$$X = \begin{cases} 1 & \mathbb{P}(Y) \\ 0 & \mathbb{P}(Y) \end{cases} \qquad Y = \begin{cases} 1/3 & \text{вер } 1/2 \text{ мъж} \\ 2/3 & \text{вер } 1/2 \text{ жена} \end{cases}$$

$$X = Z_1 \mathbb{1}\{Y = 1/3\} + Z_2 \mathbb{1}\{Y = 2/3\} \qquad Z_1 \sim Ber(1/3)$$
$$Z_2 \sim Ber(2/3)$$

$$X = f(Z_1, Z_2, Y) = \begin{cases} Z_1 & Y = 1/3 \\ Z_2 & Y = 2/3 \end{cases}$$

$$\mathbb{E}X = \mathbb{P}(X=1) = \mathbb{E}(\mathbb{E}(X|Y)) =$$

$$= \mathbb{E}(X | Y = 1/3)\mathbb{P}(Y = 1/3) + \mathbb{E}(X | Y = 2/3)\mathbb{P}(Y = 2/3) =$$

$$= \mathbb{E}(Z_1 | Y = 1/3)\frac{1}{2} + \frac{1}{2}\mathbb{E}(Z_2 | Y = 2/3) =$$

$$= \mathbb{E}Z_1 \frac{1}{2} + \mathbb{E}Z_2 \frac{1}{2} = \cancel{\mathbb{E}} \frac{1}{2} = \frac{1}{2}(1/3 + 2/3) = \frac{1}{2}$$

$V = (X_j)_{j \geq 1}$ $\quad X_j \sim Be(p)$ $\quad$ вс са независими

$N \sim Ge(r)$ $\quad$ и N не зависи от V $\qquad f(V, N) = \sum_{j=1}^{N+1} X_j$, ако $N = n$

$$\mathbb{E}\sum_{j=1}^{N+1} X_j = \mathbb{E}[f(V,N)] = \mathbb{E}[\mathbb{E}[f(V,N) | N]] =$$

$$= \mathbb{E}\sum_{n=0}^{\infty} \mathbb{E}[f(V,N) | N = n] \mathbb{1}\{N = n\} = \mathbb{E}\left[\sum_{n=0}^{\infty} \mathbb{E}[f(V,n)] \mathbb{1}\{N=n\}\right] =$$

$$= \mathbb{E}\sum_{n=0}^{\infty}\left(\mathbb{E}\sum_{j=1}^{n+1} X_j\right)\mathbb{1}_{N=n} = \sum_{n=0}^{\infty}\mathbb{E}\sum_{j=1}^{n+1} X_j \cdot \mathbb{E}\mathbb{1}_{N=n} = \sum_{n=0}^{\infty}\mathbb{E}\sum_{j=1}^{n+1} X_j \cdot \mathbb{P}(N=n) =$$

$$= \sum_{n=0}^{\infty}(n+1)p \cdot (1-r)^n r = p \cdot r \sum_{n=0}^{\infty}(n+1)(1-r)^n$$