This assignment is due on Friday, February 1 to Gradescope by Noon. You are expected to write up your solutions neatly and **use the coverpage**. Remember that you are encouraged to discuss problems with your classmates, but you must work and write your solutions on your own.

Important: On the **official CSCI 2824 cover page** of your assignment clearly write your full name, the lecture section you belong to (001 or 002), and your student ID number. You may **neatly** type your solutions for +1 extra credit on the assignment. You will lose all 5 style/neatness points if you fail to use the official cover page.

1. You decided to go on a spring break vacation! Your destination: the island of Knights & Knaves. On this island, there are only two types of native inhabitants; Knights, who always tell the truth, and Knaves, who always lie. As you are finding a nice spot on the beach to set up a picnic, you are approached by 3 of the native inhabitants. We'll call them Aramis, Bertrand, and Charleston. Aramis says, " Bertrand is a knave." Bertrand says, " Charleston is a knave or I am a knight, but not both." Charleston says, "Aramis is a knight and Bertrand is a knave." Determine the nature of each of these three inhabitants. Justify your answer with a truth table; showing all rows.

   Solution: To determine the nature of the three inhabitants, we must analyze the statements made by each of these people. We know that each inhabitant must be either lying or telling the truth. Thus, we have the following possibilities.

   Truth table:
   Define p: Aramis is a knight., q: Bertrand is a knight., r: Charleston is a knight.

p	q	r	$\neg q$	$\neg r$	$\neg r \oplus q$	$p \land \neg q$	$p \Leftrightarrow \neg q$	$q \Leftrightarrow \neg r \oplus q$	$r \Leftrightarrow p \land \neg q$
T	T	T	F	F	T	F	F	T	F
T	T	F	F	T	F	F	F	F	T
T	F	T	T	F	F	T	T	T	T
T	F	F	T	T	T	T	T	F	F
F	T	T	F	F	T	F	T	T	F
F	T	F	F	T	F	F	T	F	T
F	F	T	T	F	F	F	F	T	F
F	F	F	T	T	T	F	F	F	T

   Using row 3, we see that Aramis and Charleston are knights, and Bertrand is a knave.

   Another way to reason: Assume that Aramis is telling the truth, which means that he is a knight. This means that Bertrand is a knave, which means his statement is false. Bertrand's statement can be interpreted using an XOR gate - "Charleston is a knave XOR Bertrand is a knight". Let A(x) = "x is a knave" and B(x) = "x is a knight". Thus, Bertrand's statement is false if either A(Charleston) and B(Bertrand) are both true or if neither of the predicates are true. If both are true, then Bertrand must be a knight which is a contradiction. If neither of the predicates are true, then Charleston is a knight and his statement is true, which is in agreement with the rest of the assumptions.

   Assume that Aramis is lying, which means that he is a knave. This means that Bertrand is a knight, which means that Charleston is also a knight. However, Charleston claims that Aramis is a knight which presents a contradiction.

   Thus, the only possible solution is that Aramis is a knight, Bertrand is a knave and Charleston is a knight.

2. (a) Show that $((p \to q) \wedge (q \to r)) \to (p \to r)$ is a tautology using **both** (i) a truth table and (ii) a chain of logical equivalences. Note that you may only use logical equivalences from Table 6 (p. 27 of the Rosen textbook) and the other four starred equivalences given in lecture. At each step you should cite the name of the equivalence rule you are using, and please only use one rule per step. Is this compound proposition satisfiable? Why or why not?

(b) Show that $(p \to q) \to r$ and $p \to (q \to r)$ are not logically equivalent.

Solution:

(a) $((p \to q) \wedge (q \to r)) \to (p \to r)$

i. Truth table

p	q	r	$p \to q$	$q \to r$	$(p \to q) \wedge (q \to r)$	$p \to r$	$((p \to q) \wedge (q \to r)) \to (p \to r)$
F	F	F	T	T	T	T	T
F	F	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	T	T	T	T	T	T	T
T	F	F	F	T	F	F	T
T	F	T	F	T	F	T	T
T	T	F	T	F	F	F	T
T	T	T	T	T	T	T	T

ii. Using chain of logical equivalences

$$
\begin{array}{rcll}
((p \to q) \wedge (q \to r)) \to (p \to r) & \equiv & \neg((\neg p \vee q) \wedge (\neg q \vee r)) \wedge (\neg p \vee r) & \\
& \equiv & ((p \wedge \neg q) \vee (q \wedge \neg r)) \vee (\neg p \vee r) & \text{DeMorgan's Law} \\
& \equiv & (p \wedge \neg q) \vee (\neg p \vee r) \vee (q \wedge \neg r) & \text{Associativity} \\
& \equiv & (((\neg p \vee r) \vee p) \wedge ((\neg p \vee r) \wedge \neg q)) \vee (q \wedge \neg r) & \text{Distributive} \\
& \equiv & (((p \vee \neg p) \vee r) \wedge ((\neg p \vee r) \vee \neg q)) \wedge (q \vee \neg r) & \text{Associative} \\
& \equiv & ((T \vee r) \wedge ((\neg p \vee r) \vee \neg q)) \wedge (q \vee \neg r) & \text{Negation} \\
& \equiv & (T \wedge ((\neg p \vee r) \vee \neg q)) \wedge (q \vee \neg r) & \text{Domination} \\
& \equiv & ((\neg p \vee r) \vee \neg q) \wedge (q \vee \neg r) & \text{Identity} \\
& \equiv & (q \vee ((\neg p \vee r) \vee \neg q) \wedge \neg r \vee ((\neg p \vee r) \vee \neg q) & \text{Distributive} \\
& \equiv & ((q \vee \neg q) \vee (\neg p \vee r)) \wedge (\neg r \vee r \vee \neg p \vee \neg q) & \text{Associative} \\
& \equiv & (T \vee (\neg p \vee r)) \wedge (T \vee \neg p \vee \neg q) & \text{Negation} \\
& \equiv & T \wedge T & \text{Domination} \\
& \equiv & T & \text{Identity}
\end{array}
$$

Thus the original compound preposition is logically equivalent to True, making it a tautology (it is true, no matter what the truth values of p,q and r)

The compound preposition is satisfiable because every tautology is satisfiable.

(b) $(p \to q) \to r$ and $p \to (q \to r)$

p	q	r	$p \to q$	$q \to r$	$(p \to q) \to r$	$p \to (q \to r)$
F	F	F	T	T	F	T
F	F	T	T	T	T	T
F	T	F	T	F	F	T
F	T	T	T	T	T	T
T	F	F	F	T	T	T
T	F	T	F	T	T	T
T	T	F	T	F	F	F
T	T	T	T	T	T	T

When p=F and r=F, then the truth values of $(p \to q) \to r$ and $p \to (q \to r)$ does not match. Hence, they are not logically equivalent.

3. Suppose that the domain of the propositional function $P(x)$ consists of the integers 5, 6, 7, and 8. Express the following statements without using quantifiers, instead using negations, disjunctions, and conjunctions. [e.g. $\exists x P(x)$ would be $P(5) \vee P(6) \vee P(7) \vee P(8)$]

   (a) $\forall x P(x)$

   (b) $\neg \exists x P(x)$

   (c) $\neg \forall x P(x)$

   Solution:

   (a) $P(5) \wedge P(6) \wedge P(7) \wedge P(8)$

   (b) $\neg P(5) \wedge \neg P(6) \wedge \neg P(7) \wedge \neg P(8)$

   (c) $\neg P(5) \vee \neg P(6) \vee \neg P(7) \vee \neg P(8)$

4. We spent time in lecture talking about how to convert base-10 numbers to binary. Use the same principles to convert 163 to base-3. Make sure to show all of your steps.

   Solution:

$$\begin{aligned} 163 &= 162 + 1 \\ &= 2 \cdot 81 + 1 \\ &= 2 \cdot 3^4 + 3^0 \\ &= 2 \cdot 3^4 + 0 \cdot 3^3 + 0 \cdot 3^2 + 0 \cdot 3^1 + 1 \cdot 3^0 \end{aligned}$$

   Therefore, $(163)_{10} = (20001)_3$

   An algorithmic way:

   (a) Here, n = 163, which is the decimal number.

   (b) Let m be the number, initially empty, that we are converting to. We'll be composing it right to left.

   (c) 3 is the base of the number we are converting to.

   (d) Repeat until n becomes 0

      i. Divide n by 3, letting the result be d and the remainder be r.

      ii. Write the remainder, r, as the leftmost digit of m.

      iii. Let d be the new value of n.

   (a) 163 when divided by 3 leaves d = 54 and r = 1. n becomes 54 and m = 1

   (b) 54 when divided by 3 leaves d = 18 and r = 0. n becomes 18 and m = 01. Note that here the remainder was added as the leftmost digit of m

   (c) 18 when divided by 3 leaves d = 6 and r = 0. n becomes 6 and m = 001

   (d) 6 when divided by 3 leaves d = 2 and r = 0. n becomes 2 and m = 0001

   (e) 2 when divided by 3 leaves d = 0 and r = 2. n becomes 0 and m = 20001. Since n has reached 0, we stop the computation.

5. Consider the following **satisfiability** problem: The Scooby Doo gang: Fred, Daphne, Shaggy, Velma, and Scooby are going on vacation. However, before they can book their travel, they need to all agree on where to go. Their trip may involve one or more destinations. They must all travel together to all of the places as one group (so part of the group cannot go to one location while the others go somewhere else).

   i Shaggy wants to go to Venice, or not to Shanghai.

   ii If the gang goes to Paris, then Velma does not want to go to Venice.

   iii Daphne wants to go to Brussels if and only if the gang also goes to London and Paris.

   iv Fred does not want to go to Paris.

   v Scooby just wants to leave the house and does not care where the gang goes.

   Let $T(x)$ represent the propositional function "the trip must include destination x", where the domain for x is the set of possible travel locations: Venice (V), Shanghai (S), Paris (P), Brussels (B), and London (L). Note that statements such as "Shaggy want to go to Venice" does *not* imply that Shaggy only wants to go to Venice. For example, Shaggy would be perfectly happy going to Venice and Shanghai.

   (a) Translate each of the group's travel requirements $i - iv$ from English into a proposition using the given propositional function. [No need to translate Scooby's wishes!]

   (b) Are the group's travel wishes satisfiable? If they are, provide a list of destinations that satisfies the requirements. If they are not, provide a **concise** written argument explaining why not. Do **not** use a truth table.

   (c) What travel destination should we have included in the list? [Note, this is for fun.]

   Solution:

   (a)

   (i)$T(V) \vee \neg T(S)$

   (ii)$T(P) \to \neg T(V)$

   (iii)$T(B) \leftrightarrow (T(L) \wedge T(P))$

   (iv)$\neg T(P)$

   (b) The group's travel wishes are satisfiable. A trip to Venice would satisfy the entire group.

   We can come to this conclusion by working on problem (iv). The options for destinations can be reduced since Fred does not want to go to Paris so trip should not include Paris.

   Next considering question (iii), Daphne want to go to Brussels if and only if the group goes to London and Paris but since Fred does not want to go to Paris, Brussels is out.

   In (i), Shaggy wants to go to Venice or not to Shanghai. Suppose the gang goes to Venice. This satisfies (ii) because Paris has already been ruled out.

   (c) Varied answers.