⑦ $X \in N(\mu, \sigma^2)$, но σ^2 не се знае. Как да конструираме доверителен интервал само за μ?

$$\hat{\mu} = \overline{X_n} \qquad S^2 = \frac{1}{n-1} \cdot \sum_{j=1}^{n} (X_j - \overline{X_n})^2$$

<u>Твърдение</u> $X \in N(\mu, \sigma^2)$ $\vec{X} = (X_1 \dots X_n)$. Тогава:

а) $\mu \perp\!\!\!\perp S^2$

б) $\dfrac{(n-1)S^2}{\sigma^2} \in \chi^2(n-1)$

$$T = \frac{\dfrac{\overline{X_n} - \mu}{\sigma/\sqrt{n}} \to N(0,1)}{\sqrt{\dfrac{(n-1)}{(n-1)}\underbrace{\dfrac{S^2}{\sigma^2}}_{Y \in \chi^2_{(n-1)}}}} = \frac{\overline{X_n} - \mu}{\frac{S}{\sqrt{n}}} \text{ е у.с}$$

$$\| $$

$$\frac{Z}{\sqrt{\frac{Y}{n-1}}} \qquad Z \in N(0,1) \quad Z \perp\!\!\!\perp Y, \quad Y \in \chi^2(n-1)$$

- T е намаляваща по μ
- $T \in t(n-1)$ и не зависи от μ

$\implies T$ е у.с за μ

$$T = \frac{Z}{\sqrt{\frac{Y}{n-1}}} \qquad Z = \frac{\overline{X_n} - \mu}{\sigma/\sqrt{n}}; \quad Y = (n-1)\frac{S^2}{\sigma^2}$$

$$\gamma = \mathbb{P}\left(-q_{\frac{1}{2}+\frac{\alpha}{2}} < T < q_{\frac{1}{2}+\frac{\alpha}{2}}\right) = \mathbb{P}\left(\mu \in \left(\overline{X_n} - q_{\frac{1}{2}+\frac{\alpha}{2}}\frac{S}{\sqrt{n}}, \overline{X_n} + q_{\frac{1}{2}+\frac{\alpha}{2}}\frac{S}{\sqrt{n}}\right)\right)$$

$$I_1 = \overline{X_n} - q_{\frac{1}{2}+\frac{\alpha}{2}}\frac{S}{\sqrt{n}}; \quad I_2 = \overline{X_n} + q_{\frac{1}{2}+\frac{\alpha}{2}}\frac{S}{\sqrt{n}}$$

$$\overline{X_n} = \frac{\sum_{j=1}^{n} X_j}{n} \in N(x, y^2)$$

$$\mathbb{E}\overline{X_n} = \frac{n \cdot \mu}{n} = \mu = x$$

$$Var(\overline{X_n}) = \frac{1}{n^2}\sum_{j=1}^{n}\sigma^2 = \frac{\sigma^2}{n} = y^2$$