4) средно време на свещта, ако $X = \frac{1}{4}$?

$$\mathbb{E}(Y - X \mid X = \tfrac{1}{4}) = \mathbb{E}(Y - \tfrac{1}{4} \mid X = \tfrac{1}{4})$$

$f_{Y \mid X = \frac{1}{4}}$

	x_1	x_2	x_3
y_1	$1/8$	$1/8$	$1/8$
y_2			
y_3			

$$\mathbb{P}(X = x_1 \mid Y = y_1) = \frac{1}{3}$$

$$\frac{\mathbb{P}(X = x_1 \cap Y = y_1)}{\mathbb{P}(Y = y_1)}$$

$$f_{Y \mid X = \frac{1}{4}}(y) = \frac{f_{X,Y}(\frac{1}{4}, y)}{f_X(\frac{1}{4})} = \frac{B \cdot \frac{1}{4} y}{4(1 - \frac{1^2}{4^2}) \cdot \frac{1}{4}} = \frac{2y}{\frac{15}{16}} = \frac{32}{15} y$$

$$\mathbb{E}(Y - \tfrac{1}{4} \mid X = \tfrac{1}{4}) = \int_{1/4}^{1} \frac{32}{15} y \cdot (y - \tfrac{1}{4}) \, dy = \text{---} = \frac{27}{60}$$

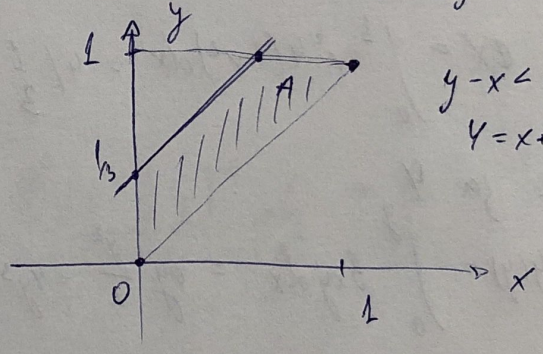
$$f_{X \mid Y = x}(x) = \frac{f_{X,Y}(x, x)}{f_Y(x)} \qquad f_{X,Y}(x \mid y) = \frac{f_{X,Y}(x, y)}{f_Y(y)}$$

при положение, че $Y = y$

5) $\mathbb{P}$(гаснене да свещта < 20 мин) = ?

$\mathbb{P}(Y - X < \frac{1}{3}) = ?$

$0 < x < y < 1$



$y - x < \frac{1}{3}$

$y = x + \frac{1}{3}$

$$\mathbb{P}((X, Y) \in A) = \iint_A cxy \, dx \, dy =$$

$$= 1 - \iint_B cxy \, dx \, dy = 1 - \int_0^{2/3} \int_{x+1/3}^{1} 8xy \, dy \, dx =$$

$$= 1 - \int_0^{2/3} 8x \left(\frac{1}{2} - \left(\frac{x + \frac{1}{3}}{2} \right)^2 \right) dx = 1 - 4 \int_0^{2/3} x - (x + \tfrac{1}{3})^2 \, dx =$$

$$= 1 - 4 \left[\frac{4}{18} - \frac{(x + \frac{1}{3})^3}{3} \Big|_0^{2/3} \right] = \text{---} \approx 0{,}67$$