

Get in small groups (about 4 students maximum) and work out these problems on the whiteboard. Ask one of the teaching assistants for help if your group gets stuck. You do **not** need to turn anything in.

1. Define the following terms in English. Also specify whether the conditions are satisfiable or unsatisfiable.
  - (a) tautology
  - (b) contradiction
  - (c) contingency
2. Translate these English sentences into 2 unique propositions. Additionally, use truth tables to show that the 2 propositions you made are logically equivalent.
  - (a) Tony and Rachel like chocolate, but neither Tucker nor Nihar do.
  - (b) If it is raining, then Nihar only brings an umbrella when he doesn't wear a jacket.
3. Reduce the following expressions to their simplest form. Also mention whether the following statements are a tautology, contradiction or contingency
  - (a)  $p \wedge (p \vee \neg p)$
  - (b)  $p \vee \neg(p \wedge \neg p)$
  - (c)  $(p \vee (q \wedge \neg q)) \wedge (q \wedge (p \vee \neg p))$
4. Determine whether each of these compound propositions is satisfiable - *Hint*: First reduce the propositions into a simple expression and then use a truth table.
  - (a)  $(p \vee \neg q) \wedge (\neg p \vee q) \wedge (\neg p \vee \neg q)$
  - (b)  $(p \rightarrow q) \wedge (p \rightarrow \neg q) \wedge (\neg p \rightarrow q) \wedge (\neg p \rightarrow \neg q)$
  - (c)  $(p \leftrightarrow q) \wedge (\neg p \leftrightarrow q)$
5. Prove the following using a proof by contraposition:
  - (a) Let  $x \in \mathbb{Z}$ ; If  $x^2 - 6x + 5$  is even, then  $x$  is odd.
  - (b) If a product of two positive real numbers is greater than 100, then at least one of the numbers is greater than 10.