$X_1 \sim N(0, \sigma_1^2)$

$X_2 \sim N(0, \sigma_2^2)$ $\quad X_1 \perp\!\!\!\perp X_2$

$$f_X(x) = f_{X_1X_2}(x_1, x_2) = \underbrace{\frac{1}{\sqrt{2\pi}\,\sigma_1} \cdot e^{\frac{-x^2}{2\sigma_1^2}}}_{f_{X_1}(x_1)} \cdot \underbrace{\frac{1}{\sqrt{2\pi}\,\sigma_2}\, e^{\frac{-x^2}{2\sigma_2^2}}}_{f_{X_2}(x_2)} =$$

$$= \frac{1}{2\pi\sigma_1\sigma_2}\, e^{\frac{-x_1^2}{2\sigma_1^2} - \frac{x_2^2}{2\sigma_2^2}} \qquad D_X = \mathbb{R}^2$$

$Y_1 = X_1 + X_2$

$Y_2 = X_1$ $\quad = Y = g(X)$

$$\Longrightarrow g(D_X) = g(\mathbb{R}^2) = \mathbb{R}^2$$

$$y = g(x) \iff x = h(y) = \begin{pmatrix} y_1 - y_2 \\ y_2 \end{pmatrix}$$

$$|J(y)| = \left| \det \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \right| = 1;$$

$$f_Y(y) = f_X(h(y)) \cdot 1 = \frac{1}{2\pi\sigma_1\sigma_2}\, e^{\frac{-(y_1-y_2)^2}{2\sigma_1^2} - \frac{y_2^2}{2\sigma_2^2}} \qquad \forall\, y = (y_1, y_2) \in \mathbb{R}^2$$

$Y_1 = X_1 + X_2 \qquad Y_2 = X_2$

$$(**)\ \frac{(y_1 - y_2)^2}{\sigma_1^2} + \frac{y_2^2}{\sigma_2^2} \overset{*}{=} (y_1^2 + (by_2 + ay_1)^2$$

$$f_{Y_1}(y_1) = \frac{1}{2\sigma_1\sigma_2} \int_{-\infty}^{\infty} e^{\frac{-(y_1-y_2)^2}{2\sigma_1^2} - \frac{y_2^2}{2\sigma_2^2}}\, dy_2 \overset{*}{=}$$

$$= \frac{e^{\frac{-y_1^2 c}{2}}}{2\pi\sigma_1\sigma_2} \int_{-\infty}^{\infty} e^{-1/2(by_1 - ay_1)^2}\, dy_2 \overset{by_2 = w}{=} \frac{e^{\frac{-y_1^2 c}{2}}}{2\pi\sigma_1\sigma_2 b} \int_{-\infty}^{\infty} e^{-1/2(w-ay)}\, dw \overset{v = w - ay_1}{=}$$

$$= \frac{e^{\frac{-y_1^2 c}{2}}}{2\pi\sigma_1\sigma_2 b} \int_{-\infty}^{\infty} e^{-1/2 v^2}\, dw = \frac{e^{-\frac{y_1^2 c}{2}}}{\sqrt{2\pi}\,\sigma_1\sigma_2 b}$$

$$f_{Y_1}(y_1) = \frac{e^{-\frac{y_1^2 c}{2}}}{\sqrt{2\pi}\,\sigma_1\sigma_2 b} = \frac{e^{\frac{-y_1^2}{2} \cdot \frac{1}{\sigma_1^2 + \sigma_2^2}}}{\sqrt{2\pi}\,\sigma_1\sigma_2 \cdot}$$

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$$(**) = \frac{y_1^2}{\sigma_1^2} - \frac{2y_1y_2}{\sigma_1^2} + \frac{y_2^2}{\sigma_1^2} + \frac{y_2^2}{\sigma_2^2} =$$

$$= y_2^2 \underbrace{\left(\frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}\right)}_{b^2} - \underbrace{\frac{2y_1y_2}{\sigma_1^2} + y_1^2}_{} + \frac{y_1^2}{\sigma_1^2} =$$

$$b^2 = \frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2} = \frac{\sigma_1^2 + \sigma_2^2}{\sigma_1^2\sigma_2^2} \qquad a^2 = \frac{1}{\sigma_1^4} \cdot \frac{1}{b^2} = \frac{\sigma_2^2}{(\sigma_1^2 + \sigma_2^2)\sigma_1^2}$$

$$= \left(y_2^2 b^2 - \frac{2y_1y_2}{\sigma_1^2} \cdot \frac{b}{b} + \frac{y_1^2}{b^2} \cdot \sigma_1^4 - \frac{y_1^2}{\sigma_1^4 b^2}\right) + \frac{y_1^2}{\sigma_1^2}$$

$$c = \frac{1}{\sigma_1^2} - \frac{\sigma_2^2}{(\sigma_1^2 + \sigma_2^2)\sigma_1^2} = \frac{1}{\sigma_1^2}\left(\frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2}\right) = \frac{1}{\sigma_1^2 + \sigma_2^2}$$

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