Mathematical modeling in science, engineering, and economics

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1 Network of springs and dashpots

We consider a network of springs and dashpots which connect some masses. Define the following physical parameters:

$$\begin{aligned}
M_k &= \text{mass at node } k \\
S_{jk} &= \text{spring constant between node } j \text{ and node } k \\
D_{jk} &= \text{dashpot constant between node } j \text{ and node } k \\
R_{jk}^0 &= \text{resting link of spring between node } j \text{ and node } k
\end{aligned}$$

For a mass k, let $N(k)$ be the index set of masses which are connected to mass k. Let $\mathbf{X}_k$ and $\mathbf{U}_k$ be the position and velocity vectors of mass k respectively. The equations of motion for this network of masses connected by springs and dashpots are

$$M_k \frac{d\mathbf{U}_k}{dt} = \sum_{j \in N(k)} T_{jk} \frac{\mathbf{X}_j - \mathbf{X}_k}{\|\mathbf{X}_j - \mathbf{X}_k\|} \tag{1}$$

$$\frac{d\mathbf{X}_k}{dt} = \mathbf{U}_k \tag{2}$$

$$T_{jk} = S_{jk} \left(\|\mathbf{X}_j - \mathbf{X}_k\| - R_{jk}^0 \right) + D_{jk} \frac{d}{dt} \|\mathbf{X}_j - \mathbf{X}_k\| \tag{3}$$

Equation (1) is a statement of force balance. Notice the total force on mass k is the sum of forces which each point in the direction from mass j to k given by the unit vector $\frac{\mathbf{X}_j - \mathbf{X}_k}{\|\mathbf{X}_j - \mathbf{X}_k\|}$, with magnitude T_{jk}.