Твърдение $X \sim NB(r, p)$. Тогава

X	$0 \ 1 \ \dots \ k \ \dots$
$\mathbb{P}$	$\binom{r+k-1}{k} p^r q^k$

$$g_X(s) = \left(\frac{p}{1-qs}\right)^r \quad k!\,\mathbb{P}(X=k) \overset{\text{знам}}{=} \frac{d^k}{ds^k} g_X(s)\Big|_{s=0} \qquad = \frac{p^r}{(1-qs)^r}$$

$$\frac{1}{(1-x)^r} = \sum_{k=0}^{\infty} \binom{r+k-1}{k} \frac{x^k}{k!} \qquad \frac{d^n}{dx^n} \cdot \frac{1}{(1-x)^r}\Big|_{x=0} = r(r+1)(r+1-n)\frac{1}{(1-x)^{r+n}}\Big|_{x=0} =$$

$$= r(r+1) \dots (r-n+1)$$

$$\Longrightarrow g_X(s) = p^r \cdot \sum_{k=0}^{\infty} \binom{r+k-1}{k} q^k \cdot s^k$$

$$\frac{d^k}{ds^k} g_X(s)\Big|_{s=0} = p^r \binom{r+k-1}{k} q^k k! = k!\,\mathbb{P}(\xi = k)$$

k - 0 нули r - 1 единици

$$\underbrace{\_\ \_\ \_\ \_\ \_}_{r+k-1}\ 1$$

$$\binom{r+k-1}{k} \cdot q^k \cdot p^{r-1} p$$

Д. Поасоново разпределение

Дефиниция: Нека $\lambda \geq 0$. Казваме, че $X \sim Poi(\lambda)$, ако $\mathbb{P}(X=k) = \frac{\lambda^k}{k!} e^{-\lambda}$, $k \geq 0$

$$? \ 1 = \sum_{1=0}^{\infty} \mathbb{P}(X=k) = \sum_{k=0}^{\infty} \frac{\lambda^k}{k!} e^{-\lambda} = e^{-\lambda} \sum_{k=0}^{\infty} \frac{\lambda^k}{k!} = e^{-\lambda} e^{\lambda} = 1$$

Твърдение $X \in Poi(\lambda)$. Тогава а) $g_X(s) = e^{-\lambda + \lambda s}$ за $|s| \leq 1$

б) $\mathbb{E}X = \mathbb{D}X = \lambda$

Док. а) $g_X(s) = \sum_{k=0}^{\infty} s^k \frac{g^k}{k!} e^{-1} = e^{-1} \sum_{k=0}^{\infty} \frac{(s\lambda)^k}{k!} = e^{-\lambda} e^{\lambda s}$

б) $\mathbb{E}X = g_X'(1) = \lambda e^{-\lambda} e^{\lambda s}\big|_{s=1} = \lambda$

$\mathbb{D}X = g_X''(1) + g_X'(1) - (g_X'(1))^2$