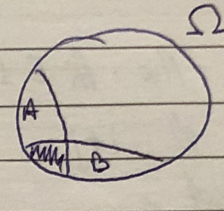
Док: а) $\overline{\Omega} = \emptyset \Rightarrow P(\emptyset) = 1 - P(\Omega) = 0$



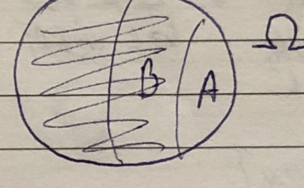
б) $P(A) + P(B) = P(A\cup B) + P(A\cap B)$

Ако $P(A\cap B) = 0 \Rightarrow P(A\cup B) = P(A) + P(B)$

в) $B = A \cup (B\cap A^c)$

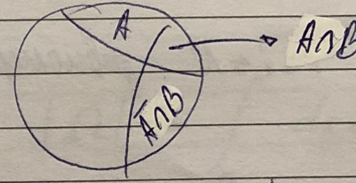
! $A^c = \bar{A}$ (допълн)

$P(B) = P(A) + P(B\cap \bar{A}) \geq P(A)$



г) $B = (A\cap B) \cup (\bar{A}\cap B)$

$P(B) = P(A\cap B) + P(\bar{A}\cap B)$



д) Ще док., че за $\forall n \geq 1$: (*) $P\left(\bigcup_{i=1}^{n} A_i\right) \leq \sum_{i=1}^{n} P(A_i)$

Индукция

$k=1$ $P(A_1) = P(A_1)$

Нека (*) е вярно за $k=n$, т.е. $P\left(\bigcup_{i=1}^{n} A_i\right) \leq \sum_{i=1}^{n} P(A_i)$

$k = n+1$, $\bigcup_{i=1}^{n+1} A_i = \bigcup_{i=1}^{n} A_i \cup A_{n+1} = B \cup A$ $\qquad B = \bigcup_{i=1}^{n} A_i$

$$P\left(\bigcup_{i=1}^{n+1} A_i\right) = P(B\cup A) \overset{б)}{=} P(B) + P(A_{n+1}) - P(A_{n+1}\cap B) \leq P(B) + P(A_{n+1}) =$$
$$= P\left(\bigcup_{i=1}^{n} A_i\right) + P(A_{n+1}) \overset{инд. k=n}{\leq} \sum_{i=1}^{n} P(A_i) + P(A_{n+1}) = \sum_{i=1}^{n+1} P(A_i)$$

$$P\left(\bigcup_{i=1}^{\infty} A_i\right) \overset{???}{=} \lim_{n\to\infty} P\left(\bigcup_{i=1}^{n} A_i\right) \leq \lim_{n\to\infty} \sum_{i=1}^{n} P(A_i) = \sum_{i=1}^{\infty} P(A_i)$$

Дискретна вероятност

Пр: $\Omega = \{0;1\}$ $\qquad P(\{0\}) = p \in [0;1]$

$\qquad P(\{1\}) = 1-p \in [0;1]$

$\Omega = \{w_1, \dots, w_N\}$, $\mathcal{A} = 2^{\Omega}$

Дефиниция: Ако $\Omega = \{w_1, \dots, w_N\}$, то вс. N-орка $(p_1, \dots, p_N)$: $\sum_{i=1}^{N} p_i = 1$ се нарича дискретна вероятност в/у 2^{Ω}, ако $p_i \geq 0$ $\forall i \in [1;N]$

$P(\{w_i\}) = p_i$ $\forall 1 \leq i \leq N$ и $\forall A \subseteq \Omega$

$P(A) = \sum_{w_i \in A} p_i$