док) а) $1 = \mathbb{P}(L)$ $\quad L = \{\lim_{n\to\infty} X_n = X\} \overset{?}{=} \bigcap_{r=1}^{\infty} \bigcup_{n=1}^{\infty} \bigcap_{k\geq n} A^c_{k,1/r}$

$\Pi_{k,r} = A^c_{k,1/r} = \{|X_k - X| \leq 1/r\}$

фикс k $\ldots \supseteq \Pi_{k,r-1} \supseteq \Pi_{k,r} \supseteq \Pi_{k,r+1} \supseteq \ldots$

$B_{n,r} = \bigcap_{k\geq n} \Pi_{k,r}$

фикс k $\ldots \supseteq B_{k,r-1} \supseteq B_{k,r} \supseteq B_{k,r+1} \supseteq \ldots$

$L \overset{?}{=} \bigcap_{r=1}^{\infty} \bigcup_{r=1}^{\infty} B_{n,r} = \bigcap_{r=1}^{\infty} C_r$

фикс r $B_{k-1,r} \subseteq B_{k,r} \subseteq B_{k+1,r} \subseteq \ldots$

$C_r = \bigcup_{n=1}^{\infty} B_{n,r}$

фикс r $\supseteq C_{r-1} \supseteq C_r \supseteq C_{r+1} \supseteq \ldots$

$L \overset{?}{=} C = \bigcap_{r=1}^{\infty} C_r$ $\quad C_r \supseteq C_{r-1}$

Нека $\bar\omega \in L \Rightarrow \forall r \geq 1\ \exists n_r\ n \geq n_r\ |X_n(\bar\omega) - X(\bar\omega)| \leq 1/r \Rightarrow$

$\Rightarrow \bar\omega \in \Pi_{n,r}\ \forall n \geq n_r \Rightarrow \bar\omega \in B_{n_r,r}$

$\Rightarrow \bar\omega \in B_{n_r,r} \subseteq \bigcup_{n=1}^{\infty} B_{n,r} = C_r \quad \forall r \geq 1$

$\Rightarrow \omega \in C = \bigcap_{r=1}^{\infty} C_r$

Нека $\bar\omega \in C \Rightarrow \bar\omega \in C_r\ \forall r \geq 1 \Rightarrow \exists n_r : \bar\omega \in B_{n_r,r} \subseteq B_{n,r}\ \forall n \geq n_r$

$\Rightarrow \bar\omega \in B_{n,r}\ \forall n \geq n_r \Rightarrow \bar\omega \in \Pi_{k,r}\ \forall k \geq n_r$

$|X_k(\bar\omega) - X(\bar\omega)| \leq 1/r \quad \forall k \geq n_r$

$L = C$ $\quad 1 = \mathbb{P}(L) = \mathbb{P}(C)$

$\Rightarrow \mathbb{P}(C_r) = 1\ \forall r \geq 1$ $\quad B_{n,r} \subseteq \Pi_{n,r}$

$C \subseteq C_r\ \forall r \geq 1$

$1 = \mathbb{P}(C_r) = \Pi\left(\bigcup_{n\geq 1}^{\infty} B_{n,r}\right) = \lim_{n\to\infty} \mathbb{P}(B_{n,r}) \leq \lim_{n\to\infty} \mathbb{P}(\Pi_{n,r}) \leq 1$

$\Rightarrow \lim_{n\to\infty} \mathbb{P}(\Pi_{n,r}) = \lim_{n\to\infty} \mathbb{P}(|X_n - X| \leq 1/r) = 1 \Rightarrow \lim_{n\to\infty} \mathbb{P}(A_{n,r}) = 0$