Ако знаем, че $Po(\lambda_2) + Po(\lambda_2) \sim Po(X_1 + X_2)$, то за 3 месеца имаме

$$Po_1(1) + Po_2(2) + Po_3(2) \stackrel{d}{=} Po(6)$$

1мес. 2мес 3мес

$$\Rightarrow \mathbb{P}(X \leq 3) = \mathbb{P}(Po(6) \leq 3) = \sum_{k=0}^{3} e^{-\lambda} \cdot \frac{\lambda^k}{k!} \text{ за } \lambda = 6$$

$$= e^{-6} \sum_{k=0}^{3} \frac{6^k}{k!}$$

~~Пораждащи функ~~

$$Po(\lambda_1) + Po(\lambda_2) \sim Po(\lambda_1 + \lambda_2)$$

Пораждащи функц

Нека X е сл.вел

$$g_X(s) = \mathbb{E} s^X$$

X	0	1	2	3	4
	p_0	p_1	p_2	p_3	

$$\mathbb{E} s^X = p_0 s^0 + p_1 s^1 + p_2 s^2 + \ldots = g_X(s)$$

$$g_X(1) = p_0 + p_1 + p_2 + \ldots = 1$$

$$g_X'(s) = p_1 \cdot 1 + p_2 \cdot 2s + p_3 \cdot 3s^2 + \ldots$$

$$g_X'(1) = p_1 \cdot 1 + p_2 \cdot 2 + p_3 \cdot 3 + \ldots = \mathbb{E}X$$

$$\frac{d}{ds} g_X(s) = \frac{d}{ds} \mathbb{E} s^X$$

$$g_X'(s) = \mathbb{E} X \cdot s^{X-1}$$

$$g_X'(1) = \mathbb{E}X$$

$$g_X''(s) = \mathbb{E} X(X-1) s^{X-2}$$

$$g_X''(1) = \mathbb{E}X^2 - \mathbb{E}X$$

$$\mathbb{E}X^2 = g_X''(1) + g_X'(1)$$

$X \sim Ber(p)$

$$g_X(s) = \mathbb{E} s^X = s^0 \cdot (1-p) + s^1 \cdot p = $$

$$= ps + 1 - p$$

$X \sim Bin(n, p)$

$X = X_1 + \ldots + X_n$, където $X_i \sim Ber(p)$

$$g_X(s) = \mathbb{E} s^X = \mathbb{E} s^{X_1 + \ldots + X_n} \stackrel{нез}{=}$$

$$= \mathbb{E} s^{X_1} \mathbb{E} s^{X_2} \ldots \mathbb{E} s^{X_n} = (ps + 1 - p)^n$$

$$g_X'(s) = n(ps + 1 - p)^{n-1} \cdot p \Rightarrow \mathbb{E}X = g_X'(1) = np$$