$$Y = \frac{X-\mu}{\sigma} \sim N(0,1)$$

$$EY = \frac{1}{\sigma} EX - \frac{\mu}{\sigma} \Rightarrow 0 = \frac{EX}{\sigma} - \frac{\mu}{\sigma} \Rightarrow EX = \mu$$

$$EY = \int_{-\infty}^{\infty} y f_Y(y)\,dy = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} y e^{-\frac{y^2}{2}} dy = 0$$

$ye^{-\frac{y^2}{2}}$ е нечетна функ.

$$DY = EY^2 = \frac{1}{\sigma^2} DX \Rightarrow DY = 1 = \frac{DX}{\sigma^2} \Rightarrow DX = \sigma^2$$

$$EY^2 = \int_{-\infty}^{\infty} y^2 f_Y(y)\,dy = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} y^2 e^{-\frac{y^2}{2}} dy = 1$$

$$1 = \frac{1}{\sqrt{2\pi}\,\sigma} \int_{-\infty}^{\infty} e^{\frac{-y^2}{2\sigma^2}} dy \quad \forall \sigma > 0$$

$$\sqrt{2\pi}\,\sigma = \int_{-\infty}^{\infty} e^{\frac{-y^2}{2\sigma^2}} dy$$

$$\sqrt{2\pi} = \int_{-\infty}^{\infty} \frac{d}{d\sigma} e^{\frac{-y^2}{2\sigma^2}} dy = \int \frac{y^2}{2} \cdot \frac{2}{\sigma^3} e^{\frac{-y^2}{2\sigma^2}} dy$$

$\sigma = 1$

$$\sqrt{2\pi} = \int_{-\infty}^{\infty} y^2 e^{-\frac{y^2}{2}} dy$$