$M_X(t) = e^{\mu t} M_Z(\sigma t) = e^{\mu t} e^{\frac{\sigma^2 t^2}{2}} \quad \forall t \in \mathbb{R}$

$\boxed{\text{Док (ЦГТ)}}$ $(X_i)_{i=1}^{\infty}$ от н.е.р. сл.вел $EX_i = \mu_i$ $DX_i = \sigma^2$ $\frac{S_n - n\mu}{\sigma\sqrt{n}} \xrightarrow[n\to\infty]{d} Z$

X_1 има функ. на мом. $M_{X_1}(t)$ е добре деф за $t \in (-\varepsilon, \varepsilon)$

$$\frac{S_n - n\mu}{\sigma\sqrt{n}} = \frac{1}{\sqrt{n}} \sum_{j=1}^{n} \frac{(X_j - \mu)}{\sigma} = \frac{1}{\sqrt{n}} \sum_{j=1}^{n} Y_j = \frac{V_n}{\sqrt{n}} : W_n$$

$Y_j = \frac{X_j - \mu}{\sigma} \quad \forall j \geq 1$

и $(Y_j)_{j=1}^{\infty}$ н.е.р. сл.вел

$EY_1 = \frac{EX_1 - \mu}{\sigma} = 0$

$DY_1 = \frac{DX_1}{\sigma^2} = 1$

$M_{Y_1}(t) = e^{-\frac{\mu}{\sigma}t} M_{X_1}\left(\frac{t}{\sigma}\right)$

$\varepsilon < \frac{t}{\sigma} < +\varepsilon$

$\Rightarrow -\sigma\varepsilon < t < \sigma\varepsilon$

M_{Y_1} е добре деф $|t| < \sigma\varepsilon$

Нека фикс t. Ще докажем, че $M_{W_n}(t) \xrightarrow[n\to\infty]{} M_Z(t) = e^{t^2/2}$

$$M_{W_n}(t) = E e^{\frac{t}{\sqrt{n}} V_n} = E e^{\frac{t}{\sqrt{n}} \sum_{j=1}^{n} Y} \overset{\text{от незав}}{=} \prod_{j=1}^{n} M_{Y_j}\left(\frac{t}{\sqrt{n}}\right) = M_{Y_1}\left(\frac{t}{\sqrt{n}}\right) \qquad Y_j \overset{d}{=} Y_1$$

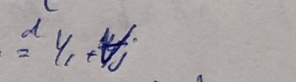
Ако $\left|\frac{t}{\sqrt{n}}\right| < \sigma\varepsilon$, то $M_{W_n}(t)$ е добре деф.

$$M_{W_n}(t) = \left(M_{Y_1}\left(\frac{t}{\sqrt{n}}\right)\right)^n = \left(1 + \frac{t^2}{2n} + \frac{t^2}{2n}\left(\frac{\rho_3}{\sqrt{n}}\right)\right)^n = \left(1 + \frac{t^2}{2n}\left(1 + \frac{\rho_3}{\sqrt{n}}\right)\right)^n \xrightarrow[n\to\infty]{} e^{t^2/2}$$

$M_{Y_1}\left(\frac{t}{\sqrt{n}}\right) = E e^{\frac{t}{\sqrt{n}} Y_1}$

$$e^{\frac{t}{\sqrt{n}} Y_1} = 1 + \frac{t}{\sqrt{n}} Y_1 + \frac{t^2}{2n} Y_1^2 + \frac{\theta(Y_1) t^3 Y_1^3}{3!\, n^{3/2}} \qquad |\theta(Y_1)| \leq 1$$

$$M_{Y_1} = E e^{\frac{t}{\sqrt{n}} Y_1} = 1 + \frac{t^2}{2n} + E\frac{\theta(Y_1) t^3 Y^3}{3!\, n^{3/2}}$$



$|\theta(Y_1) Y_1^3| \leq |Y_1|^3 \Rightarrow |E(\theta(Y_1) Y_1^3)| \leq E|Y_1|^3 = \rho_3$

$$\left|\frac{t^3}{\sigma n^{3/2}} E\,\theta(Y_1) Y_1^3\right| \leq \frac{t^3}{\sigma n^{3/2}} \cdot \rho_3 = \frac{t^2}{2n}\left(\frac{\rho_3}{3\sqrt{n}}\right)$$