Ако g е монотонна,

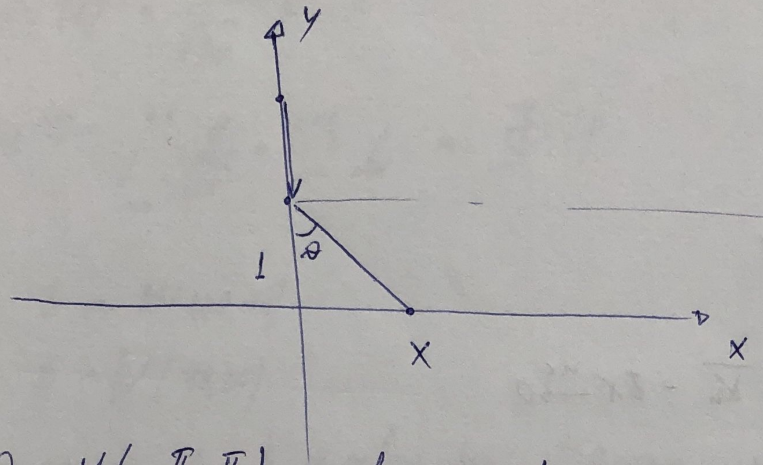
$$f_{g(X)}(t) = f_X(g^{-1}(t)) \cdot |g^{-1}(t)'|$$

1) $Y = X^{\alpha}$ $\quad \alpha > 0 \quad X = Y^{1/\alpha}$, т.е $g^{-1}(t) = t^{1/\alpha}$

$X \sim Exp(\lambda)$

$$f_Y(t) = f_X(t^{1/\alpha}) \cdot (t^{1/\alpha})' = \lambda e^{-\lambda t^{1/\alpha}} \mathbb{1}\{t^{1/\alpha} > 0\} \cdot \frac{1}{\alpha} t^{1/\alpha - 1} = \frac{\lambda}{\alpha} e^{-\lambda t^{1/\alpha}} t^{\frac{1-\alpha}{\alpha}} \mathbb{1}\{t > 0\}$$

Зад.



$$\theta \sim U\left(-\frac{\pi}{2}; \frac{\pi}{2}\right) \Rightarrow f_\theta(t) = \frac{1}{\pi} \mathbb{1}_{\{t \in (-\frac{\pi}{2}; \frac{\pi}{2})\}}$$

$X \sim ?$; $f_X(t) = ?$; $X \in (-\infty; +\infty)$

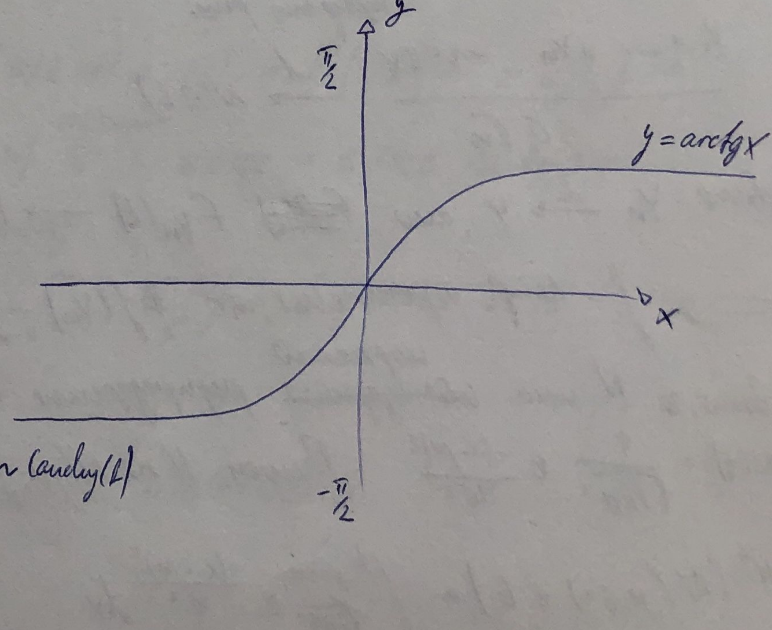
X чрез θ? $\quad X = tg\,\theta$

$$\mathbb{P}(X \le t) = \mathbb{P}(tg\,\theta \le t) = \mathbb{P}(\theta \le arctg\,t) = \int_{-\pi/2}^{arctg\,t} \frac{1}{\pi}\,ds = \frac{arctg\,t + \frac{\pi}{2}}{\pi}$$

Следов. $f_{(X)}(t) = \frac{1}{1+t^2} \cdot \frac{1}{\pi}$

* проверка

$$\int_{-\infty}^{\infty} \frac{1}{1+t^2} \cdot \frac{1}{\pi}\,dt = \frac{arctg\,t}{\pi}\Bigg|_{-\infty}^{\infty} = 1$$



$X \sim Cauchy(1)$