Formalization of AMR Inference via Hybrid Logic Tableaux

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Abstract

AMR and its extensions have become popular in semantic representation due to their ease of annotation by non-experts, attention to the predicative core of sentences, and abstraction away from various syntactic matter. An area where AMR and its extensions warrant improvement is formalization and suitability for inference, where it is lacking compared to other semantic representations, such as description logics, episodic logic, and discourse representation theory. This thesis presents a formalization of inference over a merging of Donatelli et al.'s (2018) AMR extension for tense and aspect and with Pustejovsky et al.'s (2019) AMR extension for quantification and scope. Inference is modeled with a merging of Blackburn and Marx's tableau method for quantified hybrid logic (*QHL*) and Blackburn and Jørgensen's tableau method for basic hybrid tense logic (*BHTL*). We motivate the merging of these AMR variants, present their interpretation and inference in the combination of *QHL* and *BHTL*, which we will call *QHTL* (quantified hybrid tense logic), and demonstrate *QHTL*'s soundess, completeness, and decidability.

1 Merging Quantified Hybrid Logic and Indexical Hybrid Tense Logic

1.1 Background

1.2 Quantified Hybrid Logic

1.3 Basic Hybrid Tense Logic

2 Quantified Hybrid Tense Logic - Syntax and Semantics

The syntax of *QHTL* is identical to *QHL* as given in Blackburn and Marx (2002) except uses of $\downarrow$ as in $\downarrow w.\phi$ are omitted along with $\Box$ and $\Diamond$ as in $\Box\phi$ and $\Diamond\phi$. $\Box$ and $\Diamond$ are replaced by their semantic equivalents F and G and their temporal duals P and H are added.

Atomic formulae are the same as in *QHL*, symbols in NOM and SVAR together with first-order atomic formulae generated from the predicate symbols and equality over the terms. Thus complex formulae are generated from the atomic formulae according to the following rules:

$$\neg\phi|\phi \wedge \psi|\phi \vee \psi|\phi \rightarrow \psi|\exists x\phi|\forall x\phi|F\phi|G\phi|P\phi|H\phi|@_n\phi$$

Since we want the domain of quantification to be indexed over the collection of nominals/times, we look to Fitting and Mendelsohn's (1998) treatment of first-order modal logic with varying domain semantics and use it to alter the *QHL* model definition to the following:

$$(T, R, D_t, I_{nom}, I_t)_{t\in T}$$

Thus with varying domain semantics a *QHTL* model is identical to the definition for a *QHL* model in that:

- (T, R) is a modal frame.
- I_{nom} is a function assigning members of T to nominals.

The differences manifest on the level of the model and interpretation. Namely, where $D = \cup_{t\in T} D_t$, (D, I_t) is a first-order model where:

- $I_t(q) \in D$ where q is a unary function symbol.

- $I_t(P) \in D^k$ where P is a k-ary predicate symbol.

Notice we've relaxed the requirement that $I_t(c) = I_{t'}(c)$ for c a constant and $t, t' \in T$, since the interpretation of the constant need not exist at both times. This permits us to distinguish between the domain of a frame and the domain of a time/world, in a way that prevents a variable x from failing to refer at a given time/world, even if it has no interpretation at that time. Intuitively this permits *QHTL* to handle interpretation of entities in natural language utterances, which while reasonable to refer to do not exist at a current time, e.g. previous and future presidents.

Free variables are handled similarly as in *QHL*. Where again $D = \cup_{t \in T} D_t$, a *QHTL* assignment is a function:

$$g : \textsf{SVAR} \cup \textsf{FVAR} \to T \cup D$$

Where state variables are sent to times/worlds and first-order variables are sent to D, the domain of the frame. Thus given a model and an assignment g, the interpretation of terms t denoted by $\bar{t}$ is defined as:

- $\overline{x} = g(x)$ for x a variable and $t \in T$ a time/world.
- $\overline{c} = I_t(c)$ for c a constant and some $t \in T$.
- For q a unary function symbol:
  - For n a nominal:
  $$\overline{@_n q} = I_{I_{nom}(n)}(q)$$
  - For n a state variable:
  $$\overline{@_n q} = I_{g(n)}(q)$$

With the final adjustment of having $g^x_{d,s}$ denoting the assignment which is just like g_s except $g_s(x) = d$ for $d \in g(s)$, we can proceed with the inductive definition for satisfaction of a formula. Given a model $\mathfrak{M}$, a variable assignment g, and a state s, the inductive definition is:

$$\begin{array}{ll}
\mathfrak{M}, g, s \Vdash P(t_1, \ldots, t_n) & \Longleftrightarrow \langle \overline{t_1}, \ldots, \overline{t_n} \rangle \in I_s(P) \\
\mathfrak{M}, g, s \Vdash t_i = t_j & \Longleftrightarrow \overline{t_i} = \overline{t_j} \\
\mathfrak{M}, g, s \Vdash n & \Longleftrightarrow I_{nom}(n) = s, \text{for } n \text{ a nominal} \\
\mathfrak{M}, g, s \Vdash w & \Longleftrightarrow g(w) = s, \text{for } w \text{ a state variable} \\
\mathfrak{M}, g, s \Vdash \neg\phi & \Longleftrightarrow \mathfrak{M}, g, s \nVdash \phi \\
\mathfrak{M}, g, s \Vdash \phi \wedge \psi & \Longleftrightarrow \mathfrak{M}, g, s \Vdash \phi \text{ and } \mathfrak{M}, g, s \Vdash \psi \\
\mathfrak{M}, g, s \Vdash \phi \vee \psi & \Longleftrightarrow \mathfrak{M}, g, s \Vdash \phi \text{ or } \mathfrak{M}, g, s \Vdash \psi \\
\mathfrak{M}, g, s \Vdash \phi \to \psi & \Longleftrightarrow \mathfrak{M}, g, s \Vdash \phi \text{ implies } \mathfrak{M}, g, s \Vdash \psi \\
\mathfrak{M}, g, s \Vdash \exists x \phi & \Longleftrightarrow \mathfrak{M}, g', s \Vdash \phi \text{ for some } x\text{-variant } g' \text{ of } g \text{ at } s \\
\mathfrak{M}, g, s \Vdash \forall x \phi & \Longleftrightarrow \mathfrak{M}, g', s \Vdash \phi \text{ for some } x\text{-variant } g' \text{ of } g \text{ at } s \\
\mathfrak{M}, g, s \Vdash F\phi & \Longleftrightarrow \mathfrak{M}, g, t \Vdash \phi \text{ for some } t \in T \text{ such that } Rst \\
\mathfrak{M}, g, s \Vdash G\phi & \Longleftrightarrow \mathfrak{M}, g, t \Vdash \phi \text{ for all } t \in T \text{ such that } Rst \\
\mathfrak{M}, g, s \Vdash P\phi & \Longleftrightarrow \mathfrak{M}, g, t \Vdash \phi \text{ for some } t \in T \text{ such that } Rts \\
\mathfrak{M}, g, s \Vdash H\phi & \Longleftrightarrow \mathfrak{M}, g, t \Vdash \phi \text{ for all } t \in T \text{ such that } Rts \\
\mathfrak{M}, g, s \Vdash @_n\phi & \Longleftrightarrow \mathfrak{M}, g, I_{nom}(n) \Vdash \phi \text{ for } n \text{ a nominal} \\
\mathfrak{M}, g, s \Vdash @_w\phi & \Longleftrightarrow \mathfrak{M}, g, g(w) \Vdash \phi \text{ for } w \text{ a state variable}
\end{array}$$

2.1 The Tableau Calculus

In the existential rules 8 c is a parameter new to the branch,
In the universal rules 9 c is a term grounded on the branch.

$$\frac{@_s \neg \phi}{\neg @_s \phi} [\neg] \qquad \frac{\neg @_s \neg \phi}{@_s \phi} [\neg\neg]$$

Figure 1: Negation rules

$$\frac{@_s(\phi \wedge \psi)}{\begin{array}{c} @_s \phi \\ @_s \psi \end{array}} [\wedge] \qquad \frac{\neg @_s(\phi \vee \psi)}{\begin{array}{c} \neg @_s \phi \\ \neg @_s \psi \end{array}} [\neg\vee]$$

$$\frac{\neg @_s(\phi \to \psi)}{\begin{array}{c} @_s \phi \\ \neg @_s \psi \end{array}} [\neg \to]$$

Figure 2: Conjunctive rules.

$$\frac{@_s(\phi \vee \psi)}{@_s \phi \mid @_s \psi} [\vee] \qquad \frac{\neg @_s(\phi \wedge \psi)}{\neg @_s \phi \mid \neg @_s \psi} [\neg\wedge]$$

$$\frac{@_s(\phi \to \psi)}{\neg @_s \phi \mid @_s \psi} [\to]$$

Figure 3: Disjunctive rules.

2.2 Soundness

The proof of the soundness of the tableau method for *QHTL* is adapted from the proof of soundness of the tableau method for $\mathcal{H}(@)$ given in Blackburn (2000).

We can observe from the tableau rules that every formula in a tableau is of the form $@_s \phi$ or $\neg @_s \phi$. We call formulae of these forms *satisfaction statements*. Give a set of satisfaction statements Σ and a tableau rule R we develop the notion of Σ^+ as an expansion of Σ by R as follows based on the different cases for R:

1. If R is *not* a branching rule, and R takes a single formula as input, and Σ^+ is the set obtained by adding to Σ the formulae yielded by applying R to $\sigma \in \Sigma$, then we say Σ^+ is the result of expanding Σ by R.
2. If R is a binary rule, and Σ^+ is the set obtained by adding to Σ the formulae yielded by applying R to $\sigma_1, \sigma_2, \sigma_2 \in \Sigma$, then we say Σ^+ is the result of expanding Σ by R.
3. If R is a branching rule, and Σ^+ is the set obtained by adding to Σ the formulae yielded by one of the possible outcomes of applying R to $\sigma_1 \in \Sigma$, then we say Σ^+ is the result of expanding Σ by R.
4. If a nominal s belongs to some formula in Σ, then $\Sigma^+ = \Sigma \cup @a_s s$ is the result of expanding Σ by Ref.
5. If a nominal s belongs to some formula in Σ, then $\Sigma^+ = \Sigma \cup t = t$ is the result of expanding Σ by $=$-Ref.

Definition 2.1 (Satisfiable by label)**.** Suppose Σ is a set of satisfaction statements and $\mathfrak{M} = (T, R, D_t, I_{nom}, I_t)_{t \in T}$ is a standard *QHTL* model. We say Σ is *satisfied by label* in $\mathfrak{M}$ under a *QHTL* assignment g if and and only if for all formulae in Σ:

1. If $@_s \phi \in \Sigma$ then $\mathfrak{M}, g, I_{nom}(s) \Vdash \phi$
2. If $\neg @_s \phi \in \Sigma$ then $\mathfrak{M}, g, I_{nom}(s) \nVdash \phi$

We say Σ is *satisfiable by label* if and only if there is a standard *QHTL* model and assignment in which it is satisfied by label.

$$\frac{@_s @_t \phi}{@_t \phi}\ [@] \qquad \frac{\neg @_s @_t \phi}{\neg @_t \phi}\ [\neg @]$$

$$\frac{[s \text{ on the branch}]}{@_s s}\ [\mathsf{Ref}] \qquad \frac{@_t s}{@_s t}\ [\mathsf{Sym}]$$

$$\frac{@_s Pt \qquad @_t u}{@_s Pu}\ P\text{–}\mathsf{Bridge} \qquad \frac{@_s Ft \qquad @_t u}{@_s Fu}\ F\text{–}\mathsf{Bridge}$$

$$\frac{@_s t \qquad @_s \phi}{@_t \phi}\ [\mathsf{Nom}] \qquad \frac{@_s t \qquad @_t \phi}{@_s \phi}\ [\mathsf{Nom}^{-1}]$$

$$\frac{@_s t \qquad @_t r}{@_s r}\ [\mathsf{Trans}]$$

Figure 4: @ rules

$$\frac{@_s F\phi}{\begin{array}{c}@_s Fa\\ @_a \phi\end{array}}\ [F] \qquad \frac{@_s P\phi}{\begin{array}{c}@_s Pa\\ @_a \phi\end{array}}\ [P]$$

$$\frac{\neg @_s G\phi}{\begin{array}{c}@_s Fa\\ \neg @_a \phi\end{array}}\ [\neg G] \qquad \frac{\neg @_s H\phi}{\begin{array}{c}@_s Pa\\ \neg @_a \phi\end{array}}\ [\neg H]$$

Figure 5: F and P rules

$$\frac{@_s H\phi \qquad @_s Pt}{@_t \phi}\ [H] \qquad \frac{@_s G\phi \qquad @_s Ft}{@_t \phi}\ [G]$$

Figure 6: G and H rules

$$\frac{@_s Pt}{@_t Fs}\ [P\text{–}\mathsf{Trans}] \qquad \frac{@_s Ft}{@_t Ps}\ [F\text{–}\mathsf{Trans}]$$

Figure 7: Temporal order rules.

$$\frac{@_s \exists x \phi(x)}{@_c \phi(c)}\ [\exists] \qquad \frac{@_s \neg \forall x \phi(x)}{\neg @_c \phi(c)}\ [\neg \forall]$$

Figure 8: Existential rules

$$\frac{@_s \forall x \phi(x)}{@_c \phi(c)}\ [\forall] \qquad \frac{\neg @_s \exists x \phi(x)}{\neg @_c \phi(c)}\ [\neg \exists]$$

Figure 9: Universal rules

$$\frac{}{t = t}\ [= \text{–}\mathsf{Ref}] \qquad \frac{@_n m}{@_n q = @_m q}\ [\mathsf{DD}]$$

$$\frac{@_s t = u \qquad @_s \phi(t)}{@_s \phi[u]}\ [\mathsf{RR}]$$

Figure 10: *QHTL* Equality rules

Theorem 2.1 (Soundess)**.** *If Σ is a set of satisfaction statements which is satisfiable by label, then for any tableau rule R, at least one of the sets obtainable by expanding Σ by R is satisfiable by label.*

(*Proof*). We prove soundness by induction on the tableau rules, with particular attention to "existential" rules as they are called in Blackburn (2000) which introduce a new parameter on the branch. In all cases discussed below let $\mathfrak{M} = (T, R, D_t, I_{nom}, I_t)_{t\in T}$ and g be the *QHTL* model and assignment in which Σ is satisfiable by label.

- *Non-branching Rules*

  We will take the $\wedge$ rule as an example. Beginning from $@_s\phi \wedge \psi$ we have:

  $$\mathfrak{M}, g, I_{nom}(s) \Vdash \phi \wedge \psi$$

  and consequentially

  $$\mathfrak{M}, g, I_{nom}(s) \Vdash \phi \iff @_s\phi$$
  $$\mathfrak{M}, g, I_{nom}(s) \Vdash \psi \iff @_s\psi$$

  Similarly for their negations if at least one of $@_s\phi \wedge \psi$ are not satisfied in $\mathfrak{M}$ under g. Thus the results of the application of the $\wedge$ rule, $@_s\phi$ and $@_s\psi$ are satisfiable in $\mathfrak{M}$ under g and the expansion of Σ by $\wedge$ is satisfiable by label. The proof

  for other non-branching rules are analogous.

- *Binary Rules* We will take the H rule as an example. Beginning with $@_sH\phi$ and $@_sPt$ we have from the former:

  $$\mathfrak{M}, g, t \Vdash \phi \text{ for all } t \in T \text{ such that } RtI_{nom}(s)$$

  and from the latter:

  $$\mathfrak{M}, g, t' \Vdash t \text{ for some } t' \in T \text{ such that } RtI_{nom}(s)$$

  and consequentially since Rts

  $$\mathfrak{M}, g, I_{nom}(t) \Vdash \phi \iff @_t\phi$$

  Similarly for their negations if at least one of $@_sH\phi$ and $@_sPt$ are not satisfied in $\mathfrak{M}$ under g. Thus the result of application of the H rule, $@_t\phi$ is satisfiable in $\mathfrak{M}$ under g and the expansion of Σ by H is satisfiable by label. The

  proof for other binary rules is analogous.

- *Branching Rules* We will take the $\vee$ rule as an example. Beginning from $@_s\phi \vee \psi$ if it's satisfied we have:

  $$\mathfrak{M}, g, I_{nom}(s) \Vdash \phi \vee \psi$$

  and consequentially at least one of

  $$\mathfrak{M}, g, I_{nom}(s) \Vdash \phi \iff @_s\phi$$

  or

  $$\mathfrak{M}, g, I_{nom}(s) \Vdash \psi \iff @_s\psi$$

  Similarly for their negations if $@_s\phi \vee \psi$ is not satisfied in $\mathfrak{M}$ under g. Thus at least one of the results of the application of the $\vee$ rule, $@_s\phi$ or $@_s\psi$ or their negations are satisfiable in $\mathfrak{M}$ under g and the expansion of Σ by $\vee$ is satisfiable by label. The proof for other branching rules are analogous.

- *"Existential" Rules* We will take the $\forall$ rule as an example. Beginning from $@_s\forall x\phi(x)$ if it's satisfied we have (where $s' = I_{nom}(s)$):

  $$\mathfrak{M}, g^x_{d,s'}, s' \Vdash \phi \text{ for all } d \in D_{s'} \iff @_s\phi(t')$$

  Similarly for its negation if $@_s\forall x\phi(x)$ is not satisfied in $\mathfrak{M}$ under g. In accordance with the constraints for the rule we can select t' to be any grounded term on the branch which is also a member of $D_{I_{nom}(s)}$ Thus the result of the application of the $\forall$ rule, or its negations are satisfiable in $\mathfrak{M}$ under g and the expansion of Σ by $\forall$ is satisfiable by label. The proof for $\neg\exists$ is analogous.

- *Ref rules* $t = t$ is a tautology, invariant of model or assignment. For $@_s s$, we begin with having s is the branch, as result we certainly have

$$\mathfrak{M}, g, I_{nom}(s) \Vdash I_{nom}(s) \iff @_s s$$

Thus the expansion of Σ by $= -\mathsf{Ref}$ or Ref is satisfiable by label.

□

$$\begin{array}{ll}
AT_x(p) & := Px \\
AT_x(n) & := x = n \\
AT_x(w) & := x = w \\
AT_x(\neg\phi) & := \langle \lambda x.\neg AT_x(\phi)\rangle(x) \\
AT_x(\phi \wedge \psi) & := \langle \lambda x.AT_x(\phi) \wedge AT_x(\psi)\rangle(x) \\
AT_x(G\phi) & := \langle \lambda x.\forall y(Rxy \to AT_y(\phi))\rangle(x) \\
AT_x(H\phi) & := \langle \lambda x.\forall y(Ryx \to AT_y(\phi))\rangle(x) \\
AT_x(@_n\phi) & := \langle \lambda x.\forall x(x = n \to AT_x(\phi))\rangle(x) \\
AT_x(P(t_1, \ldots, t_k)) & := P'(x, t_1, \ldots, t_k) \\
AT_x(t_i = t_j) & := \langle \lambda x.t_i = t_j\rangle(x) \\
AT_x(\forall v\phi) & := \langle \lambda x.\forall v AT_x(\phi)\rangle(x)
\end{array}$$

Figure 11: TEST

$$\begin{array}{ll}
AT^-_x(Px) & := p \\
AT^-_x(x = n) & := n \\
AT^-_x(x = w) & := w \\
AT^-_x(\langle \lambda x.\neg\phi\rangle(x)) & := \neg AT^-_x(\phi) \\
AT^-_x(\langle \lambda x.\neg\phi \wedge \psi\rangle(x)) & := AT^-_x(\phi) \wedge AT^-_x(\psi) \\
AT^-_x(\langle \lambda x.\forall y(Rxy \to \phi)\rangle(x)) & := GAT^-_y(\phi) \\
AT^-_x(\langle \lambda x.\forall y(Ryx \to \phi)\rangle(x)) & := HAT^-_y(\phi) \\
AT^-_x(\langle \lambda x.\forall x(x = n \to \phi)\rangle(x)) & := @_n AT^-_x(\phi) \\
AT^-_x(P'(x, t_1, \ldots, t_k)) & := P(t_1, \ldots, t_k) \\
AT^-_x(\langle \lambda x.t_i = t_j\rangle(x)) & := t_i = t_j \\
AT^-_x(\langle \lambda x.\forall v\phi\rangle(x)) & := \forall v AT^-_x(\phi)
\end{array}$$

Figure 12: TEST

2.3 Decidability

The proof of the tableau construction algorithm's termination is adapted from the proof given in Bolander and Braüner (2006) for the termination of the tableau construction algorithm for $\mathcal{H}(@)$ as described in Blackburn (2000) except extended with the universal modality.

Definition 2.2. When a formula $@_s\phi$ occurs in a tableau branch Θ we will write $@_s\phi \in \Theta$, and say ϕ is true at s on Θ or s makes ϕ true on Θ.

$$\begin{array}{ll}
P(t)^* & := @_t p \\
(t = u)^* & := @_t u \\
(Rst)^* & := @_s Ft \\
(\langle \lambda x.\phi \rangle (t))^* & := @_t AT_x^-(\langle \lambda x.\phi \rangle (x)) \\
(\langle \lambda y.\phi \rangle (t))^* & := @_t AT_y^-(\langle \lambda y.\phi \rangle (y)) \\
(AT_x(\phi)[t/x])^* & := @_t \phi \\
(\neg AT_x(\phi)[t/x])^* & := \neg @_t \phi \\
P'(s, t_1, \ldots, t_k)^* & := @_s P(t_1, \ldots, t_k) \\
(t_i = t_j)^* & := t_i t_j
\end{array}$$

Figure 13: TEST

Definition 2.3. Given a tableau branch Θ and a nominal or state variable s the *set of true formulae* at s on Θ, is written $T^\Theta(s)$ and defined as follows:

$$T^\Theta(s) = \phi | @_s \phi \in \Theta$$

Definition 2.4. A formula ϕ is a *quasi-subformula* of a formula ψ if one of the the following is the case:

1. ϕ is a subformula of ψ.

2. ϕ is of the form $\neg\chi$ where χ is a subformula ψ

Lemma 2.2 (Quasi-subformula Property). *Let $\mathcal{T}$ be a tableau with the formula $@_s\phi$ as root. For any formula $@_t\psi$ occuring on $\mathcal{T}$, ψ is a quasi-subformula of ϕ.*

$\begin{array}{|c|}\hline \text{T} \\ \hline \text{h} \\ \hline \end{array} \vdash \begin{array}{|c|}\hline \text{i} \\ \hline \text{s} \\ \hline \end{array}$ is verified by induction on the tableau rules

Definition 2.5.

Theorem 2.3.

Corollary 1.

Definition 2.6.

Theorem 2.4.

Definition 2.7.

Proposition 1.

Definition 2.8.

Definition 2.9.

Theorem 2.5.

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