Hi Chris and Steve

For a new project, we are considering a variant on our earlier paper on perception.

> Fagan, W. F., E. Gurarie, S. Bewick, A. Howard, S. Cantrell, and C. Cosner. 2017. Perceptual ranges, information gathering, and foraging success in dynamic landscapes. American Naturalist. 189: 474-489.

We start with this equation set from the "switching alternatives" manuscript that we have been working on with Tyler and Cole. In that paper, we consider 6 models of 'switching on gradients', all of which have the following base model

$$\begin{aligned}
\frac{\partial u(x,t)}{\partial t} &= \overbrace{D\frac{\partial^2}{\partial x^2}u}^{\substack{random\\ search}} - \overbrace{\alpha(x,t)u}^{\substack{switching\ from\\ random\ search}} + \overbrace{\beta(x,t)v}^{\substack{switching\ to\\ random\ search}} \\
\frac{\partial v(x,t)}{\partial t} &= \overbrace{\left[\overbrace{\varepsilon\frac{\partial^2}{\partial x^2}v}^{\substack{low\ level\\ random\\ movement}} + \overbrace{\theta\frac{d}{dx}(x-\mu)v}^{\substack{home\ ranging\\ central\ tendency}}\right]}^{range\ residency} + \overbrace{\alpha(x,t)u}^{\substack{switching\ to\\ range\\ residency}} - \overbrace{\beta(x,t)v}^{\substack{switching\ from\\ range\\ residency}}
\end{aligned} \qquad . \tag{1}$$

We then eliminate the switching stuff (for this newest project) and change the spatial focus of the ranging behavior from a constant μ to a situation where μ is a function of perceived food, leading to something like

$$\frac{\partial v(x,t)}{\partial t} = \overbrace{\varepsilon\frac{\partial^2}{\partial x^2}v}^{\substack{low\ level\\ random\\ movement}} + \overbrace{\theta\frac{d}{dx}\Big(x-\mu\big(m(x,t)\big)\Big)v}^{\substack{focused\ foraging\\ near\ resource\ peak}} \qquad , \tag{2}$$

where $m(x,t)$ is the resource density as in our 2017 paper. For example, we can have a Pulsed Gaussian resource

$$m_g(x,t) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(x-z)^2}{2\sigma^2}\right)\sin^2(\omega\, t/2) \qquad , \tag{3}$$

where z and σ are, respectively, the mean and standard deviation of the resource pulse and ω is the temporal frequency of the pulse. Alternatively, we can have a Pulsed Uniform resource

$$m_s(x,t) = \sin^2(\omega\, t/2) \times \begin{cases} \frac{1}{2\sqrt{3}\,\sigma} & z-\sqrt{3}\sigma \le x \le z+\sqrt{3}\sigma \\ 0 & else \end{cases} \qquad , \tag{4}$$

where the $\sqrt{3}$ term appears so that the standard deviation of the Pulsed Uniform function matches that of the Pulsed Gaussian. However, note that the formulation in (2) changes the model from an advection diffusion formulation as in our 2017 paper to one that is O-U based, but one where the focus of attraction is a resource peak instead of a nest, den site, or similar home range feature.

To make the dimensional units match up for the O-U component, we are thinking of $\mu(\ )$ as a composite function. That is, we assume that a forager at spatial location x takes a "census" of the nonlocal resource distribution via perception so that the perceived resource, $h(x,t)$, is

$$h(x,t) = \int_{-\infty}^{\infty} m(y,t)g(y-x)dy \qquad , \tag{5}$$

where $g(y-x)$ describes modifications in the forager's perception with distance. For simplicity, we choose to focus on the Top-hat detection function, which is written

$$g_u(y,x,R) = \begin{cases} \frac{1}{2R} & -R\sqrt{3} \leq x-y \leq R\sqrt{3} \\ 0 & else \end{cases} \quad , \tag{6}$$

where R is the standard deviation of the forager's detection function, or 'detection scale.' Previous work explored the effects of alternative detection functions on resource gathering in dynamic landscapes when animals move by a combination of advection and diffusion (Fagan et al (2017)).

Because animals should not instantaneously adopt a new focal area for foraging immediately as their perceived resource landscape changes, we introduce an exponential weighting function that smooths out the changes in the perceived spatial distribution of resources. We justify this by the argument that animals must think and process information before making a major movement decision. We have

$$\mu\big(m(x,t)\big) = argmax\left(\int_{t-\tau}^{t} \exp(-\lambda t) \int_{-\infty}^{\infty} m(y,t)g(y-x)dy\, dt\right) \tag{7}$$

where λ is the rate at which the animals discount past information about perceived resources and τ is the timeframe over which they do the weighted averaging. The timescale for repositioning the focus of attraction for foraging ($1/\lambda$) should be slower than the timescale for movement.

(Note A: I'm not sure this is what you mean by the 'exponential weighted average'. As far as I can find, that term usually applies to a set of discrete events, whereas we would seem to want something continuous. However, this particular continuous function is a bit sketchy to me, because the outer integral would seem to just simplify away …)

(Note B: I haven't quite figured out the mathematical notation for "argmax" but in python this is a function call that returns the coordinate location of the maximum of a function. That is what we want here. Evaluate the μ function and return the 'best location' based on the time average of the changes in resources. Make that best location the center of attraction for the O-U movement process and undertake the appropriate stochastic movement.)

Putting all the pieces together and simplifying the outer integral, we have the equation

$$\frac{\partial v(x,t)}{\partial t} = \overbrace{\varepsilon \frac{\partial^2}{\partial x^2} v}^{\substack{low\ level \\ random \\ movement}} + \overbrace{\theta \frac{d}{dx}\left(x - argmax\left(\frac{exp(-\lambda(t-\tau)) - exp(-\lambda t)}{\lambda} \int_{-\infty}^{\infty} m(y,t)g(y-x)dy\right)\right) v}^{\substack{focused\ foraging \\ near\ resource\ peak}} \quad . \tag{8}$$

Any suggestions ?

Thanks

Bill F.