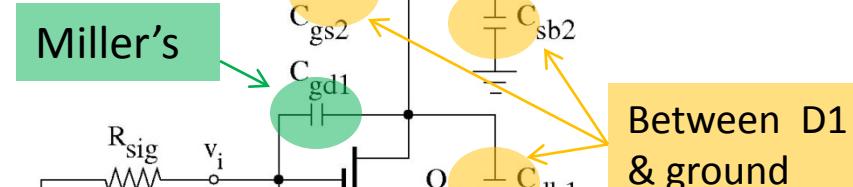


$$C_{in} = C_{gs1} + C_{gd1,i}$$

Between output & ground



Miller's



Between D1 & ground



Between input & ground

C_{sb1} is shorted out.

High- f response of cascode amplifiers

$$A_{v2} = g_{m2}(r_{o2} \parallel R_L) \quad \text{Mid-band Gains & resistances}$$

$$R_{o2} = r_{o1} + r_{o2} + g_{m2}r_{o1}r_{o2} \quad \& \quad R_{i2} = \frac{r_{o2} + R_L}{1 + g_{m2}r_{o2}} = R_{L1}$$

$$A_{v1} = -g_{m1}(r_{o1} \parallel R_{i2})$$

$$C_{gd1,i} = C_{gd1}(1 - A_{v1})$$

Miller's
Capacitors

$$C_{gd1,o} = C_{gd1}(1 - 1/A_{v1})$$

$$C'_L = C_L + C_{gd2} + C_{db2}$$

$$C_1 = C_{gd1,o} + C_{gs2} + C_{db1} + C_{sb2}$$

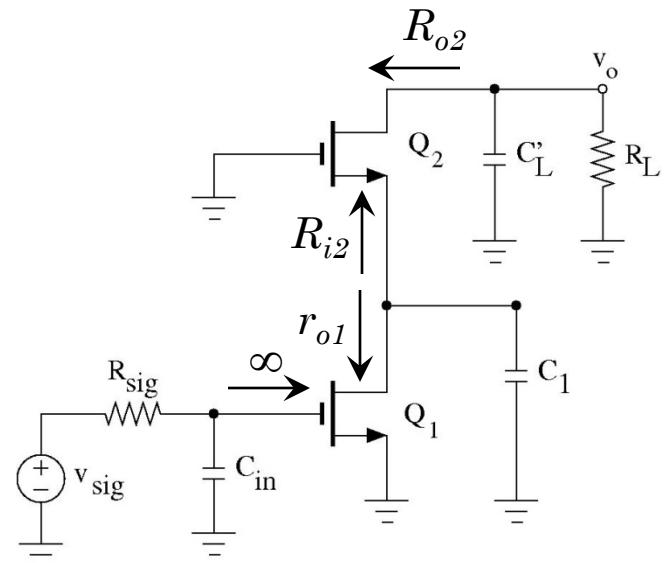
$$C_{in} = C_{gs1} + C_{gd1,i}$$

$$C_{in} : \quad R = R_{sig} \parallel \infty = R_{sig} \quad \Rightarrow \quad \tau_1 = C_{in} R_{sig}$$

$$C_1 : \quad R = r_{o1} \parallel R_{i2} \quad \Rightarrow \quad \tau_2 = C_1(r_{o1} \parallel R_{i2})$$

$$C'_L : \quad R = R_{o2} \parallel R_L \quad \Rightarrow \quad \tau_3 = C'_L(R_{o2} \parallel R_L)$$

$$\frac{1}{2\pi f_H} = b_1 = \tau_1 + \tau_2 + \tau_3$$



$$s_z = \frac{g_{m1}}{C_{gd1}}, \quad f_z = \frac{g_{m1}}{2\pi C_{gd1}}$$

High-*f* response of differential amplifiers

- For a symmetric differential amplifier (i.e. when we can use the half-circuit concept):

$$v_{o2} = -v_{o1} \rightarrow v_{o,d} = v_{o2,d} - v_{o1,d} = -2v_{o1,d}$$

$$A_d = \frac{v_{o,d}}{v_d} = -2 \times \frac{v_{o,1d}}{v_d} = \frac{v_{o,1d}}{-0.5v_d}$$

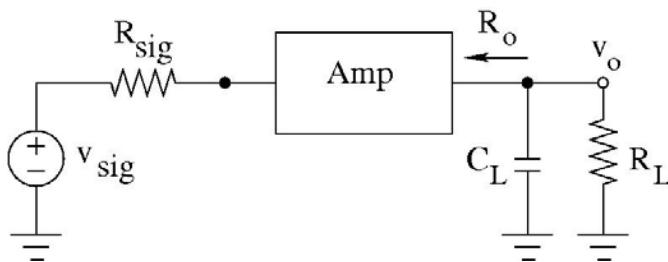
- Therefore, the poles and zero of the differential amplifier (differential gain) are the same as those of differential half circuit.
- Detailed analysis is necessary for asymmetric differential amplifier (e.g., IC differential amplifier with a single-ended output of slides 21-29 of Lecture set 7B),

Dominant Pole Compensation

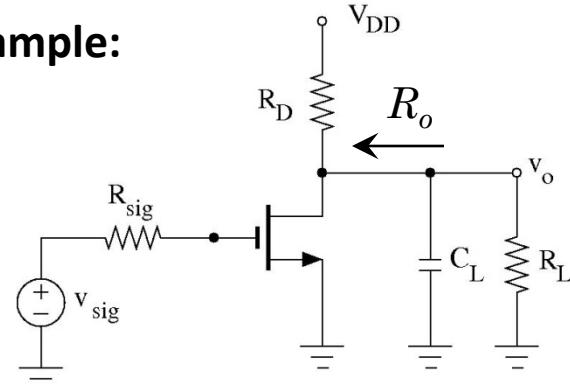
1. Very often we need to purposely introduce an additional “pole” in the circuit (in order to provide gain or phase margin in feedback amplifiers). This is called the dominant pole compensation.
2. This pole has to be the “dominant pole” (several octave below any zero or pole).
3. As such, we can ignore transistor internal capacitances in the analysis as poles introduced by these capacitances would be at higher frequencies and does not impact the dominant pole.
4. Dominant pole is introduce by the addition of a capacitor either
 - a) Between output & ground or
 - b) Between input & output of one stage (using Miller Effect)

Dominant Pole by adding C_L

Generic Form:



Example:

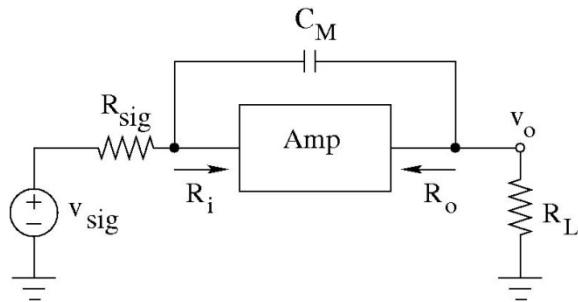


$$1/f_p = 2\pi C_L (R_L \parallel R_o)$$

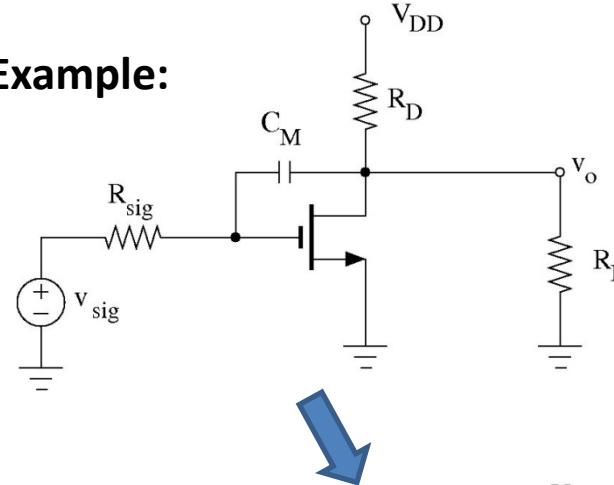
- Typically the required C_L is large and C_L is located outside the chip (i.e., between output terminal and ground).

Dominant Pole via Miller's effect

Generic Form:

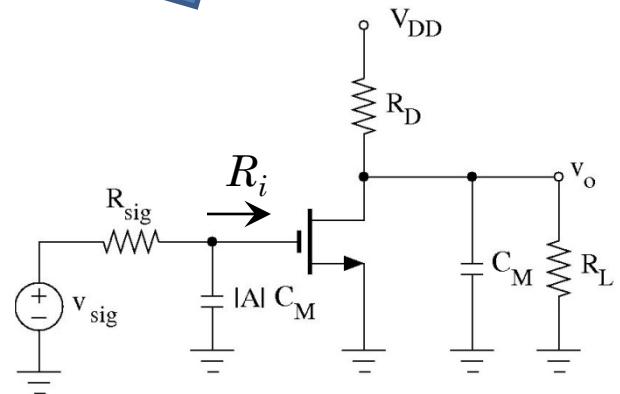


Example:



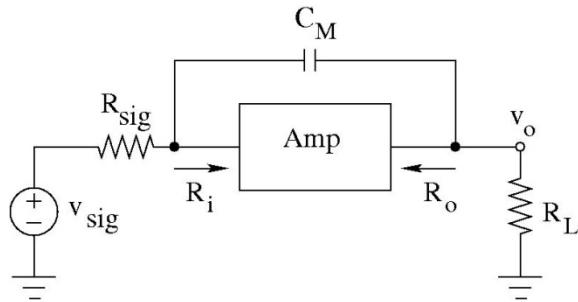
- Dominant pole is produced by $|A| C_M$ at the input (not C_M in the output). Otherwise we would have connected C_M in the output and not as a Miller capacitor!

$$1/f_p = 2\pi |A| C_M (R_{sig} \parallel R_i)$$



Dominant Pole via Miller's effect

Generic Form:



$$1/f_p = 2\pi |A| C_M (R_{sig} \parallel R_i)$$

Note: f_{p1} is proportional to R_{sig}

1. Usually not used in the first-stage as we do NOT know what R_{sig} is.
2. For second or latter stages, R_{sig} is R_o of previous stage and can be large. Because C_M appears as a large capacitor due to Miller's effect, this is the preferred method for including a capacitor inside the chip.
3. Note Miller's approximation does NOT give the correct value for poles. Simulation should be used to confirm the value of C_M . Also note that C_M introduces a zero.