Sphere-Edge intersection

EMT

Due: 30/08/2017

- Sphere S centred in $\mathbf{O} = (O_x, O_y, O_z)$ of radius R:
$$(x - O_x)^2 + (y - O_y)^2 + (z - O_z)^2 = R^2;$$
- Open segment ξ limited by $\mathbf{v_1} = (v_{1x}, v_{1y}, v_{1z})$ and $\mathbf{v_2} = (v_{2x}, v_{2y}, v_{2z})$ in parametric form:
$$\mathbf{x} = t(\mathbf{v_1} - \mathbf{v_2}) + \mathbf{v_2} = (tc_x + v_{2x}, tc_y + v_{2y}, tc_z + v_{2z}), \qquad t \in (0, 1),$$
where $(c_x, c_y, c_z) = \mathbf{C}$;
- it will be useful to call $\mathbf{D} = (\mathbf{v_1} - \mathbf{O})$.

Looking for intersections:

$$(c_x t + v_{2x} - O_x)^2 + (c_y t + v_{2y} - O_y)^2 + (c_z t + v_{2z} - O_z)^2 = R^2$$
$$\dots$$
$$t^2(||\mathbf{C}||^2) + t(2\mathbf{c} \cdot \mathbf{D}) + (||\mathbf{D}||^2 - R^2) = 0$$

In short, using trivial replacements:

$$t^2\alpha + t\beta + \gamma = 0.$$

Since β is even, we use $\frac{\Delta}{4}$.

In the code, this is implemented in *extract_ring*.