set2

October 7, 2017

```
In [2]: import numpy as np
        import matplotlib.pyplot as plt
        import random as ran
        import scipy.integrate as integrate
        from sklearn.linear_model import LinearRegression
        %matplotlib inline
```

```
In [3]: # Toss coin N times and return frequency of heads
        def tossCoin(N):
            results = []
            for i in range(0,N):
                results.append(ran.choice([0,1]))
            return np.mean(results)
```

```
In [4]: # Toss the coin for fun
        tossCoin(1000000)
```

```
Out[4]: 0.49902600000000003
```

```
In [5]: # Toss 'coins' coins N times and return the interesting coins in a 3-tuple
        def getInterestCoins(coins, N):
            results = []
            for i in range(coins):
                results.append(tossCoin(N))
            return (results[0], ran.choice(results), min(results))
```

```
In [11]: c1 = []
         c_ran = []
         c_min = []
         for i in range(100000):
             trial = getInterestCoins(1000, 10)
             c1.append(trial[0])
             c_ran.append(trial[1])
             c_min.append(trial[2])
```

```
In [10]: print("Average of c1: " + str(np.mean(c1)))
```

```
        print("Average of c_ran: " + str(np.mean(c_ran)))

        print("Average of c_min: " + str(np.mean(c_min)))
```

```
Average of c1: 0.499788
Average of c_ran: 0.499046
Average of c_min: 0.037341
```

```
In [7]: # make a line between two random points
        def getLine():
            (x1, y1) = (ran.uniform(-1,1), ran.uniform(-1,1))
            (x2, y2) = (ran.uniform(-1,1), ran.uniform(-1,1))
            line = lambda x:(y2-y1)/(x2-x1)*(x-x1)
            return line
```

```
In [8]: # Generate N unlabeled test pts
        def generatePts(N):
            pts = []
            for i in range(0,N):
                pt = ran.uniform(-1,1), ran.uniform(-1,1)
                pts.append(pt)
            return pts
```

```
In [9]: # label and return a given set of points
        def labelPts(pts, line, noise = False):
            labeled_pts = []
            for pt in pts:
                if pt[1] > line(pt[0]):
                    labeled_pts.append([1, pt[0], pt[1], 1])
                else:
                    labeled_pts.append([1, pt[0], pt[1], -1])

            if noise:
                for pt in labeled_pts:
                    # 1/10 chance we flip
                    if ran.choice(range(10)) == 1:
                        pt[3] = -pt[3]
            return labeled_pts
```

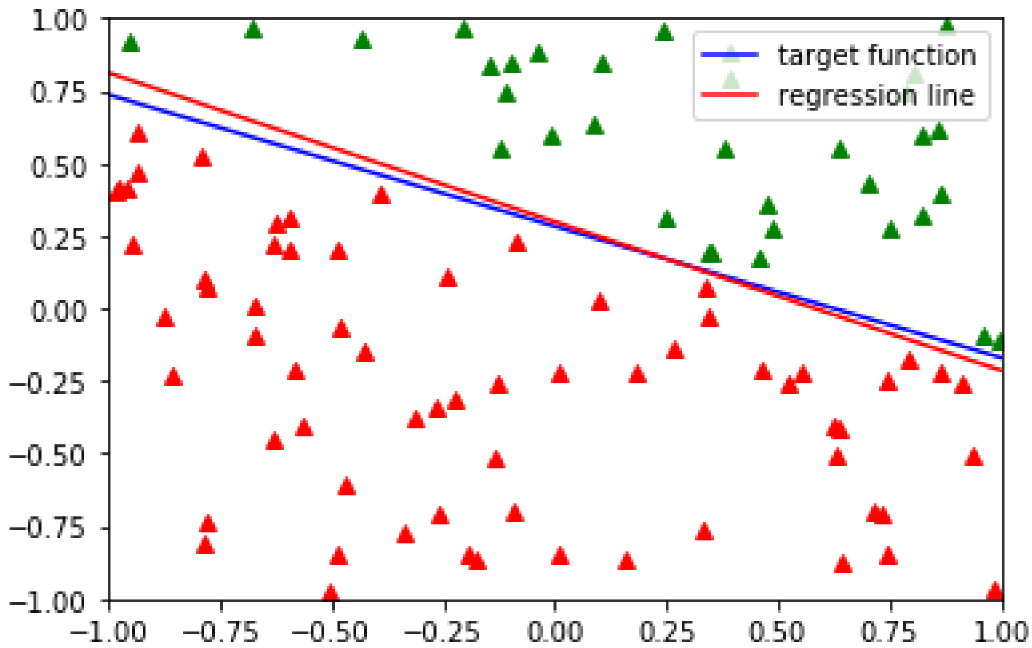
```
In [10]: # runs regression on given pts classified by given target
         # if testing is on it returns (w, E_in, E_out)
         # if testing is off it returns w, line of best fit
         def doRegression(pts, target, graph = True, test = True):
             # generate points labeled according to the target function
             line = target
             labeled_pts = labelPts(pts, line)
```

```
# format data for sklearn package
x = []
y = []
for pt in labeled_pts:
    x.append([pt[1],pt[2]])
    y.append(pt[3])

# perform regression with sklearn
lm = LinearRegression(fit_intercept=True)
lm.fit(x,y)

# compile resulting w
w = [lm.intercept_, lm.coef_[0],lm.coef_[1]]

# line of best fit:
rline = lambda x1:-(w[0]+w[1]*x1)/w[2]

# graph things if asked
if graph == True:
    X = np.linspace(-1,1,1000)
    y = line(X)
    ry = rline(X)
    plt.ylim(-1,1)
    plt.xlim(-1,1)
    plt.plot(X,y, color = "blue", label = "target function")
    plt.plot(X,ry, color = "red", label = "regression line")
    plt.legend()
    for pt in labeled_pts:
        if pt[3] == 1:
            plt.plot(pt[1],pt[2], 'g^')
        else:
            plt.plot(pt[1],pt[2], 'r^')


# approximate E_in and E_out if testing is requested
if test:
    test_pts = labelPts(generatePts(1000), line)


    E_in = 0
    for pt in labeled_pts:
        if np.sign(w[0]+w[1]*pt[1]+w[2]*pt[2]) != pt[3]:
            E_in += 1
    E_in /= float(len(labeled_pts))

    E_out = 0
    for pt in test_pts:
        if np.sign(w[0]+w[1]*pt[1]+w[2]*pt[2]) != pt[3]:
```

```
                E_out += 1

        E_out /= 1000.0
        return w, E_in, E_out


    return w, rline
```

In [11]:
```
# run a regression for fun and eye test
doRegression(generatePts(100), getLine())
```

Out[11]:
```
([-0.37371213847165796, 0.64008593379375134, 1.2481589686929375], 0.0, 0.(
```



In [12]:
```
# get E_in and approx E_out by random sampling
E_ins = []
E_outs = []
for i in range(1000):
    result = doRegression(generatePts(100),getLine(), graph = False)

    E_ins.append(result[1])
    E_outs.append(result[2])

E_in = np.mean(E_ins)
E_out = np.mean(E_outs)
```

```
    print("AVERAGE VALUES FOR 1000 ATTEMPTS:")
    print ("E_in: " + str(E_in))
    print("E_out: " + str(E_out))
```

```
AVERAGE VALUES FOR 1000 ATTEMPTS:
E_in: 0.03485
E_out: 0.04404
```

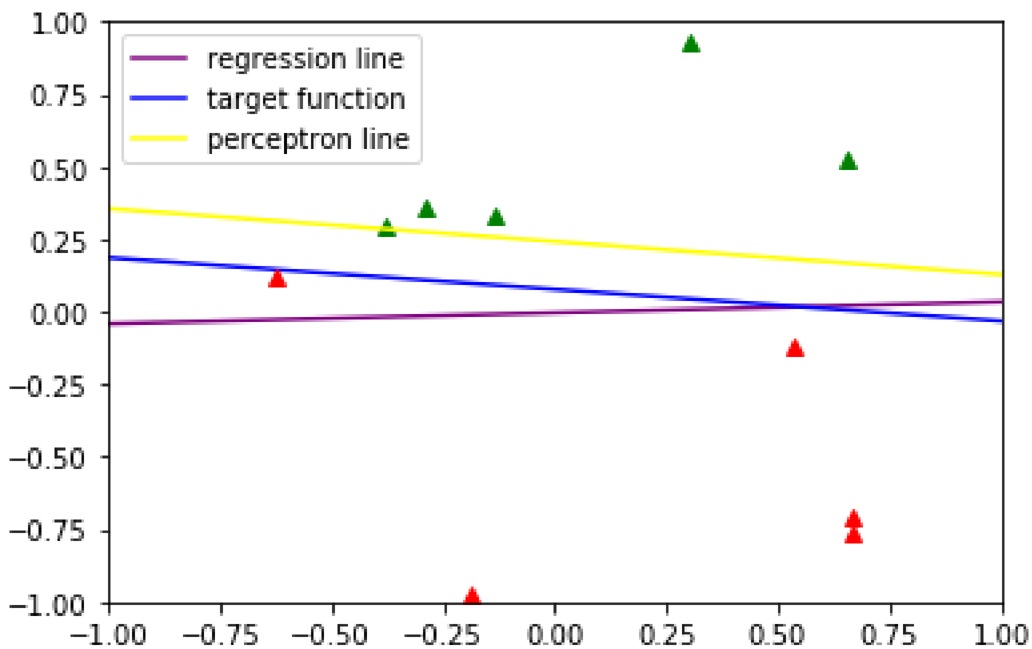
In [13]:
```
# run the PLA, graph if asked, test w/ 1000 pts if asked
# sets w based on the result of regression on the points.
# returns iterations required. If testing=True returns missed pts out of 1
def doRegPLA(N, graph = True, test = True):
    line = getLine()
    pts = generatePts(N)

    # get starting w via regression
    reg = doRegression(pts, line, test = False, graph = False)
    w = reg[0]
    rline = reg[1]

    if graph:
        X = np.linspace(-1,1,1000)
        ry = rline(X)
        plt.plot(X,ry, color = 'purple', label = "regression line")




    labeled_pts = labelPts(pts, line)



    # Graph target function and labeled pts if we want
    if graph == True:
        X = np.linspace(-1,1,1000)
        y = line(X)
        plt.ylim(-1,1)
        plt.xlim(-1,1)
        plt.plot(X,y, label = "target function", color = "blue")

        for pt in labeled_pts:
            if pt[3] == 1:
                plt.plot(pt[1],pt[2], 'g^')
            else:
                plt.plot(pt[1],pt[2], 'r^')
```

```
mis_pts = []

for pt in labeled_pts:
    if np.sign(w[0]+w[1]*pt[1]+w[2]*pt[2]) != pt[3]:
        mis_pts.append(pt)

# begin perceptron iterations and count them
iters = 0
while len(mis_pts) is not 0:
    # choose randoim missclasified point and update
    pt = ran.choice(mis_pts)

    w[0] += pt[3]*pt[0]
    w[1] += pt[3]*pt[1]
    w[2] += pt[3]*pt[2]


    # recheck for misclassified points
    mis_pts = []

    for pt in labeled_pts:
        if np.sign(w[0]+w[1]*pt[1]+w[2]*pt[2]) != pt[3]:
            mis_pts.append(pt)
    iters += 1

pline = lambda x: -(x*w[1]+w[0])/w[2]


if graph == True:
    xp = np.linspace(-1,1,1000)
    yp = pline(xp)
    plt.plot(xp, yp, "yellow", label = "perceptron line")
    plt.legend()

# if we want to test, we will test:
if test == True:
    # generate and label 10000 test points according to f(x)
    test_points = generatePts(1000)

    labeled_test_pts = []
    for pt in test_points:
        if pt[1] > line(pt[0]):
            labeled_test_pts.append([1, pt[0], pt[1], 1])
        else:
            labeled_test_pts.append([1, pt[0], pt[1], -1])

    # count misclassified test points
```

```
                mis_labeled_pts = []
                for pt in labeled_test_pts:
                    if np.sign(w[0]+w[1]*pt[1]+w[2]*pt[2]) != pt[3]:
                        mis_labeled_pts.append(pt)

                return iters, len(mis_labeled_pts)

            return iters
```

```
In [14]: # run the primed PLA for eye-test
         doRegPLA(10, test=False)
```

```
Out[14]: 21
```




```
In [15]: # average iterations over 1000 PLA runs
         iters = []
         for i in range(1000):
             iters.append(doRegPLA(10, graph = False, test = False))


         print("average Iterations: " + str(np.mean(iters)))
```

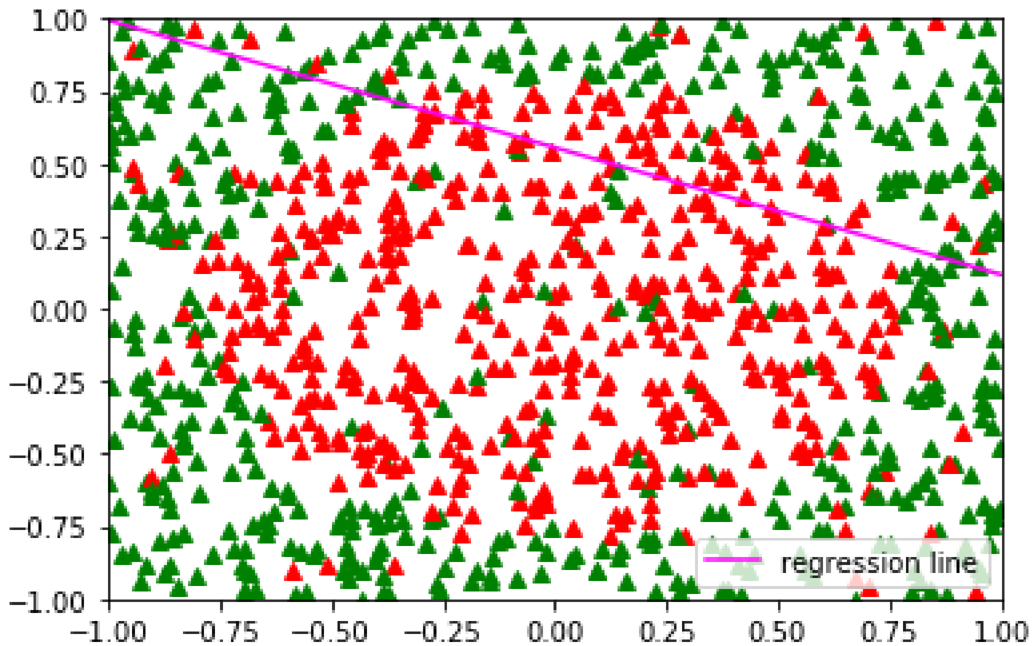
```
average Iterations: 4.214
```

```
In [124]: def doReg(data, graph = True, test = True):
              # format data for sklearn package
```

```
x = []
y = []
for pt in data:
    x.append(pt[1:len(pt)-1])
    y.append(pt[len(pt)-1])


lm = LinearRegression(fit_intercept=True)
lm.fit(x,y)

w = []
w.append(lm.intercept_)
coef = lm.coef_
for i in range(len(coef)):
    w.append(coef[i])

# w = [lm.intercept_, lm.coef_[0],lm.coef_[1]]
rline = lambda x1:-(w[0]+w[1]*x1)/w[2]

if graph:
    for pt in data:
        if pt[len(pt)-1] == 1:
            plt.plot(pt[1],pt[2], 'g^')
        else:
            plt.plot(pt[1],pt[2], 'r^')

    X = np.linspace(-1,1,1000)
    ry = rline(X)
    plt.ylim(-1,1)
    plt.xlim(-1,1)
    plt.plot(X,ry, color = "magenta", label = "regression line")
    plt.legend()

if test:
    E_in = 0
    for x in data:
        prod = 0
        for i in range(len(w)):
            prod += x[i]*w[i]
        if np.sign(prod) != x[len(x)-1]:
            E_in += 1
    E_in /= float(len(data))

    return w, E_in
return w
```

In [125]:
```
# set target function and label pts with simulated noise
# do a regression for eye-test

target = lambda x1, x2: np.sign(x1**2+x2**2-0.6)
raw = generatePts(1000)
data = []
for x in raw:
    data.append([1, x[0], x[1], target(x[0],x[1])])

for x in data:
    if ran.choice(range(10)) == 1:
        x[3] = -x[3]
doReg(data, graph = True)
```

Out[125]:
```
([0.029624295381474857, -0.0233628378857754 82, -0.053256269705368689], 0.
```



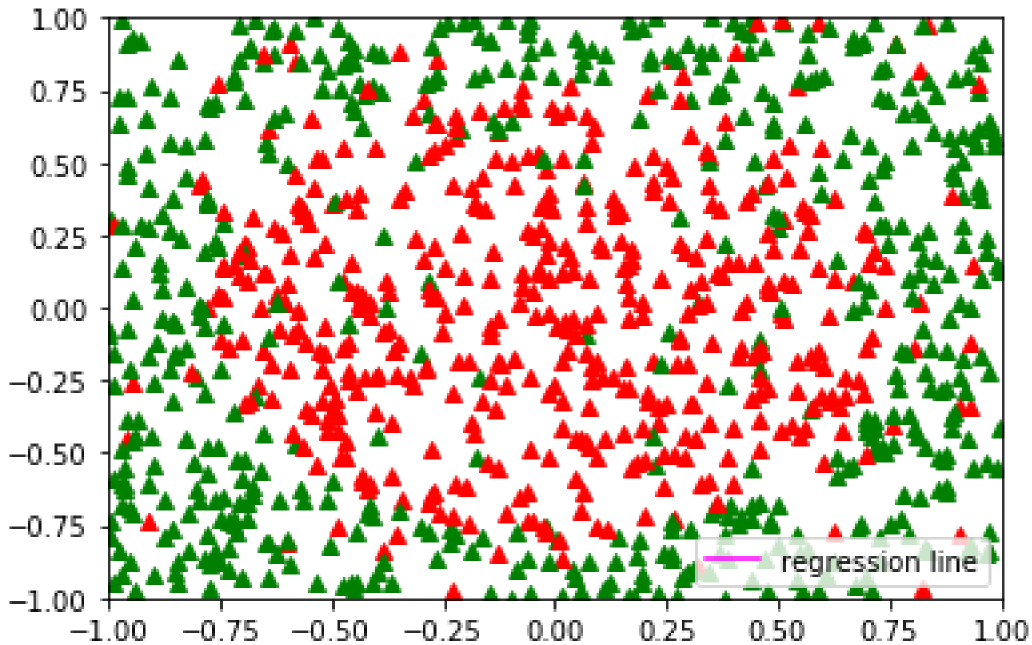
In [126]:
```
# average E_in over 1000 runs for non transformed data
E_ins = []
for i in range(1000):
    target = lambda x1, x2: np.sign(x1**2+x2**2-0.6)
    raw = generatePts(1000)
    data = []
    for x in raw:
        data.append([1, x[0], x[1], target(x[0],x[1])])
```

```
            for x in data:
                if ran.choice(range(10)) == 1:
                    x[3] = -x[3]
            E_ins.append(doReg(data, graph = False)[1])
```

In [127]:
```
print("average E_in: " + str(np.mean(E_ins)))
```

```
average E_in: 0.505405
```

In [128]:
```
target = lambda x1, x2: np.sign(x1**2+x2**2-0.6)
raw = generatePts(1000)
data = []
for x in raw:
    data.append([1, x[0], x[1], target(x[0],x[1])])

for x in data:
    if ran.choice(range(10)) == 1:
        x[len(x)-1] = -x[len(x)-1]

trans_data = []
for x in data:
    trans_data.append([1,x[1],x[2],x[1]*x[2],x[1]**2,x[2]**2,x[3]])

w = doReg(trans_data, test=False)
```



```
In [141]: ws = [[],[],[],[],[],[]]
          errors = []

          # w's for the options
          w_a = [-1,-.05,0.08,0.13,1.5,1.5]
          w_b = [-1,-.05,.08,.13,1.5,15]
          w_c = [-1,-.05,.08,.13,15,1.5]
          w_d = [-1,-1.5,.08,.13,.05,.05]
          w_e = [-1,-.05,.08,1.5,.15,.15]
          for i in range(1000):
              # generate labeled pts w/ noise
              target = lambda x1, x2: np.sign(x1**2+x2**2-0.6)
              raw = generatePts(1000)
              data = []
              for x in raw:
                  data.append([1, x[0], x[1], target(x[0],x[1])])

              for x in data:
                  if ran.choice(range(10)) == 1:
                      x[len(x)-1] = -x[len(x)-1]

              # generate transformed data set
              trans_data = []
              for x in data:
                  trans_data.append([1,x[1],x[2],x[1]*x[2],x[1]**2,x[2]**2,x[3]])

              # run regression in Z space
              w = doReg(trans_data, test=False, graph=False)

              # keep track of w's for this run
              for i in range(len(w)):
                  ws[i].append(w[i])


              # test E_out with 1000 noisy points
              raw_test=generatePts(1000)
              test_data = []
              for x in raw_test:
                  test_data.append([1, x[0], x[1], target(x[0],x[1])])

              for x in test_data:
                  if ran.choice(range(10)) == 1:
                      x[len(x)-1] = -x[len(x)-1]

              trans_test_data = []
              for x in test_data:
                  trans_test_data.append([1,x[1],x[2],x[1]*x[2],x[1]**2,x[2]**2,x[
```

```
# approx E_out for the five w's
E_in = 0
E_a = 0
E_b = 0
E_c = 0
E_d = 0
E_e = 0

for x in trans_test_data:
    prod = 0
    prod_a = 0
    prod_b = 0
    prod_c = 0
    prod_d = 0
    prod_e = 0

    for i in range(len(w)):
        prod += x[i]*w[i]
        prod_a += x[i]*w_a[i]
        prod_b += x[i]*w_b[i]
        prod_c += x[i]*w_c[i]
        prod_d += x[i]*w_d[i]
        prod_e += x[i]*w_e[i]

    if np.sign(prod) != x[len(x)-1]:
        E_in += 1

    if np.sign(prod) != np.sign(prod_a):
        E_a += 1

    if np.sign(prod) != np.sign(prod_b):
        E_b += 1

    if np.sign(prod) != np.sign(prod_c):
        E_c += 1

    if np.sign(prod) != np.sign(prod_d):
        E_d += 1

    if np.sign(prod) != np.sign(prod_e):
        E_e += 1


E_a /= float(len(trans_test_data))
E_b /= float(len(trans_test_data))
E_c /= float(len(trans_test_data))
E_d /= float(len(trans_test_data))
E_e /= float(len(trans_test_data))
```

```
            E_in /= float(len(trans_test_data))
            errors.append(E_in)
```

```
In [142]: print("AVERAGES: ")
          print("E_a: " + str(E_a))
          print("E_b: " + str(E_b))
          print("E_c: " + str(E_c))
          print("E_d: " + str(E_d))
          print("E_e: " + str(E_e))
```

```
AVERAGES:
E_a: 0.046
E_b: 0.349
E_c: 0.322
E_d: 0.381
E_e: 0.447
```

```
In [122]: print('AVERGAES')
          for i in range(len(ws)):
              print ('w_' + str(i) + ': ' + str(np.mean(ws[i])))
```

```
AVERGAES
w_0: -0.991873753407
w_1: 0.00176627230769
w_2: -0.00129835540913
w_3: -0.00120287709573
w_4: 1.55874871723
w_5: 1.55733598801
```

```
In [131]: print('AVERAGE E_out estimate: ' + str(np.mean(errors)))
```

```
AVERAGE E_out estimate: 0.127061
```