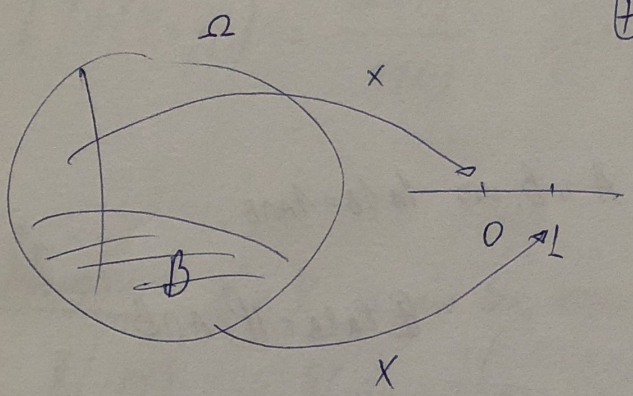
$$\min_{a,b} \mathbb{E}(X - a\mathbb{1}_A - b\mathbb{1}_{A^c})^2 = \min_{a,b} \underbrace{(\mathbb{E}X^2 + a^2\mathbb{E}\mathbb{1}_A + b^2\mathbb{E}\mathbb{1}_B - 2a\mathbb{E}X\mathbb{1}_A - 2b\mathbb{E}X\mathbb{1}_{A^c})}_{f(a,b)}$$

$$\min_{(a,b)} f(a,b) \Rightarrow \quad 0 = \frac{\partial f}{\partial a}$$

$$0 = \frac{\partial f}{\partial b}$$

$$G(Y) = \frac{\mathbb{E}X\mathbb{1}_A}{\mathbb{P}(A)}\mathbb{1}_A + \frac{\mathbb{E}X\mathbb{1}_{A^c}}{\mathbb{P}(A^c)}\mathbb{1}_{A^c} = \frac{\mathbb{E}(X,Y)}{\mathbb{P}(Y=1)}\mathbb{1}_{\{Y=1\}} + \frac{\mathbb{E}(X(1-Y))}{\mathbb{P}(Y=0)}\mathbb{1}_{\{Y=0\}}$$

⊕ $X = \mathbb{1}_B$



$$\mathbb{E}(X|Y) = \mathbb{E}(\mathbb{1}_B|Y) = \frac{\mathbb{E}\mathbb{1}_B\mathbb{1}_A}{\mathbb{P}(A)}\mathbb{1}_A + \frac{\mathbb{E}\mathbb{1}_B\mathbb{1}_{A^c}}{\mathbb{P}(A^c)}\mathbb{1}_{A^c} =$$

$$= \frac{\mathbb{P}(A\cap B)}{\mathbb{P}(A)}\mathbb{1}_A +$$

Твърдение Нека X и Y са случайни, като Y е дискретна

Y	y_1	...	y_j
$\mathbb{P}$	p_1		p_j

$$Y = \sum y_j \mathbb{1}_{A_j} \qquad A_j = \{Y = y_j\}$$

$$\mathbb{P}(A_j) = p_j$$

Тогава $\mathbb{E}[X|Y] = \sum_j \frac{\mathbb{E}X\mathbb{1}_{A_j}}{\mathbb{P}(A_j)} \cdot \mathbb{1}_{A_j}$

Количествено $\mathbb{E}(X|Y=y_j) := \frac{\mathbb{E}X_{A_j}}{\mathbb{P}(A_j)}$ е каква условно очакване на X при положение $Y = y_j$