Sample Event in Mathematical Finance

Computing volatility surface via SVI model (Stochastic Volatility Inspired)

Elezov D.N.

Supervisors: Mikhail V. Zhitlukhin, Charles-Henri Roubinet

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Introduction

In real markets and on active exchanges, options are traded only at specific strike prices and expirations. As a result, we need to explore various models that allow us to determine the value of implied volatility for arbitrary strike prices and expiration values. While there are several models for constructing the implied volatility surface, the SVI (Stochastic Volatility Inspired) model is one of the simplest, and it effectively describes individual volatility smiles.

Objectives:

- Apply the SVI model to real market data and implement a unified algorithm for constructing an implied volatility surface for a given asset;
- Develop a method of extrapolation of the SVI parameters for constructing out-of-sample implied volatility curves.

Black-76 model

Undiscounted call price at time t_0 of a European option with strike K and expiration time T:

$$\mathsf{G}(t_0, F, T, K, \Sigma^F,) = \cdot \left(F \cdot \mathcal{N}(\cdot d_1) - K \cdot \mathcal{N}(\cdot d_2)\right),$$

$$d_1 = \frac{k}{\Sigma^F \sqrt{T - t_0}} + \frac{1}{2}\Sigma^F \sqrt{T - t_0}, \quad d_2 = d_1 - \Sigma^F \sqrt{T - t_0},$$

$$k = \ln\left(\frac{F}{K}\right),$$

where:

- $\mathcal{N}()$ is the normal distribution function;
- Σ^F is the implied volatility for an option with strike K and expiration T.

Then the market price of the European option at time t_0 is calculated as Black Formula$\cdot$ $DF(t_0, T)$.

Estimating forward value

For using this model we have to compute forward price $F(t_0, T)$ by:

$$F(t_0, T) = S(t)\frac{RF(t_0, T)DivF(t_0, T)}{DF(t_0, T)},$$

where:
- $DivF(t_0, T)$ is the constant dividend yield;
- $DF(t_0, T)$ is the discount factor from 3 month SOFR forward rate;
- $RF(t_0, T)$ is the REPO factor.

After that we use **call put parity** for closest to our estimated forward value strikes to find the most accurate forward value:

$$F(t_0, T) = \frac{Call(K, T) - Put(K, T)}{DF(t_0, T)}.$$

SVI (Stochastic volatility inspired) model

The SVI (Stochastic volatility inspired) model is a non-symmetric parabolic function with a flexible set of parameters.
The **raw parameterization** is given by the following formula and parameters:

$$\omega(x) = a + b\left(\rho(x-m) + \sqrt{(x-m)^2 + \sigma^2}\right),$$

a, b, ρ, m, σ are parameters, where $a \in R, b \geq 0, |\rho| < 1, m \in R, \sigma > 0.$

Jump Wings parameters are determined from the raw parameters and have understandable meaning: $v_t -$ ATM implied variance, $\phi_t -$ ATM skew, $p_t, c_t -$ slope for left and right wings, $\widetilde{v}_t -$ minimum implied variance.

SVI parameter calibration

Consider the raw parameterization for a fixed expiration T. Then the implied total variance is:

$$\omega(x) = a + dy(x) + cz(x), \text{where } y(x) = \frac{x-m}{\sigma}.$$

$$z(x) = \sqrt{y(x)^2 + 1}, \;\; d = \rho b \sigma, \;\; c = b\sigma.$$

The calibration problem is:

$$f(a, b, \rho, m, \sigma) = \sum_{i=1}^{n} (\omega_i - \omega(x_i))^2 \to min.$$

For efficient calibration, we have to split the calibration problem into two steps: external (calibration of m and σ) and internal (calibration of a, b, ρ). We use the gradient descent method for the internal step, and SLSQP and Dual Annealing methods for the external step.

Implied volatility via SVI model

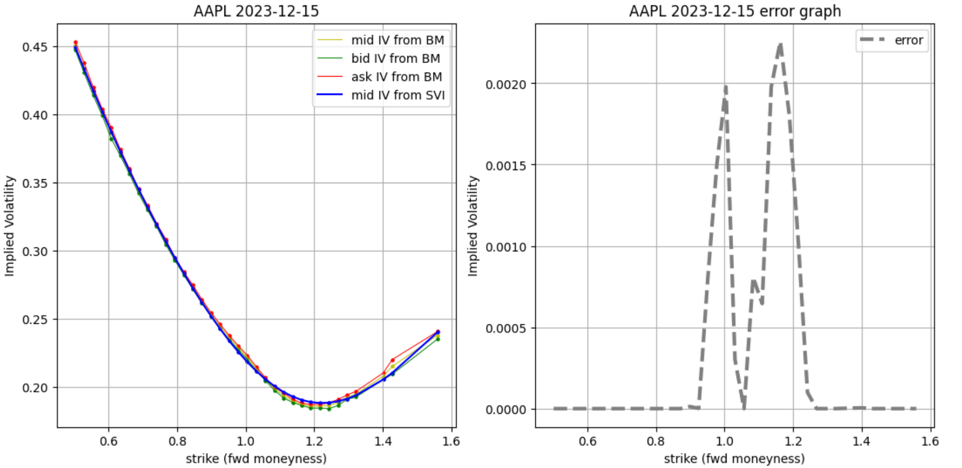

Figure: IV for Apple on 20.06.2023 for expiration on 15.12.2023 from calibrated SVI.

Calendar arbitrage

There is no calendar arbitrage only when $\partial_t W(x,t) \geq 0$, where $W(x,t)$ is the total implied variance, $x \in R$. This means that the total variance curves for different expiration dates should not intersect.

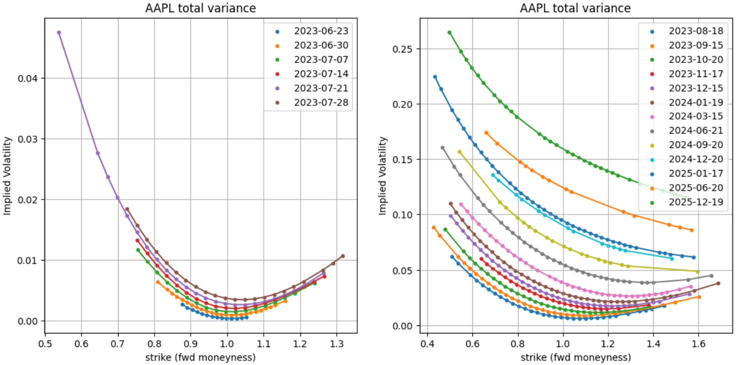

Figure: Total implied variance for Apple on 20.06.2023 for different expiration dates.

Butterfly Arbitrage

Butterfly arbitrage is absent if and only if the function $g > 0$ for all x values and all expirations. Function g: $g(x) := \left(1 - \frac{xw'(x)}{2w(x)}\right)^2 - \frac{w'(x)^2}{4}\left(\frac{1}{w(x)} + \frac{1}{4}\right) + \frac{w''(x)}{2}$.

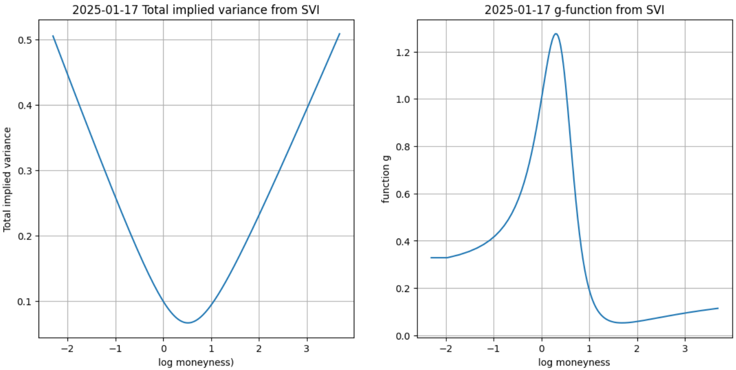

Figure: Total implied variance and g-function for Apple on 20.06.2023.

Extrapolation implied volatility beyond known expirations

Jump Wing parameters behavior is more stable than in raw parameterization.

Figure: Changes in raw and Jump Wing parameters for Apple on 08.06.2023 for different expiration dates.

Extrapolation implied volatility beyond known expirations

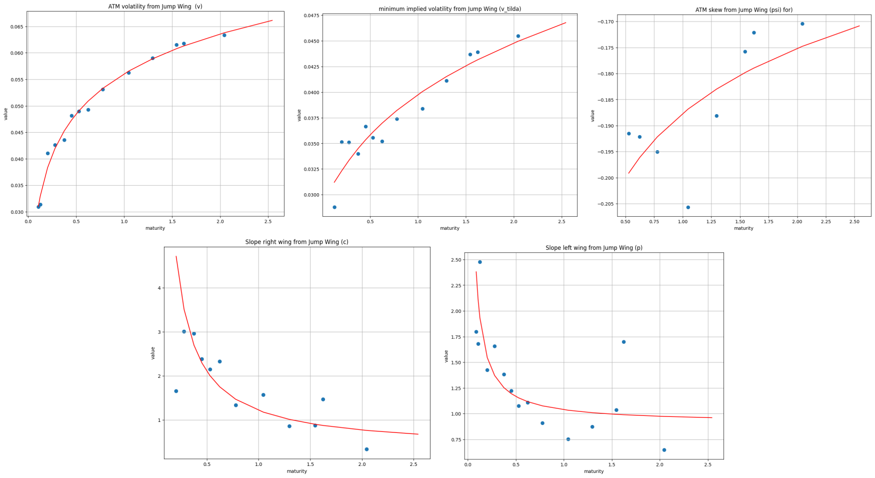

Figure: Regressions for Jump Wing parameters for Apple on 08.06.2023.

Extrapolation implied volatility beyond known expirations

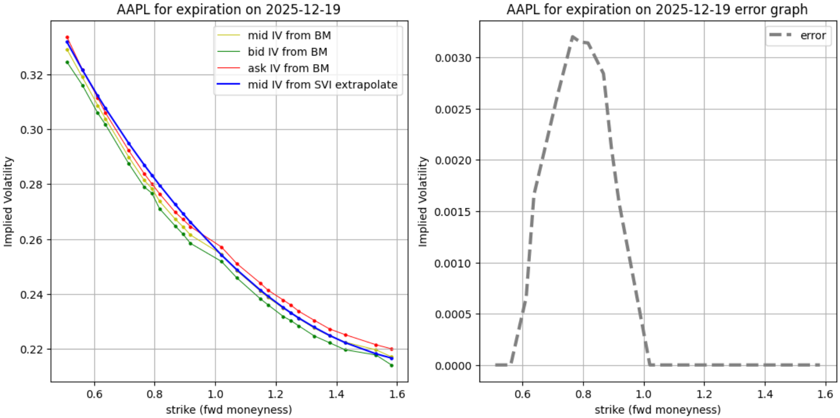

Figure: Extrapolation result for Jump Wing parameters for Apple with data on 08.06.2023. and expiration on 19.12.2025.

Implied volatility Surface with interpolation and extrapolation

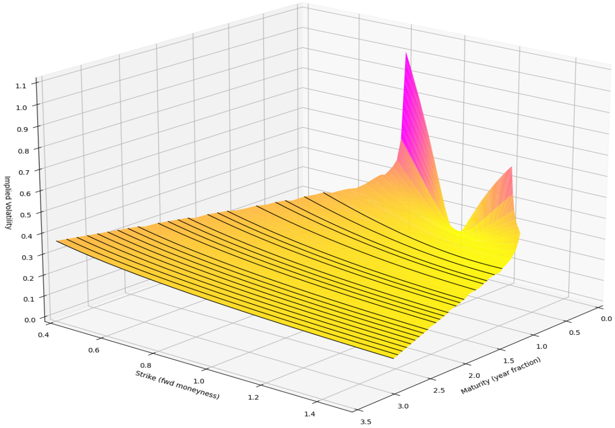

Figure: Static arbitrage free implied volatility surface for Apple for Data on 08.06.2023 with extrapolation and interpolation.