Nonlinear Systems

Assignments

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Assignment 1 Stability of equilibrium points and bifurcations

1.1 A simple population model

Question 1 The position and number of equilibrium points depends on the value of α and β. In Table 1 the different possible cases are listed together with information about the equilibrium points. When looking at the graph of $\dot{N}$ against N, a negative parabola can be observed, intersecting the N-axis in two points. When solving the quadratic equation for $\dot{N} = 0$ with α and β as unknown parameters, expressions shown in (2) for the equilibrium points ensue.

$$N_1 = 0 \tag{1}$$

$$N_2 = \frac{K(\alpha - \beta)}{\alpha} \tag{2}$$

$\alpha < \beta$	$\alpha > \beta$	$\alpha = \beta$
N_1 is stable (●) N_2 is unstable (○)	N_1 is unstable (○) N_2 is stable (●)	$N_1 = N_2$ half stable equilibrium point (◑)

Table 1: Characteristics of the different training algorithms for the given experiment, performed with and without noise.

The type of bifurcation that occurs here is called transcritical.

Question 2 For the practical example described here, the solution for $t \to \inf$ will converge to one of the equilibrium points. As $\alpha > \beta$, the second case in Table 1 is applicable. The only stable equilibrium point is $N_2 = \frac{K(\alpha-\beta)}{\alpha} = 10\,470\,086$.

1.2 Gene control model

Question 1 When taking the repression rate $r = 0$, only one fixed point is visible.