

Energy transport in trapped ion chains

This content has been downloaded from IOPscience. Please scroll down to see the full text.

2014 New J. Phys. 16 063062

(<http://iopscience.iop.org/1367-2630/16/6/063062>)

View [the table of contents for this issue](#), or go to the [journal homepage](#) for more

Download details:

IP Address: 169.229.32.36

This content was downloaded on 31/12/2014 at 17:59

Please note that [terms and conditions apply](#).

Energy transport in trapped ion chains

Michael Ramm, Thaned Pruttivarasin and Hartmut Häffner

Department of Physics, University of California, Berkeley, CA 94720, USA

E-mail: hhaeffner@berkeley.edu

Received 14 February 2014, revised 27 April 2014

Accepted for publication 21 May 2014

Published 26 June 2014

New Journal of Physics **16** (2014) 063062

doi:[10.1088/1367-2630/16/6/063062](https://doi.org/10.1088/1367-2630/16/6/063062)

Abstract

We experimentally study energy transport in chains of trapped ions. We use a pulsed excitation scheme to rapidly add energy to the local motional mode of one of the ions in the chain. Subsequent energy readout allows us to determine how the excitation has propagated throughout the chain. We observe energy revivals that persist for many cycles. We study the behavior with an increasing number of ions of up to 37 in the chain, including a zig-zag configuration. The experimental results agree with the theory of normal mode evolution. The described system provides an experimental toolbox for the study of thermodynamics of closed systems and energy transport in both classical and quantum regimes.

Keywords: energy transport, trapped ions, pulsed excitation

Chains of trapped ions have been proposed as a model system for the study of quantum transport of heat, equilibration in dissipative quantum systems, phase transitions in oscillator chain models, and friction in nanoscale systems [1–4]. In contrast to the current major efforts studying many body physics using the internal degree-of-freedom [5, 6], these proposals focus on the motional degree of freedom of the ions. While the linear regime of the chain already exhibits interesting phenomena like violations of Fourier’s law, the most intriguing physics emerges from nonlinearities in the potential. Such nonlinearities could stem from a zig-zag configuration of the chain, an additional optical lattice, or a coupling to an internal degree-of-freedom [7–9]. In particular, additional optical potentials allow to engineer phonon–phonon interactions realizing the Bose–Hubbard and Jaynes–Cummings–Hubbard models [10, 11].



Content from this work may be used under the terms of the [Creative Commons Attribution 3.0 licence](https://creativecommons.org/licenses/by/3.0/). Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.

Moreover, the dynamics of the linear to zig-zag quantum phase transition of the chain has been mapped onto an Ising chain model in a transverse field [12] and the stability of the structural transition can be modified with an optical cavity [13].

The theoretical predictions become most intriguing in the regime of many particles as they exhibit features distinct from expected thermodynamic behavior. For instance, when the extreme ions on either side of the chain are coupled to heat baths of different temperatures, there arises a nonlinear temperature distribution across the ion chain [2]. Similarly, the amount of dephasing and disorder have been shown to determine the deviation from the Fourier's law [1]. In addition, the non-uniformity of ions trapped in the harmonic potential leads to non-extensive scaling of thermodynamic quantities and to eigenmodes that differ from phonon-like waves [14, 15]. This host of interesting phenomena has not yet been observed experimentally. The realization of the proposals requires a precise level of control of ion motion, a challenging requirement for chains with a significant number of trapped ions.

In this work, we perform first experiments towards realizing the aforementioned proposals with long ion chains. We make use of the motional degree of freedom of long chains of trapped ions to study transport of energy in the system. We prepare an out-of-equilibrium state of the chain by rapidly imparting momentum onto a single ion at one end of the chain. We then monitor the energy of the ions in the chain as the initial excitation propagates, leading to multiple revivals of energy. The energy revivals persist for a surprisingly long time indicating that the system does not thermalize on the experimental timescale, an important requirement to study the aforementioned model systems. Our work extends the results obtained for two ions [16] to much longer chains of up to 37 ions. The resultant dynamics are more complex as they involve participation of a greater number of normal modes of the chain.

In order to observe the energy propagation, both the excitation and the energy readout have to be faster than the coupling between the ions. Intuitively, the dynamics can be understood in terms of the eigenmodes of the ion chain. When a single ion on the end of the chain is excited, the system is not in an eigenstate of the full coupled set of oscillators. Rather, the excitation creates a superposition of the eigenmodes which then evolve at their eigenfrequencies. The time evolution results in the local excitation transferred to other ions in the chain. Rephasing of the participating eigenmodes corresponds to energy revivals. The same reasoning applies for propagation of a single local phonon as observed in [16]. We find that the time for the excitations to travel across the whole chain remains nearly independent of the chain size. In addition, the revival patterns do not alter significantly even for chains where the center of the chain is in a zig-zag regime, indicating that the center of the ion string is decoupled from the edges.

The experiment proceeds as follows: a chain of N $^{40}\text{Ca}^+$ ions is confined in a harmonic potential of a linear Paul trap with trap frequencies $(\omega_x, \omega_y, \omega_z) = 2\pi \times (2.25, 2.0, 0.153)$ MHz. A laser at 397 nm is red-detuned with respect to the $S_{1/2}$ - $P_{1/2}$ transition to perform Doppler cooling of the whole chain. An intense beam at 397 nm is tightly focused onto a single ion on one end of the chain, as shown in figure 1.

We rapidly add energy using a technique of pulsed excitation: the intensity of a focused beam is switched off and on with the frequency of the local mode [17, 18]. This resonant process results in a quadratic increase in energy of the excited ion with the number of applied kicks [3]. After 10 μs of pulsed excitation, we turn off the excitation laser and let the system evolve freely for a time τ . Then the energy of individual ions in the chain is measured with a

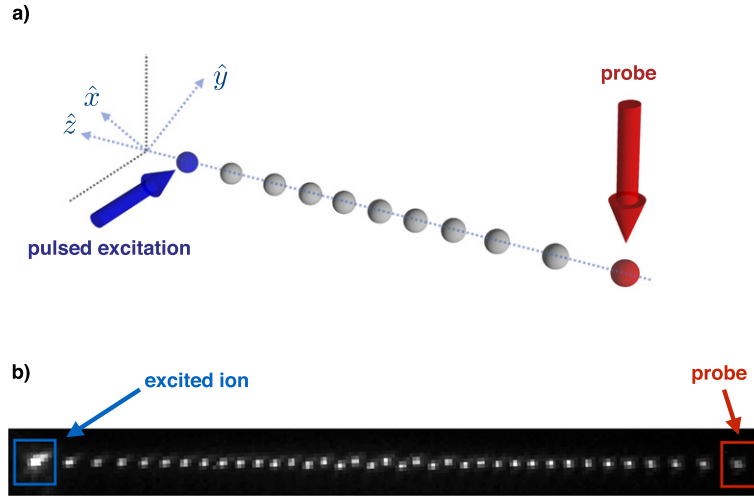


Figure 1. The schematic of the energy transport experiment. (a) The pulsed excitation beam (blue) rapidly adds energy to the ion chain. After a free time evolution, the energy can be read out anywhere along the chain with a probe at 729 nm (red). Both beams are at 45° with respect to the radial modes of motion. (b) The CCD image of the ion chain with 37 ions with the tightly focused excitation beam.

laser at 729 nm tuned to the red motional sideband of the narrow quadrupole transition between $|g\rangle = |S_{1/2}; m_j = -1/2\rangle$ and $|e\rangle = |D_{5/2}; m_j = -5/2\rangle$ [19].

We analyze the dynamics in terms of the eigenmodes of the ion chain by considering that the ions are confined in a potential V , which is the sum of the harmonic trap potential and the Coulomb interaction. Minimizing potential energy V with respect to the coordinates of every ion i , (x_i, y_i, z_i) , yields the equilibrium positions (x_i^0, y_i^0, z_i^0) . We treat each ion as an individual oscillator coupled to the other ions in the chain. Considering the motion in only one of the radial directions, x , the potential energy V can be expanded for small displacements about the equilibrium, $q_i = x_i - x_i^0$,

$$V = \sum_{i=1}^N \frac{1}{2} m \left(\omega_x^2 - \sum_{\substack{j=1 \\ j \neq i}}^N \frac{e^2}{4\pi\epsilon_0 m} \frac{1}{|z_i^0 - z_j^0|^3} \right) q_i^2 + \sum_{i=1}^N \sum_{\substack{j=1 \\ j \neq i}}^N \frac{1}{2} \frac{e^2}{4\pi\epsilon_0} \frac{1}{|z_i^0 - z_j^0|^3} q_i q_j \quad (1)$$

where e is the electron charge and m is the ion mass [20, 21]. The local oscillation frequency differs from the radial trap frequency ω_x due to the repulsive force from the other ions in the chain. The effect of the other ions drops off as the cube of the inter-ion distance. The ions are closer to each other in the center of the chain, hence the local oscillation frequency will be minimal for the middle ion. The local displacements q_i are coupled, and the system may be diagonalized in terms of the N radial normal modes of the ion chain [20]. We enumerate the normal modes \vec{v}_n in the order of decreasing eigenfrequencies such that \vec{v}_1 always refers to the center-of-mass mode with the corresponding eigenfrequency of ω_x . We model the effect of the pulsed excitation of the leftmost ion as a radial displacement of the ion from its equilibrium position. The unit displacement can be decomposed as the sum of radial normal modes with coefficients c_n : $\vec{q} = [1, 0, 0, \dots, 0] = \sum_{n=1}^N c_n \vec{v}_n$.

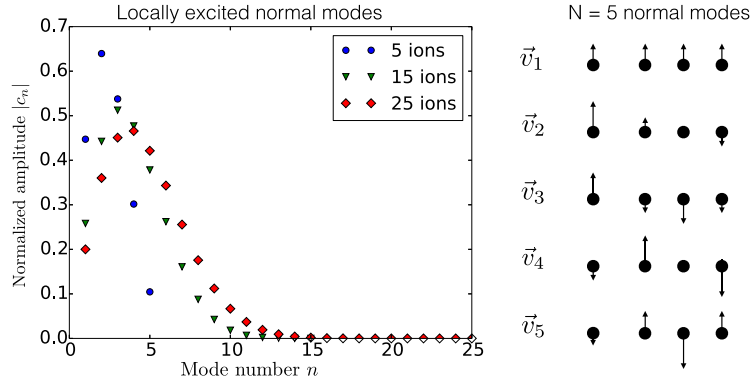


Figure 2. Shown on the left is the decomposition of a unit displacement of the leftmost ion in terms of the radial eigenmodes of the ion chain. For longer chains, only lower-ordered modes are excited. On the right are the 5 possible eigenmodes for a chain of $N = 5$ ions. Modes 4 and 5 are not excited strongly because they do not involve significant motion of the leftmost ion.

The eigenmode decomposition is shown in figure 2 for chains of increasing number of ions. We see that the pulsed excitation creates a superposition of eigenmodes of chain motion, which then evolve at their corresponding eigenfrequencies. However, even for long chains, only the first 10 modes play a large role in determining the subsequent time evolution.

In order to precisely measure the energy of the system after an evolution time τ , we consider the quantized version of the system with the potential energy described by equation (1) with the Hamiltonian H_x written in terms of local phonons creation and annihilation operators at site i denoted a_i^\dagger , a_i , respectively [11, 22]:

$$H_x = \sum_{i=1}^N \hbar \omega_{x,i} a_i^\dagger a_i + \hbar \sum_{i=1}^N \sum_{\substack{j=1 \\ j < i}}^N t_{ij} (a_i^\dagger a_j + a_i a_j^\dagger) \quad (2)$$

where we have neglected fast-rotating non-energy conserving terms. We note that this approximation is not valid in the zig-zag regime of the ion chain where the local trap frequency may be smaller than the coupling. The site-dependent oscillation frequency and the tunneling amplitude are given by:

$$\omega_{x,i} = \omega_x - \frac{1}{2} \sum_{\substack{j=1 \\ j \neq i}}^N \frac{1}{m \omega_x} \frac{e^2}{4\pi\epsilon_0} \frac{1}{|z_i^0 - z_j^0|^3} \quad (3)$$

$$t_{ij} = \frac{1}{2} \frac{1}{m \omega_x} \frac{e^2}{4\pi\epsilon_0} \frac{1}{|z_i^0 - z_j^0|^3} = \frac{1}{2} \frac{\omega_z^2}{\omega_x} \frac{1}{\Delta \bar{z}^3} \quad (4)$$

where $\Delta \bar{z} \sim O(1)$ is a dimensionless separation of ions in the units of the characteristic length scale [20]. We find that for the experimental trap frequencies, the tunneling matrix element between the leftmost ion and its neighbor, t_{12} , ranges from $2\pi \times 6.7$ kHz for a chain of 5 ions to $2\pi \times 21.1$ kHz for a chain of 25 ions.

After performing Doppler cooling, the local radial mode of every ion is in a thermal state with a temperature \bar{n} . The density matrix of the ion i , ρ_i^{th} , can be expressed in terms of the local

phonon number basis $|n_i\rangle$:

$$\rho_i^{\text{th}} = \frac{1}{\bar{n} + 1} \sum_n \left(\frac{\bar{n}}{\bar{n} + 1} \right)^n |n_i\rangle \langle n_i|. \quad (5)$$

The pulsed excitation process is modeled as a displacement operator $D(\alpha)$ applied onto the leftmost ion ($i = 1$) in the chain [23] for some complex amplitude $\alpha = |\alpha|e^{i\phi}$ resulting in a displaced thermal state of motion. Experimentally we do not control the phase of the pulsed laser ϕ , yielding a diagonal density matrix of the first ion ρ_1 after averaging over ϕ :

$$\rho_1 = \frac{1}{2\pi} \int d\phi \left(D(\alpha) \rho_1^{\text{th}} D(\alpha)^\dagger \right) = \sum_n p_n^{\text{disp}} |n_1\rangle \langle n_1| \quad (6)$$

with the occupational probabilities given by [24]:

$$p_n^{\text{disp}} = \left(\frac{1}{\bar{n} + 1} \right) \left(\frac{\bar{n}}{\bar{n} + 1} \right)^n e^{-\frac{|\alpha|^2}{\bar{n} + 1}} L_n \left(-\frac{|\alpha|^2}{\bar{n}(\bar{n} + 1)} \right) \quad (7)$$

where L_n is the Laguerre polynomial of the n^{th} degree. We measure the displacement $|\alpha|$ of the ion i by driving the red motional sideband of the transition between $|g\rangle$, and $|e\rangle$. This interaction couples the electronic and motional states of the ion in the form $|g\rangle |n_i\rangle$ and $|e\rangle |n_i - 1\rangle$ with Rabi frequency $\Omega_{n,n-1}$, which depends on the particular motional state n [19]:

$$\Omega_{n,n-1} = \Omega_0 \left| \langle n - 1 | e^{i\eta(a+a^\dagger)} | n \rangle \right| \quad (8)$$

where Ω_0 is a scale of the coupling strength and η is the Lamb Dicke parameter. In the regime of our experiment, the Rabi frequency $\Omega_{n,n-1}$ increases with the n , and the energy can be determined by monitoring the strength of the sideband interaction. Specifically, we measure the probability to find the ion in the electronic ground state $|g\rangle$:

$$P_g(t) = \frac{1}{2} \left[1 + \sum_{n=0}^{\infty} p_n^{\text{disp}} \cos(\Omega_{n,n-1} t) \right]. \quad (9)$$

First, we extract the initial temperature, \bar{n} , without pulsed excitation. The knowledge of \bar{n} , the laser intensity, and the trapping parameters allows us to numerically invert equation (9). This way, we directly convert the measured ground state probability $P_g(t)$ to the displacement $|\alpha|$. The experimental duration of the sideband interaction is fixed at $t = 7.5 \mu\text{s}$, short compared to the coupling in order to address only the local mode of motion. The short durations of both the energy readout ($7.5 \mu\text{s}$) and the pulsed excitation ($10 \mu\text{s}$) are crucial to observe the energy dynamics as these operations are much faster than the characteristic coupling time of the leftmost ion $2\pi/t_{12}$.

The results for the five-ion chain and the comparison to theory are presented in figure 3. The experimental data are in agreement with a molecular dynamics simulation where the leftmost ion is initially displaced in the radial direction. The simulation takes into account the full potential V , including nonlinearities from the Coulomb repulsion and the driven motion of ions in the Paul trap. It has no free parameters and uses independently measured trap frequencies as inputs. The simulation results show that the dynamics of the time evolution do

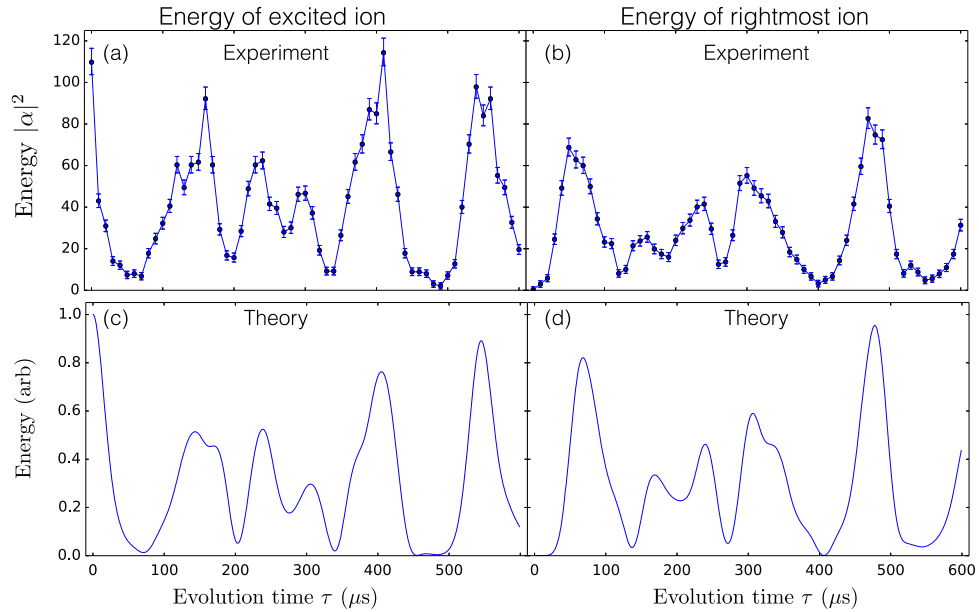


Figure 3. Energy transport in a chain of five ions. The leftmost ion is given a kick and its energy is measured after a subsequent evolution time τ as shown in (a). The energy revivals occur at the rephasing times of the eigenfrequencies and approach the energy of the initial excitation. Plot (b) shows the energy of the rightmost ion: initially unexcited the energy from the kick is rapidly transferred across the chain. The energy of both ions evolves according to the populated normal modes of motion. Each point is a result of 500 measurements resulting in the denoted statistical error bars. The points are connected to guide the eye. Plots (c) and (d) show the kinetic energies of the ions from the molecular dynamics simulations.

not change with a decreased initial excitation amplitude, confirming the absence of nonlinear effects and justifying the normal mode picture.

We repeat the experiment with progressively longer chains. The energy of the rightmost ion during the sequence of experiments is presented in figure 4. We observe clear revivals of the energy even for long chains. With the increase of the length of the ion chain, only a few additional eigenmodes are populated with the local excitation. Once again, the eigenfrequencies of the populated eigenmodes rephase resulting in observed revivals.

For all ion numbers, the time it takes for the excitation to propagate across the entire chain is comparable to the coupling rate between nearest neighbors, $2\pi/t_{12}$. This is explained by the fact that the excitation does not simply hop from one ion to another along the chain. The intuition about excitation hopping does not apply because all of the ions are coupled to each other due to the long range of the Coulomb interaction. It is more appropriate to analyze the dynamics in terms of a normal mode picture. The initial excitation creates a superposition of the normal modes, and the energy is transferred across the chain after rephasing of some of the excited modes of motion and is thus comparable to the normal-mode frequency splitting.

Interestingly, we observe that the timescale of the revivals is similar even for long ion chains. This is explained by the fact that the eigenfrequencies of the populated ten normal modes have only a weak dependence on the ion number. For example, the splitting between the eigenfrequencies of the modes \vec{v}_1 and \vec{v}_5 increases by $\sim 3\%$ as the length is increased from $N = 5$

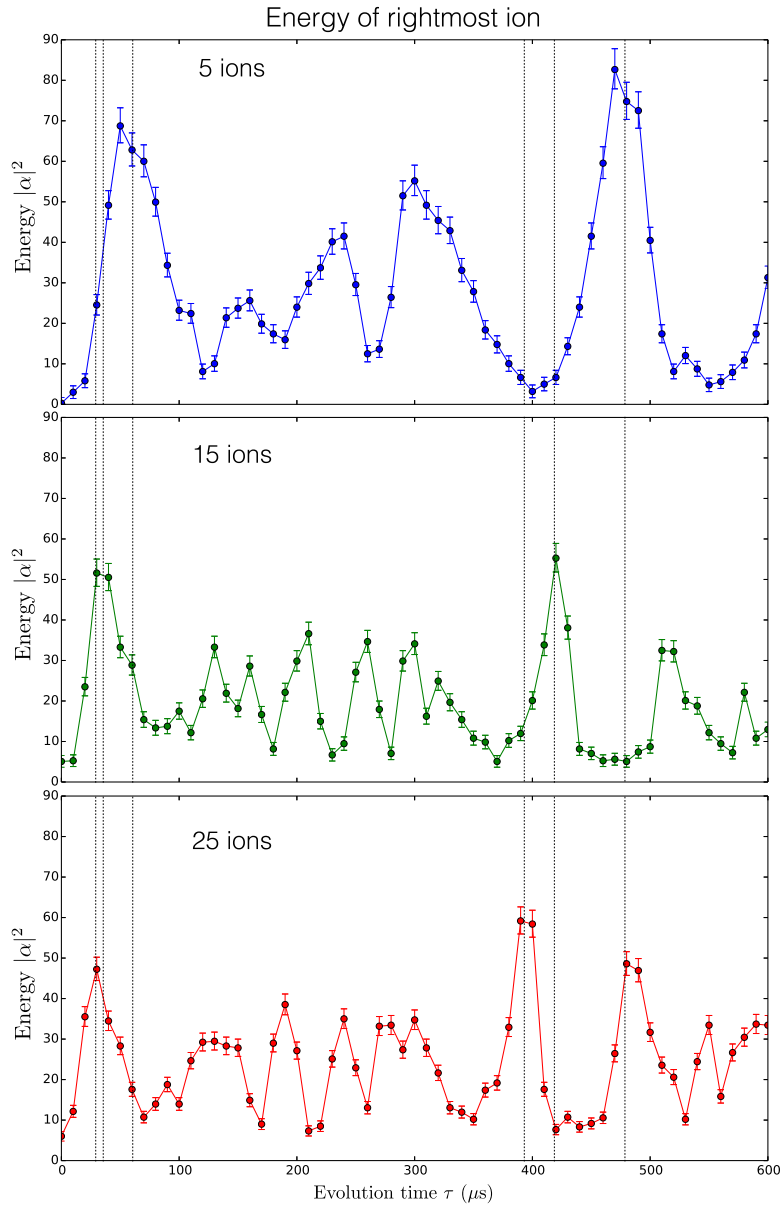


Figure 4. The energy of the rightmost ions measured for progressively longer chains. As seen by position the first peak, the rate of energy transfer across the chain increases slightly due to the reduction in the inter-ion distance. Due to the faster coupling, the feature size of the revivals decreases for longer chains. The average measured energy drops: with a greater number of participating normal modes, the excitation is distributed among more ions.

to $N = 25$. It can be seen that the revival features become sharper for longer chains: more normal modes participate in the dynamics and the ions are spaced closer together increasing the coupling rate. The faster coupling also leads to a higher energy of the rightmost ion for evolution times $\tau \sim 0 \mu\text{s}$: some energy has already transferred from the excited ion to the rightmost ion by the time the pulsed excitation is complete.

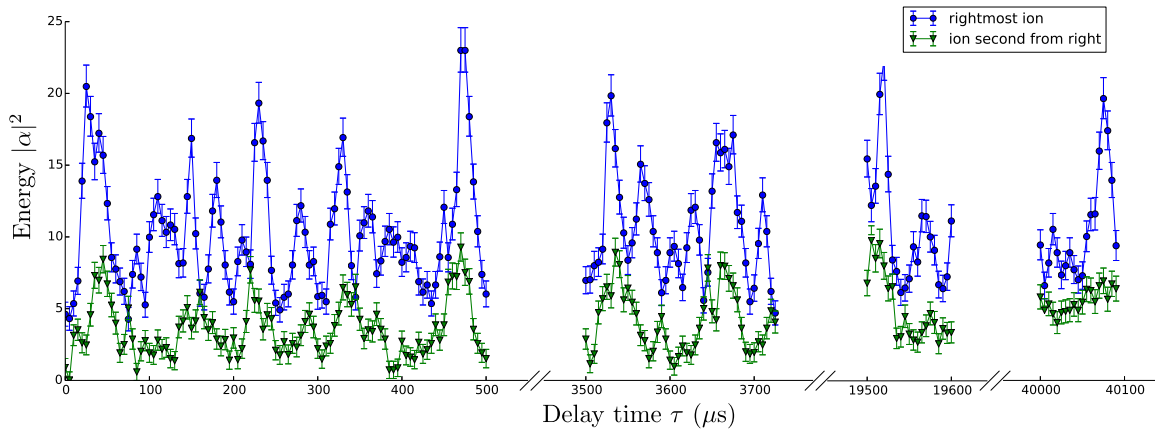


Figure 5. The energy revivals measured for the rightmost ion (circles) and ion second from the right (triangles) in a 37-long ion chain partially in the zig-zag configuration. The dynamics extend up 40 ms and are likely limited by trap frequency instabilities. The energy from the kicked ion does not efficiently transfer to the ion second from the right. The general rise in the measured energy corresponds to background heating during the free evolution.

Even for very long chains, the energy revivals persist for a long time compared to the coupling strength. This is illustrated by figure 5. This measurement was performed with 37 ions in a partially zig-zag configuration as shown in figure 1(b). The energy revivals continue even after an evolution time of $\tau = 40$ ms. For times longer than 40 ms the dynamics wash out, likely due to the instability of the trapping frequencies. Consecutive measurements apart by 12 min for the evolution time $\tau = 40$ ms revealed a $20 \mu\text{s}$ shift in the position of the revival peak, corresponding to 5×10^{-4} change in the coupling strength, ω_z^2/ω_x . The trap frequencies, particularly the radial frequency ω_x , are not expected to be stable at this level.

The measurement presented in figure 5 shows a large difference in the excitation between adjacent ions: the rightmost ion in the chain and its neighbor. While the energy efficiently transfers from the leftmost kicked ion to the rightmost ion, the ion second from the right does not get as energetic. This phenomenon follows from the normal mode decomposition of the initial excitation: the excited normal modes have a small projection onto the motion of the ion second from the right. In the local mode picture, it can also be seen that the efficient transfer of energy occurs because the kicked and the rightmost ion have the same local trap frequency leading to an on-resonant coupling. However, the local frequency of the ion second from the right is different (by approximately the next neighbor coupling), leading to an off-resonant excitation.

In summary, we have presented experimental results measuring the transport of energy in chains of trapped ions. By exciting the ions and reading out the energy faster than the coupling, we are able to observe the energy propagation from the kicked ion. The dynamics observed for five ions agree with numerical simulations and are explained by the normal mode decomposition of the initial excitation. The dynamics of the 37-ion chain persist for evolution times of up to $\tau = 40$ ms, indicating that the system is well-isolated from the environment. The demonstrated control of motion of the long ion chains, including the zig-zag configuration of the chain, is a key ingredient in realizing the outstanding theoretical proposals. Particularly, the

results represent the first experimental steps towards the study of equilibration and the effects of dephasing and disorder on the final energy distribution. In the future, we plan to investigate how the ion chain thermalizes by engineering additional nonlinearities to break with the normal mode structure. The long-range interaction among the ions can be used to study non-local propagation of correlations in interacting quantum many-body systems [25, 26]. Additionally, the presented system can be directly used to study multidimensional spectroscopic techniques as has been proposed recently [27].

Acknowledgments

We would like to acknowledge useful discussions with M Gessner. This work was supported by the NSF CAREER Program Grant No. PHY 0955650. M R was supported by an award from the US Department of Energy Office of Science Graduate Fellowship Program (DOE SCGF).

References

- [1] Bermudez A, Bruderer M and Plenio M B 2013 *Phys. Rev. Lett.* **111** 040601
- [2] Lin G D and Duan L M 2011 *New J. Phys.* **13** 075015
- [3] Pruttivarasin T, Ramm M, Talukdar I, Kreuter A and Häffner H 2011 *New J. Phys.* **13** 075012
- [4] Benassi A, Vanossi A and Tosatti E 2011 *Nat. Commun.* **2** 236
- [5] Islam R *et al* 2011 *Nat. Commun.* **2** 377
- [6] Blatt R and Roos C F 2012 *Nat. Phys.* **8** 277–84
- [7] Freitas N, Martinez E and Paz J P 2013 arXiv:1312.6644
- [8] Ruiz A, Alonso D, Plenio M B and del Campo A 2014 arXiv:1401.5480
- [9] Ai B-q, He D and Hu B 2010 *Phys. Rev. E* **81** 031124
- [10] Porras D and Cirac J I 2004 *Phys. Rev. Lett.* **93** 153902
- [11] Ivanov P A, Ivanov S S, Vitanov N V, Mering A, Fleischhauer M and Singer K 2009 *Phys. Rev. A* **80** 060301
- [12] Shimshoni E, Morigi G and Fishman S 2011 *Phys. Rev. Lett.* **106** 010401
- [13] Cormick C and Morigi G 2012 *Phys. Rev. Lett.* **109** 053003
- [14] Morigi G and Fishman S 2004 *Phys. Rev. Lett.* **93** 170602
- [15] Morigi G and Fishman S 2004 *Phys. Rev. E* **70** 066141
- [16] Haze S, Tateishi Y, Noguchi A, Toyoda K and Urabe S 2012 *Phys. Rev. A* **85** 031401
- [17] Sheridan K, Seymour-Smith N, Gardner A and Keller M 2012 *Eur. Phys. J. D* **66** 289
- [18] Lin Y W, Williams S and Odom B C 2013 *Phys. Rev. A* **87** 011402
- [19] Leibfried D, Blatt R, Monroe C and Wineland D 2003 *Rev. Mod. Phys.* **75** 281–324
- [20] James D 1998 *Appl. Phys. B* **66** 181–90
- [21] Porras D and Cirac J I 2004 *Phys. Rev. Lett.* **92** 207901
- [22] Deng X L, Porras D and Cirac J I 2008 *Phys. Rev. A* **77** 033403
- [23] Ziesel F, Ruster T, Walther A, Kaufmann H, Dawkins S, Singer K, Schmidt-Kaler F and Poschinger U G 2013 *J. Phys. B: At. Mol. Opt. Phys.* **46** 104008
- [24] Saito H and Hyuga H 1996 *J. Phys. Soc. Japan* **65** 1648–54
- [25] Richerme P, Gong Z-X, Lee A, Senko C, Smith J, Foss-Feig M, Michalakakis S, Gorshkov A V and Monroe C 2014 arXiv:1401.5088
- [26] Jurcevic P, Lanyon B P, Hauke P, Hempel C, Zoller P, Blatt R and Roos C F 2014 arXiv:1401.5387
- [27] Gessner M, Schlawin F, Häffner H, Mukamel S and Buchleitner A 2014 arXiv:1312.3365v2