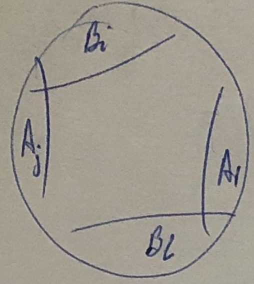
⊕ $X = \mathbb{1}_B$ $\quad \mathbb{E}[\mathbb{1}_B | Y = y_j] = \dfrac{\mathbb{E}\mathbb{1}_B \mathbb{1}_{A_j}}{\mathbb{P}(A_j)} = \dfrac{\mathbb{P}(B \cap A_j)}{\mathbb{P}(A_j)} = \mathbb{P}(B | A_j) = \mathbb{P}(X = 1 | Y = y_j)$

$B = \{X = 1\}$

⊕ $X = \sum_i x_i \mathbb{1}_{B_i}$ $\quad Y = \sum_j y_j \mathbb{1}_{A_j}$ и $p_{ij} = \mathbb{P}(X = x_i \cap Y = y_j)$



$$\mathbb{E}[X|Y] = \sum_j \frac{\mathbb{E}X\mathbb{1}_{A_j}}{\mathbb{P}(A_j)} \mathbb{1}_{A_j}$$

$$\mathbb{E}X\mathbb{1}_{A_j} = \mathbb{E}\left(\sum_i x_i \mathbb{1}_{B_i}\right)\mathbb{1}_{A_j} = \sum_i x_i \mathbb{E}\mathbb{1}_{B_i}\mathbb{1}_{A_j} = \sum x_i \mathbb{P}(B_i \cap A_j) = \sum x_i p_{ij}$$

$$\mathbb{E}(X | Y = y_j) = \frac{\sum x_i p_{ij}}{\mathbb{P}(A_j)} = \sum_i x_i \frac{\mathbb{P}(A_j \cap B_i)}{\mathbb{P}(A_i)} = \sum_i x_i \mathbb{P}(B_i | A_j) =$$
$$= \sum_i x_i \mathbb{P}(X = x_i | Y = y_j)$$

<u>Твърдение:</u> X и Y са случ. вел, където Y е дискретна. Тогава $\mathbb{E}(X|Y)$ е дискр. сл. вел

$$\mathbb{E}(X|Y) = \sum_j \mathbb{E}(X | Y = y_j)\mathbb{1}_{A_j} \qquad A_j = \{Y = y_j\}$$

$\mathbb{E}(X\|Y)$	— —	$\mathbb{E}(X\|Y = y_j)$
		$\mathbb{P}(A_j)$

<u>Теорема:</u> X, Z са сл. вел и Y е дискр. сл. вел $Y = \sum_j y_j \mathbb{1}_{A_j}$. Тогава:

а) $\mathbb{E}(aX + bZ | Y) = a\mathbb{E}(X|Y) + b\mathbb{E}(Z|Y) = \mathrm{I}$

б) $X \perp\!\!\!\perp Y$, то $\mathbb{E}(X|Y) = \mathbb{E}X = \mathrm{II}$

в) $X = g(Y)$, то $\mathbb{E}(X|Y) = g(Y) = \mathrm{III}$

г) $\mathbb{E}(\mathbb{E}(X|Y)) = \mathbb{E}X = \mathrm{IV}$

д) $\mathbb{E}(f(U,Y) | Y = y_j) = \mathbb{E}(f(U,y))$ където V

$U = X$

$U = (X_1, \dots, X_n)$ $\qquad U = (X_i)_{i \geq 1}$ ако $U \perp\!\!\!\perp Y$