$$0 = \frac{\partial}{\partial b_1}\Sigma \ldots = -2\sum_{k=1}^{n}(b_0 + b_1 x_k - y_k)x_k \Rightarrow 0 = b_0 n\bar{x} + b_1 \Sigma x_k^2 - \Sigma x_k y_k =$$

$$= n\bar{x}(\bar{y} - b\bar{x}) + b_1 \Sigma x_k^2 - \Sigma x_k y_k$$

$$b_0 = \bar{y} - b_1\bar{x} \quad (= \bar{Y} - b_1\bar{X})$$

$$b_1 = \frac{\Sigma X_k Y_k - n\bar{Y}\cdot\bar{X}}{\Sigma X_k^2 - n(\bar{X})^2} = \frac{\sum_{k=1}^{n}(Y_k - \bar{Y})(X_k - \bar{X})}{\sum_{k=1}^{n}(X_k - \bar{X})^2} = \frac{\sum_{k=1}^{n}(X_k - \bar{X})Y_k}{\sum_{k=1}^{n}(X_k - \bar{X})^2}$$

$$A = \sum_{k=1}^{n}(X_k - \bar{X})^2$$

* в първо е с малки букви,
после го написа с големи :)

$$Y_k = \beta_0 + \beta_1 x_k + \varepsilon_k$$

$(\varepsilon_i)_{i=1}^{n}$ са незав еднакво разпр сл вел като $\varepsilon_i \in N(0, \sigma^2)$ $\quad k = \overline{1, n}$

хомоскезастичност

$$Y_k \in N(\beta_0 + \beta_1 x_k, \sigma^2)$$

$$\mathbb{E}b_1 = \frac{1}{A}\sum_{k=1}^{n}(X_k - \bar{X})\mathbb{E}Y_k = \frac{1}{A}\sum_{k=1}^{n}(X_k - \bar{X})(\beta_0 + \beta_1 X_k) = \frac{\beta_0}{A}\underbrace{\Sigma(X_k - \bar{X})}_{0} + \frac{\beta_1}{\cancel{A}}\cancel{\sum_{k=1}^{n}(X_k - \bar{X})X_k} = \beta_1$$

$$\mathbb{E}b_0 = \mathbb{E}\bar{Y} - \bar{X}\,\mathbb{E}b_1 = \frac{1}{n}\sum_{k=1}^{n}\mathbb{E}Y_k - \beta_1\bar{X} = \frac{1}{n}\sum_{k=1}^{n}(\beta_0 + \beta_1 X_k) - \beta_1\bar{X} = \beta_0$$

$$Db_1 = D\frac{\sum_{k=1}^{n}(X_k - \bar{X})}{A} = \frac{1}{A^2}\sum_{k=1}^{n}(X_k - \bar{X})^2 \overbrace{DY_k}^{=\sigma^2} = \sigma^2\frac{\sum_{k=1}^{n}(X_k - \bar{X})^2}{A^2} = \frac{\sigma^2}{A}$$

$$b_1 = \hat{\beta}_1 \in N\left(\beta_1, \frac{\sigma^2}{A}\right)$$

$$Db_0 = \sigma^2\left(\frac{1}{n} + \frac{\bar{X}^2}{A}\right) \Rightarrow b_0 = \hat{\beta}_0 \in N\left(b_0, \sigma^2\left(\frac{1}{n} + \frac{\bar{X}^2}{A}\right)\right)$$