Database L
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3.2.9 Exercise for Section 3.2 from *Database Systems: The Complete Book*

Exercise 3.2.1. Consider a relation with schema $R(A, B, C, D)$ and FD's $AB \to C, C \to D, D \to A$.

a) What are all nontrivial FD's that follow from the given FD's? You should restrict yourself to FD's with single attributes on the right side.
b) What are all the keys of R?
c) What are all the superkeys for R that are not keys?

Solution:

1. $\{A\} + = \{A\}$
   $X = \{A\}$

2. $\{B\} + = \{B\}$
   $X = \{B\}$

3. $\{C\} + = \{A, C, D\}$
   $X = \{C\}\ \&\ C \to D$
   $X = \{C, D\}\ \&\ D \to A$
   $X = \{A, C, D\}$
   $C \to A$ is new functional dependency

4. $\{D\} + = \{A, D\}$
   $X = \{D\}\ \&\ D \to A$
   $X = \{A, D\}$

5. $\underline{\{A, B\}} + = \{A, B, C, D\}$ - key
   $X = \{A, B\}\ \&\ AB \to C$
   $X = \{A, B, C\}\ \&\ C \to D$
   $X = \{A, B, C, D\}$
   $AB \to D$ is new func. dep.

6. $\{A, C\} + = \{A, C, D\}$
   $X = \{A, C\}\ \&\ C \to D$
   $X = \{A, C, D\}$
   $AC \to D$ is new func. dep.

7. $\{A, D\} + = \{A, D\}$
   $X = \{A, D\}$ the coverage of $\{A, D\}$ cannot be expand

8. $\underline{\{B, C\}} + = \{A, B, C, D\}$ - key
   $X = \{B, C\}\ \&\ C \to D$
   $X = \{B, C, D\}\ \&\ D \to A$
   $X = \{A, B, C, D\}$
   $BC \to A$ and $BC \to D$ are new (nontrivial) functional dependencies

9. $\underline{\{B, D\}} + = \{A, B, C, D\}$ - key
   $X = \{B, D\}\ \&\ D \to A$
   $X = \{A, B, D\}\ \&\ AB \to C$
   $X = \{A, B, C, D\}$
   $BD \to A$ and $BD \to C$ are new functional dependencies

10. $\{C, D\} + = \{A, C, D\}$
    $X = \{C, D\}\ \&\ D \to A$
    $X = \{A, C, D\}$ is the coverage of $\{C, D\}$
    $CD \to A$ is new func. dep.

11. $\underline{\underline{\{A, B, C\}}} + = \{A, B, C, D\}$
    - superkey, since $\{A, B\} \subseteq \{A, B, C\}$ and $\underline{\{A, B\}}$ is a key.

12. $\underline{\underline{\{A, B, D\}}} + = \{A, B, C, D\}$
    - superkey, since $\{A, B\} \subseteq \{A, B, D\}$ and $\underline{\{A, B\}}$ is a key.

13. $\{A, C, D\} + = \{A, C, D\}$ there are no new attributes, which can

be added to the coverage of $\{A, C, D\}$

14. $\{B, C, D\} += \{A, B, C, D\}$ - superkey, since $\{B, C\}$ is a key and $\{B, C\} \subseteq \{B, C, D\}$.

15. $\{A, B, C, D\}$ is trivial superkey.

Exercise 3.5.2.

$S(A, B, C, D), A \to B, B \to C, B \to D$

Solution:

1. $\{A\} += \{A, B, C, D\}$ - key
$X = \{A\}$ & $A \to B$
$X = \{A, B\}$ & $B \to C$ & $B \to D$
$X = \{A, B, C, D\}$
$A \to C$ and $A \to D$ are new functional dependencies.

2. $\{B\} += \{B, C, D\}$
$X = \{B\}$ & $B \to C$ & $B \to D$
$X = \{B, C, D\}$

3. $\{C\} += \{C\}$ there are no new attributes, which can be addded to the coverage of $\{C\}$

4. $\{D\} += \{D\}$ similar to C

5. $\{A, B\} += \{A, B, C, D\}$
- superkey, since $\{A\}$ is a key and $\{A\} \subseteq \{A, B\}$

6. $\{A, C\} += \{A, B, C, D\}$ - superkey, similar to $\{A, B\}$
$AC \to B$ and $AC \to D$
new func. dependencies

7. $\{A, D\} += \{A, B, C, D\}$
- superkey, similar to $\{A, B\}$

8. $\{B, C\} += \{B, C, D\}$
$X = \{B, C\}$ & $B \to D$
$X = \{B, C, D\}$
$BC \to D$ *nfd*

9. $\{B, D\} += \{B, C, D\}$
$X = \{B, D\}$ & $B \to C$
$X = \{B, C, D\}$
$BD \to C$ *nfd*

10. $\{C, D\} += \{C, D\}$
$X = \{C, D\}$ there are no nwe attributes, which can be added to the coverage of $\{C, D\}$

11. $\{A, B, C\} += \{A, B, C, D\}$
- superkey, since $\{A\} \subseteq \{A, B, C\}$ and $\{A\}$ is a key.
$ABC \to D$ new func. dep.

12. $\{A, B, D\} += \{A, B, C, D\}$
similar to 11.

13. $\{A, C, D\} += \{A, B, C, D\}$
similar to 11.
$ACD \to B$ new dunc. dep.

14. $\{B, C, D\} += \{B, C, D\}$

15. $\{A, B, C, D\}$ trivial superkey