$X_1, X_2 \sim U(0,1) \qquad t \in (0,2)$

$$f_{X_1+X_2}(t) = \int_{-\infty}^{\infty} f_{X_1}(u) f_{X_2}(t-u)\,du$$

I сл. $t \in (0,1)$: т.к. плътността $f_{X_2}(t-u)$ е ненулева само в интервала $(0,1)$, то

$$I = \int_0^t f_{X_1}(u) f_{X_2}(t-u)\,du = \int_0^t 1\,du = t$$

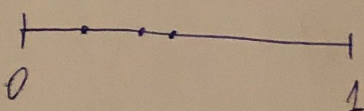
II сл. $t \in [1;2)$

$$t-u<1 \Rightarrow u > t-1$$

$$I = \int_{t-1}^{1} 1\,du = 1-(t-1) = 2-t$$

Зад. $X_1, X_2 \ldots X_n$ iid $\sim X$ за $\min(X_1 \ldots X_n)$ или $\max(X_1, \ldots X_n)$

Искаме да изразим разпределението им чрез това на X



Нека $Z = \max(X_1 \ldots X_n)$

$$\mathbb{P}(Z \le t) = \mathbb{P}(\max(X_1 \ldots X_n) \le t) \overset{\text{независ}}{=} \mathbb{P}(X_1 \le t; X_2 \le t \ldots X_n \le t) =$$
$$= \mathbb{P}(X_1 \le t)\mathbb{P}(X_2 \le t) \ldots \mathbb{P}(X_n \le t) \overset{\text{от еднакво разпр.}}{=} (\mathbb{P}(X \le t))^n$$

* $X_1, X_2 \sim Exp(\lambda) \quad X_1 \perp\!\!\!\perp X_2$

Тогава $F_Z(t) = \left(\int_0^t \lambda e^{-\lambda x} dx\right)^2 = (1-e^{-\lambda t})^2$

$f_Z(t) = 2(1-e^{-\lambda t})e^{-\lambda t}\lambda$ за $t>0$

Нека $Y = \min(X_1, X_2 \ldots X_n)$

$$\mathbb{P}(Y \le t) = \mathbb{P}(\min(X_1 \ldots X_n) \le t) = 1-\mathbb{P}(\min(X_1 \ldots X_n) > t) = 1-\mathbb{P}(X_1>t, X_2>t \ldots X_n>t) =$$
$$\overset{iid}{=} 1-(\mathbb{P}(X>t))^n = 1-(1-\mathbb{P}(X \le t))^n = 1-(1-F_X(t))^n$$