Зад: 5 бели, 8 зелени, 7 червени копчета

$A =$

$\mathbb{P}(\{$бяла преди зелена$\})$, ако

а) с връщане

б) без връщане

Решение: а) I начин

$\mathbb{P}(A) = \mathbb{P}($бяла$) + \mathbb{P}($черв бяла$) +$

$\qquad + \mathbb{P}($черв черв бяла$) + \ldots =$

$= \dfrac{5}{8+7+5} + \dfrac{7}{20}\cdot\dfrac{5}{20} + \dfrac{7}{20}\cdot\dfrac{7}{20}\cdot\dfrac{5}{20} + \ldots =$

$= \dfrac{5}{20}\left(1 + \dfrac{7}{20} + \left(\dfrac{7}{20}\right)^2 + \ldots\right) =$

$= \dfrac{5}{20}\cdot\dfrac{1}{1-\frac{7}{20}} = \dfrac{5}{20}\cdot\dfrac{20}{13} = \dfrac{5}{13}$, т.к.

$1 + x + x^2 + \ldots = \dfrac{1}{1-x}$

$\displaystyle\sum_{n=1}^{\infty} n x^n = \dfrac{x}{(1-x)^2} = 1 + 2x^2 + 3x^2 + \ldots$

за $|x| < 1$

II начин ~~$\mathbb{P}(A) \quad \mathbb{P}(\ldots)$~~

$\mathbb{P}(A) = \mathbb{P}($бяла $<$ зелена $\mid$ 1та е зелена$)\cdot\mathbb{P}($1та е зелена$) +$

$+ \mathbb{P}($бяла $<$ зелена $\mid$ 1та е бяла$)\cdot\mathbb{P}($1та е бяла$) +$

$+ \mathbb{P}(-\!\!-\!\!// -\!\!- \mid$ 1та е червена$)\cdot\mathbb{P}($1та е черв$) =$

$= 0 + 1\cdot\dfrac{5}{20} + \mathbb{P}(A)\cdot\dfrac{7}{20} \Rightarrow \dfrac{13}{20}\,\mathbb{P}(A) = \dfrac{5}{20}$, т.е. $\mathbb{P}(A) = \dfrac{5}{13}$

б) I начин

$\mathbb{P}(A) = \mathbb{P}(б) + \mathbb{P}($черв бяла$) + \mathbb{P}($червчерв бяла$) + \ldots +$

$+ \mathbb{P}(\underbrace{ч\ldots ч}_{7}\,бяла) = \dfrac{5}{20} + \dfrac{7}{20}\cdot\dfrac{5}{19} + \dfrac{7}{20}\cdot\dfrac{6}{19}\cdot\dfrac{5}{18} + \ldots + \dfrac{?}{20}$

$+ \dfrac{7}{20}\cdot\dfrac{6}{19}\cdot\ldots\cdot\dfrac{1}{14}\cdot\dfrac{5}{19} =$

$= \dfrac{5}{20}\left(1 + \dfrac{7}{19} + \dfrac{7\cdot 6}{19\cdot 18} + \ldots\right)$

$\underbrace{}_{X}$

$\mathbb{P}(\bar A) = \mathbb{P}(з) + \mathbb{P}(чз) + \ldots + \mathbb{P}(\underbrace{ч\ldots чз}_{7}) =$
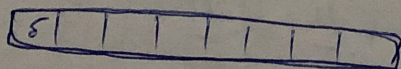
$= \dfrac{8}{20} + \dfrac{7}{20}\cdot\dfrac{8}{19} + \ldots + \dfrac{7}{20}\cdot\dfrac{6}{19}\ldots\dfrac{8}{13} =$

$= \dfrac{8}{20} X$

$\mathbb{P}(A) + \mathbb{P}(\bar A) = \dfrac{5}{20}X + \dfrac{8}{20}X = 1$

$\dfrac{13}{5}\cdot\dfrac{4}{12} \qquad \Rightarrow \dfrac{20}{13} = X$ и $\mathbb{P}(A) = \dfrac{5}{20}\cdot\dfrac{20}{13} = \dfrac{5}{13}$

III начин: Червените копчета нямат значение.

5 бели и 8 зелени

$\dfrac{\dbinom{?}{?}\ \dfrac{12!}{4!\,8!}}{\dbinom{?}{?}\ \dfrac{13!}{5!\,8!}} = \dfrac{5}{13}$