Зад. Ако знаем $f_X$, то как да намерим $f_{g(X)}$ $\quad X \sim Exp(\lambda)$

• $X \sim Exp(\lambda) \overset{def}{\Rightarrow} f_X(t) = \lambda e^{-\lambda t} \mathbb{1}_{\{t>0\}} = \begin{cases} \lambda e^{-\lambda t} & t>0 \\ 0 & t\leq 0 \end{cases}$

а) $f_Y = ?$ за $Y = -X \quad t \in \mathbb{R}$

$\mathbb{P}(Y\leq t) = \mathbb{P}(-X \leq t) = \mathbb{P}(X \geq -t) = 1 - \mathbb{P}(X \leq t)$

$-F_X'(-t).(-1)'$

Ако $X$ има плътност, то $\mathbb{P}(X=c) = \int_c^c f_X(t)dt = 0$

$\Rightarrow f_Y(t) = F_Y'(t) = (\mathbb{P}(Y\leq t))' = (1 - \underset{F_X(-t)'}{\underset{\parallel}{\mathbb{P}(X\leq t)}})' = -(F_X(-t))' = -f_X(-t).(-1) = f_X(-t) =$

$= \lambda e^{-\lambda(t)} \mathbb{1}_{\{-t>0\}} = \lambda e^{\lambda t} \mathbb{1}_{\{t<0\}}$

б) $Y = 2X - 1$

$\mathbb{P}(Y \leq t) = \mathbb{P}(2X - 1 \leq t) = \mathbb{P}(X \leq \frac{t+1}{2}) = F_X(\frac{t+1}{2}) \Rightarrow f_Y(t) = (F_X(\frac{t+1}{2}))' =$

$= F_X'(\frac{t+1}{2}) . (\frac{t+1}{2})' = f_X(\frac{t+1}{2}) . \frac{1}{2} = \lambda e^{-\lambda \frac{t+1}{2}} \mathbb{1}_{\{\frac{t+1}{2}>0\}} . \frac{1}{2}$

Ако $g$ е растяща

$Y = g(X)$

$\mathbb{P}(Y \leq t) = \mathbb{P}(g(X) \leq t) = \mathbb{P}(g^{-1}(g(X)) \leq g^{-1}(t)) = \mathbb{P}(X \leq g^{-1}(t)) = F_X(g^{-1}(t))$

Следов.

$f_Y(t) = F_Y(t)' = (F_X(g^{-1}(t)))' = F_X'(g^{-1}(t)) . g^{-1}(t)' = f_X(g^{-1}(t)) . g^{-1}(t)'$

Ако $g$ е намаляваща

$\mathbb{P}(Y \leq t) = \mathbb{P}(g(X) \leq t) = \mathbb{P}(g^{-1}(g(X)) \geq g^{-1}(t)) = 1 - \mathbb{P}(X \leq g^{-1}(t)) =$

$= 1 - F_X(g^{-1}(t))$

Следов.

$f_Y(t) = F_Y'(t) = (1 - F_X(g^{-1}(t)))' = -f_X(g^{-1}(t)) . g^{-1}(t)'$