

# Matrix projection models (MPM's)

$$M \times N^* = \lambda N^*$$

For every (**population**) matrix there is a **vector** (**age distribution**) for which the matrix transformation (**population growth process**) increases the **vector** by a fixed proportion  $\lambda$  (**population growth rate**).

- $N^*$  - is the **eigenvector** = **stable population distribution**
- $\lambda$  - is the **eigenvalue** = **population growth factor**

- Stationary (parameters don't change)

- Discrete time
- Deterministic



**EXONENTIAL GROWTH**  
(i.e. no Density Dependence)

# Age-structured Leslie Matrix

Diagonal elements all 0

$$\begin{bmatrix} n_0 \\ n_1 \\ \vdots \\ n_{\omega-1} \end{bmatrix}_{t+1} = \begin{bmatrix} f_0 & f_1 & f_2 & \dots & f_{\omega-2} & f_{\omega-1} \\ s_0 & 0 & 0 & \dots & 0 & 0 \\ 0 & s_1 & 0 & \dots & 0 & 0 \\ 0 & 0 & s_2 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & s_{\omega-2} & 0 \end{bmatrix} \begin{bmatrix} n_0 \\ n_1 \\ \vdots \\ n_{\omega-1} \end{bmatrix}_t$$

**Every age class ages out.**

*This maps exactly to a Life-History Table*

# Stage structure: Loggerhead turtles

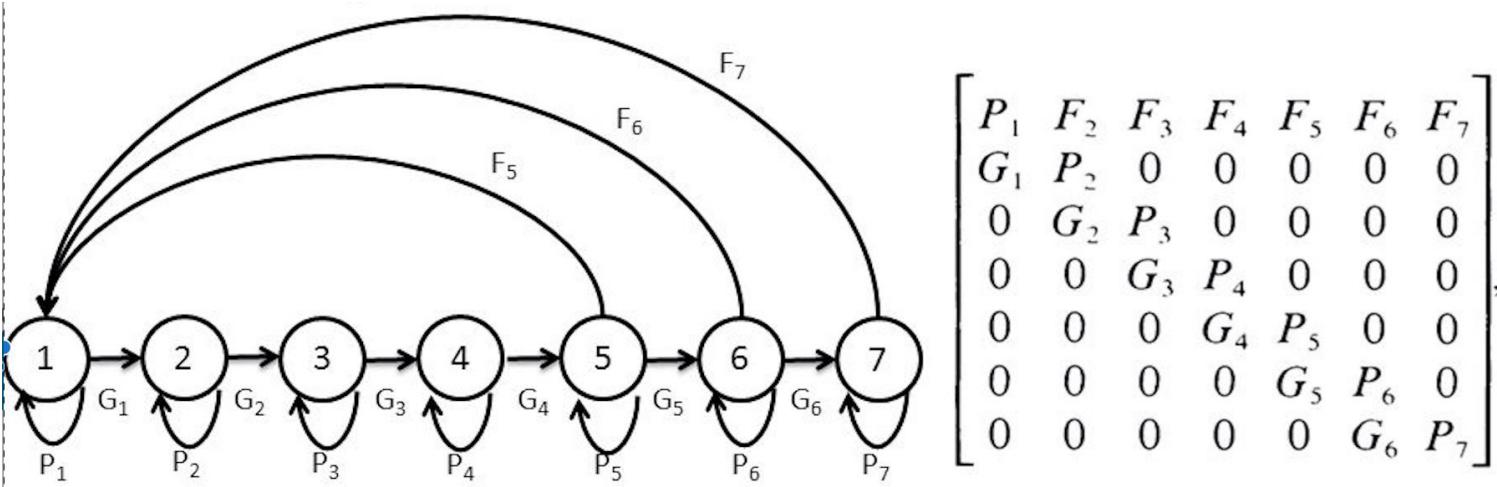


TABLE 4. Stage-class population matrix for loggerhead sea turtles based on the life table presented in Table 3. For the general form of the matrix and formulae for calculating the matrix elements see Theoretical Population Projections.

0	0	0	0	127	4	80	
0.6747	0.7370	0	0	0	0	0	
0	0.0486	0.6610	0	0	0	0	
0	0	0.0147	0.6907	0	0	0	
0	0	0	0.0518	0	0	0	
0	0	0	0	0.8091	0	0	
0	0	0	0	0	0.8091	0.8089	
Eggs/ hatchlings	Small juveniles	Large juveniles	Sub- adults	Novice breeders	1 <sup>st</sup> -year remigrants	Mature breeders	

*Some probability of staying in class at time step of population*

# Stage structure: Red-cockaded woodpecker


$$\begin{pmatrix} F_1 & F_2 & F_3 & F_4 & F_5 & F_6 \\ G_{21} & P_2 & 0 & 0 & 0 & 0 \\ G_{31} & 0 & 0 & 0 & 0 & 0 \\ G_{41} & G_{42} & G_{43} & P_4 & 0 & 0 \\ G_{51} & 0 & 0 & 0 & 0 & 0 \\ 0 & G_{62} & G_{63} & G_{64} & G_{65} & P_6 \end{pmatrix}$$

A

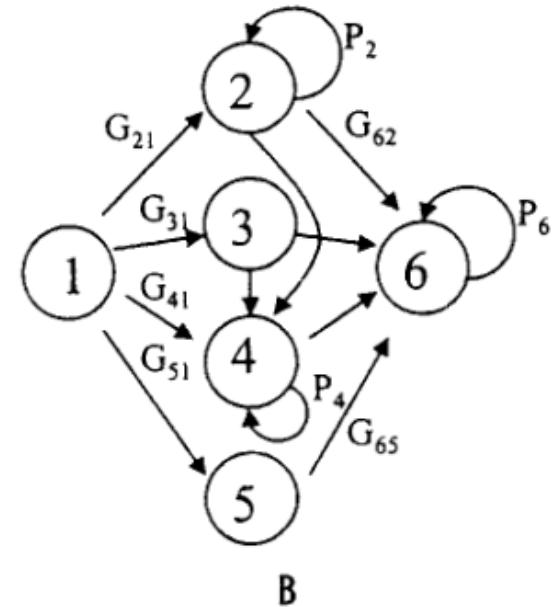
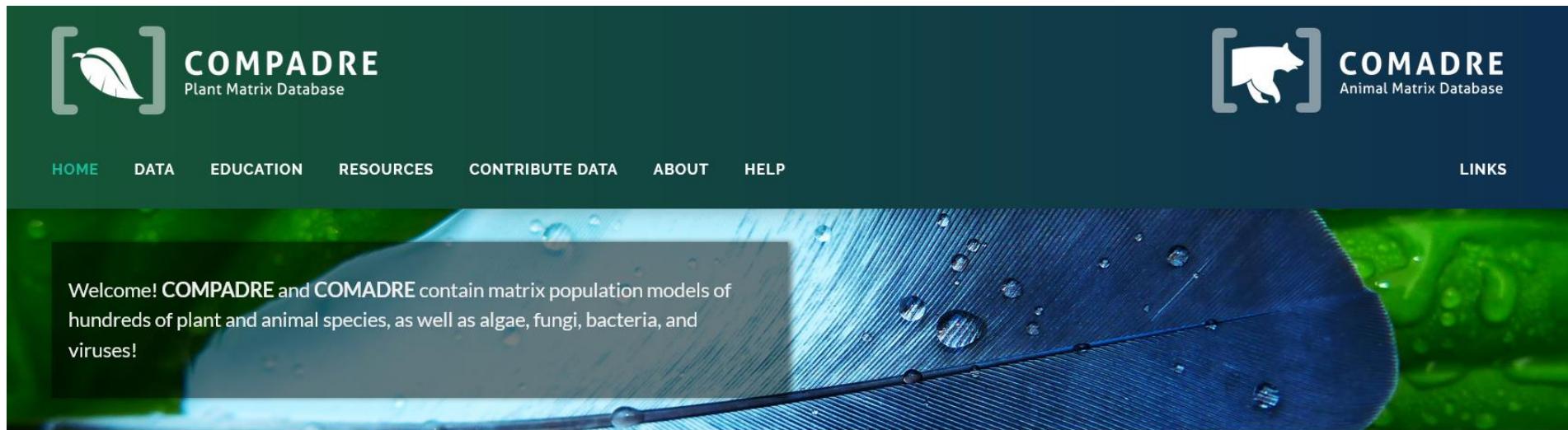


FIG. 4.4. (A) Projection matrix for male red-cockaded woodpeckers. Stages: 1, fledgling; 2, helper; 3, floater; 4, solitary; 5, 1-year-old breeder; 6, older breeder. (B) Life cycle graph for male red-cockaded woodpeckers; no fertilities and only some transition probabilities are shown.  $P_s$  represent survival probabilities;  $G_s$  represent probabilities of transition from one stage to another. (After Heppell, Walters, and Crowder 1994.)

**You can include interesting behavioral structure in a matrix model!**

# There's a whole database of MPM's!



Taxonomic Species

792

Studies

648

Matrix Population Models

8999

Taxonomic Species

430

Studies

416

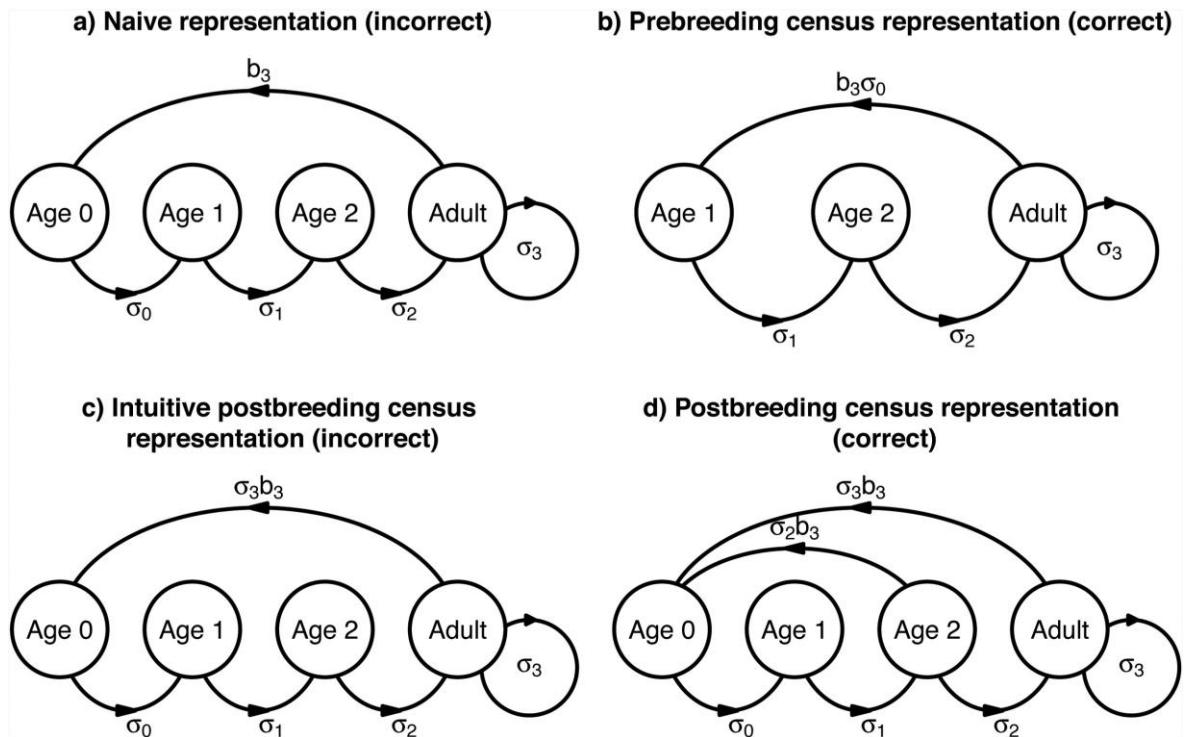
Matrix Population Models

3489

<https://compadre-db.org/>

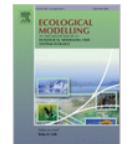
# Care needed in construction!

Especially with respect  
to Stage/Age 0.



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Persistent problems in the construction of  
matrix population models

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# Elasticity Analysis

The image shows a video player interface. On the left, there is a photograph of several deer in a forest setting. Overlaid on the photo is the title "Elasticity analysis in matrix models". Below the photo, the text "Picture: pixabay CCO" is visible. On the right side of the frame, a man with short brown hair, wearing a blue patterned shirt, is speaking. He is positioned against a solid teal background. A small white logo consisting of two overlapping L-shaped blocks is located in the top right corner of the video area. At the bottom of the screen, there is a dark blue bar containing the text "Pieter Zuidema" and "Forest Ecology and Forest Management". In the bottom left corner of the video area, there is a small circular inset showing a close-up of some fallen leaves on the ground. The bottom of the video player features a standard control bar with icons for play, volume, and other media controls, along with the text "WAGENINGEN UNIVERSITY & RESEARCH" and the timestamp "0:12 / 6:38".

Elasticity analysis in matrix models

Picture: pixabay CCO

Pieter Zuidema  
Forest Ecology and Forest Management

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0:12 / 6:38

[Elasticity Analysis Mini-Lecture](#)