8. Експоненциално разпр.

Дефиниция $X \in Exp(\lambda)$, $\lambda > 0$, ако X има плътност от вида $f_X(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & \text{иначе} \end{cases}$

$$F_X(x) = \begin{cases} 0 & x \leq 0 \\ \int_0^x \lambda e^{-\lambda y} dy = 1 - e^{-\lambda x} & x > 0 \end{cases}$$

$$F_X(x) = \mathbb{P}(X < x)$$

$$\overline{F}_X(x) = \mathbb{P}(X \geq x) = \begin{cases} e^{-\lambda x} & x > 0 \\ 1 & x \leq 0 \end{cases}$$

(опашка)

$$EX = \lambda \int_0^\infty x e^{-\lambda x} dx \overset{\text{инт. по части}}{=} \int_0^\infty e^{-\lambda x} dx = \frac{1}{\lambda}$$

$$1 = \lambda \int_0^\infty e^{-\lambda x} dx$$

$$(\lambda e^{-\lambda x}) = -(\lambda e^{-\lambda x})'$$

$$\frac{1}{\lambda} = \int_0^\infty e^{-\lambda x} dx \Big/ \frac{d}{dy}$$

$$EX^2 = \lambda \int_0^\infty x^2 e^{-\lambda x} dx \overset{\text{инт. по части}}{=} 2 \int_0^\infty x e^{-\lambda x} dx = \frac{2}{\lambda^2}$$

$$DX = \frac{2}{\lambda^2} - \frac{1}{\lambda^2} = \frac{1}{\lambda^2}$$

$t > 0 \quad s > 0$

$$\mathbb{P}(X > t+s \mid X > t) = \mathbb{P}(X > s) \qquad \text{безпаметност}$$

$$\| $$

$$\frac{\mathbb{P}(X > t+s \cap X > t)}{\mathbb{P}(X > t)} = \frac{\mathbb{P}(X > t+s)}{\mathbb{P}(X > t)} = \frac{e^{-\lambda(t+s)}}{e^{-\lambda t}} = e^{-\lambda s} = \mathbb{P}(X > 0)$$

$X = (X_1, X_2) \quad f_X \quad x = (x_1, x_2) \qquad g: \mathbb{R}^2 \longrightarrow \mathbb{R}^2 \quad y = g(x)$ се

$Y = (Y_1, Y_2) \quad f_Y \quad y = (y_1, y_2) \qquad (y_1, y_2) = g_1(x_1, x_2)$