

# Formalizing AMR Inference via Hybrid Logic Tableaux

## CL Masters Thesis Defense

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# Introduction

- ▶ Semantic representation:
  - ▶ Capture meaning of natural language content.
  - ▶ Designed for manipulation via software.
- ▶ Abstract Meaning Representation (AMR):
  - ▶ Graph-based (DAG), nodes are *concepts*, edges are *relations*.
  - ▶ Built on predicative core of a sentence.
  - ▶ Ignores syntactic differences between equivalent sentences.
  - ▶ PropBank framesets are used for concepts (entities, events, properties, states).

# Introduction

(Basic) AMR is reductionistic. This is awesome for:

- ▶ Annotation (esp. by non-experts).
- ▶ Semantic parsing (smaller target space).

This is not awesome for:

- ▶ Representing and recovering fine-grained meaning.
- ▶ Automating reasoning/inference.

# Introduction

- ▶ The trade-off between ease of generation/use and rich expressivity/inferentiability is at least as old as computing.
- ▶ AMR has made a choice that works well in data-driven NLP.
- ▶ However AMR can bridge this gap:
- ▶ AMR already does this in a modular way with extensions.
- ▶ Some extensions afford interpretation in first-order logic:
- ▶ Automated inference for logics is a rich area with lots of tools.

# Motivation

*“Why do we need formal methods? Can’t state-of-the-art language models do this already?”*

- ▶ Statistically driven techniques are unnecessarily expensive for formal inference.
- ▶ Increasing need for ability to guarantee/verify properties of software:
  - ▶ Does the software give us the right *type* of result for an input?
  - ▶ Bias in NLP.
- ▶ Machine learning (by itself) does not lend itself well to this.

# Approach

- ▶ Combine two AMR extensions for richer interpretation:
  - ▶ Scope and quantification (Pustejovsky et al., 2019)
  - ▶ Tense and aspect (Donatelli et al., 2018)
- ▶ Interpret these extended AMR into a logic that handles quantification and tense.
- ▶ Develop tableau methods for this logic:
  - ▶ General method for proving/disproving sentences in the logic.
  - ▶ Restricted method for checking if sentence holds in some model.

# AMR with Scope and Quantification

- ▶ Disambiguates scope.
- ▶ Annotates central predicate and its arguments.
- ▶ Clearest path for AMR → standard first-order predicate logic.

# AMR with Tense and Aspect

- ▶ Standard AMR structure.
- ▶ Central predicate annotated for:
  - ▶ Aspect.
  - ▶ Speech time.
  - ▶ Reference time.

# Combined Extensions

- ▶ Assume each AMR has information from both extensions.
- ▶ Attach tense and aspect information to central predicate node.
- ▶ Extract a tense-sensitive FOPL representation (details later).

# Modal Logic

- ▶ Propositional logic lets us form statements like  $p \wedge (q \vee \neg r)$ .
- ▶ Modal propositional logic extends propositional logic with an operator  $\diamond$ , read as “possible”. i.e. it is not possible that  $p$  and  $\neg p$  are the case would be:

$$\neg\diamond(p \wedge \neg p)$$

- ▶ (More) formal meaning of  $\diamond$ : There is a *possible world* where  $p$  is true, and this possible world is *accesible* from the current one.
- ▶ The problem: the “current world” is an implicit notion dependent on context. Is there something more expressive?

# Hybrid Logic

- ▶ Idea: take propositional modal logic, and add an operator  $\text{@}$ , that lets us know which world we're referring to.
- ▶  $p$  or  $r$  is possible at world  $i$ :  $\text{@}_i \Diamond(p \vee r)$
- ▶ In the above proposition  $i$  is called a nominal since it *names* some/is true at exactly one world.
- ▶ Everything true at the nominal  $j$  is true at the nominal  $i$  (they name the same world):  $\text{@}_{ij}$

# Hybrid Logic Variants

- ▶ Hybrid tense logic:
  - ▶ Two tense modalities:  $\langle F \rangle$  and  $\langle P \rangle$
  - ▶ World  $j$  is in the past of world  $i$ :  $\text{@}_i P j$
- ▶ Quantified hybrid logic:
  - ▶ Hybrid logic with first-order quantifiers, relation, and function symbols.
  - ▶ At  $n$  (now) there is a person for whom it is possible to own a car:

$$\text{@}_n (\exists x)(\text{Person}(x) \wedge (\exists y)(\text{Car}(y) \wedge \Diamond \text{Afford}(x, y)))$$

# Hybrid Logic Variants

A problem

$$\textcircled{O}_n(\exists x)(\textit{Person}(x) \wedge (\exists y)(\textit{Car}(y) \wedge \Diamond \textit{Afford}(x, y)))$$

What's the domain of quantification?

- ▶ Option 1: The domain is the same at every world: *There is someone (out of all people all people at all worlds) who can afford some car (out of all cars at all worlds).*
- ▶ Option 2: Each world has a different domain: *There is someone (out of everyone in this world) who can afford some car (out of all cars in this world).*

The first option in quantified hybrid (or modal) logic is called possibilist quantification, the second is called actualist. When worlds are interpreted as times, the former is called eternalist quantification, and the latter is called presentist. We will use presentist quantification.

# First-Order Hybrid Tense Logic

First-order Hybrid Tense Logic (*FHTL*) is:

- ▶ Quantified hybrid logic, with tense modalities  $F$  and  $P$  instead of  $\Diamond$ .
- ▶ Presentist quantification:
  - ▶ Each world has its own domain.
  - ▶ Quantification is domain sensitive.

# First-Order Hybrid Tense Logic

In logic models are interpretations of symbols and constants in the language. An *FHTL* model  $\mathfrak{M}$  is a tuple

$$(T, \mathcal{R}, (D_t)_{t \in T}, I_{nom}, (I_t)_{t \in T})$$

Where:

- ▶  $T$  is a set of times/worlds.
- ▶  $\mathcal{R}$  is the binary accessibility relation over times.
- ▶  $D_t$  is the domain of a time  $t$
- ▶  $I_{nom}$  assigns nominals to worlds.
- ▶  $I_t$  interprets the value of terms at a time  $t$ .

The satisfiability of a formula with free variables depends on a variable assignment function. A formula that does not depend on variable assignment (every variable is bound by a quantifier) is called a *sentence*.

# First-Order Hybrid Tense Logic

- ▶ Two main tasks for a sentence  $\Theta_s \varphi$ :
  - ▶ Theorem proving – is  $\Theta_s \varphi$  true regardless of the model?
  - ▶ Model checking – is  $\Theta_s \varphi$  true in a given model  $\mathfrak{M}$ ?
- ▶ For both we use versions of the tableau method.

# Tableau Method

- ▶ Tableau for a formula is a tree structure.
- ▶ A tableau calculus for a logic breaks down and transforms formulae
- ▶ Rules of the calculus use semantics of connectives/quantifiers/modifiers.
- ▶ *FHTL* tableau based on *QHL* tableau modified for tense rules.

# Tableau Method

- ▶ A tableau branch is a subtree of the main tableau tree.
- ▶ A tableau *branches* for rules involving disjunctions.
- ▶ A branch is *closed* if it contains both a formula  $\Diamond_s \varphi$  and its negation  $\Diamond_s \neg \varphi$ . Otherwise it is open.
- ▶ A branch is *saturated* if no rules of the calculus can be applied without adding a redundant formula on the branch.

# Tableau Example Rules

$$\frac{\textcircled{O}_s F \varphi}{\textcircled{O}_s Fa} (F)^{12}$$

$\textcircled{O}_a \varphi$

$$\frac{\textcircled{O}_s \textcircled{O}_t \varphi}{\textcircled{O}_t \varphi} (\textcircled{O})$$

$$\frac{\textcircled{O}_s (\exists x) \varphi}{\textcircled{O}_s \varphi [s:p/x]} (\exists)^3$$

$$\frac{\textcircled{O}_i j:t = k:s \quad \textcircled{O}_i \varphi}{\textcircled{O}_i \varphi [j:t//k:s]} (\textbf{sub})^4$$

<sup>1</sup>The nominal  $a$  is new to the branch.

<sup>2</sup>The formula  $\varphi$  is not a nominal.

<sup>3</sup> $s:p$  is new to the branch.

<sup>4</sup> $\varphi[j:t//k:s]$  is  $\varphi$  where some occurrences of  $j:t$  have been replaced by  $k:s$ .

## FHTL Tableau Example

- |      |   |
|------|---|
| (1)  |   |
| (2)  | $\textcircled{S}(\exists x)[P((\exists y)[f(x, y) = f(y, x)]) \vee \neg(\exists z)[x = z]]$ |
| (3)  | $\textcircled{S}P((\exists y)[f(s_1, y) = f(y, s_1)]) \vee \neg(\exists z)[s_1 = z]$        |
| (4)  | $\textcircled{S}P((\exists y)[f(s_1, y) = f(y, s_1)])$                                      |
| (5)  | $\textcircled{S}\neg(\exists z)[s_1 = y]$   |
| (6)  | $\textcircled{S}Pt$   |
| (7)  | $\textcircled{S}\neg[s_1 = s_1]$  |
| (8)  | $\textcircled{S}( \exists y)[f(s_1, y) = f(y, s_1)]$  |
| (9)  | $\textcircled{S}[s_1 = s_1]$  |
| (10) | $\dots$   |
| (11) | $\otimes$   |

# Tableau Proofs

- ▶ The *root formula* of a tableau is unsatisfiable if every branch of the tableau closes.
- ▶ If we want to prove  $\mathbb{Q}_s\varphi$  then we begin the tableau with  $\mathbb{Q}_s\neg\varphi$  (proof by contradiction).
- ▶ Need to show for tableau method (or any proof system) that it is:
  - ▶ Sound – if a tableau is closed then the root formula is unsatisfiable.
  - ▶ Complete – if a formula is unsatisfiable it has a closed tableau proof.

# Tableau Proofs

- ▶ Soundness is demonstrated by checking the rules.
- ▶ Completeness is demonstrated by contrapositive.
- ▶ Open tableau branch → there is a model which satisfies the root.
- ▶ Construct the model out of equivalence classes of everything that shows up on the branch.

# Tableaux for Model Checking

- ▶ So far the tableau method has been about general theorem proving.
- ▶ Problem: no tableau construction method guaranteed to terminate (if there was we'd have decidability of first-order logic).
- ▶ Most of the time however we are reasoning about an AMR/*FHTL* sentence in some local context.
- ▶ Revise tableau rules, trade completeness for termination.

# Tableaux for Model Checking

NB: Terms in *FHTL* have nominal prefixes since their value can depend on the world they are evaluated in, i.e.  $i:t$ ,  $i:j:t$ . There are three things that prevent tableau construction from terminating:

- ▶ Universal tableau rules ( $\neg\exists$ ,  $\forall$ ) – can generate an arbitrary number of conclusions.
- ▶ Term rules – can keep adding prefixes to terms.
- ▶ Nominal rules – generating redundant nominals (our complete rules take care of this).

- ▶ Idea: give tableau a reasonable chance to close, but make sure it terminates.
- ▶ Fix some  $q \in \mathbb{N}^+$  and let universal rules generate at most  $q$  conclusions. (Fitting (1988) says  $q = 1$  works surprisingly often.)
- ▶ Restrict term rules to depend on on nominals and terms already on the branch. Prevent redundant prefixes (no  $i:j:i:t$  etc.).

## Model checking procedure

- ▶ Build tableau using restricted rules.
- ▶ Check tableau tree from leaves up, skipping closed branches.
- ▶ Every formula on the branch has a set of variable and parameter assignments which satisfy it in the model.
- ▶ Solve for a formula's set based on the sets of its conclusions.
- ▶ If the root formula is satisfied in the model by every variable assignment then it holds.

# Model Checking Example

- ▶ Every desk will have a computer located there.

- ▶ AMR with quantification and tense:

(s / scope

```
:pred (b / be-located-at-91 :ongoing -
      :complete +
      :time (a / after
            :op1 (n / now))

      :ARG0 (c / computer)
      :ARG1 (d / desk
            :quant (e / every)))
```

```
:ARG0 d
:ARG1 c)
```

- ▶ FHTL translation:

$$@_{\text{now}}(\forall y)[\text{desk}(y) \rightarrow (\exists x)[\text{computer}(x) \wedge F(\text{be-located-at-91}(x, y))]]$$

# Model Checking Example

Define a small *FHTL* model  $\mathfrak{M} = (T, \mathcal{R}, (D_t)_{t \in T}, I_{nom}, (I_t)_{t \in T})$  where:

$$T = \{\text{yesterday, now, tomorrow}\}$$

$$\mathcal{R} = \{(\text{yesterday, now}), (\text{now, tomorrow}), (\text{yesterday, tomorrow})\}$$

$$I_{nom} = \{(\text{y, yesterday}), (\text{n, now}), (\text{t, tomorrow})\}$$

$$I_{\text{yesterday}}(\text{be-located-at-91}) = \dots$$

$$I_{\text{now}}(\text{be-located-at-91}) = \dots$$

$$I_{\text{tomorrow}}(\text{be-located-at-91}) = \{\langle \text{computer}_2, \text{desk}_2 \rangle, \langle \text{computer}_1, \text{desk}_1 \rangle\}$$

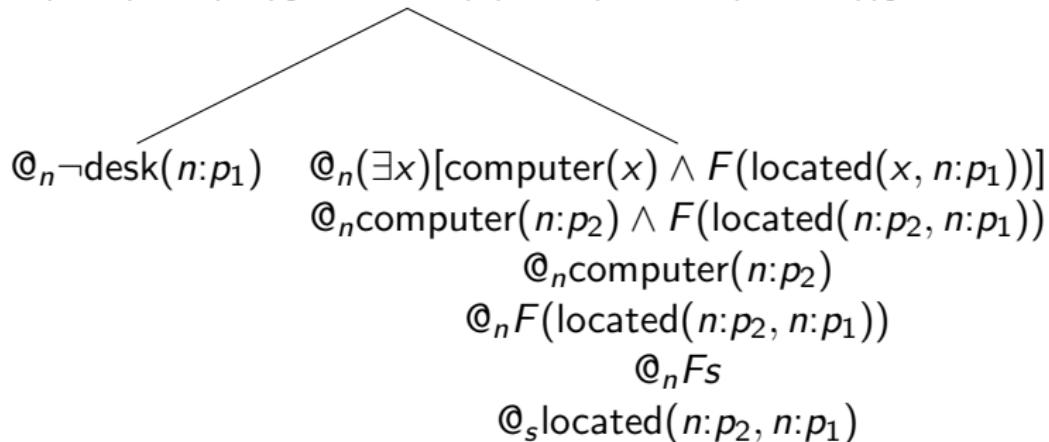
$$D_{\text{yesterday}} = \{\text{computer}_1, \text{desk}_1\}$$

$$D_{\text{now}} = \{\text{computer}_1, \text{computer}_2, \text{desk}_1, \text{desk}_2, \text{desk}_3\}$$

$$D_{\text{tomorrow}} = \{\text{computer}_1, \text{computer}_2, \text{desk}_1, \text{desk}_2\}$$

# Model Checking Example

$$\begin{aligned} @_n(\forall y)[\text{desk}(y) \rightarrow (\exists x)[\text{computer}(x) \wedge F(\text{located}(x, y))]] \\ @_n \text{desk}(n:p_1) \rightarrow (\exists x)[\text{computer}(x) \wedge F(\text{located}(x, n:p_1))] \end{aligned}$$



Where we assign  $s = t = \text{tomorrow}$ ,  $n:p_2 = \text{computer}_1$  and  $n:p_1 = \text{desk}_2$ , or  $n:p_2 = \text{computer}_2$  and  $n:p_1 = \text{desk}_1$  we see that  $\mathfrak{M}$  satisfies the *FHTL* sentence.

# Extraction

In this new annotation time information is of the form:

:<aspect> <polarity> :time (<reference> :op1 (<speech>))

- ▶ <speech> is some  $t \in T$ , so formula has the prefix  $\text{@}_t\varphi$
- ▶ <reference> (if present) is the tense, e.g.  $\text{@}_tP\psi$
- ▶ We don't make use of aspect (right now).

# Interpretation

Every AMR in this annotation is either of the form (for a connective or, and, or cond):

```
(<connective>
 :op1 (...))
 :op2 (...))
 :op3 (...))
```

Or

```
(s / scope
 :pred ...
 :ARG0 (...))
 :ARG1 (...))
 :ARG0 <primary-scope>
 :ARG1 <secondary-scope>)
```

# Interpretation Examples

```
(o / or
  :op1 (...))
  :op2 (...))
  :op3 (...))
                           [[op1]] ∨ [[op2]] ∨ [[op3]]
```

```
(s / scope
  :pred ...
    :ARG0 (...))
    :ARG1 (...))
  :ARG0 <primary-scope>
  :ARG1 <secondary-scope>)

                           [[<primary-scope>]]([[<secondary-scope>]]([[pred]]))
```

# Interpretation Examples

```
(s / scope
  :pred (b / be-located-at-91 :ongoing -
          :complete +
          :time (a / after
                 :op1 (n / now)))
  :ARG0 (c / computer)
  :ARG1 (d / desk
         :quant (e / every)))
:ARG0 d
:ARG1 c)
```

$$@_{\text{now}}(\forall y)[\text{desk}(y) \rightarrow (\exists x)[\text{computer}(x) \wedge F(\text{be-located-at-91}(x, y))]]$$

(c / computer)

$$\lambda\varphi.(\exists x)[Computer(x) \wedge \varphi(x)]$$

(d / desk

:quant (e/ every))

$$\lambda\varphi.(\forall x)[Desk(x) \rightarrow \varphi(x)]$$

# Interpretation Example

```
(:pred (b / be-located-at-91 :ongoing -  
           :complete +  
           :time (a / after  
                  :op1 (n / now))
```

$$\lambda\varphi.\lambda x.\lambda y.\varphi(\text{be-located-at-91}(x, y))$$

# Interpretation Example

We'll just do the first application.

$$\begin{aligned} & \lambda\varphi.(\lambda\varphi'.(\exists c)(\text{computer}(c) \wedge \varphi'(c)))(\lambda\psi.(\lambda\psi'.\psi'(\lambda x.\lambda y.\text{loc}(x,y))))(\lambda\gamma.\varphi(\gamma(\psi))) \triangleright_{\beta} \\ & \lambda\varphi.(\lambda\varphi'.(\exists c)(\text{computer}(c) \wedge ((\lambda\psi'.\psi'(\lambda x.\lambda y.\text{loc}(x,y))))(\lambda\gamma.\varphi(\gamma(c))))) \triangleright_{\beta} \\ & \lambda\varphi.(\lambda\varphi'.(\exists c)(\text{computer}(c) \wedge (\varphi(\lambda y.\text{loc}(c,y))))) \end{aligned}$$

# Conclusion

- ▶ We explored why AMR can benefit from formal inference.
- ▶ Described a combination of AMR extensions.
- ▶ Sketched a first-order hybrid logic variant with general proof procedure and model checking methods.
- ▶ Demonstrated how sentences are handled by these methods.
- ▶ Showed how AMR sentences can be translated into *FHTL*

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