Formalization of AMR Inference via Hybrid Logic Tableaux

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Abstract

AMR and its extensions have become popular in semantic representation due to their ease of annotation by non-experts, attention to the predicative core of sentences, and abstraction away from various syntactic matter. An area where AMR and its extensions warrant improvement is formalization and suitability for inference, where it is lacking compared to other semantic representations, such as description logics, episodic logic, and discourse representation theory. This thesis presents a formalization of inference over a merging of Donatelli et al.'s (2018) AMR extension for tense and aspect and with Pustejovsky et al.'s (2019) AMR extension for quantification and scope. Inference is modeled with a merging of Blackburn and Marx's tableau method for quantified hybrid logic (*QHL*) and Blackburn and Jørgensen's tableu method for basic hybrid tense logic (*BHTL*). We motivate the merging of these AMR variants, present their interpretation and inference in the combination of *QHL* and *BHTL*, which we will call *QHTL* (quantified hybrid tense logic), and demonstrate *QHTL*'s the soundess, completeness, and decidability.

1 Merging Quantified Hybrid Logic and Indexical Hybrid Tense Logic

1.1 Background

1.1.1 Quantified Hybrid Logic

1.1.2 Basic Hybrid Tense Logic

1.2 Quantified Hybrid Tense Logic

The syntax of $QHTL$ is identical to QHL except uses of $\downarrow$ as in $\downarrow w.\phi$ are omitted along with $\Box$ and $\Diamond$ as in $\Box\phi$ and $\Diamond\phi$. $\Box$ and $\Diamond$ are replaced by their semantic equivalents F and G and their temporal duals P and H are added.

Atomic formulae are the same as in QHL, symbols in NOM and SVAR together with first-order atomic formulae generated from the predicate symbols and equality over the terms. Thus complex formulae are generated from the atomic formulae according to the following rules:

$$\neg\phi|\phi \wedge \psi|\phi \vee \psi|\phi \rightarrow \psi|\exists x\phi|\forall x\phi|F\phi|G\phi|P\phi|H\phi|@_n\phi$$

Since we want the domain of quantification to be indexed over the collection of nominals/times, we alter the QHL model definition to a structure:

$$(T, R, D_w, I_{nom}, I_w)_{w\in W}$$

Identical to the definition for a QHL model in that:

- (T, R) is a modal frame.
- I_{nom} is a function assigning members of T to nominals.

The differences manifest on the level of the model and interpretation. That is, for every $t \in T$, (D_t, I_t) is a first-order model where:

- $I_t(q) \in D_t$ where q is a unary function symbol.
- $I_t(P) \subseteq^k D_t$ where P is a k-ary predicate symbol.

Notice we've relaxed the requirement that $I_t(c) = I_{t'}(c)$ for c a constant and $t, t' \in T$, since the interpretation of the constant need not exist at both times.

Free variables are handled similarly as in QHL. A $QHTL$ assignment is a function:

$$g : \mathsf{SVAR} \cup \mathsf{FVAR} \to T \cup D$$

Where state variables are sent to times/worlds and first-order variables are sent to D_t where t is the time assigned to the state variable by g. Thus given a model and an assignment g, the interpretation of terms t denoted by $\bar{t}$ is defined as:

- $\overline{x} = g_t(x)$ for x a variable and the relevant $t \in T$.
- $\overline{c} = I_t(c)$ for c a constant and some $t \in T$.
  - For q a unary function symbol:

    For n a nominal:

$$\overline{@_n q} = I_{I_{nom}}(n)$$

  - For n a state variable:

$$\overline{@_n q} = I_{g(n)}(q)$$

With the final adjustment of having $g^x_{d,s}$ denoting the assignment which is just like g_s except $g_s(x) = d$ for $d \in g(s)$, we can proceed with the inductive definition for satisfaction of a formula give a model $\mathfrak{M}$, a variable assignment g, and a state s. The inductive definition is:

$$\begin{aligned}
\mathfrak{M}, g, s \Vdash P(t_1, \ldots, t_n) &\iff \langle \overline{t_1}, \ldots, \overline{t_n} \rangle \in I_s(P) \\
\mathfrak{M}, g, s \Vdash t_i = t_j &\iff \overline{t_i} = \overline{t_j} \\
\mathfrak{M}, g, s \Vdash n &\iff I_{nom}(n) = s, \text{for } n \text{ a nominal} \\
\mathfrak{M}, g, s \Vdash w &\iff g(w) = s, \text{for } s \text{ a state variable} \\
\mathfrak{M}, g, s \Vdash \neg\phi &\iff \mathfrak{M}, g, s \nVdash \phi \\
\mathfrak{M}, g, s \Vdash \phi \wedge \psi &\iff \mathfrak{M}, g, s \Vdash \phi \text{ and } \mathfrak{M}, g, s \Vdash \psi \\
\mathfrak{M}, g, s \Vdash \phi \vee \psi &\iff \mathfrak{M}, g, s \Vdash \phi \text{ or } \mathfrak{M}, g, s \Vdash \psi \\
\mathfrak{M}, g, s \Vdash \phi \to \psi &\iff \mathfrak{M}, g, s \Vdash \phi \text{ implies } \mathfrak{M}, g, s \Vdash \psi \\
\mathfrak{M}, g, s \Vdash \exists x\phi &\iff \mathfrak{M}, g^x_{d,s}, s \Vdash \phi \text{ for some } d \in D_s \\
\mathfrak{M}, g, s \Vdash \forall x\phi &\iff \mathfrak{M}, g^x_{d,s}, s \Vdash \phi \text{ for all } d \in D_s \\
\mathfrak{M}, g, s \Vdash F\phi &\iff \mathfrak{M}, g, t \Vdash \phi \text{ for some } t \in T \text{ such that } Rst \\
\mathfrak{M}, g, s \Vdash G\phi &\iff \mathfrak{M}, g, t \Vdash \phi \text{ for all } t \in T \text{ such that } Rst \\
\mathfrak{M}, g, s \Vdash P\phi &\iff \mathfrak{M}, g, t \Vdash \phi \text{ for some } t \in T \text{ such that } Rts \\
\mathfrak{M}, g, s \Vdash H\phi &\iff \mathfrak{M}, g, t \Vdash \phi \text{ for all } t \in T \text{ such that } Rts \\
\mathfrak{M}, g, s \Vdash @_n\phi &\iff \mathfrak{M}, g, I_{nom}(n) \Vdash \phi \text{ for } n \text{ a nominal} \\
\mathfrak{M}, g, s \Vdash @_w\phi &\iff \mathfrak{M}, g, g(w) \Vdash \phi \text{ for } w \text{ a state variable}
\end{aligned}$$

1.3 The Tableau Calculus

Non-branching rules:

Branching rules:

Binary rules:

$$\dfrac{@_s\neg\phi}{\neg @_s\phi}\ [\neg] \qquad \dfrac{\neg @_s\neg\phi}{@_s\phi}\ [\neg\neg]$$

$$\dfrac{@_s @_t\phi}{@_t\phi}\ [@] \qquad \dfrac{\neg @_s @_t\phi}{\neg @_t\phi}\ [\neg @]$$

$$\dfrac{@_s F\phi}{\begin{array}{c}@_s Fa\\ @_a\phi\end{array}}\ [F] \qquad \dfrac{@_s P\phi}{\begin{array}{c}@_s Pa\\ @_a\phi\end{array}}\ [P]$$

$$\dfrac{\neg @_s G\phi}{\begin{array}{c}@_s Fa\\ \neg @_a\phi\end{array}}\ [\neg G] \qquad \dfrac{@_s H\phi}{\begin{array}{c}@_s Pa\\ \neg @_a\phi\end{array}}\ [\neg H]$$

$$\dfrac{@_s Pt}{@_t Fs}\ P\text{-trans} \qquad \dfrac{@_s Ft}{@_t Ps}\ F\text{-trans}$$

$$\dfrac{@_s\exists x\phi(x)}{@_c\phi(c)}\ [\exists] \qquad \dfrac{@_s\neg\forall x\phi(x)}{\neg @_c\phi(c)}\ [\neg\forall]$$

$$\dfrac{@_s\forall x\phi(x)}{@_c\phi(c)}\ [\forall] \qquad \dfrac{\neg @_s\exists x\phi(x)}{\neg @_c\phi(c)}\ [\neg\exists]$$

$$\dfrac{[s\text{ on the branch}]}{@_s s}\ [\text{ Ref }] \qquad \dfrac{@_t s}{@_s t}\ [\text{ Sym }]$$

$$\dfrac{}{t = t}\ [\text{ Ref }] \qquad \dfrac{@_n m}{@_n q = @_m q}\ [\text{ DD }]$$

$$\dfrac{@_n(t_i = t_j)}{t_i = t_j}\ @= \qquad \dfrac{\neg @_n(t_i = t_j)}{\neg(t_i = t_j)}\ \neg @=$$

Figure 1: Non-Branching Rules.

$$\dfrac{@_s(\phi\vee\psi)}{@_s\phi \mid @_s\psi}\ \vee \qquad \dfrac{\neg @_s(\phi\wedge\psi)}{\neg @_s\phi \mid \neg @_s\psi}\ \neg\wedge$$

$$\dfrac{@_s(\phi\to\psi)}{\neg @_s\phi \mid @_s\psi}\ \to$$

$$\dfrac{@_s H\phi \qquad @_s Pt}{@_t\phi}\ H \qquad \dfrac{@_s G\phi \qquad @_s Ft}{@_t\phi}\ G$$

$$\dfrac{@_s Pt \qquad @_t u}{@_t Pu}\ P\text{-bridge} \qquad \dfrac{@_s Ft \qquad @_t u}{@_t Fu}\ F\text{-bridge}$$

$$\dfrac{@_s t \qquad @_s\phi}{@_t\phi}\ \text{Nom} \qquad \dfrac{@_s t \qquad @_t\phi}{@_s\phi}\ \text{Nom}^{-1}$$

$$\dfrac{@_s t \qquad @_t r}{@_s r}\ \text{Trans}$$

(Sketch)

The main issue with the tableau rules for the merged logics is treatment of the quantification rules, for the existential rule, the quantifer is removed and a parameter new on the branch is substituted for the formerly bound variable, and in the universal case, the bound variable in the formula is substituted for a term already grounded on the branch (a first-order consant, parameter, or grounded definite description). What is now at issue is unlike QHL we are not using a fixed domain semantics, thus we must find a way to integrate the constraint that for universal quantification, the grounded term needs to have a known interpretation at the current world/state/time. NB: Other than the issue of encoding this constraint I see no reason why the same approach of merging would not work here as well.

1.4 Soundness and Completeness

1.4.1 Soundness

The proof of the soundness of the tableau method for QHTL is adapted from the proof of soundness of the tableau method for $\mathcal{H}(@)$ given in Blackburn (2000).

We can observe from the tableau rules that every formula in a tableau is of the form $@_s\phi$ or $\neg @_s\phi$. We call formulae of these forms *satisfaction statements*. Give a set of satisfaction statements Σ and a tableau rule R we develop the notion of Σ^+ as an expansion of Σ by R as follows based on the different cases for R:

1. If R is *not* a branching rule, and R takes a single formula as input, and Σ^+ is the set obtained by adding to Σ the formulae yielded by applying R to $\sigma \in \Sigma$, then we say Σ^+ is the result of expaning Σ by R.
2. If R is a binary rule, and Σ^+ is the set obtained by adding to Σ the formulae yielded by applying R to $\sigma_1, \sigma_2, \sigma_2 \in \Sigma$, then we say Σ^+ is the result of expaning Σ by R.
3. If R is a branching rule, and Σ^+ is the set obtained by adding to Σ the formulae yielded by one of the possible outcomes of applying R to $\sigma_1 \in \Sigma$, then we say Σ^+ is the result of expaning Σ by R.
4. If a nominal s belongs to some formula

Definition 1.1 (Satisfiable by label)**.** Testing testing

Theorem 1.1 (Soundess)**.** *The tableau method for QHTL is sound.*

(*Proof*). We prove soundness by induction on the tableau rules, with particular attention to "existential" rules as they are called in Blackburn (2000) which introduce a new parameter on the branch.

- *Non-branching Rules*
- *Binary Rules*
- *Branching Rules*
- *"Existential" Rules*
- *Ref*

□

1.4.2 Completeness

In either of the cases mentioned below, the proof of soundess would likely proceed by checking satisfaction in a way Blackburn and Jørgensen (2012) refers to being demonstrated in Blackburn (2000) For integrating basic hybrid tense logic rather than indexical hybrid logic, the completeness proof seems merely to be an issue of integrating AT_x translations $F\phi$, $P\phi$ and AT_x^- translations of their images under AT_x into the completeness proof of QHL in Blackburn and Marx (2002). NB: To my current understanding it's less clear how to give a completeness proof for QHL with full indexical hybrid tense logic, although since the proof does not seem to make much use of the structure of formulae outside of tense, it's also not clear to me that adding quantification would cause many/any issues, and if so the completeness proof would be adapted mostly from Blackburn and Jørgensen (2012) rather than Blackburn and Marx (2002).

$$
\begin{aligned}
AT_x(p) &:= Px\\
AT_x(n) &:= x = n\\
AT_x(w) &:= x = w\\
AT_x(\neg\phi) &:= \langle \lambda x.\neg AT_x(\phi)\rangle(x)\\
AT_x(\phi \wedge \psi) &:= \langle \lambda x.AT_x(\phi) \wedge AT_x(\psi)\rangle(x)\\
AT_x(G\phi) &:= \langle \lambda x.\forall y(Rxy \to AT_y(\phi))\rangle(x)\\
AT_x(H\phi) &:= \langle \lambda x.\forall y(Ryx \to AT_y(\phi))\rangle(x)\\
AT_x(@_n\phi) &:= \langle \lambda x.\forall x(x = n \to AT_x(\phi))\rangle(x)\\
AT_x(P(t_1,\dots,t_k)) &:= P'(x,t_1,\dots,t_k)\\
AT_x(t_i = t_j) &:= \langle \lambda x.t_i = t_j\rangle(x)\\
AT_x(\forall v\phi) &:= \langle \lambda x.\forall v AT_x(\phi)\rangle(x)
\end{aligned}
$$

Figure 2: TEST

$$
\begin{aligned}
AT_x^-(Px) &:= p\\
AT_x^-(x = n) &:= n\\
AT_x^-(x = w) &:= w\\
AT_x^-(\langle \lambda x.\neg\phi\rangle(x)) &:= \neg AT_x^-(\phi)\\
AT_x^-(\langle \lambda x.\neg\phi \wedge \psi\rangle(x)) &:= AT_x^-(\phi) \wedge AT_x^-(\psi)\\
AT_x^-(\langle \lambda x.\forall y(Rxy \to \phi)\rangle(x)) &:= GAT_y^-(\phi)\\
AT_x^-(\langle \lambda x.\forall y(Ryx \to \phi)\rangle(x)) &:= HAT_y^-(\phi)\\
AT_x^-(\langle \lambda x.\forall x(x = n \to \phi)\rangle(x)) &:= @_n AT_x^-(\phi)\\
AT_x^-(P'(x,t_1,\dots,t_k)) &:= P(t_1,\dots,t_k)\\
AT_x^-(\langle \lambda x.t_i = t_j\rangle(x)) &:= t_i = t_j\\
AT_x^-(\langle \lambda x.\forall v\phi\rangle(x)) &:= \forall v AT_x^-(\phi)
\end{aligned}
$$

Figure 3: TEST

$$
\begin{aligned}
P(t)^* &:= @_t p\\
(t = u)^* &:= @_t u\\
(Rst)^* &:= @_s Ft\\
(\langle \lambda x.\phi\rangle(t))^* &:= @_t AT_x^-(\langle \lambda x.\phi\rangle(x))\\
(\langle \lambda y.\phi\rangle(t))^* &:= @_t AT_y^-(\langle \lambda y.\phi\rangle(y))\\
(AT_x(\phi)[t/x])^* &:= @_t\phi\\
(\neg AT_x(\phi)[t/x])^* &:= \neg @_t\phi\\
P'(s,t_1,\dots,t_k)^* &:= @_s P(t_1,\dots,t_k)\\
(t_i = t_j)^* &:= t_i t_j
\end{aligned}
$$

Figure 4: TEST

1.5 Decidability

The proof of the tableau construction algoritheorem's termination is adapted from the proof given in Bolander and Braüner (2006) for the termination of the tableau construction algoritheorem for $\mathcal{H}(@)$ as described in Blackburn (2000) except extended with the universal modality.

Lemma 1.2 (Quasi-subformula property)**.**

Definition 1.2.

Theorem 1.3.

Corollary 1.

Definition 1.3.

Theorem 1.4.

Definition 1.4.

Proposition 1.

Definition 1.5.

Definition 1.6.

Theorem 1.5.

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