Твърдение: Нека X и Y са две независими дискр. сл. вел. Тогава $D(X+Y) = DX + DY$

Док: $Z = X+Y \quad DZ = \mathbb{E}(Z-\mathbb{E}Z)^2 = \mathbb{E}(X+Y-\mathbb{E}(X+Y))^2 =$

$$= \mathbb{E}((X-\mathbb{E}X+Y-\mathbb{E}Y)^2) = \mathbb{E}\left((X-\mathbb{E}X)^2 + (Y-\mathbb{E}Y)^2 + 2(X-\mathbb{E}X)(Y-\mathbb{E}Y)\right) =$$

$$= \mathbb{E}(X-\mathbb{E}X)^2 + \mathbb{E}(Y-\mathbb{E}Y)^2 + 2\mathbb{E}(X-\mathbb{E}X)(Y-\mathbb{E}Y) =$$

$$= DX + DY + 2\mathbb{E}(XY - Y\mathbb{E}Y - Y\mathbb{E}X + \mathbb{E}X\mathbb{E}Y) =$$

$$= DX + DY + 2\mathbb{E}XY - 2\mathbb{E}Y\mathbb{E}Y \cancel{- 2\mathbb{E}X\mathbb{E}Y + 2\mathbb{E}X\mathbb{E}Y} =$$

$$= DX + DY + \cancel{2\mathbb{E}X\mathbb{E}Y} - \cancel{2\mathbb{E}X\mathbb{E}Y} \quad \#$$

Пораждаща функция

В тази тема X винаги ще е целочислена и неотрицателна $X \in \mathbb{N}^+$

Дефиниция: Нека $X \in \mathbb{N}^+$, тогава функ. $g_X(s) = \mathbb{E}s^X = \sum_{k=0}^{\infty} s^k \,\mathbb{P}(X=k)$, за $|s| \le 1$ се нарича пораждаща функция на X.

⊕

X	0	1
$\mathbb{P}$	$1-p$	p

$$g_X(s) = \sum_{k=0}^{\infty} s^k \,\mathbb{P}(X=k) = (1-p)\cdot s^0 + p s^1 = 1-p+ps$$

⊕

X	1	2	...	n
$\mathbb{P}$	$1/n$	$1/n$		$1/n$

$$g_X(s) = \sum_{k=1}^{n} s^k \frac{1}{n} = \frac{s(1-s^n)}{n(1-s)}$$

Свойства

1) $g_X^{(n)}(0) = n!\,\mathbb{P}(X=n)$

$$g_X^{(n)}(s)\Big|_{s=0} = \sum_{k=0}^{\infty} (s^k)^{(n)}\,\mathbb{P}(X=k)\Big|_{s=0} = (s^n)^{(n)}\,\mathbb{P}(X=n) = n!\,\mathbb{P}(X=n)$$

2) $g_X'(1) = \mathbb{E}X$

$$\frac{d}{ds} g_X(s) = \frac{d}{ds}\mathbb{E}s^X = \mathbb{E}\frac{d}{ds}s^X = \mathbb{E}Xs^{X-1}$$

$$g_X'(1) = \mathbb{E}X \quad \#$$

3) $DX = g_X''(1) + g_X'(1) - (g_X'(1))^2$

$\quad = \mathbb{E}X^2 - (\mathbb{E}X)^2$