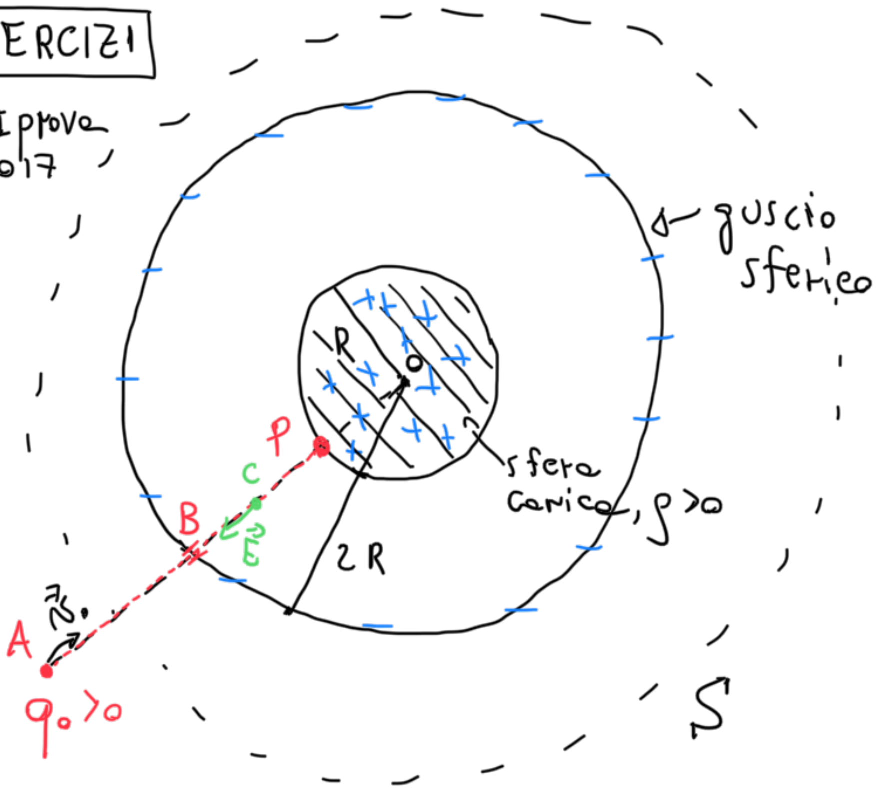
ESERCIZI

P. 4 II prova
5/6/2017



Vogliamo che $\vec{E}$ fuori dal guscio sia nullo

$\Rightarrow$ quanto vale σ?

$$\oint_{\vec{E}}(S)=0$$

Legge di Gauss: $\oint_{\vec{E}}(S)=\frac{Q_{int}}{\varepsilon_0}$

$$Q_{int}(S)=0$$

$$Q_{int}(S)=0 \iff \underline{Q_{guscio}+Q_{sfera}=0}$$

$$Q_{guscio}=\sigma\cdot S_{guscio}=\sigma\cdot 4\pi(2R)^2=16\pi R^2\sigma$$

$$Q_{sfera}=\rho\cdot V_{sfera}=\rho\cdot\frac{4}{3}\pi R^3=\frac{4}{3}\pi R^3\rho$$

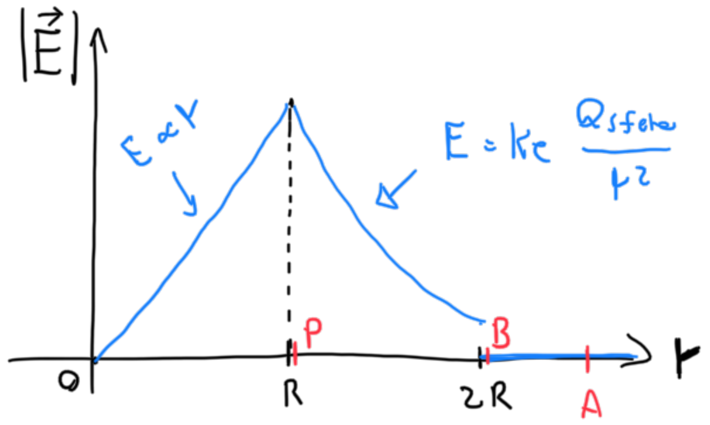
$$16\cancel{\pi}\cancel{R^2}\sigma+\frac{4}{3}\cancel{\pi}R^{\cancel{3}}\rho=0 \quad\Rightarrow\quad 16\sigma=-\frac{4}{3}R\rho$$

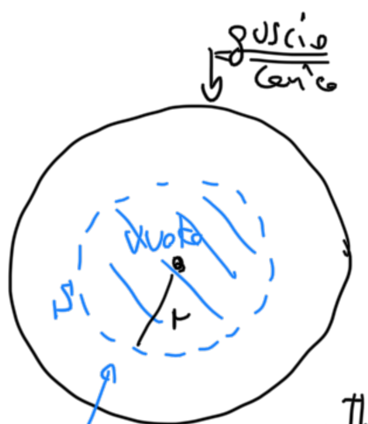
$$\sigma=-\frac{\cancel{4}}{3\cdot\cancel{16}_4}R\rho=\ominus\frac{1}{12}\rho R.$$

se $\rho>0 \Rightarrow \sigma<0$

quanto vale $\Delta K(q_0)$

se va da un punto fuori dal guscio, fino alla superficie della sfera?





$\oint_{\vec{E}}(S)=\frac{Q_{int}}{\varepsilon_0}=0$ Th. Gauss

$\oint_{\vec{E}}(S)=E\cdot 4\pi r^2$ def. di flusso

$\Rightarrow E=0$

$$\Delta K=K_P-K_A=$$

$$=L_{\vec{E}}=\cancel{L_{\vec{E}}(A\to B)}+\underline{L_{\vec{E}}(B\to P)}=$$

perché $\vec{E}=0$

$$=\int_B^P\vec{F}_{el}\cdot d\vec{s}=\int_B^P(\underbrace{q_0\vec{E}}_{\vec{F}_{el}})\cdot d\vec{s}=q_0\int_B^P\vec{E}\cdot d\vec{s}=$$

$\vec{E}$ ha la stessa direzione di $d\vec{s}$

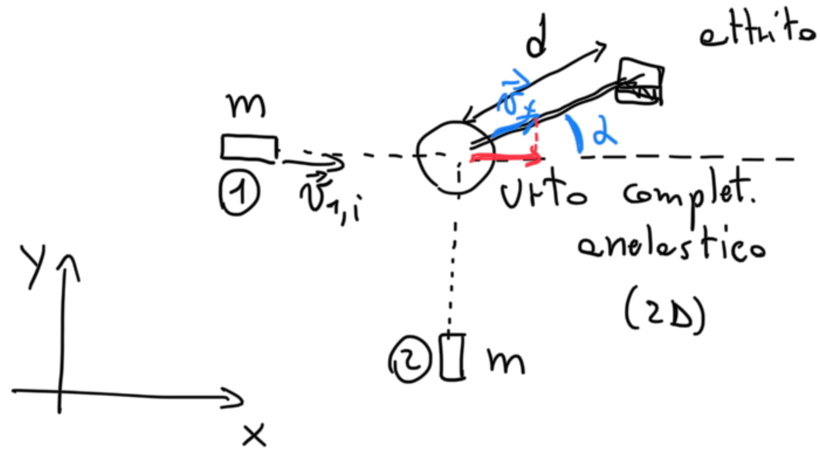
$$= -q_0 \int_B E\,dr = -q_0 \int_B k_e \frac{Q_{sfera}}{r^2}\,dr =$$

$$= -q_0 k_e Q_{sfera}\cdot\left(-\frac{1}{r}\Big|_{B=2R}^{P=R}\right) = -q_0 k_e Q_{sfera}\cdot\Big(\overbrace{-\frac{1}{R}+\frac{1}{2R}}^{-\frac{1}{2R}}\Big) = \frac{q_0 k_e Q_{sfera}}{2R}$$

$$\Rightarrow \Delta K = \frac{q_0 k_e}{\cancel{2}\cancel{R}}\cdot\frac{\cancel{4}^{2}}{3}\pi R^{\cancel{3}^{2}}\rho = \frac{2}{3}\pi k_e q_0 \rho R^2 .$$

(Conservazione energia: $\Delta K + \Delta U_{el} = 0 \Rightarrow \Delta K = -\Delta U_{el}$)

P.5 R II prova (13/6/2017)



attrito dinamico μ_d

determinare
- $\vec{v}_{2,i}$
- α

$$\text{II}\left[\; m\vec{v}_{1,i} + m\vec{v}_{2,i} = 2m\vec{v}_f \quad\Rightarrow\quad \begin{cases} m v_{1,i} = 2m v_{f,x} = 2m v_f \cos\alpha \\ m v_{2,i} = 2m v_{f,y} = 2m v_f \sin\alpha \end{cases}\right.$$

moto dopo l'urto:

$$\begin{cases} \bullet\; s(t) = s_0 + v_f t - \frac{1}{2} a t^2 \\ \bullet\; v(t) = v_f - at \end{cases}$$

(moto uniformemente decelerato)

$$|\vec{a}| = \frac{F_d}{2m} = \frac{\mu_d N}{2m} = \frac{\mu_d (2m\cdot g)}{2m} = \mu_d g$$

T tempo impiegato per percorrere la distanza d $\Rightarrow v(T) = 0 \Rightarrow 0 = v_f - aT$

$$T = \frac{v_f}{a} = \frac{v_f}{\mu_d g}$$

$$s(T) = s_0 + v_f T - \frac{1}{2} a T^2$$

$$= s_0 + v_f\cdot\frac{v_f}{\mu_d g} - \frac{1}{2}\cancel{\mu_d g}\cdot\frac{v_f^2}{\mu_d^{\cancel{2}} g^{\cancel{2}}} = s_0 + \frac{v_f^2}{\mu_d g} - \frac{1}{2}\frac{v_f^2}{\mu_d g} = s_0 + \frac{v_f^2}{2\mu_d g}$$

$$\Rightarrow d = \frac{v_f^2}{2\mu_d g} \Rightarrow v_f = \sqrt{2\mu_d g\, d}$$

$$v_f^2 = v_{f,x}^2 + v_{f,y}^2$$

$$\begin{cases} \cancel{m} v_{1,i} = 2\cancel{m} v_f \cos\alpha \\ \cancel{m} v_{2,i} = 2\cancel{m} v_f \sin\alpha \end{cases} \Rightarrow \begin{cases} v_{1,i} = 2v_{f,x} \rightarrow v_{f,x} = \frac{1}{2} v_{1,i} \\ v_{2,i} = 2v_{f,y} \end{cases} *$$

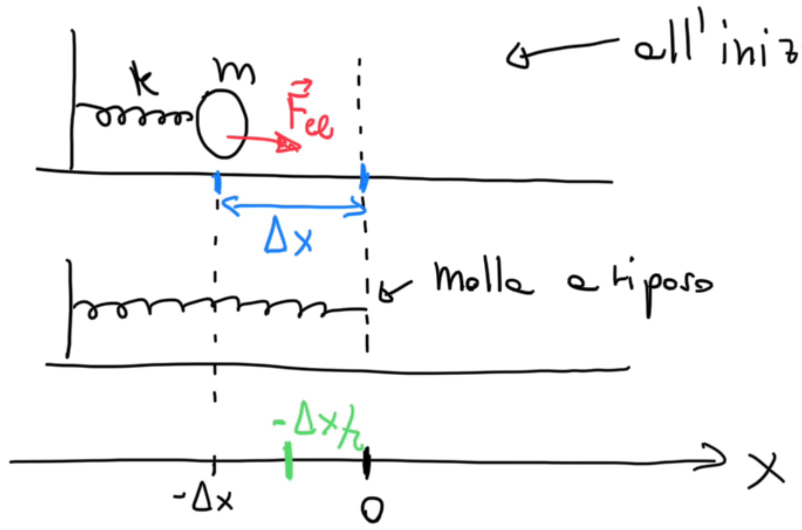
$\Rightarrow v_{f,y} = \frac{1}{2} v_{2,i}$

$$* \; 4 v_f^2 = \cancel{4} \left(v_{1,i}^2 + v_{2,i}^2 \right)$$

$$\Rightarrow v_{2,i}^2 = 4 v_f^2 - v_{1,i}^2 \quad \Rightarrow v_{2,i} = \sqrt{4(2\mu_d g d) - v_{1,i}^2}\,.$$

$$\begin{cases} v_f \sin\alpha = v_{2,i} \\ v_f \cos\alpha = v_{1,i} \end{cases} \Rightarrow \tan\alpha = \frac{v_{2,i}}{v_{1,i}} \Rightarrow \alpha = \arctan\left(\frac{\sqrt{8\mu_d g d - v_{1,i}^2}}{v_{1,i}} \right).$$

P.5 R II (14/6/2018)



← all'inizio

← molla a riposo

- determinare v quando siamo a metà dell'ampiezza delle oscillazioni
- dove sta il corpo dopo un tempo $\frac{1}{8}$ del periodo?

$$m \frac{d^2 x}{dt^2} = -k x \Rightarrow x(t) = A \cos(\omega t + \phi)$$

$A, \phi \to$ cond. iniziali

$$\Updownarrow$$

$$\boxed{\frac{d^2 x}{dt^2} + \omega^2 x = 0}$$

eq. differenziale

$$\omega = \sqrt{\frac{k}{m}}$$

$$\begin{cases} x(t=0) = -\Delta x \\ v(t=0) = 0 \end{cases}$$

$$\Downarrow$$

$$x(t) = A\cos(\omega t + \phi) \quad \downarrow \left(\frac{d}{dt}\right) \quad v(t) = -A\omega \sin(\omega t + \phi)$$

$$\begin{cases} -\Delta x = A\cos\phi \\ 0 = -A\omega \sin\phi \end{cases}$$

$\phi = 0$; $A = -\Delta x$

$$\Rightarrow \underline{x(t) = -\Delta x \cos\left(\sqrt{\frac{k}{m}}\, t\right)}$$

$$v(t) = +\Delta x\, \omega \sin(\omega t)$$

trovare t^* tale che $x(t^*) = -\frac{\Delta x}{2}$

$$x(t^*) = -\Delta x \cos(\omega t^*) = -\frac{\Delta x}{2} \Rightarrow \cos(\omega t^*) = \frac{1}{2}$$

quando $\boxed{\omega t^* = \frac{\pi}{3}} = 60°$

$$v(t^*) = \Delta x \,\omega \sin(\omega t^*)$$

$$= \Delta x \sqrt{\frac{k}{m}} \cdot \underbrace{\sin\left(\frac{\pi}{3}\right)}_{\sqrt{3}/2} = \frac{\Delta x}{2}\sqrt{\frac{3k}{m}} \quad (*)$$

$$\left[\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}\right]$$

Oppure: Cons. energia tra istante iniziale ($x = -\Delta x$) e istante in cui $x = -\frac{\Delta x}{2}$

$$\cancel{\frac{1}{2}} k \Delta x^2 = \cancel{\frac{1}{2}} m v^2 + \cancel{\frac{1}{2}} k \left(\frac{\Delta x}{2}\right)^2$$

$$m v^2 = k \Delta x^2 - k \cdot \frac{\Delta x^2}{4} = \frac{3}{4} k \Delta x^2 \Rightarrow v = \sqrt{\frac{3}{4}\frac{k}{m}\Delta x^2} \quad (*)$$

- dove sta la massa a $\bar{t} = \frac{T}{8}$? T periodo $\left(T = \frac{2\pi}{\omega} \text{ per definizione, noto } \omega\right)$

$$x\left(\bar{t} = \frac{T}{8}\right) = -\Delta x \cos(\omega \bar{t}) =$$

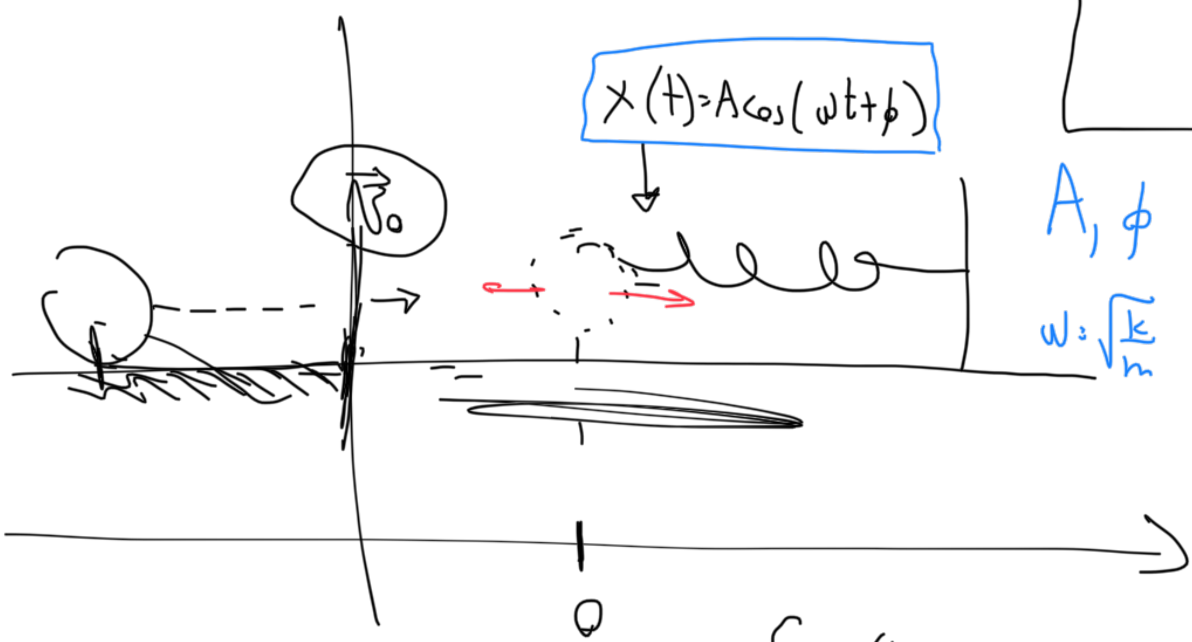
$$= -\Delta x \cos\left(\omega \cdot \frac{T}{8}\right) = -\Delta x \cos\left(\cancel{\omega} \frac{2\pi}{8\cancel{\omega}}\right) =$$

$$= -\Delta x \cos\left(\frac{\pi}{4}\right) = \underbrace{-\frac{\sqrt{2}}{2}\Delta x}$$

rispetto alla posizione iniziale: $x(\bar{t}) - x(t=0) = -\frac{\sqrt{2}}{2}\Delta x - (-\Delta x)$

$$= \Delta x \left(1 - \frac{\sqrt{2}}{2}\right).$$

$\boxed{x(t) = A\cos(\omega t + \phi)}$



A, ϕ

$\omega = \sqrt{\frac{k}{m}}$

$$\begin{cases} x(t=0) = 0 & \rightarrow 0 = A\cos\phi \\ v(t=0) = v_0 & \rightarrow v_0 = -A\omega \sin\phi \end{cases}$$

$\phi = \frac{\pi}{2}$

$v_0 = -A\omega$ ...

$A w$

$\cos\phi = 0$

$\sin\phi = 1$

$\Downarrow$

$A = -\frac{v_0}{w}$

$$x(t) = -\frac{v_0}{w}\cos\left(wt + \frac{\pi}{2}\right) =$$

$$= \frac{v_0}{w}\sin(wt)$$