

Get in small groups (about 4 students maximum) and work out these problems on the whiteboard. Ask one of the teaching assistants for help if your group gets stuck. You do **not** need to turn anything in.

1. Classify the following things as either continuous or discrete. Can you explain why each fits into its respective category? **Note:** *some of these are open to interpretation - be creative! There is not just one correct answer. The point is to start talking to your classmates and discussing things in groups.*
  - (a) A crowd of people entering an amusement park
  - (b) A cloud
  - (c) A chocolate candy bar full of almonds
  - (d) Soup
2. There are 40 people in a workshop, and each must be assigned a unique identification number. You and two of your fellow workshop organizers, Megan and Murray, are typing up ID badges but some of your keyboards do not work quite right.
  - (a) All ten of your keys function properly. How many **digits** must be used in order to assign all 40 people a unique decimal ID number?
  - (b) On Megan's keyboard, only the 0 and the 1 function properly. How many **bits** must be used in order to assign all 40 people a unique binary ID number?
  - (c) On Murray's keyboard, only the 0, 1 and 2 keys function properly. How many **trits** (ternary bits) must be used in order to assign all 40 people a unique ternary ID number?
3. A game called "Nim" is played between two players as follows. Two piles of objects are set up, and each pile initially contains  $n$  objects. Player 1 starts by taking any number of objects from a single pile; they can take all of the objects from that pile, or only one object, but cannot take zero objects, and they can only take from one pile. Player 2 then takes any number of objects from a single pile of their choosing; the same rules apply. The game proceeds like this until someone takes the last object. Whoever takes the last object wins the game. Is there some strategy one of the players can follow so they always win?
4. On the Island of Knights and Knaves live two types of people: Knights who always tell the truth and Knaves who always lie. As you are exploring the Island of Knights and Knaves you encounter two people named Tony and Rachel. Consider the following two independent situations, and see if you can classify each of Tony and Rachel as either a knight or a knave.
  - (a) Rachel tells you that "Tony is a knave". Tony says "Neither Rachel nor I are knaves."
  - (b) Tony tells you "Both of us are knights." Rachel tells you "That is not true."

5. Here is a map of part of the United States:



Let's represent each of the states as a vertex in a graph, depicted as bold dots. Let's also represent whether or not two states are touching by connecting the vertices with edges in the graph, depicted as a line between connected vertices. Now in graph form, our map looks like this:



What is the minimum number of colors you need in order to color the map such that states that are touching are always different colors? What is the maximum number of colors you need?