Твърдение: (безпаметност) Нека $X \in Ge(p)$. Тогава $\forall\, m \geq 0$ и $k \geq 1$ $\mathbb{P}(X \geq m+k / X \geq m) =$

$$= \mathbb{P}(X \geq k) = q^k \quad , k \geq 0$$

Док) $\mathbb{P}(X \geq l) = \sum_{j=l}^{\infty} p \cdot q^j = p \cdot q^l \sum_{j=0}^{\infty} q^j = \frac{pq^l}{1-q} = q^l$

Тогава $\mathbb{P}(X \geq m+k / X \geq m) = \frac{\mathbb{P}(X \geq m+k \cap X \geq m)}{\mathbb{P}(X \geq m)} = \frac{\mathbb{P}(X \geq m+k)}{\mathbb{P}(X \geq m)} =$

$$= \frac{q^{m+k}}{q^m} = q^k = \mathbb{P}(X \geq k) \quad \#$$

$\mathbb{P}(X \geq 5 / X \geq 3) = \mathbb{P}(X \geq 2) \qquad X \sim Ge\left(\frac{18}{37}\right)$

Г. Отрицателен бином

Дефиниция $X \in NB(r,p)$ и $X = \min\{j \geq 1 : \sum_{i=1}^{j} X_i = r\} - r$

или броя неуспехи до $r^{\text{ти}}$ успех

⊕ $r = 1 \quad NB(1,p) = Ge(p)$

Твърдение $X \in NB(r,p)$, то $X = \sum_{j=1}^{r} Y_j$, където $Y_j \in Ge(p)$ $1 \leq j \leq r$ и са независими в съвкупност

$r=2 \quad X \in NB(2,p) \qquad \underbrace{0001}_{Y_1}\underbrace{001}_{Y_2}$

$X \overset{?}{=} \sum_{j=1}^{2} Y_j$

Ще проверим, че $Y_1 \perp\!\!\!\perp Y_2$. Ясно е, че $X = Y_1 + Y_2$

Ясно е, че $Y_1 \in Ge(p)$ и $Y_2 \in Ge(p)$

$\mathbb{P}(Y_1 = l \cap Y_2 = m) \overset{?}{=} \mathbb{P}(Y_1 = l) \cdot \mathbb{P}(Y_2 = m) \quad \forall\, l, m \geq 0$

$\mathbb{P}(X_1 = 0, X_2 = 0, X_{l+1} = 1, X_{l+2} = 0, \dots X_{l+m} = 0, X_{l+m+1} = 1) =$

$= \dots\dots\dots = \mathbb{P}(Y_1 = l)\,\mathbb{P}(Y_2 = m) \Rightarrow$ независими

Твърдение: $X \sim NB(r,p)$, то $g_X(s) = \left(\frac{p}{1-qs}\right)^r$, $\mathbb{E}X = \frac{rq}{p}$, $DX = \frac{rq}{p^2}$

Док: $\mathbb{E}X = \mathbb{E}\sum_{j=1}^{r} Y_j = \sum_{j=1}^{r} \mathbb{E}Y_j = r \cdot \frac{q}{p}$

$DX = D\sum_{j=1}^{r} Y_j \overset{\text{нез.}}{=} \sum_{j=1}^{r} DY_j = r \cdot \frac{q}{p^2}$

$g_X(s) \overset{\text{твър}}{=} \prod_{j=1}^{r} g_{Y_j}(s) = \left(\frac{p}{1-qs}\right)^r$