Math 350 HW 1

See Canvas for due date

Problems

1. Two children, Maria and Norbert, are going to get ice cream cones. Each child gets their own cone, and can choose from vanilla, rocky road, and strawberry. Our "experiment" will be to observe the flavors they choose.

   Some events that we will work with:

   - A: Both children choose the same flavor.
   - B: Neither child chooses strawberry.
   - C: Maria chooses rocky road.

   (a) Suppose your friend suggested that a good sample space for this experiment would be $\{m, n, v, r, s\}$. Explain in words why that doesn't fit with this experiment. Can you describe a different experiment for which it would work?

   (b) Suppose a different friend suggested that a good sample space for this experiment would be $\{mv, mr, ms, nv, nr, ns\}$. Explain in words why that doesn't fit with this experiment either. Can you describe a different experiment for which it would work?

   (c) Write down a set that would work well as a sample space for this experiment. Pick two of the elements of your sample space and explain in words what outcomes they represent.

   (d) For each of the following events, give a description in words. Also write it out as a set of outcomes. (Check that your description fits for each of the outcomes in the set.)

      i. $A \cup C$

      ii. $A \cap C^c$

      iii. $(A \cap B)^c$

      iv. $(A \cap C) \setminus B$

2. Almanzo is going to surprise a group of their friends by cooking a randomly selected dinner. It could be burgers (B), tacos (T), or something else. Whatever it is, it might or might not be spicy (S).

   Suppose we know that:

   - The probability of having burgers is .47
   - The probability of having spicy burgers is .25

- The probability of having non-spicy food is .40
- The probability of having tacos is .40
- The probability of having either tacos or a spicy dish is .70

Solve each of the following problems. For each one, give a complete explanation of your solution process, with justification of your steps. Each time you use an axiom or theorem about probability, mention it (e.g. "by the additivity axiom", "by the inclusion-exclusion theorem"); if it has assumptions, explain why its assumptions are satisfied. Use set operations when appropriate to describe the events you are working with; for example, the event of getting a non-spicy burger is $B \cap S^c$.

(a) Edwina likes burgers, as well as all kinds of spicy food. What is the probability that Edwina will be happy about the meal?

(b) Also attending is Fred, who hates burgers, tacos, and spicy food. What's the probability that Fred will enjoy the dinner?

(c) Gina likes spicy tacos and non-spicy burgers, and nothing else. What's the probability that she will like what is served?

(d) Hermione insists on eating only spicy food, but dislikes tacos. What is the probability that Hermione will be pleased?

3. (a) Come up with an example of a real-life experiment with events A, B, C for which all of the following are true.

   - A and B are positively correlated
   - B and C are positively correlated
   - A and C are negatively correlated

   Then explain briefly why it makes sense that this combination is possible.

   You don't have to define precisely the probabilities of the events, but make sure their correlations are something that's intuitive from common sense or general knowledge. For example, if our experiment is to select a random UNC student and observe what courses they take, the events "they take probability theory" and "they take differential equations" should be a good example of positively correlated events.

   (b) Similarly to the previous problem, now come up with an example of a real-life experiment with events A, B, C for which all of the following are true:

   - A and B are negatively correlated
   - B and C are negatively correlated
   - A and C are negatively correlated

   Then explain briefly why it makes sense that this combination is possible.

   (c) Similarly, now come up with an example of a real-life experiment with events A, B, C for which all of the following are true:

   - A and B are independent
   - B and C are independent
   - A and C are negatively correlated

   Then explain briefly why it makes sense that this combination is possible.