Write **clearly** and **in the box**:

CSCI 2824
Midterm Exam 1
Spring 2019

Name:
Student ID:
Section number:

Read the following:

- **RIGHT NOW**! Write your name, student ID and section number on the top of your exam.
- You are allowed one 3×5-in index card of notes (both sides). No magnifying glasses!
- You may use a calculator provided that it cannot access the internet or store large amounts of data.
- You may **NOT** use a smartphone as a calculator.
- Clearly mark answers to multiple choice questions in the provided answer box.
- Mark only one answer for multiple choice questions. If you think two answers are correct, mark the answer that **best** answers the question. No justification is required for multiple choice questions.
- If you do not know the answer to a question, skip it and come back to it later.
- For free response questions you must clearly justify all conclusions to receive full credit. A correct answer with no supporting work will receive no credit.
- If you need more space for free-response questions, there are blank pages at the end of the exam. If you choose to use the extra pages, make sure to **clearly** indicate which problem you are continuing.
- You have **90 minutes** for this exam.

Page	Points	Score
3	9	
4	12	
5	9	
6	10	
7	20	
9	20	
11	20	
Total	100	

.

Multiple choice problems: Write your answers in the boxes, or they will not be graded!

1. (3 points) Convert $(11011)_2$ in binary to a decimal number.
   A. 14

   B. 28

   C. 27

   D. 46

   E. 47

C

2. (3 points) Decide whether the following argument is valid or invalid and choose the appropriate answer below.
   If the moon is made of cheese, then 8 is an irrational number. The moon is made of cheese. Therefore, 8 is an irrational number.

   A. This argument is **valid**; Modus Ponens.

   B. This argument is **valid**; Modus Tollens.

   C. This argument is **valid**; Disjunctive Syllogism.

   D. This argument is **valid**; Hypothetical Syllogism.

   E. This argument is **invalid**; Fallacy of Denying the Hypothesis.

   F. This argument is **invalid**; Fallacy of Affirming the Conclusion.

A

3. (3 points) Decide whether the following argument is valid or invalid and choose the appropriate answer below.
   If the moon is full, then the coyotes will howl. If the coyotes howl, then the baby will cry. Therefore, if the moon is full, then the baby will cry.

   A. This argument is **valid**; Modus Ponens.

   B. This argument is **valid**; Modus Tollens.

   C. This argument is **valid**; Disjunctive Syllogism.

   D. This argument is **valid**; Hypothetical Syllogism.

   E. This argument is **invalid**; Fallacy of Denying the Hypothesis.

   F. This argument is **invalid**; Fallacy of Affirming the Conclusion.

D

Multiple choice problems: Write your answers in the boxes, or they will not be graded!

4. (3 points) Decide whether the following argument is valid or invalid and choose the appropriate answer below.
   Murray doesn't wear his snow boots. If it doesn't snow, then Murray won't wear his snow boots. Therefore, it doesn't snow.

   A. This argument is **valid**; Modus Ponens.

   B. This argument is **valid**; Modus Tollens.

   C. This argument is **valid**; Disjunctive Syllogism.

   D. This argument is **valid**; Hypothetical Syllogism.

   E. This argument is **invalid**; Fallacy of Denying the Hypothesis.

   F. This argument is **invalid**; Fallacy of Affirming the Conclusion.

   F

5. (3 points) Choose the logical equivalent of $\neg\forall\epsilon\exists\delta((f(x) \neq \epsilon) \wedge (x \leq \delta))$

   A. $\forall\epsilon\forall\delta((f(x) \neq \epsilon) \wedge (x > \delta))$

   B. $\exists\epsilon\forall\delta((x \leq \delta) \implies (f(x) = \epsilon))$

   C. $\exists\epsilon\neg\exists\delta((f(x) = \epsilon) \vee (x \leq \delta))$

   D. $\exists\epsilon\neg\exists\delta((f(x) \neq \epsilon) \vee (x > \delta))$

   E. $\forall\epsilon\exists\delta((f(x) = \epsilon) \vee (x > \delta))$

   B

6. (3 points) Which of the following compound propositions is **not** satisfiable?

   A. $(p \wedge q) \wedge (\neg p \vee q) \wedge (q \to p)$

   B. $(\neg p \wedge q) \wedge (p \vee q) \wedge (p \to q)$

   C. $(\neg p \wedge q) \wedge (p \vee q) \wedge (p \to \neg q)$

   D. $(p \wedge q) \wedge (\neg p \vee q) \wedge (p \to \neg q)$

   D

7. (3 points) When E.T. phones home, he talks to his family in his native language. His native language has an alphabet with 41 letters. We would like to use n bits to uniquely encode any given single letter of this alphabet. What is the minimum number of bits required to do this?

   A. 3

   B. 4

   C. 5

   D. 6

   E. 7

   D

Multiple choice problems: Write your answers in the boxes, or they will not be graded!

8. (3 points) Which of the following compound propositions is logically equivalent to $p \wedge (q \to r)$
   A. $(p \wedge q) \to (p \wedge r)$
   B. $(p \wedge \neg q) \vee \neg(p \wedge r)$
   C. $\neg((p \to q) \wedge (p \to \neg r))$
   D. $(p \to q) \wedge (p \to r)$

$\boxed{\text{C}}$

9. (3 points) Select the English translation that best matches the following quantifier statement.

$$\forall x \exists y L(x, y)$$

   Let $L(x, y)$ denote " x likes y", and let the domain for x be all people and the domain for y to be all things.
   A. Not everyone likes everything.
   B. There is something that everyone likes.
   C. Someone likes everything.
   D. Everyone likes something.

$\boxed{\text{D}}$

10. (3 points) Assume that x is a real number. Consider the equation $x^3 + 1 = 0$. What can you say about the solution to this equation?
   A. A solution exists and is unique.
   B. A solution exists but is not unique.
   C. A solution does not exist.
   D. Unable to be determined.

$\boxed{\text{A}}$

Short-answer problems: If your answers do not fit in the given box, MAKE A NOTE of where the work is continued or it will NOT be graded!

11. (4 points) Convert the decimal number 305 to binary.

Solution: You may use the even/odd algorithm that we discussed in class, or:

$$305 = 256 + 32 + 16 + 1$$
$$= 1 \cdot 2^8 + 0 \cdot 2^7 + 0 \cdot 2^6 + 1 \cdot 2^5 + 1 \cdot 2^4 + 0 \cdot 2^3 + 0 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0$$

Thus

$$305_{10} = (100110001)_2$$

12. (6 points) Given the following claim: "If $6 \geq n - 3$, then $n > 4$ or $n > 10$.", write out the starting assumption(s) for the following types of proof:

(i) a direct proof

(ii) a contrapositive proof

(iii) a proof by contradiction

Note: You are not trying to prove or disprove this claim. You are **only** specifying what the starting assumptions would be **IF** you were going to prove it.

Solution:

(i) Assumption: $6 \geq n - 3$
(ii) Assumption: $n \leq 4$ and $n \leq 10$
(iii) Assumption: $6 \geq n - 3$, $n \leq 4$, and $n \leq 10$

13. (20 points) Prove the following using the permitted logical equivalences and rules of inference. Be cure to cite which rules or equivalences you use on each line of your proofs, and **use only one rule/equivalence per line**. Recall that there is only a specific set of equivalences and rules of inference that you are permitted to use. You may **not** use truth tables.

(a) Suppose p, q, and r are all propositions. Prove that the following logical equivalence is a tautology.

$$(p \land q) \to (p \lor q)$$

(b) Suppose we have propositional functions $P(x)$, $Q(x)$, and $R(x)$, for variable x defined on some domain. Prove that the following argument is valid. Be sure to cite which rules of inference or logical equivalences you use on each line of your proof, indicate which line numbers are involved in each step, and use only one rule/equivalence per line.

$$\begin{array}{rl} \text{Premise:} & \forall x(P(x) \to Q(x)) \\ \text{Premise:} & \exists x(\neg Q(x) \lor R(x)) \\ \text{Premise:} & \forall x(\neg Q(x)) \\ \hline \text{Conclusion:} & \exists x(\neg P(x) \land (P(x) \to R(x))) \end{array}$$

Solution:

(a)

$$\begin{array}{rll} (p \land q) \to (p \lor q) & \equiv \neg(p \land q) \lor (p \lor q) & \text{RBI} \\ & \equiv (\neg p \lor \neg q) \lor (p \lor q) & \text{De Morgan's} \\ & \equiv \neg p \lor \neg q \lor p \lor q & \text{Associativity} \\ & \equiv \neg p \lor p \lor \neg q \lor q & \text{Commutativity} \\ & \equiv (\neg p \lor p) \lor (\neg q \lor q) & \text{Associativity} \\ & \equiv \mathbb{T} \lor \mathbb{T} & \text{Negation Law} \\ & \equiv \mathbb{T} & \end{array}$$

(b) Note that some of these steps may be done in different orders, with a still-valid resulting proof.

1. $\forall x(P(x) \to Q(x))$	Premise
2. $\exists x(\neg Q(x) \lor R(x))$	Premise
3. $\forall x(\neg Q(x))$	Premise
4. $\neg Q(c) \lor R(c)$	existential instantiation (2) (some specific c in the domain)
5. $P(c) \to Q(c)$	universal instantiation (1)
6. $\neg Q(c)$	universal instantiation (3)
7. $\neg Q(c) \to \neg P(c)$	contrapositive of (5)
8 $\neg P(c)$	modus ponens (6), (7)
9. $Q(c) \to R(c)$	RBI (4)
10. $P(c) \to R(c)$	hypothetical syllogism (5), (9)
11. $\neg P(c) \land (P(c) \to Q(c))$	Conjunction (8), (10)
12. $\exists x(\neg P(x) \land (P(x) \to R(x)))$	existential generalization (11)

14. (20 points) On the Island of Knights and Knaves live two types of people: Knights who always tell the truth and Knaves who always lie. As you are exploring the Island, you encounter two people named A and B. A tells you "If I am a knight, then B is a knight." B tells you "I am a knave or A is a knight."

(a) Define propositional variables and translate the statements of A and B into symbolic logic.

(b) Use a **truth table** to determine the natures of A and B. Clearly state and justify your conclusions.

Solution:

(a) Define the propositions: p: A is a knight. and q: B is a knight.

Then their statements can be translated as:

A: $p \to q$

B: $\neg q \vee p$

(b) A's statement is true if and only if he is a knight; same goes for B. So we must test the satisfiability of the compound proposition:

$$(p \iff (p \to q)) \wedge (q \iff (\neg q \vee p))$$

p	q	$\neg q$	$p \to q$	$\neg q \vee p$	$p \iff (p \to q)$	$q \iff (\neg q \vee p)$	$(p \iff (p \to q)) \wedge (q \iff (\neg q \vee p))$
T	T	F	T	T	T	T	T
T	F	T	F	T	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	T	F	F	F

From the truth table above, we see that the only possible outcome occurs in the first row. This corresponds to both p and q being true. **Thus, both A and B are knights.**

15. (20 points) Prove or disprove the following claims. Be sure to indicate which type(s) of proof you are using (i.e. Direct Proof, Proof by Contrapositive, Proof by Contradiction, Proof by Cases, Proof of Existence/Uniqueness, Counterexample).

   (a) Prove that if a divides b and a divides c, then a divides $b - c$.

   (b) Prove that if $(n+1) \times (m-1)$ is even, then at least one of n or m is odd.

Solution:

(a)

Proof. We will use a direct proof. Suppose a divide b and a divides c. Then:

$$b = ak, \quad \text{for some integer } \ k$$

$$c = am, \quad \text{for some integer } \ m$$

Therefore,

$$b - c = ak - am = a(k - m)$$

Since $n - m$ is an integer, this proves that a divides $b - c$.

□

(b)

Proof. For a proof by contrapositive, assume that neither n nor m is odd. Thus, both n and m must be even. Let $n = 2k$ and $m = 2l$ for some integers k and l. Then;

$$\begin{aligned}(n+1) \times (m-1) &= (2k+1) \times (2l-1) \\ &= 4kl + 2l - 2k - 1 \\ &= 2(2kl + l - k) - 1 \\ &= 2s - 1\end{aligned}$$

where $s = 2kl + l - k$ is some integer. We've found that $(n+1) \times (m-1)$ is odd when n and m are even. Thus by contraposition, the claim is proven.

□

Alternative proof by contradiction:

Proof. Assume that $(n+1) \times (m-1)$ is even and that both n and m are even for a contradiction. Let $n = 2k$ and $m = 2l$. Then;

$$\begin{aligned}(n+1) \times (m-1) &= (2k+1) \times (2l-1) \\ &= 4kl + 2l - 2k - 1 \\ &= 2(2kl + l - k) - 1 \\ &= 2s - 1\end{aligned}$$

where $s = 2kl + l - k$ is some integer. We've found that $(n+1) \times (m-1)$ is odd which contradicts our assumption that the product is even. Thus, it must be that if $n+1) \times (m-1)$ is even, then at least one of n or m is odd.

□