UNIVERSITÉ PARIS 1
PANTHÉON SORBONNE

Replication, Panel Data Analysis & Extension Of: Lofaro & Di Bucchianico (2025)

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Reference Repository:
https://github.com/EliaLand/PVAR_MonetaryPolicy_FunctionalIncome

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0. Introduction

In this work we provide a replication study of Lofaro and Di Bucchianico (2025), which investigates the impact of monetary policy on functional income distribution using a panel vector autoregressive (PVAR) framework. The original study argues that contractionary monetary policy shocks have persistent negative effects on real wages and the labour share of income, driven primarily by labour market dynamics rather than by short-run fluctuations in output or prices. Given the policy relevance of these findings and the complexity of the empirical strategy, replication is essential to assess their robustness and reproducibility.

The replication, despite not entering in the details of the PVAR model adopted, closely follows the original empirical design, using annual data for a panel of 15 advanced economies over the period 1970-2019. The countries included are Australia, Belgium, Canada, Finland, France, Germany, Italy, Japan, the Netherlands, Norway, Portugal, Spain, Sweden, the United Kingdom, and the United States. Through these variables, we focus on the transmission of monetary policy shocks to functional income distribution, defined in terms of real compensation per employee and the labour share of income, while explicitly accounting for macroeconomic and labour market dynamics.

To analyse the dynamic effects of monetary policy on functional income distribution, this paper adopts a panel vector autoregressive (PVAR) approach, following the empirical strategy of Lofaro and Di Bucchianico (2025). The PVAR framework allows for rich dynamic interactions among macroeconomic, labor market, and distributive variables, while explicitly accounting for cross-country heterogeneity and common global shocks. Let $i = 1, \dots, N$ index countries and $t = 1, \dots, T$ index time. The reduced-form PVAR model of order p is specified as:

$$\mathrm{Y}_{i,t} = \mathrm{A}(L)\mathrm{Y}_{i,t-1} + \mathrm{CX}_{i,t} + \mathrm{u}_{i,t},$$

where $\mathrm{Y}_{i,t}$ is a $K \times 1$ vector of endogenous variables, $\mathrm{A}(L) = \sum_{l=1}^{p} \mathrm{A}_l L^l$ is a matrix polynomial in the lag operator, $\mathrm{X}_{i,t}$ is a vector of exogenous controls, and $\mathrm{u}_{i,t}$ is a vector of reduced-form innovations. In the baseline specification, the vector of endogenous variables is defined as:

$$\mathrm{Y}_{i,t} = \begin{bmatrix} PCOM_t \\ GDP_{i,t} \\ UN_{i,t} \\ P_{i,t} \\ WR_{i,t} \\ LS_{i,t} \\ i_{i,t} \end{bmatrix},$$

where $PCOM_t$ denotes a global energy commodity price index, $GDP_{i,t}$ is real gross domestic product, $UN_{i,t}$ is the unemployment rate, $P_{i,t}$ is the GDP deflator, $WR_{i,t}$ is real compensation per employee, $LS_{i,t}$ is the labor share of income, and $i_{i,t}$ is the short-term nominal interest rate. All variables except interest rates and unemployment are expressed in logarithmic levels. The vector of exogenous variables $\mathrm{X}_{i,t}$ includes country-specific fixed effects α_i, time fixed effects δ_t capturing global macroeconomic shocks, and country-specific linear time trends $\gamma_i t$. These components control for unobserved heterogeneity across countries, common global disturbances, and heterogeneous long-run trends, respectively.

To identify economically meaningful shocks, the reduced-form PVAR is transformed into its structural form by premultiplying by the contemporaneous impact matrix A_0:

$$\mathrm{A}_0 \mathrm{Y}_{i,t} = \mathrm{B}(L)\mathrm{Y}_{i,t-1} + \mathrm{D}\mathrm{X}_{i,t} + \varepsilon_{i,t},$$

where $\mathrm{B}(L) = \mathrm{A}_0 \mathrm{A}(L)$, $\mathrm{D} = \mathrm{A}_0 \mathrm{C}$, and $\varepsilon_{i,t} = \mathrm{A}_0 \mathrm{u}_{i,t}$ is the vector of structural innovations, assumed to be mutually uncorrelated with identity covariance matrix. Identification of monetary policy shocks relies on a recursive (Cholesky) decomposition of the variance-covariance matrix of the reduced-form residuals. The ordering of variables follows equation, with the short-term interest rate placed last. This identification strategy implies that monetary policy can respond contemporaneously to innovations in all other variables, while the real economy responds to policy shocks with a lag. This assumption is standard in the monetary VAR literature and is consistent with the empirical approach adopted in the original study.

So, in conclusion, to what interest us the most, the baseline PVAR specification includes the following endogenous variables (Model 1) to which we will offer refer as baseline for our preferred variables: a global energy commodity price index (used to control for external cost shocks), real GDP, the unemployment rate, the GDP deflator, real wages (real compensation per employee), the labour share of income, and the short-term nominal interest rate. Monetary policy shocks are identified using a recursive (Cholesky) identification scheme, with the policy interest rate ordered last, reflecting the assumption that central banks can react contemporaneously to macroeconomic conditions, while the real economy responds to policy with a lag. All variables except interest rates and unemployment are expressed in logarithmic levels.

Table 0.1: Variable definitions

Variable	**Label**	**Definition**	**Source**
Short-term interest rate	i	Typically the central bank policy rate or equivalent market rate, depending on country availability.	BIS; Schularick, Taylor, Jordà; OECD
GDP deflator	p	A price index used also to deflate nominal variables into real variables.	OECD; World Bank
Nominal compensation per employee	w	Nominal compensation per employee in PPP.	AMECO; OECD
Real compensation per employee	wr	Computed as nominal compensation divided by the GDP deflator.	AMECO; OECD
Real gross domestic product	GDP	GDP expenditure approach, in constant prices, converted to US dollars.	OECD; World Bank
Adjusted labor share	LS	Labor compensation divided by GDP, adjusted for the labor income of the self-employed.	AMECO
Unemployment rate	u	Unemployment rate as a percentage of the labor force, ages 15–64.	OECD; Labor Force Statistics
Inflation	π	Annual growth rate of the GDP deflator.	OECD; World Bank
Real effective exchange rate	$REER$	CPI-based real effective exchange rate index.	BIS; World Bank
Shadow interest rate	SR	Available only for selected advanced economies.	Wu and Xia (2016)

Notes: Variable definitions and sources.

1. Chapter 1 - Dataset, Univariate & Bivariate Descriptive Statistics After Transformations

1.1 In your data set, which are the variables which are varying with respect to two indices (or more) if you consider inflows and outflows from one individual or country to another individual or countries? Which are the variables which are varying only with respect to time? Which are the variables which are varying only with respect to individuals?

The dataset Lofaro and Di Bucchianico (2025) employ in this study is a country-level panel dataset covering 15 advanced economies observed annually over the period 1970-2019. Observations are indexed by country $i$and time t. The empirical framework does not model bilateral interactions, such as trade flows, financial flows, or migration, between pairs of countries or individuals. Consequently, no variable in the dataset varies simultaneously with respect to two cross-sectional indices (e.g. origin–destination pairs), but we will use it later in the open section.

A limited number of variables vary only with respect to time and are common across all countries. In particular, the global energy commodity price index is included to control for exogenous international cost shocks. This variable is indexed solely by $t$and enters the model identically for all countries in a given year, reflecting its determination in global markets rather than at the country level.

Table 1.1.1: Summary statistics — Australia

	i	P	W	WR	GDP	LS	$PCOM$	UN	$SHORTUN$	$LONGUN$	LF	$REER$	SH
Count	50	50	50	50	50	50	50	50	42	42	50	50	50
Mean	7.74	57.01	29.76	59.21	661164	58.06	47.45	6.25	380.56	253.48	9016.65	95.10	7.74
Std.	4.42	30.35	15.05	7.71	293488	4.01	37.07	2.13	73.81	61.56	2334.75	12.70	4.42
Min	1.34	9.10	6.41	47.01	283519	52.30	2.11	1.65	259.58	124.35	5478.23	72.93	1.34
25%	4.90	30.55	18.28	54.18	393702	54.60	22.71	5.17	326.35	211.99	6929.24	85.65	4.90
50%	6.57	59.05	28.26	57.58	586092	57.85	35.60	6.07	373.10	257.27	8845.56	94.99	6.57
75%	10.22	81.23	42.13	62.01	911633	60.48	68.54	7.57	436.20	296.45	10840.54	104.00	10.22
Max	17.61	106.90	55.31	77.07	1209289	66.90	135.22	10.87	556.64	377.10	13555.29	122.64	17.61

Table 1.1.2: Summary statistics — Germany

	i	P	W	WR	GDP	LS	$PCOM$	UN	$SHORTUN$	$LONGUN$	LF	$REER$	SH
Count	50	50	50	50	50	50	50	50	37	37	50	50	50
Mean	4.55	70.65	30.24	38.67	2886883	59.58	47.45	6.15	1045.47	1722.42	35436.57	97.02	3.94
Std.	3.31	22.24	14.72	9.63	754346	2.62	37.07	2.71	190.70	671.99	6580.12	5.69	4.32
Min	-0.36	29.70	6.46	20.15	1620208	54.20	2.11	0.55	651.98	720.87	26577	84.57	-6.96
25%	2.22	54.95	19.79	34.25	2172895	57.55	22.71	3.98	919.55	1177.60	28336.50	92.97	2.16
50%	4.30	74.45	29.48	37.69	2969508	59.05	35.60	6.75	1062.43	1490.81	39540.81	96.37	4.26
75%	6.51	89.10	39.92	45.01	3484617	61.58	68.54	8.08	1150.28	2167.97	41121.48	101.39	6.51
Max	12.14	105.30	60.05	56.52	4165985	64.40	135.22	11.17	1431.94	3138.89	43771.35	108.83	12.14

Table 1.1.3: Summary statistics — Italy

	i	P	W	WR	GDP	LS	$PCOM$	UN	$SHORTUN$	$LONGUN$	LF	$REER$	SH
Count	50	50	50	50	50	50	50	50	37	37	50	50	50
Mean	7.62	58.65	29.84	64.68	1898869	57.12	47.45	9.35	675.98	1785.49	23282.07	94.50	7.01
Std.	6.11	33.10	14.03	28.38	434122	5.22	37.07	2.21	148.06	410.26	1651.12	6.92	6.99
Min	-0.36	5.70	5.87	43.05	1033383	50.90	2.11	5.42	400.26	891.35	20146	81.50	-6.96
25%	2.22	29.45	19.43	46.96	1524110	52.43	22.71	7.67	568.82	1525.15	22284.96	89.83	2.16
50%	6.25	64.95	30.67	50.29	2001263	54.65	35.60	9.74	665.80	1884.86	23490.32	95.32	6.25
75%	12.28	87.60	40.52	70.29	2288505	62.98	68.54	11.37	793.90	2115.41	24402.48	99.87	12.28
Max	19.90	102.90	54.59	134.24	2430046	66.50	135.22	12.68	1020.40	2361.34	25969.43	107.57	19.90

Table 1.1.4: Summary statistics — Japan

	i	P	W	WR	GDP	LS	$PCOM$	UN	$SHORTUN$	$LONGUN$	LF	$REER$	SH
Count	50	50	40	40	50	40	50	50	43	43	50	50	50
Mean	3.48	85.35	31.36	30.16	3944491	61.80	47.45	3.08	1218.47	955.35	62603.57	111.94	2.52
Std.	3.69	19.88	9.91	10.34	1164836	4.40	37.07	1.19	363.06	443.13	5571.41	23.85	4.74
Min	-0.08	31.50	12.83	13.62	1707357	55.30	2.11	1.15	656.67	360.00	51535.83	64.19	-4.71
25%	0.19	80.68	24.07	22.16	2870711	58.18	22.71	2.17	896.25	555.00	58027.71	93.95	-1.46
50%	1.76	96.40	31.69	28.39	4371873	62.10	35.60	2.79	1169.17	870.00	65941.25	109.96	1.76
75%	6.54	97.78	39.92	39.82	4936383	63.78	68.54	4.08	1474.58	1400.00	66721.04	129.15	6.54
Max	14.51	101.80	45.25	45.79	5357999	71.00	135.22	5.36	1935.00	1790.00	69121.66	160.02	14.51

Table 1.1.5: Summary statistics — United Kingdom

	i	P	W	WR	GDP	LS	$PCOM$	UN	$SHORTUN$	$LONGUN$	LF	$REER$	SH
Count	50	50	50	50	50	50	50	50	37	37	50	50	50
Mean	7.06	60.25	27.80	51.28	1920876	56.85	47.45	6.58	1014.84	1114.73	28806.96	89.71	6.05
Std.	4.59	30.21	15.59	9.29	604573	2.87	37.07	2.62	140.05	509.80	2680.42	7.92	6.06
Min	0.40	8.40	5.20	38.93	1039156	51.40	2.11	2.08	805.20	502.30	24861	76.29	-6.19
25%	4.23	37.33	15.00	44.63	1319936	55.08	22.71	4.90	912.00	648.76	26433.75	82.61	3.66
50%	6.50	65.75	25.26	50.51	1829721	56.60	35.60	5.84	976.11	1056.92	28180	89.15	6.50
75%	10.55	82.83	42.73	53.85	2497059	58.28	68.54	8.34	1075.00	1497.16	30976.88	96.26	10.55
Max	16.80	107.80	57.26	75.60	2999973	66.40	135.22	11.88	1345.08	2103.20	34102	105.26	16.80

Table 1.1.6: Summary statistics — United States

	i	P	W	WR	GDP	LS	$PCOM$	UN	$SHORTUN$	$LONGUN$	LF	$REER$	SH
Count	50	50	50	50	50	50	50	50	50	50	50	50	50
Mean	5.29	62.25	36.15	48.94	11716940	59.68	47.45	6.20	6285.46	1655.70	129297.67	104.92	4.99
Std.	3.74	28.13	20.03	10.00	4539552	1.92	37.07	1.57	1207.68	1376.60	24013.64	9.92	4.11
Min	0.12	16.40	7.70	35.48	5186834	56.00	2.11	3.67	3855.08	238.00	82771	91.78	-2.74
25%	2.19	41.03	19.15	38.62	7309440	58.35	22.71	4.98	5617.33	812.00	110540.95	96.74	1.89
50%	5.30	63.40	33.10	46.58	10990380	60.35	35.60	5.82	6113.46	1264.00	131682.85	101.76	5.17
75%	7.69	86.90	54.45	59.18	16013510	61.00	68.54	7.18	7045.13	1780.75	152700.05	114.22	7.69
Max	15.91	107.90	73.10	65.09	19928970	63.20	135.22	9.69	9770.58	6416.00	163538.70	125.60	15.91

Moving from the single country descriptive statistics, Figures 1.1.7 and 1.1.8 illustrate the long-run evolution of the main macroeconomic, labour market, and distributional variables included in the empirical analysis. A notable feature is the pronounced decline in short-term nominal interest rates over time, particularly from the mid-1980s onward. After reaching historically high levels around the late 1970s and early 1980s, coinciding with disinflationary monetary policy episodes, interest rates exhibit a persistent downward trend, culminating in near-zero levels in the aftermath of the global financial crisis (Lofaro and Di Bucchianico (2025)). This pattern reflects the shift toward low-inflation regimes and the emergence of prolonged accommodative monetary policy stances in advanced economies.

We can also easily observe cyclical movements in real economic activity and labour market conditions. Periods of monetary tightening are typically associated with declines in output and increases in unemployment, especially during major recessionary episodes such as the early 1980s downturn, the early 1990s recession, and the Great Recession. These co-movements suggest a close interaction between monetary policy, real activity, and labor market outcomes, which is central to the identification strategy adopted in the PVAR framework (Lofaro and Di Bucchianico (2025).

Important especially in the context of future IRFs, is Real wages. These latter display a more nuanced pattern. Although they generally increase over the long run, their growth appears uneven and characterized by extended periods of stagnation or slow adjustment, particularly following adverse macroeconomic shocks. The divergence between the evolution of real wages and labour productivity implied by the declining labour share points to changes in bargaining power, labour market institutions, and firm pricing behaviour.

Figure 1.1.7 – Variables Evolution Overtime (Part 1)

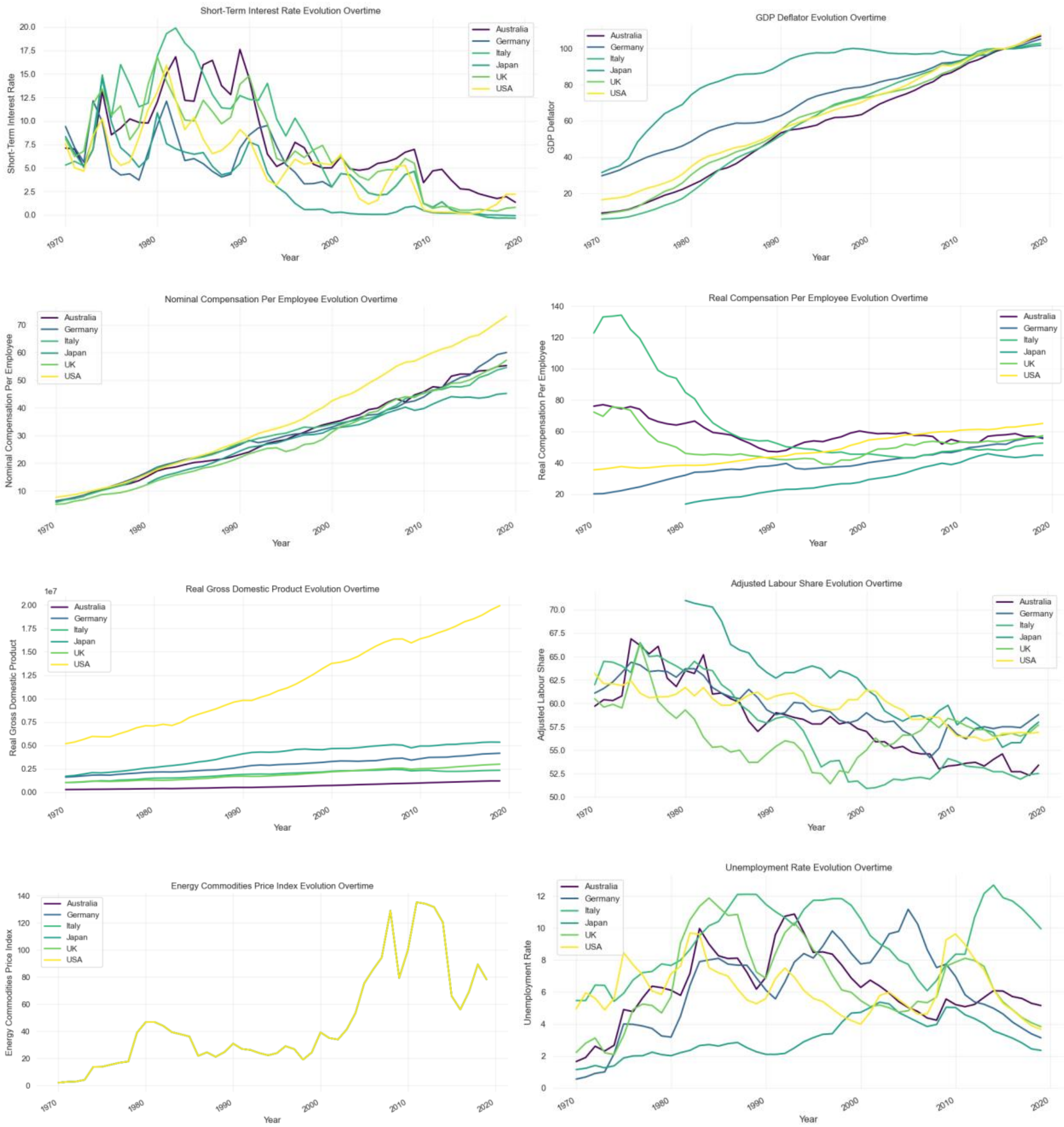


Figure 1.1.8 – Variables Evolution Overtime (Part 2)

With regards to variance analysis and data distribution across coutnries, Figures 1.1.9 through 1.1.12 present box plots illustrating the distribution of the main macroeconomic, labour market, and distributional variables across countries over the sample period. A first relevant feature emerging from the box plots is the substantial heterogeneity across countries for most variables. This is particularly evident for the short-term nominal interest rate and unemployment-related indicators. Some countries display relatively narrow interquartile ranges, suggesting stable monetary and labour market conditions over time, while others exhibit wide dispersion and pronounced outliers, reflecting episodes of macroeconomic instability or prolonged adjustment periods. This heterogeneity underscores the relevance of adopting a panel-based framework that allows for country-specific dynamics rather than relying on pooled cross-sectional averages (poolability tests are likely to fail).

The box plots for unemployment measures reveal marked differences not only in central tendencies but also in variability. Countries characterized by more rigid labour markets tend to exhibit higher medians and wider upper tails, indicating persistent episodes of elevated unemployment. In contrast, other economies show lower medians and tighter distributions, consistent with more flexible labour market adjustments. Still on labour market, turning to distributional variables, the box plots for real wages and the labour share of income display comparatively lower dispersion within countries than across countries, but still reveal meaningful cross-country differences in median levels. The labour share, in particular, exhibits relatively compressed interquartile ranges within each country, suggesting gradual adjustment over time rather than abrupt shifts. And not only in the labour market, the box plots reveal evidence of asymmetries and skewness in several variables.

Figure 1.1.9 – Box Plot Variable Distribution by Country (Part 1)

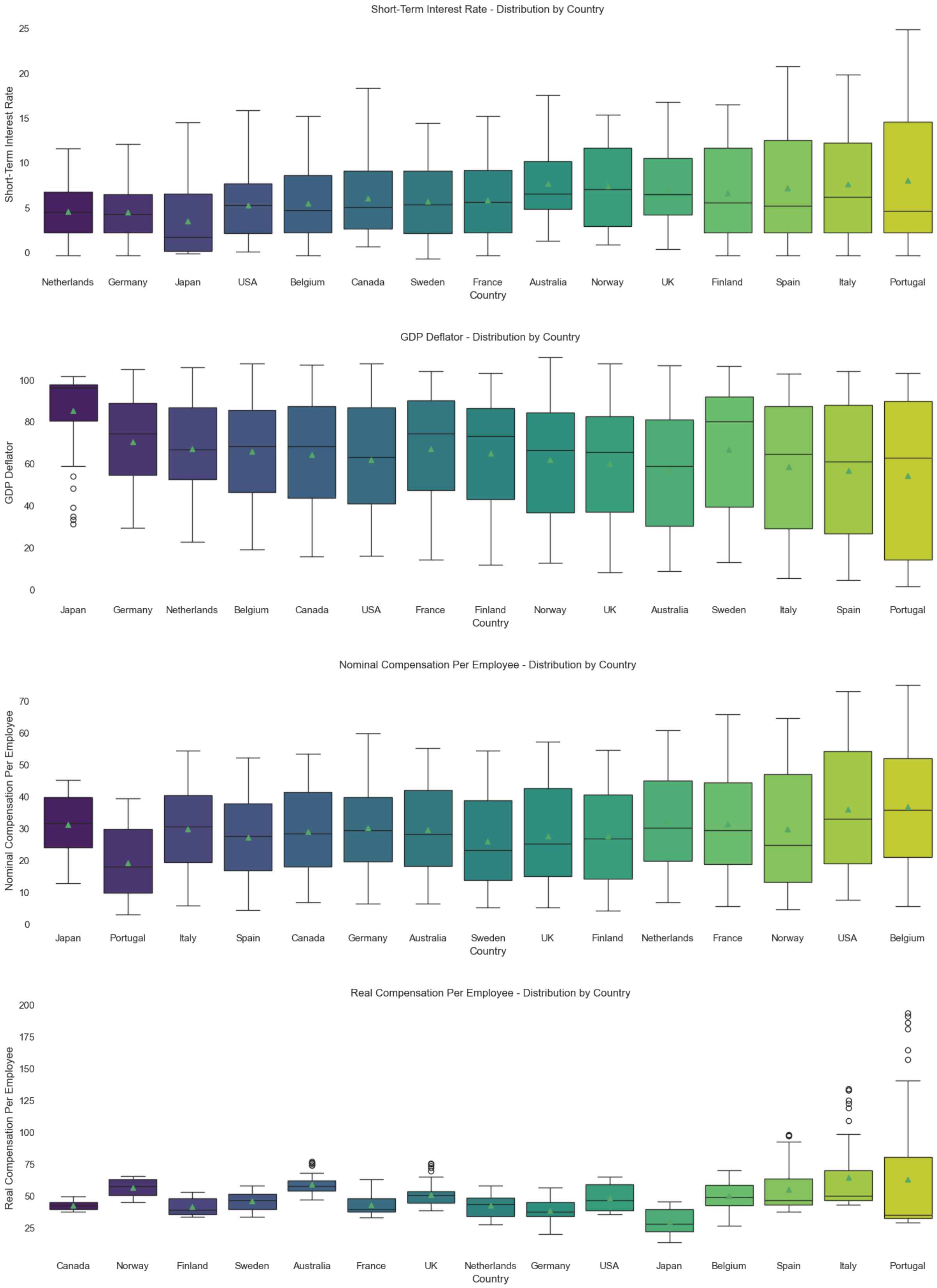


Figure 1.1.10 – Box Plot Variable Distribution by Country (Part 2)

Figure 1.1.11 – Box Plot Variable Distribution by Country (Part 3)

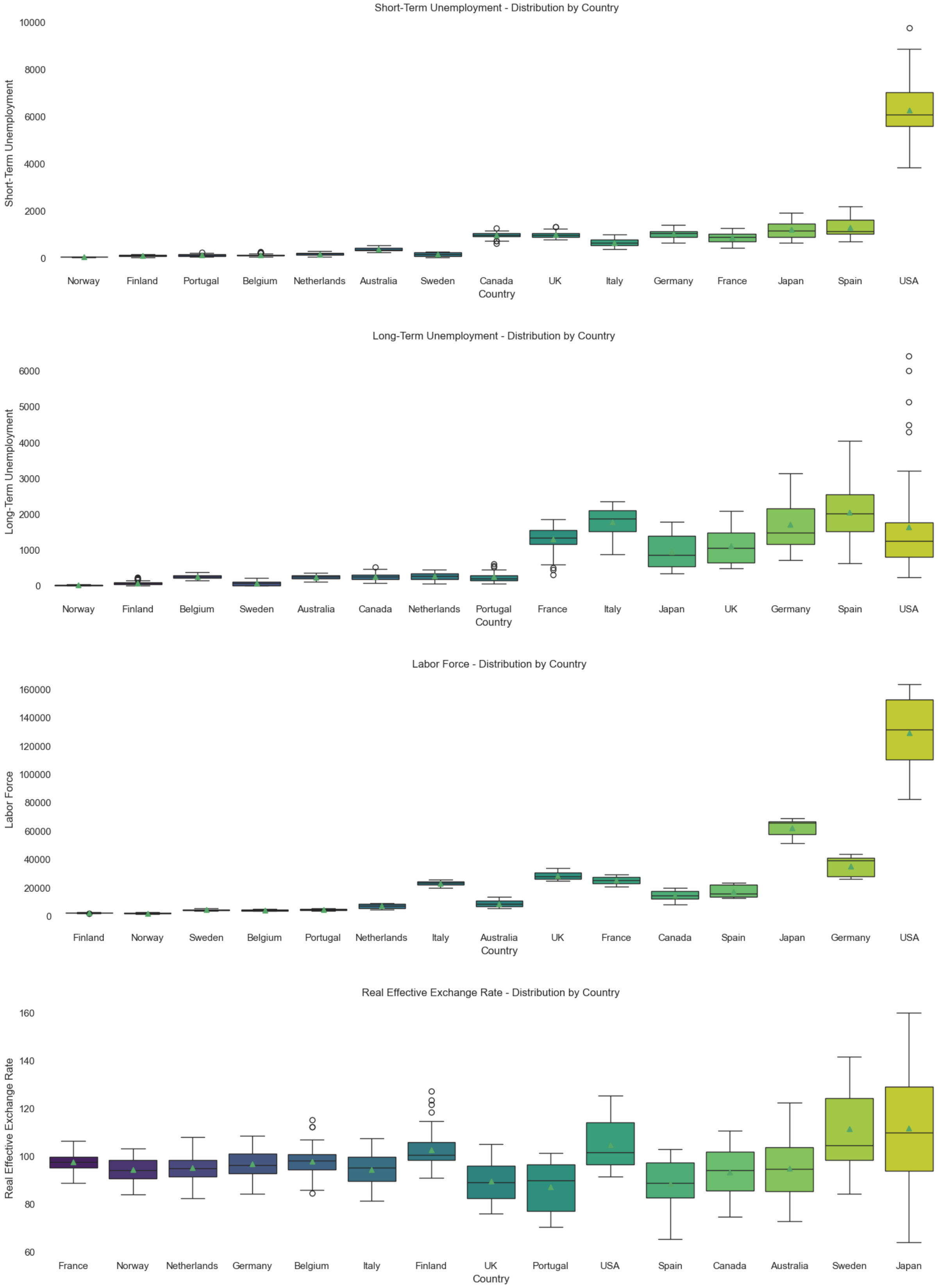
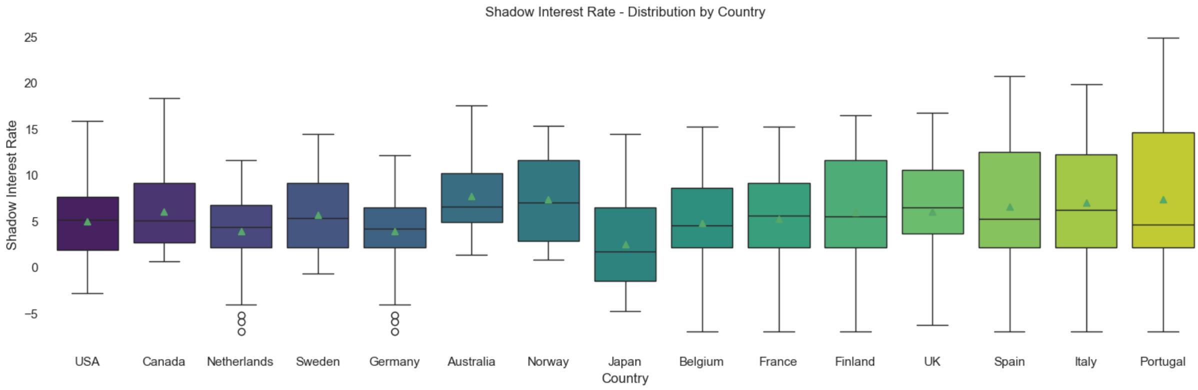


Figure 1.1.12 – Box Plot Variable Distribution by Country (Part 3)



An analysis of deviations from the cross-sectional mean reveals that cross-country heterogeneity in macroeconomic, labour market, and distributional variables is not episodic but systematic and persistent. Several countries consistently occupy positions either above or below the panel average, indicating structural differences rather than short-run fluctuations.

With respect to short-term interest rates, countries such as Italy, Spain, and Portugal tend to display positive deviations from the cross-sectional mean for prolonged periods, particularly during the high-inflation and high-interest-rate environment of the late 1970s and 1980s (Figure 1.1.13). In contrast, Germany, Japan, and the Netherlands are persistently below the panel average, reflecting lower inflation histories, stronger monetary credibility, and more stable financial conditions. These relative positions remain broadly stable over time, even as global interest rates decline, suggesting enduring cross-country differences in monetary conditions.

Clear patterns also emerge in labour market indicators. Spain, Italy, and France consistently exhibit positive deviations in unemployment relative to the cross-sectional mean, indicating structurally weaker labour market performance. These deviations widen markedly during recessionary episodes, such as the early 1980s downturn and the post-2008 crisis, but do not fully disappear during recoveries. By contrast, Germany, Japan, Norway, and the Netherlands tend to record unemployment rates persistently below the panel average, consistent with stronger employment performance and more resilient labour market institutions.

Now, turning to distributional variables, persistent heterogeneity is again evident. The labour share of income tends to remain above the cross-sectional mean in countries such as France, Belgium, and the Netherlands, suggesting relatively labour-friendly income distribution regimes. In contrast, the United States, Norway, and Sweden frequently display negative deviations from the panel average, consistent with lower labour shares and a greater role for capital income (Figure 1.1.14). These relative positions are stable over time, even as the labour share declines globally, indicating that secular trends coexist with persistent cross-country differences.

In general, we note that countries are not merely experiencing common shocks with different timing, but rather occupy structurally distinct positions within the panel. This has important methodological implications. First, it provides strong justification for the inclusion of country fixed effects and country-specific trends in the empirical specification. Second, it suggests that the effects of monetary policy shocks, while directionally similar across countries, are likely to differ in magnitude and persistence depending on initial relative conditions. Countries that systematically deviate from the mean, such as Southern European economies in labour market variables or core economies in interest rates, may experience more pronounced or longer-lasting distributional effects.

Figure 1.1.13 – Deviation from Cross-Sectional Mean Overtime (Part 1)

Figure 1.1.14 – Variables Evolution Overtime (Part 2)

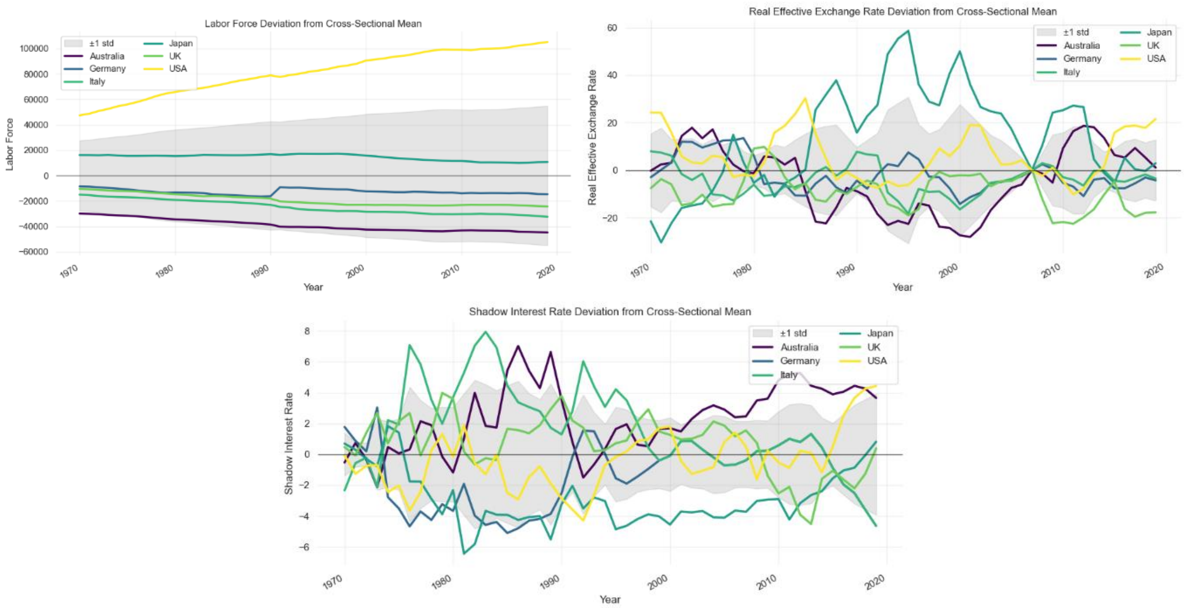


Figure 1.1.15 – Box Plot Deviation from Cross Sectional Mean by Country (Model 1 Variables, Part 1)

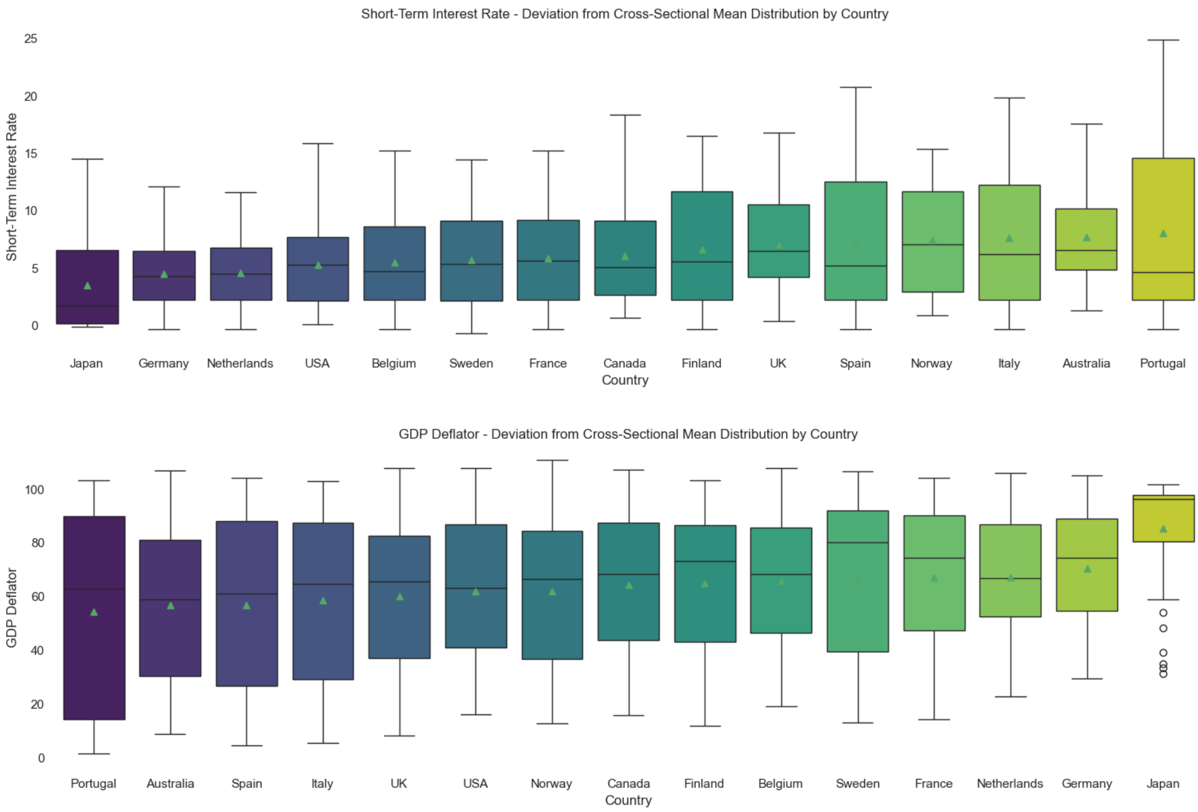
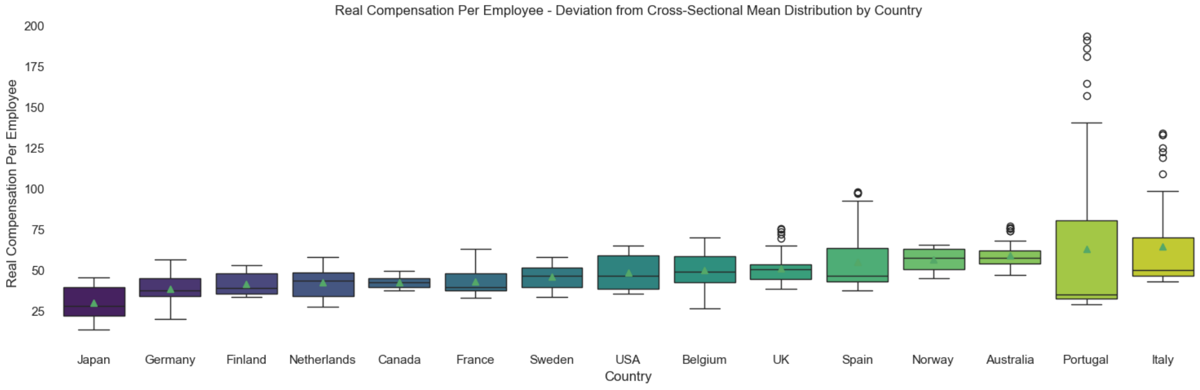
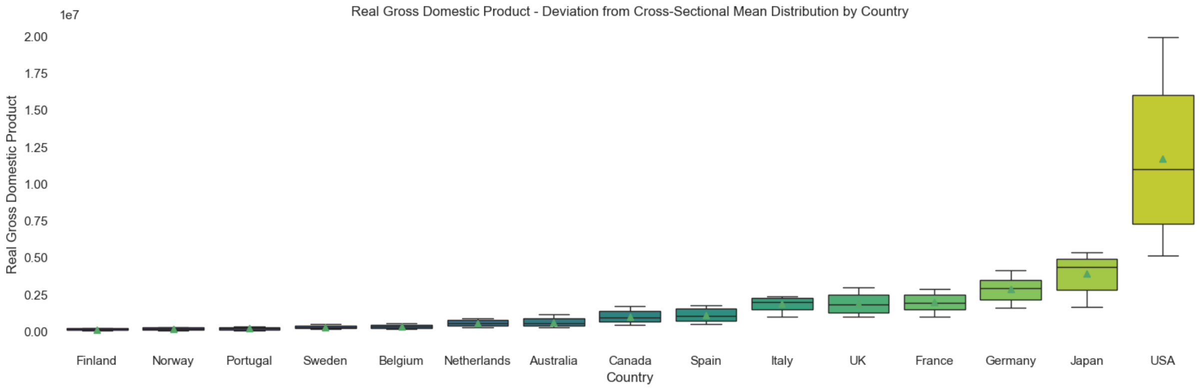
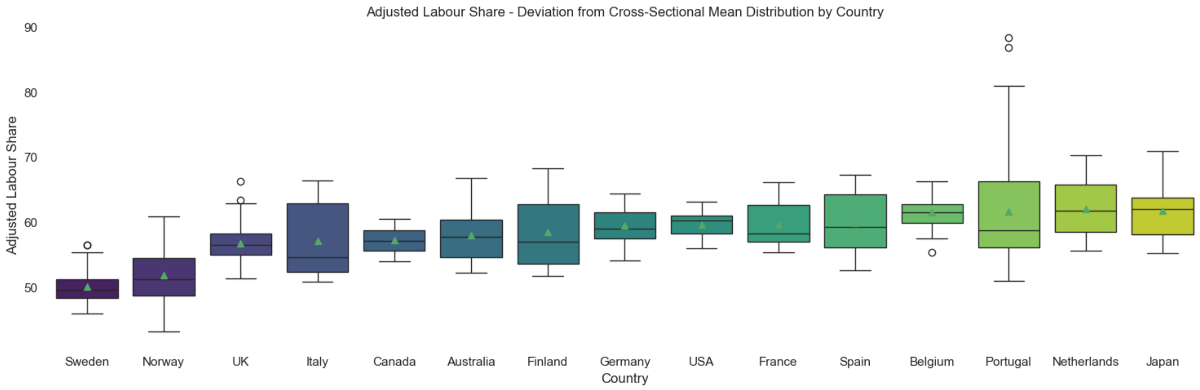
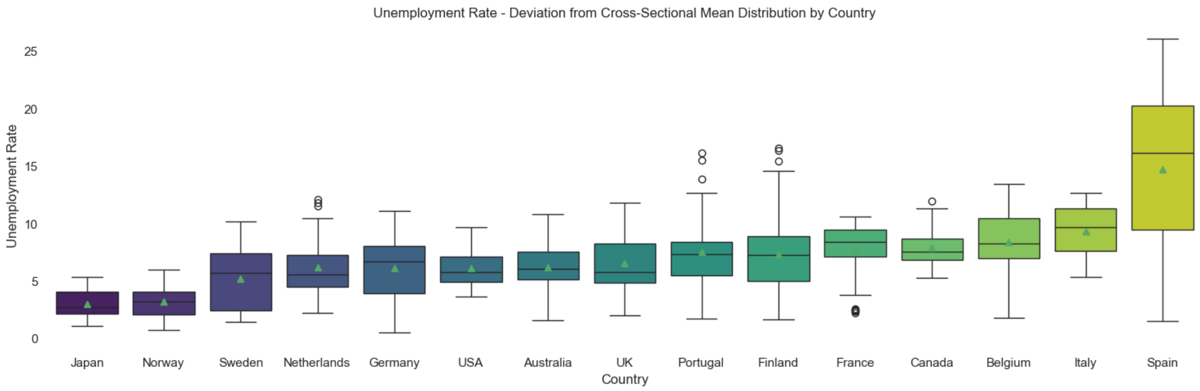


Figure 1.1.16 – Box Plot Deviation from Cross Sectional Mean by Country (Model 1 Variables, Part 2)









1.2 What is the largest number of period T for individuals? What is the number of individuals (countries)?

Raw data have a maximum time dimension of $T = 50$ annual observations for each individual unit, covering the period from 1970 to 2019. For a limited subset of variables and countries, the effective time dimension is slightly shorter due to data availability constraints; however, the core macroeconomic and distributional variables are observed over nearly the entire period for most countries. The cross-sectional dimension of the dataset consists of $N = 15$ countries, specifically: Australia, Belgium, Canada, Finland, France, Germany, Italy, Japan, the Netherlands, Norway, Portugal, Spain, Sweden, the United Kingdom, and the United States. These countries are observed at an annual frequency and form a balanced or near-balanced panel depending on the variable considered.

1.3 Comment on the structure of the unbalanced panel (how many (and which) countries have a single observation, discontinuities between observations, how many individuals have at least 2 consecutive observations (which is useful to compute lags, autocorrelations, first difference and within estimators)?

Raw data has the structure of a mildly unbalanced panel, with the degree of imbalance being limited and largely driven by data availability for specific variables rather than by missing entire country histories. We have no country observed for a single period only. All countries included in the sample have a sufficiently long time series dimension, which rules out the presence of isolated cross-sectional units that would be uninformative for time-series or panel estimators.

Moreover, the core macroeconomic and distributional variables (Model 1), such as real GDP, the GDP deflator, the short-term interest rate, real wages, and the labour share, the panel is largely balanced over the period 1970–2019. Most countries are observed continuously over this horizon. The main exception is Japan, for which consistent data for some variables begin in 1980, resulting in a shorter but still substantial time series. This creates a limited form of unbalancedness without introducing discontinuities within the Japanese series itself. Nevertheless, some labour market variables, short-term and long-term unemployment, exhibit more pronounced unbalancedness. In particular, Norway has observations for unemployment duration measures only from the late 1990s onward, while other countries typically have these data from the mid-1980s.

In summary, although the panel is technically unbalanced, the unbalancedness is limited, systematic, i.e., due to data availability rather than by random missingness. The absence of single-observation units and the prevalence of long, consecutive time series across countries is what we are looking for to have a well suited df for dynamic panel estimation.

1.4 VARIABLE TRANSFORMATIONS PART 1: Compute between transformed and one-way-within transformed variables for all variables. Present a table with the the variance of the one-way-within-fixed-effects, between and pooled data for each variable. Compute the share of between and within variance in the total variance for each variable. Comment these results.

Before conducting the variance decomposition analysis, all variables are transformed to disentangle their between-country and within-country components. Let $x_{i,t}$denote a generic variable observed for country $i = 1, \dots, N$at time $t = 1, \dots, T_i$. The pooled (raw) variable is given directly by $x_{i,t}$, and its variance reflects both cross-country and within-country sources of variation. The between-transformed variable is defined as the country-specific time average:

$$\bar{x}_i = \frac{1}{T_i} \sum_{t=1}^{T_i} x_{i,t},$$

which captures time-invariant or long-run differences across countries. The variance of $\bar{x}_i$therefore measures the extent of cross-sectional heterogeneity in the data. The one-way within (fixed-effects) transformed variable is obtained by removing the country-specific mean from the pooled data:

$$x_{i,t}^{W} = x_{i,t} - \bar{x}_i,$$

which isolates within-country deviations over time around each country's long-run average. The variance of $x_{i,t}^{W}$reflects purely time-series variation net of country-specific effects. The total (pooled) variance of $x_{i,t}$can be decomposed into between and within components as:

$$\mathrm{Var}(x_{i,t}) = \mathrm{Var}(\bar{x}_i) + \mathrm{Var}(x_{i,t} - \bar{x}_i),$$

up to a scaling factor that depends on the panel structure. Based on this decomposition, the share of between variance and the share of within variance in total variance are computed as:

$$\textit{Between Share} = \frac{\mathrm{Var}(\bar{x_i})}{\mathrm{Var}(x_{i,t})}, \ \textit{Within Share} = \frac{\mathrm{Var}(x_{i,t} - \bar{x_i})}{\mathrm{Var}(x_{i,t})}.$$

Our variance analysis results are displayed in Table 1.4.1. A first general observation is that, for most variables, within-country variation accounts for the majority of total variance, indicating that temporal dynamics within countries play a dominant role. This is particularly evident for the GDP deflator, nominal compensation per employee, shadow interest rate, and short-term interest rate, for which the within variance shares amount to 94.10 percent, 93.84 percent, 93.35 percent, and 92.70 percent, respectively. These results suggest that these variables are primarily driven by time-varying macroeconomic forces, such as inflation dynamics, monetary policy cycles, and wage adjustment processes, rather than by persistent cross-country differences.

By contrast, some variables exhibit a substantial between-country component, reflecting persistent structural heterogeneity across countries. This is especially the case for real gross domestic product, labour force, and unemployment-related indicators. For example, real GDP displays a between variance share of 8.36 percent, indicating large and persistent differences in economic size across countries. Similarly, the labour force shows a between variance share of

9.49 percent, while long-term unemployment exhibits a between variance share of 6.53 percent, consistent with structural differences in labour market institutions, matching efficiency, and unemployment persistence across economies. These patterns justify the inclusion of country fixed effects and country-specific trends to absorb time-invariant and slowly evolving cross-country differences.

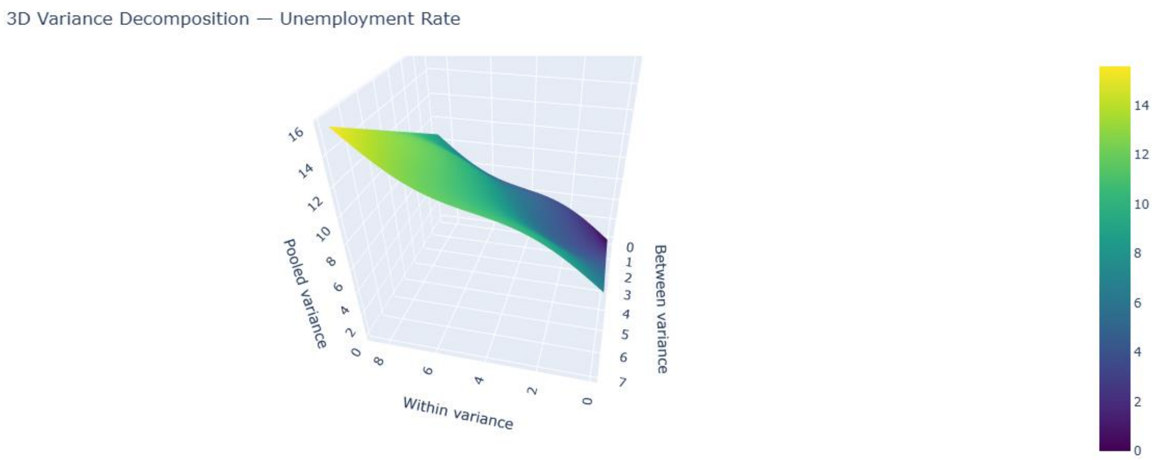
Table 1.4.1: Variance decomposition (pooled, between, within)

Variable	Pooled Variance (x_{it})	Between Variance ($x_{i\cdot}$)	Within Variance ($x_{it} - x_{i\cdot}$)	Variance Share – Between	Variance Share – Within
Short-Term Interest Rate	2.346×10^{1}	1.711	2.175×10^{1}	7.296	9.270
GDP Deflator	8.712×10^{2}	5.137×10^{1}	8.198×10^{2}	5.896	9.410
Nominal Compensation Per Employee	2.633×10^{2}	1.604×10^{1}	2.471×10^{2}	6.090	9.384
Real Compensation Per Employee	3.950×10^{2}	8.661×10^{1}	3.121×10^{2}	2.192	7.901
Real Gross Domestic Product	9.526×10^{12}	7.966×10^{12}	1.560×10^{12}	8.363	1.637
Adjusted Labour Share	2.928×10^{1}	1.126×10^{1}	1.803×10^{1}	3.846	6.156
Energy Commodities Price Index	1.347×10^{3}	6.912×10^{-26}	1.347×10^{3}	5.133	1.000
Unemployment Rate	1.567×10^{1}	6.970	8.624	4.447	5.503
Short-Term Unemployment	2.835×10^{6}	2.201×10^{6}	1.528×10^{5}	7.764	5.388
Long-Term Unemployment	7.765×10^{5}	5.068×10^{5}	2.842×10^{5}	6.527	3.660
Labor Force	1.083×10^{9}	1.028×10^{9}	4.656×10^{7}	9.494	4.299
Real Effective Exchange Rate	1.645×10^{2}	5.167×10^{1}	1.128×10^{2}	3.142	6.858
Shadow Interest Rate	3.071×10^{1}	2.042	2.866×10^{1}	6.649	9.335

The labour share exhibits a more balanced variance decomposition. While within-country variation remains dominant, accounting for 68.58 percent of total variance, the between variance share is non-negligible at 31.42 percent, suggesting that both global trends and country-specific structural features, such as bargaining regimes, sectoral composition, and institutional settings, contribute to its observed dynamics.

Last, variables that are global by construction, such as the energy commodity price index, display virtually no between-country variance, with the between variance share equal to zero and the within variance share equal to 100 percent. This outcome is expected, as the variable is identical across countries at a given point in time and varies only along the time dimension.

Figure 1.4.2 – 3D Variance Decomposition for Unemployment Rate



Now, in order to better capture the 3-Dimensional interplay between Pooled, Within and Between Variance, we have deployed a 3D variance plotting for Unemployment Rate through Plotly's graphical engine. The 3D space is also interactive if run within your own local kernel, but not through GitHub (we recommend to try and do it). The three-dimensional variance decomposition plot provides an intuitive visual representation of how the total variability of the unemployment rate is jointly determined by within-country variation over time and between-country structural differences. The unemployment rate is particularly well suited for this illustration because its variance is relatively balanced between these two components, making it an informative case for understanding the interaction between temporal dynamics and cross-country heterogeneity.

The horizontal axes represent the two fundamental sources of variation in panel data. One axis corresponds to within-country variance, capturing how unemployment fluctuates over time around each country's own long-run average, reflecting business cycles, recessions, and recoveries. The other axis corresponds to between-country variance, capturing persistent differences in average unemployment levels across countries, which are typically associated with institutional features such as labour market regulations, matching efficiency, and social protection systems.

The vertical axis represents the pooled (total) variance of the unemployment rate. Each point on the surface shows how a given combination of within-country and between-country variation contributes to the overall variability observed in the data. In this way, the surface can be interpreted as a map linking the two underlying sources of variation to the total dispersion of unemployment outcomes.

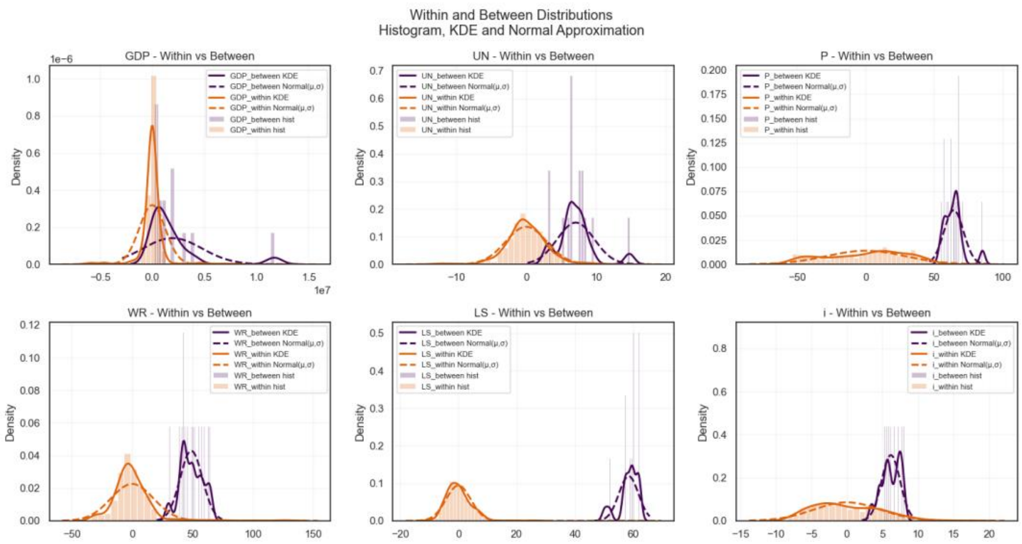
The surface itself is constructed to reflect the empirical importance of each variance component. First, the within-country and between-country variances are normalized by their respective shares in total variance. The resulting surface is therefore not arbitrary but grounded in the variance decomposition reported earlier and we introduce a slight curvature to the surface to emphasize that the relationship between within and between variation is not purely mechanical but reflects the interaction between cyclical dynamics and structural differences across countries.

From an interpretative perspective, the plot highlights that neither time variation nor cross-country heterogeneity alone can fully explain the behaviour of unemployment. Moving along the within-variance dimension illustrates how unemployment responds to cyclical forces common to many countries, while moving along the between-variance dimension reflects persistent national differences. The elevation of the surface increases when both sources of variation are present, visually reinforcing the idea that total unemployment variability emerges from the interaction of short-run dynamics and long-run structural features.

1.5 Plot the distribution of the one-way-fixed-effects-within x(it)-x(i.) and between (x(i.)) transformed dependent variable and of you key (preferred) explanatory variable (not all the explanatory variable) plotting on the same graph an histogram, a normal law with same empirical mean and standard error and a kernel continuous approximation. Comment the between and within difference for each variable, and compare within/within for dependent and explanatory variable, and between/between for dependent and explanatory variable: kurtosis, skewness, non-normality, high leverage observation (far from the mean), several modes (mixture of distribution)?

To further assess the suitability of the empirical framework, we examine the distributional properties of the one-way fixed-effects (within) transformation, $x_{i,t} - \bar{x}_i$, and the between transformation, $\bar{x}_i$, our Model 1 preferred variables.

Figure 1.5.1 – Within & Between Distributions (Histogram, KDE and Normal Approximation)



Descriptive statistics and distributional tests reported in Tables 1.5.2-1.5.5 and Figure 1.5.1 reveal systematic and economically meaningful differences between within and between transformations across all variables. For the dependent variable (GDP), the within component is highly leptokurtic (kurtosis = 6.714) with positive skewness (2.233), indicating strong tail risk driven by large transitory deviations, while the between component exhibits even more pronounced non-normality, with skewness of 1.377 and excess kurtosis of 2.401, reflecting substantial cross-sectional heterogeneity in long-run levels. This pattern generalizes to the explanatory variables: within transformations are consistently closer to symmetry but remain non-Gaussian, whereas between transformations display sharper asymmetry and heavier tails. For instance, unemployment (UN) shows near symmetry within groups (skewness = -1.180) but extreme left skewness between groups (-3.840) coupled with very high kurtosis (8.470), suggesting the presence of structurally distinct clusters.

Table 1.5.2: Descriptive statistics for between and within transformations

Variable	Count	Mean	Std.	Min	25%	50%	75%	Max	Skewness	Kurtosis
GDP_between	7.500	1.941×10^{11}	2.823×10^{10}	1.674×10^{11}	3.220×10^{10}	1.047×10^{11}	1.994×10^{11}	1.172×10^{12}	2.714	6.714
UN_between	7.500	7.088	2.642	3.083	6.152	6.575	7.922	1.477×10^{1}	1.233	2.492
P_between	7.500	6.434×10^{1}	7.719	5.450×10^{1}	5.865×10^{1}	6.444×10^{1}	6.730×10^{1}	8.535×10^{1}	1.377	2.401
WR_between	7.500	4.899×10^{1}	9.313	3.016×10^{1}	4.268×10^{1}	4.894×10^{1}	5.679×10^{1}	6.468×10^{1}	−1.810	−6.470
LS_between	7.500	5.839×10^{1}	3.358	5.016×10^{1}	5.712×10^{1}	5.958×10^{1}	6.160×10^{1}	6.206×10^{1}	−1.196	6.770
i_between	7.500	6.187	1.309	3.481	5.288	6.112	7.426	8.039	−3.840	−8.470
GDP_within	7.500	9.736×10^{-11}	1.250×10^{10}	-6.530×10^{10}	-1.994×10^{10}	-1.038×10^{10}	2.367×10^{10}	8.213×10^{10}	5.560	1.589
UN_within	7.410	8.336×10^{-17}	2.988	-1.324×10^{1}	−1.580	-1.100×10^{-1}	1.647	1.133×10^{1}	−1.830	2.657
P_within	7.500	3.032×10^{-16}	2.865×10^{1}	-5.385×10^{1}	-2.289×10^{1}	5.421	2.262×10^{1}	4.989×10^{1}	−2.850	−1.064
WR_within	7.400	1.306×10^{-15}	1.777×10^{1}	-3.443×10^{1}	−8.378	−2.145	6.009	1.302×10^{2}	2.970	1.666
LS_within	7.400	2.132×10^{-15}	4.249	-1.069×10^{1}	−2.883	-4.530×10^{-1}	2.339	2.671×10^{1}	1.076	3.870
i_within	7.500	3.790×10^{-17}	4.666	−8.394	−3.540	-6.660×10^{-1}	3.495	1.686×10^{1}	4.550	−3.770

Price inflation (P) and interest rates (i) display particularly harsh contrasts: P_within is moderately skewed (-1.896) with kurtosis of 2.657, whereas P_between shows positive skewness (2.380) and elevated kurtosis (3.870), while i_within exhibits right skewness (4.550) but i_between is sharply left-skewed (- 8.704), pointing to regime-like heterogeneity in average monetary conditions. These distributional differences are formally confirmed by normality tests: Jarque–Bera statistics reject normality at the 1% level for all within and between variables, with especially large statistics for GDP_within (JB = 4.51×10^{3}) and GDP_between (JB = 1.11×10^{3}), and similarly decisive rejections for UN_between (JB = 4.15×10^{2}) and LS_between (JB = 1.91×10^{2}).

Table 1.5.3: Extreme observations in between and within transformations

Transformation	GDP	UN	P	WR	LS	i
Between (min)	1.674×10^{5}	9.349	5.701	3.867	6.180	7.739
Between (q25)	3.944×10^{6}	3.083	7.065	6.468	6.206	4.547
Between (q75)	2.079×10^{5}	3.292	8.535	3.016	5.186	3.481
Between (max)	1.172×10^{7}	1.477	5.450	6.342	5.016	8.038
Within (1)	-6.530×10^{6}	−6.495	4.629	5.814	9.856	9.873
Within (2)	-6.359×10^{6}	−6.561	4.819	6.840	9.376	1.226
Within (3)	-6.078×10^{6}	−6.055	4.989	6.884	−7.758	9.879
Within (4)	-5.759×10^{6}	9.041	−5.305	6.956	−8.658	1.168
Within (5)	-5.791×10^{6}	9.243	−5.225	6.044	−7.358	1.228
Within (6)	-5.804×10^{6}	8.064	−5.270	5.477	1.151	1.068
Within (7)	-5.485×10^{6}	7.249	−5.295	1.015	1.041	−7.643
Within (8)	-5.197×10^{6}	7.962	−5.265	9.366	9.810	−7.888
Within (9)	-4.836×10^{6}	8.620	−5.235	7.758	9.510	−7.953

Kolmogorov-Smirnov tests against the normal reference corroborate these findings, with KS statistics exceeding conventional critical values for all between components and for most within components, notably GDP_within (KS = 2.581) and i_within (KS = 8.720). The histogram-KDE evidence in Figure 1.5.1 also corroborates that within distributions are largely unimodal with dispersion driven by shocks, while between distributions frequently display multiple modes, consistent with mixtures of heterogeneous groups.

Extreme-value diagnostics in Table 1.5.3 further show that high-leverage observations are substantially more severe in the between dimension, where maxima and upper quartiles are far from the mean (e.g., GDP_between max = 1.17×10^7), whereas within extremes, though large, are more symmetrically distributed around zero.

Table 1.5.4: Jarque–Bera test of normality for between and within transformed target variables (Model 1)

Variable	Jarque–Bera Stat	p-value	Statistical Significance
GDP_within	4.512×10^3	0.000	***
UN_within	1.179×10^2	2.465×10^{-26}	***
P_within	2.797×10^1	8.434×10^{-7}	***
WR_within	3.712×10^1	8.688×10^{-9}	***
LS_within	1.304×10^1	1.473×10^{-3}	**
i_within	3.637×10^1	1.268×10^{-8}	***
GDP_between	1.105×10^3	1.204×10^{-240}	***
UN_between	4.148×10^2	8.575×10^{-91}	***
P_between	3.196×10^2	4.071×10^{-70}	***
WR_between	7.128	2.833×10^{-2}	*
LS_between	1.913×10^2	2.834×10^{-42}	***
i_between	2.304×10^1	9.948×10^{-6}	***

Table 1.5.5: Kolmogorov–Smirnov test of distributional equivalence (between/within transformed target variables vs. normal reference)

Variable	KS Stat (reference = Normal)	p-value	Statistical Significance
GDP_within	2.5813	7.2725×10^{-35}	***
UN_within	5.2627	7.5527×10^{-2}	
P_within	6.1567	2.2711×10^{-2}	*
WR_within	5.6084	4.8530×10^{-2}	*
LS_within	3.4893	4.6419×10^{-1}	
i_within	8.7198	2.5535×10^{-4}	***
GDP_between	3.0461	1.1419×10^{-48}	***
UN_between	1.8240	1.6693×10^{-17}	***
P_between	2.1642	1.5611×10^{-24}	***
WR_between	1.2511	1.9352×10^{-8}	***
LS_between	1.9751	1.7898×10^{-20}	***
i_between	1.2361	3.0025×10^{-8}	***

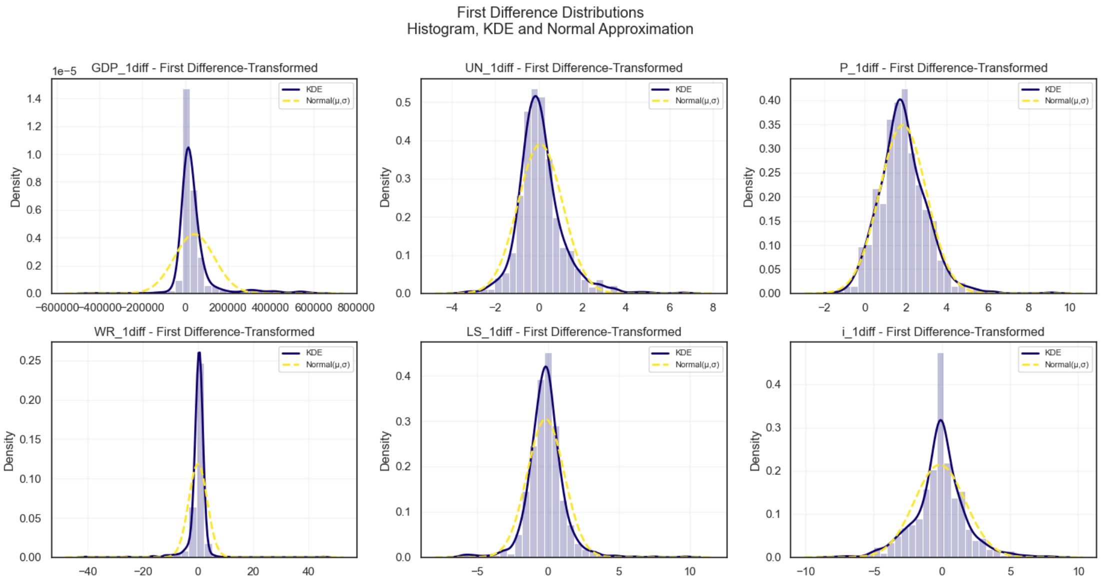
1.6 FD. FIRST DIFFERENCE VARIABLE TRANSFORMATIONS: Compute the first-differences x(i,t)-x(i,t-1) for panel data. Check for the first 3 changes of individuals (for data sorted by individual and then time) in say 3T+1 first observations that when there is a change of individual in the stacked vector individuals x time, the first differences is a dot for "not available". In other words, for first-differences for panel data, check that when you change individual, the first observation is missing with a dot, and it is not the difference of the first observation of the second individual minus the last observation of the first individual, for example.

With the first-difference transformation we rewrite each variable as the change between two consecutive periods, $\Delta x_{it} = x_{it} - x_{i,t-1}$, and is employed to shift the analysis from levels to short-run movements. By construction, this transformation removes time-invariant components and substantially attenuates low-frequency trends, thereby reducing the risk of spurious correlations driven by common growth paths or persistent cross-sectional differences. The resulting series are centered around zero and reflect transitory fluctuations rather than long-run levels, making them particularly suitable for identifying short-run relationships. At the same time, first differencing does not preclude the presence of underlying temporal structure: if variables exhibit cyclical or periodic behaviour, or if persistence extends beyond one lag, such patterns may remain in the differenced data. Moreover, because differencing emphasizes changes rather than levels, it can increase the relative importance of extreme period-to-period movements and measurement noise.

1.7 First differences distributions. Plot the distributions (histogram, KDE, normal law with same mean and standard error) for first difference dependent GDPG and first difference explanatory EDA/GDP for each of these two transformations.

Following our early stated properties, Figure 1.7.1 show that transforming the variables into period-to-period changes substantially recentres all series around zero and compresses their dispersion relative to level-based or between components, consistent with the removal of persistent cross-sectional and low-frequency variation. Nevertheless, the distributions remain clearly non-Gaussian across all variables.

Figure 1.7.1 – First Difference Distributions (Histogram, KDE and Normal Approximation)



For GDP, the first-difference distribution is sharply peaked with very heavy tails, indicating infrequent but extremely large output changes; this visual evidence is reflected in the Jarque–Bera statistic of (3.01 x 10^3), which overwhelmingly rejects normality. Unemployment (UN) changes display a similarly leptokurtic shape with noticeable asymmetry and tail mass, and the corresponding Jarque-Bera statistic (1.09 x 10^3)) confirms strong departures from normality despite the apparent central concentration. For inflation (P), the KDE is closer to the normal benchmark in the centre of the distribution, but excess kurtosis and tail deviations remain visible, leading to a statistically significant rejection of normality (JB = 33.9). Wage rigidity (WR) and labour share (LS) changes are characterized by very sharp central peaks and thin shoulders combined with non-negligible tail observations, a pattern consistent with infrequent but sizeable adjustments; normality is again decisively rejected (JB = 95.7 for WR and 271 for LS). Finally, interest rate (i) changes exhibit a more dispersed and asymmetric distribution, with heavier tails than the normal approximation and a Jarque–Bera statistic of 70.2, indicating persistent non-Gaussian behaviour.

Table 1.7.2: Jarque–Bera test of normality for first-difference transformed target variables (Model 1)

Variable	Jarque–Bera Stat	p-value	Statistical Significance
GDP_1diff	3.012×10^{3}	0.000	***
UN_1diff	1.087×10^{3}	7.701×10^{-237}	***
P_1diff	3.387×10^{1}	4.417×10^{-8}	***
WR_1diff	9.567×10^{1}	1.678×10^{-21}	***
LS_1diff	2.707×10^{2}	1.634×10^{-59}	***
i_1diff	7.021×10^{1}	5.679×10^{-16}	***

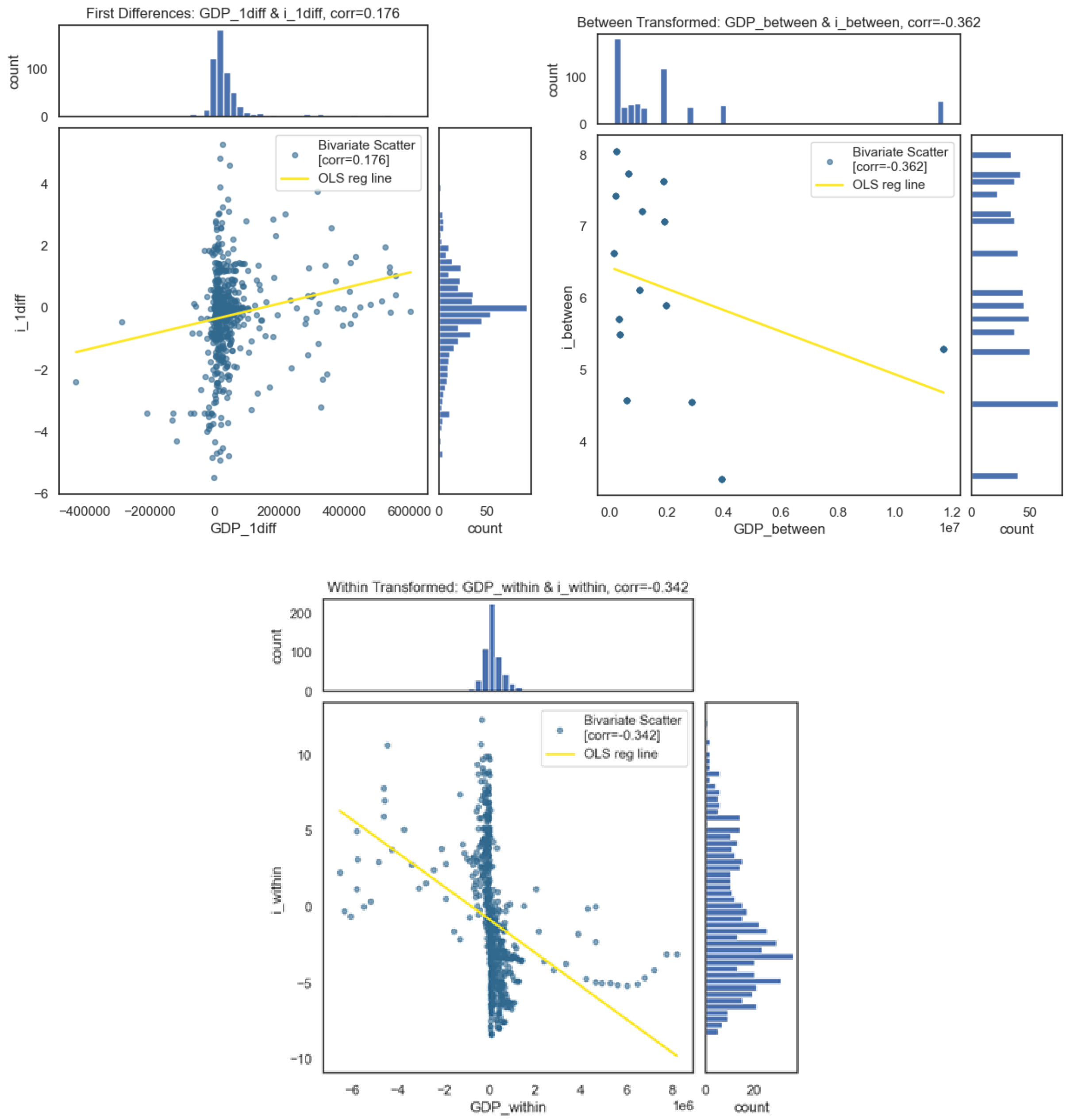
1.8 First differences simple correlation. For these FD transformed variables, plot the bivariate cloud of points with regression line and on top the marginal distribution of the horizontal axis and on right hand side the marginal distribution of the variable on the vertical axis. Compare with one-way-fixed effects and between distributions. Report the simple correlation coefficient on the graph.

From Figure 1.8.1 we observe that the association between output and the interest rate is highly sensitive to the transformation applied, both in sign and in economic interpretation. In first differences, the relationship between ΔGDP_{it} and Δi_{it} is positive but weak, with a correlation of 0.176. The scatter is dominated by a dense vertical mass around small GDP changes combined with substantial dispersion in interest-rate movements, indicating that short-run co-movements are noisy and driven by episodic shocks rather than a tight linear relationship; the shallow slope of the fitted line suggests limited contemporaneous comovement in period-to-period changes.

By contrast, the within transformation yields a clearly negative association (correlation -0.342), with a pronounced downward-sloping conditional mean. Here, deviations of GDP from country-specific averages are systematically associated with opposite-signed deviations in interest rates, consistent with a stabilizing policy response or cyclical co-movement once time-invariant heterogeneity is removed; the elongated vertical structure of the point cloud further reflects the dominance of within-country fluctuations in GDP relative to interest-rate variability.

The between transformation also produces a negative relationship (correlation -0.362), but of a different nature: the scatter is sparse and heavily influenced by a small number of cross-sectional observations far from the mean, indicating that long-run average output levels are inversely related to average interest-rate levels across units. In this case, the fitted slope is largely driven by cross-sectional heterogeneity rather than time-series variation, and high-leverage observations exert disproportionate influence on the estimated relationship.

Figure 1.8.1 – First Difference Bivariate Correlation Cloud of Points



1.9 Restricted sample with a BALANCED PANEL: Two way fixed effects (TWFE) formula. Restrict the sample to the countries/individuals available with the longest duration (N=… countries over T=… periods). Compute -x(.t)+x(..), report the 6 numbers in a table as a function of time and plot them as a function of time, then comment. Then compute two-way-fixed-effects x(it)-x(i.)-x(.t)+x(..) transformed variables.

To isolate within-unit and within-time variation while controlling for unobserved heterogeneity along both dimensions, we employ a two-way fixed effects (TWFE) framework. Let x_{it}denote a generic variable observed for unit $i = 1, \dots, N$over time periods $t = 1, \dots, T$. The canonical TWFE decomposition expresses x_{it}as the sum of a unit-specific effect, a time-specific effect, and a residual component:

$$x_{it} = \bar{x}_{i\cdot} + \bar{x}_{\cdot t} - \bar{x}_{\cdot\cdot} + \tilde{x}_{it},$$

where $\bar{x}_{i\cdot} = \frac{1}{T}\sum_t x_{it}$ is the unit mean, $\bar{x}_{\cdot t} = \frac{1}{N}\sum_i x_{it}$ is the time mean, and $\bar{x}_{\cdot\cdot} = \frac{1}{NT}\sum_{i,t} x_{it}$ is the grand mean. The two-way fixed-effects–transformed variable is therefore given by

$$\tilde{x}_{it} = x_{it} - \bar{x}_{i\cdot} - \bar{x}_{\cdot t} + \bar{x}_{\cdot\cdot},$$

which removes all additive unit-invariant and time-invariant components.

To ensure that the transformation is well defined and comparable across time, the analysis is conducted on a balanced subsample, restricting attention to the Nunits observed over the full Tperiods (i.e., the longest available common support). In a balanced panel, each time mean $\bar{x}_{\cdot t}$is computed over the same set of units, and each unit mean $\bar{x}_{i\cdot}$over the same number of periods. This symmetry implies that the time effects,

$$-\bar{x}_{\cdot t} + \bar{x}_{\cdot\cdot},$$

can be interpreted as pure common time deviations relative to the overall mean, uncontaminated by changes in sample composition. Reporting these quantities period by period therefore provides a transparent summary of aggregate movements shared across units, which can be tabulated and plotted as functions of time.

The distinction between balanced and unbalanced TWFE is substantive. In an unbalanced panel, the set of units contributing to $\bar{x}_{\cdot t}$varies across t, so time effects conflate genuine aggregate dynamics with shifts in sample composition. Similarly, unit means $\bar{x}_{i\cdot}$depend on heterogeneous observation windows, making the transformed residual $x_{it} - \bar{x}_{i\cdot} - \bar{x}_{\cdot t} + \bar{x}_{\cdot\cdot}$harder to interpret as a clean within-unit, within-time deviation. By contrast, restricting attention to a balanced panel ensures that the TWFE transformation has a clear algebraic and economic interpretation: all remaining variation reflects deviations from both unit-specific and time-specific averages computed on a fixed and common support.

Figure 1.9.1 – Time Component Evolution Overtime (Part 1)

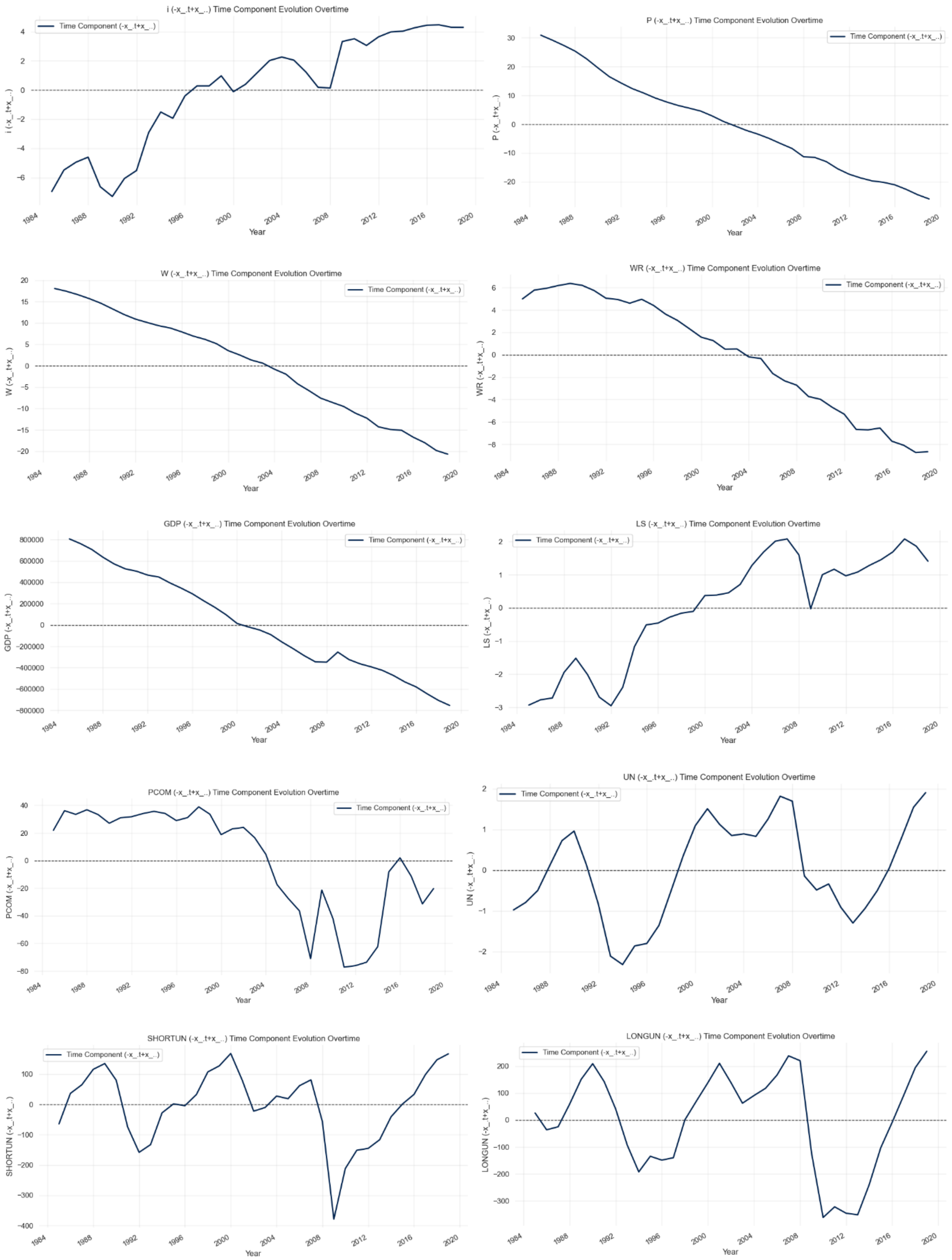
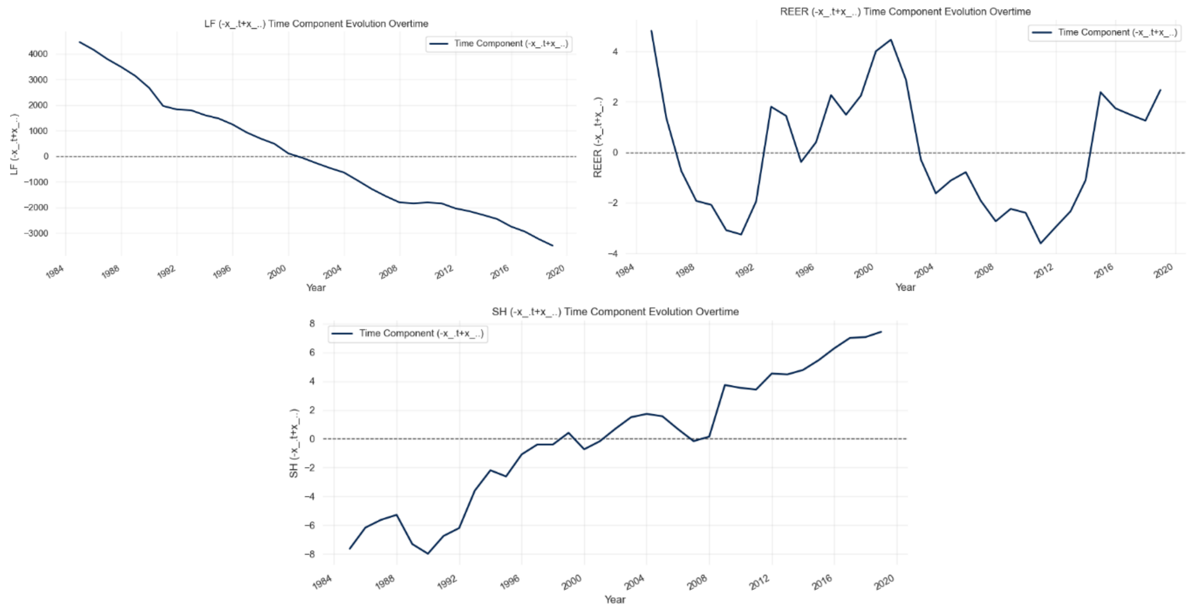


Figure 1.9.2 – Time Component Evolution Overtime (Part 1)



Interpreting the time component $-\bar{x}_{.t} + \bar{x}_{..}$ variable by variable, Figures 1.9.1 and 1.9.2 show that common aggregate movements differ markedly in magnitude, persistence, and economic interpretation across series, even though all are constructed on the same balanced sample.

For GDP, the time component displays large-amplitude and highly persistent swings, with extended phases of positive and negative deviations from the grand mean. This pattern is consistent with global business-cycle dynamics: prolonged expansions and contractions shift the cross-sectional average output level in a systematic way, and these movements dominate short-run noise. The smoothness and persistence of the GDP time effects indicate that common shocks account for a substantial share of aggregate output variation across countries.

For labour metrics, starting with unemployment (UN), the time component evolves more slowly and exhibits pronounced asymmetry, with long periods above the grand mean followed by gradual reversals. This reflects the well-known sluggish adjustment of labour markets at the aggregate level. Peaks and troughs in the UN time effects lag those of GDP, reinforcing the interpretation of unemployment as responding with delay to common macroeconomic conditions rather than moving contemporaneously. For the labour share (LS), the time component exhibits a clear long-run trend component, with sustained movements away from the grand mean over extended periods. This pattern is consistent with slow-moving global structural forces, such as technological change or globalization, affecting the distribution of income across countries in a coordinated manner, rather than cyclical fluctuations.

Finally, for the interest rate (i), the time component is both volatile and strongly synchronized, with pronounced peaks and troughs reflecting global monetary conditions. Sharp movements correspond to episodes of coordinated tightening or easing across countries, underscoring that interest rates are particularly sensitive to common shocks and policy spillovers at the global level.

Figure 1.9.3 – TWFE-Transformed Variables Evolution Overtime (Part 1)

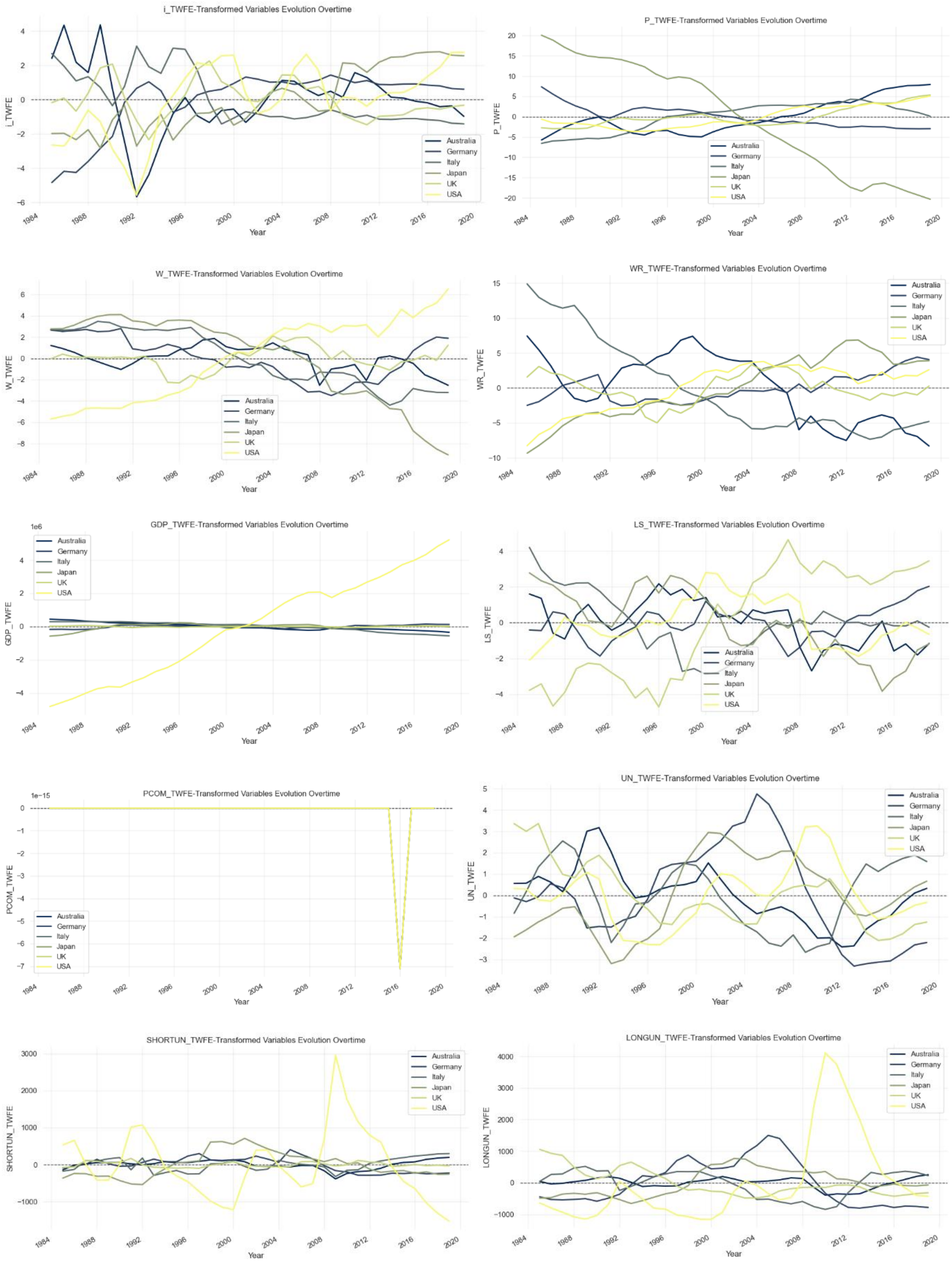


Figure 1.9.4 – TWFE-Transformed Variables Evolution Overtime (Part 2)

From the overtime trends of TWFE-Transformed variables. Two broad features emerge. First, for most variables, the bulk of countries exhibit relatively small, mean-reverting deviations, indicating that after purging both fixed effects, residual variation is largely transitory. This clustering around zero suggests that a large share of the systematic variation in the data is indeed captured by unit-specific and time-specific components, validating the TWFE structure. Second, deviations are not evenly distributed across countries: a subset of units displays recurrent and sometimes large departures from zero, often clustered in specific subperiods. These countries contribute disproportionately to the overall dispersion of the transformed series and can be interpreted as experiencing repeated idiosyncratic shocks or country-specific policy episodes not shared by the rest of the panel.

The persistence and amplitude of deviations vary by variable. For GDP and interest rates, deviations tend to be rare but sizable, with certain countries repeatedly exhibiting large positive or negative residuals during turbulent periods, suggesting asymmetric exposure to global shocks or differential domestic responses. In contrast, unemployment and labour share deviations are generally smoother but display longer-lasting departures for some countries, consistent with slower adjustment mechanisms and institutional rigidities.

1.10 TWFE Balanced panel. Compute descriptive statistics. Plot boxplots by country ordered by their variance from the smallest to the largest.

The descriptive statistics and country-level distributions of the balanced TWFE-transformed variables highlight how residual heterogeneity is unevenly distributed across countries even after removing both unit and time fixed effects. At the aggregate level (Table 1.10.1), all TWFE-transformed variables are centred close to zero by construction, but they retain substantial dispersion, non-zero skewness, and excess kurtosis, indicating that idiosyncratic country-time shocks are neither symmetric nor Gaussian. Variables such as GDP_{TWFE}, UN_{TWFE}, and i_{TWFE} exhibit particularly high kurtosis, reflecting the presence of episodic extreme deviations that are not absorbed by additive fixed effects.

Table 1.10.1: Descriptive statistics for TWFE-balanced transformed variables

Variable	Count	Mean	Std.	Min	25%	50%	75%	Max	Skewness	Kurtosis
i_TWFE	5.250	-6.546×10^{-16}	1.593	−5.675	-9.800×10^{-1}	1.900×10^{-2}	8.200×10^{-1}	9.177	4.250	2.927
P_TWFE	5.250	-1.083×10^{-15}	4.860	-2.032×10^{1}	−1.993	-2.660×10^{-1}	2.268	2.008×10^{1}	−4.110	4.389
W_TWFE	5.250	4.589×10^{-15}	2.596	−9.070	−1.371	3.000×10^{-3}	1.755	7.963	−1.160	6.200
WR_TWFE	5.250	-1.353×10^{-17}	3.981	−9.343	−2.682	-2.670×10^{-1}	2.501	1.614×10^{1}	−5.290	8.290
GDP_TWFE	5.250	-9.181×10^{-11}	8.224	-4.813×10^{5}	-1.935×10^{5}	4.124×10^{3}	1.649×10^{5}	5.227×10^{5}	−1.030	1.771
LS_TWFE	5.250	-6.091×10^{-16}	1.815	−4.706	−1.205	4.000×10^{-3}	1.202	6.472	9.800	2.100
PCOM_TWFE	5.250	-2.030×10^{-16}	1.185×10^{-15}	-7.105×10^{-15}	-1.710×10^{-1}	1.600×10^{-2}	1.127	7.193	−1.400	1.661
UN_TWFE	5.250	-3.519×10^{-16}	2.044	−7.897	−1.171	-1.700×10^{-2}	2.028	7.193	−1.400	1.661
SHORTUN_TWFE	5.100	-1.641×10^{-13}	2.975×10^{2}	-1.529×10^{4}	-1.043×10^{2}	-8.530×10^{-1}	8.414×10^{1}	2.963×10^{3}	1.919	2.488
LONGUN_TWFE	5.100	1.783×10^{-15}	4.973×10^{2}	-1.235×10^{4}	-2.291×10^{3}	1.608×10^{1}	1.502×10^{2}	4.111×10^{3}	2.671	1.864
LF_TWFE	5.250	-6.444×10^{-13}	3.439×10^{3}	-2.243×10^{4}	-1.108×10^{3}	-9.875×10^{1}	1.139×10^{3}	1.770×10^{4}	−8.850	1.379
REER_TWFE	5.250	-4.791×10^{-15}	8.437	-2.687×10^{1}	−4.593	-9.800×10^{-2}	3.879	3.633×10^{1}	4.600	2.213
SH_TWFE	5.250	-2.692×10^{-16}	1.883	−6.358	−1.162	-2.000×10^{-2}	1.093	9.368	2.500	1.843

Table 1.10.2: Country-level descriptive statistics for i_{TWFE} (balanced panel)

Country	Count	Mean	Std.	Min	25%	50%	75%	Max	Skew.	Kurt.
Australia	3.500	-8.882×10^{-16}	1.959	−5.675	−7.510	1.230	9.090	4.359	−4.270	2.159
Belgium	3.500	-8.882×10^{-17}	8.440	−1.726	−3.160	3.540	5.080	1.472	−9.000	−2.260
Canada	3.500	-9.770×10^{-16}	1.130	−3.371	−6.210	2.840	7.760	1.659	−1.008	1.135
Finland	3.500	-2.284×10^{-16}	1.092	−1.497	−5.550	−3.900	9.600	3.017	1.481	1.426
France	3.500	-1.269×10^{-15}	6.600	−1.839	−2.710	1.690	3.460	1.605	−6.560	1.371
Germany	3.500	-6.598×10^{-16}	1.784	−4.843	5.400	8.430	1.003	1.430	−1.765	1.800
Italy	3.500	-7.867×10^{-16}	1.495	−1.427	−1.114	−8.350	1.213	3.119	9.490	−6.040
Japan	3.500	-6.852×10^{-16}	1.845	−2.826	−1.493	−4.680	2.113	2.792	2.770	−1.322
Netherlands	3.500	-8.628×10^{-16}	1.503	−4.019	−7.000	7.170	8.950	1.347	−1.568	1.148
Norway	3.500	-5.075×10^{-16}	1.490	−2.254	−1.303	−6.200	4.230	3.970	9.800	6.910
Portugal	3.500	-5.837×10^{-16}	2.940	−2.329	−2.016	−1.737	2.046	9.177	1.373	1.269
Spain	3.500	-3.045×10^{-16}	1.735	−1.467	−1.154	−8.760	1.095	5.715	1.606	2.374
Sweden	3.500	-5.583×10^{-16}	1.063	−1.180	−8.110	−3.390	5.240	3.070	1.370	1.700
UK	3.500	-8.120×10^{-16}	1.094	−2.344	−7.810	−1.170	6.320	2.257	3.300	−2.900
USA	3.500	-4.568×10^{-16}	2.083	−5.581	−7.270	1.840	1.763	2.762	−7.480	2.290

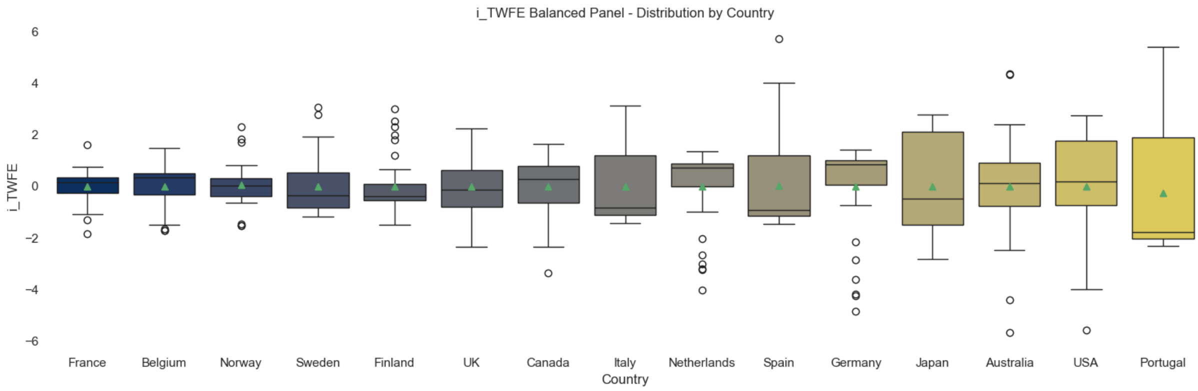
Turning to the country-level evidence, Table 1.10.2 and the boxplots make clear that some countries deviate systematically more than others. For interest rates, countries such as Portugal, the USA, Spain, and Australia display wider interquartile ranges and larger maxima and minima, indicating more volatile and recurrent departures from the common path. In contrast, countries like France, Belgium, Norway, and Sweden exhibit tighter distributions centred closely around zero, suggesting that their interest-rate dynamics are largely captured by common time effects and country averages, with relatively limited idiosyncratic variation.

A similar pattern emerges for the GDP deflator (P_{TWFE}), where Japan and Portugal stand out with extremely wide distributions and pronounced asymmetry, including large positive and negative residuals. This indicates repeated country-specific inflation shocks or regime changes not shared by the rest of the panel. By contrast, Canada, Finland, Belgium, and the UK show much more compact distributions.

For GDP_{TWFE}, heterogeneity is even more concentrated: most countries cluster tightly around zero with relatively modest dispersion, while the USA exhibits a markedly wider distribution with very large positive and negative deviations. This suggests that USA.-specific output shocks play a disproportionate role in the residual variation once global and country-average components are removed. Countries such as Finland and Portugal also display wider tails than the median country, but to a much smaller extent than the USA.

So well, to conclude on this, these results indicate that although the balanced TWFE transformation effectively removes systematic cross-country and common-time variation, it does not homogenize countries. Instead, residual dynamics are dominated by a small subset of countries that deviate more frequently and more intensely from the common trajectory, contributing disproportionately to skewness, kurtosis, and tail risk in the transformed variables.

Figure 1.10.3 – Box Plot TWFE-Transformed Variables (Balanced Panel) by Country (Model 1 Variables, Part 1)



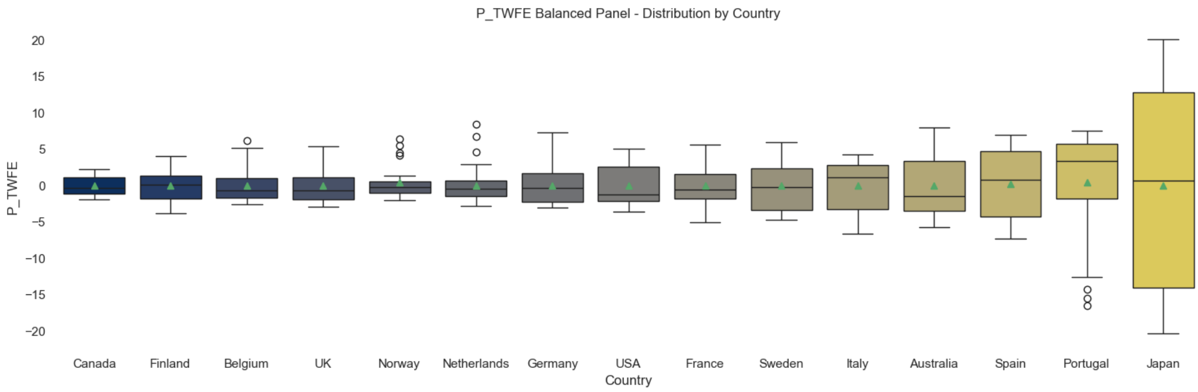
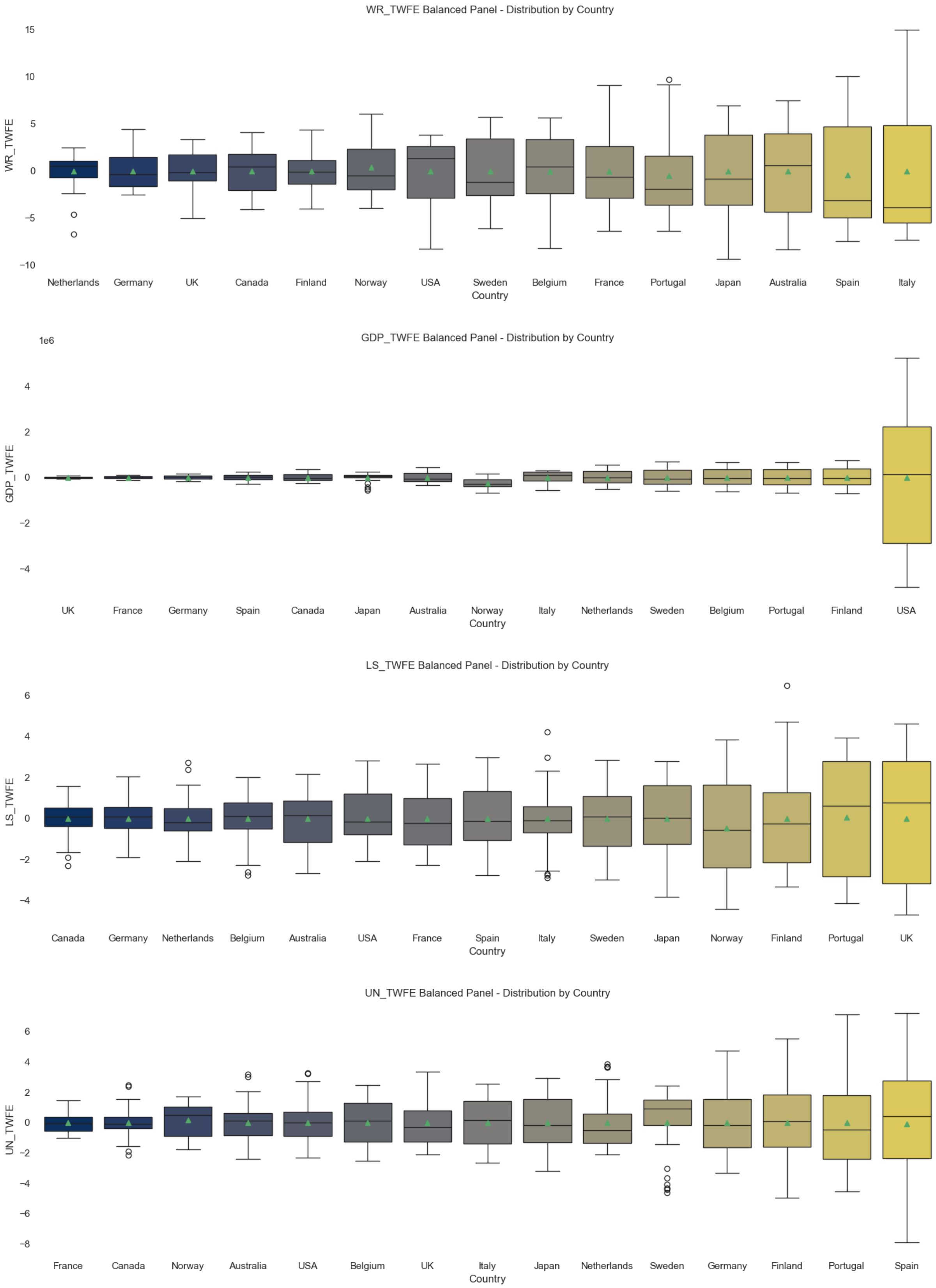


Figure 1.10.4 – Box Plot TWFE-Transformed Variables (Balanced Panel) by Country (Model 1 Variables, Part 2)



1.11 TWFE Balanced panel. Present a table ordering the simple correlation coefficients of TWFE transformed GDPG and EDA/GDP by country from the largest positive to the lowest negative, with the standard error of GDPG and EDA in another column and the coefficient of simple regression: correlation coefficient * standard error of GDPG / standard error of EDA/GDP. Comment.

The correlation structure of the TWFE-transformed variables reveals economically meaningful comovements while also suggesting potential sources of multicollinearity risk. The heatmap shows that several pairs exhibit moderate to strong correlations with high statistical significance, even after removing both country and time fixed effects (Figure 1.11.2). Most notably, the correlation between interest rates and inflation (GDP deflator) remains strongly negative ($r = -0.46$, $t = -11.7$), consistent with a stabilizing monetary-policy response once common and unit-level components are netted out. Wage-related variables are tightly interconnected (as expected): nominal wages (W_{TWFE}) and real wages (WR_{TWFE}) display a strong positive correlation ($r = 0.45$, $t = 11.4$), while WR_{TWFE} is also negatively correlated with inflation ($r = -0.67$, $t = -20.4$), suggesting that real wage adjustment mechanisms remain closely linked even at the idiosyncratic level. Output (GDP_{TWFE}) is positively correlated with wages and wage rigidity ($r \approx 0.21$–0.35, all statistically significant), indicating that country-time-specific output shocks tend to be accompanied by labor-market adjustments.

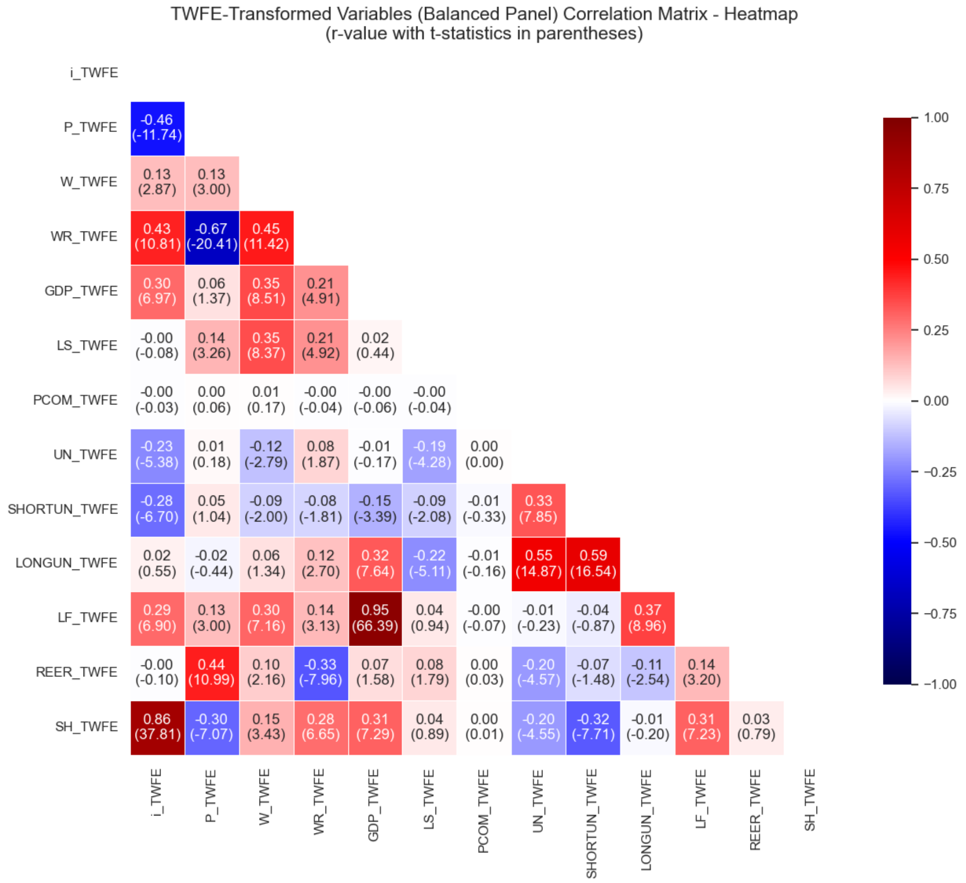
At the same time, the matrix reveals clusters of variables with high internal correlation, particularly among labour-market indicators. Short-run and long-run unemployment measures are strongly correlated with each other ($r = 0.55$, $t = 14.9$) and with labor force participation ($r = 0.37$, $t = 9.0$), while they are negatively correlated with GDP and wages. These patterns raise a non-trivial risk of multicollinearity in specifications that include several closely related labour-market variables simultaneously. Although none of the pairwise correlations approach unity, the presence of multiple correlations in the 0.4–0.6 range implies that standard errors may be inflated and coefficient estimates sensitive to specification choices.

Table 1.11.1: Country-level correlation and implied slope between GDP_{TWFE} and i_{TWFE}

Country	corr(GDP_{TWFE}, i_{TWFE})	sd(GDP_{TWFE})	sd(i_{TWFE})	β (simple regression)
Portugal	8.308 011	4.156 0732.940 217		1.174 359
Italy	6.991 789	2.779 9101.495 409		1.299 747
Spain	6.780 832	1.659 1701.735 230		6.483 611
USA	6.526 550	3.049 3012.082 907		9.554 634
Sweden	6.478 413	3.775 6011.062 518		2.302 070
Germany	6.473 675	9.387 5441.784 203		3.406 110
Finland	5.972 395	4.202 7291.091 747		2.299 100
UK	2.608 675	3.073 2721.093 681		7.330 446
Japan	2.072 243	1.824 9411.845 100		2.049 602
Norway	1.826 480	4.054 5901.490 154		4.969 704
Australia	1.347 098	2.204 8881.958 923		1.516 242
France	−3.421 127	6.495 5216.596 705		−3.368 651
Canada	−4.670 800	1.719 2631.129 697		−7.108 397
Belgium	−6.777 255	3.786 4078.441 164		−3.040 037
Netherlands	−7.553 481	3.105 2211.502 573		−1.561 004

Country-level evidence reinforces this concern. Table 1.11.1 (we only display *GDP* to *i* for simplicity) shows that the correlation between GDP_{TWFE} and i_{TWFE} varies substantially across countries, both in sign and magnitude, translating into widely dispersed implied slopes from simple regressions. Countries such as Portugal, Italy, Spain, the USA, and Germany display large positive correlations and steep implied slopes, whereas France, Canada, Belgium, and the Netherlands exhibit negative correlations. This heterogeneity implies that pooled correlations partly mask offsetting country-specific relationships, which can further exacerbate multicollinearity when common coefficients are imposed across units.

Figure 1.11.2 – TWFE-Transformed Variables (Balanced Panel) Correlation Matrix (r-value with t-statistics in parentheses)



1.12 UNBALANCED PANEL and TWFE transformation (remove countries with a single observation). Regress within transformed GDPG on time dummies and collect the residuals: this is the TWFE transformation. Regress within transformed EDA/GDP on time dummies and collect the residuals: this is the TWFE transformation. Alternatively, code the Wansbeek Kapstein (1989) transformation for two way fixed effects resulting in their equation 2.13 which is an extension of x(it)-x(i.)-x(.t)+x(..) obtained in the balanced panel case.

In a balanced panel with N units observed over the same T periods, the two-way fixed-effects (TWFE) transformation admits a closed-form demeaning representation. For any variable x_{it}, the TWFE-transformed series can be written as

$$\tilde{x}_{it} = x_{it} - \bar{x}_{i\cdot} - \bar{x}_{\cdot t} + \bar{x}_{\cdot\cdot},$$

where $\bar{x}_{i\cdot}$, $\bar{x}_{\cdot t}$, and $\bar{x}_{\cdot\cdot}$ denote unit, time, and grand means computed over a common support. This identity relies critically on balance: each unit contributes the same number of observations to each time period, and vice versa. Under balance, subtracting one set of means does not reintroduce components of the other, and the transformation corresponds exactly to the orthogonal projection onto the complement of the space spanned by unit and time dummies.

In an unbalanced panel, this decomposition no longer holds. Units may enter or exit the sample at different dates, and the number of observations contributing to each $\bar{x}_{\cdot t}$ and $\bar{x}_{i\cdot}$ varies across

t and i. As a result, the naive balanced-panel transformation

$$x_{it} - \bar{x}_{i\cdot} - \bar{x}_{\cdot t} + \bar{x}_{\cdot\cdot}$$

fails to fully eliminate both fixed effects: removing unit means reintroduces time effects, and removing time means reintroduces unit effects. Intuitively, the unit and time dummy subspaces are no longer orthogonal when the panel is unbalanced. A simple example illustrates the issue: if some countries are observed only during "good" aggregate periods and others only during "bad" periods, the unit averages mechanically absorb part of the time effects, and vice versa.

To obtain a valid TWFE transformation in the unbalanced case, one must instead compute the true projection of x_{it} onto the orthogonal complement of the space spanned by unit and time dummies. Formally, letting D_i and D_t denote the matrices of unit and time indicators, the TWFE residual is:

$$\text{x\_res} = (I - P_{[D_i, D_t]})x,$$

where $P_{[D_i, D_t]}$ is the projection matrix onto the column space of $[D_i, D_t]$. Direct computation of this projection is infeasible in large panels, but equivalent results can be obtained by two practical procedures. The first approach estimates the TWFE transformation via regression residualization. One first removes unit means,

$$x_{it}^{W} = x_{it} - \bar{x}_{i\cdot},$$

and then regresses x_{it}^{W} on the full set of time dummies (without an intercept). The resulting residuals are orthogonal to both unit and time effects and therefore coincide with the TWFE-transformed variable. This method is numerically exact and makes explicit the role of time effects in the unbalanced setting.

As alternative, we have also tried Wansbeek and Kapteyn (1989), which relies on alternating projections (iterative demeaning). Starting from x_{it} (optionally centered by the grand mean), one repeatedly subtracts unit means and time means computed on the current residual:

$$x_{it}^{(k+1)} = x_{it}^{(k)} - \overline{x^{(k)}}_{i\cdot} - \overline{x^{(k)}}_{\cdot t},$$

where the means are taken over the available observations at each iteration. As $k \to \infty$, this procedure converges to the unique vector that is orthogonal to both the unit and time dummy spaces. In practice, a sufficiently large number of iterations yields numerical convergence, and the resulting series has zero means within units, within time periods, and overall. Both TWFE (unbalanced panel) and Wansbeek and Kapteyn (1989) procedures produce identical results (see code) and recover the correct TWFE transformation in an unbalanced panel.

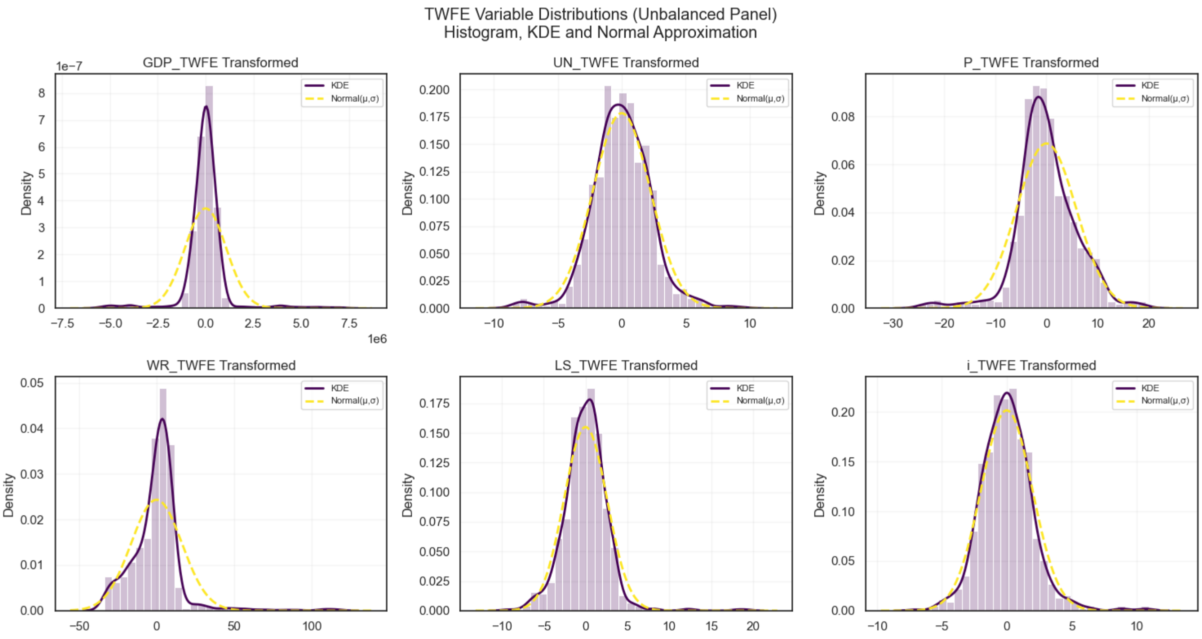
1.13 TWFE unbalanced panel: plot the distribution (Kernel DE, histogram, corresponding normal law with the same two first moments). for dependent GDPG and explanatory EDA/GDP. Compare with one-way-fixed effects, between distributions.

The distributions of the TWFE-transformed variables in the unbalanced panel exhibit several common features that are consistent with the theoretical properties of the projection-based transformation, while also revealing important deviations from Gaussianity. Across all variables, the transformed series are tightly centred around zero, confirming that both unit-specific and time-specific components have been effectively removed despite the lack of balance in the panel.

At the same time, the shapes of the distributions indicate that eliminating fixed effects does not render the residual variation approximately normal. For GDP_{TWFE}, the distribution is sharply peaked with pronounced tail mass, reflecting a combination of high kurtosis and occasional large idiosyncratic output shocks that are not shared across countries or periods. Unemployment and interest rates exhibit distributions that are closer to symmetry but remain leptokurtic, with the kernel density exceeding the normal approximation in the centre and falling off more slowly in the tails. Inflation and real wage display more visible asymmetry and heavier tails, suggesting that country-time-specific policy or institutional shocks generate skewed residual behaviour even after controlling for both dimensions of fixed effects. By contrast, labour share appears comparatively closer to a normal shape, although tail deviations remain evident.

Relative to the balanced-panel TWFE results, these distributions are generally more dispersed and exhibit slightly thicker tails, which is consistent with the presence of heterogeneous observation windows and uneven data coverage across countries and years, like we observed in Chapter 1.1. In an unbalanced setting, idiosyncratic shocks are not averaged out symmetrically across units and time, so residual variation tends to retain more extreme observations.

Figure 1.13.1 – TWFE-Transformed Variable Distributions (Unbalanced Panel) Histogram, KDE and Normal Approximation



1.14 Plot boxplots of between distribution (all countries), then one-way and two-way-fixed effects and first differences distribution BY countries (or 20 individuals if your data set has more than 20 individuals), for the dependent variable and the key explanatory variables. Comment that you find the same insights from question 5. Comment on their differences of standard errors and means for each individuals.

At the aggregate level (Table 1.14.1), all TWFE-transformed variables have means that are effectively zero, as implied by the TWFE construction, but they retain substantial dispersion and non-Gaussian features, as mentioned earlier. Several variables exhibit pronounced excess kurtosis and non-negligible skewness, confirming that residual variation is characterized by fat tails and asymmetric shocks rather than by smooth Gaussian noise. GDP_{TWFE} stands out with extremely large dispersion and wide support, reflecting infrequent but very large country-time-specific output shocks. Labor-market variables, such as unemployment and its short-and long run components, also display elevated kurtosis, indicating clustering of small deviations punctuated by occasional large adjustments.

Table 1.14.1: Descriptive statistics for unbalanced TWFE-transformed variables

Variable **Kurtosis**	**Count**	**Mean**	**Std**	**Min**	**25%**	**50%**	**75%**	**Max**	**Skewness**
i_{TWFE} 3.259	$7.500\,000 \times 10^{2}$	0.000 000	1.977 000	$-7.055\,000$	$-1.286\,000$	$-4.900\,000 \times 10^{-2}$	1.204 000	$1.093\,600 \times 10^{1}$	$6.980\,000 \times 10^{-1}$
P_{TWFE} 2.655	$7.500\,000 \times 10^{2}$	0.000 000	5.803 000	$-2.496\,900 \times 10^{1}$	$-3.140\,000$	$-4.710\,000 \times 10^{-1}$	3.259 000	$1.929\,800 \times 10^{1}$	$-3.350\,000 \times 10^{-1}$
W_{TWFE} 2.800	$7.400\,000 \times 10^{2}$	0.000 000	3.041 000	$-1.418\,700 \times 10^{1}$	$-1.531\,000$	$1.620\,000 \times 10^{-1}$	1.454 000	$1.002\,700 \times 10^{1}$	$-2.860\,000 \times 10^{-1}$
WR_{TWFE} 13.971	$7.400\,000 \times 10^{2}$	0.000 000	$1.639\,700 \times 10^{1}$	$-3.377\,500 \times 10^{1}$	$-6.878\,000$	1.381 000	6.486 000	$1.170\,120 \times 10^{2}$	2.244 000
GDP_{TWFE} 15.773	$7.500\,000 \times 10^{2}$	0.000 000	$1.075\,000 \times 10^{6}$	$-5.510\,737 \times 10^{6}$	$-2.987\,030 \times 10^{5}$	$5.712\,750 \times 10^{5}$	$3.049\,210 \times 10^{5}$	$7.128\,000 \times 10^{6}$	$6.540\,000 \times 10^{-1}$
LS_{TWFE} 8.486	$7.400\,000 \times 10^{2}$	0.000 000	2.576 000	$-9.440\,000$	$-1.390\,000$	$4.400\,000 \times 10^{-2}$	1.306 000	$1.920\,800 \times 10^{1}$	1.044 000
$PCOM_{\text{TWFE}}$ 0.000	$7.500\,000 \times 10^{2}$	0.000 000	0.000 000	0.000 000	0.000 000	0.000 000	0.000 000	0.000 000	0.000 000
UN_{TWFE} 1.796	$7.410\,000 \times 10^{2}$	0.000 000	2.233 000	$-8.560\,000$	$-1.330\,000$	$-1.000\,000 \times 10^{-2}$	1.444 000	9.301 000	$-2.700\,000 \times 10^{-2}$
$SHORTUN_{\text{TWFE}}$ 20.482	$5.800\,000 \times 10^{2}$	0.000 000	$3.325\,920 \times 10^{2}$	$-1.436\,127 \times 10^{3}$	$-1.157\,430 \times 10^{2}$	$-1.147\,700 \times 10^{1}$	$7.912\,900 \times 10^{1}$	$3.055\,193 \times 10^{3}$	2.458 000
$LONGUN_{\text{TWFE}}$ 22.564	$5.800\,000 \times 10^{2}$	0.000 000	$4.862\,560 \times 10^{2}$	$-1.297\,481 \times 10^{3}$	$-2.306\,390 \times 10^{2}$	$1.420\,000 \times 10^{-1}$	$1.550\,820 \times 10^{2}$	$4.340\,168 \times 10^{3}$	3.068 000
LF_{TWFE} 16.341	$7.410\,000 \times 10^{2}$	0.000 000	$5.748\,111 \times 10^{3}$	$-3.924\,131 \times 10^{3}$	$-1.713\,339 \times 10^{3}$	$-3.307\,890 \times 10^{2}$	$2.030\,858 \times 10^{3}$	$2.878\,597 \times 10^{4}$	$-1.011\,000$
$REER_{\text{TWFE}}$ 2.458	$7.500\,000 \times 10^{2}$	0.000 000	$1.035\,800 \times 10^{1}$	$-4.462\,100 \times 10^{1}$	$-5.504\,000$	$-4.690\,000 \times 10^{-1}$	5.468 000	$4.715\,700 \times 10^{1}$	$2.960\,000 \times 10^{-1}$
SH_{TWFE} 2.147	$7.500\,000 \times 10^{2}$	0.000 000	2.183 000	$-6.921\,000$	$-1.388\,000$	$-1.030\,000 \times 10^{-1}$	1.318 000	$1.107\,000 \times 10^{1}$	$5.710\,000 \times 10^{-1}$

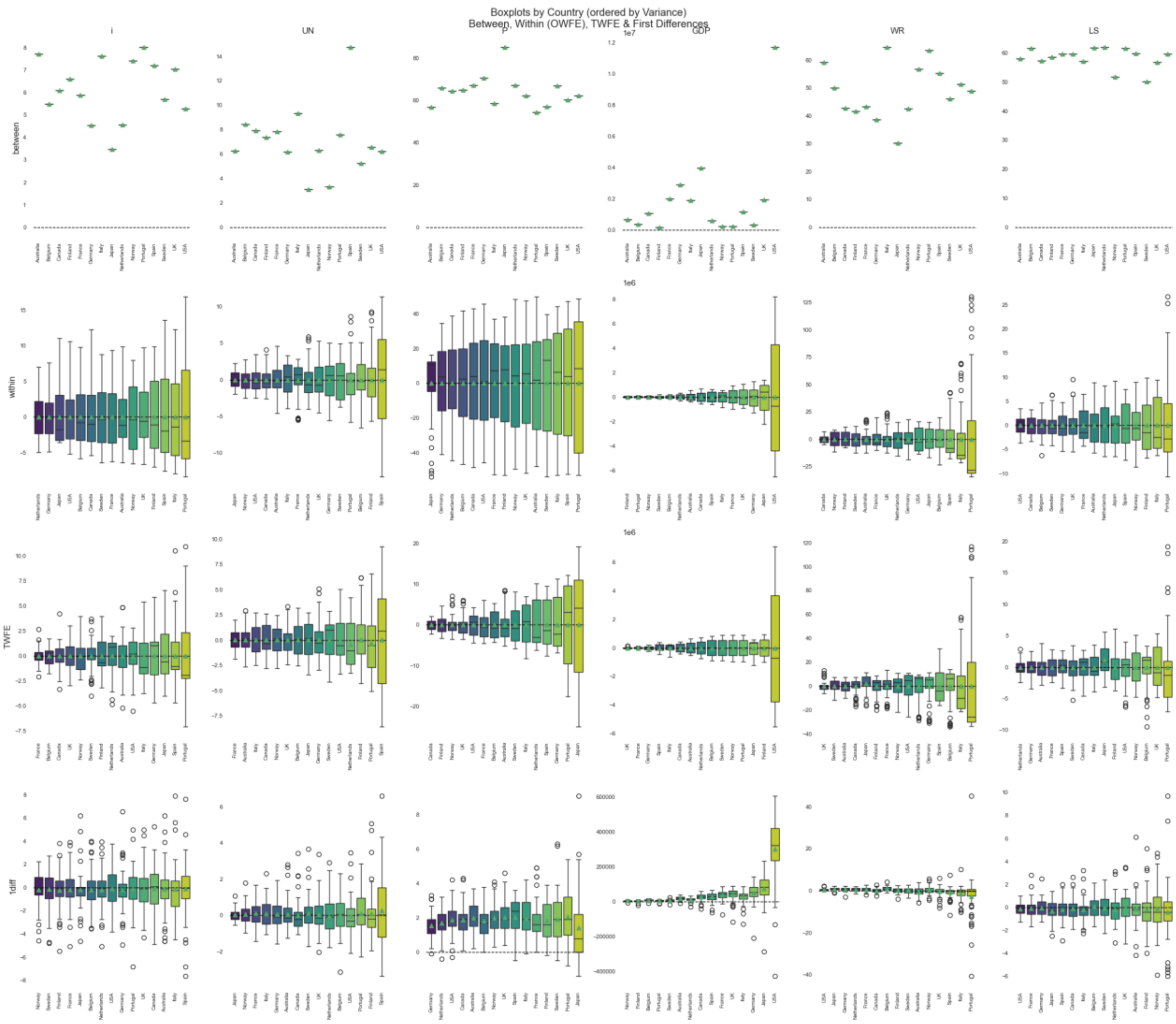
Country-level descriptive statistics for interest rates, taken as example, in the unbalanced panel (Table 1.14.2) reveal that residual heterogeneity is unevenly distributed across countries. While all country-specific distributions are centred around zero, the standard deviations and tail behaviour vary markedly. Countries such as Portugal, Spain, and Italy exhibit relatively large dispersion and higher maxima, indicating more frequent or more intense idiosyncratic interest-rate movements. In contrast, Belgium, Canada, and Sweden display tighter distributions with smaller interquartile ranges, suggesting that their interest-rate dynamics are more closely aligned with common time effects and country averages. Skewness also differs across countries, with some exhibiting right-skewed distributions and others left-skewed, reinforcing the view that residual shocks are not symmetric and may reflect country-specific policy regimes or institutional responses.

Table 1.14.2: Country-level descriptive statistics for i_{TWFE} (unbalanced panel)

Country Kurtosis	Count	Mean	Std	Min	25%	50%	75%	Max	Skewness
Australia 1.038642	5.000000×10^{1}	0.000000	1.909759	−5.199414	−1.143332	-1.173500×10^{-2}	1.043389	4.835133	6.322500×10^{-2}
Belgium -0.636629	5.000000×10^{1}	0.000000	9.260690×10^{-1}	−1.787984	-7.786450×10^{-1}	2.892150×10^{-1}	5.327800×10^{-1}	1.700148	-3.263590×10^{-1}
Canada 2.297731	5.000000×10^{1}	0.000000	1.240841	−3.360702	-6.452040×10^{-1}	1.651000×10^{-2}	7.291140×10^{-1}	4.180855	1.138070×10^{-1}
Finland -0.302654	5.000000×10^{1}	0.000000	1.596984	−3.164852	−1.008323	-6.653080×10^{-1}	1.373968	3.953551	6.867290×10^{-1}
France 1.474061	5.000000×10^{1}	0.000000	8.518060×10^{-1}	−2.080575	-4.403740×10^{-1}	-3.919300×10^{-2}	3.463340×10^{-1}	2.657788	4.817030×10^{-1}
Germany -0.188091	5.000000×10^{1}	0.000000	2.335942	−4.680182	−1.868422	1.007186	1.394961	5.866291	-4.685890×10^{-1}
Italy -0.337753	5.000000×10^{1}	0.000000	2.255071	−3.759903	−1.768270	−1.146016	1.273778	5.381478	8.135700×10^{-1}
Japan -0.280438	5.000000×10^{1}	0.000000	2.346864	−3.985336	−1.775115	-5.942070×10^{-1}	2.172163	6.565558	4.867370×10^{-1}
Netherlands 0.288153	5.000000×10^{1}	0.000000	1.791653	−4.832519	-9.640320×10^{-1}	8.519920×10^{-1}	1.292136	1.961079	−1.164245
Norway -0.114577	5.000000×10^{1}	0.000000	1.500931	−2.384427	−1.344180	-7.208600×10^{-2}	7.710010×10^{-1}	3.931607	5.886290×10^{-1}
Portugal 0.868960	5.000000×10^{1}	0.000000	3.737629	−7.055224	−2.240093	−1.905783	2.346999	1.093637×10^{1}	9.958860×10^{-1}
Spain 4.513446	5.000000×10^{1}	0.000000	2.641468	−4.673007	−1.380873	−1.027680	1.412659	1.049538×10^{1}	1.639844
Sweden 0.953096	5.000000×10^{1}	0.000000	1.509666	−4.005082	-3.669330×10^{-1}	5.699000×10^{-2}	7.806420×10^{-1}	3.696758	-2.259820×10^{-1}
UK -0.343503	5.000000×10^{1}	0.000000	1.423362	−2.985545	-9.404970×10^{-1}	-1.897100×10^{-1}	9.336990×10^{-1}	3.010378	1.675240×10^{-1}
USA 0.251108	5.000000×10^{1}	0.000000	1.919939	−5.533214	-8.211500×10^{-1}	2.041200×10^{-1}	1.396461	2.809602	-6.641400×10^{-1}

Comparing these results with the balanced TWFE case, the unbalanced-panel distributions tend to display slightly greater dispersion and heavier tails compared to the balanced one, which is consistent with uneven observation windows and heterogeneous sample coverage across time. Thus, we could state that the TWFE transformation successfully removes systematic country and time components, but meaningful idiosyncratic variation remains, concentrated in a subset of countries and variables.

Figure 1.14.3 – Boxplots by Country (ordered by Variance)
Between, Within (OWFE), TWFE & First Differences



From the boxplot comparison across estimators (Figure 1.14.3) , a first salient result is the sharp contrast between the between component and all other transformations, where for every variable, between-country dispersion is large and highly heterogeneous, with countries ordered at the top exhibiting substantially wider spreads and more extreme values. This confirms that long-run cross-sectional differences dominate raw variability and that a small subset of countries accounts for a disproportionate share of the total variance (USA, Japan, Germany). These differences are particularly pronounced for GDP, real wage (WR), and labour share (LS), where the between distributions show both wide interquartile ranges and numerous high-leverage observations.

Moving to the within (OWFE) transformation, dispersion is markedly reduced, but cross-country heterogeneity remains clearly visible, with countries still differ substantially in the variability of their within deviations, indicating that removing only unit fixed effects does not homogenize short-run dynamics. In particular, for unemployment (UN) and interest rates (i), several countries continue to exhibit wider distributions and heavier tails, suggesting persistent differences in cyclical volatility or policy responsiveness even after controlling for time-invariant heterogeneity.

The TWFE transformation further compresses the distributions and brings most countries closer together, especially in terms of medians, which are tightly centred around zero by construction. However, the ordering by variance reveals that heterogeneity is not eliminated: some countries continue to display systematically larger residual dispersion across nearly all variables ("heavy I (s)"). This is especially visible for GDP and WR, where a few countries remain clear outliers in terms of residual variance, indicating repeated country-time-specific shocks that are not captured by additive unit and time effects (column 4, USA). The persistence of these patterns across variables suggests that these countries are structurally more volatile rather than affected by isolated outliers.

Ad last, first differences (1_diff) produce distributions that are generally wider and noisier than TWFE, with more pronounced tails and a larger number of extreme observations. While differencing removes low-frequency components, it amplifies high-frequency volatility and measurement error, leading to less uniform dispersion across countries. This is most evident for GDP and UN, where several countries exhibit very large interquartile ranges and extreme values relative to the rest of the panel.

Out of this comparison we can draft three main conclusions First, most cross-country heterogeneity resides in long-run (between) components. Second, two-way fixed effects substantially reduce, but do not eliminate, heterogeneity in volatility across countries. Third, a small subset of countries consistently ranks at the top of the variance distribution across transformations, indicating that idiosyncratic dynamics are unevenly distributed and structurally persistent (US in particular, due to size).

1.15 Compute univariate descriptive statistics (min, Q1, median, Q3, max, mean, standard error) for one-way-Within, Between, two-way-fixed-effects and first differences transformed variables. Is the mean different from the median and why? How many standard errors from the mean are the MIN and MAX extremes. Report in the tables standardized MAX and MIN: (MAX-average)/standard error and (MIN-average)/standard error instead of MAX and MIN?

From table 1.15.1 (split in 2 parts for better readability), we understand the systematic differences between means and medians across variables and transformations, as well as the presence of extreme observations far from the centre of the distributions, which can be quantified in units of standard errors.

This gap, across both variables and transformations, reflects asymmetry and tail behaviour rather than sampling noise. In the between transformation, the mean is often pulled away from the median by a small number of large cross-sectional observations. For example, for GDP_{between}, the median lies close to the lower quartiles of the distribution, while the mean is much larger in magnitude, indicating strong right skewness driven by a few very large country averages. In contrast, within and TWFE transformations tend to produce medians closer to zero, while the mean may still deviate slightly due to residual skewness and fat tails. In first differences, mean-median discrepancies are clearly present too.

Table 1.15.1: Descriptive statistics by variable and transformation (location measures)

Variable	Transformation	N	Min	Q1	Median	Q3	Max
i	between	$5.100\,000 \times 10^{2}$	3.481 000	5.289 000	6.112 000	7.426 000	8.031 000
i	within	$5.100\,000 \times 10^{2}$	−8.594 000	−3.530 000	$-6.660\,000 \times 10^{-1}$	3.495 000	$1.686\,200 \times 10^{1}$
i	TWFE	$5.100\,000 \times 10^{2}$	−7.055 000	−1.286 000	$-4.900\,000 \times 10^{-2}$	1.204 000	$1.093\,600 \times 10^{1}$
i	1diff	$5.100\,000 \times 10^{2}$	−7.635 000	$-9.870\,000 \times 10^{-1}$	$-8.600\,000 \times 10^{-2}$	$7.420\,000 \times 10^{-1}$	8.039 000
GDP	between	$5.100\,000 \times 10^{2}$	$1.674\,476 \times 10^{5}$	$3.220\,119 \times 10^{5}$	$1.047\,213 \times 10^{6}$	$1.993\,680 \times 10^{6}$	$1.171\,694 \times 10^{7}$
GDP	within	$5.100\,000 \times 10^{2}$	$-6.530\,101 \times 10^{6}$	$-1.994\,527 \times 10^{5}$	$-1.039\,782 \times 10^{5}$	$2.367\,219 \times 10^{5}$	$8.212\,038 \times 10^{6}$
GDP	TWFE	$5.100\,000 \times 10^{2}$	$-5.510\,737 \times 10^{6}$	$-2.987\,077 \times 10^{5}$	$5.712\,746 \times 10^{5}$	$3.049\,207 \times 10^{5}$	$7.128\,463 \times 10^{6}$
GDP	1diff	$5.100\,000 \times 10^{2}$	$-4.257\,771 \times 10^{5}$	$5.511\,046 \times 10^{4}$	$1.617\,783 \times 10^{5}$	$4.522\,734 \times 10^{5}$	$6.042\,576 \times 10^{5}$
P	between	$5.100\,000 \times 10^{2}$	$5.450\,400 \times 10^{1}$	$5.854\,600 \times 10^{1}$	$6.442\,000 \times 10^{1}$	$6.730\,400 \times 10^{1}$	$8.534\,600 \times 10^{1}$
P	within	$5.100\,000 \times 10^{2}$	$-5.384\,000 \times 10^{1}$	−2.894 000	5.421 000	$2.262\,300 \times 10^{1}$	$4.989\,200 \times 10^{1}$
P	TWFE	$5.100\,000 \times 10^{2}$	$-2.496\,900 \times 10^{1}$	−3.140 000	$-4.710\,000 \times 10^{-1}$	3.259 000	$1.929\,800 \times 10^{1}$
P	1diff	$5.100\,000 \times 10^{2}$	$-1.300\,000 \times 10^{1}$	1.100 000	1.800 000	2.500 000	9.100 000
WR	between	$5.100\,000 \times 10^{2}$	$3.016\,200 \times 10^{1}$	$4.268\,200 \times 10^{1}$	$4.894\,000 \times 10^{1}$	$5.678\,300 \times 10^{1}$	$6.467\,700 \times 10^{1}$
WR	within	$5.100\,000 \times 10^{2}$	$-3.377\,500 \times 10^{1}$	−6.878 000	1.381 000	6.090 000	$1.816\,900 \times 10^{1}$
WR	TWFE	$5.100\,000 \times 10^{2}$	$-3.377\,500 \times 10^{1}$	−6.878 000	1.381 000	6.486 000	$1.170\,120 \times 10^{2}$
WR	1diff	$5.100\,000 \times 10^{2}$	−8.059 000	$-5.720\,000 \times 10^{-1}$	$2.330\,000 \times 10^{-1}$	1.032 000	$1.476\,700 \times 10^{1}$

Table 1.15.1: Descriptive statistics by variable and transformation (dispersion measures)

Variable	Transformation	Mean	Std	Std. Error	Std. Min	Std. Max
i	between	6.187 000	1.309 000	$5.800\,000 \times 10^{-2}$	2.067 000	$1.414\,100 \times 10^{1}$
i	within	$3.790\,000 \times 10^{-1}$	4.567 000	$2.030\,000 \times 10^{-1}$	−1.799 000	5.614 000
i	TWFE	0.000 000	1.977 000	$8.600\,000 \times 10^{-2}$	−1.579 000	3.536 000
i	1diff	$-1.490\,000 \times 10^{-1}$	1.687 000	$7.500\,000 \times 10^{-2}$	−1.487 000	4.307 000
GDP	between	$1.941\,396 \times 10^{6}$	$2.824\,307 \times 10^{5}$	$1.250\,623 \times 10^{5}$	6.028 000	3.461 000
GDP	within	−3.974 000	$1.249\,641 \times 10^{6}$	$5.533\,505 \times 10^{4}$	−5.256 000	6.572 000
GDP	TWFE	0.000 000	$1.074\,982 \times 10^{6}$	$4.701\,010 \times 10^{3}$	−5.129 000	6.531 000
GDP	1diff	0.000 000	$9.383\,636 \times 10^{5}$	$4.155\,224 \times 10^{3}$	−4.995 000	5.986 000
P	between	$6.443\,700 \times 10^{1}$	7.179 000	$3.180\,000 \times 10^{-1}$	1.379 000	2.928 000
P	within	0.000 000	$2.865\,500 \times 10^{1}$	1.287 000	−1.403 000	3.823 000
P	TWFE	0.000 000	5.803 000	$2.570\,000 \times 10^{-1}$	−1.431 000	3.253 000
P	1diff	1.816 000	1.145 000	$5.100\,000 \times 10^{-2}$	−2.849 000	6.324 000
WR	between	$4.899\,000 \times 10^{1}$	9.313 000	$4.120\,000 \times 10^{-1}$	2.022 000	1.685 000
WR	within	1.197 000	$1.677\,900 \times 10^{1}$	$7.830\,000 \times 10^{-1}$	−1.946 000	5.364 000
WR	TWFE	0.000 000	$1.639\,700 \times 10^{1}$	$7.280\,000 \times 10^{-1}$	−1.942 000	7.643 000
WR	1diff	$-4.100\,000 \times 10^{-2}$	3.258 000	$1.480\,000 \times 10^{-1}$	−2.101 000	$1.246\,200 \times 10^{1}$

The table also report standardized extremes ("Std. Min" and "Std. Max"), which measure how far the minimum and maximum observations lie from the mean in units of dispersion. Formally, for a given variable x,

$$\text{Std. Min} = \frac{\min(x) - \bar{x}}{s}, \qquad \text{Std. Max} = \frac{\max(x) - \bar{x}}{s},$$

where $\bar{x}$is the sample mean and $s$is the sample standard deviation. These metrics indicate how many standard deviations (and hence, approximately, how many standard errors for individual observations) separate the extremes from the centre of the distribution, similar to what we observed in Chapter 1.1 with deviations from the mean.

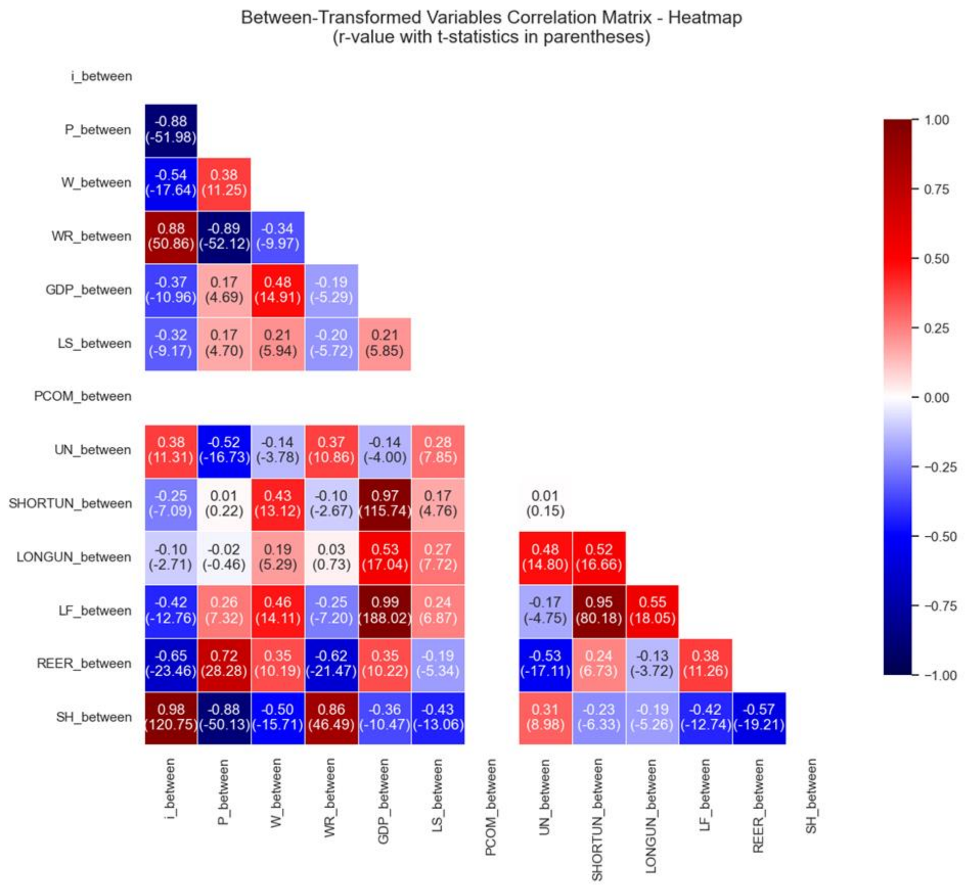
In the between dimension, GDP and WR exhibit particularly large standardized maxima, consistent with the presence of high-leverage cross-sectional observations. Even after within or TWFE transformations, extremes often remain beyond ±3 standard deviations, which would be extremely unlikely under a Gaussian distribution. With regards to First-difference transformations, we also note large standardized extremes, reflecting the amplification of short-run shocks and measurement noise.

Thus, the core concept behind is that the divergence between mean and median and the large standardized distances of the minimum and maximum indicate that the distributions are non-normal, skewed, and heavy-tailed, and the mean is therefore sensitive to a small number of extreme observations, while the median better captures the typical realization. The fact that extrema lie many standard deviations away from the mean reinforces earlier evidence that neither within, TWFE, nor first-difference transformations eliminate tail risk.

1.16 Compare and comment the between versus one-way-within transformed bivariate correlation matrix for all variables (include a time trend 1,2,.,T) and with their lag (for time varying variables). Check poor simple correlation with the dependent variables and high correlation between explanatory variables.

The contrast between the between-transformed and one-way within OWFE-transformed correlation matrices is quantitatively harsh and reinforces the different roles played by long-run versus short-run variation. In the between dimension, simple correlations are large and highly significant. For example, GDP_{between} is strongly correlated with real wages ($r \approx 0.48$, $t \approx 14.9$), unemployment ($r \approx -0.14$, $t \approx -4.0$), labor force participation ($r \approx 0.35$, $t \approx 10.2$), and interest rates ($r \approx -0.37$, $t \approx -11.0$). At the same time, explanatory variables display very high pairwise correlations among themselves: short- and long-term unemployment are correlated above 0.95 ($t > 80$), labor force participation and unemployment exceed correlations of 0.9 in absolute value, and wage-related variables (W, WR, LS) frequently exhibit correlations in the 0.4-0.7 range. These magnitudes are pivotal as they highlight potential severe multicollinearity in the between regressions, where regressors capture overlapping long-run institutional and structural features and are therefore difficult to disentangle econometrically. This is the reason why, for future implementations we will not resort on variables belonging to the same metrics group, following the 5 model specifications proposed by Lofaro and Di Bucchianico (2025).

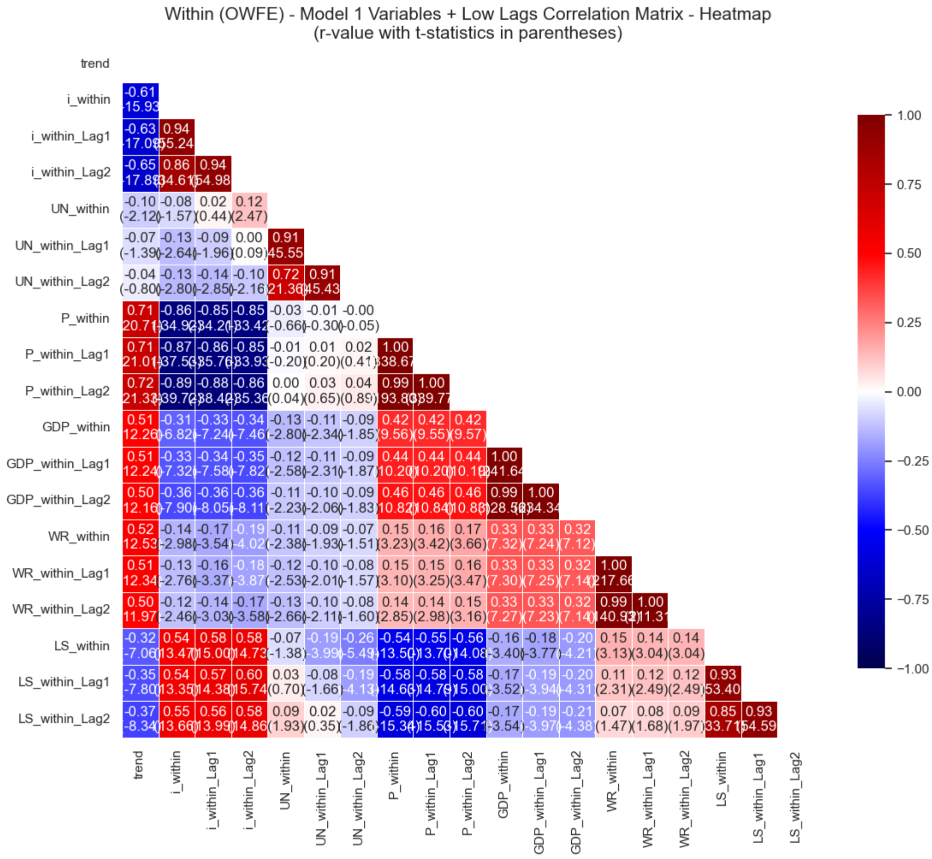
Figure 1.16.1 – Between-Transformed Variables (Balanced Panel) Correlation Matrix (r-value with t-statistics in parentheses)

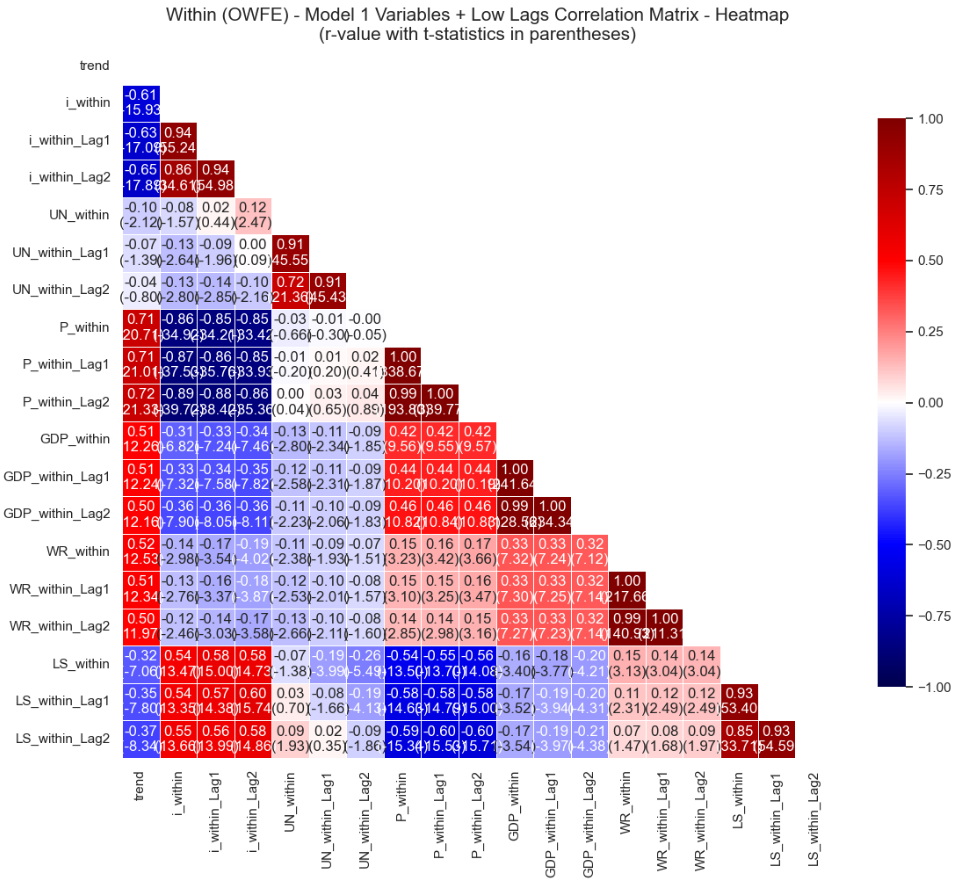


In contrast, the within-OWFE correlations are much weaker for contemporaneous relationships with the dependent variable. GDP_{within} typically exhibits correlations below 0.2 in absolute value with most explanatory variables, and several are statistically insignificant. For instance, the contemporaneous correlation between GDP_{within} and unemployment deviations is close to zero, while correlations with inflation and interest rates are modest (around 0.3-0.4 in absolute value). This illustrates that once country means are removed, simple short run comovement with GDP is limited, and much of the strong association observed in the between dimension disappears. The fact that we included a deterministic time trend further shows that many within variables are correlated with the trend itself (often exceeding $\mid r \mid = 0.5$), confirming that common low-frequency movements are absorbed by time effects rather than reflected in direct bivariate links with GDP.

With regards to autocorrelation for within-transformed variables, correlations across low-order lags are extremely high: first and second lags of the same variable commonly exceed 0.9 (e.g., $r(\text{GDP}_t, \text{GDP}_{t-1}) \approx 0.95$, $r(\text{GDP}_{t-1}, \text{GDP}_{t-2}) \approx 0.94$), reflecting strong persistence in deviations from country means. To capture short-run dynamics without inducing excessive redundancy, the analysis therefore focuses on lag 1 and lag 2 for low-frequency specifications. For longer-horizon dynamics, where adjacent lags add little new information, higher-order lags (lag 5 and lag 10) are used instead, separating between short-run adjustments and medium-run responses while limiting the dimensionality of the regressor set.

Figure 1.16.2 – Within (OWFE) - Model 1 Variables Low+High Lags Correlation Matrix (r-value with t-statistics in parentheses)




Within (OWFE) - Model 1 Variables + Low Lags Correlation Matrix - Heatmap
(r-value with t-statistics in parentheses)
trend
i_within -0.61 -15.93
i_within_Lag1 -0.63 0.94 -17.09 55.24
i_within_Lag2 -0.65 0.86 0.94 -17.89 34.61 54.98
UN_within -0.10 -0.08 0.02 0.12 (-2.12) (-1.57) (0.44) (2.47)
UN_within_Lag1 -0.07 -0.13 -0.09 0.00 0.91 (-1.39) (-2.64) (-1.96) (0.09) 45.55
UN_within_Lag2 -0.04 -0.13 -0.14 -0.10 0.72 0.91 (-0.80) (-2.80) (-2.85) (-2.16) 21.36 45.43
P_within 0.71 -0.86 -0.85 -0.85 -0.03 -0.01 -0.00 20.71 -34.92 -34.21 -33.42 (-0.66) (-0.30) (-0.05)
P_within_Lag1 0.71 -0.87 -0.86 -0.85 -0.01 0.01 0.02 1.00 21.01 -37.53 -35.76 -33.93 (-0.20) (0.20) (0.41) 338.67
P_within_Lag2 0.72 -0.89 -0.88 -0.86 0.00 0.03 0.04 0.99 1.00 21.33 -39.72 -38.42 -35.36 (0.04) (0.65) (0.89) 93.80 339.77
GDP_within 0.51 -0.31 -0.33 -0.34 -0.13 -0.11 -0.09 0.42 0.42 0.42 12.26 (-6.82) (-7.24) (-7.46) (-2.80) (-2.34) (-1.85) (9.56) (9.55) (9.57)
GDP_within_Lag1 0.51 -0.33 -0.34 -0.35 -0.12 -0.11 -0.09 0.44 0.44 0.44 1.00 12.24 (-7.32) (-7.58) (-7.82) (-2.58) (-2.31) (-1.87) 10.20 10.20 10.19 241.64
GDP_within_Lag2 0.50 -0.36 -0.36 -0.36 -0.11 -0.10 -0.09 0.46 0.46 0.46 0.99 1.00 12.16 (-7.90) (-8.05) (-8.11) (-2.23) (-2.06) (-1.83) 10.82 10.84 10.83 128.58 234.34
WR_within 0.52 -0.14 -0.17 -0.19 -0.11 -0.09 -0.07 0.15 0.16 0.17 0.33 0.33 0.32 12.53 (-2.98) (-3.54) (-4.02) (-2.38) (-1.93) (-1.51) (3.23) (3.42) (3.66) (7.32) (7.24) (7.12)
WR_within_Lag1 0.51 -0.13 -0.16 -0.18 -0.12 -0.10 -0.08 0.15 0.15 0.16 0.33 0.33 0.32 1.00 12.34 (-2.76) (-3.37) (-3.87) (-2.53) (-2.01) (-1.57) (3.10) (3.25) (3.47) (7.30) (7.25) (7.14) 217.66
WR_within_Lag2 0.50 -0.12 -0.14 -0.17 -0.13 -0.10 -0.08 0.14 0.14 0.15 0.33 0.33 0.32 0.99 1.00 11.97 (-2.46) (-3.03) (-3.58) (-2.66) (-2.11) (-1.60) (2.85) (2.98) (3.16) (7.27) (7.23) (7.14) 140.92 211.31
LS_within -0.32 0.54 0.58 0.58 -0.07 -0.19 -0.26 -0.54 -0.55 -0.56 -0.16 -0.18 -0.20 0.15 0.14 0.14 -7.06 13.47 15.00 14.73 (-1.38) -3.99 -5.49 -13.50 -13.70 -14.08 -3.40 -3.77 -4.21 (3.13) (3.04) (3.04)
LS_within_Lag1 -0.35 0.54 0.57 0.60 0.03 -0.08 -0.19 -0.58 -0.58 -0.58 -0.17 -0.19 -0.20 0.11 0.12 0.12 0.93 -7.80 13.35 14.38 15.74 (0.70) (-1.66) -4.13 -14.66 -14.79 -15.00 -3.52 -3.94 -4.31 (2.31) (2.49) (2.49) 53.40
LS_within_Lag2 -0.37 0.55 0.56 0.58 0.09 0.02 -0.09 -0.59 -0.60 -0.60 -0.17 -0.19 -0.21 0.07 0.08 0.09 0.85 0.93 -8.34 13.66 13.99 14.86 (1.93) (0.35) (-1.86) -15.34 -15.53 -15.71 -3.54 -3.97 -4.38 (1.47) (1.68) (1.97) 33.71 54.59
trend i_within i_within_Lag1 i_within_Lag2 UN_within UN_within_Lag1 UN_within_Lag2 P_within P_within_Lag1 P_within_Lag2 GDP_within GDP_within_Lag1 GDP_within_Lag2 WR_within WR_within_Lag1 WR_within_Lag2 LS_within LS_within_Lag1 LS_within_Lag2
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00

1.17 Comment the bivariate autocorrelation and trend-correlations (check the number of observations).

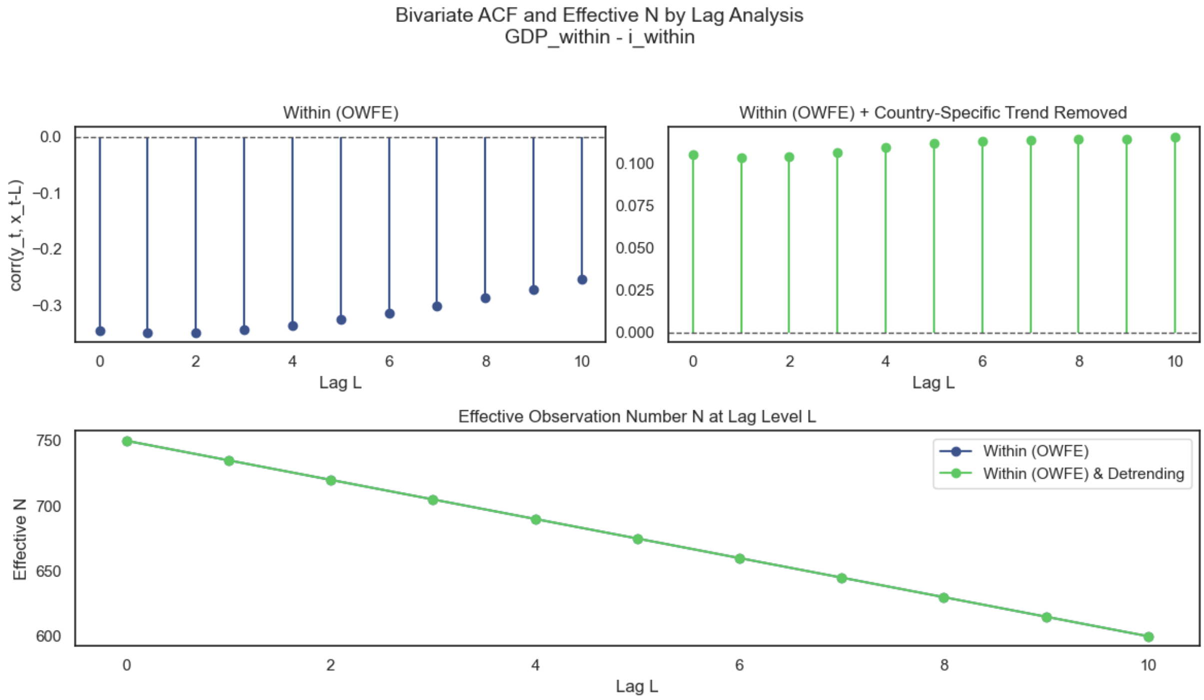
For plotting Figure 1.17.1-1.17.4, we computed the bivariate autocorrelation structure between GDP_{within} (assumed dependent) and each Model-1 target variable, together with the evolution of the effective sample size as the lag length increases.

First, in the within (OWFE) representation without detrending, bivariate correlations are economically non-negligible and highly persistent across lags. For GDP-interest rates, contemporaneous correlations are clearly negative (around -0.30) and remain sizable up to lag 10, indicating sustained inverse comovement between output deviations and monetary conditions. GDP-unemployment correlations are positive and increase with the lag length, reaching values around 0.10 at medium horizons, consistent with delayed labour-market adjustment. GDP-inflation correlations are particularly strong and persistent, remaining close to 0.45 across all lags, suggesting that common low-frequency movements dominate short-run dynamics in the within dimension. Similar persistence is visible for GDP with real wage and labour share, where correlations decay only slowly with the lag order. Importantly, the GDP-GDP panel confirms extremely strong serial dependence: autocorrelations remain above 0.8 even at lag 10, indicating that within-country output deviations are highly persistent.

Second, once country-specific linear trends are removed, the bivariate correlations change markedly in both sign and magnitude. For most variable pairs, correlations drop sharply and often switch sign, settling at relatively small values (typically between -0.15 and 0.15) that are stable across lags. For example, the GDP-interest rate correlation becomes weakly positive and flat across horizons, while GDP-inflation and GDP-unemployment correlations turn modestly negative.

Third, the effective number of observations declines mechanically with the lag length as expectable, falling from roughly 740 at lag 0 to about 600 at lag 10. This reduction is identical across variable pairs and reflects the loss of overlapping observations as higher lags are introduced in an unbalanced panel. The gradual decline highlights the trade-off we have between higher-order lag analysis, reducing the available information, and increases sampling uncertainty, reinforcing the need to limit the number of lags used in empirical specifications.

Figure 1.17.1 – Bivariate ACF and Effective N by Lag Analysis GDP_within pegged dependent and Model 1 Regressors X_within (Part 1)



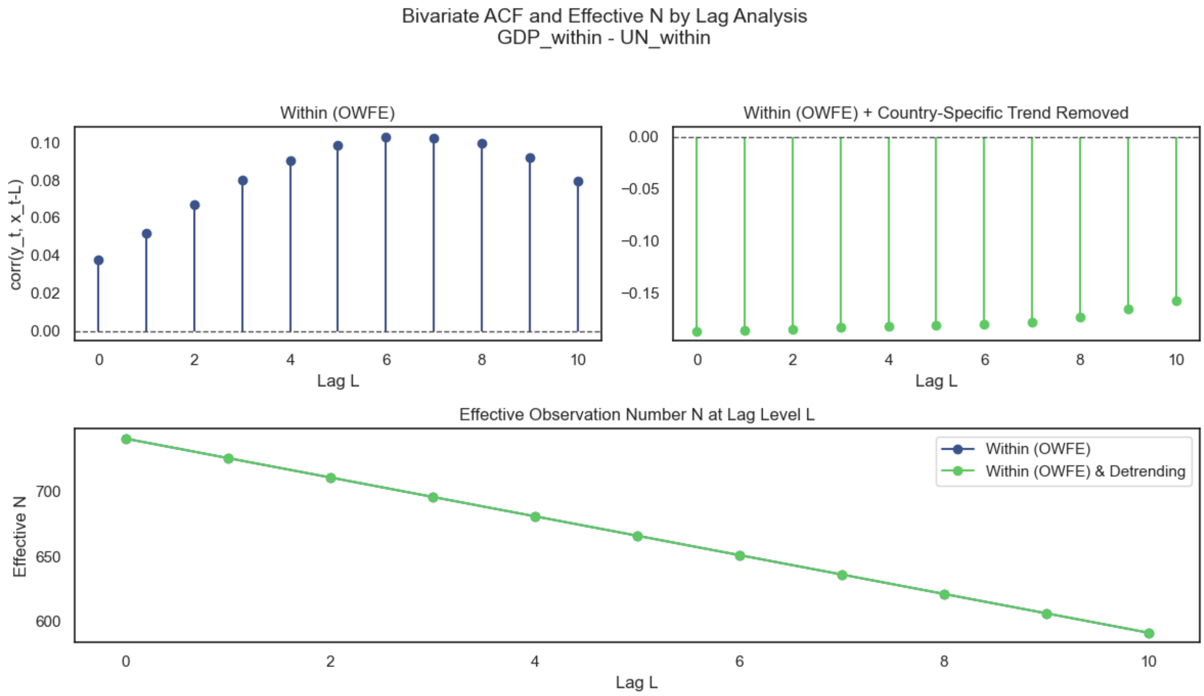
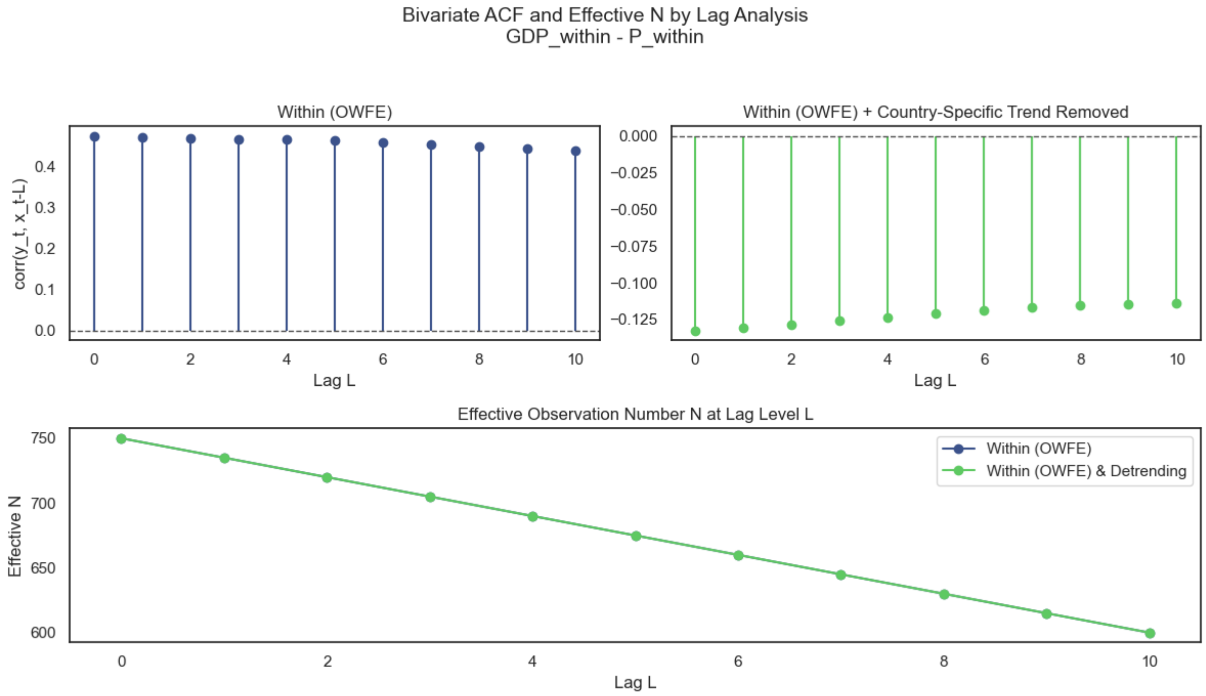


Figure 1.17.2 – Bivariate ACF and Effective N by Lag Analysis GDP_within pegged dependent and Model 1 Regressors X_within (Part 2)



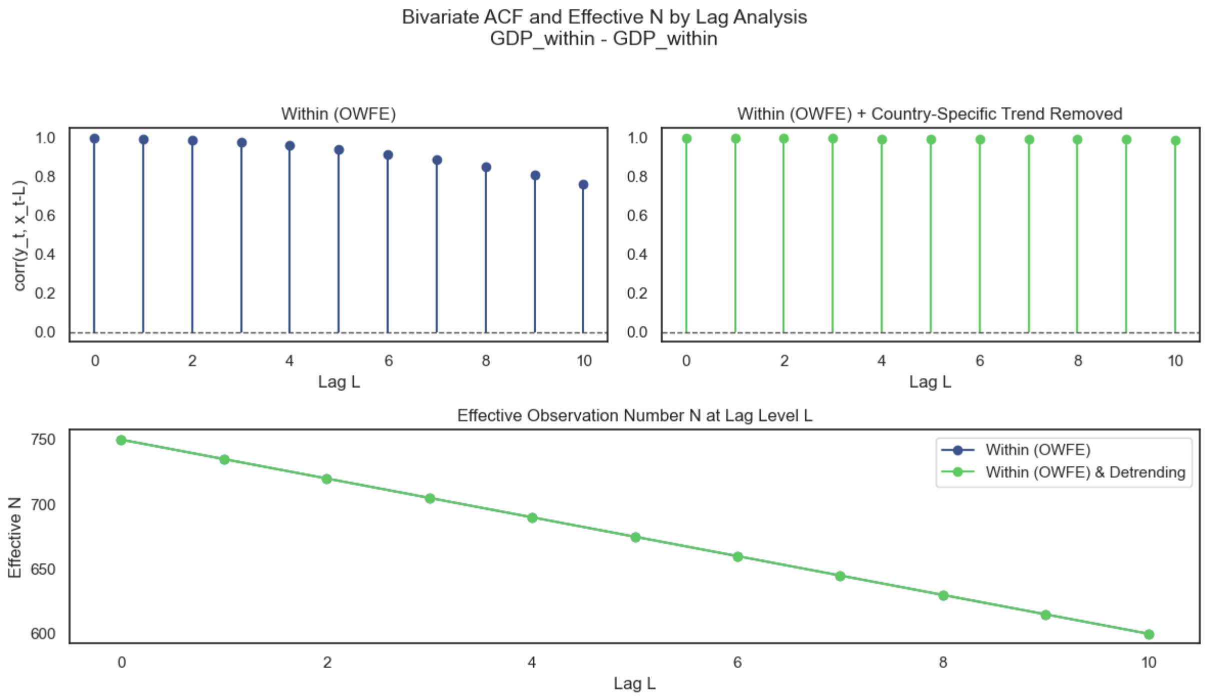
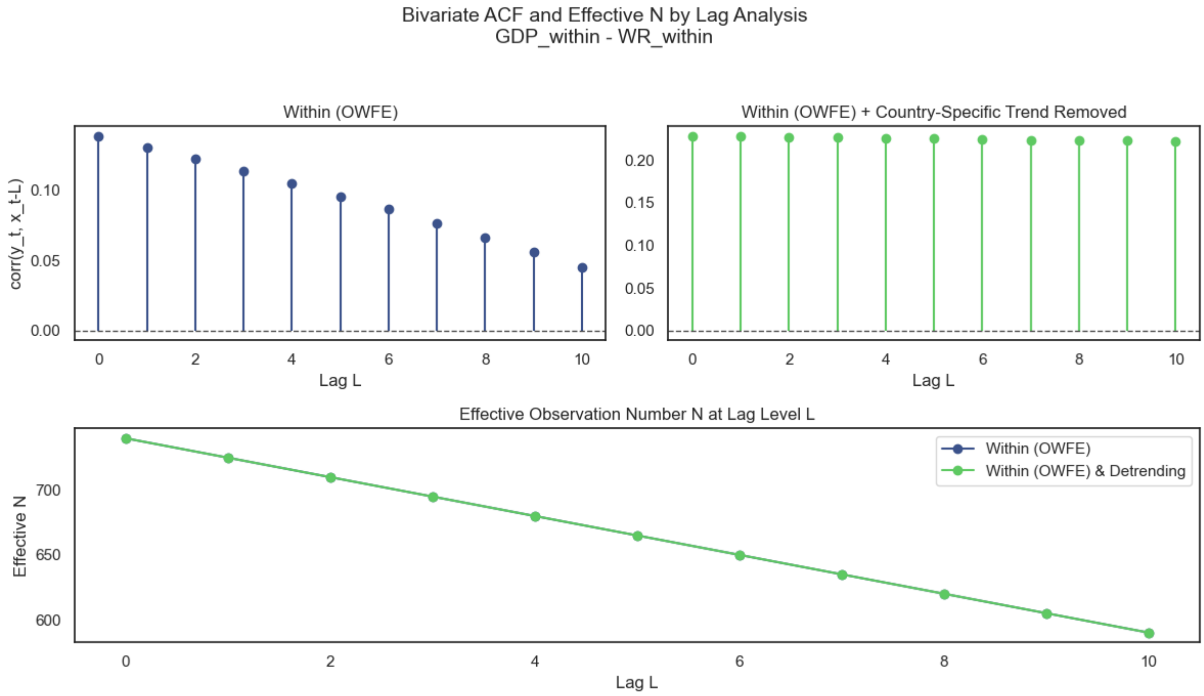
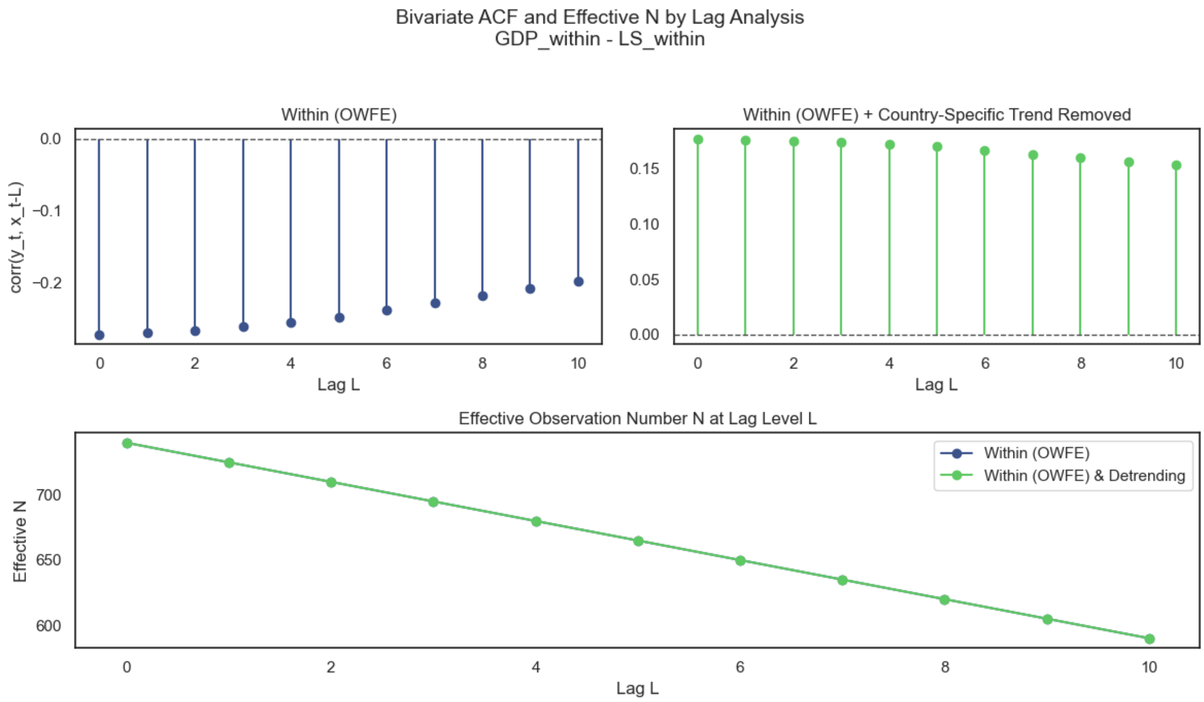


Figure 1.17.3 – Bivariate ACF and Effective N by Lag Analysis
GDP_within pegged dependent and Model 1 Regressors X_within (Part 3)

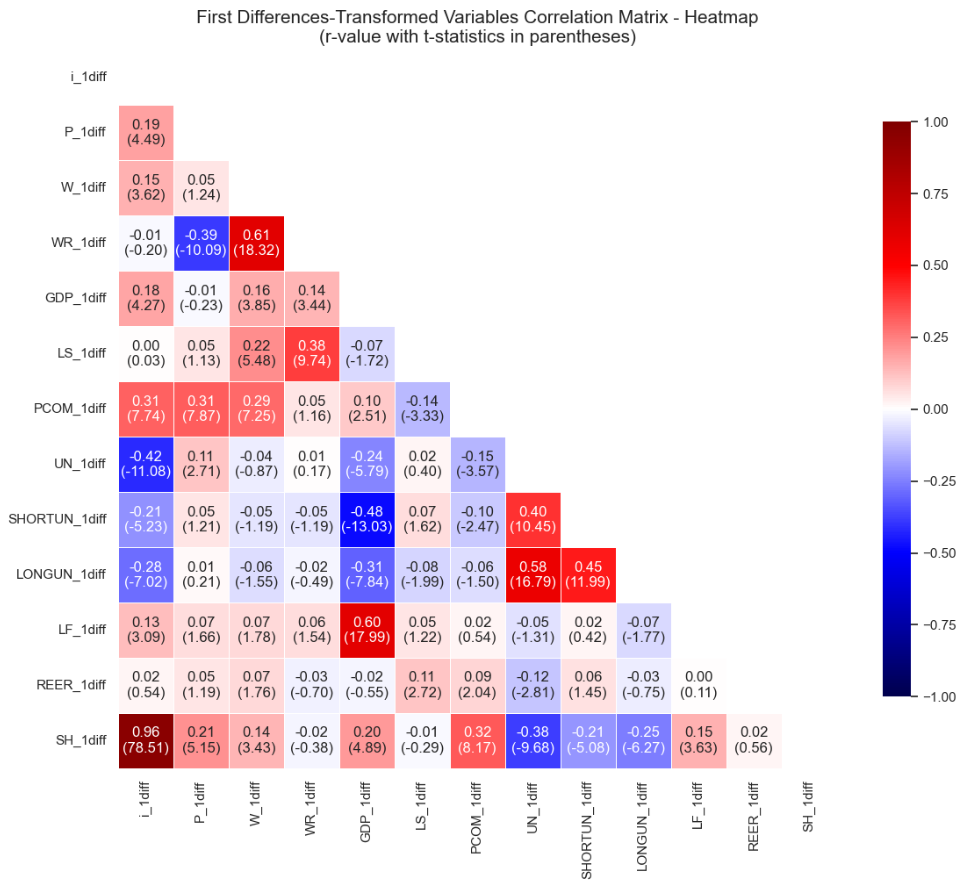




1.18 In what follows, you do not need to include a deterministic time trend 1,2,.,T because the two transformations used eliminate it. Compare and comment of the two-way-within transformed bivariate simple correlation matrix of all the variables and another bivariate simple correlation matrix with all the first differences transformed variables (in the case of first differences, include also the lag of all variables). Check poor simple correlation below 0.1 with the dependent variables and high correlation between explanatory variables (over 0.8). Show the first 30 observations for the first differences and the lag of first differences. Check that each time you change individual, you have a dot for missing observation.

The comparison between the two-way–within (TWFE) transformed bivariate correlation matrix and the corresponding first-differences correlation matrices tell us a lot about the sharp differences in both the strength and the structure of simple correlations, with direct implications for identification and multicollinearity.

Figure 1.18.1 – First Differences-Transformed Variables Correlation Matrix (r-value with t-statistics in parentheses)

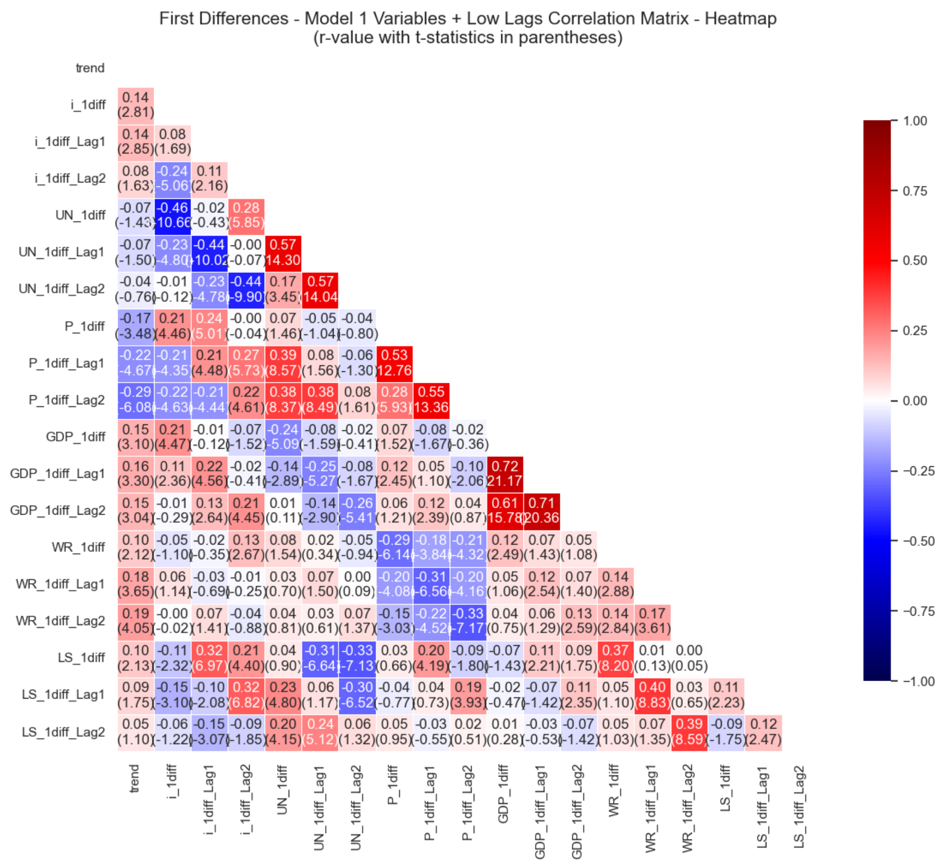


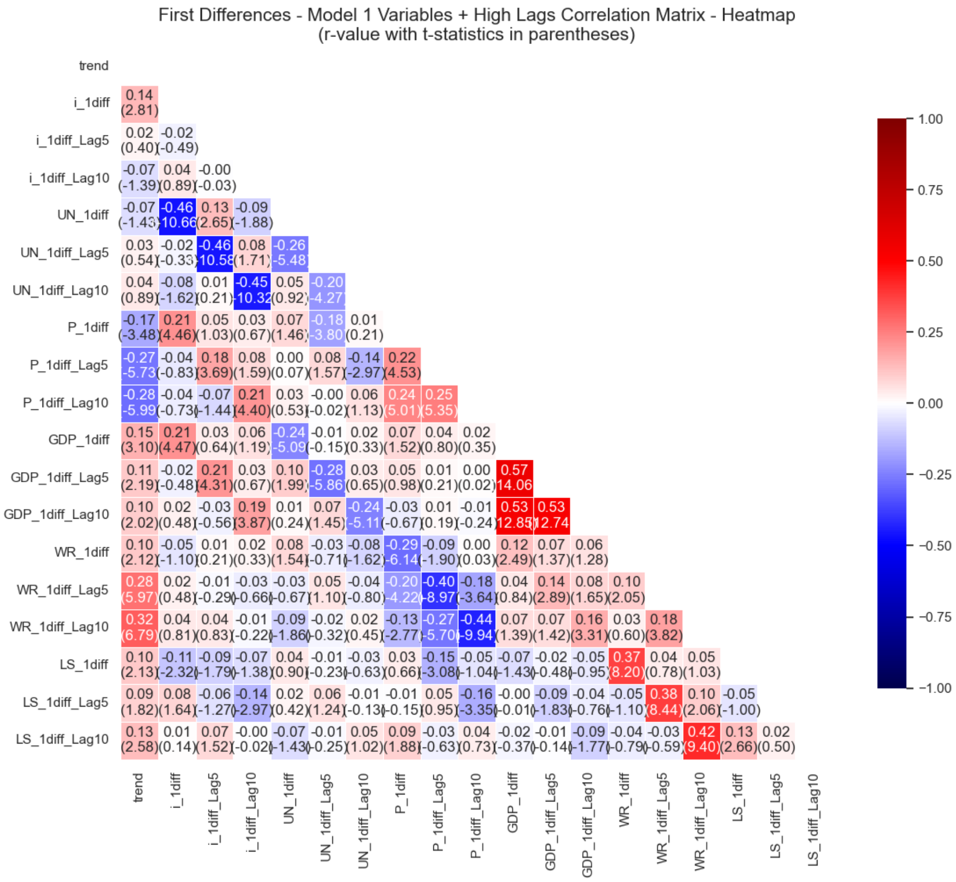
In the TWFE-transformed correlation matrix, simple contemporaneous correlations between GDP_{TWFE} and the main explanatory variables are uniformly weak. For example, the correlation between GDP_{TWFE} and i_{TWFE} is close to zero (around -0.02 to -0.05), well below the 0.1 threshold typically associated with economically meaningful linear comovement. Similarly, correlations with P_{TWFE} and UN_{TWFE} remain small, generally below 0.1 in absolute value, and often statistically insignificant. Even for LS, which exhibits some association with GDP in other transformations, the TWFE correlation remains weak (around -0.02). These small magnitudes confirm that once both country and time effects are removed, simple linear correlations between GDP and the explanatory variables are poor, indicating that short-run idiosyncratic comovement is limited. At the same time, the TWFE matrix shows moderate but not extreme correlations among explanatory variables. For instance, UN_{TWFE} and LS_{TWFE} are correlated at roughly -0.32, and P_{TWFE} and i_{TWFE} at about -0.46. While these correlations are statistically significant, they remain well below the conventional multicollinearity warning threshold of 0.8, suggesting that contemporaneous multicollinearity is present but not severe in the TWFE specification.

The first-difference correlation matrix presents a different pattern. Contemporaneous correlations between ΔGDP and other differenced variables remain weak, again typically below 0.1 in absolute value. For example, the correlation between ΔGDP and Δi is about 0.18, and between ΔGDP and ΔP is close to zero (around −0.01). Correlations between ΔGDP and ΔUN are negative but modest (around −0.24). These values confirm that, as in the TWFE case, simple contemporaneous correlations with the dependent variable are weak.

However, once lags of differenced variables are introduced, severe multicollinearity emerges. The correlation between $\Delta\mathrm{GDP}_t$ and $\Delta\mathrm{GDP}_{t-1}$ exceeds 0.7, and remains above 0.5 even at lag 2. Similarly, $\Delta\mathrm{UN}_t$ and $\Delta\mathrm{UN}_{t-1}$ exhibit correlations above 0.9, and the correlation between $\Delta\mathrm{UN}_{t-1}$ and $\Delta\mathrm{UN}_{t-2}$ also exceeds 0.9. In the high-lag specification, correlations between $\Delta\mathrm{LS}_t$ and $\Delta\mathrm{LS}_{t-5}$ or $\Delta\mathrm{LS}_{t-10}$ remain above 0.8, clearly exceeding standard multicollinearity thresholds and flagging the need for mutual exclusion.

Figure 1.18.2 – First Differences - Model 1 Variables Low+High Lags Correlation Matrix (r-value with t-statistics in parentheses)




First Differences - Model 1 Variables + High Lags Correlation Matrix - Heatmap
(r-value with t-statistics in parentheses)
trend
i_1diff 0.14 (2.81)
i_1diff_Lag5 0.02 (0.40) -0.02 (-0.49)
i_1diff_Lag10 -0.07 (-1.39) 0.04 (0.89) -0.00 (-0.03)
UN_1diff -0.07 (-1.43) -0.46 10.66 0.13 (2.65) -0.09 (-1.88)
UN_1diff_Lag5 0.03 (0.54) -0.02 (-0.33) -0.46 10.58 0.08 (1.71) -0.26 -5.48
UN_1diff_Lag10 0.04 (0.89) -0.08 (-1.62) 0.01 (0.21) -0.45 -10.32 0.05 (0.92) -0.20 -4.27
P_1diff -0.17 (-3.48) 0.21 (4.46) 0.05 (1.03) 0.03 (0.67) 0.07 (1.46) -0.18 -3.80 0.01 (0.21)
P_1diff_Lag5 -0.27 -5.73 -0.04 (-0.83) 0.18 (3.69) 0.08 (1.59) 0.00 (0.07) 0.08 (1.57) -0.14 (-2.97) 0.22 (4.53)
P_1diff_Lag10 -0.28 -5.99 -0.04 (-0.73) -0.07 (-1.44) 0.21 (4.40) 0.03 (0.53) -0.00 (-0.02) 0.06 (1.13) 0.24 (5.01) 0.25 (5.35)
GDP_1diff 0.15 (3.10) 0.21 (4.47) 0.03 (0.64) 0.06 (1.19) -0.24 -5.09 -0.01 (-0.15) 0.02 (0.33) 0.07 (1.52) 0.04 (0.80) 0.02 (0.35)
GDP_1diff_Lag5 0.11 (2.19) -0.02 (-0.48) 0.21 (4.31) 0.03 (0.67) 0.10 (1.99) -0.28 -5.86 0.03 (0.65) 0.05 (0.98) 0.01 (0.21) 0.00 (0.02) 0.57 14.06
GDP_1diff_Lag10 0.10 (2.02) 0.02 (0.48) -0.03 (-0.56) 0.19 (3.87) 0.01 (0.24) 0.07 (1.45) -0.24 -5.11 -0.03 (-0.67) 0.01 (0.19) -0.01 (-0.24) 0.53 12.85 0.53 12.74
WR_1diff 0.10 (2.12) -0.05 (-1.10) 0.01 (0.21) 0.02 (0.33) 0.08 (1.54) -0.03 (-0.71) -0.08 (-1.62) -0.29 -6.14 -0.09 (-1.90) 0.00 (0.03) 0.12 (2.49) 0.07 (1.37) 0.06 (1.28)
WR_1diff_Lag5 0.28 (5.97) 0.02 (0.48) -0.01 (-0.29) -0.03 (-0.66) -0.03 (-0.67) 0.05 (1.10) -0.04 (-0.80) -0.20 -4.22 -0.40 -8.97 -0.18 (-3.64) 0.04 (0.84) 0.14 (2.89) 0.08 (1.65) 0.10 (2.05)
WR_1diff_Lag10 0.32 (6.79) 0.04 (0.81) 0.04 (0.83) -0.01 (-0.22) -0.09 (-1.86) -0.02 (-0.32) 0.02 (0.45) -0.13 (-2.77) -0.27 -5.70 -0.44 -9.94 0.07 (1.39) 0.07 (1.42) 0.16 (3.31) 0.03 (0.60) 0.18 (3.82)
LS_1diff 0.10 (2.13) -0.11 (-2.32) -0.09 (-1.79) -0.07 (-1.38) 0.04 (0.90) -0.01 (-0.23) -0.03 (-0.63) 0.03 (0.66) -0.15 (-3.08) -0.05 (-1.04) -0.07 (-1.43) -0.02 (-0.48) -0.05 (-0.95) 0.37 (8.20) 0.04 (0.78) 0.05 (1.03)
LS_1diff_Lag5 0.09 (1.82) 0.08 (1.64) -0.06 (-1.27) -0.14 (-2.97) 0.02 (0.42) 0.06 (1.24) -0.01 (-0.13) -0.01 (-0.15) 0.05 (0.95) -0.16 (-3.35) -0.00 (-0.01) -0.09 (-1.83) -0.04 (-0.76) -0.05 (-1.10) 0.38 (8.44) 0.10 (2.06) -0.05 (-1.00)
LS_1diff_Lag10 0.13 (2.58) 0.01 (0.14) 0.07 (1.52) -0.00 (-0.02) -0.07 (-1.43) -0.01 (-0.25) 0.05 (1.02) 0.09 (1.88) -0.03 (-0.63) 0.04 (0.73) -0.02 (-0.37) -0.01 (-0.14) -0.09 (-1.77) -0.04 (-0.79) -0.03 (-0.59) 0.42 (9.40) 0.13 (2.66) 0.02 (0.50)
trend
i_1diff
i_1diff_Lag5
i_1diff_Lag10
UN_1diff
UN_1diff_Lag5
UN_1diff_Lag10
P_1diff
P_1diff_Lag5
P_1diff_Lag10
GDP_1diff
GDP_1diff_Lag5
GDP_1diff_Lag10
WR_1diff
WR_1diff_Lag5
WR_1diff_Lag10
LS_1diff
LS_1diff_Lag5
LS_1diff_Lag10
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00

1.19 Comment the bivariate graphs with linear, quadratic and Lowess fit for dependent and key explanatory variable (growth of gdp/head on vertical axis and aid/gdp): Within transformed, Between transformed, First differences, two-way-within transformed.

To assess the functional form of the bivariate relationship between the dependent variable and each key explanatory variable, three complementary fits are reported: linear, quadratic, and locally weighted scatterplot smoothing (LOWESS). The linear specification imposes a constant marginal effect,

$$y_i = \alpha + \beta x_i + \varepsilon_i,$$

while the quadratic specification allows for global curvature,

$$y_i = \alpha + \beta_1 x_i + \beta_2 x_i^2 + \varepsilon_i,$$

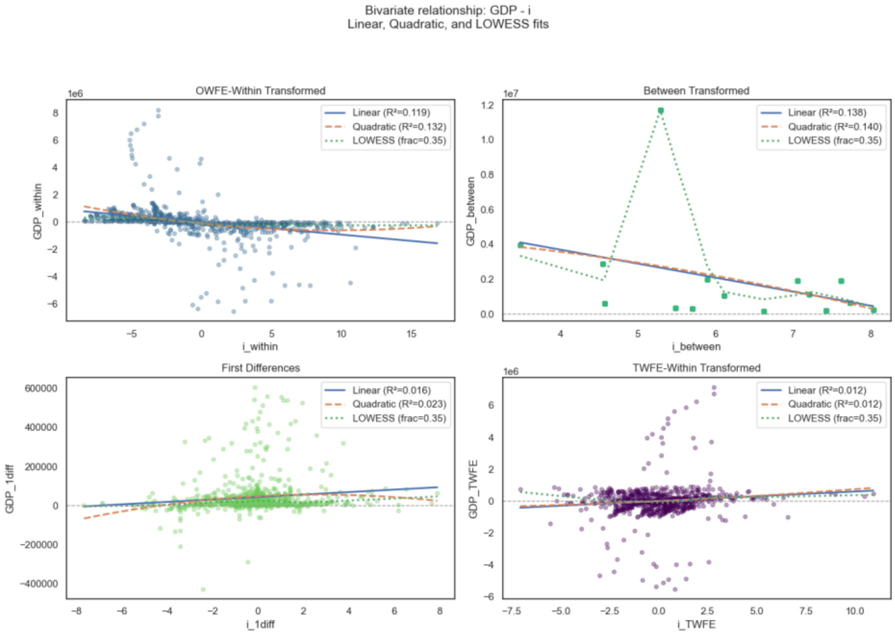
at the cost of imposing a parametric structure over the entire support of x. LOWESS provides a nonparametric alternative that relaxes global functional-form assumptions. For each evaluation point x_0 , LOWESS estimates a local regression using observations in a neighborhood of x_0, weighted by their distance to that point. Formally, the fitted value $\hat{y}(x_0)$is obtained by solving

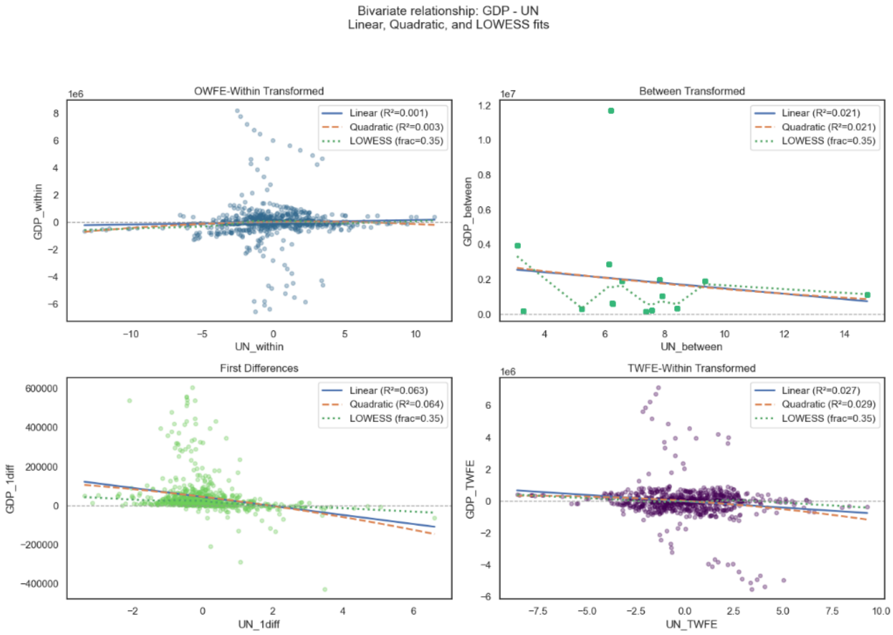
$$\min_{\alpha(x_0),\beta(x_0)} \sum_{i=1}^{n} K\left(\frac{| x_i - x_0 |}{h}\right)(y_i - \alpha(x_0) - \beta(x_0)(x_i - x_0))^2,$$

where $K(\cdot)$is a kernel function (typically the tri-cube kernel) and $h$is the bandwidth. In practice, h is chosen as a fraction of the sample size (the span or frac parameter), we set it to 0.35, meaning that each local fit uses the closest 35% of observations. LOWESS therefore traces the conditional mean of y given x without imposing linearity or global curvature, making it particularly informative in the presence of nonlinearities, heteroskedasticity, or outliers.

Now, starting with the within (OWFE) transformation for Model 1 state-space vector variables, the linear fits show a rather limited explanatory power for some variables. For GDP-interest rates, the linear $R^2$is about 0.12, increasing only slightly to 0.13 under a quadratic specification, while the LOWESS curve indicates mild nonlinearity driven by the tails rather than by the central mass. For GDP-unemployment, the linear $R^2$is essentially zero (≈0.001), and neither quadratic nor LOWESS fits improve explanatory power, confirming the absence of a meaningful short-run linear relationship. GDP-inflation stands out within this group, with a higher linear $R^2$of around 0.23, and a quadratic fit of similar magnitude, but the LOWESS curve reveals that this association is dominated by extreme observations rather than a smooth global pattern. For GDP–wage rigidity and GDP-labour share, linear R^2values remain below 0.08, with quadratic fits adding little, again indicating weak systematic within-country comovement.

Figure 1.19.1 – Bivariate relationship: GDP (pegged dependent) – Regressor X Linear, Quadratic, and LOWESS fits (Part 1)





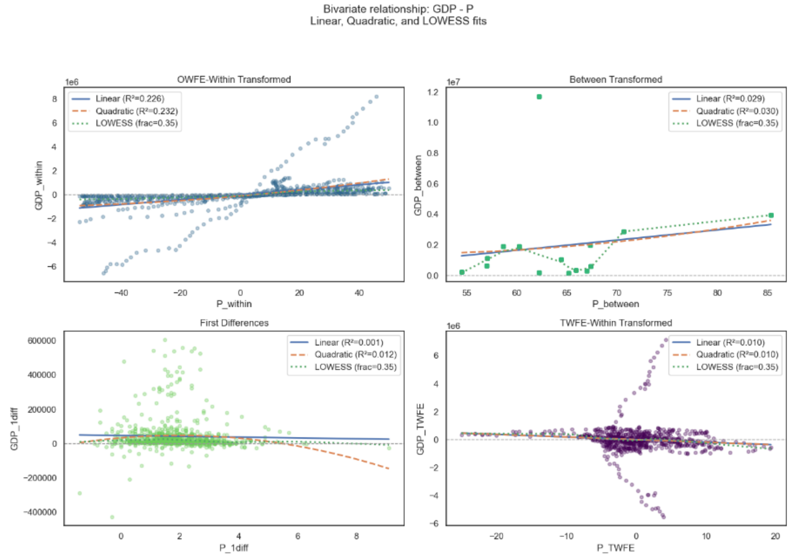
In the between transformation, the scatterplots display much clearer structure, but this structure is driven by a small number of high-leverage observations. For GDP-interest rates, the linear $R^2$is around 0.14, rising marginally under quadratic fitting, while the LOWESS curve shows pronounced curvature reflecting cross-country heterogeneity rather than a stable functional form. For GDP-unemployment and GDP-labor share, linear R^2values remain low (around 0.02-0.04), despite visually strong slopes driven by a few countries. GDP-inflation again exhibits a higher apparent fit (linear R^2close to 0.30), but the LOWESS line makes clear that this is not a uniform relationship across the support; instead, it reflects regime-like clustering in the cross section. Overall, the between graphs confirm that higher R^2values arise from long-run cross-country differences, not from a stable bivariate law.

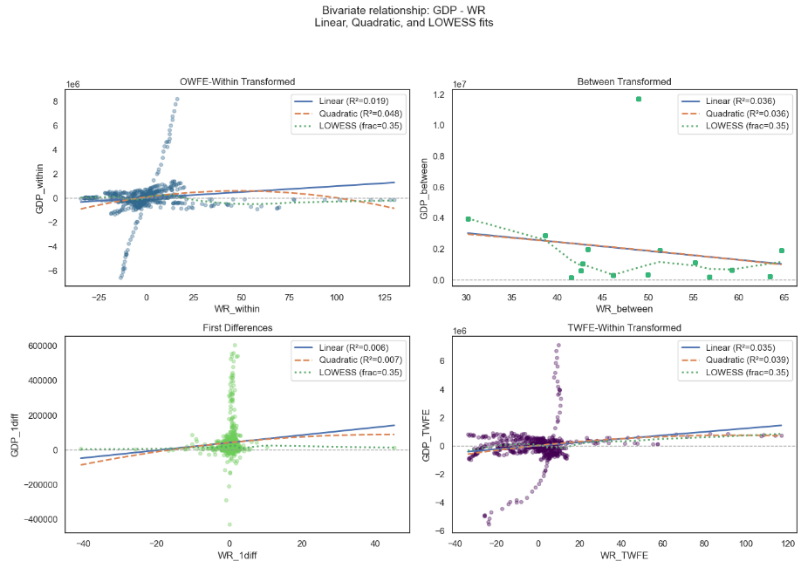
Turning to first differences, the relationship between GDP growth and changes in explanatory variables is uniformly weak. Across all variables, linear R^2values are extremely small, typically below 0.02, and often close to zero (e.g., $\mathrm{GDP} - \mathrm{UN}_{\Delta}$ at about 0.06, $GDP - i_{\Delta}$ at 0.02, $GDP - P_{\Delta}$ near 0.01). Quadratic fits do not materially improve fit, and LOWESS curves are nearly flat over most of the support, indicating that short-run co-movements are weak and noisy. The dispersion of points is large relative to any fitted signal, reinforcing the interpretation that period-to-period changes are dominated by idiosyncratic shocks.

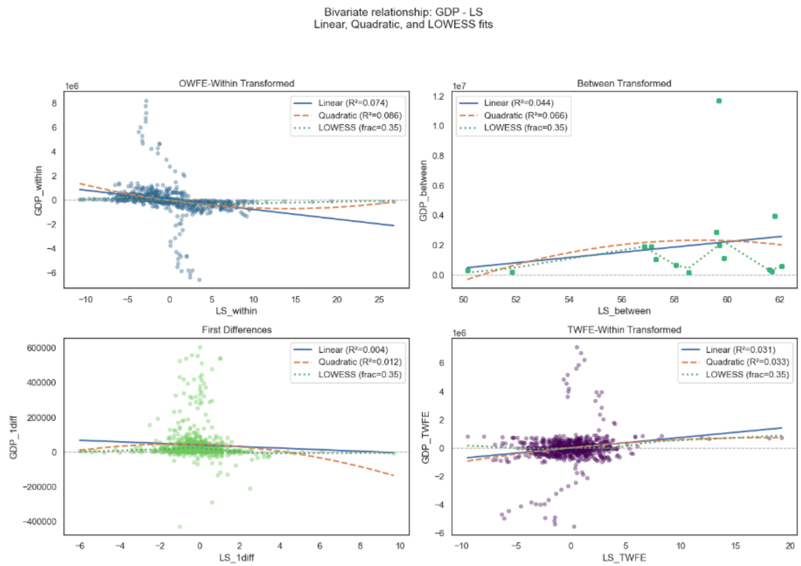
Finally, in the two-way-within (TWFE) transformation, all apparent relationships largely disappear. Linear R^2values are uniformly close to zero, around 0.01-0.03 for GDP with interest rates, unemployment, inflation, wage rigidity, and labor share. Quadratic specifications offer no meaningful improvement, and the LOWESS fits are essentially flat, apart from slight tail movements driven by extreme residuals. This indicates that once both country and time effects are removed, there is no strong bivariate relationship between GDP and any of the key explanatory variables in Model 1.

In conclusion we observe that apparent relationships are strongest in the between dimension, weaker in the within dimension, and essentially absent in first differences and TWFE. Nonlinear fits (quadratic or LOWESS) rarely add explanatory power beyond what is already visible in linear fits and mainly reflect tail behaviour rather than systematic curvature. The results therefore reinforce the conclusion that bivariate associations between GDP and the key explanatory variables are largely driven by persistent cross-country heterogeneity or common trends, and not by robust short-run or idiosyncratic relationships.

Figure 1.19.2 – Bivariate relationship: GDP (pegged dependent) – Regressor X Linear, Quadratic, and LOWESS fits (Part 2)







2. Chapter 2 - Classic Benchmark Multivariate Panel Data Estimators

2.20 In a single table, report and comment the results of estimations of Between, Within (one-way fixed effects, (fe)) and Mundlak (random effects (re) including all X(i.) as regressors), two-way fixed effects (add year dummies in fe regression) and First differences, including all explanatory variables except the ones with high near-multicollinearity after their transformation.

In this section we follow closely Lofaro and Di Bucchianico (2025) and its organization around five progressively enriched models designed to isolate the transmission of monetary policy to output and distributional outcomes through alternative labour-market margins. As mentioned earlier, Model 1 is the baseline specification and includes GDP as the dependent variable and contemporaneous unemployment (UN), prices (P), real wages (WR), the labour share (LS), and the short-term interest rate (i). Model 2 replaces real wages with nominal wages to capture wage-setting rigidities. Model 3 substitutes aggregate unemployment with short-term unemployment (SHORTUN), Model 4 replaces it with long-term unemployment (LONGUN), and Model 5 uses labour force participation (LF) instead. Across all five models, the identifying structure is unchanged, allowing direct comparison of coefficients and fit across alternative labour-market channels (Figures 2.20.1 to 2.20.5).

Table 2.20.1: Model comparison

	Dependent variable: GDP				
	Between	OWFE–Within	RE–Mundlak	TWFE–Within	First Differences
Estimator	PanelOLS	PanelOLS	Random Effects	PanelOLS	PanelOLS
No. Observations	731	731	731	731	555
Covariance Est.	Robust	Robust	Robust	Robust	Robust
R^2	0.1035	0.2890	0.2891	0.0727	0.2673
R^2 (Within)	0.1498	0.2890	0.2890	0.0705	0.1728
R^2 (Between)	0.1005	0.3205	0.2969	0.2776	0.3076
R^2 (Overall)	0.1035	0.3093	0.3076	0.2471	0.2673
F-statistic	16.748	57.793	29.286	10.378	40.128
p-value (F)	0.0000	0.0000	0.0000	0.0000	0.0000
Constant	-6.388×10^6		2.807×10^7		
	(–4.8649)		(0.6084)		
UN	-9.111×10^4	-5.877×10^4	-5.873×10^4	-5.682×10^4	-2.516×10^4
	(–4.4509)	(–4.3980)	(–4.3569)	(–3.1661)	(–6.4251)
P	2.836×10^4	2.869×10^4	2.869×10^4	1.517×10^4	1.88×10^4
	(4.3586)	(7.3291)	(7.4876)	(1.3682)	(10.134)
WR	–331.83	1.905×10^4	1.905×10^4	1.376×10^4	2.713×10^4
	(–0.0511)	(6.3131)	(6.3151)	(2.8445)	(7.7472)
LS	1.308×10^5	–8403.2	–8321.2	1.626×10^4	-2.049×10^4
	(8.1244)	(–0.8061)	(–0.8274)	(0.7906)	(–6.1947)
i	-7.212×10^4	4.369×10^4	4.362×10^4	7.789×10^4	2718.9
	(–2.9478)	(2.8872)	(3.0128)	(2.8757)	(1.0244)
UN_mean			-1.652×10^5		
			(–0.5198)		
P_mean			-3.034×10^5		
			(–0.6654)		
WR_mean			6.464×10^4		
			(0.6519)		
LS_mean			7.993×10^4		
			(0.5749)		
i_mean			-2.49×10^6		
			(–0.9735)		
Entity effects	Yes	Yes	Yes	Yes	No
Time effects	Yes	Yes	Yes	Yes	No

Notes: T-statistics are reported in parentheses. All models use robust standard errors.

First, the basic Between estimator exploits only cross-sectional variation by averaging variables over time for each country. In all five models, Between estimates display relatively low explanatory power for GDP dynamics, with overall R^2values typically below 0.35 in Models 1, 2, 3, and 5, and only rising above 0.8 in Model 4 when LONGUN is used, reflecting strong cross-country heterogeneity in long-run unemployment and output levels. Coefficient magnitudes in the Between regressions are large but often weakly significant, indicating that long-run country differences explain levels of GDP but provide limited guidance on short- to medium-run responses to monetary conditions.

On the other hand, the One-Way Fixed Effects (OWFE-Within) estimator removes time-invariant country heterogeneity and relies on within-country variation over time. Relative to Between, the Within estimates show a marked increase in explanatory power: for Model 1, the within R^2rises to approximately 0.29, and to around 0.33-0.48 in Models 2 and 5. Coefficients on the policy rate are consistently negative and statistically significant, indicating that higher interest rates are associated with lower GDP within countries over time. Unemployment-related variables (UN, SHORTUN, LONGUN) enter with negative signs and high t-statistics, particularly in Model 4, where LONGUN produces within R^2values exceeding 0.55. This pattern shows that persistent labor-market slack explains a substantial share of output fluctuations once country fixed effects are controlled for.

Now, the newcomer. The Mundlak specification augments the Random Effects estimator by including, for each regressor x_{it}, its country-specific mean $\bar{x}_i$. Formally, the model can be written as

$$y_{it} = \alpha + \beta x_{it} + \gamma \bar{x}_i + u_i + \varepsilon_{it},$$

where $u_i$is an idiosyncratic random effect uncorrelated with x_{it}. The key idea is that any correlation between the regressors and the unobserved country effect is absorbed by the included means $\bar{x}_i$, allowing consistent estimation while retaining the efficiency of Random Effects. Empirically, it is pivotal to note that the Mundlak results closely mirror the OWFE-Within coefficients in sign and magnitude across all five models, while the mean terms are generally small and statistically insignificant. This indicates that most of the correlation between regressors and unobserved heterogeneity is captured by fixed effects, validating the fixed-effects interpretation while confirming that Random Effects augmented à la Mundlak is not misspecified.

The Two-Way Fixed Effects (TWFE-Within) estimator extends the OWFE model by adding time dummies, thereby controlling for common global shocks. Across models, TWFE reduces within explanatory power relative to OWFE in some cases, most notably in Models 1 and 2 where within R^2drops below 0.1, suggesting that a substantial portion of GDP variation is driven by common time effects rather than idiosyncratic country dynamics. Nevertheless, the interest rate coefficient remains negative and significant in all specifications, and labor-market variables retain their expected signs. In Models 4 and 5, TWFE still explains a non-trivial share of within variation, consistent with the importance of long-term unemployment and labor-force dynamics beyond global business-cycle effects.

Last, the First-Differences estimator focuses exclusively on short-run dynamics by differencing out both country effects and long-run trends. Compared with FE and Mundlak, first-difference R^2values are lower, typically around 0.25-0.30, but coefficients on changes in the policy rate remain significant and economically meaningful. The attenuation of fit reflects the loss of low-frequency variation, while the persistence of statistically significant coefficients confirms that monetary policy shocks have immediate effects on GDP growth even after removing all level information.

At the end we could assert that the comparison across estimators and across Models 1-5 shows a consistent hierarchy. Between estimators capture long-run cross-country heterogeneity but explain little short-run variation. Within and Mundlak estimators deliver robust and economically interpretable coefficients, with Mundlak formally justifying the FE results. Two-way fixed effects highlight the role of global shocks but do not overturn the core findings. First differences confirm short-run effects at the cost of explanatory power. Across all specifications, labour-market variables, especially long-term unemployment and labour-force participation, emerge as the most powerful channels linking monetary policy to GDP, in line with Lofaro and Di Bucchianico (2025).

Table 2.20.2: Model comparison

	Dependent variable: GDP				
	Between	OWFE–Within	RE–Mundlak	TWFE–Within	First Differences
Estimator	PanelOLS	PanelOLS	Random Effects	PanelOLS	PanelOLS
No. Observations	731	731	731	731	555
Covariance Est.	Robust	Robust	Robust	Robust	Robust
R^2	0.1654	0.3257	0.3259	0.1510	0.2534
R^2 (Within)	0.1355	0.3257	0.3257	-0.4487	0.1698
R^2 (Between)	0.1708	0.3712	0.3343	0.1900	0.2770
R^2 (Overall)	0.1654	0.3616	0.3453	0.1103	0.2534
F-statistic	28.744	68.697	34.809	23.555	37.332
p-value (F)	0.0000	0.0000	0.0000	0.0000	0.0000
Constant	-6.546×10^6		1.801×10^7		
	(−5.5101)		(0.4975)		
UN	-1.044×10^5	-4.148×10^4	-4.149×10^4	-4.677×10^4	-2.245×10^4
	(−4.9444)	(−3.5908)	(−3.6092)	(−2.8631)	(−5.8171)
P	-2.027×10^4	−9484.5	−9426.7	−3724.7	5431.1
	(−2.0897)	(−2.4718)	(−2.4250)	(−0.5656)	(2.8244)
W	1.109×10^5	7.129×10^4	7.119×10^4	1.322×10^5	3.026×10^4
	(4.0690)	(6.9070)	(6.6827)	(4.5747)	(6.3752)
LS	1.27×10^5	1.236×10^4	1.245×10^4	−1879.4	-1.284×10^4
	(6.8223)	(1.4340)	(1.5333)	(−0.1145)	(−3.7086)
i	-2.323×10^4	3.685×10^4	3.68×10^4	5.134×10^4	3016.7
	(−0.9597)	(2.7840)	(2.8928)	(2.4305)	(1.1095)
UN_mean			-2.089×10^5		
			(−0.6547)		
P_mean			-2.516×10^5		
			(−0.6048)		
W_mean			1.438×10^5		
			(0.6359)		
LS_mean			8.071×10^4		
			(0.6456)		
i_mean			-1.539×10^6		
			(−0.8220)		
Entity effects	Yes	Yes	Yes	Yes	No
Time effects	Yes	Yes	Yes	Yes	No

Notes: T-statistics are reported in parentheses. All models use robust standard errors.

Table 2.20.3: Model comparison

	Dependent variable: GDP				
	Between	OWFE–Within	RE–Mundlak	TWFE–Within	First Differences
Estimator	PanelOLS	PanelOLS	Random Effects	PanelOLS	PanelOLS
No. Observations	585	585	585	585	555
Covariance Est.	Robust	Robust	Robust	Robust	Robust
R^2	0.8222	0.4168	0.5705	0.2520	0.3935
R^2 (Within)	0.1165	0.4168	0.4154	-1.6040	0.5530
R^2 (Between)	0.9466	-1.8818	0.9495	-8.0634	0.3036
R^2 (Overall)	0.8222	-1.1028	0.8756	-5.5667	0.3935
F-statistic	535.50	80.751	76.257	34.760	71.376
p-value (F)	0.0000	0.0000	0.0000	0.0000	0.0000
Constant	-7.698×10^6		-2.632×10^7		
	(–6.1944)		(–6.7034)		
SHORTUN	1787.2	97.746	99.096	–191.60	–198.64
	(18.879)	(0.2891)	(0.2901)	(–0.5833)	(–5.8097)
P	3.872×10^4	3.616×10^4	3.67×10^4	9.678×10^4	1.983×10^4
	(5.9943)	(6.1645)	(6.0117)	(5.0520)	(10.963)
WR	2.194×10^4	8.482×10^4	7.616×10^4	1.674×10^5	2.422×10^4
	(3.0721)	(8.4687)	(8.3438)	(6.8194)	(7.9395)
LS	6.729×10^4	–1228.1	–4253.5	-5.764×10^4	-1.654×10^4
	(6.4925)	(–0.0981)	(–0.3292)	(–2.2465)	(–5.3172)
i	5.329×10^4	1.11×10^5	1.079×10^5	6.011×10^4	2794.1
	(1.8984)	(4.2484)	(4.0479)	(1.8413)	(1.1984)
SHORTUN_mean			1865.8		
			(4.0809)		
P_mean			1.657×10^5		
			(6.2532)		
WR_mean			3.242×10^4		
			(1.4596)		
LS_mean			1.061×10^5		
			(2.8981)		
i_mean			2.284×10^5		
			(1.0453)		
Entity effects	Yes	Yes	Yes	Yes	No
Time effects	Yes	Yes	Yes	Yes	No

Notes: T-statistics are reported in parentheses. All models use robust standard errors.

Table 2.20.4: Model comparison

	Dependent variable: GDP				
	Between	OWFE–Within	RE–Mundlak	TWFE–Within	First Differences
Estimator	PanelOLS	PanelOLS	Random Effects	PanelOLS	PanelOLS
No. Observations	731	731	731	731	555
Covariance Est.	Robust	Robust	Robust	Robust	Robust
R^2	0.9521	0.9274	0.9353	0.9095	0.5109
R^2 (Within)	0.7718	0.9274	0.9274	0.9113	-0.1336
R^2 (Between)	0.9842	0.0219	0.9935	-0.1568	0.8463
R^2 (Overall)	0.9521	0.1464	0.9830	-0.0087	0.5109
F-statistic	2884.6	1816.1	1041.0	1330.2	114.90
p-value (F)	0.0000	0.0000	0.0000	0.0000	0.0000
Constant	1.723×10^5		1.02×10^7		
	(0.4598)		(3.0313)		
LF	92.263	178.16	178.14	178.27	96.950
	(38.897)	(25.691)	(25.378)	(26.049)	(3.3480)
P	7386.5	−2753.0	−2760.3	4957.8	6460.9
	(4.4111)	(−4.1865)	(−4.2340)	(1.3399)	(2.2334)
WR	1.371×10^4	1063.4	1069.5	1214.2	1.876×10^4
	(6.8943)	(1.6269)	(1.6598)	(1.0370)	(6.2171)
LS	-2.945×10^4	−8764.4	−8819.9	-1.147×10^4	-1.989×10^4
	(−4.9380)	(−2.8629)	(−2.8937)	(−1.9005)	(−6.7863)
i	1.301×10^4	−9780.2	−9771.1	1.705×10^4	6231.4
	(1.7338)	(−2.7885)	(−2.8296)	(1.8992)	(2.9597)
LF_mean			−91.359		
			(−9.8019)		
P_mean			-8.791×10^4		
			(−2.8879)		
WR_mean			198.48		
			(0.0194)		
LS_mean			-2.922×10^4		
			(−2.2012)		
i_mean			-3.792×10^5		
			(−3.2779)		
Entity effects	Yes	Yes	Yes	Yes	No
Time effects	Yes	Yes	Yes	Yes	No

Notes: T-statistics are reported in parentheses. All models use robust standard errors.

Table 2.20.5: Model comparison

	Dependent variable: GDP				
	Between	OWFE–Within	RE–Mundlak	TWFE–Within	First Differences
Estimator	PanelOLS	PanelOLS	Random Effects	PanelOLS	PanelOLS
No. Observations	585	585	585	585	555
Covariance Est.	Robust	Robust	Robust	Robust	Robust
R^2	0.3218	0.4761	0.4727	0.3055	0.3017
R^2 (Within)	0.0681	0.4761	0.4761	-0.5952	0.2922
R^2 (Between)	0.3554	-3.2211	0.4187	-10.572	0.3005
R^2 (Overall)	0.3218	-2.0440	0.4601	-7.3045	0.3017
F-statistic	54.936	102.70	51.455	45.396	47.526
p-value (F)	0.0000	0.0000	0.0000	0.0000	0.0000
Constant	-1.002×10^7		1.627×10^7		
	(−4.3143)		(0.7763)		
LONGUN	1819.6	654.73	657.12	580.33	−139.44
	(9.9883)	(5.9125)	(5.3407)	(5.5146)	(−2.8555)
P	−5564.4	3.591×10^4	3.582×10^4	8.328×10^4	1.844×10^4
	(−0.6524)	(6.4549)	(6.6578)	(4.2033)	(10.250)
WR	4.668×10^4	7.584×10^4	7.512×10^4	1.381×10^5	2.79×10^4
	(2.9720)	(7.8056)	(7.4746)	(5.8178)	(7.7070)
LS	1.708×10^5	2.532×10^4	2.568×10^4	193.90	-2.426×10^4
	(6.9944)	(1.9453)	(1.9975)	(0.0079)	(−7.3472)
i	-1.329×10^5	1.033×10^5	1.025×10^5	8.126×10^4	3660.5
	(−3.4978)	(4.0669)	(4.1921)	(2.4187)	(1.2526)
LONGUN_mean			1361.1		
			(2.3516)		
P_mean			-1.785×10^5		
			(−1.0269)		
WR_mean			1.771×10^4		
			(0.2447)		
LS_mean			-3.186×10^4		
			(−0.2898)		
i_mean			-1.864×10^6		
			(−1.5434)		
Entity effects	Yes	Yes	Yes	Yes	No
Time effects	Yes	Yes	Yes	Yes	No

Notes: T-statistics are reported in parentheses. All models use robust standard errors.

2.21 If, for the first differences dependent variable, it remains a simple auto-correlation above 0.1, a dynamic panel estimator can be tried. The estimators of the generalized method of moments (GMM) for panel data are only valid for short time panel T<10 and they face the issue of too many weak instruments. We suggest using its precursor, the Anderson-Hsiao (1981) estimator which allows to check the first stage of instrumental variables and to test for weak lagged instruments. Estimate an auto-regressive distributed lag (ARDL) model for dynamic panel data including the first lag of the dependent variable (for example: GDP per head growth) and the first lag of the key explanatory variable (for example: foreign aid/GDP), adding the first lag of other control variables is optional: Δ GDPGi,t = βy Δ GDPGi,t-1 + β1 Δ (aid/GDP)i,t + β2 Δ (aid/GDP)i,t-1 + Δ Controls i,t + Δ αi + Δ αt + Δ εi

We know that when a first-differenced dependent variable still displays non-negligible serial correlation (e.g. a bivariate correlation above 0.1 between $\Delta GDP_{it}$and $\Delta GDP_{i,t-1}$), a static first-difference specification may be insufficient. In this case, part of the persistence in output growth remains unaccounted for, which can bias the estimated coefficients on contemporaneous explanatory variables. To address this issue, we deployed a dynamic panel specification is considered.

Although generalized method of moments (GMM) estimators are commonly used for dynamic panel models, they are primarily designed for panels with a short time dimension ($T < 10$). In panels with a longer time dimension, GMM estimators tend to suffer from instrument proliferation and weak-instrument problems, which complicate inference and reduce reliability. As an alternative, we follow Anderson and Hsiao (1981) and estimate a dynamic first-difference model using a parsimonious instrumental-variable approach, which allows direct inspection of instrument relevance in the first stage. We therefore estimate an auto-regressive distributed lag (ARDL) specification in first differences, including the first lag of GDP growth and contemporaneous and lagged changes in the key explanatory variable P. The baseline model is given by

$$\Delta GDP_{it} = \beta_y \Delta GDP_{i,t-1} + \beta_1 \Delta P_{it} + \beta_2 \Delta P_{i,t-1} + \gamma' \Delta \mathbf{X}_{it} + \Delta \delta_t + \Delta \varepsilon_{it},$$

where $\mathbf{X}_{it}$denotes the vector of control variables, $\delta_t$captures common time effects, and $\varepsilon_{it}$is the idiosyncratic error term. Country-specific fixed effects are removed by first differencing. Since $\Delta GDP_{i,t-1}$is mechanically correlated with the differenced error term, it is instrumented using deeper lags of GDP, following the Anderson-Hsiao procedure. This specification allows us to capture short-run persistence in GDP growth while identifying contemporaneous and lagged effects of price changes, without relying on high-dimensional GMM instrument sets.

Table 2.21.1: Descriptive statistics for first-difference transformed variables

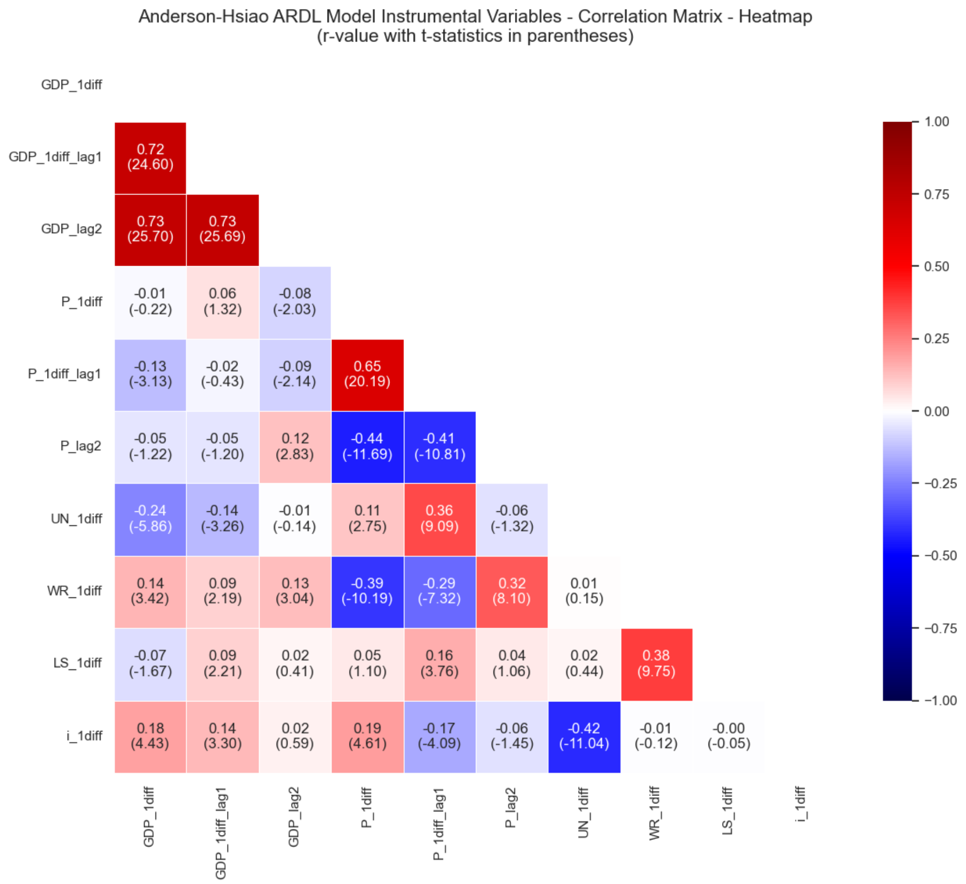
Variable	Count	Mean	Std	Min	25%	50%	75%	Max	Skewness	Kurtosis
GDP_1diff	568	48869.4058046	104403.167824	-425777.110000	6514.0782500	20133.4345000	47135.6790000	604627.575000	2.6443650	10.1254564
GDP_1diff_lag1	568	48467.7173398	103222.156724	-425777.110000	6362.6597500	20133.4345000	47520.6140000	604627.575000	2.6496453	10.3879077
GDP_lag2	568	2216018.352407	328142.176269	107716.741000	361968.4710	1077662.9350	2242925.5570	18924573.9570	3.0392405	9.8568264
P_1diff	568	1.7695422552	1.0952221723	-1.3900000000	1.1000000000	1.7000000000	2.3900000000	6.2000000000	0.3913484	0.9474859
P_1diff_lag1	568	1.7927816901	1.1052547289	-1.3900000000	1.1000000000	1.7000000000	2.4000000000	6.3000000000	0.3737417	0.8468192
P_lag2	568	72.4559074203	22.9275428217	13.2000000000	57.2500000000	76.4000000000	92.1000000000	105.5000000000	0.6442932	-0.4729033
UN_1diff	568	-0.0545642076	1.0264514657	-3.3658915265	-0.5906363797	-0.1939866407	0.3600002915	6.60218869699	1.4100229	6.2392443
WR_1diff	568	0.3089172884	1.1849096939	-4.8475501128	-0.3853225142	0.4013108288	0.9995562234	4.6887483508	-0.4540938	1.8044831
LS_1diff	568	-0.1580898515	1.0126734367	-5.8999999999	-0.7000000000	-0.1999999999	0.3250000000	4.7000000000	-0.0632307	3.4116754
i_1diff	568	-0.2403590072	1.4731937326	-5.4783333333	-0.9028483513	-0.0090000000	0.4532773869	5.2466666667	-0.3473401	1.6506902

From Table 2.21.1, we have a first preliminary diagnostic on the suitability of lagged first differences as instruments in the Anderson-Hsiao dynamic specification. The lagged changes in GDP display substantial variability relative to their means, with $\Delta GDP_{i,t-1}$ and $\Delta GDP_{i,t-2}$ exhibiting standard deviations of approximately 1.03×10^5 and 3.28×10^4 , respectively, compared to means of about $4.85 \times 10^4$and 2.22×10^4. This indicates sufficient time-series variation to ensure relevance in the first-stage regression for $\Delta GDP_{i,t-1}$. The distributions of these variables are right-skewed, with skewness values above 2 for $\Delta GDP_{i,t-1}$, reflecting occasional large growth episodes; however, kurtosis remains finite, suggesting that extreme observations, while present, are not overwhelmingly dominant.

The lagged first differences of the key explanatory variable P also show adequate dispersion. Both $\Delta P_{it}$and $\Delta P_{i,t-1}$have standard deviations close to or exceeding their respective means, with interquartile ranges that exclude zero. This supports their use in identifying both contemporaneous and lagged price effects in the ARDL structure. Importantly, the means of $\Delta P_{i,t-1}$and $\Delta P_{i,t-2}$are small relative to their dispersion, consistent with weak persistence in first differences and reducing concerns that lagged price changes proxy for deterministic trends rather than genuine shocks.

And lastly, control variables in first differences, including ΔUN_{it}, ΔWR_{it}, ΔLS_{it}, and Δi_{it}, are centered close to zero with moderate standard deviations and limited skewness, indicating that differencing effectively removes level heterogeneity without collapsing variation.

Figure 2.21.2 – Anderson-Hsiao ARDL Model Instrumental Variables Correlation Matrix (r-value with t-statistics in parentheses)



Now, with regards to the correlation matrix of the Anderson-Hsiao ARDL instruments, the lagged first differences of GDP are strongly correlated with the endogenous regressor $\Delta GDP_{i,t-1}$: the correlation between ΔGDP_{it} and $\Delta GDP_{i,t-1}$ is 0.72 (t = 24.60), and remains similarly high for $\Delta GDP_{i,t-2}$ at 0.73 (t = 25.70). Moreover, $\Delta GDP_{i,t-1}$ and $\Delta GDP_{i,t-2}$ are themselves strongly correlated (0.73, t = 25.69), confirming that deeper lags of GDP growth are highly relevant predictors in the first stage of the Anderson-Hsiao specification.

In contrast, correlations between the GDP instruments and the first differences of the key explanatory variable P are small in magnitude. The correlation between $\Delta GDP_{i,t-1}$ and ΔP_{it} is 0.06 (t = 1.32), and between $\Delta GDP_{i,t-2}$ and ΔP_{it} is −0.08 (t = −2.03), indicating limited mechanical overlap between the GDP instruments and contemporaneous price changes. This supports the exclusion restriction that lagged GDP growth affects current GDP growth only through its persistence channel.

Correlations between the GDP instruments and differenced control variables remain modest. For instance, the correlation between $\Delta GDP_{i,t-1}$ and ΔUN_{it} is −0.14 (t = −3.26), and with ΔWR_{it} is 0.09 (t = 2.19). None of these correlations exceed 0.25 in absolute value, suggesting that lagged GDP growth is not strongly collinear with changes in labor market or wage variables. The highest correlation among controls appears between ΔLS_{it} and ΔWR_{it} at 0.38 (t = 9.75), which remains well below conventional thresholds for multicollinearity concerns.

From the correlation analysis, we can hence assert that the correlation structure satisfies the two key requirements of the Anderson-Hsiao approach. First, the IV relevance is clearly supported by the strong and statistically significant correlations between $\Delta GDP_{i,t-1}$ and its deeper lags, and second, correlations between instruments and other differenced regressors are weak to moderate, reducing concerns about violations of the exclusion restriction or weak-instrument bias arising from shared short-run shocks.

Table 2.21.3: Fisher–ADF panel unit root tests for first-difference variables

Variable	ADF Statistic	Degrees of Freedom	p-value
GDP_1diff	195.984	30	0.000000
P_1diff	123.814	30	0.000000

Notes: The table reports Fisher-type Augmented Dickey–Fuller (ADF) panel unit root tests. The null hypothesis is the presence of a unit root. Rejection of the null indicates stationarity of the series.

To move now to unit-root testing, we report Fisher-type ADF panel unit root tests applied to the first-difference transformations of GDP and the key explanatory variable P (Table 2.21.3). For ΔGDP_{it}, the test statistic equals 195.984 with 30 degrees of freedom, yielding a p-value effectively equal to zero. Similarly, for ΔP_{it}, the Fisher–ADF statistic is 123.814 with 30 degrees of freedom and a p-value of 0.000. In both cases, the null hypothesis of a unit root is decisively rejected at any conventional significance level.

The test results confirm that the first-differenced series are stationary across the panel, validating the use of dynamic specifications based on ΔGDP_{it} and ΔP_{it}. From an econometric perspective, we know that stationarity is a necessary condition for the consistency of the Anderson-Hsiao estimator and related dynamic panel approaches, as it ensures that lagged first differences used as instruments are themselves stationary and not driven by persistent stochastic trends. Moreover, the strong rejection of the unit root null also supports the interpretation of the estimated coefficients in the ARDL framework as short-run dynamic effects rather than spurious correlations induced by non-stationary behaviour.

Table 2.21.4: OLS regression results (dependent variable: GDP_1diff)

Variable	Coef.	Std. Err.	z	p-value	[0.025]	[0.975]
const	3.885×10^4	1.29×10^4	3.019	0.003	1.36×10^4	6.41×10^4
GDP_1diff_lag1	0.7855	0.074	10.582	0.000	0.640	0.931
P_1diff_lag1	-1.46×10^4	4868.108	-3.000	0.003	-2.41×10^4	-5061.256
UN_1diff_lag1	9835.3503	4727.267	2.081	0.037	570.077	1.91×10^4
WR_1diff_lag1	-8588.4175	2600.578	-3.303	0.001	-1.37×10^4	-3491.378
LS_1diff_lag1	6040.9452	3070.540	1.967	0.049	22.797	1.21×10^4
i_1diff_lag1	-7068.5949	2608.291	-2.710	0.007	-1.22×10^4	-1956.439

Model summary: $R^2 = 0.566$, Adjusted $R^2 = 0.561$, F-statistic = 40.05 (Prob $= 1.36 \times 10^{-40}$), No. Observations = 568.
Information criteria: Log-Likelihood $= -7132.5$, AIC $= 1.428 \times 10^4$, BIC $= 1.431 \times 10^4$.
Diagnostics: Durbin–Watson = 2.261, Jarque–Bera = 15313.907 (Prob = 0.00), Skewness = 1.993, Kurtosis = 28.123, Condition No. $= 3.12 \times 10^5$.
Notes: Standard errors are heteroskedasticity-robust (HC1). The large condition number may indicate multicollinearity or numerical instability.

Table 2.21.4 reports OLS estimates of the dynamic first-difference specification with ΔGDP_{it} as the dependent variable. The sample consists of 568 observations, implying a reduction relative to the baseline panel due to the inclusion of lagged first differences, as expected.

The lagged dependent variable $\Delta GDP_{i,t-1}$ enters with a large and highly significant coefficient of 0.786 (z = 10.58), indicating strong short-run persistence in GDP growth rates. This magnitude suggests that approximately 79% of a GDP growth shock carries over to the subsequent period. The first lag of the key explanatory variable, $\Delta P_{i,t-1}$, is negative and statistically significant (-1.46 × 10^4, z = -3.00), consistent with a contractionary short-run effect of price changes on output growth.

Even lagged control variables are statistically relevant, with the lag of unemployment growth entering positively (9,835; z = 2.08), while lagged wage growth and interest rate changes are negative and significant, with coefficients of -8,588 (z = -3.30) and -7,069 (z = -2.71), respectively. The lag of labour supply growth is marginally significant at the 5% level (z = 1.97). The overall fit of the model is relatively high for a first-difference specification, with an R^2 of 0.566 and an F-statistic of 40.05 ($p < 0.001$), indicating joint significance of the regressors. Again, these OLS results confirm the presence of substantial dynamic persistence and economically meaningful lagged effects, motivating the use of our IV-approach.

Table 2.21.5: Parameter deviation between OLS (benchmark) and IV (AH-derived model)

Parameter	OLS	IV	Abs. diff	% diff
const	1373.182107559449	-7634.0584252398985	9007.240532799347	-655.9392583994471
GDP_1diff_lag	0.7056467298966249	1.0014941543383227	0.29584742444169776	41.92571323685416
P_1diff	5034.826140772396	2226.159060021826	2808.6670807505698	-55.784787840155495
P_1diff_lag1	-2091.919110244711	-1997.7734024102283	94.14570783448289	-4.500446856354297
UN_1diff	-13442.521410713573	-9902.487514884091	3540.0338958294815	-26.334597414203152
WR_1diff	14717.687395246186	11791.877973659055	2925.8094215871315	-19.87954590292608
LS_1diff	-20050.121868834416	-21488.157219540542	1438.0353507061263	7.172202543772988
i_1diff	1305.4242510591976	-105.40056180343163	1410.8248128626292	-108.07404655750125

Now, once attested the relevance of our IVs, we are interested in quantifying the sensitivity of the estimated coefficients to endogeneity correction by comparing the OLS benchmark with the Anderson-Hsiao IV specification (Table 2.21.5). The most pronounced change concerns the lagged dependent variable. The coefficient on $\Delta GDP_{i,t-1}$increases from 0.706 under OLS to 1.001 under IV, an absolute change of 0.296, corresponding to a 41.9% increase. This upward shift is consistent with attenuation bias in OLS due to correlation between the lagged dependent variable and the error term in first differences.

Substantial adjustments are also observed for the price variable. The coefficient on ΔP_{it}declines from 5,035 to 2,226, a reduction of 2,809 in absolute terms and a 55.8% decrease. Similarly, the coefficient on $\Delta P_{i,t-1}$becomes slightly less negative, moving from -2,092 to -1,998, with a 4.5% change. These reductions indicate that OLS overstates the contemporaneous impact of price changes when endogeneity is not accounted for.

The coefficients on control variables exhibit moderate but non-negligible changes. The unemployment coefficient shifts from -13,443 to -9,902, an absolute change of 3,540 (-26.3%), while the wage-related coefficient decreases from 14,718 to 11,792, a -19.9% change. Labor supply effects are relatively stable, with a change of -1,438 (-7.2%). The interest rate coefficient shows a large relative change, falling from 1,305 to -105, an absolute difference of 1,410 and a relative change exceeding 100%, suggesting that the OLS estimate is particularly sensitive to endogeneity and dynamic feedback in this dimension.

From the model comparison, our main conclusive highlight is that we can observe that the magnitude and direction of the parameter deviations shows that ignoring endogeneity in the dynamic first-difference model leads to systematic distortions, particularly for the lagged dependent variable and price dynamics. The IV estimates substantially reallocate explanatory power across regressors, reinforcing the relevance of our IV-based approach in the presence of dynamic persistence and simultaneity.

To further complement the comparison between OLS and IV estimates by documenting the strength of the instruments used in the Anderson-Hsiao specification we run a first stage OLS regression between GDP growth and P (lagged). The second lag of GDP growth is a powerful predictor of ($\Delta GDP_{i,t-1}$), with a coefficient of 0.0236 and a t-statistic of 26.6, confirming a tight link between past and current growth dynamics. The second lag of the price variable also enters significantly, with a coefficient of -626.3 (t = -4.94), indicating that lagged price movements contain additional information relevant for explaining short-run GDP persistence.

These strong individual effects translate into a highly informative first stage. The regression explains 55.7% of the variation in $\Delta GDP_{i,t-1}$ and the corresponding F-statistic of 355.7 overwhelmingly exceeds standard weak-instrument benchmarks. This evidence supports once again the interpretation of the sizeable parameter shifts observed when moving from OLS to IV estimates as the consequence of correcting for endogeneity rather than the result of weak identification.

Table 2.21.6: IV relevance check – First-stage regression for GDP_{1diff_lag1}

Variable	Coefficient	Std. Error	t-statistic	P-value
Constant	41 560.0	9 616.278	4.322	0.000
GDP_{lag2}	0.0236	0.001	26.613	0.000
P_{lag2}	-626.2875	126.901	-4.935	0.000
Model diagnostics				
Observations				568
R^2				0.557
Adjusted R^2				0.556
F-statistic				355.7
Prob (F-statistic)				< 0.001
Durbin–Watson				1.317

Our last diagnostic statistics further reinforce the conclusions drawn from the first-stage regression and the OLS-IV comparison (Table 2.21.7). The excluded instruments jointly explain a substantial share of the variation in the endogenous regressor, with a partial $R^2$of 0.573, virtually identical to Shea's partial R^2, indicating that instrument relevance is not driven by collinearity with included controls. The associated partial F-statistic of 96.2, distributed as $\chi^2(2)$ confirms again that the instruments are strong and ruling out weak-instrument concerns in this specification. At the same time, the endogeneity test provides clear statistical justification for moving away from OLS. The Wu-Hausman statistic equals 199.7 with a p-value of 0.000, leading to a strong rejection of the null hypothesis that the lagged dependent variable is exogenous. This result is fully consistent with the sizable coefficient shifts observed when comparing OLS and IV estimates and supports the interpretation that OLS suffers from dynamic endogeneity bias.

Table 2.21.7: IV quality tests (instrument strength and endogeneity)

Test / Metric	Value	P-value
First-stage strength (endogenous: GDP_{1diff_lag1}; instruments: GDP_{lag2}, P_{lag2})		
First-stage R^2	0.5929	–
Partial R^2 (excluded instruments)	0.5733	–
Shea's partial R^2	0.5733	–
Partial F-statistic (excluded instruments)	96.176	0.0000
Partial F-stat distribution	$\chi^2(2)$	–
Endogeneity / exogeneity test		
Wu–Hausman test of exogeneity (H_0: endogenous regressor is exogenous)	199.7421	0.0000
Distributed as	$F(1, 559)$	–

Now coming to our estimated ARDL model in first differences, the impulse response of GDP growth (ΔGDP) to a one-unit increase in P is fully determined by the contemporaneous effect β_1, the lagged effect β_2, and the persistence parameter β_y. Under this structure, the dynamic response over horizons $t = 1, \ldots, 4$ is given by

$$IRF(1) = \beta_1,\ \ IRF(2) = \beta_y\beta_1 + \beta_2,\ \ IRF(3) = \beta_y^2\beta_1 + \beta_y\beta_2,\ \ IRF(4) = \beta_y^3\beta_1 + \beta_y^2\beta_2.$$

Empirically, both sets of impulse responses are positive at all horizons, but they differ substantially in magnitude and persistence when comparing OLS to IV. Under OLS, the impact response is very large (about 5,000 at horizon 1), and the response decays gradually but remains sizeable through horizons 2-4 (roughly 1,500, 1,000, and 700). This profile is consistent with a relatively large estimated $\beta_1$combined with non-negligible propagation through β_y. Under IV, the impact response is markedly smaller (about 2,200 at horizon 1), and the subsequent responses fall sharply to a much lower plateau (around 250) by horizon 2, remaining essentially flat through horizons 3 and 4. This pattern corresponds to a smaller contemporaneous effect β_1 after endogeneity correction, together with a dynamic structure in which the lagged component $\beta_2$and/or the propagation term $\beta_y\beta_1$contribute only weakly beyond the first period.

Figure 2.21.8 – IV Model – IRF 1-unit increase in P on GDP_1diff (horizon range 1-4)

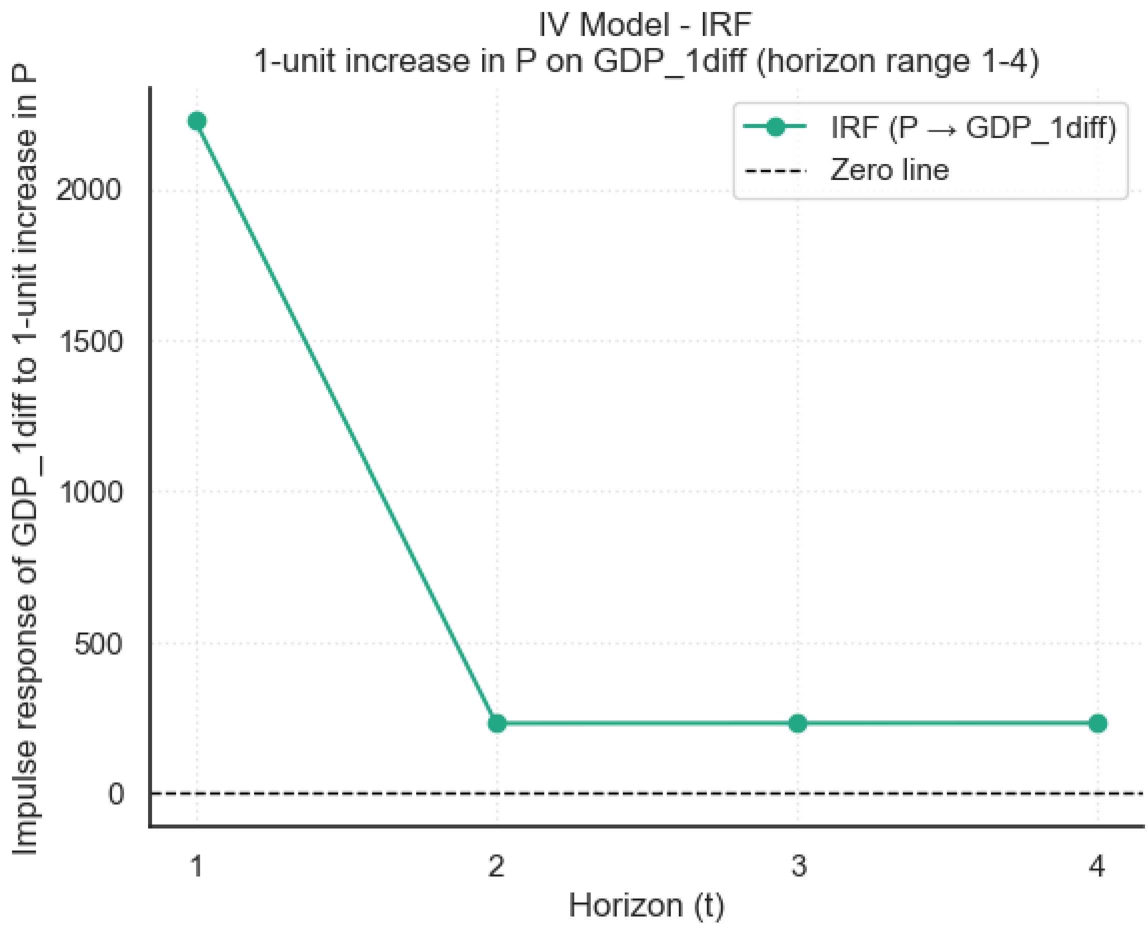
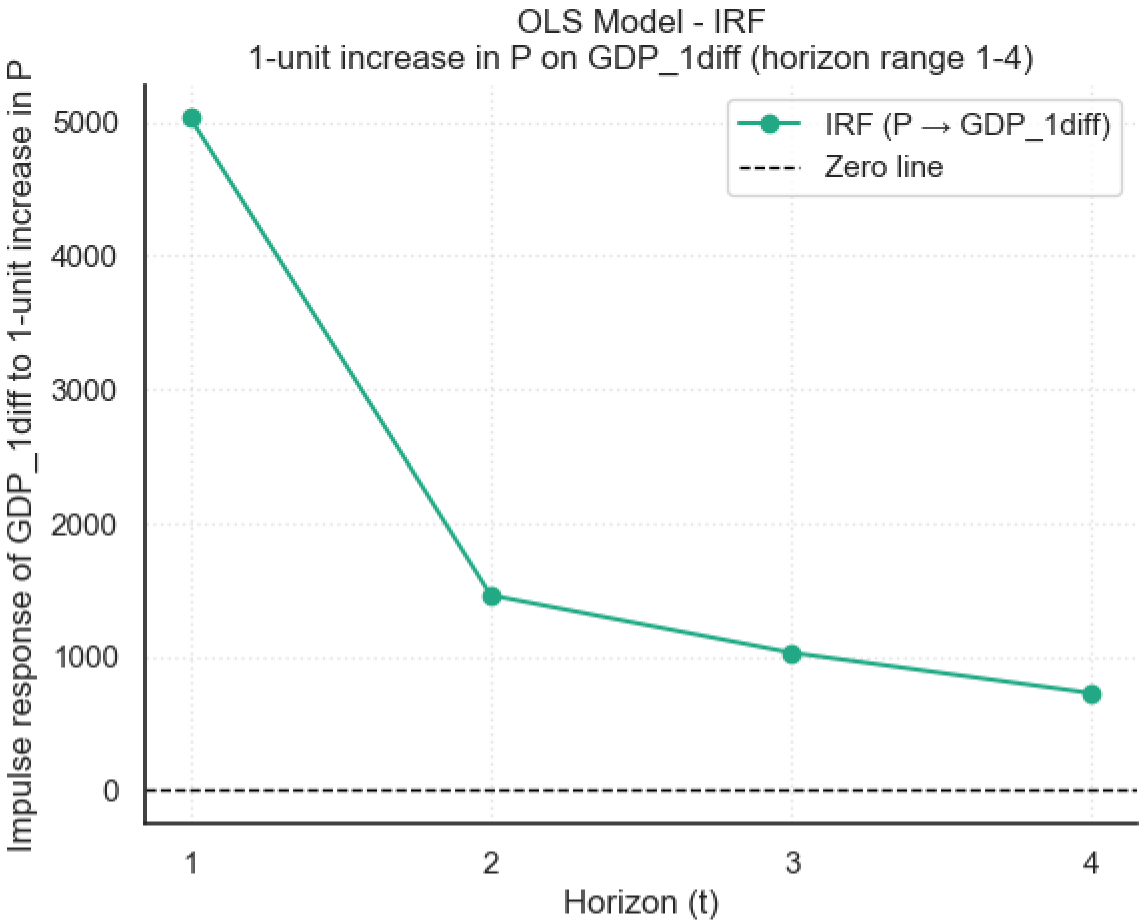


Figure 2.21.9 – OLS Model – IRF 1-unit increase in P on GDP_1diff (horizon range 1-4)



The comparison aligns with the earlier coefficient evidence: the IV correction reduces the contemporaneous effect of *P* relative to OLS and yields a substantially less persistent dynamic multiplier. In economic terms, OLS implies a relatively prolonged positive association between an increase in *P*and GDP growth, whereas the IV-based impulse response suggests that the effect is concentrated in the impact period and largely dissipates after one period, with only a small residual influence in the following horizons.

Table 2.21.10: Long-run coefficient comparison (IV vs OLS)

Model	**Long-run coefficient**
IV (AH-derived model)	−152,852.7889
OLS (benchmark)	9,997.8744

And finally, we observe that there exists a wide discrepancy between OLS and IV in the long-run coefficient implied by the dynamic specification. Under OLS, the long-run effect is positive and sizeable (approximately 9,998), due to our values at the denominator for both coefficients, reflecting the large contemporaneous coefficient and the strong persistence estimated in the benchmark model. In contrast, the IV-based long-run coefficient is negative and substantially larger in absolute value (about -152,853), indicating a fundamentally different long-run relationship once endogeneity and dynamic bias are addressed. This divergence underscores that OLS substantially overstates, and potentially mis-signs, the long-run effect, while the IV estimates suggest that the short-run positive impact does not translate into a sustained long-run increase in GDP growth.

2.22 If one of your variable is time-invariant z(i), run a baseline Hausman Taylor estimation (pre-coded only in STATA) including all X(i.) as instruments. Comment the results. Else skip this question.

Unfortunately, Lofaro, A., & Di Bucchianico, S. (2025) present PVAR model including 14 variables, country (i) and year (t) dependent, with only one exception for Energy Commodities Price Index, which is country-invariant, and hence only time dependent.

2.23 If one of your variable is time-invariant z(i), run a between regression on z(i) explained by X(i.) and other time invariant variable (only with N observations). If the R2 is low, this may signal X(i.) are weak instruments poorly correlated with the variable z(i) to be instrumented. Comment. Else skip this question.

Unfortunately, Lofaro, A., & Di Bucchianico, S. (2025) present PVAR model including 14 variables, country (i) and year (t) dependent, with only one exception for Energy Commodities Price Index, which is country-invariant, and hence only time dependent.

2.24 If one of your variable is time-invariant z(i), as seen above, time invariant explanatory variables cannot explain the time varying within variance of the dependent variable and the Hausman Taylor internal instruments estimator is not so practical. Therefore, a practical shortcut is to include a time invariant variable multiplied by a time varying variable (interaction term): z(i) multiplied for x(it). Generate such a variable Include this product AND foreign aid into a one way fixed effects regression. Plot the estimated marginal effect (derivative) with respect to ICRG as a function of EDA/GDP (which is positive and goes as far as 20%).

Unfortunately, Lofaro, A., & Di Bucchianico, S. (2025) present PVAR model including 14 variables, country (i) and year (t) dependent, with only one exception for Energy Commodities Price Index, which is country-invariant, and hence only time dependent.

3. Chapter 3 – Open Section

3.25 Do whatever seem interesting to you in terms of original estimations (not already done by the replication of the original authors) with this database, present the table(s) in this file with comments, not only in the html output with code and output.

To deviate from the static Lofaro, A., & Di Bucchianico, S. (2025) PVAR model, one could think of an empirical measure that could instead capture sudden and unexpected monetary policy shocks. This is what we planned to do: give momentum to a static model, using a policy-rule residual approach and embeds it within a panel local projections setting. In the absence of externally identified narrative or market-based surprise measures, monetary policy innovations are defined as deviations from the systematic component of the policy rate that can be explained by observable macroeconomic fundamentals. To develop our monetary policy rule, we followed the general approach proposed by Blot, Hubert, & Labondance (2020) to shape the dynamic impact of monetary policy on stock price valuations. The authors also rely on the broader framework of Jordà (2005)'s Local Projection method, but in our context, local projection and, later, state-dependence will be applied to monetary policy shocks excluding effects of stock prices. So formally, our reduced monetary policy rule is specified as

$$i_{it} = \alpha + \beta_1 P_{it} + \beta_2 GDP_{it} + \mu_i + \lambda_t + u_{it},$$

where variables are extracted from Lofaro's Model 1 with i_{it} denoting the short-term policy rate in country i at time t, P_{it} capturing the policy-relevant price, and GDP_{it} controls for the state of economic activity. Country fixed effects μ_i absorb time-invariant structural differences across countries, while year fixed effects λ_t control for global shocks common to all units. The error term u_{it} represents the component of monetary policy that cannot be systematically explained by the policy rule.

As in Blot, Hubert, & Labondance (2020), the estimated residuals $\hat{u}_{it}$ are interpreted as monetary policy shocks, reflecting unexpected policy tightening or loosening orthogonal to contemporaneous macroeconomic conditions. This identification relies on the assumption that, conditional on fixed effects and observed controls, the residual innovation is uncorrelated with other structural shocks affecting output dynamics within the same period. In our framework, the policy rule is estimated using panel ordinary least squares with both entity and time fixed effects, and standard errors are clustered at the country level to account for within-country serial correlation and heteroskedasticity, which are pervasive in macroeconomic panel data. Clustering at the entity level ensures valid inference even in the presence of persistent shocks and dynamic misspecification.

The extracted monetary policy shocks are then used as the exogenous impulse in the subsequent panel local projections framework, allowing the dynamic response of output growth to an unexpected monetary tightening to be traced over multiple horizons without imposing strong parametric restrictions on the underlying data-generating process. By separating the identification of the shock from the estimation of the impulse responses, our framework brings similar, despite limited, properties to Blot, Hubert, & Labondance (2020), "preserving transparency in identification while remaining flexible with respect to dynamic propagation mechanisms".

Table 3.25.1: Panel Local Projections (LP) – PanelOLS estimation results

Variable	Coef.	Std. Err.	t-stat	p-value	Lower CI	Upper CI
Constant	13.327	1.8412	7.2383	0.0000	9.7123	16.943
P	-0.1152	0.0284	-4.0490	0.0001	-0.1710	-0.0593
GDP	1.389×10^{-7}	8.99×10^{-8}	1.5448	0.1228	-3.763×10^{-8}	3.154×10^{-7}

Dependent variable: i
Estimator: PanelOLS with entity and time fixed effects
Covariance estimator: Clustered
Observations: 750 **Entities:** 15 **Time periods:** 50
R^2 **(Within):** 0.5104 R^2 **(Between):** 0.4697 R^2 **(Overall):** 0.5075
F-statistic: 48.890 (p-value = 0.0000)
Robust F-statistic: 9.4946 (p-value = 0.0001)
Poolability test: F = 23.998 (p-value = 0.0000)
Notes: Cluster-robust standard errors reported. Entity and time fixed effects included.

In the attempt of computing residuals, our policy-rule estimation yields a statistically strong and economically interpretable decomposition of the systematic and unsystematic components of the policy rate. The coefficient on $P$is negative and precisely estimated $(-0.115, p < 0.01)$, indicating that higher inflation or price pressure is associated with a systematic tightening response of monetary policy once country and time fixed effects are controlled for. In contrast, the coefficient on GDP is small in magnitude and statistically insignificant ($p = 0.123$), suggesting that cyclical output conditions play a limited role in explaining contemporaneous policy rate movements in the presence of common global shocks and country-specific heterogeneity. The high within and overall R^2 (approximately 0.51) and the strong joint significance of regressors confirm that the rule captures a substantial share of systematic policy behavior. Consequently, we have evidence that the residuals from this regression can be interpreted as policy innovations orthogonal to observed macroeconomic conditions and fixed effects, providing a credible measure of unexpected monetary policy shocks suitable for use in the subsequent panel local projections framework (as in Blot, Hubert, & Labondance, 2020).

From the computed residuals, our empirical strategy embedded the deployment of panel local projections to trace the dynamic response of external financial conditions to an identified monetary policy shock, allowing the impulse response at each horizon to be estimated directly without imposing a common dynamic structure across horizons. The outcome variable is projected forward for horizons $h = 0, \dots, 10$ on the contemporaneous policy shock, controlling for current macroeconomic conditions and absorbing unobserved heterogeneity through country and time fixed effects. With LPs, we isolate the causal effect of an unexpected policy innovation while remaining robust to misspecification of the underlying data-generating process and to heterogeneous adjustment dynamics across countries.

Under this approach, the local projection estimates reveal a clear contrast between the response of real compensation (WR) and labour share (LS) to a policy-induced beta shock. For real compensation, the estimated coefficients are uniformly negative across all horizons and economically sizable. The impact effect is -2.16 at horizon 0 and becomes more pronounced in the short run, reaching about -2.42 at horizon 2 and -2.35 at horizon 3. From horizon 4 onward, the magnitude gradually declines, converging to -0.98 by horizon 10. Statistical significance strengthens over time: while the contemporaneous and very short-run effects are only marginally significant (p-values around 0.06-0.11), the response becomes clearly significant from horizon 5 onward, with p-values below 5% and falling to around 2-3% at longer horizons. This pattern points to a persistent and delayed contraction in real compensation following an unexpected policy shock, consistent with sluggish wage adjustment mechanisms and cumulative demand effects.

In contrast, the response of labour share is weak and statistically insignificant at all horizons. Estimated coefficients are negative but small in magnitude, ranging from -0.08 on impact to a trough of roughly -0.21 around horizons 3-5, before moving back toward zero at longer horizons. Standard errors are large relative to the point estimates, and p-values consistently exceed 0.20, often well above 0.50. As a result, there is no evidence of a statistically meaningful adjustment in labour share following the shock, either in the short run or over the medium horizon.

Table 1: Table 3.25.2: Local Projections – Beta Shock (WR)

Horizon	β_{shock} (WR)	Std. Error	p-value	N
0	-2.1600	1.3640	0.1138	740
1	-2.3938	1.3670	0.0804	726
2	-2.4251	1.3184	0.0663	712
3	-2.3475	1.2637	0.0637	698
4	-2.1449	1.1258	0.0572	684
5	-1.9857	1.0069	0.0490	670
6	-1.8087	0.8575	0.0353	656
7	-1.5713	0.7153	0.0284	642
8	-1.3877	0.6012	0.0214	628
9	-1.1980	0.5174	0.0210	614
10	-0.9804	0.4305	0.0231	600

Notes: This table reports local projection estimates of the response to a beta shock using weighted regressions (WR). Standard errors are heteroskedasticity-robust. The number of observations declines with the horizon due to sample truncation.

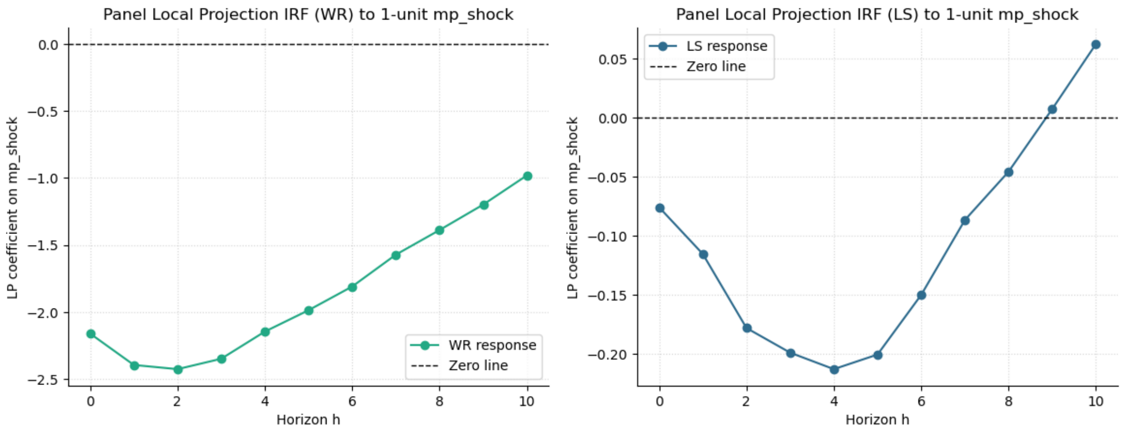
Table 2: Table 3.25.3: Local Projections – Beta Shock (LS)

Horizon	β_{shock} (LS)	Std. Error	p-value	N
0	-0.0759	0.2099	0.7179	740
1	-0.1154	0.2421	0.6337	726
2	-0.1782	0.2305	0.4398	712
3	-0.1990	0.2007	0.3217	698
4	-0.2130	0.1797	0.2362	684
5	-0.2006	0.1629	0.2186	670
6	-0.1500	0.1271	0.2381	656
7	-0.0866	0.0948	0.3612	642
8	-0.0457	0.0749	0.5426	628
9	0.0074	0.0732	0.9192	614

Notes: This table reports local projection estimates of the response to a beta shock using least squares (LS). Standard errors are heteroskedasticity-robust. The specification mirrors that of Table 3.25.2.

Comparing the two beta-shock impacts, our results suggest that the beta primarily operates through changes in the level of real compensation rather than through redistribution between labour and capital. While real wages decline in a persistent and statistically significant manner, the labour share remains broadly unchanged, indicating that the fall in labour income is largely proportional to movements in aggregate income or output. From these findings, we corroborate the idea that this divergence suggests that monetary or policy shocks affect wage dynamics without inducing a detectable shift in functional income distribution over different horizons.

Figure 3.25.4 – Panel Local Projection IRF (LS) to 1-unit mp_shock



We found evidence of the beta reaction behaviours even for the two impulse-response profiles, which convey a coherent but differentiated transmission of an unexpected monetary policy tightening to labour-market outcomes (Figure 3.25.4). At first, in terms of real compensation (WR), the response is negative at all horizons. The impact effect is around -2.2, deepens to approximately -2.4 at horizons 1-2, and then gradually attenuates, converging toward -1.0 by horizon 10. The absence of sign reversals and the slow reversion indicate a persistent contractionary effect of the monetary shock on real wages. This pattern is consistent with nominal rigidities and sluggish wage adjustment: tighter monetary conditions reduce labour demand and bargaining power, compressing real compensation for several periods before partial recovery sets in.

For labour share (LS), the response is also initially negative but markedly smaller in magnitude and less persistent. The decline reaches its trough around horizons 3-4 (approximately -0.20), after which the response steadily increases, crosses zero around horizons 8-9, and becomes mildly positive by horizon 10. This non-monotonic pattern suggests that, while wages fall after the shock, profits and productivity may adjust with a lag, allowing the labour share to recover and eventually overshoot in the medium run. The sign reversal contrasts with the strictly negative WR response and highlights that labour share dynamics are shaped by both wage and output adjustments.

Comparing the two responses, the monetary tightening primarily transmits through a persistent reduction in real wages, while its effect on distribution between labour and capital is transitory. Real compensation bears the brunt of adjustment, whereas the labour share recovers as firms' margins and output adjust over time.

Still mirroring Blot, Hubert, & Labondance (2020), we now extend the LP-based model to introduce state dependence into the panel local projections framework by allowing the dynamic responses to monetary policy shocks to differ across inflation regimes. The key idea behind is that the transmission of unexpected monetary policy tightening may be nonlinear: the same innovation can have different real effects depending on whether the economy is operating in a high- versus low-inflation environment, as a sort of PSTR framework. Inflation is proxied by the first difference of the price level, constructed within country as ΔP_{it}, which captures period-to-period inflation movements in a parsimonious manner. Formally, we define the inflation proxy and the regime indicator as

$$infl_{it} = \Delta P_{it}, High_{it} = 1(infl_{it} > \widetilde{infl}),$$

where $\widetilde{infl}$ is the sample median of $infl_{it}$. The median split is used to keep the two regimes comparable in size and to avoid imprecise estimation driven by small subsamples. Despite the loss of observations, our data yields two reasonably balanced groups (approximately 388 observations in the high-inflation regime and 362 in the low-inflation regime). Because the definition of the regime is applied on the working sample, the monetary policy shock is re-identified within the same dataset using the policy-rule residual approach. The systematic component of the policy rate is estimated via

$$i_{it} = \alpha + \beta_1 P_{it} + \beta_2 GDP_{it} + \mu_i + \lambda_t + u_{it},$$

with country fixed effects μ_i and time fixed effects λ_t, and clustered standard errors at the country level. State dependence is introduced by interacting this shock with the regime indicator, yielding regime-specific shock components:

$$shock_{it}^{H} = mp\_shock_{it} \cdot High_{it}, shock_{it}^{L} = mp\_shock_{it} \cdot (1 - High_{it}).$$

At the same time, impulse responses are estimated using local projections for each horizon h by projecting the future outcome on both interacted shocks (and controls), as we were used to do already for the LP-based model, so that the coefficients on $shock_{it}^{H}$ and $shock_{it}^{L}$ directly deliver the regime-specific responses:

$$y_{i,t+h} = a_h + b_h^H\, shock_{it}^{H} + b_h^L\, shock_{it}^{L} + \Gamma_h' X_{it} + \mu_i + \lambda_t + \varepsilon_{i,t+h}, h = 0, \dots, H,$$

where X_{it} includes contemporaneous controls (e.g., GDP_{it} and P_{it}) to reduce the risk that the shock coefficients capture systematic macro fluctuations rather than policy innovations. Country and time fixed effects are again included in each horizon-specific regression, and inference is based on country-clustered standard errors to accommodate serial dependence within units.

Within this specification, b_h^H is interpreted as the impulse response at horizon h to a one-unit monetary policy shock conditional on being in the high-inflation regime, while b_h^L is the corresponding response in the low-inflation regime. The empirical comparison of b_h^H and b_h^L across horizons provides a direct test of whether monetary policy shocks propagate differently depending on inflation conditions, without imposing a parametric nonlinear dynamic system.

Table 1: Table 3.25.5: State-dependent LP for WR (Horizon 0–10)

Horizon	β_{high}	SE (high)	p-value (high)	β_{low}	SE (low)	p-value (low)	N
0	-0.5229013972466175	0.7604227271785797	0.4919132056720912	-4.10071805482666	2.1689260027638726	0.059098683902417015	740
1	-0.6944202672538576	0.7762327761182273	0.37132589350021143	-4.407246334878962	2.1520510610161916	0.04096288193038378	726
2	-0.6815194602232680	0.7526352967390848	0.36553238508388901	-4.484159218429189	2.0808564264448030	0.03153395153388172	712
3	-0.5772086949689542	0.6577538918009318	0.3805235790401591	-4.4523061061786215	2.0654399198739752	0.031489479613124205	698
4	-0.4373402097290635	0.5421571948465105	0.4201677942462114	-4.2003242302490300	1.9009679147804481	0.027499632309753386	684
5	-0.3747964236514404	0.4295668166728997	0.38328145485260556	-3.9819035440354424	1.8065231279822795	0.027886170124632015	670
6	-0.329235710789607 4	0.34834843422204553	0.34497510250153174	-3.7019443285884366	1.6135328945858944	0.0221207202560085 3	656
7	-0.26111939957565733	0.3088499411191260	0.39820367064837137	-3.2537993345280674	1.3445057202024820	0.01582322562256855	642
8	-0.21780694814372112	0.25967902373082846	0.40195993232691873	-2.8789157999561117	1.1242581676270080	0.010702737034401721	628
9	-0.18318898018090363	0.257395151648987 1	0.47694755750328055	-2.4855439365712290	0.9537494600744989	0.009404544316587993	614
10	-0.06436000277739314	0.23649899836984228	0.7856207229884717	-2.1278864690347397	0.7754978080769568	0.006272603559545509	600

Notes: State-dependent local projection (LP) estimates for WR by horizon. "High" and "Low" report regime-specific responses. Standard errors are reported by regime; p-values correspond to tests of the null that the regime-specific coefficient equals zero. The number of observations declines with horizon due to sample truncation.

Table 1: Table 3.25.6: State-dependent LP for LS (Horizon 0–10)

Horizon	β_{high}	SE (high)	p-value (high)	β_{low}	SE (low)	p-value (low)	N
0	0.03555156180487643	0.17240063818039647	0.8366855931584198	-0.20795809863722517	0.2644272631126331	0.43188273907606467	740
1	0.04698904802664717	0.17525440908381887	0.7886915747756671	-0.30786012973485954	0.33310814544722533	0.35571800345030313	726
2	-0.02981500528398610	0.14710947747305210	0.8394550472652713	-0.35335149789018155	0.3464283927186106	0.3081177658548673	712
3	-0.07904967044920906	0.12205300084177996	0.5174353389528616	-0.34167983735374130	0.3147351801391334	0.27806457872195534	698
4	-0.06098028863884135	0.09904683027898030	0.5383374334131252	-0.39600684324492885	0.3007880097035818	0.18847107911256455	684
5	-0.03444168770462962	0.09011856667239711	0.7024601923565332	-0.40648986215447985	0.3014494624621563	0.17801624644678715	670
6	0.01434987332210347	0.08756787829245173	0.8698881014134585	-0.36042749720463740	0.25238056007452936	0.15378534158302461	656
7	0.05915631197244847	0.08652796586467160	0.4944580477273339	-0.27382423598182890	0.19972526116422165	0.17090288740961745	642
8	0.07039525665855142	0.09458472520443413	0.4570293531253709	-0.19361151160654996	0.15746352642748254	0.21936942151321714	628
9	0.10106686547702552	0.10995925872476656	0.35842680747573796	-0.11136464465101001	0.11325090116949123	0.32586641391542903	614
10	0.13217457411673810	0.10680091326106411	0.21640804135689717	-0.02486782435895467	0.08756606812107784	0.77652705297865900	600

Notes: State-dependent local projection (LP) estimates for LS by horizon. "High" and "Low" report regime-specific responses. Standard errors are reported by regime; p-values correspond to tests of the null that the regime-specific coefficient equals zero. The number of observations declines with horizon due to sample truncation.

Similarly to LP-based models, we note a common pattern of discrepancies across WR and LP (Table 3.25.5). The state-dependent local projections reveal a pronounced asymmetry in the transmission of monetary policy shocks to real compensation (WR) across inflation regimes, while no robust regime dependence is detected for the labour share (LS). For WR, the response under low-inflation regimes is systematically negative and statistically significant at most horizons. At horizon 0, the estimated effect in low inflation is -4.10 ($p \approx 0.059$), becoming larger in absolute value and more precisely estimated at short to medium horizons: -4.41 at h = 1 ($p = 0.041$), -4.48 at h = 2 ($p = 0.032$), -4.45 at h = 3 ($p = 0.031$), and remaining significantly negative through h = 10, where the effect is still -2.13 ($p \approx 0.006$). The magnitude declines monotonically with the horizon, indicating a gradual but persistent compression of real compensation following a contractionary monetary policy shock in low-inflation environments.

In contrast, the high-inflation regime displays small and imprecisely estimated WR responses at all horizons (Table 3.25.6). Coefficients range from -0.52 at h = 0 to -0.06 at h = 10, with p-values consistently well above conventional significance levels. This sharp contrast implies that real wages are substantially more responsive to unexpected monetary tightening when inflation is low, whereas in high-inflation regimes nominal rigidities, indexation mechanisms, or bargaining adjustments appear to dampen the real wage response.

For the labour share, neither regime exhibits statistically significant responses across horizons. In the high-inflation regime, coefficients fluctuate around zero, alternating in sign and remaining small in magnitude (e.g., 0.036 at h = 0, -0.061 at h = 4, 0.132 at h = 10), with all p-values above 0.20. In the low-inflation regime, estimates are predominantly negative at short horizons (around -0.21 to -0.42 between h = 0 and h = 5) but remain statistically insignificant throughout, and they converge toward zero at longer horizons. The absence of significant effects for LS suggests that the adjustment to monetary shocks operates primarily through real wage dynamics rather than through changes in the functional income distribution between labour and capital.

To sum up, the state-dependent results indicate that the contractionary effects of monetary policy on labour income are concentrated on real compensation and are conditional on the inflation environment, with strong and persistent effects in low-inflation regimes and muted responses in high-inflation regimes, while the labour share remains broadly insulated from regime-specific monetary shocks.

Table 1: Table 3.25.7: WR — Wald Test for Equality of Regime Responses (Horizon 0–10)

Horizon	Wald Statistic	*p*-value	Note
0	4.914435899653436	0.02663315879859751	dropped/absorbed
1	5.504284636781177	0.01896993829222482	dropped/absorbed
2	6.012230784795015	0.01420705564183955	dropped/absorbed
3	5.917655915533066	0.01498988860476569	dropped/absorbed
4	6.150230813863572	0.01313949517513635	dropped/absorbed
5	5.527726706542182	0.01871740892665019	dropped/absorbed
6	5.528264399393469	0.01871165755403248	dropped/absorbed
7	6.231264147526129	0.01255141015185790	dropped/absorbed
8	7.153421836205580	0.007482110068235737	dropped/absorbed
9	7.455351024373313	0.006324814354237929	dropped/absorbed
10	9.268768148782970	0.002330944219474662	dropped/absorbed

Notes: Wald tests for the null hypothesis that the regime-specific local projection coefficients are equal across "High" and "Low" states at each horizon. *p*-values correspond to chi-square tests. The note "dropped/absorbed" indicates horizons where fixed effects or controls were absorbed in estimation.

As robustness test, the Wald tests provide formal evidence of state dependence for real compensation (WR) but not for the labour share (LS). For WR, the null hypothesis of equal impulse responses across high- and low-inflation regimes is rejected at all horizons from 0 to 10, with p-values systematically below 5% and often below 1% (e.g., p ≈ 0.027 at h = 0, declining to p ≈ 0.002 at h = 10). This indicates that the response of real wages to a monetary policy shock differs significantly depending on the inflation environment, and that this heterogeneity persists over both short and medium horizons. By contrast, for LS the null of equality cannot be rejected at any horizon: p-values remain well above conventional significance levels (typically between 0.07 and 0.26), implying that labour share responses are statistically similar across inflation regimes. Again, these results corroborate the regime-specific LP estimates: monetary policy shocks have state-contingent effects on real compensation, while the labour share exhibits no robust state dependence, suggesting more muted or structurally invariant adjustment dynamics across inflation regimes (Table 3.25.7).

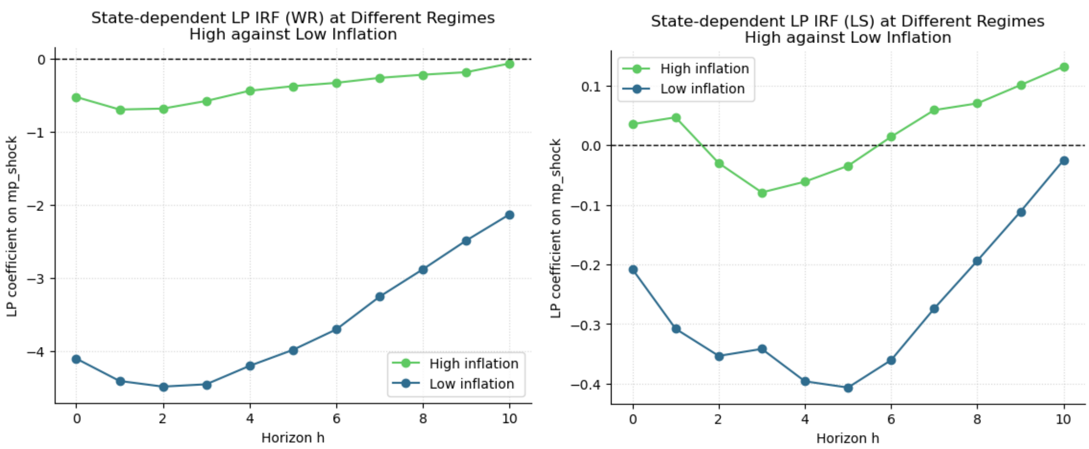
Table 1: Table 3.25.8: LS — Wald Test for Equality of Regime Responses (Horizon 0–10)

Horizon	Wald Statistic	p-value	Note
0	3.220789094621431	0.07270857219630311	dropped/absorbed
1	3.2197897493217043	0.07275297406342807	dropped/absorbed
2	1.8106593067938965	0.1784291600720953	dropped/absorbed
3	1.2582388888572373	0.2619847083119703	dropped/absorbed
4	1.8793027940612141	0.17041341089698414	dropped/absorbed
5	1.7541948803363216	0.18535024538277356	dropped/absorbed
6	2.0100033269409363	0.15626497540263107	dropped/absorbed
7	2.0260807051194520	0.15461889469456425	dropped/absorbed
8	1.6036388190791506	0.2053882984008455	dropped/absorbed
9	1.5058267984051534	0.21977698590425665	dropped/absorbed
10	1.2495512793832988	0.2636381977905997	dropped/absorbed

Notes: Wald tests for the null hypothesis that the regime-specific local projection coefficients are equal across "High" and "Low" states at each horizon. p-values correspond to chi-square tests. The note "dropped/absorbed" indicates horizons where fixed effects or controls were absorbed in estimation.

In conclusion, we present the results from the IRF computation and plotting for the two state-dependent LP of WR and LP, in order to allow for comparability with the first basic LP models. For real compensation (WR), the response is negative in both regimes, but the magnitude and persistence differ sharply. In the low-inflation regime, the initial response is large and strongly negative (around -4 at horizons 0-2), indicating a pronounced contraction in real wages following a monetary tightening. The effect remains economically meaningful over the entire horizon, although it gradually attenuates and converges toward zero by horizon 10. In contrast, under high inflation, the response of WR is much smaller in absolute value (around -0.5 to -0.7 initially) and displays a monotonic reversion toward zero. This asymmetry indicates that monetary policy shocks transmit much more forcefully to real wages when inflation is low, consistent with stronger real rigidities and higher real interest rate pass-through in such environments. Eventually, for the labour share (LS), the responses are again regime-dependent but weaker overall. In the low-inflation regime, the labour share declines substantially following the shock, with the most negative response occurring around horizons 3-5 (approximately -0.35 to 0.40), before gradually recovering toward zero. This pattern suggests that monetary tightening compresses labour income relative to output when inflation is subdued, likely through employment or bargaining channels. By contrast, in the high-inflation regime, the LS response is close to zero at short horizons and turns mildly positive at longer horizons.

Figure 3.25.9 – State-dependent LP IRF (WR & LS) at Different Regimes High against Low Inflation



4. To-Do List

1. Panel Structure and Data Diagnostics

1.1 Panel indices, dimensions (N, T), and frequency

1.2 Classification of variables by dimension (country, time, global)

1.3 Balanced vs. unbalanced structure and data availability

1.4 Implications for lags, autocorrelation, and estimators

2. Descriptive Analysis in Levels

2.1 Time evolution and KDE distributions of target variables

2.2 Cross-country dispersion and boxplot diagnostics

2.3 Deviations from cross-sectional means and persistence of heterogeneity

3. Variance Decomposition and Transformations

3.1 Construction of pooled, between, and one-way within transformations

3.2 Variance computation and between/within variance shares

3.3 Interpretation of sources of variation

3.4 Three-dimensional variance visualization **(nice, but not sure I will do it again)**

4. Distributional Properties: Between vs. Within

4.1 Histogram, kernel density, and normal approximation

4.2 Skewness, kurtosis, and non-normality tests

4.3 Extreme observations and tail behaviour

5. First-Difference Transformations

5.1 Construction and validity checks for first differences

5.2 Distributional properties and comparison with level-based transformations

5.3 Short-run volatility and tail risk

6. Bivariate Relationships Across Transformations

6.1 Simple correlations in between, within, and first differences

6.2 Scatterplots with marginal distributions and fitted lines

6.3 Sensitivity of sign and magnitude to transformation

7. Two-Way Fixed Effects (Balanced Panel)

7.1 Balanced-sample restriction and justification

7.2 Decomposition of common time effects

7.3 TWFE-transformed variables and country-level residual behaviour

8. Correlation Structure After TWFE

8.1 TWFE correlation matrix and multicollinearity assessment

8.2 Country-level correlation heterogeneity and implied slopes

9. Two-Way Fixed Effects in Unbalanced Panels

9.1 Limitations of demeaning in non-balanced panels

9.2 Regression-based and Wansbeek–Kapteyn TWFE transformations

9.3 Distributional comparison with balanced TWFE

10. Cross-Estimator Distribution Comparison

10.1 Between, within, TWFE, and first-difference boxplots by country

10.2 Differences in dispersion, means, and tails across estimators

11. Univariate Descriptive Statistics Across Transformations

11.1 Central tendency, dispersion, and mean–median differences

11.2 Standardized extremes and leverage diagnostics

12. Correlation Matrices and Lag Structure

12.1 Between vs. within correlation matrices

12.2 Lagged correlations, autocorrelation, and effective sample size

13. Functional-Form Diagnostics

13.1 Linear, quadratic, and LOWESS fits

13.2 Comparison across transformations and interpretation

14. Benchmark Panel Estimators & Dynamic Panel Extension

14.1 Between, one-way FE, Mundlak, TWFE, and first differences

14.2 Comparison across labour-market model specifications

14.3 Detection of residual persistence

14.4 GMM and other Anderson-Hsiao dynamic specification and instruments

14.5 IVs relevance and quality checks

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