Project Report for ECE 351

Lab 11: Z - Transform Operations

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ECE351 Code Repository:
$https://github.com/ElfinPeach/ECE351\_Code.git$

ECE351 Report Repository:
$https://github.com/ElfinPeach/ECE351\_Report.git$

Contents

1 Objective

The objective of this lab is to analyze a discrete system using Python's built-in functions and a function developed by Christopher Felton.

2 Equation

The equation analyzed in this lab is:

$$y[k] = 2x[k] - 40x[k-1] + 10y[k-1] - 16y[k-2]$$

2.1 Hand Derivation of H(z)

Below is the hand calculated z-transform of the function:

$$Y(z) = 2X(z) - 40(z^{-1}X(z) - x[-1] + 10(z^{-1}Y(z) - y[-1]) - 16(Y(z)z^{-2} - z^{-1}y[-1] - y[-2])$$

Knowing that the system is initially at rest, the function can be further simplified to:

$$Y(z) = 2X(z) - 40z^{-1}X(z) + 10z^{-1}Y(z) - 16z^{-2}Y(z)$$

$$(40z^{-1} - 2)X(z) = (-16z^{-2} + 10z^{-1} - 1)Y(z)$$

$$\frac{Y(z)}{X(z)} = H(z) = \frac{40z^{-1} - 2}{-16z^{-2} + 10z^{-1} - 1}$$

$$H(z) = \frac{2z^2 - 40z}{(z-8)(z-2)}$$

2.2 h[k] Hand Derivation

Below is the steps used to go beck to the h[k] domain.

$$H(z) = \frac{2z^2 - 40z}{(z-8)(z-2)}$$

$$\frac{H(z)}{z} = \frac{2z - 40}{(z-8)(z-2)}$$

Using partial fraction decomposition it can be found that:

$$\frac{H(z)}{z} = \frac{-4}{z-8} + \frac{6}{z-2}$$

Multiplying by z the result is:

$$H(z) = 6\frac{z}{z-8} - 4\frac{z}{z-2}$$

The inverse Z transform of H(z) is:

$$h[k] = (6(2)^k - 4(8)^k)u[k]$$

2.3 Computer Results

Below are the results fro the almighty computer code that verify partial fraction expansion.

```
Res:  [ 6.  -4.]
Poles:  [2.  8.]
Coefficients:  []
```

2.4 Figures

Below are the figures related to the equation.

Figure 1: ZPlane Plot

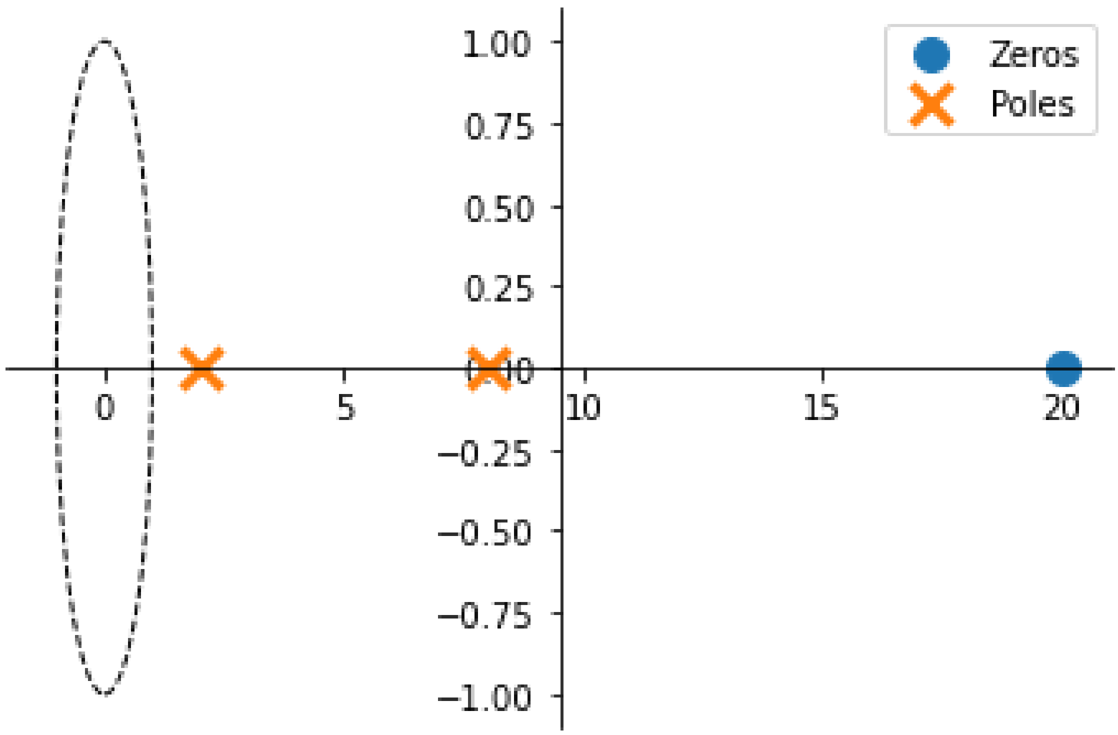
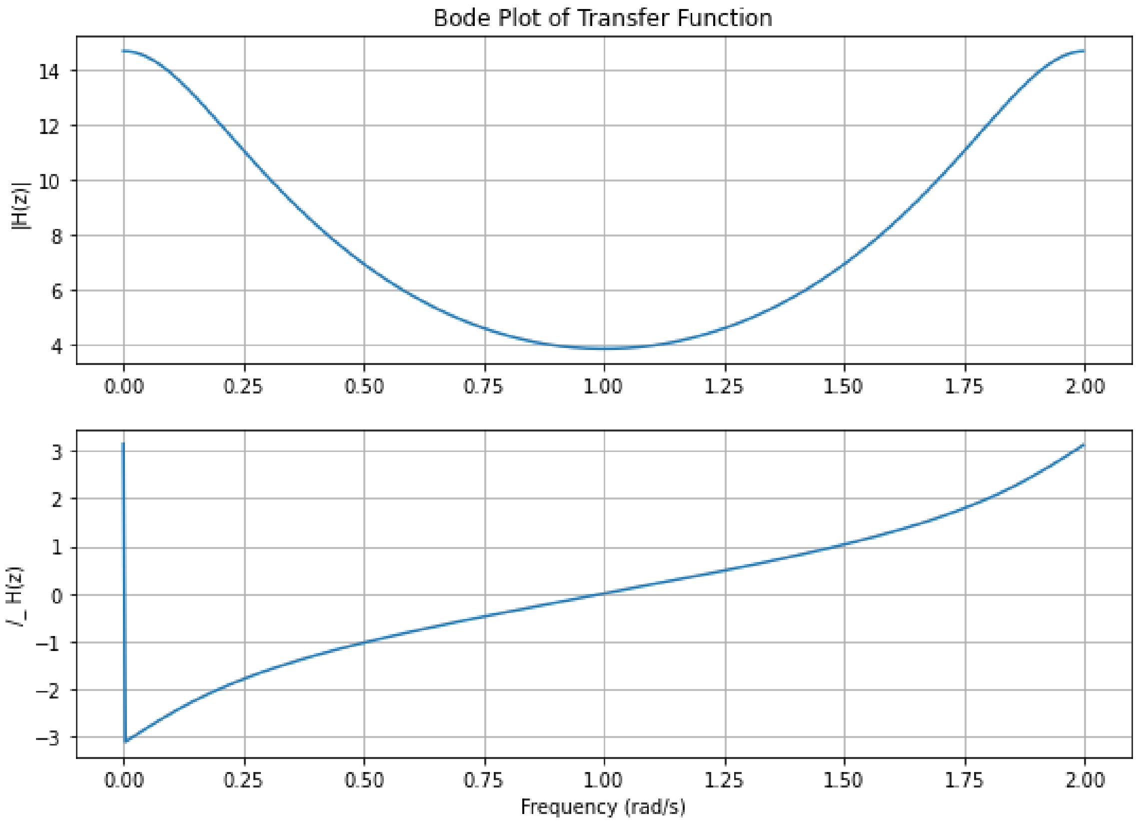


Figure 2: Bode Plot



3 Question

Looking at the plot generated in Task 4, is H(z) stable? Explain why or why not.

Since the poles (the X's on the ZPlane plot) are not within the unit circle (the dashed ellipse on the left side of the ZPlane plot), this system is unstable.