Get in small groups (about 4 students maximum) and work out these problems on the whiteboard. Ask one of the teaching assistants for help if your group gets stuck. You do **not** need to turn anything in.

1. **Predicates/Quantifiers warm up:** Translate these sentences into predicates that include nested quantifiers. Be sure to define any variables and propositional functions that you use.

   (a) All engineering students are tired

   (b) Not every person in Boulder enjoys quinoa

   (c) You can't finish all of your homework all of the time

   (d) Some CSCI 2830 students attend every session

2. *Predicates/Quantifiers - satisfiability*

   (a) Consider the domain to be all people at CU Boulder. Let $S(x)$ be: x is a student at CU Boulder. Let $P(x)$ be: x uses public transportation to get to Boulder. Let $W(x)$ be: x works at CU Boulder. Determine the truth of each statement. If the statement is false, give an example of someone (faculty, student, supporting staff, etc.) in the domain who is not covered by the statement.

      i. $\exists x\ (\neg S(x) \to W(x))$

      ii. $\forall x\ (S(x) \lor P(x) \lor W(x))$

      iii. $\forall x\ (\neg S(x) \land \neg P(x))$

      iv. $\exists x\ ((S(x) \land P(x)) \to W(x))$

   (b) Consider the domain of all real numbers. Let $P(x, y)$ be: $x+y = 0$; Let $Q(x, y)$ be: $x^2-2x+1 = y$. Determine the truth of each statement.

      i. $\forall x \exists y\ P(x, y)$

      ii. $\exists y \forall x\ Q(x, y)$

      iii. $\forall y \exists x\ (P(x, y) \land Q(x, y))$

      iv. $\forall y \exists x\ P(x, y) \land Q(x, y)$

3. Suppose p, q and r are all propositions. Prove that the following are logically equivalent. Use either logical equivalences or a truth table. If you use logical equivalences(recommended for practice), be sure to cite which rules you use.

$$p \land (q \to r) \equiv \neg((p \to q) \land (p \to \neg r))$$

4. What is wrong with this proof?

   Theorem: If n^2 is positive, then n is positive.

   Proof: Suppose that n^2 is positive. Because the conditional statement "If n is positive, then n^2 is positive" is true, we can conclude that n is positive.

5. Suppose A is a proposition that does not depend on the variable x, but the propositional function $P(x)$ does depend on x. Establish these logical equivalences, using logical equivalences for compound propositions and quantifiers.

   (a) $\forall x\ (P(x) \to A) \equiv \exists x\ P(x) \to A$

   (b) $\exists x\ (P(x) \to A) \equiv \forall x\ P(x) \to A$

   Hint: Recall that $\forall x\ P(x) \equiv P(a) \land P(b) \land \ldots$ and $\exists x\ P(x) \equiv P(a) \lor P(b) \lor \ldots$, if the domain for x is $a, b, \ldots$

6. Prove that the following rules of inference are tautologies. Try to name the rules of inference without looking at the textbook.

   (a) $(p \wedge (p \rightarrow q)) \rightarrow q$
   (b) $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$
   (c) $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$
   (d) $((p \vee q) \wedge \neg p) \rightarrow q$
   (e) $p \rightarrow (p \vee q)$
   (f) $(p \wedge q) \rightarrow p$
   (g) $((p) \wedge (q)) \rightarrow (p \wedge q)$
   (h) $((p \vee q))$

7. Determine the truth value of the statement $\exists x \forall y (x \leq y^2)$ if the domain for the variables consists of

   (a) the positive real numbers.
   (b) the integers
   (c) the nonzero real numbers.

8. Show that if n is an integer and $n^3 + 5$ is odd, then n is even using

   (a) a proof by contraposition.
   (b) a proof by contradiction.

9. *Take home coding problem (this week's concept is fall-through cases)*

   Tony is looking for someone who can code up a basic function that analyzes the truth values of an input. With your python coding expertise you step-up to that challenge. Write a function that takes 2 inputs, the first one is a string argument that represents what logical operation you need to perform. The valid strings are `AND`, `OR`, `CONDITIONAL`, `BICONDITIONAL`. Your second argument is a list of n truth values that you have to evaluate to a final `True` or `False` value which you will return. 2 example test cases are given below -

   (a) `function_name('AND', [True,False]) -> False`
   (b) `function_name('OR', [True,False,False]) -> True`

   For the biconditional case you have to check to make sure there are only 2 values in the list. Return `False` along with an error message if otherwise.