CSCI 2830-800: Discrete Structures Workgroup – Fall 2018 **Worksheet 05**

Get in small groups (about 4 students maximum) and work out these problems on the whiteboard. Ask one of the teaching assistants for help if your group gets stuck. You do **not** need to turn anything in. Since this worksheet contains a lot of problems, a good strategy would be to first skim the worksheet and then discuss and solve the problems which you think are difficult.

1. Prove or disprove each claim:
   (a) Every positive integer can be expressed as the sum of two perfect squares. (A perfect square is the square of an integer. 0 may be used in the sum.)
   (b) For all rational numbers a and b, a^b is also rational.

2. Express the negations of each of these statements so that all negation symbols immediately precede predicates
   (a) $\neg\forall x\forall y P(x, y)$
   (b) $\neg\forall y\exists x P(x, y)$
   (c) $\neg\forall y\forall x(P(x, y) \lor Q(x, y)$
   (d) $\neg(\exists x\exists y\neg P(x, y) \land \neg\forall x\forall y Q(x, y))$
   (e) $\neg\forall x(\exists y\forall z P(x, y, z) \land \exists z\forall y P(x, y, z))$

3. Determine the truth value of each of these statements if the domain for n consists of all integers.
   (a) $\forall n\ (n + 1 > n)$
   (b) $\exists n\ (n = -n)$
   (c) $\exists n\ (2n = 3n)$
   (d) $\forall n\ (3n \leq 4n)$

4. Determine a truth value for these quantifier statements where the domain for all variables consists of all integers. If they are false, then find a counterexample.
   (a) $\forall x\ (x^2 \geq x)$
   (b) $\forall x\ (x > 0 \lor x < 0)$
   (c) $\forall x\ (x = 1)$

5. Show that $\exists x\ P(x) \land \exists x\ Q(x)$ and $\exists x(P(x) \land Q(x))$ are not logically equivalent.

6. On the Island of Knights and Knaves live two types of people: Knights who always tell the truth and Knaves who always lie. Consider the following situations, and see if you can classify each of the inhabitants as either a knight or a knave **using truth tables**.
   (a) Nihar says "Tucker is a knave. Tucker says "Nihar and I are knights.
   (b) Tony says "Rachel and I are not the same. Rachel says "Of Tony and I, exactly one is a knight.
   (c) Rachel says "Either Tucker is a knight or I am a knight. Tucker says that Rachel is a knave.
   (d) Tony says "I am a knight or Nihar is a knave. Nihar says "Of Tony and I, exactly one is a knight.

7. Use truth tables to show that these are logically equivalent and name the associated logical equivalence(s):

(a) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

(b) $\neg(p \wedge q \wedge r) \equiv (\neg p) \vee (\neg q) \vee (\neg r)$

(c) $p \to q \equiv \neg p \vee q$

8. Decide whether the following pairs of statements are equivalent or not and name the logical equivalence(s) used or misused:

   (a) $p \vee (q \wedge r)$ and $(p \vee q) \wedge r$

   (b) $(p \to q) \vee r$ and $\neg((p \wedge \neg q) \wedge \neg r)$

9. Prove that 4 does not divide $n^2 - 3$ where $n \in \mathbb{Z}$.

10. **Valid vs. Sound arguments**

    A reminder of the definitions of **sound** and **valid** arguments:

    - A valid argument is one in which ***if*** the premises are true, then the conclusion must also be true.
    - A sound argument is one that is valid, ***and*** the premises are indeed true.
    - So, a sound argument is always valid, but a valid argument need not be sound.

    Suppose we have the following premises and conclusion:

Premise 1: If a number is even, then it is prime.

Premise 2: If a number is a multiple of 6, then it is even.

Conclusion: If a number is a multiple of 6, then it is prime.

The argument is valid, because there is a coherent structure to the argument. In fact, this is one of our rules of inference, which we know to be valid "mini-arguments" (which rule of inference is it?). ***If*** Premises 1 and 2 are true, then based on these the Conclusion must be true.

The argument, however, is **not** sound, because Premise 1 is false. Here, saying that a number is prime if it is a multiple of 6 is clearly false.

Come up with your own argument that is **valid** but is **not sound**.