$\mathbb{P}(X=3) = \frac{3}{8} \cdot \frac{2}{7} \cdot \frac{1}{6} \cdot \frac{5}{5} = \frac{1}{56}$

$\mathbb{E}X = 0 \cdot \frac{5}{8} + 1 \cdot \frac{15}{56} + 2 \cdot \frac{15}{56} + 3 \cdot \frac{1}{56} = \frac{28}{56} = \frac{1}{2}$

$\mathbb{D}X = \mathbb{E}X^2 - (\mathbb{E}X)^2 = \left(0^2 \cdot \frac{5}{8} + 1^2 \cdot \frac{15}{56} + 2^2 \frac{15}{56} + 3^2 \cdot \frac{1}{56}\right) - \left(\frac{1}{2}\right)^2 = \frac{15}{28}$

$X \sim Ge(p) \longrightarrow$ вер. за успех p

1) # неуспехи до 1ви успех

$\mathbb{P}(X=k) = (1-p)(1-p) \cdots (1-p)p = (1-p)^k p \quad k \geq 0$

2) # опити до 1ви успех

$\mathbb{P}(X=k) = (1-p)^{k-1} p \quad k \geq 0 \quad \mathbb{E}X = \frac{1}{p}$

$X \sim Ge(p)$, ако $\mathbb{P}(X=k) = (1-p)^{k-1} p$

$\mathbb{E}X = \sum_{k=0}^{\infty} p_k k = 1 \cdot p + 2 \cdot \overbrace{(1-p)p}^{p2(1-p)^2} + \overbrace{3(1-p)^2 \cdot p}^{p3(1-p)^3} + \dots = \frac{p}{1-p} \sum_{k=0}^{\infty} (1-p)^k \cdot k = \frac{1-p}{(1-(1-p))^2}$

$= \frac{p}{1-p} \cdot \frac{1-p}{p^2} = \frac{1}{p}$

$1 + x + x^2 + \dots = \sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \quad |x| < 1$

$\sum_{n=0}^{\infty} n x^{n-1} = \frac{1}{(1-x)^2} \Longrightarrow \sum_{n=0}^{\infty} n x^n = \frac{x}{(1-x)^2}$

$\mathbb{D}X = \mathbb{E}X^2 - \mathbb{E}X$

$\mathbb{E}X^2 = 1^2 \cdot p + 2^2 \cdot \overbrace{(1-p)p}^{p2^2(1-p)^2} + \overbrace{3^2(1-p)^2 p}^{p3^2(1-p)^3} = \frac{p}{1-p} \sum_{k=0}^{\infty} k^2 (1-p)^k$

$\sum_{n=0}^{\infty} x^n \cdot n = \frac{x}{(1-x)^2}$

$\sum_{n=0}^{\infty} x^{n-1} \cdot n^2 = \frac{(1-x)^2 + x \cdot 2(1-x)}{(1-x)^4}$

$\sum_{n=0}^{\infty} x^n n^2 = \underline{x(1-x+2x)}$

15.