Tema 4

2. Rezolvați următoarele sisteme de ecuații liniare:

a) $\begin{cases} x+4y=1 \pmod 9 \\ 5x+7y=1 \pmod 9 \end{cases} \Big/ \cdot 5 \Leftrightarrow \begin{cases} 5x+20y=5 \pmod 9 \\ 5x+7y=1 \pmod 9 \end{cases}$

$\Leftrightarrow \begin{cases} 5x+2y=5 \pmod 9 \\ 5x+7y=1 \pmod 9 \end{cases} \quad (-)$

$/(-5) \quad y = 4 \pmod 9$

$4y = 4 \pmod 9$

$y = 4^{-1} \cdot 4 \pmod 9$

$X_9 = (1, 0) \quad X_4 = (0, 1)$

$9 = 4 \cdot 2 + 1 \quad X_1 = X_9 - 2X_4 = (1, -2)$

$y = (-2) \cdot 4 \pmod 9$

$y = 7 \cdot 4 \pmod 9$

$y = 28 \pmod 9$

$y = 1 \pmod 9 \Rightarrow \begin{cases} x+4=1 \pmod 9 \Rightarrow \\ 5x+7=1 \pmod 9 \end{cases}$

$\Rightarrow X \equiv -3 \pmod 9$

$X \equiv 6 \pmod 9$

$R: X = 6, y = 1$

b) $\begin{cases} 17X + 11y \equiv 7 \pmod{29} \\ 13X + 10y \equiv 8 \pmod{29} \end{cases} \Leftrightarrow$ (/·11)

$\Leftrightarrow \begin{cases} 170X + 110y \equiv 70 \pmod{29} \\ 143X + 110y \equiv 88 \pmod{29} \end{cases}$

$\Leftrightarrow \begin{cases} 25X + 23y \equiv 12 \pmod{29} \\ 27X + 23y \equiv 1 \pmod{29} \quad (-) \end{cases}$

$-2X + 0 \equiv 11 \pmod{29}$

$27X \equiv 11 \pmod{29}$

$X \equiv 27^{-1} \cdot 11 \pmod{29}$

$X_{29} = (1, 0) \quad X_{27} = (0, 1)$

$29 = 27 \cdot 1 + 2 \Rightarrow X_2 = X_{29} - X_{27} = (1, -1)$

$27 = 2 \cdot 13 + 1 \Rightarrow X_1 = X_{27} - 13X_2 = (-13, 14)$

$X \equiv 14 \cdot 11 \pmod{29}$

$X \equiv 154 \equiv 9 \pmod{29} \Rightarrow$

$$\Rightarrow \begin{cases} 17 \cdot 9 + 11y = 7 \pmod{29} \\ 13 \cdot 9 + 10y = 8 \pmod{29} \end{cases}$$

$$\Rightarrow \begin{cases} 8 + 11y = 7 \pmod{29} \\ 1 + 10y = 8 \pmod{29} \Rightarrow \end{cases}$$

$$\Rightarrow 10y = 7 \pmod{29}$$

$$y = 10^{-1} \cdot 7 \pmod{29}$$

$X_{29} = (1, 0) \quad X_{10} = (0, 1)$

$29 = 10 \cdot 2 + 9 \quad X_9 = X_{29} - 2X_{10} = (1, -2)$

$10 = 9 \cdot 1 + 1 \quad X_1 = X_{10} - X_9 = (-1, 3)$

$$y = 3 \cdot 7 \pmod{29}$$

$$y = 21 \pmod{29}$$

R: $X = 9 \quad y = 21$