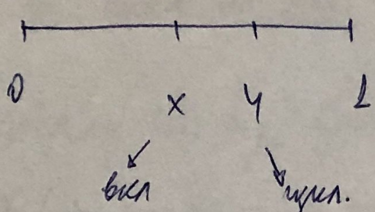
Зад.



$f_{X,Y}(x,y) = cxy$, ако $0<x<y<1$

1) Колко е c? Имаме

$$\iint\limits_{0<x<y<1} cxy\,dx\,dy = 1$$

$$\int_0^1\int_x^1 cxy\,dy\,dx = c\int_0^1 x\left(\frac{1}{2}-\frac{x^2}{2}\right)dx = \frac{c}{2}\left[\frac{1}{2}-\frac{1}{4}\right] = \frac{c}{8}$$

Следователно $c=8$

2) $f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dy = \int_x^1 8xy\,dy = 8x\left(\frac{1}{2}-\frac{x^2}{2}\right) = 4x-4x^3 = 4x(1-x^2)$

След. $\mathbb{E}X = \int_0^1 x^2\cdot 4(1-x^2)\,dx = 4\left(\frac{1}{3}-\frac{1}{5}\right) = \frac{8}{15}$

Анал. за Y

$f_Y(y) = \int_0^y 8xy\,dx = 8y\frac{y^2}{2} = 4y^3$

$\mathbb{E}Y = \int_0^1 y\,4y^3\,dy = 4\cdot\frac{1}{5} = \frac{4}{5}$

3) $\mathbb{P}\left(X\le \frac{3}{4},\ Y\le X+\frac{1}{6}\right) = \int_0^{3/4}\int_x^{x+1/6} 8xy\,dy\,dx = \int_0^{3/4} 8x\left(\frac{(x+\frac{1}{6})^2}{2}-\frac{x^2}{2}\right)dx =$

$= \int_0^{3/4} 4x\left(\frac{x}{3}+\frac{1}{36}\right)dx$ ----