

# Formalization of AMR Inference via Hybrid Logic Tableaux

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## Abstract

AMR and its extensions have become popular in semantic representation due to their ease of annotation by non-experts and attention to the predicative core of sentences, abstracting away from syntactic differences. An area where AMR and its extensions warrant improvement is formalization and suitability for inference, where it is lacking compared to other semantic/knowledge representations such as description logics, episodic logic, and discourse representation theory. This thesis presents a formalization of inference over AMR variants annotated for tense and aspect along with quantification and scope, via Blackburn and Marx's tableaux method for quantified hybrid logic, and Blackburn and Jørgensen's tableaux method for basic hybrid tense logic. Hybrid logic's nominals are used to handle tense (and non-habitual aspect) in AMR via Blackburn's treatment of Reichenbach tenses for hybrid logic. Quantification, negation, and modality are handled natively in quantified hybrid logic. We motivate the merging of these AMR variants, present their interpretation and inference in the combined quantified hybrid logic and basic hybrid tense logic, and demonstrate the soundness, completeness, and decidability of the combined logics.

## 1 Introduction

## 2 Related Work

We draw from a number of areas which motivate this approach, namely designing semantic representations for inferentiality, the history and goals of AMR and its different annotations, and hybrid logic with its variants and their accompanying tableaux methods for proof.

### 2.1 Inference in Semantic Representation

Semantic representation is the task of representing meaning at the sentential and potentially the discourse levels of language in a formally specifiable way.

## 2.1.1 Discourse Representation Theory

## 2.1.2 Description Logics

## 2.1.3 Situation Semantics Influenced – TTR – Robin Cooper / Jonathan Ginzburg / Massimiliano Poesio

## 2.1.4 Episodic Logic

## 2.2 Semantic Features in AMR and Possibility of Inference

## 2.3 Hybrid Logic and Our Chosen Semantic Features

## 3 Merging AMR Annotations

### 3.1 AMR Annotated for Tense and Aspect

### 3.2 AMR Annotated for Scope and Quantification

## 4 Merging Quantified Hybrid Logic and Indexical Hybrid Tense Logic

### 4.1 Background

#### 4.1.1 Quantified Hybrid Logic

#### 4.1.2 Basic Hybrid Tense Logic

### 4.2 Quantified Hybrid Tense Logic

The syntax of *QHTL* is identical to *QHL* except uses of  $\downarrow$  as in  $\downarrow w.\phi$  are omitted along with  $\Box$  and  $\Diamond$  as in  $\Box\phi$  and  $\Diamond\phi$ .  $\Box$  and  $\Diamond$  are replaced by their semantic equivalents  $F$  and  $G$  and their temporal duals  $P$  and  $H$  are added.

Atomic formulae are the same as in *QHL*, symbols in **NOM** and **SVAR** together with first-order atomic formulae generated from the predicate symbols and equality over the terms. Thus complex formulae are generated from the atomic formulae according to the following rules:

$$\neg\phi|\phi \wedge \psi|\phi \vee \psi|\phi \rightarrow \psi|\exists x\phi|\forall x\phi|F\phi|G\phi|P\phi|H\phi|\@_n\phi$$

Since we want the domain of quantification to be indexed over the collection of nominals/times, we alter the *QHL* model definition to a structure:

$$(T, R, D_w, I_{nom}, I_w)_{w \in W}$$

Identical to the definition for a *QHL* model in that:

- $(T, R)$  is a modal frame.
- $I_{nom}$  is a function assigning members of  $T$  to nominals.

The differences manifest on the level of the model and interpretation. That is, for every  $t \in T$ ,  $(D_t, I_t)$  is a first-order model where:

- $I_t(q) \in D_t$  where  $q$  is a unary function symbol.
- $I_t(P) \subseteq^k D_t$  where  $P$  is a  $k$ -ary predicate symbol.

Notice we've relaxed the requirement that  $I_t(c) = I_{t'}(c)$  for  $c$  a constant and  $t, t' \in T$ , since the interpretation of the constant need not exist at both times.

Free variables are handled similarly as in *QHL*. A *QHTL* assignment is a function:

$$g : \text{SVAR} \cup \text{FVAR} \rightarrow T \cup D$$

Where state variables are sent to times/worlds and first-order variables are sent to  $D_t$  where  $t$  is the time assigned to the state variable by  $g$ . Thus given a model and an assignment  $g$ , the interpretation of terms  $t$  denoted by  $\bar{t}$  is defined as:

- $\bar{x} = g_t(x)$  for  $x$  a variable and the relevant  $t \in T$ .

- $\bar{c} = I_t(c)$  for  $c$  a constant and some  $t \in T$ .

- For  $q$  a unary function symbol:

For  $n$  a nominal:

$$\overline{@_n q} = I_{I_{nom}}(n)$$

- \* For  $n$  a state variable:

$$\overline{@_n q} = I_{g(n)}(q)$$

With the final adjustment of having  $g_{d,s}^x$  denoting the assignment which is just like  $g_s$  except  $g_s(x) = d$  for  $d \in g(s)$ , we can proceed with the inductive definition for satisfaction of a formula give a model  $m$ , a variable assignment  $g$ , and a state  $s$ . (Will give this in full soon)

## 4.3 The Tableaux Calculus

(Sketch)

The main issue with the tableaux of the merged logics is treatment of the quantification rules, for the existential rule, the quantifier is removed and a parameter new on the branch is substituted for the formerly bound variable, and in the universal case, the bound variable in the formula is substituted for a term already grounded on the branch (a first-order constant, parameter, or grounded definite description). What is now at issue is unlike *QHL* we are not using a fixed domain semantics, thus we must find a way to integrate the constraint that for universal quantification, the grounded term needs to have a known interpretation at the current world/state/time. NB: Other than the issue of encoding this constraint I see no reason why the same approach of merging would not work here as well.

## 4.4 Soundness and Completeness

(Sketch)

In either of the cases mentioned below, the proof of soundness would likely proceed by checking satisfaction in a way Blackburn and Jørgensen (2012) refers to being demonstrated in Blackburn (2000). For integrating basic hybrid tense logic rather than indexical hybrid logic, the completeness proof seems merely to be an issue of integrating  $AT_x$  translations  $F\phi$ ,  $P\phi$  and  $AT_x^-$  translations of their images under  $AT_x$  into the completeness proof of *QHL* in Blackburn and Marx (2002). NB: To my current understanding it's less clear how to give a completeness proof for *QHL* with full indexical hybrid tense logic, although since the proof does not seem to make much use of the structure of formulae outside of tense, it's also not clear to me that adding quantification would cause many/any issues, and if so the completeness proof would be adapted mostly from Blackburn and Jørgensen (2012) rather than Blackburn and Marx (2002).

## 4.5 Decidability

### 4.5.1 Decidability of the Merged Logic

(Proof Sketch)

While  $\mathcal{H}(\downarrow @)$  is not decidable,  $\mathcal{H}(@)$  is (Areces et al., 1999). Quantified hybrid logic makes use of  $\downarrow$  (Blackburn and Marx, 2002), but modulo  $\downarrow$  and quantification over first-order variables does not differ from  $\mathcal{H}(@)$ . Indexical hybrid tense logic does not make use of  $\downarrow$  (Blackburn and Jørgensen, 2012), and differs only from  $\mathcal{H}(@)$  in replacing the  $\Box$  and  $\Diamond$  with  $F$  and  $P$ , which respectively have the same semantics and similar semantics (the direction of the accessibility relation is changed) to  $\Diamond$ . Similarly for  $G$  and  $H$  respectively in relation to  $\Box$ . Thus given the absence of  $\downarrow$  in the merged logic, replacing  $\Box$  and  $\Diamond$  with  $F$  and  $P$  (and by extension  $G$  and  $H$ ) will not have negatively affect the decidability given the analogous complexity of  $F$  and  $P$  to  $\Diamond$ . This is especially the case for us because nominals are document creation times, of which there will necessarily be a finite number, all totally ordered, which in turn will make checking accessibility more efficient. Keeping quantification over first-order variables will not affect decidability since we take the domain of quantification to be objects that exist at a particular world, that is at the time indicated by the nominal which picks out that world. That is we use presentist quantification as opposed to eternalist quantification.

#### 4.5.2 Termination of the Tableaux/Decision Procedure

Bolander and Braüner (2006) seems like the clearest place to start from for termination of the tableaux. Norgèla and Šalaviejenė (2007) and Norgèla (2012) seem potentially useful but they discuss things in terms of sequents rather than tableaux.

## 5 AMR Interpretation in Hybrid Logic

### 5.1 Examples

- (1) a. Carl submitted the forms and everyone will sign up again tomorrow.

b.

```
(a / and
  :op1 (s / scope
    :pred (f / fill-out-03 :ongoing - :complete + :time (b / before :op1 (n / now))
      :ARG0 (p / person
        :name (n2 / name
          :op "Carl"))
      :ARG1 (f2 / form))
    :ARG0 p
    :ARG1 f2)
  :op2 (s2 / scope
    :pred (m / submit-01 :ongoing - :complete + :time (a2 / after :op1 n)
      :ARG0 (p2 / person
        :mod (a3 / all))
      :ARG1 f2)
    :ARG0 p2
    :ARG1 f2))
```

c. It was impossible not to notice the license plate.

d.

```
(s / scope
  :pred (p / possible-01
    :ARG0 (n / notice-01 :ongoing - :complete + :time (b / before :op1 (n2 / now))
      :polarity (n3 / not)
      :ARG1 (c / car)
      :polarity (n4 / not))
    :ARG0 n4
    :ARG1 p))
```

NB: Will complete these translations in full.

### 5.2 Extraction Steps

With the chosen annotation, the root node can consist of either a logical connective (`and`, `or`, or `cond`) linking two AMR graphs, or a `scope` node with its following predicate and arguments.

### 5.3 General Extraction Algorithm

## 6 Future Work

### 6.1 ↓ and Quantification over Nominals

Main points, at the cost of undecidability with adding  $\downarrow$  some additional things can be done, and at the cost of the integration of generalized quantifiers you can ostensibly handle even things like habitual aspect.

## 6.2 AMR Reentrancy and Non-Temoral Nominals

There are some difficulties with maintaining the usual notion of possible worlds being maximal with this idea, but there seems to be a direct sympathy between the predicative core of an AMR sentence and in general reentrancy of the nodes with the idea of a nominal as a “point of view” rather than the “name” of a world. Maybe things like epistemic logic could be helpful here.

## 6.3 Automated Inference and HTab

HTab (Hoffmann and Areces, 2009) provides an implementation of  $\mathcal{H}(@, A)$ , which does not natively provide a way to reason with  $P$ ,  $H$ , or first-order quantification. The effort required in making the needed changes to handle these remains to be determined.

## 6.4 The Future of AMR and Parsing for Semantic Features

To what extent can current AMR parsers extract the needed semantic features to make full use of automated inference? Of UMR, Dialogue-AMR, and the AMR annotation variants we’ve used, which logically has the best outlook?

## 7 Conclusion

## References

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