Practical Programming and Numerical Methods 2020

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June 21, 2020

Exam project: mod(54,22) = 10 → Adaptive integrator with subdivision into three subintervals

Link to repository: `https://github.com/ElhamAmr/ppnm/tree/master/Exam`

1 Adaptive integration

Adaptive quadrature is an algorithm where the integration interval is subdivided into adaptively refined subintervals until given accuracy goals in reached. In order to subdivide the interval into three and then use the quadrature on the half-intervals the following points, x_i were chosen

$$x_i = \frac{1}{6}\left\{1, 3, 5\right\},$$

from the points x_i the weights w_i are calculated using the trapezium rule. The wights can be found easily by solving the following:

$$\begin{bmatrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \\ x_1^2 & x_2^2 & x_3^2 \end{bmatrix} \cdot \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{1}{2} \\ \frac{1}{3} \end{bmatrix}$$

which gives

$$w_i = \frac{1}{8}\left\{3, 2, 3\right\} \text{ (trapezium rule)},$$

$$v_i = \frac{1}{3}\left\{1, 1, 1\right\} \text{ (rectangle rule)}.$$

Subdivision into three subintervals(Q_i) between the limits a and b in a uniform partition becomes

$$\int_a^b f(x)\mathrm{d}x = \underbrace{\int_a^{\left(a+\left(\frac{b-a}{3}\right)\right)} f(x)\mathrm{d}x}_{\text{Q1}} + \underbrace{\int_{\left(a+\left(\frac{b-a}{3}\right)\right)}^{\left(a+2\cdot\left(\frac{b-a}{3}\right)\right)} f(x)\mathrm{d}x}_{\text{Q2}} + \underbrace{\int_{\left(a+2\cdot\left(\frac{b-a}{3}\right)\right)}^{b} f(x)\mathrm{d}x,}_{\text{Q3}}, \tag{1}$$

where

$$a < a + \left(\frac{b-a}{3}\right) < a + 2\cdot\left(\frac{b-a}{3}\right) < b.$$