Formalization of AMR Inference via Hybrid Logic Tableaux

Eli Goldner

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Abstract

AMR and its extensions have become popular in semantic representation due to their ease of annotation by non-experts, attention to the predicative core of sentences, and abstraction away from various syntactic matter. An area where AMR and its extensions warrant improvement is formalization and suitability for inference, where it is lacking compared to other semantic representations, such as description logics, episodic logic, and discourse representation theory. This thesis presents a formalization of inference over a merging of Donatelli et al.'s (2018) AMR extension for tense and aspect and with Pustejovsky et al.'s (2019) AMR extension for quantification and scope. Inference is modeled with a merging of Blackburn and Marx's tableaux method for quantified hybrid logic (*QHL*) and Blackburn and Jørgensen's tableux method for basic hybrid tense logic (*BHTL*). We motivate the merging of these AMR variants, present their interpretation and inference in the combination of *QHL* and *BHTL*, which we will call *QHTL* (quantified hybrid tense logic), and demonstrate *QHTL*'s the soundess, completeness, and decidability.

1 Merging Quantified Hybrid Logic and Indexical Hybrid Tense Logic

1.1 Background

1.1.1 Quantified Hybrid Logic

1.1.2 Basic Hybrid Tense Logic

1.2 Quantified Hybrid Tense Logic

The syntax of $QHTL$ is identical to QHL except uses of $\downarrow$ as in $\downarrow w.\phi$ are omitted along with $\Box$ and $\Diamond$ as in $\Box\phi$ and $\Diamond\phi$. $\Box$ and $\Diamond$ are replaced by their semantic equivalents F and G and their temporal duals P and H are added.

Atomic formulae are the same as in QHL, symbols in NOM and SVAR together with first-order atomic formulae generated from the predicate symbols and equality over the terms. Thus complex formulae are generated from the atomic formulae according to the following rules:

$$\neg\phi|\phi \wedge \psi|\phi \vee \psi|\phi \rightarrow \psi|\exists x\phi|\forall x\phi|F\phi|G\phi|P\phi|H\phi|@_n\phi$$

Since we want the domain of quantification to be indexed over the collection of nominals/times, we alter the QHL model definition to a structure:

$$(T, R, D_w, I_{nom}, I_w)_{w\in W}$$

Identical to the definition for a QHL model in that:

- (T, R) is a modal frame.
- I_{nom} is a function assigning members of T to nominals.

The differences manifest on the level of the model and interpretation. That is, for every $t \in T$, (D_t, I_t) is a first-order model where:

- $I_t(q) \in D_t$ where q is a unary function symbol.
- $I_t(P) \subseteq^k D_t$ where P is a k-ary predicate symbol.

Notice we've relaxed the requirement that $I_t(c) = I_{t'}(c)$ for c a constant and $t, t' \in T$, since the interpretation of the constant need not exist at both times.

Free variables are handled similarly as in QHL. A $QHTL$ assignment is a function:

$$g : \mathsf{SVAR} \cup \mathsf{FVAR} \to T \cup D$$

Where state variables are sent to times/worlds and first-order variables are sent to D_t where t is the time assigned to the state variable by g. Thus given a model and an assignment g, the interpretation of terms t denoted by $\bar{t}$ is defined as:

- $\overline{x} = g_t(x)$ for x a variable and the relevant $t \in T$.
- $\overline{c} = I_t(c)$ for c a constant and some $t \in T$.
  - For q a unary function symbol:

    For n a nominal:

$$\overline{@_n q} = I_{I_{nom}}(n)$$

  - For n a state variable:

$$\overline{@_n q} = I_{g(n)}(q)$$

With the final adjustment of having $g^x_{d,s}$ denoting the assignment which is just like g_s except $g_s(x) = d$ for $d \in g(s)$, we can proceed with the inductive definition for satisfaction of a formula give a model $\mathfrak{M}$, a variable assignment g, and a state s. The inductive definition is:

$$
\begin{array}{ll}
\mathfrak{M}, g, s \Vdash P(t_1, \ldots, t_n) & \iff \langle \overline{t_1}, \ldots, \overline{t_n} \rangle \in I_s(P) \\
\mathfrak{M}, g, s \Vdash t_i = t_j & \iff \overline{t_i} = \overline{t_j} \\
\mathfrak{M}, g, s \Vdash n & \iff I_{nom}(n) = s, \text{for } n \text{ a nominal} \\
\mathfrak{M}, g, s \Vdash w & \iff g(w) = s, \text{for } s \text{ a state variable} \\
\mathfrak{M}, g, s \Vdash \neg\phi & \iff \mathfrak{M}, g, s \nVdash \phi \\
\mathfrak{M}, g, s \Vdash \phi \wedge \psi & \iff \mathfrak{M}, g, s \Vdash \phi \text{ and } \mathfrak{M}, g, s \Vdash \psi \\
\mathfrak{M}, g, s \Vdash \phi \vee \psi & \iff \mathfrak{M}, g, s \Vdash \phi \text{ or } \mathfrak{M}, g, s \Vdash \psi \\
\mathfrak{M}, g, s \Vdash \phi \to \psi & \iff \mathfrak{M}, g, s \Vdash \phi \text{ implies } \mathfrak{M}, g, s \Vdash \psi \\
\mathfrak{M}, g, s \Vdash \exists x\phi & \iff \mathfrak{M}, g^x_{d,s}, s \Vdash \phi \text{ for some } d \in D_s \\
\mathfrak{M}, g, s \Vdash \forall x\phi & \iff \mathfrak{M}, g^x_{d,s}, s \Vdash \phi \text{ for all } d \in D_s \\
\mathfrak{M}, g, s \Vdash F\phi & \iff \mathfrak{M}, g, t \Vdash \phi \text{ for some } t \in T \text{ such that } Rst \\
\mathfrak{M}, g, s \Vdash G\phi & \iff \mathfrak{M}, g, t \Vdash \phi \text{ for all } t \in T \text{ such that } Rst \\
\mathfrak{M}, g, s \Vdash P\phi & \iff \mathfrak{M}, g, t \Vdash \phi \text{ for some } t \in T \text{ such that } Rts \\
\mathfrak{M}, g, s \Vdash H\phi & \iff \mathfrak{M}, g, t \Vdash \phi \text{ for all } t \in T \text{ such that } Rts \\
\mathfrak{M}, g, s \Vdash @_n\phi & \iff \mathfrak{M}, g, I_{nom}(n) \Vdash \phi \text{ for } n \text{ a nominal} \\
\mathfrak{M}, g, s \Vdash @_w\phi & \iff \mathfrak{M}, g, g(w) \Vdash \phi \text{ for } w \text{ a state variable}
\end{array}
$$

$$
\begin{array}{ll}
AT_x(p) & := Px \\
AT_x(n) & := x = n \\
AT_x(\neg\phi) & := \langle \lambda x. \neg AT_x(\phi) \rangle (x) \\
AT_x(\phi \wedge \psi) & := \langle \lambda x. AT_x(\phi) \wedge AT_x(\psi) \rangle (x) \\
AT_x(G\phi) & := \langle \lambda x. \forall y (Rxy \rightarrow AT_y(\phi)) \rangle (x) \\
AT_x(H\phi) & := \langle \lambda x. \forall y (Ryx \rightarrow AT_y(\phi)) \rangle (x) \\
AT_x(@_n\phi) & := \langle \lambda x. \forall x (x = n \rightarrow AT_x(\phi)) \rangle (x) \\
AT_x(P(t_1, \ldots, t_k)) & := P'(x, t_1, \ldots, t_k) \\
AT_x(t_i = t_j) & := \langle \lambda x. t_i = t_j \rangle (x) \\
AT_x(\forall v \phi) & := \langle \lambda x. \forall v AT_x(\phi) \rangle (x)
\end{array}
$$

Figure 1: TEST

$$\frac{@_s \neg\phi}{\neg @_s \phi} [\neg] \qquad \frac{\neg @_s \neg\phi}{@_s \phi} [\neg\neg]$$

$$\frac{@_s @_t \phi}{@_t \phi} [@] \qquad \frac{\neg @_s @_t \phi}{\neg @_t \phi} [\neg @]$$

$$\frac{@_s F\phi}{\begin{array}{c} @_s Fa \\ @_a \phi \end{array}} [F] \qquad \frac{@_s P\phi}{\begin{array}{c} @_s Pa \\ @_a \phi \end{array}} [P]$$

$$\frac{\neg @_s G\phi}{\begin{array}{c} @_s Fa \\ \neg @_a \phi \end{array}} [\neg G] \qquad \frac{@_s H\phi}{\begin{array}{c} @_s Pa \\ \neg @_a \phi \end{array}} [\neg H]$$

$$\frac{@_s Pt}{@_t Fs} P\text{-trans} \qquad \frac{@_s Ft}{@_t Ps} F\text{-trans}$$

$$\frac{@_s \exists x \phi(x)}{@_c \phi(c)} [\exists] \qquad \frac{@_s \neg \forall x \phi(x)}{\neg @_c \phi(c)} [\neg \forall]$$

$$\frac{@_s \forall x \phi(x)}{@_c \phi(c)} [\forall] \qquad \frac{\neg @_s \exists x \phi(x)}{\neg @_c \phi(c)} [\neg \exists]$$

$$\frac{[s \text{ on the branch}]}{@_s s} [\text{ Ref }] \qquad \frac{@_t s}{@_s t} [\text{ Sym }]$$

$$\frac{}{t = t} [\text{ Ref }] \qquad \frac{@_n m}{@_n q = @_m q} [\text{ DD }]$$

$$\frac{@_n (t_i = t_j)}{t_i = t_j} @ = \qquad \frac{\neg @_n (t_i = t_j)}{\neg (t_i = t_j)} \neg @ =$$

Figure 2: Non-Branching Rules.

$$\frac{@_s (\phi \vee \psi)}{@_s \phi \mid @_s \psi} \vee \qquad \frac{\neg @_s (\phi \wedge \psi)}{\neg @_s \phi \mid \neg @_s \psi} \neg \wedge$$

$$\frac{@_s (\phi \rightarrow \psi)}{\neg @_s \phi \mid @_s \psi} \rightarrow$$

$$\frac{@_s H\phi \qquad @_s Pt}{@_t \phi}\,H \qquad\qquad \frac{@_s G\phi \qquad @_s Ft}{@_t \phi}\,G$$

$$\frac{@_s Pt \qquad @_t u}{@_t Pu}\,P\text{-bridge} \qquad\qquad \frac{@_s Ft \qquad @_t u}{@_t Fu}\,F\text{-bridge}$$

$$\frac{@_s t \qquad @_s \phi}{@_t \phi}\,\text{Nom} \qquad\qquad \frac{@_s t \qquad @_t \phi}{@_s \phi}\,\text{Nom}^{-1}$$

$$\frac{@_s t \qquad @_t r}{@_s r}\,\text{Trans}$$

1.3 The Tableaux Calculus

Non-branching rules:

Branching rules:

Binary rules:

(Sketch)

The main issue with the tableaux of the merged logics is treatment of the quantification rules, for the existential rule, the quantifer is removed and a parameter new on the branch is substituted for the formerly bound variable, and in the universal case, the bound variable in the formula is substituted for a term already grounded on the branch (a first-order consant, parameter, or grounded definite description). What is now at issue is unlike QHL we are not using a fixed domain semantics, thus we must find a way to integrate the constraint that for universal quantification, the grounded term needs to have a known interpretation at the current world/state/time. NB: Other than the issue of encoding this constraint I see no reason why the same approach of merging would not work here as well.

1.4 Soundness and Completeness

The proof of the soundness of the tableaux method for QHTL is adapted from the proof of soundness of the tableaux method for $\mathcal{H}(@)$ given in Blackburn (2000).

We can observe from the tableaux rules that every formula in a tableaux is of the form $@_s\phi$ or $\neg @_s\phi$. We call formulae of these forms *satisfaction statements*. Give a set of satisfaction statements Σ and a tableaux rule R we develop the notion of Σ^+ as an expansion of Σ by R as follows based on the different cases for R:

1. If R is *not* a branching rule, and R takes a single formula as input, and Σ^+ is the set obtained by adding to Σ the formulae yielded by applying R to $\sigma \in \Sigma$, then we say Σ^+ is the result of expaning Σ by R.
2. If R is a binary rule, and Σ^+ is the set obtained by adding to Σ the formulae yielded by applying R to $\sigma_1, \sigma_2, \sigma_2 \in \Sigma$, then we say Σ^+ is the result of expaning Σ by R.
3. If R is a branching rule, and Σ^+ is the set obtained by adding to Σ the formulae yielded by one of the possible outcomes of applying R to $\sigma_1 \in \Sigma$, then we say Σ^+ is the result of expaning Σ by R.
4. If a nominal s belongs to some formula

1.5 Decidability

1.5.1 Decidabililty of the Merged Logic

(Proof Sketch)

While $\mathcal{H}(\downarrow @)$ is not decidable, $\mathcal{H}(@)$ is (Areces et al., 1999). Quantified hybrid logic makes use of $\downarrow$ (Blackburn and Marx, 2002), but modulo $\downarrow$ and quantification over first-order variables does not differ from $\mathcal{H}(@)$. Basic hybrid tense logic does not make use of $\downarrow$ (Blackburn and Jørgensen, 2012), and differs only from $\mathcal{H}(@)$ in replacing the $\Box$ and $\Diamond$ with F and P, which respectively have the same sematics and similar semantics (the direction of the accesibility relation is changed) to $\Diamond$. Similarly for G and H respectively in relation to $\Box$. Thus given the absence of $\downarrow$ in the merged logic, replacing

$\Box$ and $\Diamond$ with F and P (and by extension G and H) will not have negatively affect the decidability given the analogous complexity of F and P to $\Diamond$. This is especially the case for us because nominals are document creation times, of which there will necessarily be a finite number, all totally ordered, which in turn will make checking accessibility more efficient. Keeping quantification over first-order variables will not affect decidability since we take the domain of quantification to be objects that exist at a particular world, that is at the time indicated by the nominal which picks out that world. That is we use presentist quantification as opposed to eternalist quantification.

1.5.2 Termination of the Tableaux/Decision Procedure

Bolander and Braüner (2006) seems like the clearest place to start from for termination of the tableaux. Norgėla and Šalaviejienė (2007) and Norgėla (2012) seem potentially useful but they discuss things in terms of sequents rather than tableaux.

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