

Get in small groups (about 4 students maximum) and work out these problems on the whiteboard. Ask one of the teaching assistants for help if your group gets stuck. You do **not** need to turn anything in.

1. For the following problems, use counterexamples to disprove each statement.
  - (a) If  $x, y \in R$ , then  $|x + y| = |x| + |y|$
  - (b) If  $x$  is a real number, then  $2^x \geq x + 1$
  
2. Prove the following statements.
  - (a) Use a proof by contradiction to show that there is no rational number  $r$  for which  $r^3 + r + 1 = 0$ .
  - (b) Use a direct proof to show that if  $a$  is an odd integer, then  $a^2 + 3a + 5$  is odd.
  - (c) Given an integer  $a$ , then  $a^2 + 4a + 5$  is odd if and only if  $a$  is even.
  
3. Show that if  $n$  is an integer and  $n^3 + 5$  is odd, then  $n$  is even using
  - (a) a proof by contraposition.
  - (b) a proof by contradiction.
  
4. Use rules of inference to show that the hypotheses below imply the following conclusion
  - (a) “Randy works hard,” “If Randy works hard, then he is a dull boy,” and “If Randy is a dull boy, then he will not get the job” imply the conclusion “Randy will not get the job.”
  - (b) “If it does not rain or if it is not foggy, then the sailing race will be held and the lifesaving demonstration will go on,” “If the sailing race is held, then the trophy will be awarded,” and “The trophy was not awarded” imply the conclusion “It rained.”
  
5. Show that the argument form with premises  $(p \wedge t) \rightarrow (r \vee s)$ ,  $q \rightarrow (u \wedge t)$ ,  $u \rightarrow p$  and  $\neg s$  and conclusion  $q \rightarrow r$  is valid by using rules of inference.
  
6. Determine whether these are valid arguments.
  - (a) If  $x$  is a positive real number, then  $x^2$  is a positive real number. Therefore, if  $a^2$  is positive, where  $a$  is a real number, then  $a$  is a positive real number.
  - (b) If  $x^2 \neq 0$ , where  $x$  is a real number, then  $x \neq 0$ . Let  $a$  be a real number with  $a^2 \neq 0$ ; then  $a \neq 0$ .
  
7. Justify the rule of universal modus tollens by showing that the premises  $\forall x(P(x) \rightarrow Q(x))$  and  $\forall(\neg Q(a))$  for a particular element  $a$  in the domain, imply  $\neg P(a)$ .
  
8. Determine whether this argument is valid -
 

If Batman were able and willing to prevent evil, he would do so. If Batman were unable to prevent evil, he would be impotent; if he were unwilling to prevent evil, he would be malevolent. Batman does not prevent evil. If Batman exists, he is neither impotent nor malevolent. Therefore, Batman does not exist.