$\mathbb{P}(X \in A) = \mathbb{E}\mathbb{1}_{\{X \in A\}} = \int_A f_X(t)\,dt$

$\mathbb{P}(A) = \sum_i \mathbb{P}(A|H_i) \cdot \mathbb{P}(H_i)$

total law of expectation

$\mathbb{E}X = \mathbb{E}\left(\mathbb{E}(X|Y)\right)$

$\mathbb{P}(k(A, AB) \in (0,1)) = \mathbb{E}\mathbb{1}_{\{k(A,AB) \in K\}} = \mathbb{E}\left(\mathbb{E}\left(\mathbb{1}_{\{k(A,AB \in K\}} \| OA\right)\right) = \mathbb{E}(1 - OA^2)$ $\cancel{OA \sim U[0,1]}$

$= \int_0^1 (1-x)^2 dx = 1/3$ $OA \sim U([0,1])$

$\mathbb{E}g(X) = \int_{\mathbb{R}} g(t) f_X(t)\,dt$

$X \sim \mathbb{E}xp$

$\mathbb{E}X = ?$

$\mathbb{E}X = \int_{-\infty}^{\infty} x f(x) dx = \int_{-\infty}^{\infty} x\, d F_X(x)$

интеграл на Лебег-Стилтес

$\int x\, d\sin x = \int x \cos x\, dx$

$X \sim Exp(\lambda) \xleftrightarrow{def} f_X(x) = \lambda e^{-\lambda x} \mathbb{1}_{\{x>0\}}$

$\int_0^{\infty} f_X(t)dt = \int_0^{\infty} \lambda e^{-\lambda t} dt = \int_0^{\infty} e^{-y} dy = \left[e^{-y}\right]_0^{\infty} = \frac{1}{-\lambda t}$ $y = \lambda t$

$\int_0^{\infty} x f(x) dx = \int_0^{\infty} t \cdot \lambda e^{-\lambda t} dt = \frac{1}{\lambda} \int_0^{\infty} y e^{-y} dy$

$\int e^{-y} dy = -e^{-y}$

$\int y e^{-y} dy = \int y\, d(e^{-y}) = -e^{-y} y + \int e^{-y} dy =$

$= e^{-y}(y+1)$