Formalization of AMR Inference via Hybrid Logic Tableaux

Eli Goldner

June 9, 2021

Abstract

AMR and its extensions have become popular in semantic representation due to their ease of annotation by non-experts, attention to the predicative core of sentences, and abstraction away from various syntactic matter. An area where AMR and its extensions warrant improvement is formalization and suitability for inference, where it is lacking compared to other semantic representations, such as description logics, episodic logic, and discourse representation theory. This thesis presents a formalization of inference over a merging of Donatelli et al.'s (2018) AMR extension for tense and aspect and with Pustejovsky et al.'s (2019) AMR extension for quantification and scope. Inference is modeled with a merging of Hansen's (2007) tableau method for first-order hybrid logic with varying domain semantics (*FHL*) and Blackburn and Jørgensen's (**?**) tableau method for basic hybrid tense logic (*BHTL*). We motivate the merging of these AMR variants, present their interpretation and inference in the combination of *FHL* and *BHTL*, which we will call *FHTL* (quantified hybrid tense logic), and demonstrate *FHTL*'s soundess, completeness, and decidability.

1 Merging Quantified Hybrid Logic and Indexical Hybrid Tense Logic

1.1 Background

1.2 Quantified Hybrid Logic

1.3 Basic Hybrid Tense Logic

2 Quantified Hybrid Tense Logic - Syntax and Semantics

The syntax of *FHTL* is identical to *FHL* as given in Hansen (2007) except uses of $\downarrow$ as in $\downarrow w.\phi$ are omitted along with $\Box$ and $\Diamond$ as in $\Box\phi$ and $\Diamond\phi$. $\Box$ and $\Diamond$ are replaced by their semantic equivalents F and G and their temporal duals P and H are added.

Atomic formulae are the same as in *FHL*, symbols in NOM and SVAR together with first-order atomic formulae generated from the predicate symbols and equality over the terms. Thus complex formulae are generated from the atomic formulae according to the following rules:

$$\neg\phi|\phi \wedge \psi|\phi \vee \psi|\phi \rightarrow \psi|\exists x\phi|\forall x\phi|F\phi|G\phi|P\phi|H\phi|@_n\phi$$

Since we want the domain of quantification to be indexed over the collection of nominals/times, we look to Fitting and Mendelsohn's (1998) treatment of first-order modal logic with varying domain semantics and use it to alter the *FHL* model definition to the following:

$$(T, R, D_t, I_{nom}, I_t)_{t\in T}$$

Thus with varying domain semantics a *FHTL* model is identical to the definition for a *FHL* model in that:

- (T, R) is a modal frame.
- I_{nom} is a function assigning members of T to nominals.

The differences manifest on the level of the model and interpretation. Namely, where $D = \cup_{t\in T} D_t$, (D, I_t) is a first-order model where:

- $I_t(q) \in D$ where q is a unary function symbol.

- $I_t(P) \in D^k$ where P is a k-ary predicate symbol.

Notice we've relaxed the requirement that $I_t(c) = I_{t'}(c)$ for c a constant and $t, t' \in T$, since the interpretation of the constant need not exist at both times. This permits us to distinguish between the domain of a frame and the domain of a time/world, in a way that prevents a variable x from failing to refer at a given time/world, even if it has no interpretation at that time. Intuitively this permits *FHTL* to handle interpretation of entities in natural language utterances, which while reasonable to refer to do not exist at a current time, e.g. previous and future presidents.

Free variables are handled similarly as in *FHL*. Where again $D = \cup_{t \in T} D_t$, a *FHTL* assignment is a function:

$$g : \mathsf{SVAR} \cup \mathsf{FVAR} \to T \cup D$$

Where state variables are sent to times/worlds and first-order variables are sent to D, the domain of the frame. Thus given a model and an assignment g, the interpretation of terms t denoted by $\overline{t}$ is defined as:

- $\overline{x} = g(x)$ for x a variable.
- $\overline{c} = I_t(c)$ for c a constant and some $t \in T$.
- For q a unary function symbol:
  - For n a nominal:
$$\overline{@_n q} = I_{I_{nom}(n)}(q)$$
  - For n a state variable:
$$\overline{@_n q} = I_{g(n)}(q)$$

Finally we say an assignment g' is an x-variant of g if g' and g on all variables except possiblly x. In particular, we say g' is an x-variant of g at t, a time, if g' and g on all variables except possiblly x and $g'(x) \in D_t$. Given a model $\mathfrak{M}$, a variable assignment g, and a state s, the inductive definition is:

$$
\begin{array}{ll}
\mathfrak{M}, g, s \models P(t_1, \ldots, t_n) & \Longleftrightarrow \langle \overline{t_1}, \ldots, \overline{t_n} \rangle \in I_s(P) \\
\mathfrak{M}, g, s \models t_i = t_j & \Longleftrightarrow \overline{t_i} = \overline{t_j} \\
\mathfrak{M}, g, s \models n & \Longleftrightarrow I_{nom}(n) = s, \text{for } n \text{ a nominal} \\
\mathfrak{M}, g, s \models w & \Longleftrightarrow g(w) = s, \text{for } w \text{ a state variable} \\
\mathfrak{M}, g, s \models \neg\phi & \Longleftrightarrow \mathfrak{M}, g, s \not\models \phi \\
\mathfrak{M}, g, s \models \phi \wedge \psi & \Longleftrightarrow \mathfrak{M}, g, s \models \phi \text{ and } \mathfrak{M}, g, s \models \psi \\
\mathfrak{M}, g, s \models \phi \vee \psi & \Longleftrightarrow \mathfrak{M}, g, s \models \phi \text{ or } \mathfrak{M}, g, s \models \psi \\
\mathfrak{M}, g, s \models \phi \to \psi & \Longleftrightarrow \mathfrak{M}, g, s \models \phi \text{ implies } \mathfrak{M}, g, s \models \psi \\
\mathfrak{M}, g, s \models \exists x\phi & \Longleftrightarrow \mathfrak{M}, g', s \models \phi \text{ for some } x\text{-variant } g' \text{ of } g \text{ at } s \\
\mathfrak{M}, g, s \models \forall x\phi & \Longleftrightarrow \mathfrak{M}, g', s \models \phi \text{ for every } x\text{-variant } g' \text{ of } g \text{ at } s \\
\mathfrak{M}, g, s \models F\phi & \Longleftrightarrow \mathfrak{M}, g, t \models \phi \text{ for some } t \in T \text{ such that } Rst \\
\mathfrak{M}, g, s \models G\phi & \Longleftrightarrow \mathfrak{M}, g, t \models \phi \text{ for all } t \in T \text{ such that } Rst \\
\mathfrak{M}, g, s \models P\phi & \Longleftrightarrow \mathfrak{M}, g, t \models \phi \text{ for some } t \in T \text{ such that } Rts \\
\mathfrak{M}, g, s \models H\phi & \Longleftrightarrow \mathfrak{M}, g, t \models \phi \text{ for all } t \in T \text{ such that } Rts \\
\mathfrak{M}, g, s \models @_n\phi & \Longleftrightarrow \mathfrak{M}, g, I_{nom}(n) \models \phi \text{ for } n \text{ a nominal} \\
\mathfrak{M}, g, s \models @_w\phi & \Longleftrightarrow \mathfrak{M}, g, g(w) \models \phi \text{ for } w \text{ a state variable}
\end{array}
$$

2.1 The Tableau Calculus

For Nom we have the constraint that if the premise $@_t\phi$ are of the form $@_t Xc$ where $X \in \{F, P, \neg G, \neg H\}$ and c is a nominal or state variable, then $@_t\phi$ is a root subformula. Similarly for Nom^{-1} and the premise $@_s\phi$.

$$\frac{@_s @_t \phi}{@_t \phi}\ [@] \qquad \frac{\neg @_s @_t \phi}{\neg @_t \phi}\ [\neg @] \qquad \frac{[s \text{ on the branch}]}{@_s s}\ [\mathsf{Ref}] \qquad \frac{@_t s}{@_s t}\ [\mathsf{Sym}]$$

$$\frac{@_s Pt}{@_t Fs}\ [P\text{–}\mathsf{Trans}] \qquad \frac{@_s Ft}{@_t Ps}\ [F\text{–}\mathsf{Trans}] \qquad \frac{@_s Pt \quad @_t u}{@_s Pu}\ P\text{–}\mathsf{Bridge} \qquad \frac{@_s Ft \quad @_t u}{@_s Fu}\ F\text{–}\mathsf{Bridge}$$

$$\frac{@_s t \quad @_s \phi}{@_t \phi}\ [\mathsf{Nom}] \qquad \frac{@_s t \quad @_t \phi}{@_s \phi}\ [\mathsf{Nom}^{-1}] \qquad \frac{@_s t \quad @_t r}{@_s r}\ [\mathsf{Trans}]$$

Figure 1: @ rules

$$\frac{@_s F\phi}{\begin{array}{c} @_s Fa \\ @_a \phi \end{array}}\ [F] \qquad \frac{@_s P\phi}{\begin{array}{c} @_s Pa \\ @_a \phi \end{array}}\ [P] \qquad \frac{\neg @_s G\phi}{\begin{array}{c} @_s Fa \\ \neg @_a \phi \end{array}}\ [\neg G] \qquad \frac{\neg @_s H\phi}{\begin{array}{c} @_s Pa \\ \neg @_a \phi \end{array}}\ [\neg H]$$

Figure 2: F and P rules

In all rules in fig. 2, the nominal a is new to the branch. We have the additional constraint that if ϕ in the premise is a nominal or state variable, then the premise must be a root subformula in order for the rule to be applicable.

Following Fitting and Mendelsohn (1998) we assume for each nominal or state variable s, there is an infinite list of parameters, where parameters are free variables which are never quantified over, arranged in such a way that different nominals/state variables never share the same parameter. Informally we write p_s to indicate a parameter is associated with a nominal/state variable s.

We also introduce the notion of a grounded term. A grounded term is either a first-order constant, a parameter, or a grounded definite description, i.e. a term of the form $@_n q$ for n a nominal and q a unary function symbol.

$$\frac{@_s \exists x \phi(x)}{@_s \phi(p_s)}\ [\exists] \qquad \frac{@_s \neg \forall x \phi(x)}{\neg @_s \phi(p_s)}\ [\neg \forall]$$

Figure 3: Existential rules

In the existential rules fig. 3, p_s is a parameter associated with the nominal s, with the constraint that it is new to the branch. Since parameters are never quantified over, p_s is free in $\phi[p_s/x]$.

$$\frac{@_s \forall x \phi(x)}{@_c \phi(t)}\ [\forall] \qquad \frac{\neg @_s \exists x \phi(x)}{\neg @_c \phi(t)}\ [\neg \exists]$$

Figure 4: Universal rules

In the universal rules fig. 4 t is a grounded term on the branch which exists at D_s.

2.2 Soundness

The proof of the soundness of the tableau method for *FHTL* is adapted from the proof of soundness of the tableau method for $\mathcal{H}(@)$ given in Blackburn (2000).

$$\frac{}{t = t}\ [\mathsf{TermRef}] \qquad \frac{@_n m}{@_n q = @_m q}\ [\mathsf{DD}] \qquad \frac{t = u \qquad @_s\phi(t)}{@_s\phi[u]}\ [\mathsf{RR}]$$

$$\frac{@_n(t_i = t_j)}{t_i = t_j}\ [@ =] \qquad \frac{\neg @_n(t_i = t_j)}{\neg(t_i = t_j)}\ [\neg @ =]$$

Figure 5: *FHTL* Equality rules

$$\frac{@_s\neg\phi}{\neg @_s\phi}\ [\neg] \qquad \frac{\neg @_s\neg\phi}{@_s\phi}\ [\neg\neg]$$

Figure 6: Negation rules

We can observe from the tableau rules that every formula in a tableau is of the form $@_s\phi$ or $\neg @_s\phi$. We call formulae of these forms *satisfaction statements*. Give a set of satisfaction statements Σ and a tableau rule R we develop the notion of Σ^+ as an expansion of Σ by R as follows based on the different cases for R:

1. If R is *not* a branching rule, and R takes a single formula as input, and Σ^+ is the set obtained by adding to Σ the formulae yielded by applying R to $\sigma \in \Sigma$, then we say Σ^+ is the result of expanding Σ by R.
2. If R is a binary rule, and Σ^+ is the set obtained by adding to Σ the formulae yielded by applying R to $\sigma_1, \sigma_2, \sigma_2 \in \Sigma$, then we say Σ^+ is the result of expanding Σ by R.
3. If R is a branching rule, and Σ^+ is the set obtained by adding to Σ the formulae yielded by one of the possible outcomes of applying R to $\sigma_1 \in \Sigma$, then we say Σ^+ is the result of expanding Σ by R.
4. If a nominal s belongs to some formula in Σ, then $\Sigma^+ = \Sigma \cup \{@_s s\}$ is the result of expanding Σ by Ref.
5. If a nominal s belongs to some formula in Σ, then $\Sigma^+ = \Sigma \cup \{@_s t = t\}$ is the result of expanding Σ by $=$-Ref.

Definition 2.1 (Satisfiable by label). Suppose Σ is a set of satisfaction statements and $\mathfrak{M} = (T, R, D_t, I_{nom}, I_t)_{t\in T}$ is a standard *FHTL* model. We say Σ is *satisfied by label* in $\mathfrak{M}$ under a *FHTL* assignment g if and and only if for all formulae in Σ:

1. If $@_s\phi \in \Sigma$ then $\mathfrak{M}, g, I_{nom}(s) \models \phi$
2. If $\neg @_s\phi \in \Sigma$ then $\mathfrak{M}, g, I_{nom}(s) \not\models \phi$

We say Σ is *satisfiable by label* if and only if there is a standard *FHTL* model and assignment in which it is satisfied by label.

Theorem 2.1 (Soundness). *If Σ is a set of satisfaction statements which is satisfiable by label, then for any tableau rule R, at least one of the sets obtainable by expanding Σ by R is satisfiable by label.*

$$\frac{@_s(\phi \wedge \psi)}{\begin{array}{c}@_s\phi\\ @_s\psi\end{array}}\ [\wedge] \qquad \frac{\neg @_s(\phi \vee \psi)}{\begin{array}{c}\neg @_s\phi\\ \neg @_s\psi\end{array}}\ [\neg\vee] \qquad \frac{\neg @_s(\phi \to \psi)}{\begin{array}{c}@_s\phi\\ \neg @_s\psi\end{array}}\ [\neg \to]$$

Figure 7: Conjunctive rules.

$$\frac{@_s(\phi \vee \psi)}{@_s\phi \mid @_s\psi}\,[\vee] \qquad \frac{\neg @_s(\phi \wedge \psi)}{\neg @_s\phi \mid \neg @_s\psi}\,[\neg\wedge] \qquad \frac{@_s(\phi \to \psi)}{\neg @_s\phi \mid @_s\psi}\,[\to]$$

Figure 8: Disjunctive rules.

$$\frac{@_sH\phi \qquad @_sPt}{@_t\phi}\,[H] \qquad \frac{@_sG\phi \qquad @_sFt}{@_t\phi}\,[G]$$

Figure 9: G and H rules

(*Proof*) We prove soundness by induction on the tableau rules, with particular attention to rules which introduce nominals new to the branch, namely $\{F, P, \neg G, \neg H\}$ fig. 2, and rules which introduce new parameters to the branch, namely the universal rules fig. 4 and existential rules fig. 3. In all cases discussed below let

$$\mathfrak{M} = (T, R, D_t, I_{nom}, I_t)_{t \in T}$$

be *FHTL* model and g the assignment in which Σ is satisfiable by label.

- *Non-branching Rules*

  We will take the $\wedge$ rule as an example. Beginning from $@_s\phi \wedge \psi$ we have:

  $$\mathfrak{M}, g, I_{nom}(s) \vDash \phi \wedge \psi$$

  and consequentially

  $$\mathfrak{M}, g, I_{nom}(s) \vDash \phi \iff @_s\phi$$

  $$\mathfrak{M}, g, I_{nom}(s) \vDash \psi \iff @_s\psi$$

  Similarly for their negations if at least one of $@_s\phi \wedge \psi$ are not satisfied in $\mathfrak{M}$ under g. Thus the results of the application of the $\wedge$ rule, $@_s\phi$ and $@_s\psi$ are satisfiable in $\mathfrak{M}$ under g and the expansion of Σ by $\wedge$ is satisfiable by label. The proofs for other non-branching rules are analogous.

- *Binary Rules* We will take the H rule as an example. Beginning with $@_sH\phi$ and $@_sPt$ we have from the former:

  $$\mathfrak{M}, g, t \vDash \phi \text{ for all } t \in T \text{ such that } RtI_{nom}(s)$$

  and from the latter:

  $$\mathfrak{M}, g, t' \vDash t \text{ for some } t' \in T \text{ such that } RtI_{nom}(s)$$

  and consequentially since Rts

  $$\mathfrak{M}, g, I_{nom}(t) \vDash \phi \iff @_t\phi$$

  Similarly for their negations if at least one of $@_sH\phi$ and $@_sPt$ are not satisfied in $\mathfrak{M}$ under g. Thus the result of application of the H rule, $@_t\phi$ is satisfiable in $\mathfrak{M}$ under g and the expansion of Σ by H is satisfiable by label. The proofs for other binary rules is analogous.

- *Branching Rules* We will take the $\vee$ rule as an example. Beginning from $@_s\phi \vee \psi$ if it's satisfied we have:

  $$\mathfrak{M}, g, I_{nom}(s) \vDash \phi \vee \psi$$

  and consequentially at least one of

  $$\mathfrak{M}, g, I_{nom}(s) \vDash \phi \iff @_s\phi$$

or

$$\mathfrak{M}, g, I_{nom}(s) \vDash \psi \iff @_s\psi$$

Similarly for their negations if $@_s\phi \vee \psi$ is not satisfied in $\mathfrak{M}$ under g. Thus at least one of the results of the application of the $\vee$ rule, $@_s\phi$ or $@_s\psi$ or their negations are satisfiable in $\mathfrak{M}$ under g and the expansion of Σ by $\vee$ is satisfiable by label. The proofs for other branching rules are analogous.

- *Existential and Universal Rules* We will take the $\forall$ rule as an example. Beginning from $@_s\forall x\phi(x)$ if it's satisfied we have (where $s' = I_{nom}(s)$):

  $$\mathfrak{M}, g', s' \vDash \phi \text{ for every } x\text{-variant of } g \text{ at } s$$

  That is for every c in $D_{I_{nom}(s)}$, $\phi[t/x]$ is satisfied in $\mathfrak{M}$ under g, similarly for $\neg\phi[t/x]$ if $@_s\forall x\phi(x)$ is not satisfied in $\mathfrak{M}$ under g. In accordance with the constraints for the rule we can select t to be any grounded term on the branch which is also a member of $D_{I_{nom}(s)}$ Thus the result of the application of the $\forall$ rule, or its negations are satisfiable in $\mathfrak{M}$ under g and the expansion of Σ by $\forall$ is satisfiable by label. The proofs for $\exists$, $\neg\exists$, and $\neg\forall$ are analogous.

- *Rules Introducing a Nominal to the Branch*

  We will take the F rule as am example. Beginning from $@_sF\phi$, if it's satisfied we have:

  $$\mathfrak{M}, g, I_{nom}(s) \vDash \phi \text{ for some } t \in T \text{ such that } RI_{nom}(s)t$$

  Let a denote a nominal such that $RI_{nom}(s)I_{nom}(a)$ as above. As a result we have:

  $$\mathfrak{M}, g, I_{nom}(s) \vDash a \iff$$

  $$\mathfrak{M}, g, t \vDash a \text{ for some } t \in T \text{ such that } RI_{nom}(s)t \iff$$

  $$\mathfrak{M}, g, I_{nom}(s) = Fa \iff @_sFa$$

  and

  $$\mathfrak{M}, g, I_{nom}(a) \vDash \phi \iff @_a\phi$$

  Similarly for their negations if $@_sF\phi$ is not satisfied in $\mathfrak{M}$ under g.

- *Ref rules* $t = t$ is a tautology, invariant of model or assignment. For $@_ss$, we begin with having s is the branch, as result we certainly have

  $$\mathfrak{M}, g, I_{nom}(s) \vDash I_{nom}(s) \iff @_ss$$

  Thus the expansion of Σ by $=-\mathsf{Ref}$ or Ref is satisfiable by label.

  Using this we have demonstrated that the results of application of a tableau rule to one or more premises reflect the validity or non-validity of the premises. □

2.3 Completeness

Definition 2.2 (σ_Θ)**.** We fix a function σ that to each tableau branch and each non-empty subset $N \subseteq Nom_\Theta$ picks out the element of N which denotes the earliest time under the accessibility relation R which is a total order. We write the function value as $\sigma_\Theta N$.

Definition 2.3 (Urfathers). Let Θ be a tableau branch in the *FHTL* calculus and let a be a nominal/state variable occuring on Θ. The *urfather* of a on Θ written $u_\Theta(a)$ is defined by:

$$u_\Theta(a) = \begin{cases} \sigma_\Theta\{b | @_ab \in \Theta\}, & \text{if } \{b | @_ab \in \Theta\} \neq \emptyset \\ a, & \text{otherwise.} \end{cases}$$

a nominal a is called an *urfather* on Θ if $a = u_\Theta(b)$ for some nominal b.

Lemma 2.2. *Where Θ is a saturated tableau branch in the FHTL calculus, we have the following:*

- *If $@_a\phi$ is a root-subformula not of the form $@_aFc$ or $@_aPc$ then $u_\Theta(a) \in \Theta$.*
- *If $@_ab \in \Theta$ then $u_\Theta(a) = u_\Theta(b)$*
- *If a is an urfather on Θ then $u_\Theta(a) = a$*

Definition 2.4 ($\mathfrak{M}^\Theta$)**.** Where Θ is a saturated tableau branch in the *FHTL* calculus, we have following model definition:

$$\mathfrak{M}^\Theta = (T^\Theta, R^\Theta, D_t, I_{nom_\Theta}, I_t)_{t \in T^\Theta}$$

from:

$$\begin{aligned} T^\Theta &= \{u_\Theta(a) | a \text{ is a nominal/state variable occuring on } \Theta\} \\ R^\Theta &= \{(a, u_\Theta(b)) \in T^2 | @_aFb \in \Theta \text{ or } @_bPa \in \Theta\} \\ I_{nom_\Theta}(a) &= I_{nom}(u_\Theta(a)) \end{aligned}$$

Finally where $\mathfrak{M}$ had the assignment g, we give $\mathfrak{M}^\Theta$ the assignment g^Θ which is exactly like g except for a state variable s, we have $g^\Theta(s) = g(u_\Theta(s))$. Interpretation of terms is adjusted accordingly.

Theorem 2.3 (Completeness).

2.4 Decidability

The proof of the tableau construction algorithm's termination is adapted from the proof given in Bolander and Blackburn (2009) for the termination of the tableau construction algorithm for $\mathcal{H}(@)$ as described in Blackburn (2000) except extended with the universal modality.

Definition 2.5. When a formula $@_s\phi$ occurs in a tableau branch Θ we will write $@_s\phi \in \Theta$, and say ϕ is true at s on Θ or s makes ϕ true on Θ.

Definition 2.6. Given a tableau branch Θ and a nominal or state variable s the *set of true formulae* at s on Θ, is written $T^\Theta(s)$ and defined as follows:

$$T^\Theta(s) = \{\phi \mid @_s\phi \in \Theta\}$$

Definition 2.7 (Quasi-subformula). A formula ϕ is a *quasi-subformula* of a formula ψ if one of the the following is the case:

1. ϕ is a subformula of ψ modulo substitution of free variables in ϕ for grounded terms.
2. ϕ is of the form $\neg\chi$ where χ modulo substitution of free variables in χ for grounded terms.

Altering the definition to allow grounded terms being substituted for free variables ensures compatibility of the following proofs with the universal, existential, and RR rules.

Definition 2.8 (Accessibility formula). A formula of the form $@_sFt$ or $@_sPt$ on Θ is called an *accessibility formula* if it is the first conclusion of an application of F, P, $\neg G$, or $\neg H$. Additionally we say any formula of the form $@_ii$ is an accessibility formula if it is the conclusion of the **nom ref** rule.

The intended interpretation of $@_sFt$ is that the time denoted by t is accessible from the time denoted by s and vice versa in the case of $@_sPt$. While $@_ii$ is not a modal accessibility formula in the same way as $@_sFt$ or $@_sPt$ since its meaning does not involve the accessibility relation R of the model (and modal accessibility relations need not be reflexive), intuitively every time/world relates to itself in a similar way to how different times relate to each other, especially since we will take for granted that the collection of times is totally ordered. The main motivation for considering these formulae as one kind is that it supports the termination proof in ways we will soon see.

Definition 2.9 (Equality formula). A formula of the form $@_it = s$ or $@_i\neg t = s$ where s, t are extended terms is called an equality formula if it is a conclusion of **ref**, **:1**, **:2**, **:3**, **:fix 3**, or **:fix 4**.

Definition 2.10 (Root-subformula). Where the root formula of a tableau Θ is written $root_\Theta$, a formula $@_s\phi$ occuring on a tableau Θ is called a *root-subformula* on Θ if it is a quasi-subformula of $root_\Theta$.

Lemma 2.4 (Subformula Property). *Where Θ is a tableau branch in the FHTL calculus, any formula $@_s\phi$ occuring on Θ is either a root-subformula, accessibility formula, or equality formula.*

Proof. This is verified by induction on the tableau rules beginning with $root_\Theta$ as a base case. □

Definition 2.11 ($\prec_\Theta$)**.** Where Θ is a tableau branch in the *FHTL* calculus, if a nominal a is introduced to the branch by application of of F, P, $\neg G$, or $\neg H$ to a premise $@_s\phi$, we say a is *generated* by s on Θ and write $s \prec_\Theta a$. We write $\prec^*_\Theta$ to denote the reflexive and transitive closure of $\prec_\Theta$.

Definition 2.12 (Nom_Θ)**.** The set of nominals and state variables which occur on Θ is written Nom_Θ

Lemma 2.5. *Where Θ is a tableau branch in the FHTL calculus, the graph $G = (Nom_\Theta, \prec_\Theta)$ is a wellfounded finitely branching tree.*

Proof. Each aspect is proved below:

- *Wellfoundedness of trees in G*

  We have that if $a \prec_\Theta b$ then the first occurence of a on Θ is before the first occurence of b, thus by induction any subset of Nom_Θ under the relation $\prec_\Theta$ has a least element and each tree in G is wellfounded.

- *G is a tree*

  Every nominal in Nom_Θ can be generated by at most one other nominal, and every nominal in Nom_Θ must have one of the finitely many nominals in the root formula as an ancestor.

- *G is finitely branching*

  We show G is finitely branching by showing that given a nominal a, there can only be finitely many distinct nominals b such that $a \prec_\Theta b$. Each nominal b such that $a \prec_\Theta b$ is generated by applying one of the F, P, $\neg H$, $\neg G$ rules to a premise of the form $@_iF\phi$, $@_iP\phi$, $\neg @_iG\phi$, or $\neg @_iG\phi$ respectively, where by our restrictions, either ϕ is not a nominal, or the entire premise is a root subformula. Since there can only be finitely many root subformulae of the form of one of the the possible premises, where i is the prefix nominal in each case, only finitely many new nominals have been generated from i. Thus G is finitely branching.

□

Lemma 2.6. *Where Θ is a tableau branch in the FHTL calculus, Θ is infinite if and only if there exists an infinite chain of nominals and state variables $a_1 \prec_\Theta a_2 \prec_\Theta \ldots \prec_\Theta a_n \prec_\Theta \ldots$*

Proof. The 'if' direction follows from the fact that such an infinite chain of nominals entails there are an infinite number of conclusions/premises with distinct nominals on the branch. □

Lemma 2.7. *Where Θ is a tableau branch in the FHTL calculus, if $@_st \in \Theta$ where each of s and t is a nominal or a state variable then t is a root nominal/state variable.*

Lemma 2.8. *Where Θ is a tableau branch in the FHTL calculus, if $@_sFt \in \Theta$ or $@_sPt \in \Theta$ and t is not a root nominal/state variable then $s \prec_\Theta t$ or s and t denote the same time.*

Definition 2.13 (m_Θ and d_Θ)**.** Where Θ is a tableau branch in the *FHTL* calculus, a is a nominal/state variable occuring on Θ, and $|@_s\phi|$ denotes the length of the formula $@_s\phi$, we define $m_\Theta(a)$ as:

$$m_\Theta(a) = \max\{|@_s\phi| : @_s\phi \in \Theta \text{ and } @_s\phi \text{ is a root subformula}\}$$

If there are no root subformulae $@_a\phi$ on Θ then $m_\Theta(a) = -\infty$. The *depth* of the nominal/state variable a with regard to Θ is the length of the unique path in $(Nom_\Theta, \prec_\Theta)$ which connects the root nominal/state variable to a.

Lemma 2.9. *Where Θ is a tableau branch in the FHTL calculus, for any nominal/state variable on Θ, $m_\Theta(a) \leq |root_\Theta| - d_\Theta(a)$*

Lemma 2.10. *Where Θ is a tableau branch in the FHTL calculus, if for every nominal/state variable a in $root_\Theta$:*

$$m_\Theta(a) \leq |root_\Theta| - d_\Theta(a)$$

then Θ is finite.

Lemma 2.11. *Any tableau in the FHTL calculus is finite.*

Theorem 2.12. *The satisfiability of a finite set of FHTL sentences in a FHTL model is decidable.*

References

Patrick Blackburn. 2000. Internalizing labelled deduction. *Journal of Logic and Computation*, 10(1):137–168.

Patrick Blackburn and Klaus Frovin Jørgensen. 2012. Indexical hybrid tense logic. *Advances in Modal Logic*, 9:144–60.

Thomas Bolander and Patrick Blackburn. 2009. Terminating tableau calculi for hybrid logics extending k. *Electronic Notes in Theoretical Computer Science*, 231:21–39.

Lucia Donatelli, Michael Regan, William Croft, and Nathan Schneider. 2018. Annotation of tense and aspect semantics for sentential AMR. In *Proceedings of the Joint Workshop on Linguistic Annotation, Multiword Expressions and Constructions (LAW-MWE-CxG-2018)*, pages 96–108, Santa Fe, New Mexico, USA. Association for Computational Linguistics.

Melvin Fitting and Richard L Mendelsohn. 1998. *First-Order Modal Logic*, volume 277. Springer Science & Business Media.

Jens Ulrik Hansen. 2007. A tableau system for a first-order hybrid logic. In *Proceedings of the International Workshop on Hybrid Logic (HyLo 2007)*, pages 32–40.

James Pustejovsky, Ken Lai, and Nianwen Xue. 2019. Modeling quantification and scope in Abstract Meaning Representations. In *Proceedings of the First International Workshop on Designing Meaning Representations*, pages 28–33, Florence, Italy. Association for Computational Linguistics.