$X_n \xrightarrow[n\to\infty]{\mathbb{P}} X \overset{?}{\Longrightarrow} X_n \xrightarrow[n\to\infty]{d} X \qquad \forall \varepsilon>0 \quad \lim_{n\to\infty} \mathbb{P}(A_{n,\varepsilon})=0 \qquad A_{n,\varepsilon}=\{|X_n-X|>\varepsilon\}$

$\varepsilon = \frac{1}{r}$

$\varepsilon \in \left(\frac{1}{r}, \frac{1}{r-1}\right)$

$F_{X_n}(x) \longrightarrow F_X(x) \quad \forall x \in C_{F_X}$

$A_{n,\frac{1}{r-1}} \subseteq A_{n,\varepsilon} \subseteq A_{n,\frac{1}{r}}$

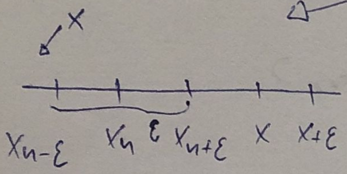
Избираме $x \in C_{F_X}$ и целим $\lim\limits_{n\to\infty} \mathbb{P}(X_n<x)=\mathbb{P}(X<x)$

$A_{n\varepsilon}^c = \{|X_n-X|\le\varepsilon\}$

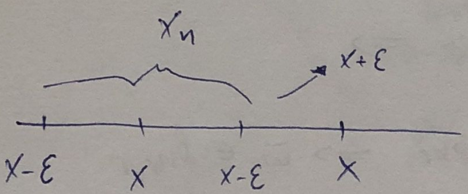
фикс $\varepsilon>0$

$\{X<x-\varepsilon\}\cap A_{n,\varepsilon}^c \subseteq \{X_n<x\}\cap A_{n,\varepsilon}^c \subseteq \{X_n<x\} = \{X_n<x\}\cap \underbrace{(A_{n,\varepsilon}\cup A_{n,\varepsilon}^c)}_{=\Omega} =$

$= \{X_n<x\}\cap A_{n,\varepsilon}^c \cup \{\cancel{X_n<x}\}\cap A_{n,\varepsilon}$



$\subseteq \{X<x+\varepsilon\}\cup A_{n,\varepsilon}$



$\{X<x-\varepsilon\}\cap A_{n\varepsilon}^c \subseteq \{X_n<x\} \subseteq \{X<x+\varepsilon\}\cup A_{n\varepsilon}$

$\mathbb{P}(X<x-\varepsilon\cap A_{n\varepsilon}^c) < \mathbb{P}(X_n<x) \le \mathbb{P}(X<x+\varepsilon)+\mathbb{P}(A_{n\varepsilon})$

$\|$

$\mathbb{P}(X<x-\varepsilon) - \mathbb{P}(X\cancel{<x-\varepsilon}\cap A_{n,\varepsilon})$

$\cup|$

$\mathbb{P}(X<x-\varepsilon) - \mathbb{P}(A_{n,\varepsilon})$

$\mathbb{P}(X<x-\varepsilon) - \underbrace{\mathbb{P}(A_{n,\varepsilon})} \le \underbrace{\mathbb{P}(X_n<x)} \le \mathbb{P}(X<x+\varepsilon) + \underbrace{\mathbb{P}(A_{n,\varepsilon})}$

$F_X(x-\varepsilon) \qquad F_{X_n}(x) \qquad F_X(x+\varepsilon)$

$= F_X(x-\varepsilon) = \mathbb{P}(X<x-\varepsilon) \le \lim\limits_{n\to\infty} \mathbb{P}(X_n<x) \le \mathbb{P}(X<x+\varepsilon) = F_X(x+\varepsilon)$

При $\varepsilon\to 0 \quad F_X(x) \le \lim \mathbb{P}(X_n<x) \le F_X(x) \qquad F_{X_n}(x) \longrightarrow F_X(x)$

$\|$