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Optimizing Water Distribution Infrastructure - Final Report

Introduction:

Water is a valuable resource that is necessary for life and it is increasingly becoming a rare commodity [1]. The domestic water consumption has favorably (fortunately) decreased during the last century thanks, in part, to the invention of washing and dishwashing machines and to showers [2]. However, we wanted to take the next step and simulate what ideal infrastructure would be needed to optimize (or minimize) water extraction and storage.

To do so, we combine our programming knowledge with the computing power of the computer, to create a model for residential water consumption. Our goal was to modelize (hourly) water consumption accurately to an average North American household. Then, optimize the pumping power required from the water pump (extraction) and the size of the water tower (storage).

Our program(s) will:

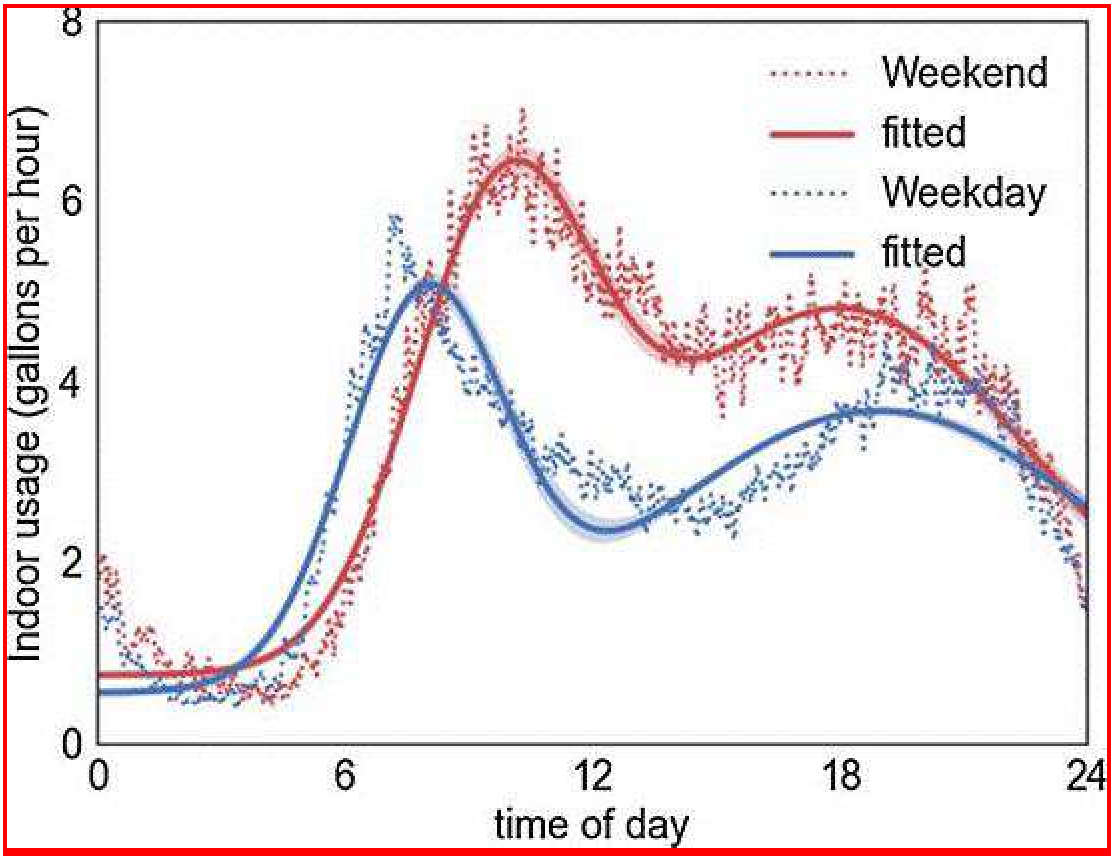
1. Simulate household water consumption (hourly, for every house).
2. Find an optimal pumping power (minimize).
3. Find the optimal volume for the water tower (storage).

Description of the scientific method:

For this project, there are three parts to be considered. First, we consider the hourly consumption of water of all the houses in a given time frame. Second, we consider the optimal pumping power of the pumping station which will provide for the consumption. Third, we consider the optimal volume of the water tower which will store the excess water during the low consumption periods of the day.

Our first task is to calculate water consumption, for a given number of houses, over a given interval of time. This part is complicated and cannot be easily predicted. For this reason, we must use (large scale) averages. However, these averages have to also be representative of the situation at hand, meaning yearly, monthly, or even daily averages will not be sufficient. We have to rely on hourly household consumption averages, because they vary a lot. In fact, water consumption rates follow a generally cyclical path from day to day, hitting their minima during the night, and their maxima in the morning. Thus, the rate of water consumption over time (in hours) can be described by a wave function [2]. However, such a function would be too complicated to implement, with too many variables for us (at our current level of education), and would still have a significant margin of error, because of the unpredictability of water consumption. This means that we must resort to somewhat randomized probabilities of water consumption per house, based on statistical averages set differently according to the hour of the day, so that the result generally mimics this function. This way, we capture both the general shape of the function and the unpredictable factor of water consumption. More specifically, we set up an hourly probability of a house to consume water, repeated for every single house, at every hour, which is meant to simulate the following graph. (It has to be noted that the "randomness" is not really significant at larger scales, as at such scales, thus, the rate of water consumption ends up following the statistical averages, which is our goal.)

Water Consumption as a function of time for an average household [2]



Our next task is to calculate an optimal pumping power for the desired number of houses which will consume water.

For the pump,we are assuming a constant and uninterrupted pumping rate. We want the smallest pumping power that would satisfy the consumption. Thus we want the pumping power that yields the minimal amount of water in the water tower. But we also want to make sure that the volume in the tower never goes to zero (we don't want to run out of water). We can visualize it as a graph of the smallest (minimal) volume of water in the water tower with respect to the amount of pumping power. We want to find the root of this function. To do so, we use root finding, where the desired pumping power is the root. The volume of the water tower by one contributing house can be described by the following function:

$$Vtf = Vti + P - C(t)$$

where Vt final and initial symbolise the volume of water in the tower, P symbolises the pumping power per house per hour of the pump, a constant value which we are trying to find, and C(t) symbolises the consumption of water by a single house with respect to the hour of the day t. As already mentioned, C(t) is a stochastic function. Since this function describes only the volume of water in the tower with only one house contributing, it is necessary to add the contribution of every single house:

$$\sum_{1}^{n} Vtf = \sum_{1}^{n} (Vti + P - C(t))$$
$$\sum_{1}^{n} Vtf = Vti + \sum_{1}^{n} (P - C(t))$$

where n represents the number of houses (predefined). P and C(t) are per house, which means the sum depends on it, while it does not on Vti, which is why we can pull it out of the sum. However, this model is also wrong as it depicts the volume of the water tower for all of the houses, but only for a day. It is thus necessary to add a sum for the time:

$$\sum_{1}^{m} \sum_{1}^{n} Vtf = \sum_{1}^{m} [Vti + \sum_{1}^{n} (P - C(t))]$$
$$\sum_{1}^{m} \sum_{1}^{n} Vtf = Vti + \sum_{1}^{m} [\sum_{1}^{n} (P - C(t))]$$

where m is the number of hours the simulation is run for (predefined). P and C(t) are also per hour, which means the second sum also depends on them, but not on the initial volume.

The root finding is thus between the minimal Vtf obtained by finding what is the minima of this function (as long as it is not negative), and P (more on this in the computational methods section).

Our last task is to calculate the optimal volume of the water tower. The volume of water in the tower will also resemble a wave function on the daily, with an increase of volume of water in the tower over time when the pumping power is greater than the consumption, and a decrease over time when the opposite is true. When the pumping power P is too high, the volume of the tower is ever increasing. When the pumping power is too low, the tower runs out of water. However, it is impossible to find an optimal pumping power, because the volume of the tower depends on the stochastic function that is C(t). This means that if a day when there is much more consumption occurs, the tower might get negative volume. Given such circumstances, it is understandable to overestimate the optimal pumping power, than to underestimate it, as it would not be acceptable to not have water. Thus, for a longer period of time analysing the volume of water in the tower, the more obvious it is to see how the volume of the tower is ever increasing. The optimal volume of the tower more closely resembles the first few days (we picked the first 5 days) then, as it is closer to the minimum volume for the tower necessary. The optimal volume is calculated by subtracting the minimum volume of the tower from the maximum volume in the given time frame.

Description of the computational methods:

The consumption of water by a house is the C(t) part of our function. C(t) is basically a method, which returns a volume consumed, taking in the hour of the day. When this method is called, it looks which hour of the day it is, and there is a given probability that water will be consumed at that time, which varies from hour to hour. This method is made to represent household consumption, based on the previously shown graph (it mimics that graph of consumption). There are three varying elements in this method: the base consumption value, the random consumption value, and the number people active. These are designed to fit the curve of the graph. The base consumption value is the area under the curve of the graph. It is the minimum value that must be consumed in order to have a realistic simulation. The random consumption value has two purposes. The first purpose is to mimic (reasonably) the randomness associated with human water consumption that is unpredictable. The second purpose is to simulate the deviation of the data points on the graph. In some points (especially in the peaks), the individual data points (individual households) diverge significantly from the average. Thus to represent that we implemented a higher chance of random water consumption during these hours. The humans active is a feature to logically (and smoothly) simulate the important slopes in the graph to better fit the variations.

Then, to find the optimal pumping power, we are looking for a minimal pumping power which will satisfy the consumption. For this, we use root finding in our main method. The program starts with a higher and a lower value of P, evaluates the function at these points, and adjusts the value of P accordingly, so it closes in on the root. The function is evaluated in a separate method, where the value of P is taken in. The way the function is evaluated is that is it looped through two while loops, one for the time, and one for the number of houses, which represent the two sigmas. This method returns the minimum volume of the run, since this is the y-variable we are finding the root (pumping) of (already explained). Once the value of P is within a certain tolerance, the program stops the root finding, and this value represents the optimal pumping power per house per hour.

With this optimal pumping power, the function is evaluated one last time in a different method, which not only evaluates it, but also prints all of the points of time and volume of water in the tower in a text file called "Stored Data", which permits us to graph it in excel for a visual representation of the result (shown in the test case presentation). The optimal volume of the tower should be of the maximum volume of water it achieves minus the minimum volume of water it achieves, using the optimal pumping power.

Test case presentation

So far, our water model simulates an average value of 275 L/person in a household. For our model, the average household will have 4 persons (we need an integer number for our program to work), so 1100 L/household. Estimates vary a lot. The Canadian government estimates a daily water consumption of 251 L/person [3]. The U.S government estimates 300 to 380 Liters per person [4]. In the scientific paper we use, they estimate an average of 1136 L/household [2]. This discrepancy gives us some flexibility in our numbers, since our goal is to model an average north american household (and since there is no such thing as The Average).

The paper from which we collected our data uses an average that includes both american and canadian households. Our number (daily consumption value) is a little lower than their values because we are assuming in our program that every day is a weekday. As you can see in the graph above, the consumption in the weekend is a little higher, and the consumption peek is later in the day (another method would be needed for simulating a weekend, and that's a lot of work for a little change in the average which will be mostly insignificant).

We are well aware that assuming an average household size of 4 person is much higher than reality (average is 2.3-2.6 person per house) [5]. However, the goal of our program is to accurately simulate the behavior of water consumption throughout the day (which it does very well, qualitatively and quantitatively). The number of persons is used as a variable which allows us to make our program simulation closer to real life values.

The City of Guelph

To concretise our findings we went on and took the data of a real city. The city of Guelph is a good example because it is an average sized north american town (120 - 130 thousand citizens) and mostly residential (no heavy water consuming industries).

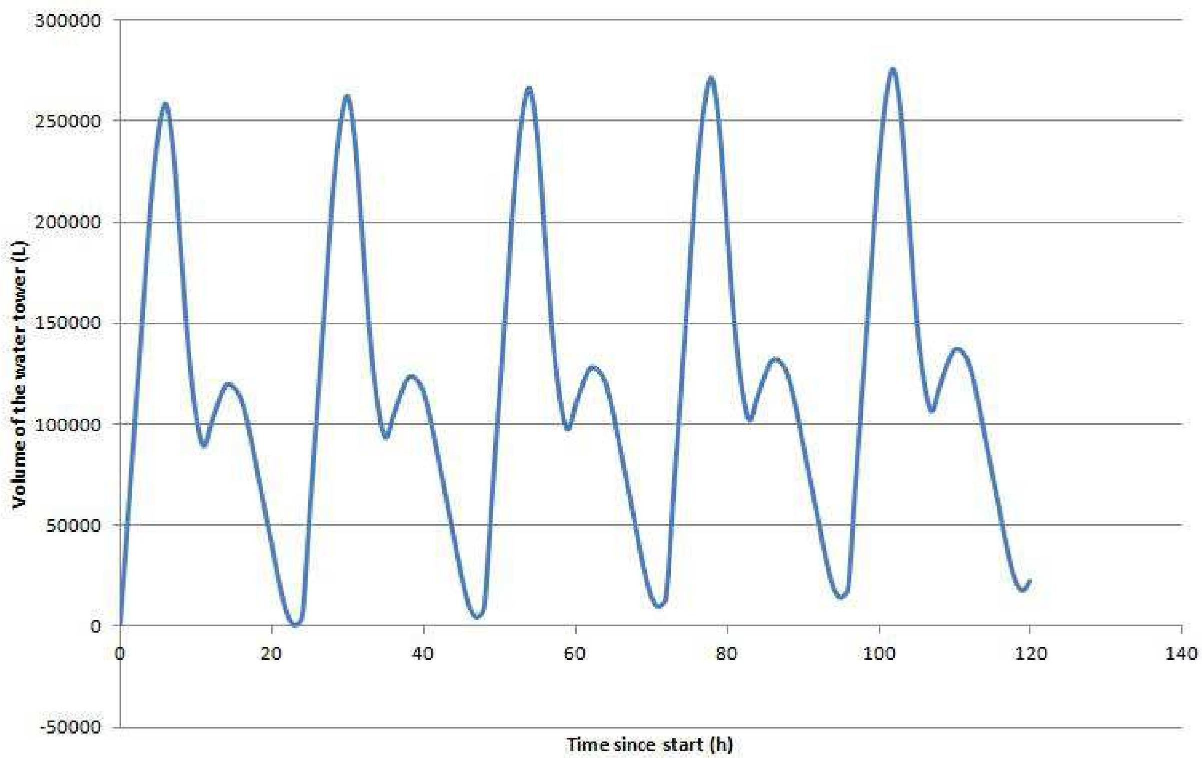
This city was estimated to consume 513 L/s [6]. 48 115 houses are reported to be regular consumers [7].

This results in:
921 L/day for every household. Our value was 1100 L and the average north american average is 1136 L [2].
An average of 38.88 L/household consumed every hour. Our model estimated a pumping power of 45.347 L/ household every hour.

This might seem like it is very far from the consumption value. It is noteworthy to say that Guelph has a lake in close proximity and that there are still functioning wells in the city, which might explain why their consumption is a little lower than north american average.

Furthermore, 38.88 is an average value while our pumping power takes in account the peak hours of consumption. Thus we can safely say that our results are not only close to the real life values but that we indeed succeeded in our optimization problem.

Volume of Water Tower as a Function of Time (Our Results)



Even so, our pumping power value is indeed divergent, if we run the simulation for a very long time, we can notice that the volume of the water tower keeps rising constantly.

One of the reasons for that is that optimization is much more complex when random values are in play. In a context where human beings needs are concerned, an assumption of uninterrupted pumping power is wrong.

Although this assumption is required when finding the optimal pumping power, this assumption has to be dropped when the simulation starts. Because of the random factor, a pumping rate will always be higher than what will bring the volume of water tower to (almost) zero at the end of every day (because the pump has to account for extra random consumption values during the hot hours that sometimes unluckily stack for many of houses).

Thus, after we find the optimal pumping power, when we run the simulation we have no choice but to make the pump stop working when the water in the water tower reaches the maximum capacity (which is more realistic).

The result for the optimal water tower volume averaged around 185 L per house. While a house consumes on average 1100 L a day, only about a quarter of that volume is required for the storage tank, thanks to the optimal pumping power. This shows how powerful and efficient this optimization tool is.

Note : To find the data collection method, please refer to our repository in the folder data at :
https://github.com/Elfelsoufim/term_project_360420_w2019-section2-elfelsoufi-stefanov/tree/master/data

References:

[1] Dowey, James W. "Water on Earth is becoming rare." *Earth Times*, 15 April 2015, http://www.earthtimes.org/politics/water-earth-rare/2868/

[2] Mostafavi, N et al. "Residential Water Consumption Modeling in the Integrated Urban Metabolism Analysis Tool (IUMAT)" *ScienceDirect*, Resources, Conservation & Recycling, April 2018. https://doi.org/10.1016/j.resconrec.2017.12.019

[3] "Residential water use." *Government of Canada*, 10 April 2017, https://www.canada.ca/en/environment-climate-change/services/environmental-indicators/residential-water-use.html

[4] "How much water does the average person use at home per day?" *U.S Geological Survey*, U.S Department of the Interior. https://water.usgs.gov/edu/qa-home-percapita.html

[5] "Household Size and Composition Around the World 2017" *United Nations*, https://www.un.org/en/development/desa/population/publications/pdf/ageing/household_size_and_composition_around_the_world_2017_data_booklet.pdf

[6] ''Guelph Water and Wastewater Servicing Master Plan-FINAL REPORT.'' City of Guelph, 2008. https://guelph.ca/wp-content/uploads/WWSMP_Sections1-8.pdf

[7] ''Guelph - City, Ontario, Canada.'' City-Data.com, Advameg Inc., http://www.city-data.com/canada/Guelph-City.html