Твърдение $X \in Ge(p)$. Тогава

а)

X	0	1	2	$\dots$	k	$\dots$
$\mathbb{P}$	p	pq	pq^2	$\dots$	pq^k	$\dots$

$\Rightarrow \mathbb{P}(X=k) = pq^k$

б) $\cancel{g}$ $g_X(s) = \dfrac{p}{1-qs}$ $\quad |s| < 1$

Док.) а) $\mathbb{P}(X=0) = \mathbb{P}(X_1 = 1) = p$ $\quad 1$

$\mathbb{P}(X=1) = \mathbb{P}(X_1 = 0, X_2 = 1) = pq$ $\quad 01$

$\vdots$

$\mathbb{P}(X=k) = \mathbb{P}(X_1 = 0, X_2 = 0 \dots X_k = 0, X_{k+1} = 1)\ q^k p$ $\quad \underbrace{00\dots0}_{k}1$

$g_X(s) = \mathbb{E}s^X = \sum_{k=0}^{\infty} s^k \cdot pq^k = p \sum_{k=0}^{\infty} s^k q^k = \dfrac{p}{1-qs}$

Следствие $X \sim Ge(p) \Rightarrow \mathbb{E}X = \dfrac{q}{p}$

$DX = \dfrac{q}{p^2} = \mathbb{E}X^2 - (\mathbb{E}X)^2$

Док.) $g_X(s) = \dfrac{p}{1-qs}$ $\quad \mathbb{E}X = g'_X(1) = \dfrac{p \cdot q}{(1-qs)^2}\Big|_{s=1} = \dfrac{pq}{(1-q)^2} = \dfrac{pq}{p^2} = \dfrac{q}{p}$

$g''_X(1) = \dfrac{2pq^2}{(1-qs)^3}\Big|_{s=1} = \dfrac{2pq^2}{(1-q)^3} = \dfrac{2q^2}{p^2}$

$DX = \dfrac{2q^2}{p^2} + \dfrac{q}{p} - \left(\dfrac{q}{p}\right)^2 = \dfrac{q^2}{p^2} + \dfrac{q}{p} = \dfrac{q^2+qp}{p^2} = \dfrac{q}{p^2}$

⊕ брой ези до първо тура $p = \frac{1}{2}$

⊕ брой неуспешни опити до първи успех