Limits on Population Growth: Part II

EFB 370: Population Ecology

Dr. Elie Gurarie

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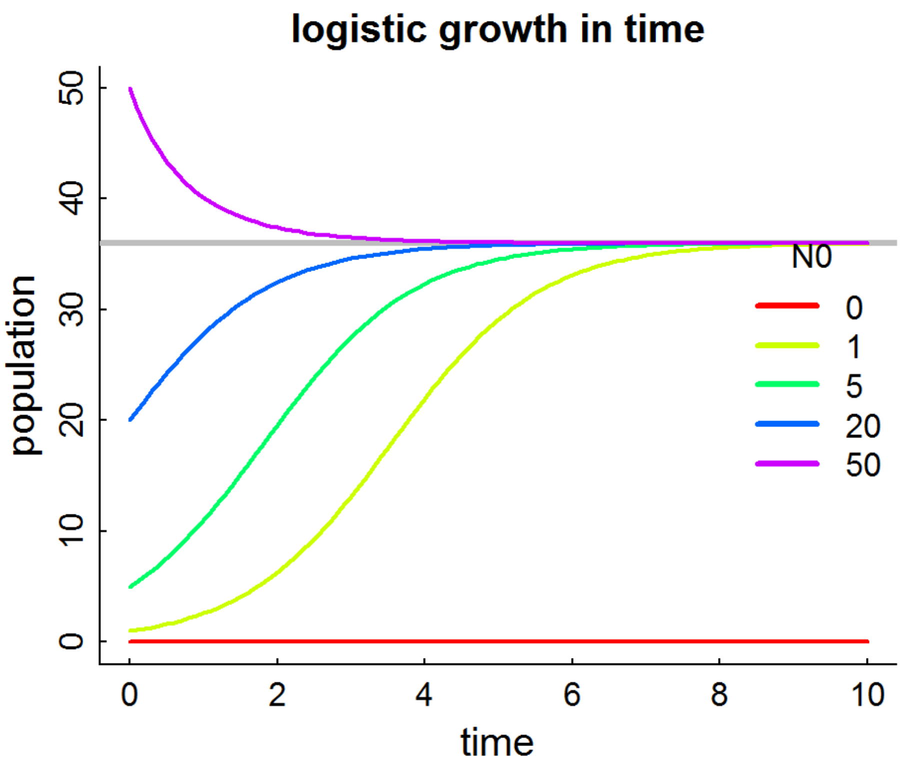
Continuous time logistic growth curves

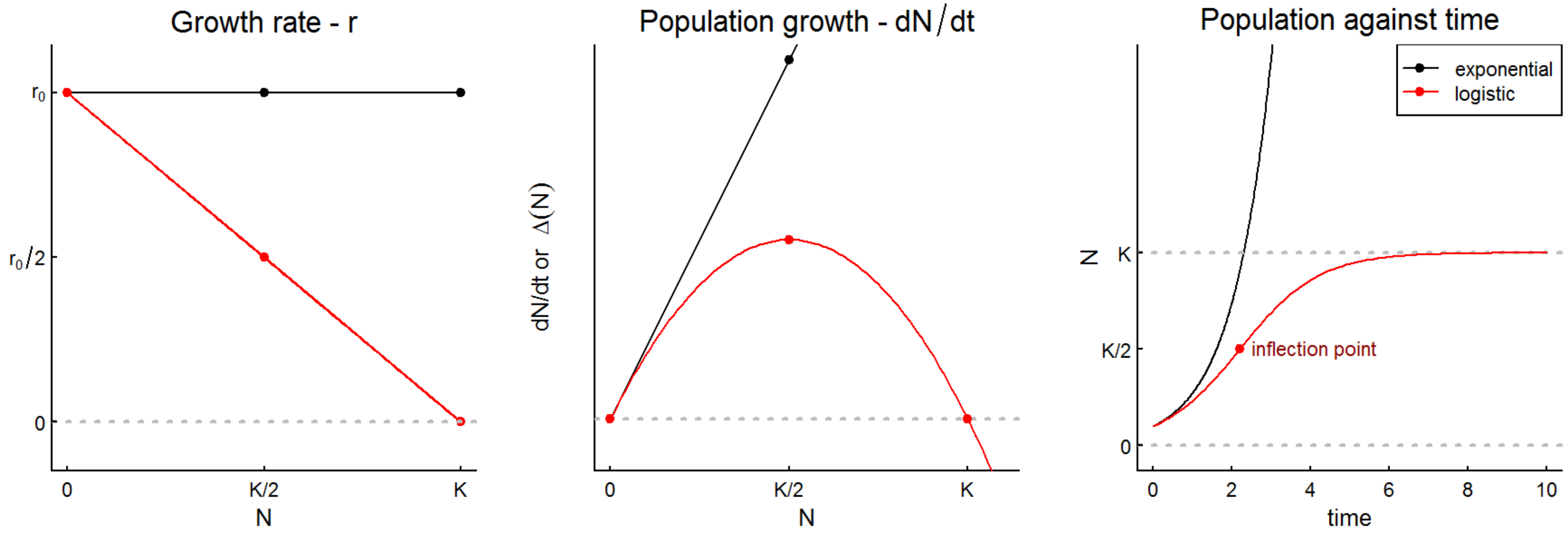


- Exponential growth at low N_0
- Typical "S" shape (**sigmoidal**) coming from below … Slows down at K
- Decays to K if $N_0 > K$

Exponential vs. Logistic Growth in Three Graphs

	Exponential Growth
differential equation:	$\frac{dN}{dt} = rN$
solution:	$N(t) = N_0 e^{rt}$

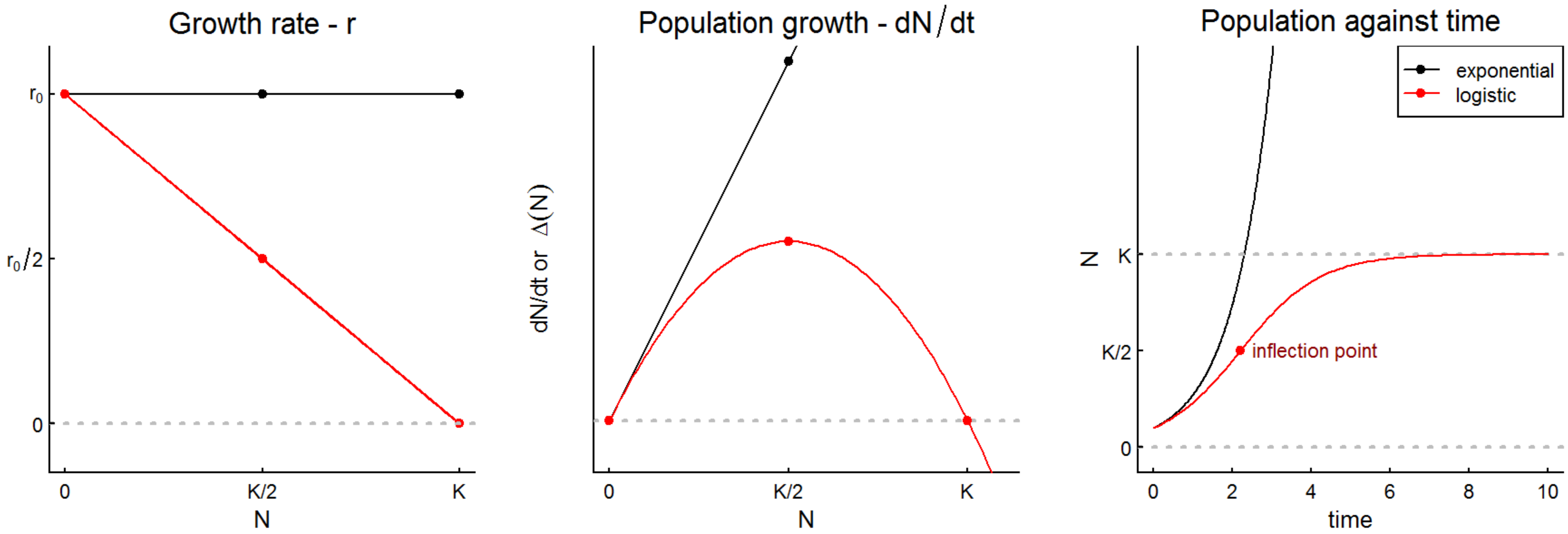
	Logistic Growth
differential equation:	$\frac{dN}{dt} = r_0 N \left(1 - \frac{N}{K}\right)$
solution:	$N(t) = \frac{K}{1+\left(\frac{K-N_0}{N_0}\right)e^{-r_0 t}}$



Properties

Inflection point - where rate of growth is maximum - occurs at $N(t^*) = K/2$.

$$t^* = \frac{1}{r}\log\left(\frac{K - N_0}{N_0}\right)$$



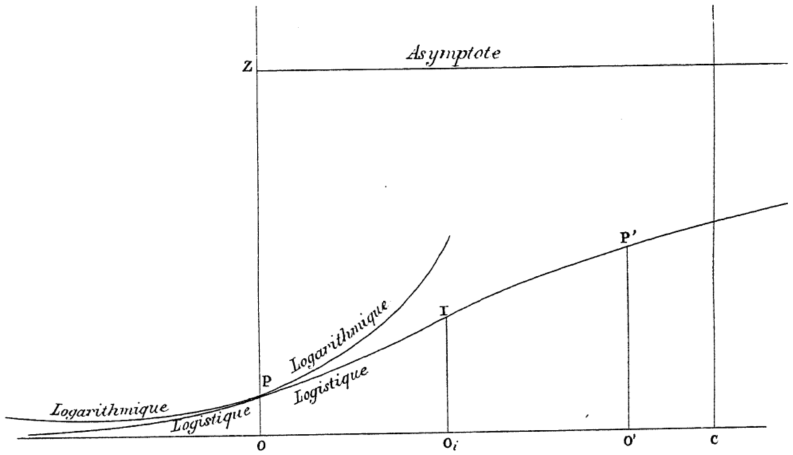
Pierre François Verhulst

(1804-1849)

Sur la loi d'accroissement de la population - 1844

$$t = \frac{1}{m} \log. \left[\frac{p(m - nb)}{b(m - np)}\right] \quad (4)$$

Nous donnerons le nom de *logistique* à la courbe (*voyez la figure*)

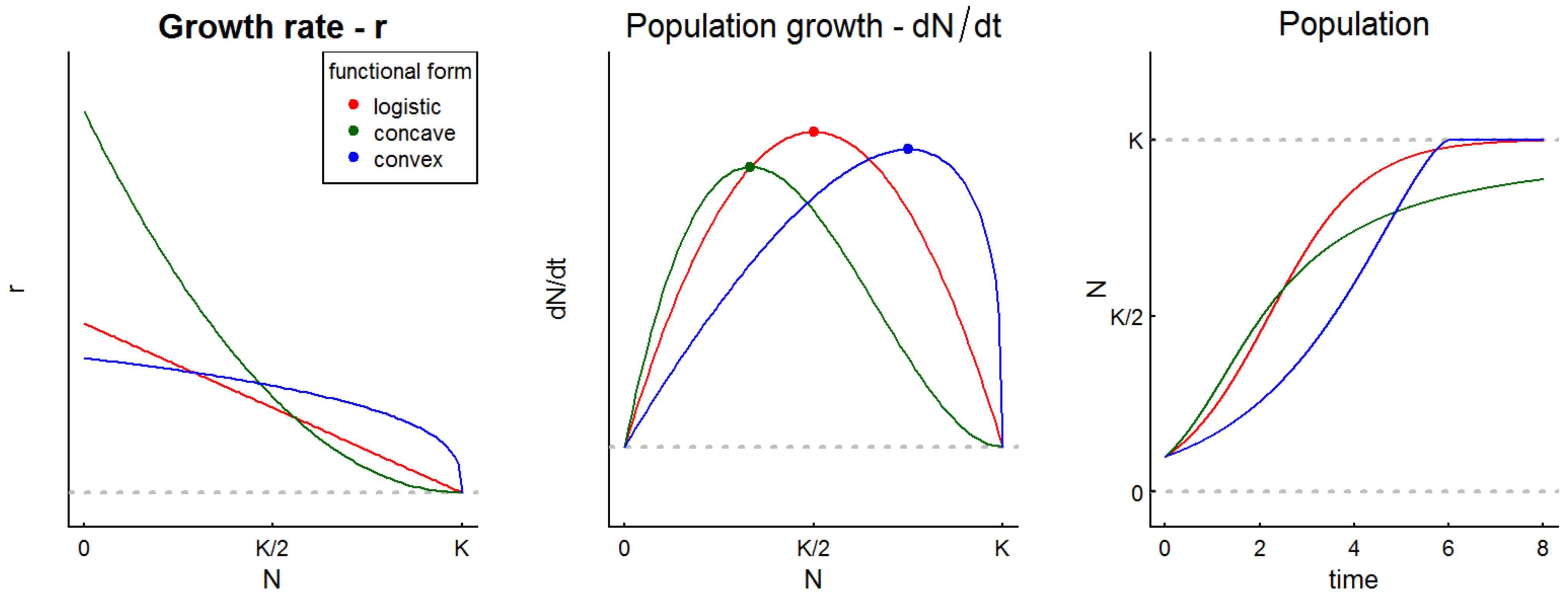


At the end of paper makes predictions - based purely on extrapolating from the inflection point:

The extreme limit of the population of France is 40,000,000 souls, and of Belgium is 6,600,000 souls.

Actual population (in 2020): 65.5 million, and 11.7 million. **Not bad at all!!**

Different functional forms



Discrete logistic growth

Take discrete population growth equation:

$$N_{t+1} = \lambda N_t$$

break up into "stable" and "growth" components ($\lambda = 1 + r_d$):

$$N_{t+1} = N_t + r_d N_t$$

add the density dependence "brake" on the growth:

$$N_{t+1} = N_t + r_d N_t \left(1 - \frac{N_t}{K}\right)$$

Discrete Logistic: Two formulations

Compute change within a time step:

$$\Delta N_t = r_d N_t \left(1 - \frac{N_t}{K}\right)$$

Compute change of a population:

$$N_{t+1} = N_t + r_d N_t \left(1 - \frac{N_t}{K}\right)$$

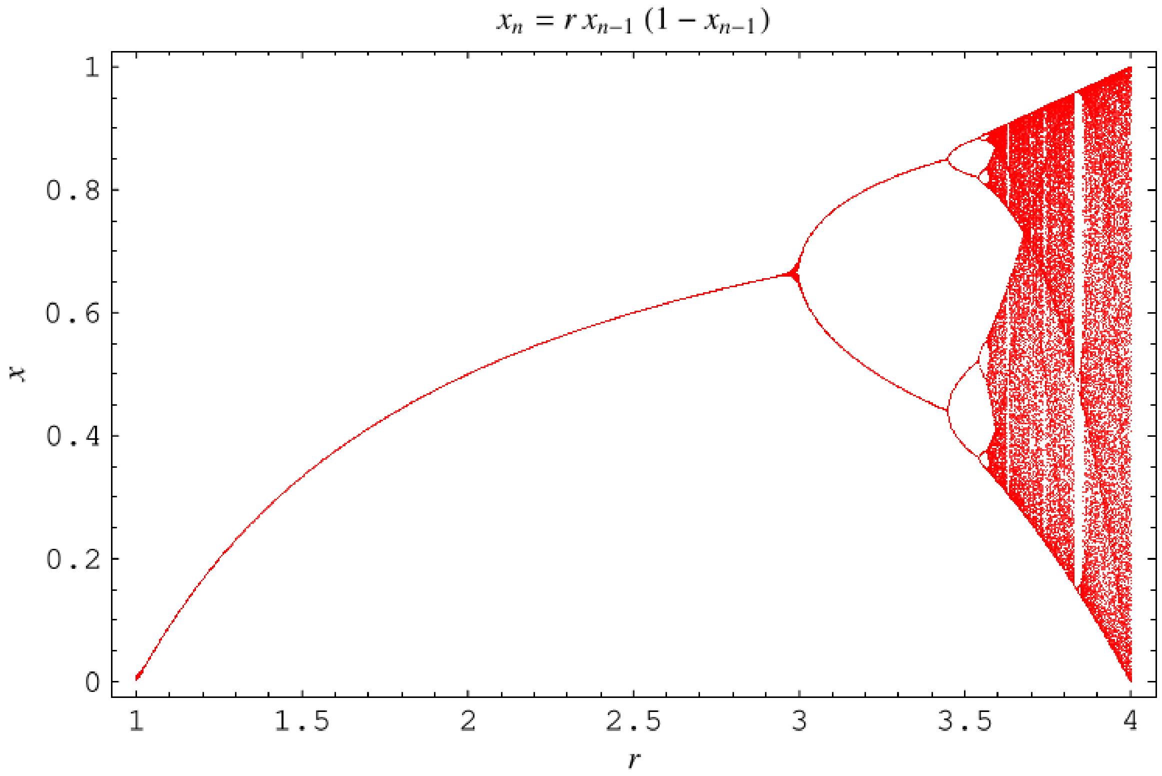
These are *difference* (as opposed to *differential*) equations. And … unlike the differential version … there is NO "analytical" solution of N_t for any N_t.

In fact, this little equation **blew up math as we know it!**

Visit this: https://egurarie.shinyapps.io/DiscreteLogisticMapping/

Unexpected aspects of chaos theory ....

- It emerged from a basic (and old) *deterministic* population model.
- It is extremely sensitive to initial conditions
- The patterns are fractal



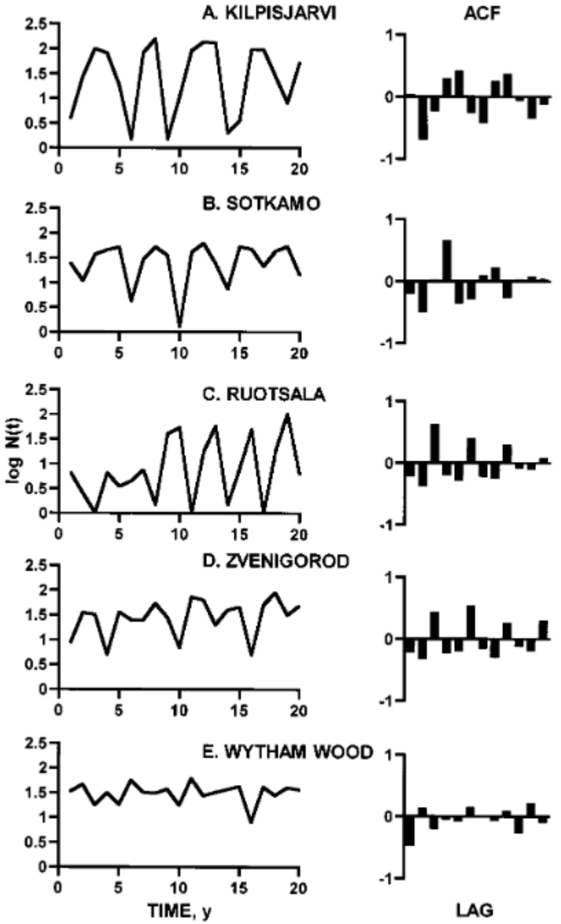
Very nice video on "logistic mapping"

This equation will change how you see the world (the logistic map)

Single population dynamics: Small Rodents

Many, relatively easy long-term studies of small-mammal populations.

Observed that populations fluctuate with higher amplitude and different periods / randomness across latitudinal gradients.

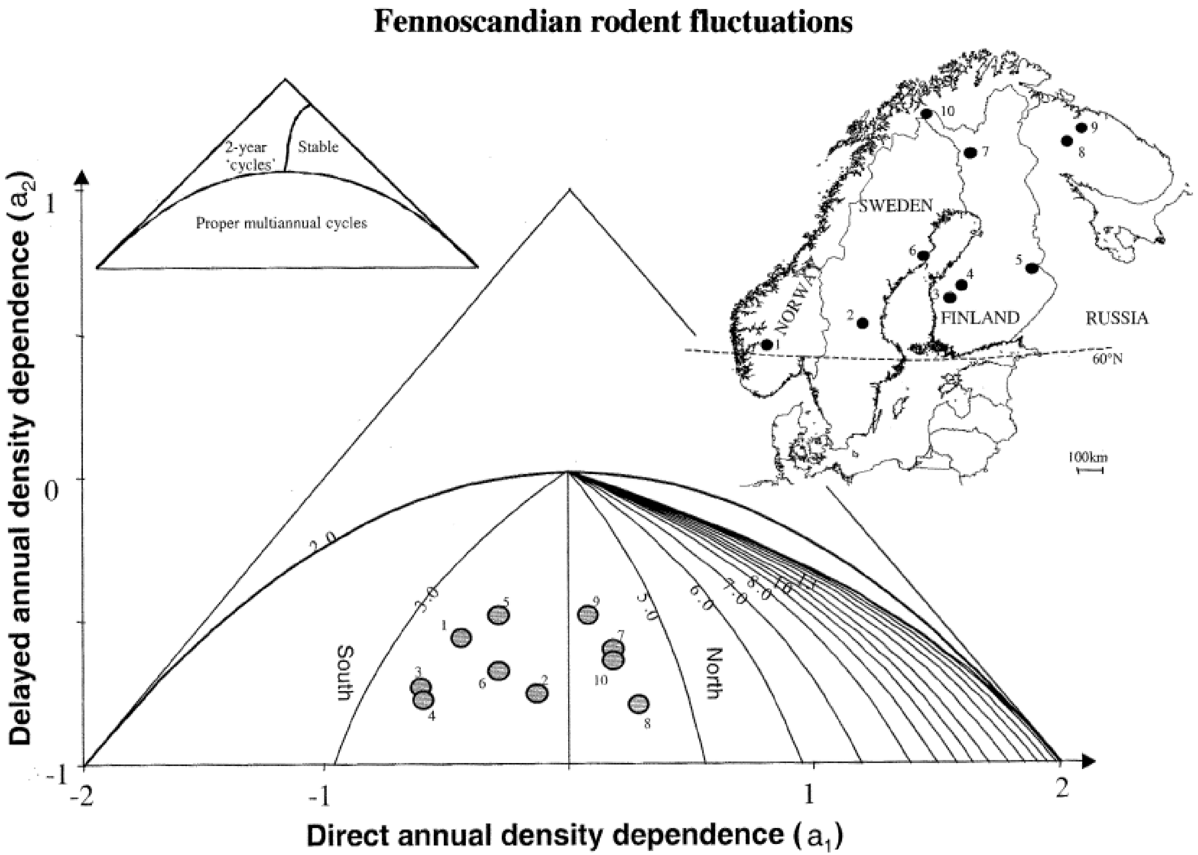


Norway lemming *(Lemmus lemmus)* Tundra vole *(Microtus oeconomus)*

Turchin and Hanski (1997)

Delayed Density Dependence

$$\log(N_t) = a_0 + a_1 \log(N_{t-1}) + a_2 \log(N_{t-2})$$

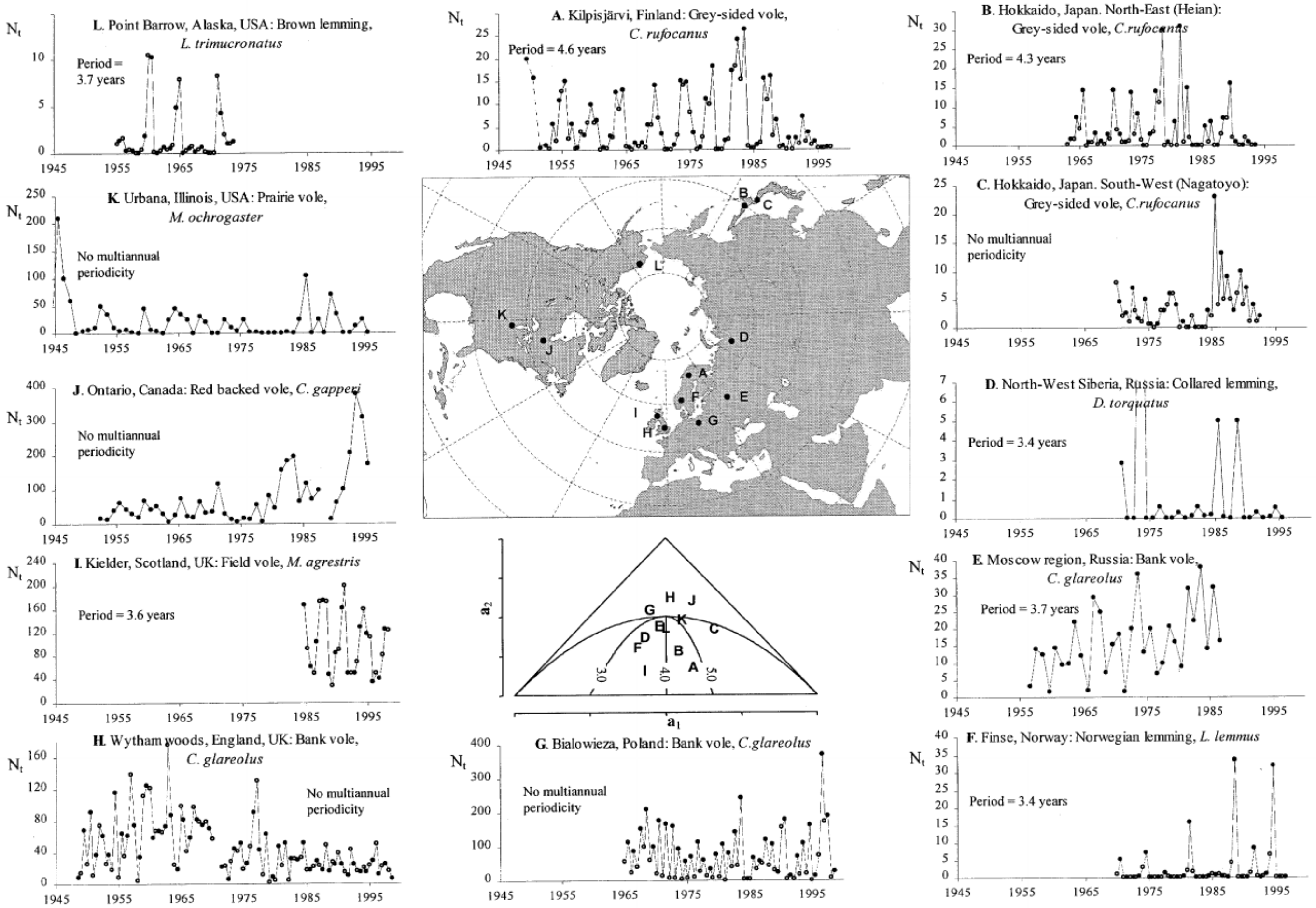


Lots of data and lots of debate

Are we observing deterministic **chaos** or are we observing **stochasticity**?

What *IS* chaos in a stochastic world?

But … methodologically … it is ALL done by fitting **simple linear models**!



Stenseth (1999) *Oikos*

Top 10 subsection heading of all time

OIKOS 87: 427–461. Copenhagen 1999

Population cycles in voles and lemmings: density dependence and phase dependence in a stochastic world

Nils Chr. Stenseth

Digression I: Are ecologists working on voles and lemmings an argumentative bunch whose debates have little relevance to other ecologists – or, is the enigma of the vole and lemming cycles an important but difficult problem to solve?

Ever since Elton's first p
rodent cycles have been
gists in scientific journals
Gaines et al. 1991; see al
as well as in referee repor
of the discussion amon
ideological quarrels rat
Through the peer review

CODA: The enigma of the vole and lemming cycle remains one of the largest unsolved – and most important – issues within the field of population ecology. Even though ecologists working on voles and lemmings are often seen as an argumentative bunch of people, much of the work on the small rodent cycle has been of great general interest and relevance to population ecology as such – and will continue to be so in the years to come.

My conclusion:

Argumentative or not, vole/lemming population ecologists they're certainly have a high **self-regard**!

Next week (and beyond) ....

We blow up:

$$N_t$$

into:

- sex / age classes
- multiple sub-populations
- multiple species (competitors / predator-prey)
- infected, susceptible, recovered

relying on:

- difference and differential equations
- probability and stochasticity
- visualization and simulation
- statistics (esp. linear modeling)
- ***natural history and biological intuition!***