MATLAB

Task 1: Kinematics

Recall some theory

A mobile log crane forms a kinematic chain of the bodies interconnected through the revolute and prismatic joints. The numbering of the joints is indicated in the Fig. 1. The crane has four independent coordinates: the angle of the pillar rotation, the angles of rotation of the lifting boom and extension boom, and the length of the extension boom. In a planar case, the angles of rotation of the main and extension booms can be chosen as independent coordinates.

The kinematics of the crane is rather simple. Every point of the crane in a global space is represented by a 3x1 position vector $r = [x, y, z]^T$.

The common practice for kinematic chains is to provide each link with the local coordinate system with the origin located in some joint. For example, the origin of the coordinate system associated with the first boom is located in the Joint 1. Using the angles of rotation of the booms the global coordinates of each point of the crane can be obtained from its local coordinates in the following way.

In the planar case, orientation of the body relative to some coordinate system is defined by the 3x3 rotation matrix A:

$$A = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (1)$$

where θ is the angle of rotation.

The position of any point of the boom number j can be represented in the local coordinate system associated with the joint number i as follows:

$$r^i = R^j + A^{ij}u^j \qquad (2)$$

where r^i is the position vector of the point relative to the joint i, R^j is the position vector of the origin of the local coordinate system associated with the joint j, relative to the joint i, A^{ij} is the rotation matrix of boom j relative to boom i, u^j is the position vector of the point relative to the joint j.

Translation and rotation together can be represented by 4x4 transformation matrix T_{ij}:

$$T_{ij} = \begin{bmatrix} A^{ij} & R^j \\ 0 & 1 \end{bmatrix} \qquad (3)$$

Using the transformation matrix, the position vector of the point can be represented as follows:

$$r_4^j = T_{ij}u_4^j \qquad (4)$$

where $r_4^j = [r_x^j, r_y^j, r_z^j, 1]^T$ and $u_4^j = [u_x^j, u_y^j, u_z^j, 1]^T$.

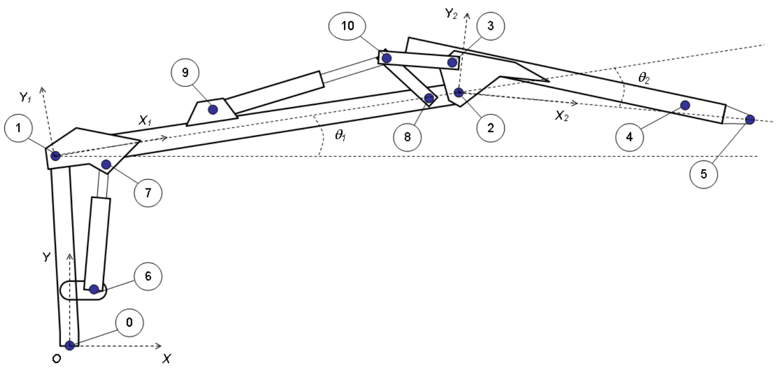

Fig. 1. The numbering of the joints of the crane.

The notation described above allows to define the transformation matrices between the coordinate systems associated with the joints. If the origin of the global coordinate system is located in the Joint 0 (point O in Fig.1), the transformation matrices for the joints 0, 1, 2 and 4 are defined as follows:

$$T_{01} = \begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 & R_x^{Jp1} \\ \sin(\theta_1) & \cos(\theta_1) & 0 & R_y^{Jp1} \\ 0 & 0 & 1 & R_z^{Jp1} \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$T_{12} = \begin{bmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & r_x^{Jp2} \\ \sin(\theta_2) & \cos(\theta_2) & 0 & r_y^{Jp2} \\ 0 & 0 & 1 & r_z^{Jp2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{24} = \begin{bmatrix} 1 & 0 & 0 & r_x^{Jp4} \\ 0 & 1 & 0 & r_y^{Jp4} \\ 0 & 0 & 1 & r_z^{Jp4} \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (12)$$

where $R^{Jp1} = [R_x^{Jp1}, R_y^{Jp1}, R_z^{Jp1}]$ is the global position vector of the joint 1, $r^{Jp2} = [r_x^{Jp2}, r_y^{Jp2}, r_z^{Jp2}]$ is the position vector of the joint 2 in the coordinate system associated with the joint 1 and $r^{Jp4} = [r_x^{Jp4}, r_y^{Jp4}, r_z^{Jp4}]$ is the position vector of the joint 4 in the coordinate system associated with the joint 2.

Using these matrices, it is easy to calculate the global position vector of any point of the crane. For example, the global position vectors of the joint 2 and joint 4 are calculated as follows:

$$R^{Jp2} = T_{01}T_{12}Z,$$

$$R^{Jp4} = T_{01}T_{12}\, T_{24}Z \qquad (13)$$

where $Z = [0, 0, 0, 1]^T$.

Task:

Using the MATLAB script, which is provided with the task, calculate the global position of a point given its local coordinates.

Vectors r00, r01, r12, r23 define the local coordinates of the origin of each coordinate system associated with some joint. Each vector, rNM, is the coordinate of the joint M in the coordinate system associated with the joint N. Vector r00 defines the coordinates of the point O (see Fig.1) in the global coordinate system.

Variables thK define the angles of rotation of the coordinate systems associated with the joints ($K \in [0, 1, 2, 3]$).

Define the matrices T_{ij} and use matrix multiplication similar to (13) to calculate the answer.

Assign the calculated value to the variable ANSWER of the provided script and see the point with the calculated coordinates at the plot. The point should match one of the red squares in the picture.