Computing volatility surface via SVI model

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Introduction

In the real market and active exchanges, options are traded only with certain strikes and expirations, therefore we need to search for various models that would allow us to determine the value of the implied volatility for arbitrary strike and expiration values. There are a number of models for constructing the implied volatility surface, one of the simplest and at the same time quite well describing individual volatility smiles is the SVI (Stochastic Volatility Inspired) model [4].

Objectives

- Applying the SVI model to real market data and implementing a unified algorithm for constructing an implied volatility surface for given asset.
- Finding the invariant behavior for different parameterizations for SVI model to extrapolate the value of implied volatility beyond existed expirations.

Computing Implied Volatility

For computing implied volatility from real market data on American option is used solution of inverse problem respect to $\Sigma^F(K,T)^*$ of the equation $V(t_0, F(t_0,T)) = V(F(t_0,T), K, t_0, T, \Sigma^F(K,T)^*)$ in the Black−76 model [1]. For using this model we have to compute forward value $F(t_0,T)$ by:

$$F(t_0,T) = S(t)\frac{RF(t_0,T)DivF(t_0,T)}{DF(t_0,T)}.$$

Where:
- $DivF(t_0,T)$ is the constant dividend yield.
- $DF(t_0,T) = \exp(-\int_{t_0}^{T} r_s ds)$ is the discount factor from 3 month SOFR forward rate.
- $RF(t_0,T)$ is the REPO factor.

After that we use **call put parity** for closest to our estimated forward value strikes to find the most accurate forward value:

$$F(t_0,T) = \frac{Call(K,T) - Put(K,T)}{DF(t_0,T)}$$

SVI model

The SVI (Stochastic volatility inspired) model is a non-symmetric parabolic function with a flexible set of different parameters model [4].

The **raw parameterization** is given by the following formula and parameters:

$$\omega(x) = a + b\left(\rho(x-m) + \sqrt{(x-m)^2 + \sigma^2}\right).$$

a, b, ρ, m, σ are parameters, where $(a \in R, b \geq 0, |\rho| < 1, m \in R, \sigma > 0)$.

Jump Wings parameters are determined from the parameters of the raw parameterization and have understandable meaning: v_t- ATM implied variance, ϕ_t- ATM skew, p_t, c_t- slope for left and right wings, $\widetilde{v_t}-$ minimum implied variance.

SVI parameter calibration

Consider a raw parameterization for a fixed expiration T. Then the implied total variance is:

$$\omega(x) = a + dy(x) + cz(x), \text{where: } y(x) = \frac{x-m}{\sigma}.$$

$$z(x) = \sqrt{y(x)^2 + 1}, \quad d = \rho b\sigma, \quad c = b\sigma.$$

We define the calibration task as follows:

$$f(a,b,\rho,m,\sigma) = \sum_{i=1}^{n}(\omega_i - \omega(x_i))^2 \to min.$$

For good calibration all parameters [3] we have to split the calibration into two steps: external (when we calibrate m and σ) and internal (when we calibrate a, b, ρ). For internal step is used gradient descent method, for external is used SLSQP and Dual Annealing methods from python library 'Optimize'.

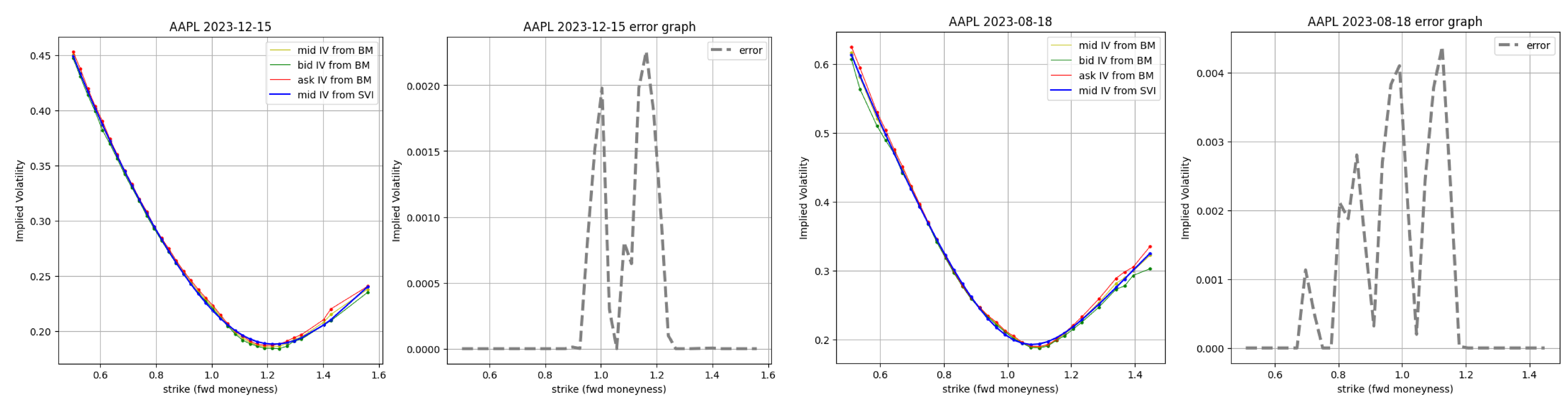

Figure 1: SVI calibration results.

Checking static arbitrage

Each time for computed volatility smiles from SVI we have to check 3 conditions of static arbitrage absence.

Calendar arbitrage is absent, when $\partial_t W(x,t) \geq 0$, where $W(x,t)$ is the total implied variance, $x \in R$.

Butterfly arbitrage is absent, when the function $g > 0$ for all x values and all expirations:

$$g(x) := \left(1 - \frac{xw'(x)}{2w(x)}\right)^2 - \frac{w'(x)^2}{4}\left(\frac{1}{w(x)} + \frac{1}{4}\right) + \frac{w''(x)}{2}.$$

Asymptotic curve behavior, Lee formula [2] is satisfied, when $(1 + |\rho| b \leq 2)$ for a raw parameterization.

Static arbitrage is absent for presented on figure 2 implied volatility surface for Apple.

Overall result

The black smiles on the figure 2 are interpolation or extrapolation. The last available expiration is 2.5 years, using extrapolated parameters, it is possible to extrapolate implied volatility values to any maturities, for example 3.5 years as on the figure.

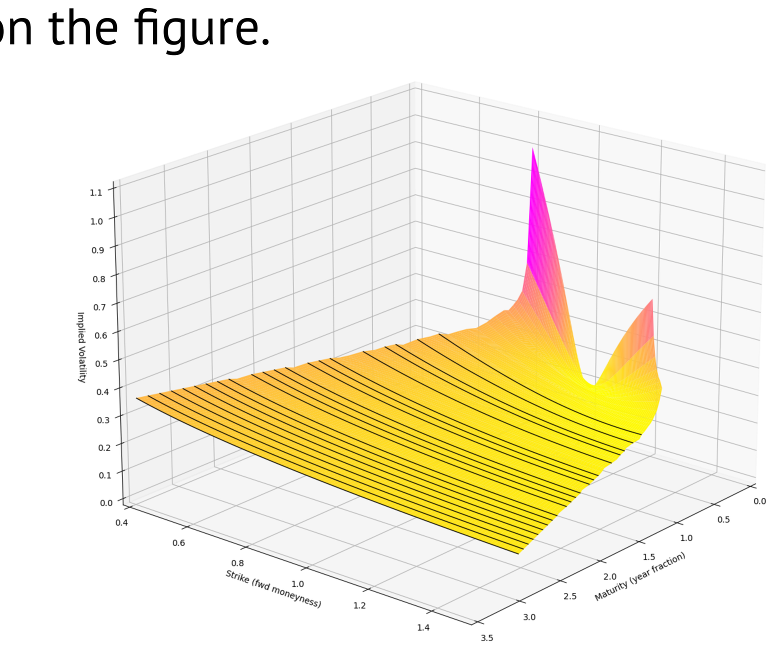

Figure 2: Implied volatility surface for Apple with extrapolation and interpolation.

Extrapolation

The extrapolation possibility derives from the some invariant behavior of Jump Wing parameters from one expiration to another. It means that we can use available information about parameters to build a regressions and extrapolate this values beyond known expirations. The following regression types were considered in work: linear, polynomial, exponential, logarithmic, reciprocal, the last two most accurately describe the dependence in the parameters changing. To check the quality of the regressions, the last expiration was used as a test. The sample values were taken without data about last expiration in order to build a regression on the data for each parameter and extrapolate it to the last expiration. Based on the obtained values of the extrapolated parameters, construct an SVI curve of implied volatility and compare it with the available bid-ask spread from Black−76 model.

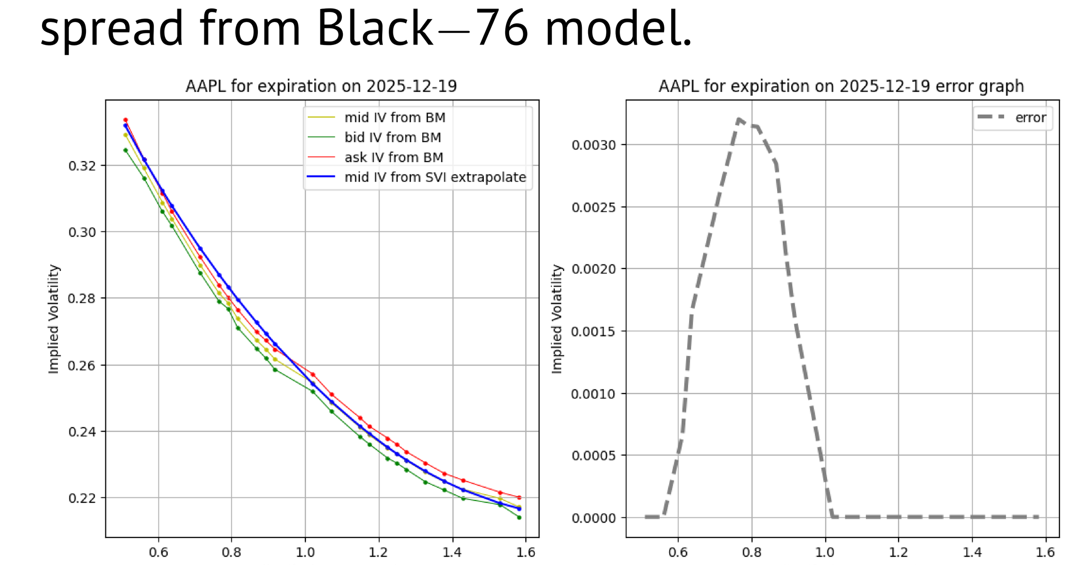

Figure 3: Extrapolation result for Jump Wing parameters for Apple with expiration on 19.12.2025.

References

[1] Fischer Black. "The pricing of commodity contracts". In: *Journal of Financial Economics* 3 (1976), pp. 167–179.

[2] Roger W. Lee. "The Moment Formula for Implied Volatility at Extreme Strikes". In: *Mathematical Finance* (2003).

[3] Zeliade Systems. "Quasi-Explicit Calibration of Gatheral's SVI mode". In: *Zeliade White Paper* ZWP-0005 (2009).

[4] Antoine Jacquiery Jim Gatheral. "Arbitrage-free SVI volatility surfaces". In: *SSRN-id2033323* (2013).