CSCI 2830-800: Discrete Structures Workgroup – Fall 2018 **Worksheet 07**

Get in small groups (about 4 students maximum) and work out these problems on the whiteboard. Ask one of the teaching assistants for help if your group gets stuck. You do **not** need to turn anything in. Since this worksheet contains a lot of problems, a good strategy would be to first skim the worksheet and then discuss and solve the problems which you think are difficult.

1. Let $A = \{1, 2, \{3, 4\}, 5\}$. Determine whether each of the following are true or false, and explain why.
   (a) $\{3, 4\} \subset A$
   (b) $\{3, 4\} \in A$
   (c) $\{\{3, 4\}\} \subset A$
   (d) $1 \in A$
   (e) $1 \subset A$
   (f) $\{1, 2, 5\} \subset A$
   (g) $\{1, 2, 5\} \in A$
   (h) $\{1, 2, 3\} \subset A$
   (i) $\emptyset \in A$

2. Consider the following functions $f(n)$, where the domain for n is the set of integers, $\mathbb{Z}$. Fill in the table regarding whether or not each function is one-to-one and/or onto.

$f(n)$	One-to-One?	Onto?
$f(n) = n^2$		
$f(n) = n + 3$		
$f(n) = \lfloor \sqrt{n} \rfloor$		
$f(n) = \begin{cases} n - 1 & n \text{ odd} \\ n + 1 & n \text{ even} \end{cases}$		

3. Prove that $\{9^n \mid n \in \mathbb{Z}\} \subseteq \{3^n \mid n \in \mathbb{Z}\}$, but $\{9^n \mid n \in \mathbb{Z}\} \neq \{3^n \mid n \in \mathbb{Z}\}$

4. Prove the following if A, B, C are sets.
   (a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$, using set builder notation
   (b) $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$

5. Uncountable/countable sets
   (a) Prove directly from the definition of countable/uncountable that the set of natural numbers that are multiples of 3 or multiples of 4 is countable.
   (b) Prove that the set of real numbers in the interval $[4, 5]$ is uncountable.
   (c) Suppose A and B are both countable sets. Prove whether the Cartesian product $A \times B$ is countable or uncountable.

6. Show that if A, B, C, and D are sets with $|A| = |B|$ and $|C| = |D|$, then $|A \times C| = |B \times D|$.

7. Show that if A and B are sets and $A \subset B$then $|A| \leq |B|$.

8. Find $f \circ g$ and $g \circ f$, where $f(x) = x^2 + 1$ and $g(x) = x + 2$, are functions from R to R.

9. Prove or disprove each of these statements about the floor and ceiling functions
   (a) $\lfloor \lceil x \rceil \rfloor$ for all real numbers x.
   (b) $\lfloor x + y \rfloor = \lfloor x \rfloor + \lfloor y \rfloor$ for all real numbers x and y.
   (c) $\lfloor \sqrt{\lceil x \rceil} \rfloor = \lfloor \sqrt{x} \rfloor$ for all positive real numbers x.

10. Find the inverse of the function $f(x) = -3 \times 5^x + 6$.