Док) $g\uparrow$ $\quad \varrho(y) = f(h(y))\,h'(y)$; $g\downarrow$ — същото ама са разменени a и b

$$\mathbb{P}(Y\in(a,b)) = \mathbb{P}(g(X)\in(a,b)) \overset{g\uparrow}{=} \mathbb{P}(X\in(g^{-1}(a),\, g^{-1}(b))) \overset{h=g^{-1}}{=} \mathbb{P}(X\in(h(a),\, h(b))) =$$

$$\| \qquad \int_a^b ?$$

$$= \int_{h(a)}^{h(b)} f(x)\,dx \overset{x=h(v)}{=} \int_a^b f(h(v))\,dh(v) = \int_a^b f(h(v))\,h'(v)\,dv = \int_a^b \varrho(v)\,dv$$

$h(a) = h(v) \Rightarrow v = a$

$h(b) = h(v) \Rightarrow v = b$

$$= -\int_a^b f(h(v))\,h'(v)\,dv = \int_a^b \underbrace{f(h(v))(-h'(v))}_{\varrho}\,dv$$

във вс. случай $\varrho(y) = f(h(y))|h'(y)|$

Математическо очакване

X е нсв, то $\mathbb{E}X = \int_{-\infty}^{+\infty} x\, f_X(x)\,dx$, ако $\int_{-\infty}^{+\infty} |x|\, f_X(x)\,dx < \infty$

X е дискр сл вел $\mathbb{E}X = \sum_i x_i p_i$

- $\mathbb{E}cX = c\mathbb{E}X \qquad g(x) = cx, \quad Y = cX$

$\|$

$\mathbb{E}Y = \int_{-\infty}^{\infty} y\, f_Y(y)\,dy$

$Y = g(X)$, то $\mathbb{E}Y = \mathbb{E}g(X) =$
$= \int_{-\infty}^{\infty} g(x) f_X(x)\,dx$, където $g\uparrow$ и $\downarrow$

- $\mathbb{E}[X+Y] = \mathbb{E}X + \mathbb{E}Y$
- $\mathbb{E}XY = \mathbb{E}X + \mathbb{E}Y$

$g\uparrow$, то $f_Y(y) = f_X(h(y))\,h'(y)$

$h(y) = g^{-1}(y)$

По деф.

$$\mathbb{E}Y = \int_{-\infty}^{\infty} y\, f_Y(y)\,dy = \int_{-\infty}^{\infty} y\, \underbrace{f_X(h(y))\,h'(y)\,dy} =$$

$$= \int_{-\infty}^{\infty} g(x)\, f_X(x)\, \underbrace{h'(g(x))\cdot g'(x)}_{1}\,dx = \int_{-\infty}^{\infty} g(x)\, f_X(x)\,dx$$