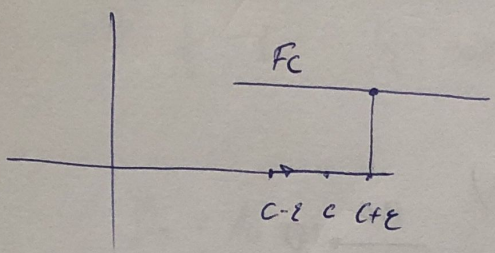
Твърдение Ако $X_n \xrightarrow{d} c$, тогава $X_n \xrightarrow{\mathbb{P}} c$

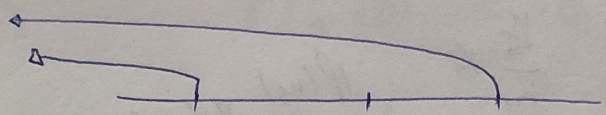
Док: $F_{X_n}(x) \xrightarrow[n\to\infty]{} F_c(x) \quad \forall x \in \mathbb{R}\setminus\{c\}$ Цел: $\forall \varepsilon > 0$

$$\mathbb{P}(|X_n - c| \le \varepsilon) \xrightarrow[n\to\infty]{} 1$$



~~$\mathbb{P}(|X_n - c| \le \varepsilon) = \mathbb{P}(X_n - c \le \varepsilon) + \mathbb{P}(X_n > c - \varepsilon)$~~

$$\mathbb{P}(|X_n - c| \le \varepsilon) = \mathbb{P}(X_n \le c+\varepsilon) - \mathbb{P}(X_n < c-\varepsilon)$$



$\downarrow n \to \infty$

$F_c(x-\varepsilon) = 0$

$$\lim_{n\to\infty} \mathbb{P}(|X_n - c| \le \varepsilon) = \lim_{n\to\infty} \mathbb{P}(X_n \le c+\varepsilon)$$

$$\mathbb{P}(X_n \le c+\varepsilon) \ge \mathbb{P}(X_n < c+\varepsilon) \xrightarrow[n\to\infty]{} 1 = F_c(x+\varepsilon)$$

Неравенство на Чебишов

Твърдение (Чебишов) Нека X е сл.вел с $\mathbb{E}X$ и $\mathbb{D}X$. Нека $a > 0$. Тогава:

$$\mathbb{P}(|X - \mathbb{E}X| > a) \le \frac{\mathbb{D}X}{a^2} \qquad 1 = 1_A + 1_{A^c}$$

Док: $A = \{|X - \mathbb{E}X| > a\} = \{(X - \mathbb{E}X)^2 > a^2\}$ $\quad \mathbb{P}(A) \le ?$

$$\mathbb{D}X = \mathbb{E}(X - \mathbb{E}X)^2 \cdot 1 \overset{\ge}{=} \mathbb{E}(X - \mathbb{E}X)^2 1_A + \cancel{\mathbb{E}(X - \mathbb{E}X)^2 \cdot 1_{A^c}} \ge a^2 \mathbb{E}1_A = a^2 \mathbb{P}(A) \Rightarrow$$

$$\Rightarrow \mathbb{P}(A) \le \frac{\mathbb{D}X}{a^2}$$

$$a = b\sqrt{\mathbb{D}X} \Rightarrow \mathbb{P}(|X - \mathbb{E}X| > b\sqrt{\mathbb{D}X}) \le \frac{1}{b^2}$$