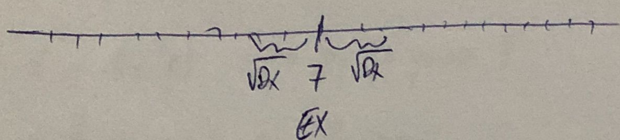
$\mathbb{E}X_1 = \frac{1}{4}(1+6) + \frac{1}{8}(2+3+4+5) = 3{,}5$

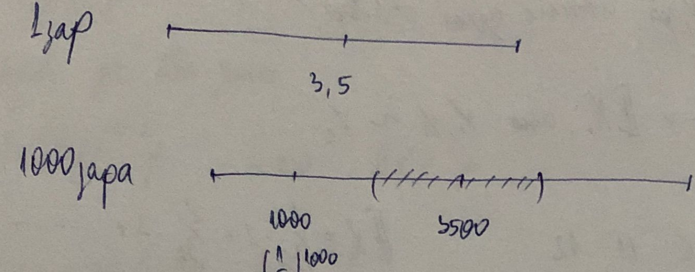
$\mathbb{D}X_1 = \mathbb{E}X_1^2 - (\mathbb{E}X_1)^2 = \frac{1}{4}(1^2+6^2) + \frac{1}{8}(2^2 + \dots + 5^2) - (3.5)^2 = \frac{15}{4}$

$\Longrightarrow \mathbb{E}X = 2 \cdot 3{,}5 = 7$

$\mathbb{D}X = 2 \cdot \frac{15}{4} = 7{,}5 \quad \sqrt{\mathbb{E}(X - \mathbb{E}(X))^2}$

$\sqrt{\mathbb{D}X}$ - стандартно отклонение





$\mathbb{P}(|X - \mathbb{E}X| \geq a) \leq \frac{\mathbb{D}X}{a^2}$

неравенство на Чебишов

$\mathbb{P}(|X| \geq a) \leq \frac{\mathbb{E}|X|}{a} \qquad a > 0$

$X = |X - \mathbb{E}X|^2 \longrightarrow$ Чебишов

$a \longrightarrow a^2$

$X = X_1 + \dots + X_{1000}$

1) $\mathbb{E}X = 1000 \cdot 3{,}5 = 3500$

$\mathbb{D}X = 1000 \cdot \frac{35}{12} \approx 2900$ (16)

$\mathbb{P}(X \geq 3700)$

$\mathbb{P}(X - 3500 \geq 200)$

$\mathbb{P}(|X - 3500| \geq 200) \overset{\text{прибл.}}{\leq} \frac{2900}{(200)^2} = \frac{2900}{40000} = \frac{29}{400}$ малко!

$\mathbb{P}(X - 3500 \geq 200) \leq \mathbb{P}(|X - 3500| \geq 200)$

Зад. 5 бели, 3 черни, вадим до бяла

X = „# изтеглени черни"

1) без връщане 2) с връщане

* 1000 пъти. Оценка да са изв. 900

$X_1 + \dots + X_{1000}$ $\quad \mathbb{P}(X_1 + \dots + X_{1000} \geq 900)$

Решение

X	0	1	2	3
$\mathbb{P}(X=k)$	5/8	15/56		

$\mathbb{P}(X=0) = 5/8$

$\mathbb{P}(X=1) = \frac{3}{8} \cdot \frac{5}{7} = \frac{15}{56}$

$\mathbb{P}(X=2) = \frac{3}{8} \cdot \frac{2}{7} \cdot \frac{5}{6} = \frac{5}{56}$