04.01. СЕМ упр.

1зад. 2000 пъти монета, ези $= 3/4$

$\#\text{ези} = X$

$\mathbb{P}(X \in [1475, 1535]) = ?$

$X \sim Bin(2000, 3/4)$

$X = X_1 + \dots + X_{2000}$

$$\frac{X_1 + \dots + X_n - n\,\mathbb{E}X_1}{\sigma\sqrt{n}} \xrightarrow{d} N(0,1)$$

$X_1, \dots, X_n$ iid

$X \approx N(2000 \cdot 3/4;\ 2000 \cdot \frac{3}{4} \cdot \frac{1}{4})$

$\mathbb{P}(1475 \le X \le 1535) \approx$

$\approx \mathbb{P}(1475 \le N(1500, 375) \le 1535) =$

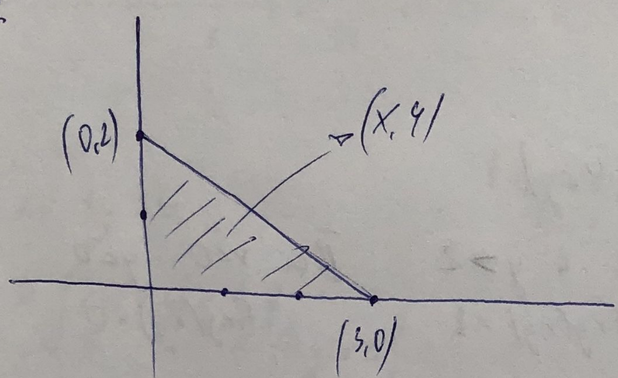
$$= \mathbb{P}\left(\frac{1475 - 1500}{\sqrt{375}} \le N(0,1) \le \frac{1535 - 1500}{\sqrt{375}}\right) =$$

$= \mathbb{P}(\alpha_1 \le N(0,1) \le \alpha_2) =$

$= \Phi(\alpha_2) - \Phi(\alpha_1)$

$\Phi(x) = \mathbb{P}(N(0,1) \le x)$

2зад.



$f_{X,Y};\ F_{X,Y};\ f_X;\ f_Y;\ \mathbb{E}X;\ \mathbb{E}Y \dots$

$f_X = F_X'$

$$f_{X,Y}(x,y) = \begin{cases} 0 & \text{извън } \Delta \\ \text{const} & \text{в } \Delta \end{cases}$$

$\text{const} = 1/3$, т.е. $f_{X,Y}(x,y) = \frac{1}{3} \mathbb{1}_{\{(x,y) \in \Delta\}}$

- Съвместна функ. на разпределение на X и Y наричаме $F_{X,Y} : \mathbb{R}^2 \longrightarrow \mathbb{R}$ $([0,1])$

  $F_{X,Y}(x,y) = \mathbb{P}(X \le x, Y \le y)$

  $(x,y) \mapsto \mathbb{P}(X \le x, Y \le y)$

- Съвместна плътност на X и Y, наричаме функ., за която $\mathbb{P}((X,Y) \in A) = \iint_A f_{X,Y}(x,y)\,dx\,dy$

$$\frac{\partial^2 F(x,y)}{\partial x \partial y} = f_{X,Y}(x,y)$$