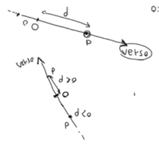
Prerequisiti Matematici

Sistemi di riferimento

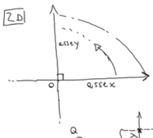
1D



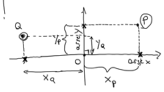
O: origine

- P sta a destra di O $\quad d>0$
- P sta a sinistra di O $\quad d<0$

2D



asse x si ruota di 90°
e in quel modo coincide con y



O: origine

$P \equiv (x_p, y_p)$ $\quad x_p > 0$, $y_p > 0$

coordinate cartesiane

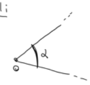
$Q \equiv (x_Q, y_Q)$

$x_Q < 0$

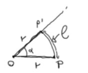
$y_Q > 0$

2 coordinate

Angoli

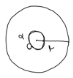


$$\alpha = \frac{\ell}{r}$$



$\overline{OP} = r$

$\overline{OP'} = r$



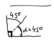
$$\alpha_{giro} = \frac{\ell_{circ.}}{r}$$

$\ell_{circ.} = 2\pi r \quad \Rightarrow \quad \alpha_{giro} = \frac{2\pi \cancel{r}}{\cancel{r}} = 2\pi$

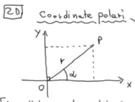
angoli misurati in radianti (rad)

$$360^\circ : 2\pi = \alpha\,(\text{in gradi}) : \alpha\,(\text{in rad})$$

Es. $\alpha = 45^\circ$ $\Rightarrow$ $\alpha\,(\text{in rad}) = \dfrac{2\pi \cdot \alpha\,(\text{in gradi})}{360} = \dfrac{2\pi \cdot 45}{360}$ rad $= \dfrac{90\pi}{360}$ rad $= \dfrac{\pi}{4}$ rad



2D Coordinate polari $\rightarrow$ $P \equiv (r, \alpha)$

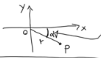


[$P \equiv (x, y)$ coordinate cartesiane]

- r: distanza tra origine O e P
- α: angolo tra OP e asse x

$r \geq 0$ sempre

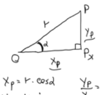
$\alpha > 0$ se asse x ruota in senso antiorario per sovrapporlo a OP

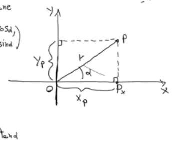


$\alpha < 0$ se asse x … in senso orario …

Coord. polari $\Rightarrow$ cartesiane

$$(r, \alpha) \longrightarrow \begin{pmatrix} x_p = r\cos\alpha, \\ y_p = r\sin\alpha \end{pmatrix}$$





$$\begin{cases} x_p = r \cdot \cos\alpha \\ y_p = r \cdot \sin\alpha \end{cases} \qquad \frac{y_p}{x_p} = \frac{\sin\alpha}{\cos\alpha} = \tan\alpha$$

Coord. cartesiane $\Rightarrow$ polari

$$(x_p, y_p) \longrightarrow (r, \alpha)$$

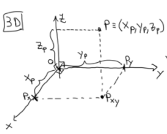
$r^2 = x_p^2 + y_p^2 \Rightarrow r = \sqrt{x_p^2 + y_p^2}$

$\alpha = \arctan\left(\dfrac{y_p}{x_p}\right)$

- $x_p > 0 \Rightarrow \alpha = \arctan \dfrac{y_p}{x_p}$
- $x_p < 0 \Rightarrow \alpha = \arctan\left(\dfrac{y_p}{x_p}\right) + \pi$
- $x_p = 0 \Rightarrow \begin{cases} y_p > 0 & \alpha = \pi/2 \\ y_p < 0 & \alpha = \dfrac{3\pi}{2} \end{cases}$

$\alpha \in [0, 2\pi)$

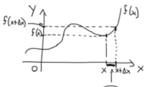
3D



$P \equiv (x_p, y_p, z_p)$

Versi orientati in modo da determinare una terna destrorsa di assi

DERIVATA $y = f(x)$

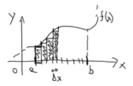


$$y' = \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$\frac{d}{dx}(x^p) = p x^{p-1} \quad (p \neq 0)$$

$$\frac{d}{dx}\left(\alpha \cdot f(x)\right) = \alpha \frac{d}{dx} f(x)$$

INTEGRALE



$$\int_a^b f(x)\,dx = \lim_{\Delta x \to 0} \left[\sum_{i=1}^{\frac{b-a}{\Delta x}} f(x_i) \cdot \Delta x \right]$$

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

$$f(x) = \frac{dF(x)}{dx}$$

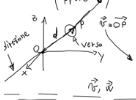
integrale è l'inverso della derivata

Algebra Vettoriale (cenni) (Capitolo 2)

- grandezze scalari (numero ± errore + unità di misura)
  es. massa, tempo, temperatura, lavoro, ...
- grandezze vettoriali → $\begin{cases} \text{valore scalare (MODULO)} \\ \text{direzione} \\ \text{verso} \end{cases}$

sempre positivo $|\vec{v}|$ oppure v

$\vec{v}$ oppure $\underline{v}$ oppure $\mathbf{v}$

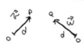


$\vec{v} = \vec{OP}$ vettore posizione del punto P rispetto al sistema di riferimento disegnato

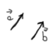
$|\vec{v}| = v = \underline{d}$

$\vec{v}, \vec{w}$ $\quad \vec{v} + \vec{w} \stackrel{?}{=} |\vec{v}| + |\vec{w}|$ dipende (in genere NO)

$\boxed{\vec{v} = \vec{w}} \Rightarrow |\vec{v}| = |\vec{w}|$ ($\not\Leftarrow$)



$|\vec{v}| = |\vec{w}|$ MA $\vec{v} \neq \vec{w}$



$\vec{a} = \vec{b}$

$P \equiv (x_p, y_p, z_p)$

$\vec{OP} \equiv \vec{v}$

$\vec{v} \begin{cases} |\vec{v}| \\ \text{dir.} \\ \text{verso} \end{cases}$ oppure (x_p, y_p, z_p)

$\vec{v} = \vec{w} = (x_Q, y_Q, z_Q)$, $\vec{v} = (x_p, y_p, z_p)$ $\iff \begin{cases} x_p = x_Q \\ y_p = y_Q \\ z_p = z_Q \end{cases}$

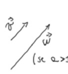
$|\vec{v}| = \sqrt{x_p^2 + y_p^2 + z_p^2}$ modulo (Pitagora)

Operazioni con i vettori

- prodotto di $\vec{v}$ per uno scalare $a \in \mathbb{R}$ → è un vettore $\vec{w}$

$\vec{w} = a\vec{v}$ $\quad \vec{v} = (v_x, v_y, v_z) \Rightarrow \vec{w} = (a v_x, a v_y, a v_z)$

$\vec{v} = \begin{cases} |\vec{v}| \text{ modulo} \\ \text{dir.} \\ \text{verso} \end{cases} \Rightarrow \vec{w} = \begin{cases} |a| \cdot |\vec{v}| \\ \text{dir.} = \text{a quella di } \vec{v} \\ \text{verso} \begin{cases} \text{uguale a quello di } \vec{v} \text{ se } a > 0 \\ \text{opposto a quello di } \vec{v} \text{ se } a < 0 \end{cases} \end{cases}$



(se $a > 1$)

$\vec{u} = (u_x, u_y, u_z)$

$\vec{v} = (v_x, v_y, v_z)$

$\Rightarrow \boxed{\vec{u}+\vec{v} = (u_x+v_x,\ u_y+v_y,\ u_z+v_z)}$

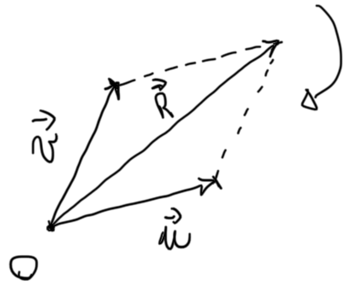
$\neq |\vec{u}|+|\vec{v}| = \sqrt{u_x^2+u_y^2+u_z^2} + \sqrt{v_x^2+v_y^2+v_z^2}$

~~$\vec{v}$ + e~~ NO!

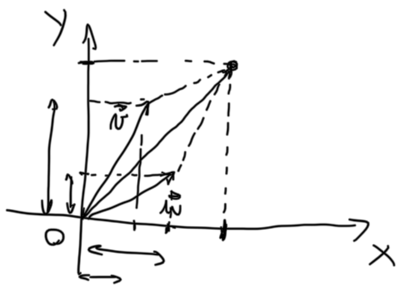
$\vec{v}+\vec{u}$ sì

è un vettore (vettore RISULTANTE) $\vec{R} = \vec{u}+\vec{v}$

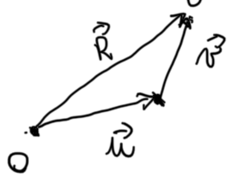
→ metodo del parallelogramma



$\vec{R}$ è la diagonale



→ metodo triangolare (o punta-coda)



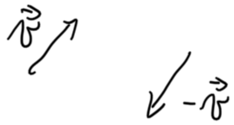
proprietà della somma di vettori

commutativa: $\vec{u}+\vec{v} = \vec{v}+\vec{u}$

associativa: $\vec{u}+(\vec{v}+\vec{w}) = (\vec{u}+\vec{v})+\vec{w}$

Sottrazione di vettori

$\vec{u}-\vec{v} = \vec{u}+(-\vec{v})$ vettore opposto di $\vec{v}$



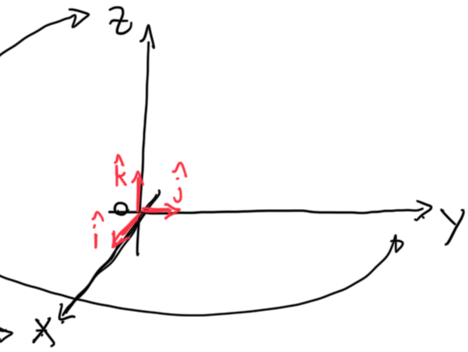
vettore opposto

Vettore scomposto in componenti

$\vec{u} = (u_x, u_y, u_z)$

definisco i versori $\hat{i}, \hat{j}, \hat{k}$

con modulo = 1



$\vec{u} = \underbrace{u_x\hat{i}}_{\equiv\ \vec{u}_x} + \underbrace{u_y\hat{j}}_{\equiv\ \vec{u}_y} + \underbrace{u_z\hat{k}}_{\equiv\ \vec{u}_z}$

simbolo ^ usato per i versori

" → " vettori