Simulations of road traffic

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Abstract

Cellular automata are computational models that process data on dynamically updated grids, evolving according to fixed rules over discrete time intervals [1]. This report builds on previous models of traffic flow, using cellular automata, by including a graphical user interface, more human-like braking and allows for 2D layouts.
By varying velocities, it is shown how different car densities correspond to different phases of traffic: wide moving jam, synchronous flow, and free flow, therefore supporting B.S. Kerner's 'Three Phase Traffic' model [2]. The variation of arrival times across the transition from congested to free flow is analysed and the critical density dividing these phases is found to be ~ 32 cars/km or $\sim 24\%$ total occupation. This value indicates the efficiency of a layout. If the density exceeds the critical threshold, the system will likely encounter phases of moving jams, suggesting that the layout should be reevaluated.
Roundabouts and traffic lights are also analysed to determine the most efficient parameters to minimise congestion and it is found that, in congested flow a roundabout is far more efficient than crossroads. After transitioning to freer flow, then the crossroads are marginally more efficient. This is an unexpected result which is a consequence of the cars having perfect vision around corners.

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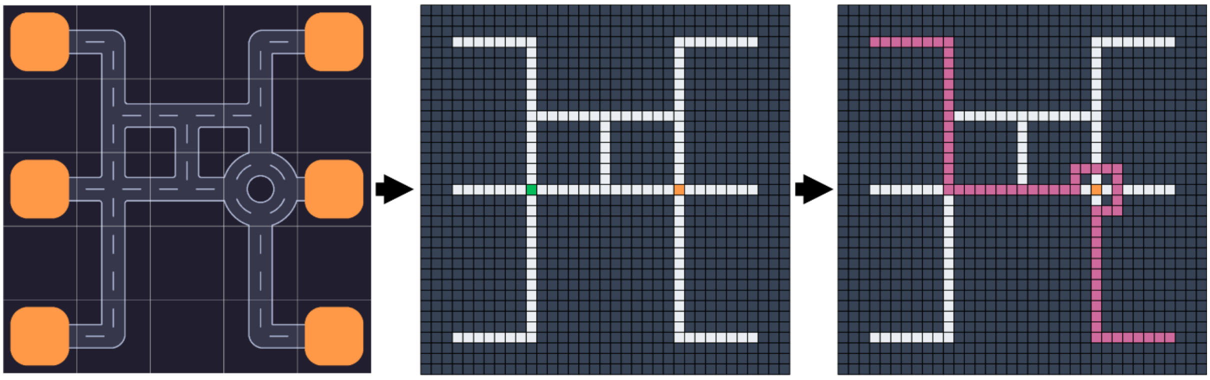


Figure 1: The left image shows the graphical interface after user input, orange cells correspond to houses; the middle shows the subdivided grid; the right shows an example of a path highlighted in pink.

1. Introduction

Optimisation of traffic flow is a pivotal issue in the modern age with ramifications extending far beyond being forced to sit in rush hour traffic. Transport is the largest emitting sector of greenhouse gases in the UK, accounting for 26% of total emissions [1]. Controlling congestion and lowering travel times are essential in cutting down carbon emissions and air pollution [2]. Moreover, congestion causes a significant financial toll, with studies estimating its annual cost to the UK economy to be approximately £20 billion [3].

Due to environmental and financial factors, we have been forced to move away from intuitive solutions, such as widening existing roads and building new ones. And so, in search of a more scientific solution, Nagel and Schreckenberg were the first to use the cellular automaton (CA) algorithm to model traffic flow [4]. While initial assessments against empirical data revealed a relatively accurate portrayal of behaviours at junctions, their model fell short when compared to motorway traffic [5].

CA is a powerful tool for simulating traffic flow along with other many-particle systems, as the simplicity allows for large-scale simulations of high complexity, whilst retaining computer performance. This is due to the algorithm being able to run at the level of single bits and have its parameters easily varied for specific use.

This report uses the CA model to explore the behaviours and efficiency of traffic flow when varying parameters such as average velocity, car density and reaction distance threshold (RDT). This data is then compared to real-world data to assess the model's strengths and limitations.

2. Cellular Automata Model

When running the programme, the user is prompted with an interface created using the pygame library, it displays a grid of tiles where they can input a combination of roads, which automatically connect, as well as houses. A house is a tile in the grid that can generate and receive cars. Houses are placed on any road tiles except crossroads and roundabouts. Once the user validates their design, the code subdivides the grid by seven, creating 49 cells per tile as seen in Figure 1.

Dividing the tiles into cells allows for a higher resolution in the simulation and makes the road layout easily recognizable for the algorithm. A tile contains a picture and its rotational data. Cells take integer values defining their properties; 0 is no path, 1 is a path, 2 and 3 are crossroads, and 4 is a roundabout. In Figure 1: 0 is dark, 1 is white, 2 is green and 4 is orange. Crossroads oscillate between values of two and three allowing vertical or horizontal flow of cars. A car's route will be modified to circulate clockwise when it encounters a roundabout. Once a roundabout is filled up with 21 cars, it creates a roadblock as each car is being stopped by the car in front, forming a loop. To prevent a gridlock, a roundabout has a maximum capacity of 14 cars; any additional cars must wait before entering.

The right side of Figure 1 depicts a possible path a car might take, moving from the top left to the bottom right, as can be determined from the rotation direction around the roundabout. To find the shortest path between two houses, cars use the A* algorithm [6]. The A* algorithm is a pathfinding technique that finds the shortest route from a starting cell to a target cell. It uses a heuristic function that estimates the cost of each path by prioritizing cells that appear to lead most directly to the target. Here the function is,

$$|g_x - x| + |g_y - y|,$$

where g_x and g_y are the final cells x and y values (the goal values) and x and y are the current values. The algorithm minimises this function which, as seen in Figure 2, leads to prioritizing down rather than left even if both paths have the same length.

2.1. Units

Throughout this paper there are two units, distance and time represented by cells and frames. A cell is one of the 49 subdivisions of a tile. For example, a straight road tile is 7 cells long. A frame represents a single update of the entire subdivided grid; the frame count increments with each recalculation.

The goal of a simulation is to predict real life events, we can get an idea of what these quantities represent using empirical data. Since neighbouring cells can be occupied simultaneously, the corresponding cell size must consider both the size of the car and the margin of space surrounding it. This paper will agree with Nagel and Schreckenberg and adopt a size of $7.5\ \mathrm{m}$ per cell [4]. The discrete nature of the model means there is a minimum non-zero velocity of $7.5\ \mathrm{m/frame}$. To match this to a reasonable velocity for an urban environment, we assume $1\ \mathrm{cell/frame} = 10\ \mathrm{km/h}$. Solving for time we get $0.75\mathrm{s/frame}$.

2.2. Cars

Cars are tuples with four parameters: their previous velocity, upcoming path, direction, and time alive. The previous velocity is the car's displacement from the previous frame to the current one. The upcoming path is the list of coordinates the car must travel to its destination, where the first element is always the current position. The direction is the difference between the next and current x,y positions. The time alive is the number of frames the car has existed.

Having the path allows for easy extraction of the next positions for the car. The code looks through the coordinates for an obstacle, once found, the distance is the index of the list, this makes the problem one dimensional as no matter if the coordinates describe a junction or a loop the process will be the same.

Thus, interactions can be defined by six parameters, the car's previous velocity v_n, the obstacle's previous velocity v_o, the distance d, the average reaction distance threshold (RDT) r_μ and its associated uncertainty r_σ and range r_R. The range defines for which values of the distance the distribution is defined, $r_\mu - r_R \le d \le r_\mu + r_R$. The integrals required to normalise the distribution and calculate the areas under the curve are done at the beginning of the simulation and stored in the random-access memory (RAM) for better performance. The integrals were solved using the finite difference method [7] and the uncertainties are negligible thanks to the output being quantised.

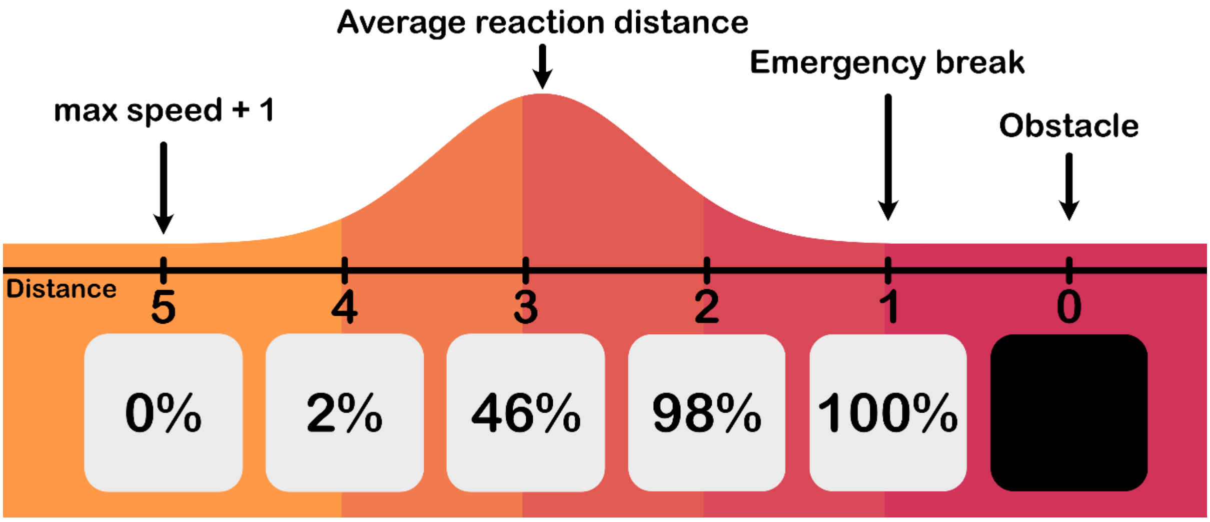


Figure 2: The probability of a car reacting to an obstacle, where $r_\mu = 2.95$, $r_\sigma = 0.45$ and $r_R = 1.5$

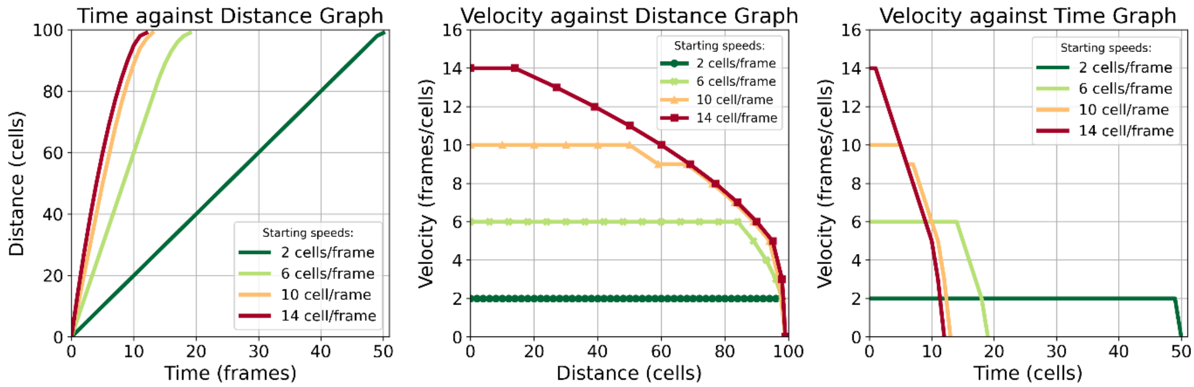


Figure 3: Graphs of Equation 1 for different starting velocities $v = \{2,6,10,14\}$ with $d = 100$

Figure 2 shows the probability distribution of a car reacting to an obstacle; as an example, at $d = 3$, the car is 46% likely to slow down. $d = 5$ is outside the range of reaction and so the probability is null. $d = 1$ is always 100% likely to avoid collisions. The probabilistic nature of cars reacting model's human-like behaviour.

Once a car reacts, its deceleration is defined by the other three parameters as follows,

$$v_{n+1} = \min\left(\text{ceil}\left[v_n + v_n \frac{v_o - v_n}{2d_n} \times 1\right], v_{max}\right), \qquad [1]$$

where v_{max} is one minus the maximum distance and 1 represents time in frames, however since we are interested in the speed of the next frame it is always one. The minimum allows the car to never overshoot an obstacle and the addition of time allows for the units to match as the right-hand side of the ceil is $(cell/frame)^2 \times cell^{-1} \times frame = cell/frame$.

Figure 3 shows the deceleration of multiple values of v_n with $v_o = 0$, each dot on the middle graph is one frame and shows all the data that the car has when slowing down.

Free acceleration, no obstacle, $d \to \infty$ is done by a similar series to Equation 1 just with v_n equal to the speed parameter and $2d$ replaced by 3.

2.3. Variables

The simulation is defined by 12 parameters. Grid size defines the number of tiles in a side of grid, car count is the number of cars spawned during the simulation and light switch interval is the number of frames for crossroad lights to switch. Speed, departure and RDT are truncated normal distributions defined by three parameters: average, standard deviation, and range. The speed and departure distribution are used to generate random numbers that define, the speed

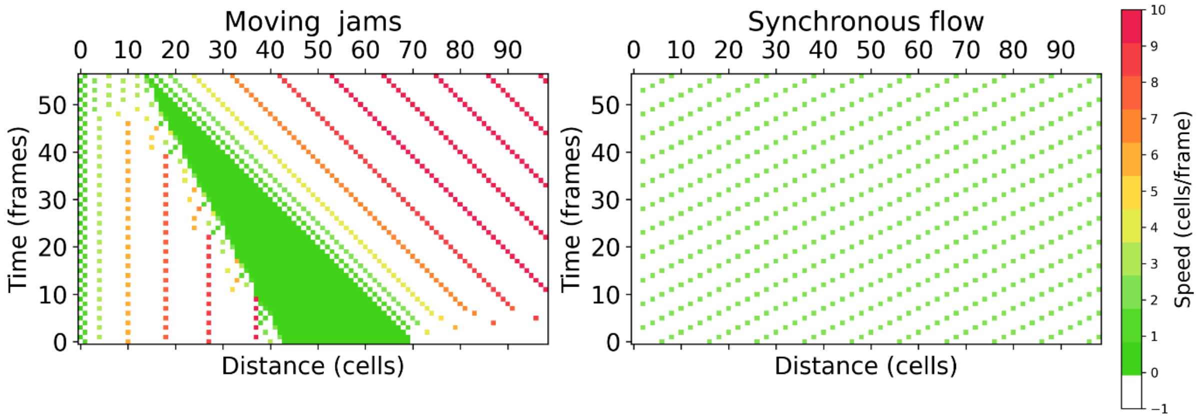


Figure 3: Position-time graphs showing a moving jam (left) with a stationary line of cars which propagates in the opposite direction to car flow, and synchronous flow (right) in which the cars each travel at the same, reduced speed. Each car is represented by a point with a colour indicating the speed.

of a car experiencing free flow and the interval between car spawns. Finally, RDT describes the Gaussian function of Figure 2.

2.4. Limitations

This setup has two unavoidable shortcomings.
The first comes from flattening the data into one dimension to calculate the distance to an obstacle. While this leads to massive performance gains and allowed the large-scale data gathering in this document, it also means that cars can see around corners. This leads to cars not slowing down when turning which has ramifications that will become apparent later.
The second is the nature of cellular automata being discontinuous. When a car brakes, v_o is the obstacles previous speed, thus cars slow down for ghost images of other cars.
Even after removing the visuals, using the multiprocessing module to get the code running on every thread of the central computational unit (CPU) and writing custom modules, (Appendix 1) the data gathered in this document took over 150 hours of computations on various computers.

3. Three Phase Traffic Model

In 1955, M.J. Lighthill proposed a new theory, where traffic was observed to have similar behaviours to water in backwards-travelling 'kinematic waves' of congestion [8]. In later years, the fluid nature of traffic flow was theorised to show two distinct phases: free flow (F) and wide moving jam (J) [9]. F occurs at high speeds or low traffic density, causing vehicles to move at a uniform speed with minimal interactions. J occurs spontaneously past some critical density, or because of an obstruction. Due to the higher density of cars in this phase, the jam propagates 'upstream' against the flow of traffic [10]. The main drawback of this model is that there is no empirical evidence for a spontaneous F → J phase transition [11]. The only way this transition can occur is through a 'boomerang effect' whereby congestion, caused by a random fluctuation in car speed, initially travels downstream, then increases in amplitude and begins propagating upstream [11].
In 1996, Boris Kerner realised a new phase was needed to explain this effect and therefore created the three-phase traffic model with the addition of synchronised flow (S) [10]. S is reminiscent of free flow as the cars in this phase have coherent speeds and the flow rate of cars per second is the same, however, it is distinguished from free flow by a higher traffic density and lower relative speed. The two phases of congested flow are shown in Figure 3.

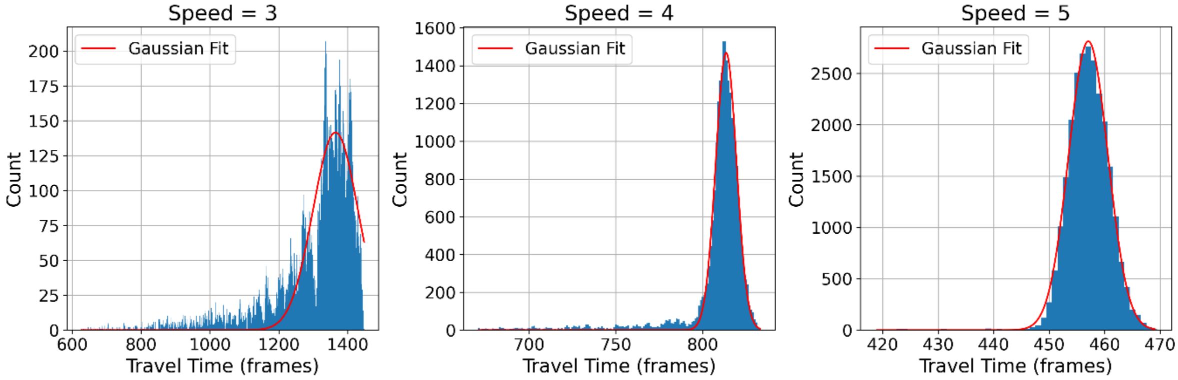


Figure 4: Frequency graphs of travel times (frames) for simulated cars. Average speed = 3, 4, 5 respectively with a constant rate of departure of 1 car per frame. A gaussian fit has been performed for each, shown by a red line.

4. Straight Road

The simplest arrangement to observe traffic behaviours, is a straight road without junctions or obstructions. In ideal conditions: a large RDT with fixed speed and departure time; the outcomes become highly deterministic. Cars can travel at their maximum speed without interaction, leading to negligible spread in the travel times. For an average speed of one cell per frame, all cars arrive with a travel time equal to the cell length of road provided. In real systems, this is unlikely to be the case as a driver's speed fluctuates when reacting to outside stimuli, such as the gap between cars or potential hazards on or around the road. To account for this, the cars are simulated to have a RDT, and random chance of accelerating and decelerating through the speed parameter.
It is important to note that throughout this paper, traffic density will be mentioned in terms of speed and departure time. Since cars are spawned at equal intervals of time, the lower the speed, the less distance is travelled before the next is spawned. This results in smaller gaps between cars and therefore a higher density. The same logic applies to departure time and vice versa for both.

4.1. Variation of Average Speed and it's Standard Deviation

When examining the variation in average speed, a clear pattern emerges, with three stages as shown in Figure 4. At average speeds ranging from 1 to 3, the leftmost graph shows a coarse distribution of travel times, with a gaussian fit giving a large standard deviation of 65.4 frames, therefore causing a low peak frequency of 142. This distribution likely emerges due to the high traffic density of around 0.34 cars per cell causing frequent interactions between vehicles. This stage is distinguished from the others by its low peak frequency and large standard deviation. In this stage, a large amount of traffic moves in either: wide moving jam phase, where one sees stationary cars and disturbances propagating upstream; or synchronised flow phase, where one sees sections of traffic moving at a similar speed. This behaviour is analogous to a low-velocity section of motorway, where densely packed cars cause movement to propagate upstream, resembling a wave through a fluid. This behaviour is shown on the left in Figure 3.

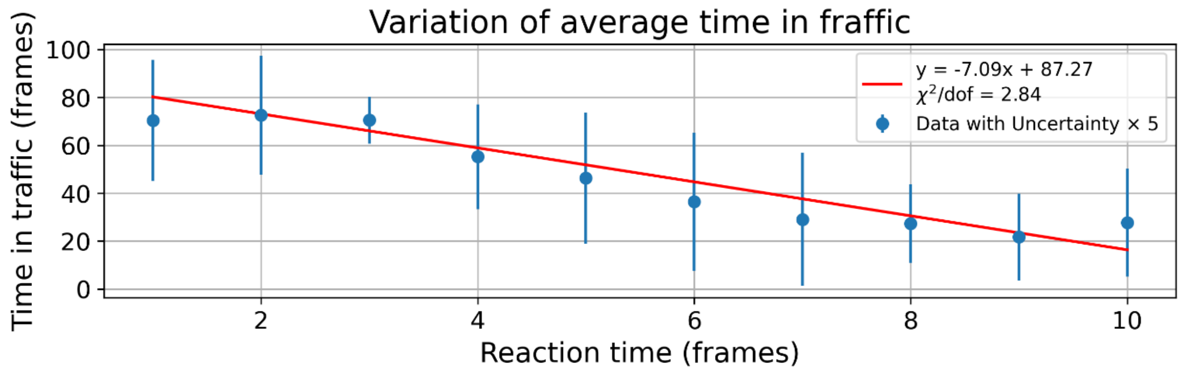


Figure 5: The trend in average time in traffic (frames) as RDT is increased with a constant average speed parameter of 7. A linear fit has been applied as shown in red.

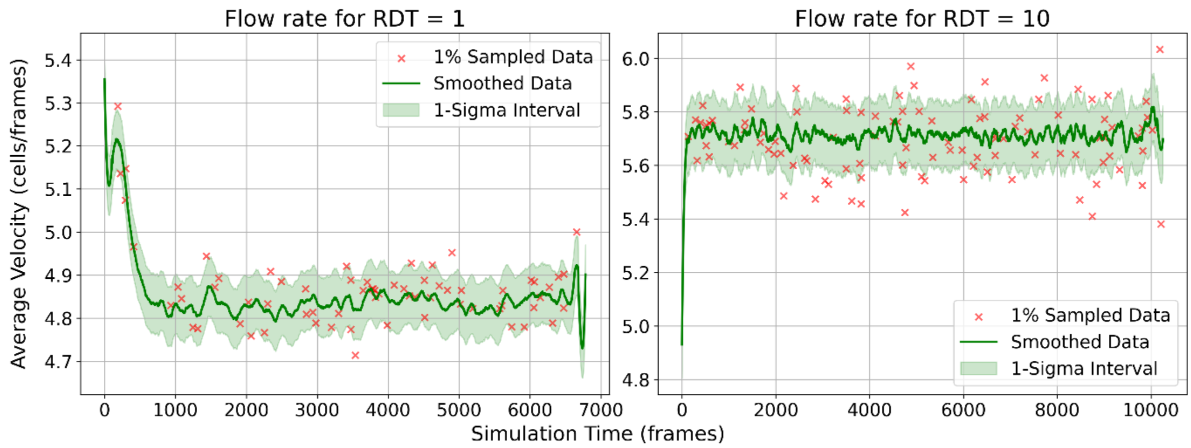


Figure 6: A representation of the data showing how average velocity varies throughout the simulation with RDT 1 and 10 respectively. The dark green line shows a smoothed variation of the data, and the light green area shows one standard deviation. Full data in Appendix 2

Increasing from an average speed of 3 to 4 cells/frame, the central graph of Figure 4 displays a more defined distribution with a drastically reduced standard deviation of around 6.0 frames. Whilst more refined, when compared to the higher speeds, it is less fitting to a gaussian distribution and displays much more noise. Despite the standard deviation being small, this behaviour is still distinct from the higher speeds as there is a large range between the fastest and longest arrival times.

Higher speeds yield much smoother, more predictable traffic flow, as can be seen on the rightmost graph of Figure 4. With increased speed, the gaps between cars get larger which lowers the density and minimises interactions between vehicles. This is shown when the speed increases from 4 to 5, the standard deviation decreases to 3.5 frames and the peak frequency greatly increases from 1468 to 2814 with an overall better fit.

These distributions show the transition from congested flow on the left, to free flow on the right. The central graph shows a 'critical density' dividing them at approximately 0.24 cars/cell which corresponds to 32 cars/km. This critical density falls within the range of values empirically proven by preceding papers [13][4]

In summary, when experiencing congestion, the cars' speeds vary greatly and unpredictably. However, when in free flow, they become more stochastic and follow a gaussian distribution. It makes sense for the data to tend towards a gaussian since, when the density is low enough,

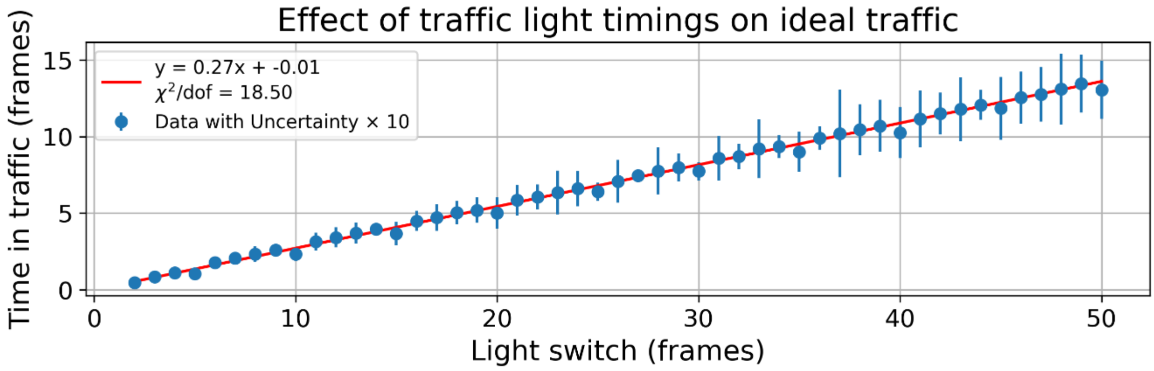


Figure 7: A graph showing the linear relationship between ideal traffic and traffic light switch time. The 0^{th} point has been removed.

the only factor affecting the speed is the independent, random event of a change in velocity as determined by the speed parameter. Although this parameter is modelled as a gaussian in the simulation, it is entirely plausible that this distribution would still be present with empirical data since it reflects the perturbations a driver would experience [12]. With a large enough data set, the central limit theorem asserts that the data would tend towards a normal distribution since the chance of slowing is an independent event [15].

4.2. Variation of RDT

As shown in Figure 5, the variation of RDT causes a linear decrease in time spent in traffic. This is due to the larger threshold giving the cars a longer time to decelerate causing a 'cushion' effect. This results in more free flow along the road, as perturbations in traffic flow are smoothed when cars react to obstacles further away from them. This behaviour is analogous to wave propagation in fluids of varying compressibility [16].
For a RDT of 1, the average car density was 0.16, whereas, with a higher RDT, smoother flow was achieved, resulting a density of 0.09. Figure 6 shows how having a smoother flow and lower density allowed the cars' average velocity to increase from 4.72 to 5.71 cells/frame therefore demonstrating the impact congestion has on traffic flow. Unintuitively, the higher flow for RDT = 10 leads to $\sigma = 0.125$ which is much larger than RDT = 1 with $\sigma = 0.069$, These are calculated from the standard deviation of five identical simulations. This is explained by the parameter 'speed'. As density decreases, the car is free to accelerate past its average speed then, as a result, is more likely to decelerate. This causes more frequent fluctuations. However, this does not cause congestion due to the 'cushion' effect softening the perturbations.

4.3. Busses

Buses operate similarly to single-file cars, with the primary distinction being their frequent stops. These stops can trigger cascading traffic jams, causing delays that accumulate and exacerbate congestion for following buses. Moreover, the inherently slower speed of buses often acts as a bottleneck, prompting many cities to implement dedicated bus lanes to minimize interference with other traffic.

5. Four-way Junction

Various methods of controlling traffic flow have been introduced to urban areas, two of the most common are roundabouts and traffic lights. This next section will explore each and compare their efficiency and evaluate what traffic flow each setup is suited to.

5.1. Efficiency of Traffic Light Timing at Crossroads

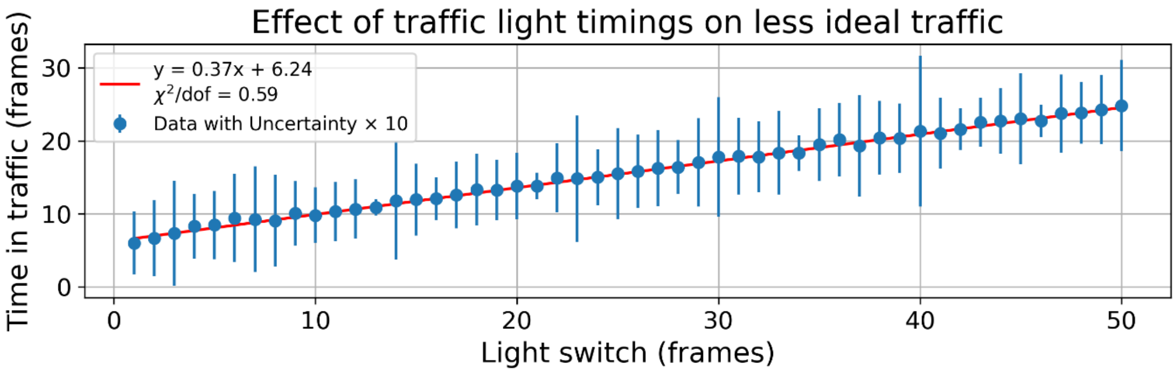
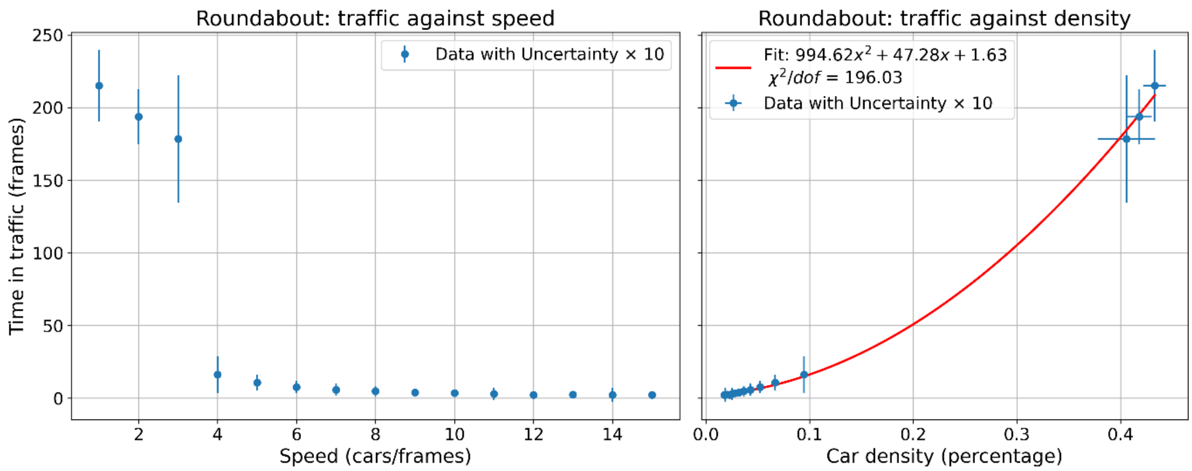


Figure 8: A graph showing the linear relationship between less idealised traffic and traffic light switch time.



Left: Figure 9: A scatter graph showing the average amount of frames of traffic experienced for different average speeds.
Right: Figure 10: A scatter graph showing the same data as figure 9 but with the average densities instead of speeds. It includes a parabolic fit. The large chi-squared value is more to do with small errors than a bad fit.

In the ideal case, varying traffic light timings behaves as expected: a linear relationship with an intercept at the origin, as seen in Figure 7. Since the traffic is idealised, there are minimal interactions between cars and the only traffic is caused by the lights themselves. Notably, every 5th point spends slightly less time in traffic as each car was travelling at 5 cells per frame and so was in phase with the lights switching.
Each data point on Figures 7 and 8 represents an average from 5 simulations of 5000 cars each. The error bars represent the standard deviation across them. The error gets larger as time in traffic increases since, similarly to the straight road, behaviour becomes more erratic the more traffic density present so there is more spread across simulations.
When introducing RDT, there is little change in the data aside from greater variance between simulations. Figure 8 shows a new intercept corresponding to the average time in traffic not caused by the traffic lights. The steeper gradient is likely due to the cars slowing down as they react to the jam in front of them, therefore causing more frames of traffic. This data again implies the most efficient use of traffic lights is not implementing them at all which is evidently not true. This discrepancy is caused by the algorithm's detection method. Namely the cars have perfect vision and so do not need to slow down or give way when approaching a junction, which makes traffic lights obsolete.

5.2. Variation of Speeds at a Roundabout

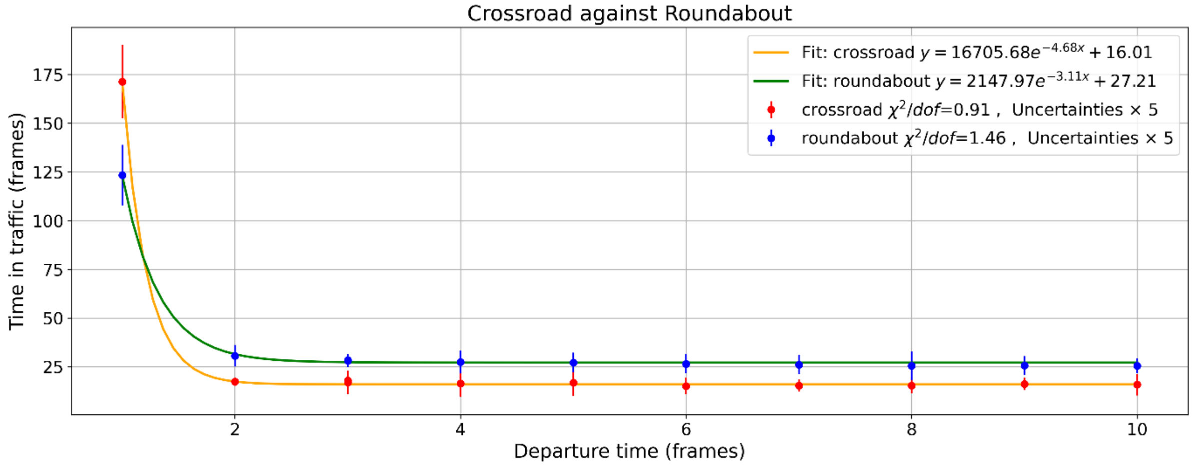


Figure 11: Lines corresponding to speed = 3 for a roundabout (green with blue dots) and crossroads (beige with red dots), showing how the average number of frames in traffic progresses when changing departure time.

Similarly to the straight road simulation, this data is much more intuitive to think of in terms of densities. At average speeds ranging from 1 to 3 cells/frame, the roundabout becomes densely occupied at approximately 0.45 cars/cell. This causes high congestion which brings many cars to a standstill, resulting in an average velocity of 0.5 cells/frame. At the average speed of 4 cells/frame – corresponding to a density of 0.1 cars/cell – there is a significant reduction in the number of frames spent in traffic. From this it can be assumed that the roundabout also has a critical density. This value will lie in the range of 0.095 to 0.41 cars/cells which corresponds to 13 to 53 cars/km. This transition resembles a phase shift in the traffic flow, causing the graph to change from one curve to another as it moves from more congested to freer flow, shown in Figure 10 by a reduction in density. When traffic around the roundabout is dense and slow moving, it is less likely for a car to exit the queue and enter the roundabout. Conversely, at higher speeds and lower densities, the roundabout experiences less congestion and clears faster.

When the time in traffic is plotted against car density, it shows the parabolic relationship

$$T = 848\rho^2 + 76\rho + 0.97$$

which suggests there exists a maximum number of frames that can be spent in traffic. The maximum possible traffic density of 1 corresponds to 925 frames in traffic. In reality, a maximum cannot be possible since traffic is a closed system with a chance of gridlock, however, this simulation is open since cars are removed when they reach their destination cell. This model also deviates from reality at higher speeds, whilst there is a physical limit of 14 cars, there is no speed limit. This results in the traffic tending to 0 as speed increases, instead of slowly building back up as arrival rates overtake the roundabouts capacity.

5.3. Efficiency of Traffic Lights against Roundabouts

Whilst traffic lights and roundabouts are both designed to control traffic flow, they have different efficiencies depending on the traffic flow. Simulation results in Figure 11 show that, for high densities, the roundabout causes considerably less traffic. When density is lower, the crossroads cause less traffic but by a much smaller margin. Despite negligible errors, the reduced chi-squared value is almost perfect.

6. Conclusion

This report has built on the Nagel and Schreckenberg CA model [4] but implemented a more natural method of braking, reacting to gaps between cars and allowing 2D systems. As a result, the data from the straight-line simulation not only verified the existence of a critical point dividing free to congested flow in traffic, but reproduced a value that is close to values of 25 – 30 carskm^{-1} empirically verified by multiple preceding papers [13][4]. This critical density also explains the spontaneous formation of traffic jams seemingly without cause. Once the critical density is passed, the traffic enters a metastable state where even a tiny fluctuation in speed can cause a state change to congested flow. Critical density was also shown in traffic flow around a roundabout which suggests there will also be a value for crossroads which this simulation has failed to reproduce. The importance of awareness when driving was shown by varying RDT. When low, the larger density causes congested flow and therefore lower speeds. In contrast, when the RDT is high, the average velocity is increased as the large distance threshold causes a 'cushion' effect, dampening perturbations and increasing velocity. The theorised three phases of traffic flow were also verified when examining the space-time diagram of figure 3, from this it is evident that wide moving jam and synchronised flow are distinct phases of traffic. It can be assumed that the varying distributions obtained at different speeds are a direct result of the consequent changing of traffic density. The changing density affects the probability of traffic experiencing separate flows, more congested flows at higher density and more free flow at low density. The more congested flow present, the larger the standard deviation of arrival times; the freer flow present, the closer the data tends to a gaussian fit which displays the random, independent fluctuations in speed exhibited by real vehicles.

6.1. Evaluation of Model and Recommendations for Improvement

Whilst the model reproduced traffic behaviour with no junctions relatively accurately, major flaws were present in other setups. Modelling the cars' paths as 1 dimensional gave them 'perfect vision', allowing them to see around bends so there was no need to slow down when approaching a junction unless there was congestion. This resulted in traffic lights being obsolete as, when increasing traffic light switch time, arrival times followed a positive linear relationship. This caused the minimum non-negative value for light switch time to be 0, meaning having no lights at all. This is obviously not true for physical traffic in which the optimum time for urban traffic lights to switch has been found to be between 60-90s [17]. For roundabouts, the model was relatively accurate for lower speed simulations, giving a critical density somewhere in the region between 53 and 13 cars/km. Physical observations in the USA have found the critical density for congestion at a roundabout to be 30 cars/km [18]. When simulating higher average speeds, the data deviates from expected values as there is no speed limit upon entering or approaching the roundabout. By this logic, the roundabout will always be more efficient at higher speeds.

With the rise of very fast computers, it currently makes more sence to model cars as continuous entities rather than as discrete grid values, this would allow implementation of line of sight and more realistic changes in velocity. However, the current model can be improved by adding a pseudo line of sight by artificially slowing down cars at junctions. This change would likely produce maxima for efficiency where this model did not. Another key change would be the ability to vary density as a parameter itself rather than as a result of changing other parameters. This would allow for more detailed analysis of the fundamental flow-density relationship that is pivotal in traffic flow theory.

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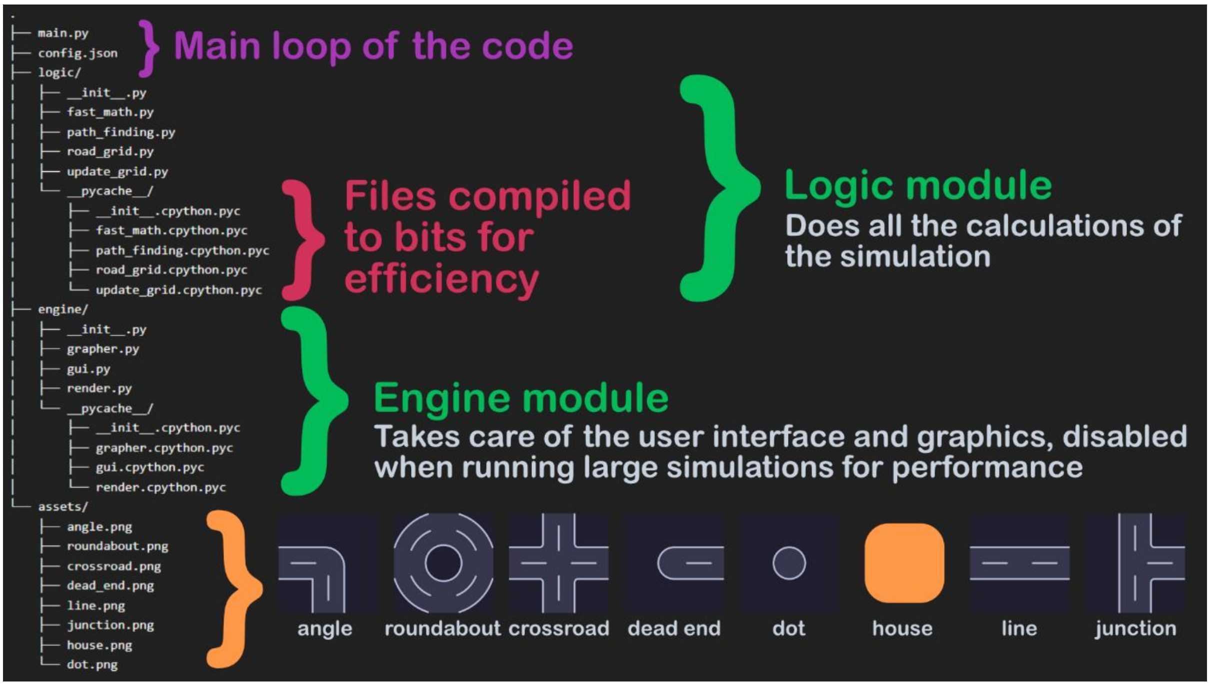
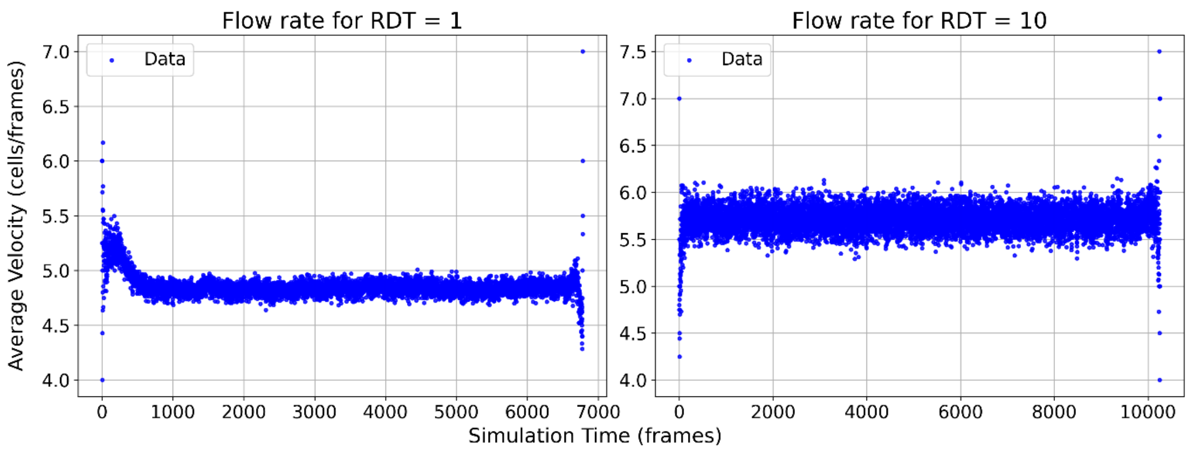
Appendix



Appendix 1: Graphic showing the structure of the code for the simulation.



Appendix 2: Full data showing how the average velocity varies throughout the simulation with RDT = 1 and 10 respectively for a straight line.