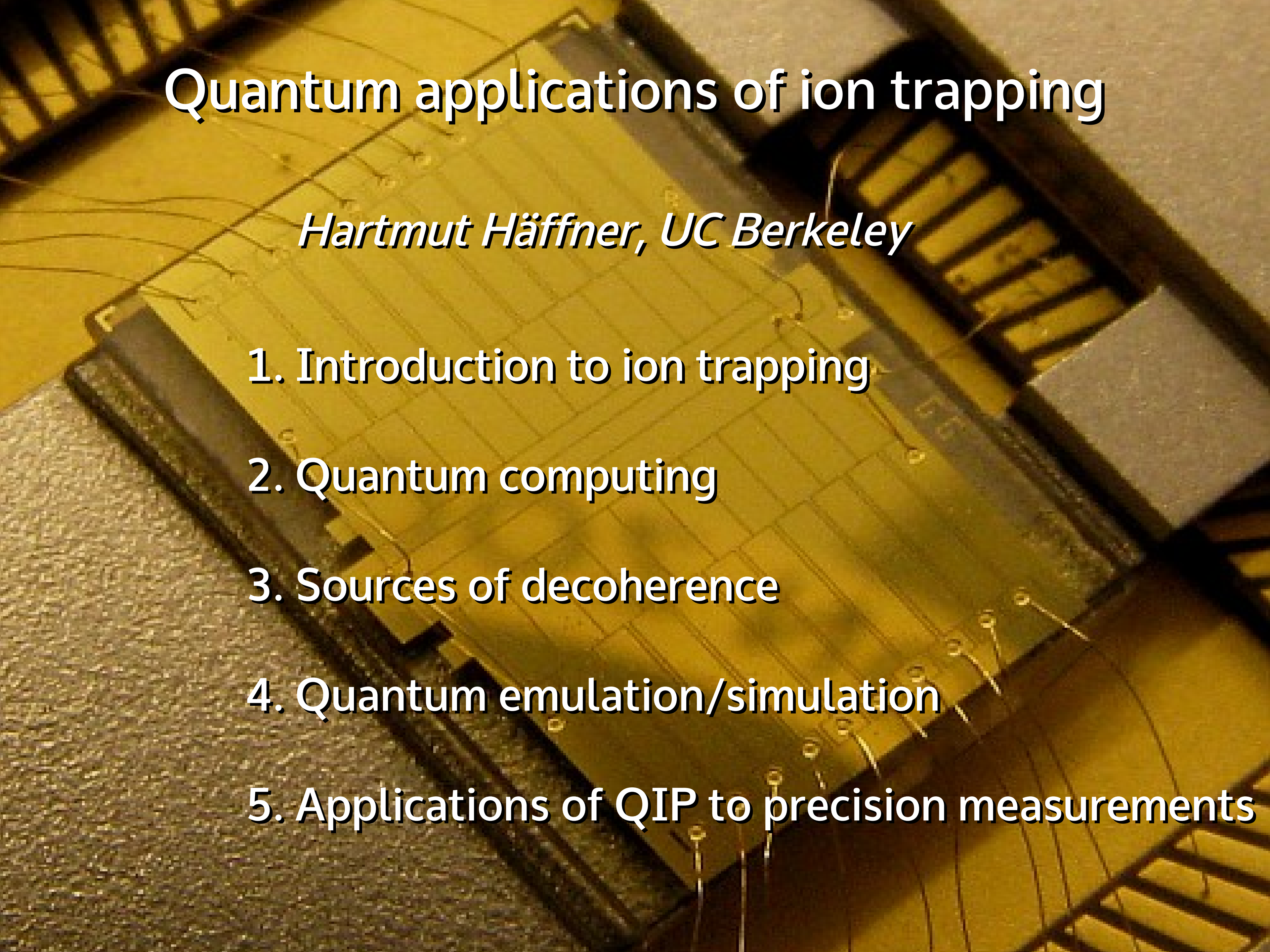


Quantum applications of ion trapping

Hartmut Häffner, UC Berkeley

1. Introduction to ion trapping
2. Quantum computing
3. Sources of decoherence
4. Quantum emulation/simulation
5. Applications of QIP to precision measurements

Plan

Lecture #1: Introduction
- Paul traps
- Laser ion-interaction

Lecture #2: Quantum computing
- Quantum gates
- Quantum state engineering

Lecture #3: Decoherence I/Scaling
- Qubit decoherence
- Scaling

Lecture #4: Decoherence II/Quantum/emulation
- Anomalous heating
- Quantum emulation

Lecture #5: Applications
- Atomic clocks
- Fundamental symmetry tests

Plan

Lecture #1: Introduction
- Paul traps
- Laser ion-interaction

Lecture #2: Quantum computing
- Quantum gates
- Quantum state engineering

Lecture #3: Decoherence I/Scaling
- Qubit decoherence
- Scaling

Lecture #4: Decoherence II/Quantum emulation
- Anomalous heating
- Quantum emulation

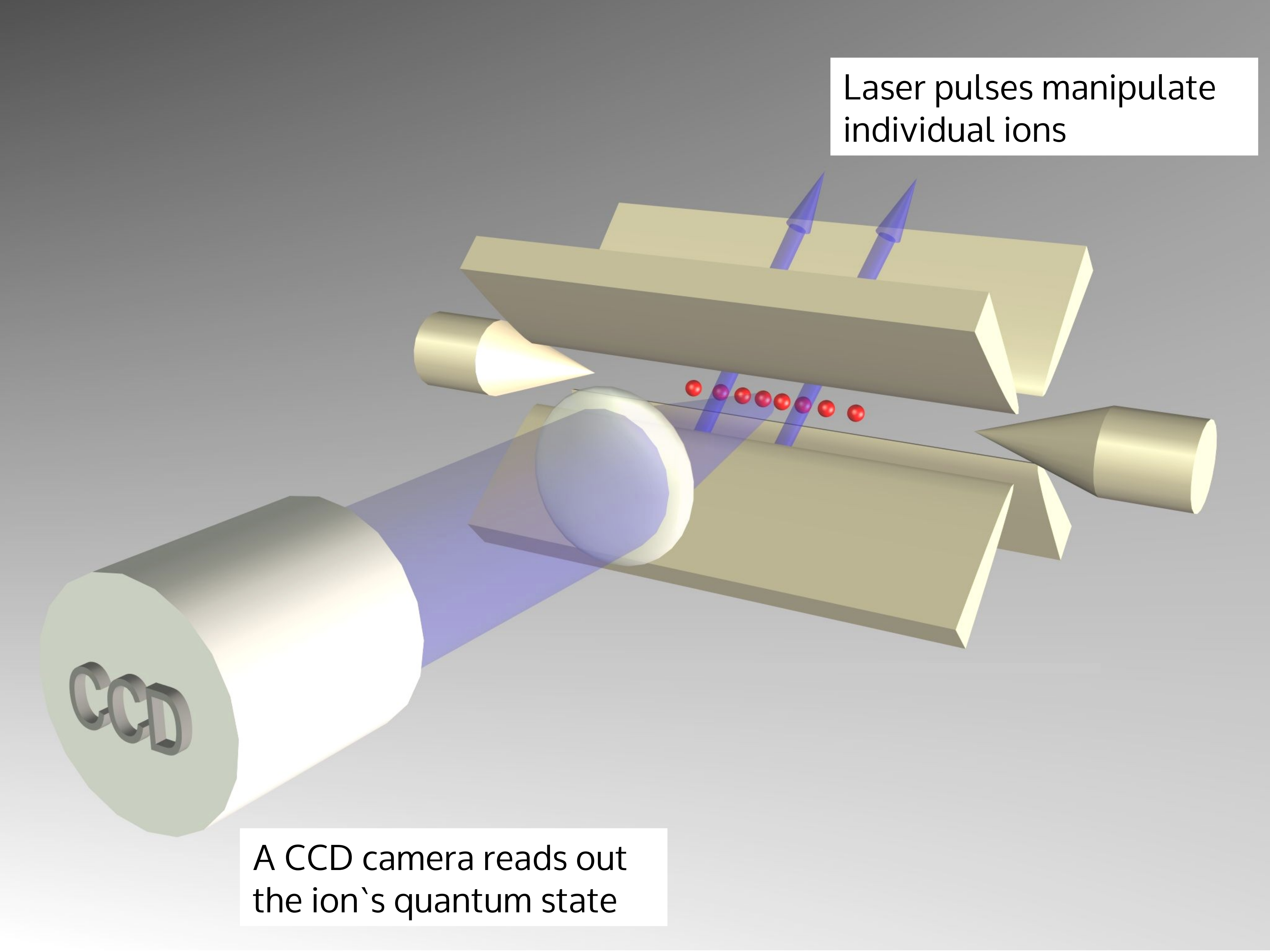
Lecture #5: Applications
- Atomic clocks
- Fundamental symmetry tests

"In the first place it is fair to state that we are not experimenting with single particles, anymore than we can raise Ichtyosauria in the zoo."

"...we *never* experiment with just *one* electron or atom or (small) molecule. In thought-experiments we sometimes assume that we do; this invariably entails ridiculous consequences."

E. Schrödinger (1952)

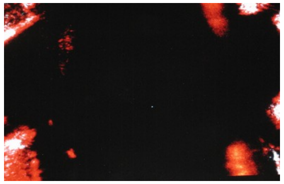
Photo credit: Warren Nagourney at the University of Washington, c. 2000
retrieved from: https://www.flickr.com/photos/fatllama/43155091/


Laser pulses manipulate individual ions
CCD
A CCD camera reads out the ion`s quantum state

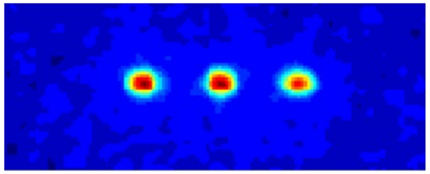
Motivation for trapped ions

Special trait: individually controllable quantum systems

- far away from noise sources

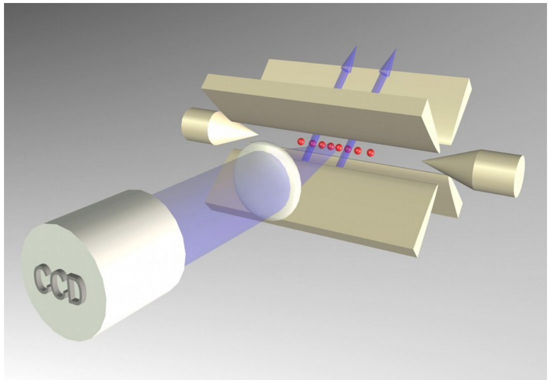


- positive charge separates them



- switchable control via laser-light

  and 100% read-out fidelity



Literature

“Quantum dynamics of single trapped ions“
D. Leibfried, R. Blatt, C. Monroe, D. Wineland
Rev. Mod. Phys. **75**, 281-324 (2003)

“Experimental Issues in Coherent Quantum-State Manipulation of Trapped Atomic Ions“
D. Wineland *et al.*
J. Res. Natl. Inst. Stand. Technol. **103**, 259-328 (1998)

“Quantum Computing with Trapped Ions“
H. Häffner, C. Roos, R. Blatt
Physics Reports **469**, 155-203 (2008).

Trapping

Trapping ions

Charged particles $\Longrightarrow$ Confinement by electric (and magnetic) fields

Ion confinement requires a binding force in three dimensions:

harmonic binding force $F \sim -r \;\Rightarrow\; F = eE = -e\nabla\Phi \Rightarrow \Phi \sim r^2$

quadrupole potential $\Phi = \Phi_0(\alpha x^2 + \beta y^2 + \gamma z^2)$

$\Delta\Phi = 0 \longrightarrow \alpha + \beta + \gamma = 0$ Laplace theorem

Electrostatic potential with rotational symmetry:

$$\boxed{\Phi(r) = \frac{\Phi_0}{r_0^2}(x^2 + y^2 - 2z^2)}$$

confinement in one or two dimensions

Penning trap : $\Phi(r)$ + axial magnetic field

Paul trap : $\Phi_{dc}(r) + \Phi_{ac}(r)\cos(\Omega t)$

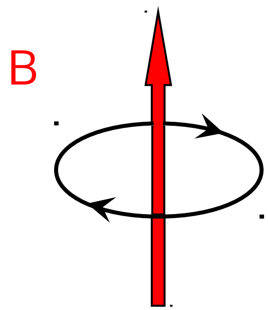
Penning trap

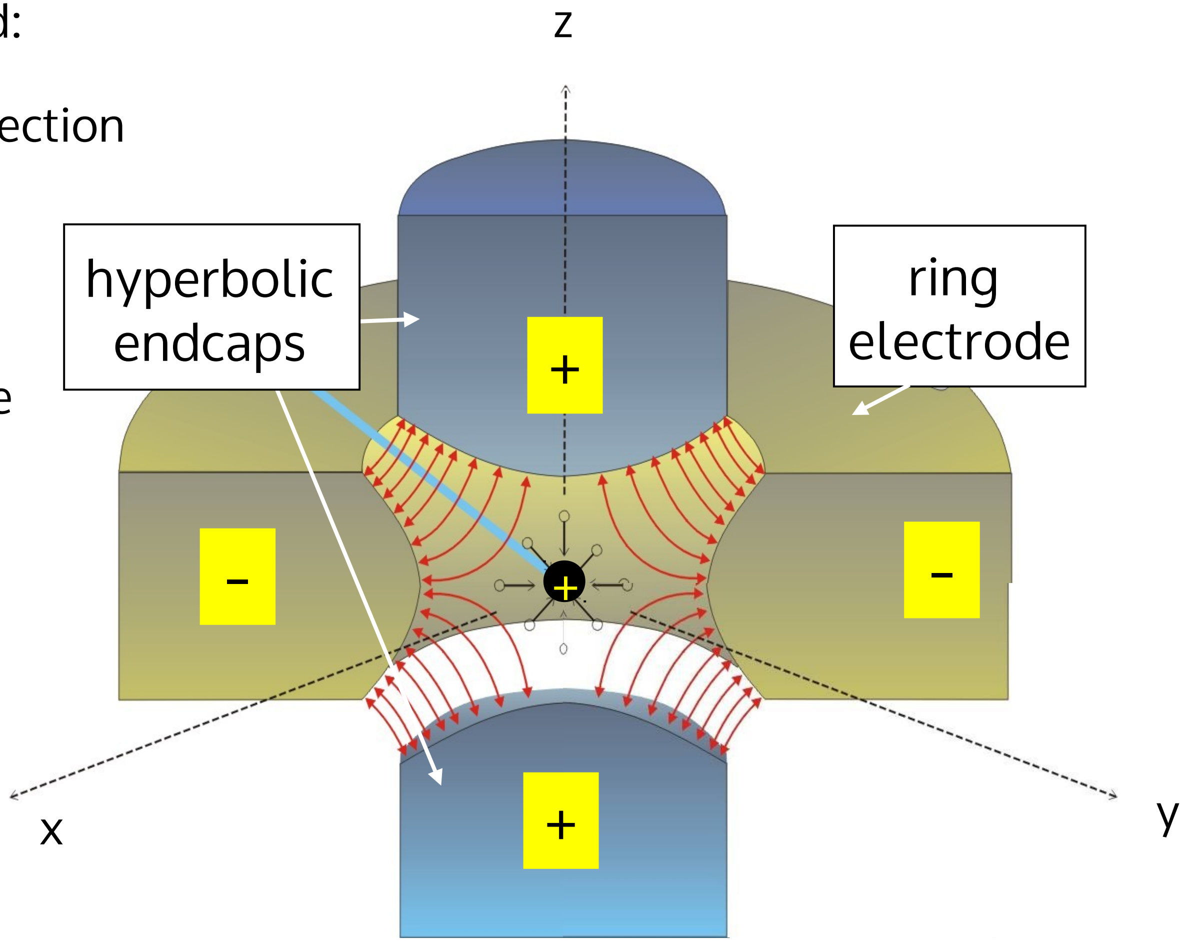
Static quadrupole electric field:

confinement in axial (z) direction

Axial magnetic field:

confinement in x/y plane





Paul or RF trap

Static and dynamic quadrupole electric field:

confinement in all directions
for suitably chosen parameters

Mechanical analogue:
rotating saddle potential

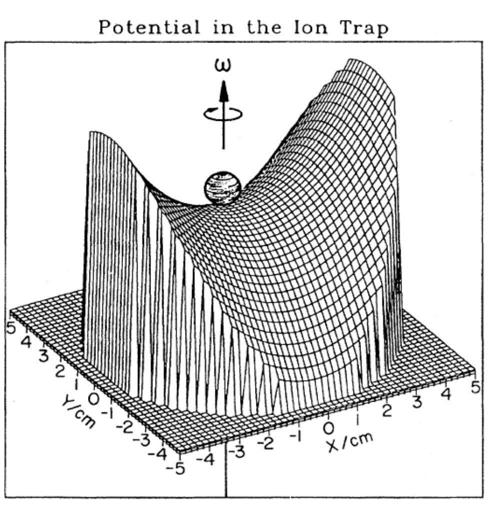
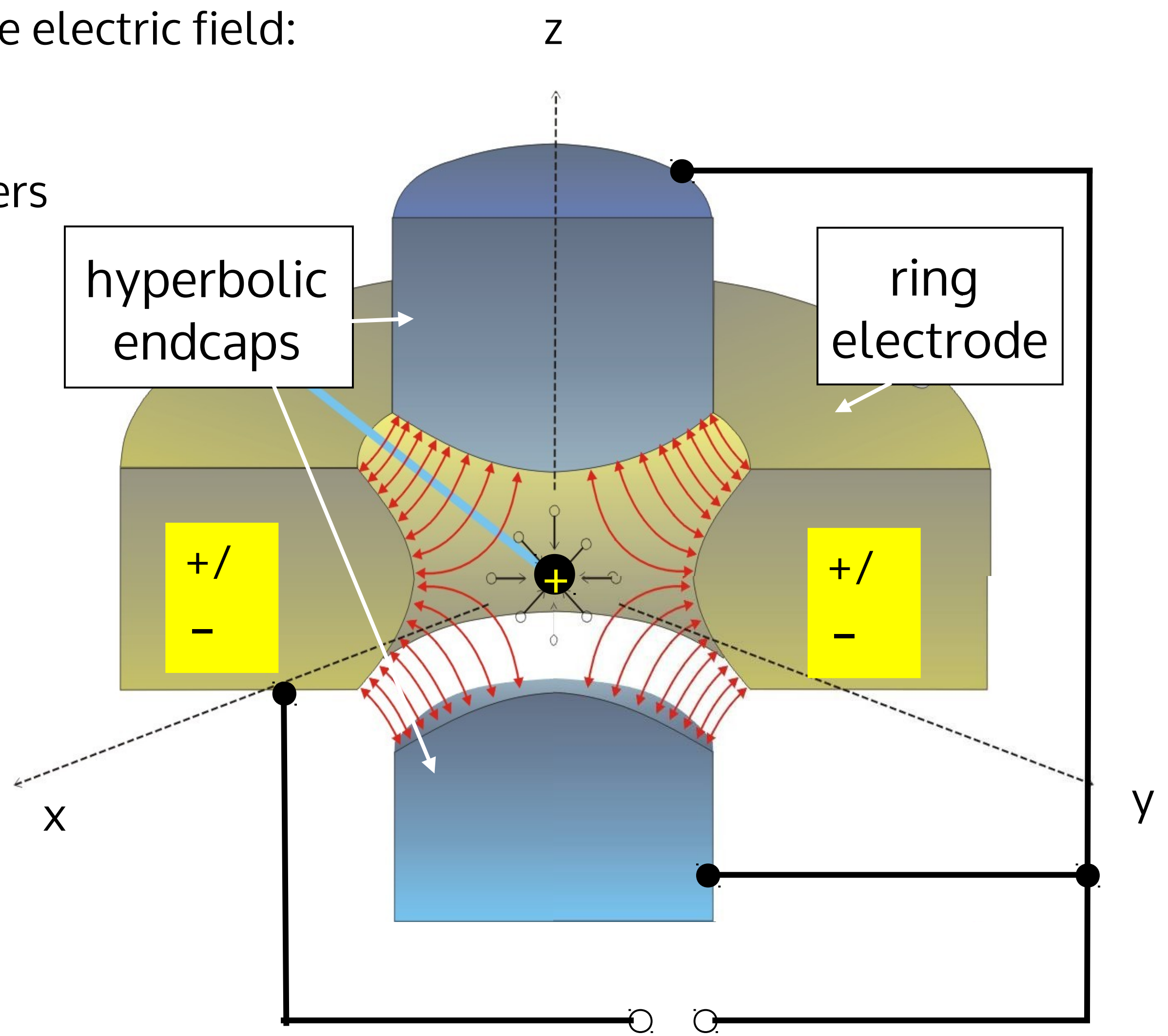


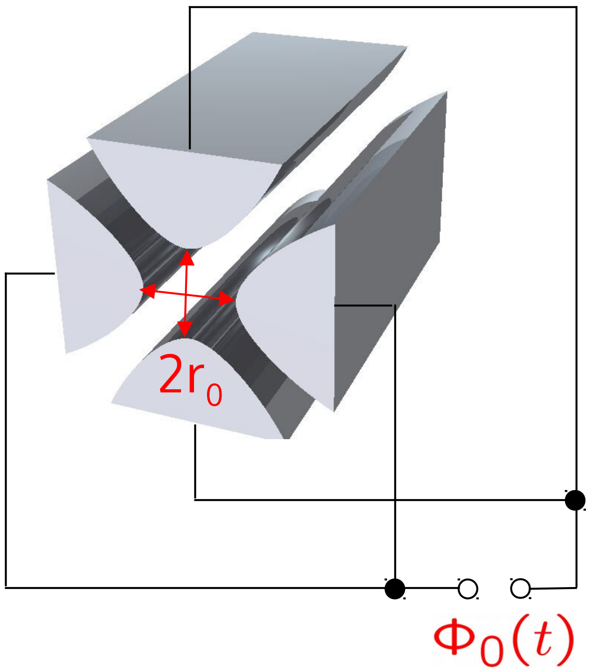
FIG. 8. Mechanical analogue model for the ion trap with steel-ball as "particle."

(W.Paul, RMP **62**,531 (1990)



$$U_{dc}(r) + U_{ac}(r)\cos(\Omega t)$$

2D trapping



$$\boxed{\Phi(r) = \Phi_0(t)\frac{x^2 - y^2}{2r_0^2}}$$

$$\Phi_0(t) = U + U_{rf}\cos(\Omega t)$$

$$E_x = -\Phi_0(t)\frac{x}{r_0^2} \qquad E_y = \Phi_0(t)\frac{y}{r_0^2}$$

Equations of motion:
$$\begin{cases} \ddot{x} + \frac{e}{mr_0^2}(U + U_{rf}\cos(\Omega t))x = 0 \\ \ddot{y} - \frac{e}{mr_0^2}(U + U_{rf}\cos(\Omega t))y = 0 \end{cases}$$

Mathieu equations

$$\frac{d^2x}{d\tau^2} + (a - 2q\cos(2\tau))x = 0$$

$$\frac{d^2y}{d\tau^2} - (a - 2q\cos(2\tau))y = 0$$

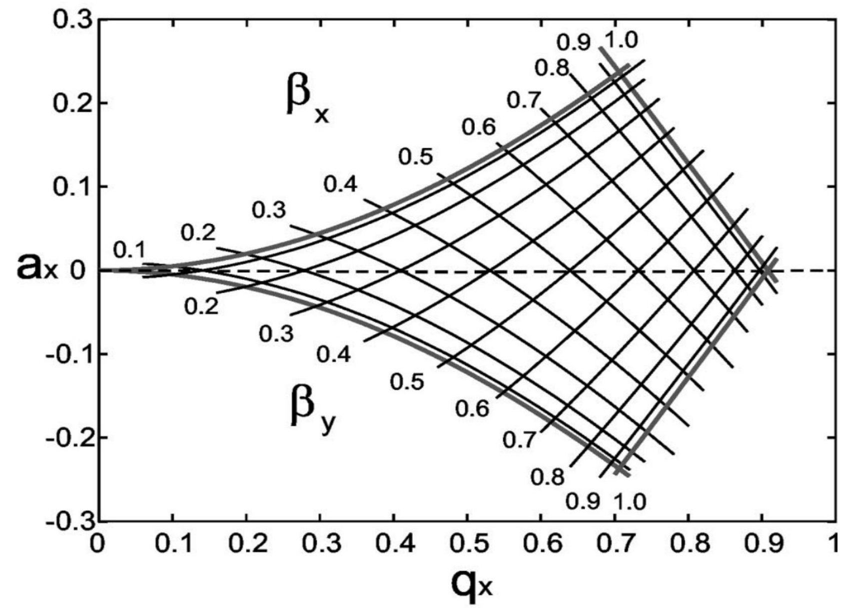
$$q = \frac{2eU_{rf}}{mr_0^2\Omega^2} \qquad a = \frac{4eU}{mr_0^2\Omega^2}$$

$$\tau = \frac{\Omega t}{2}$$

q - and a - parameter

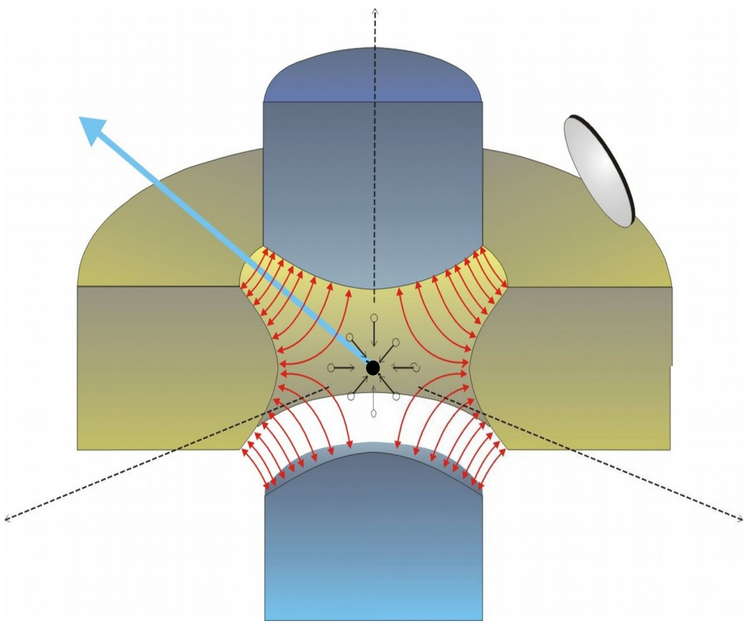
Stability diagram

see D. Leibfried et al., Rev. Mod. Phys. 75, 281 (2003)

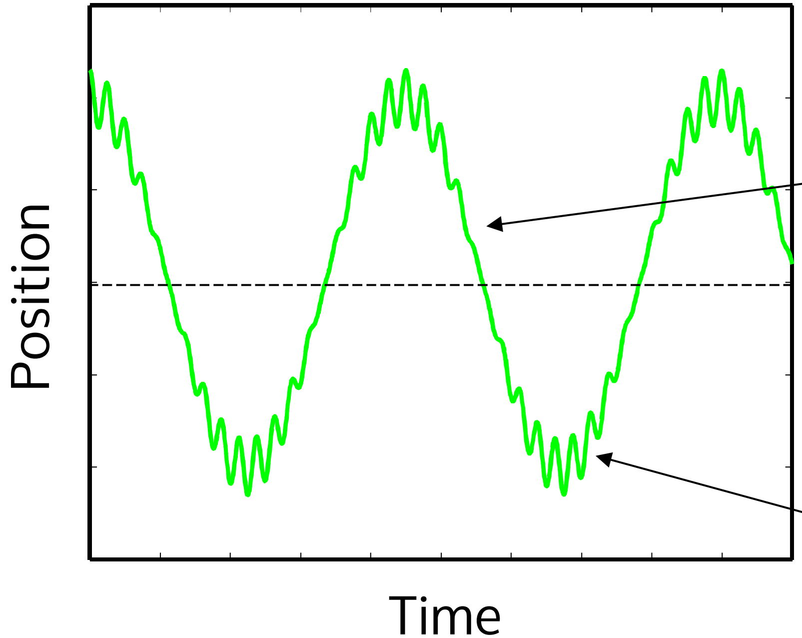


Micromotion

$$x(t) = x_0 \cos(\nu_x t)\left[1 - \frac{q_x}{2}\cos(\Omega t)\right]$$



1D-solution of Mathieu equation



ν : macromotion
(motion in pseudopotential)

$$\nu \ll \Omega$$

Ω : micromotion (driven motion)

All you need, most of the time

Quantum mechanical motion

Quantization of the ion motion:

$$x(t) = x_0 \cos(\nu_x t) \left[1 - \frac{q_x}{2} \cos(\Omega t)\right]$$

Secular approximation:

Neglects micromotion and interprets motion as generated by "pseudo-potential" (see D. Leibfried et al, Rev. Mod. Phys. 75, 281 (2003))

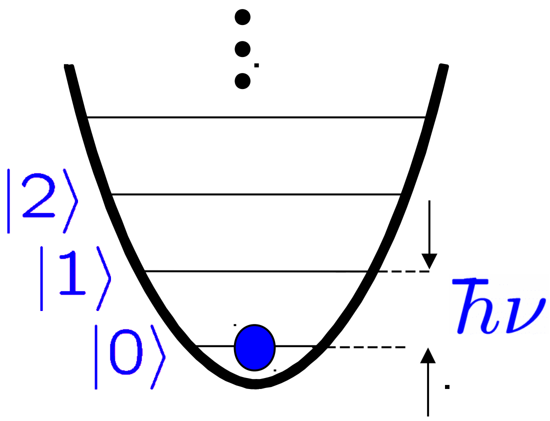
$$H = \sum_i \frac{p_i^2}{2m} + \frac{1}{2} \sum_i m \nu_i^2 x_i^2, i \in \{x, y, z\}$$

Typical numbers

Extension of the ground state:

$$x = \sqrt{\frac{\hbar}{2m\nu}}(a + a^\dagger)$$

$$\langle 0|x^2|0\rangle = \frac{\hbar}{2m\nu}\langle 0|(a + a^\dagger)^2|0\rangle = \frac{\hbar}{2m\nu}$$



$$\left.\begin{array}{l} \nu = (2\pi)1\,\mathrm{MHz} \\ \mathrm{m}{=}40\ \mathrm{u} \end{array}\right\} \quad \langle x^2\rangle^{1/2} = \sqrt{\frac{\hbar}{2m\nu}} \approx 11\mathrm{nm}$$

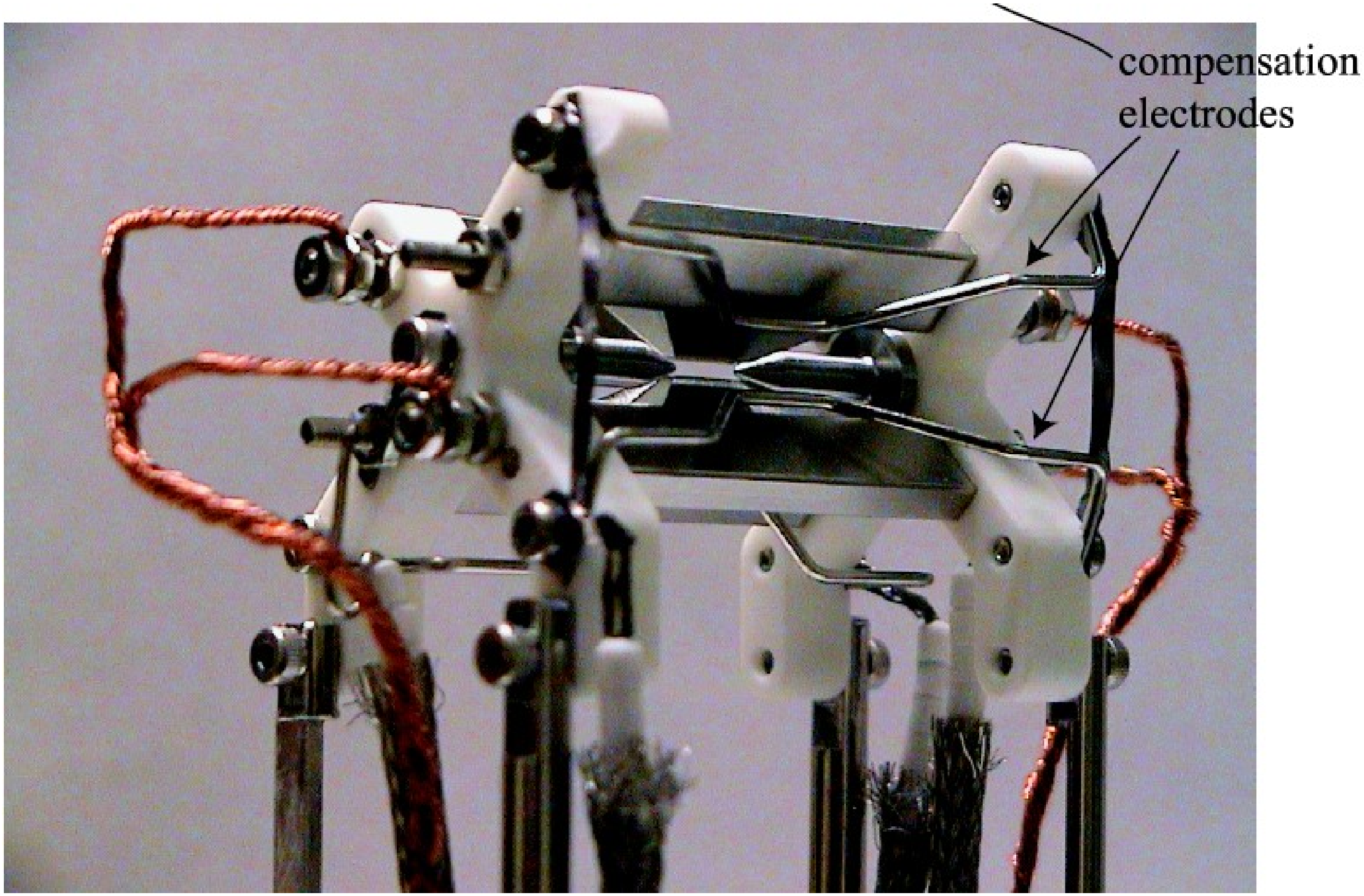
Size of the wave packet << wavelength of visible light

Energy scale of interest:

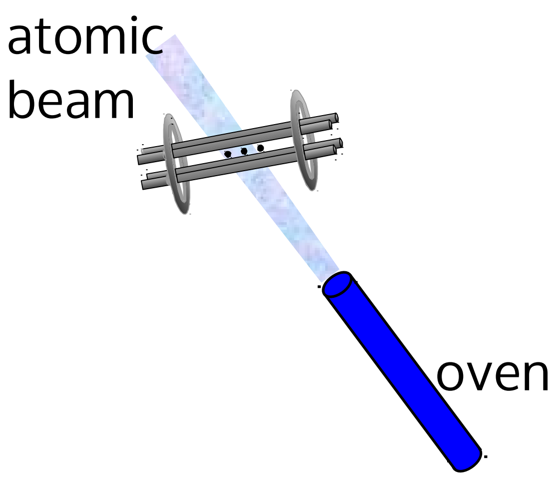
$$\hbar\nu = k_B T \longrightarrow T = \frac{\hbar\nu}{k_B} \approx 50\mu\mathrm{K}$$

"Classic" linear trap

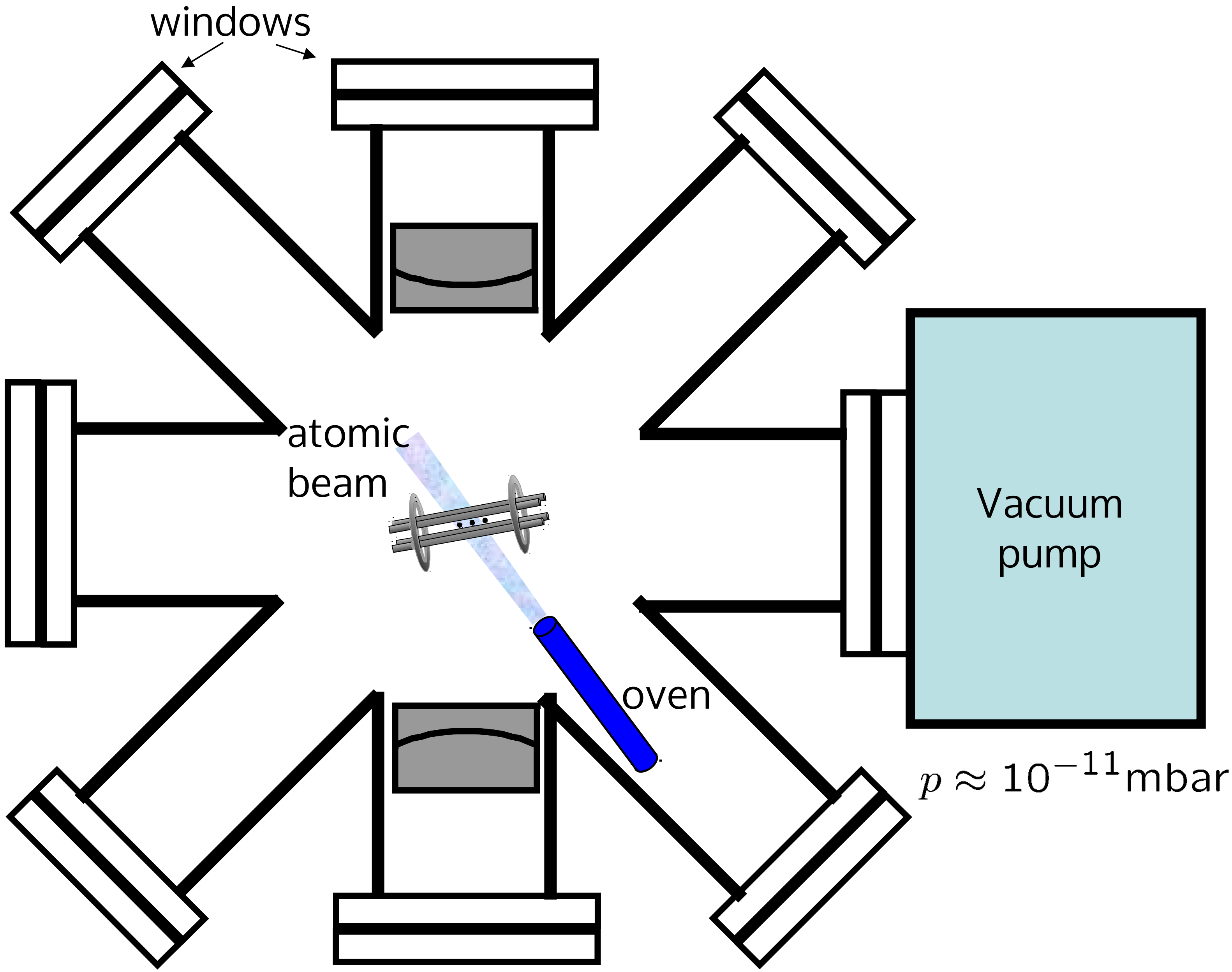
Compensation electrodes: used to null residual electric stray fields in the transverse directions



Experimental set-up



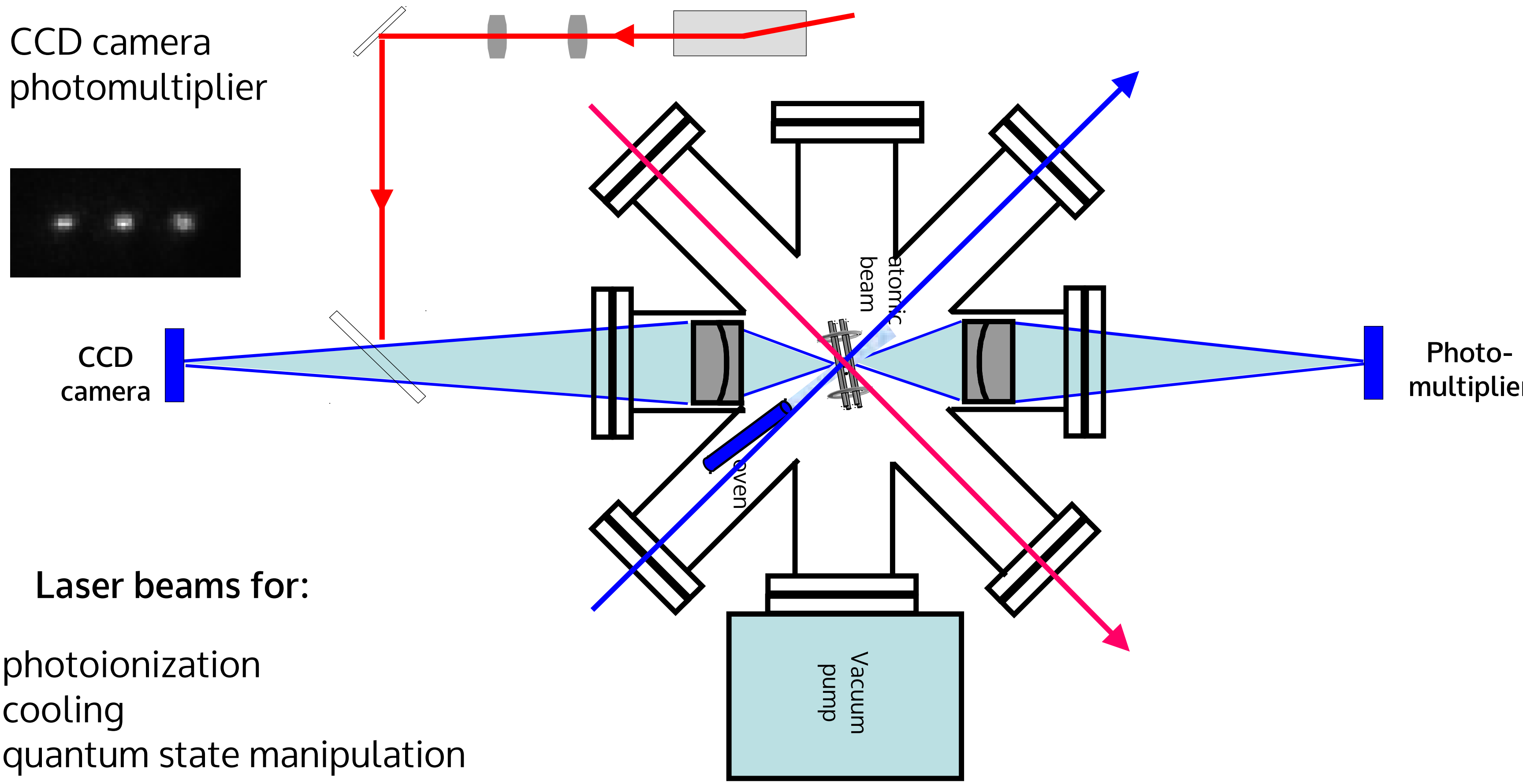
Experimental set-up



Experimental set-up

Fluorescence detection by

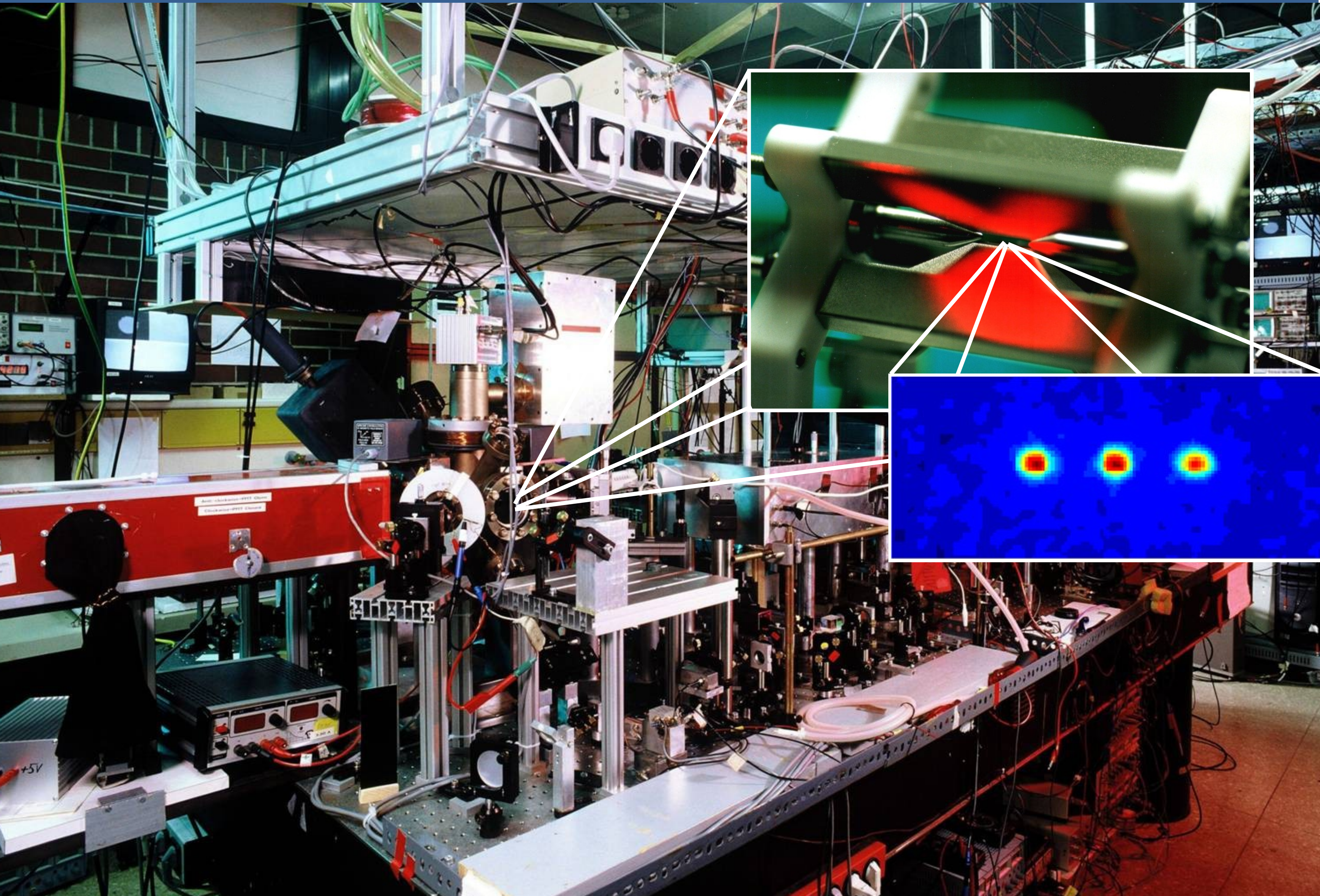
CCD camera
photomultiplier



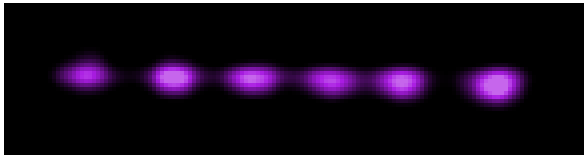
Laser beams for:

photoionization
cooling
quantum state manipulation
fluorescence excitation

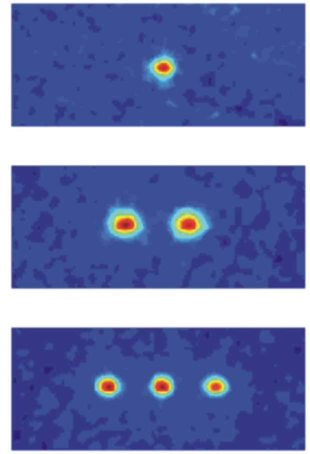
The hardware

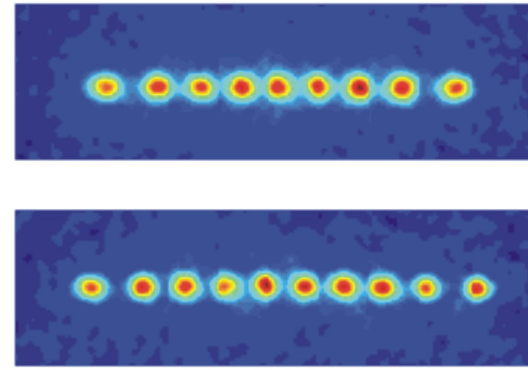


Cold ion crystals



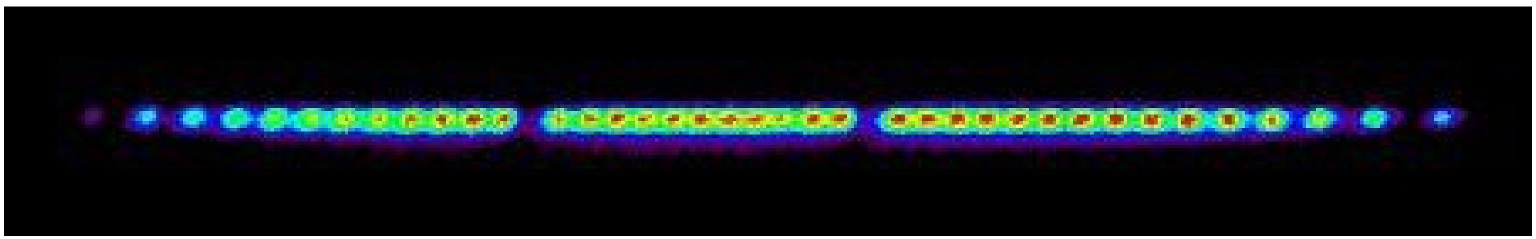
Oxford, England: $^{40}Ca^+$

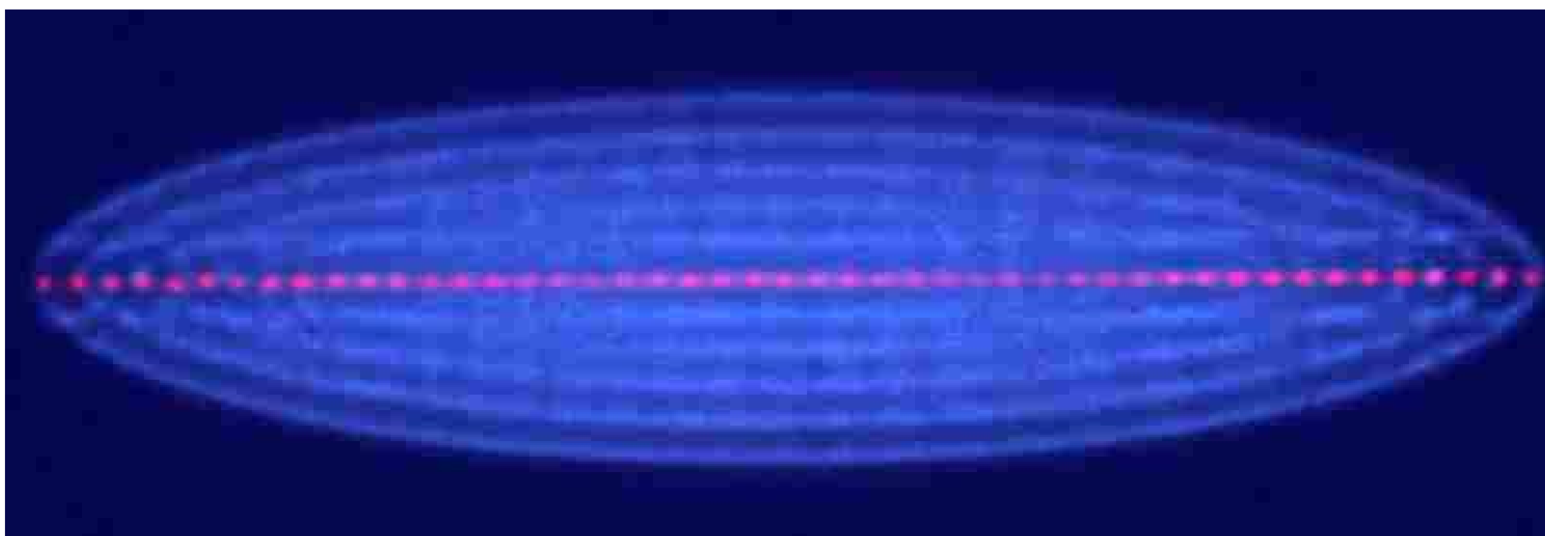




Innsbruck, Austria: $^{40}Ca^+$

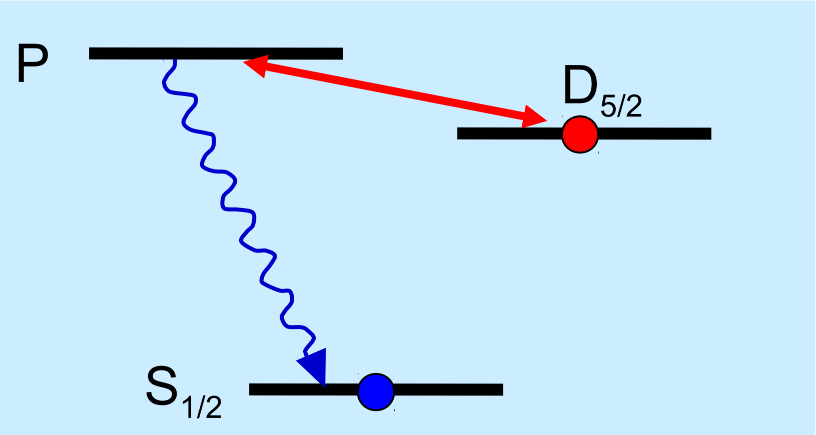
Boulder, USA: Hg^+





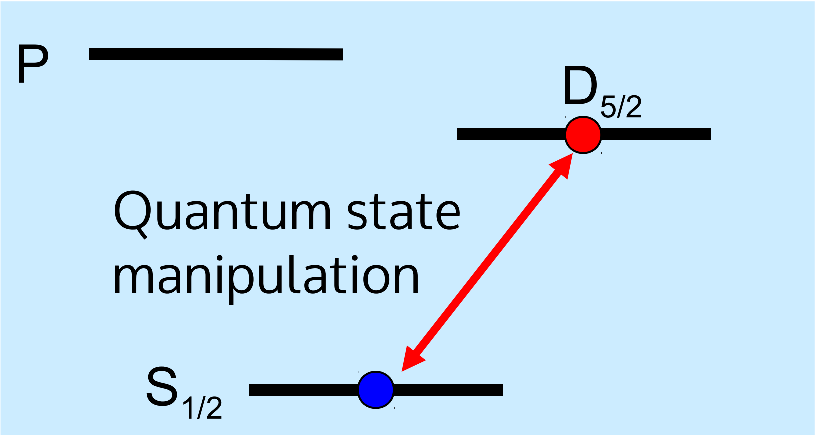
Aarhus, Denmark: $^{40}Ca^+$ (red) and $^{24}Mg^+$ (blue)

Experimental procedure



1. Initialization in a pure quantum state

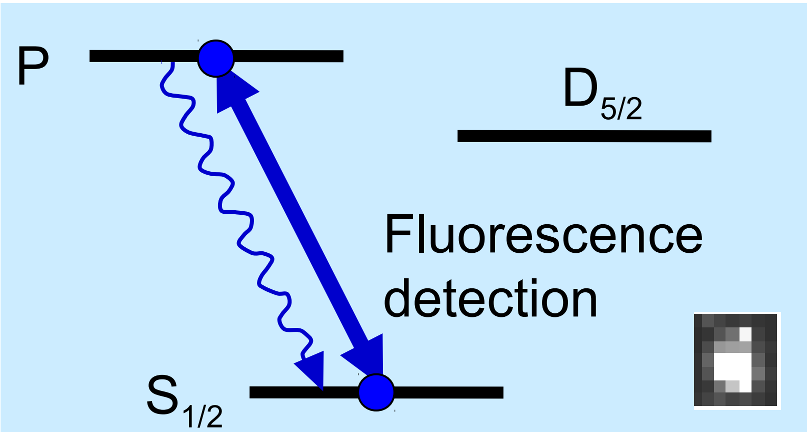
Experimental procedure



1. Initialization in a pure quantum state

2. Quantum state manipulation on $S_{1/2}$ – $D_{5/2}$ transition

Experimental procedure

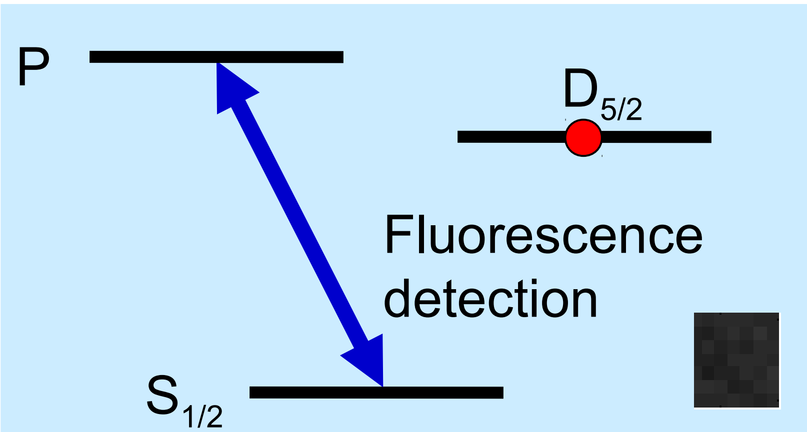


1. Initialization in a pure quantum state

2. Quantum state manipulation on $S_{1/2}$ – $D_{5/2}$ transition

3. Quantum state measurement by fluorescence detection

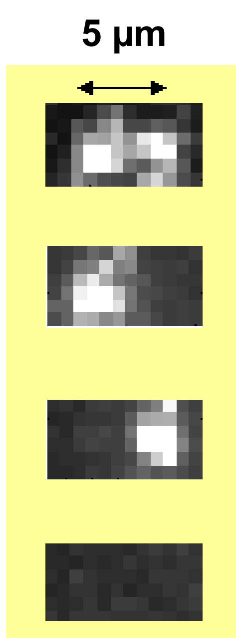
Experimental procedure



1. Initialization in a pure quantum state
2. Quantum state manipulation on $S_{1/2} - D_{5/2}$ transition
3. Quantum state measurement by fluorescence detection

<u>Two ions:</u>

Spatially resolved detection with CCD camera



$|SS\rangle$

$|SD\rangle$

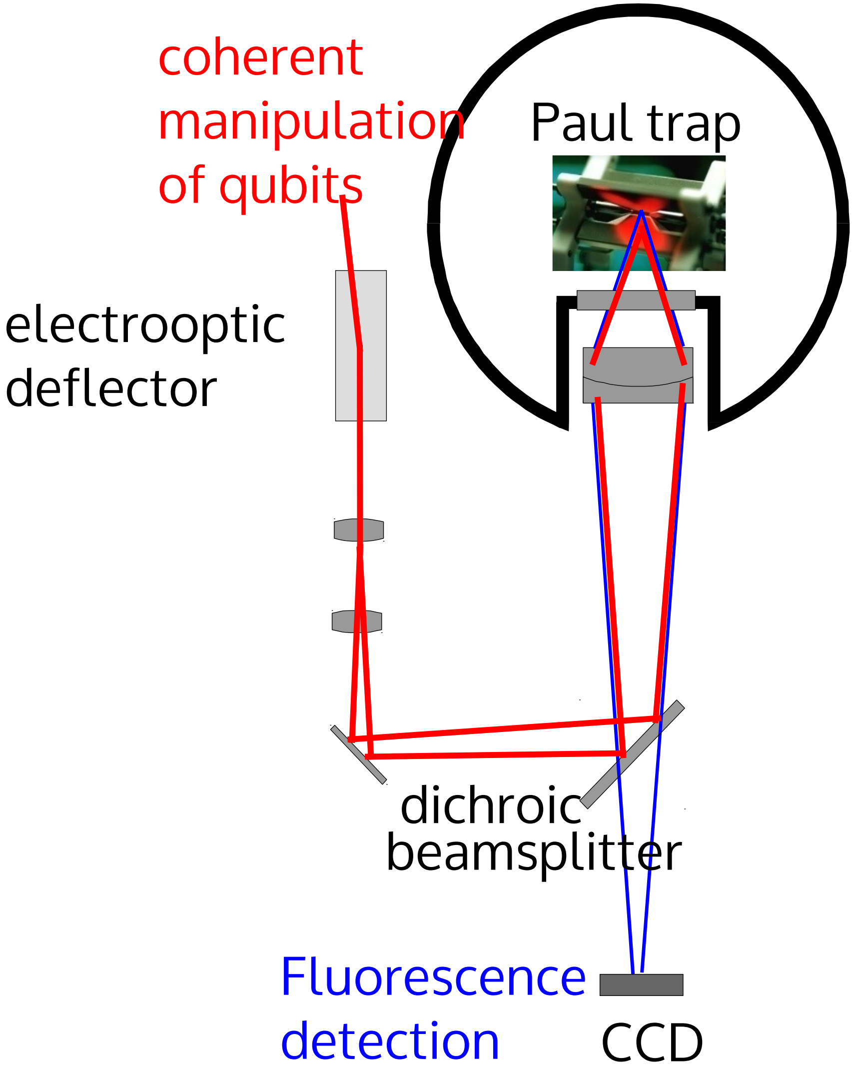
$|DS\rangle$

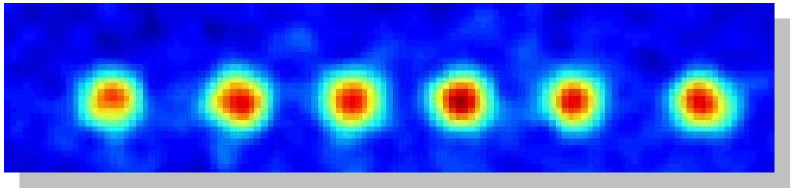
$|DD\rangle$

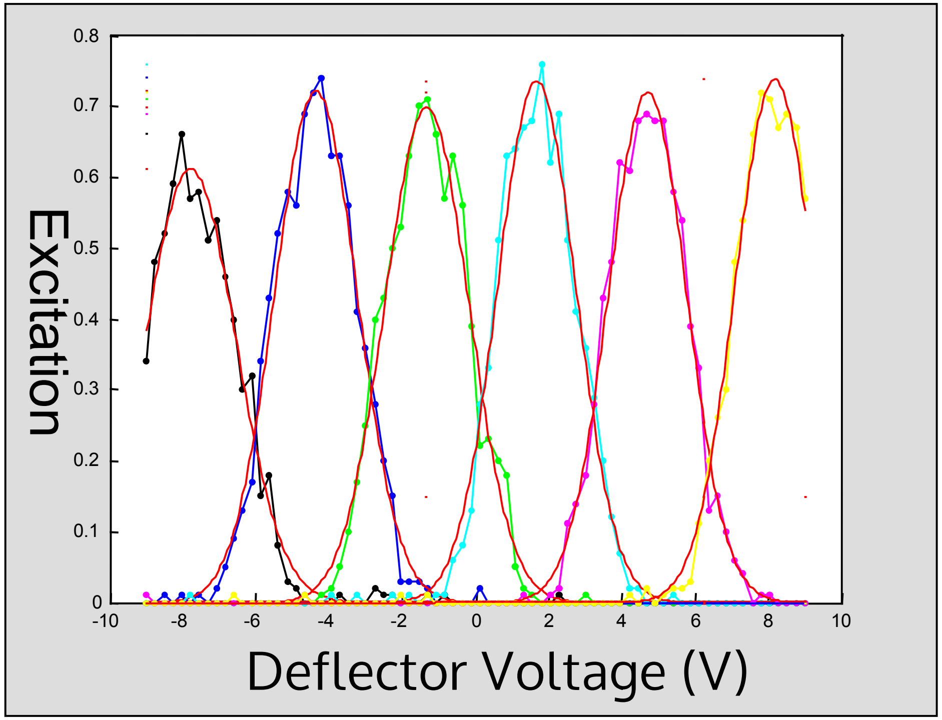
50 experiments / s

Repeat experiments 100-200 times

Adressing single ions



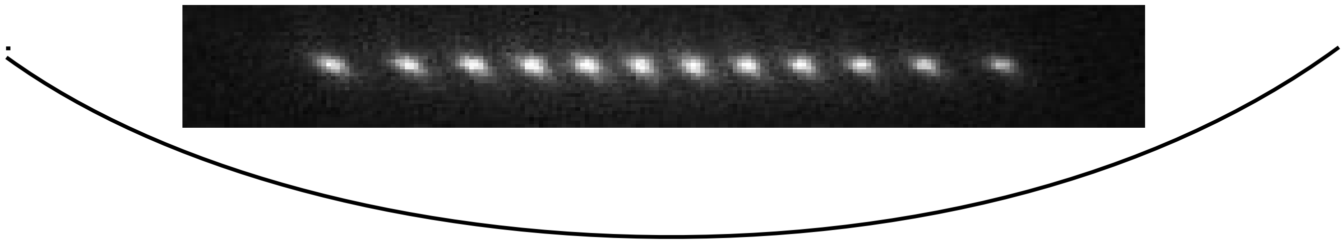




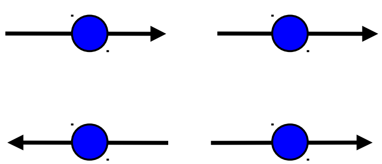
- inter ion distance: ~ 4 um
- addressing waist: ~ 2 um

< 0.1% intensity on neighbouring ions

Motion of ion crystals



2 ions:

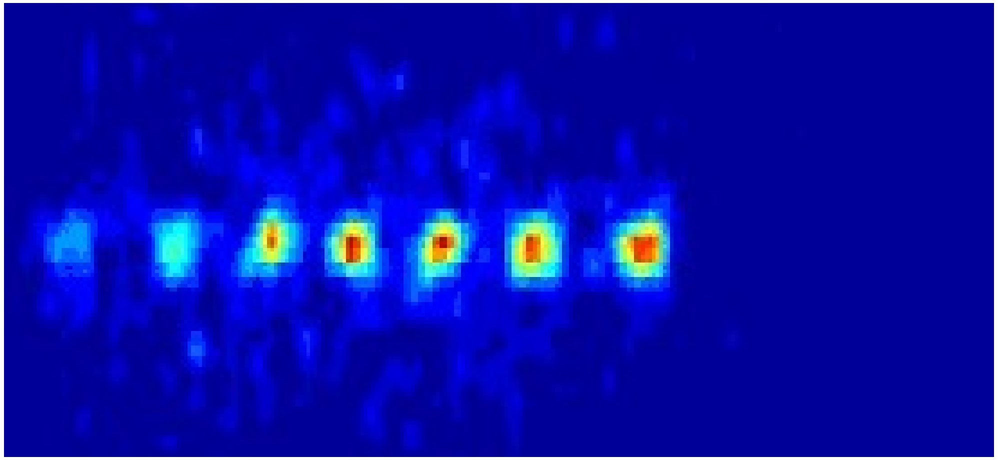


center of mass mode $\nu_1 = \nu_z$

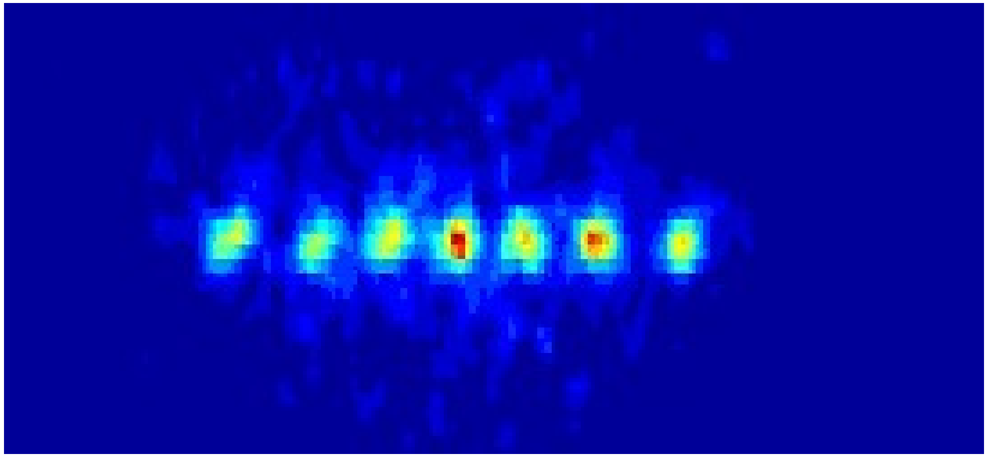
breathing mode $\nu_2 = \sqrt{3}\nu_z$

Normal modes

„center-of-mass mode“

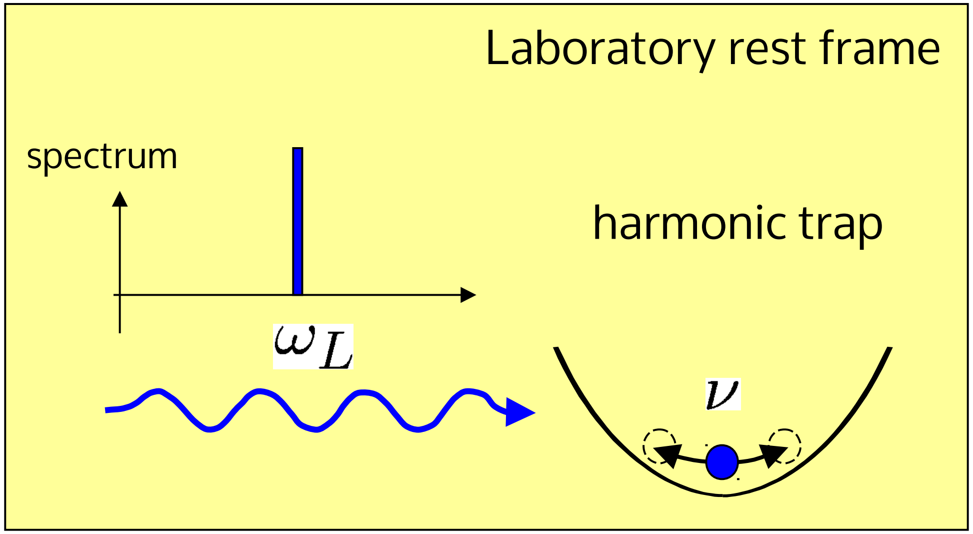


„stretch mode“



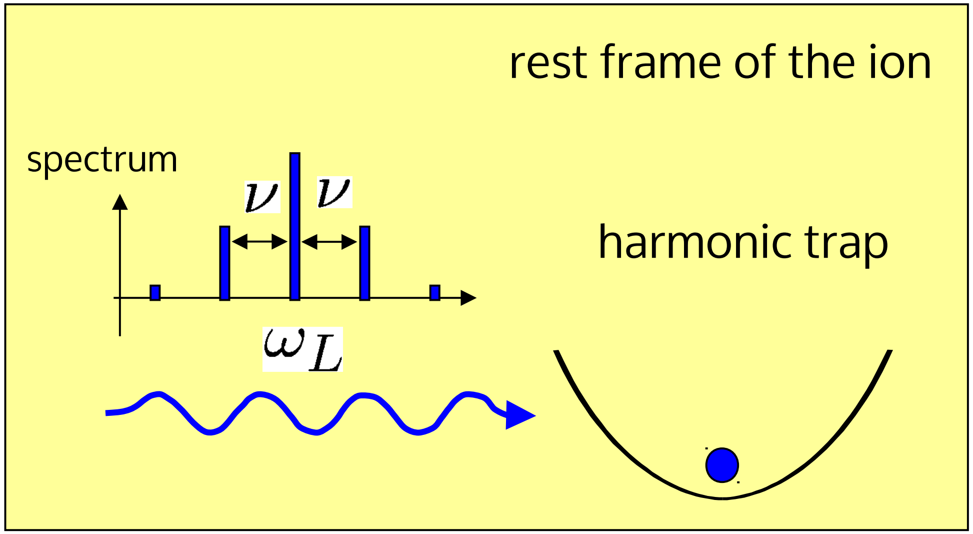
See: H.C. Nägerl, *et al.*, Opt. Express 3, 89 (1998)

How does the motion modify the spectrum ?

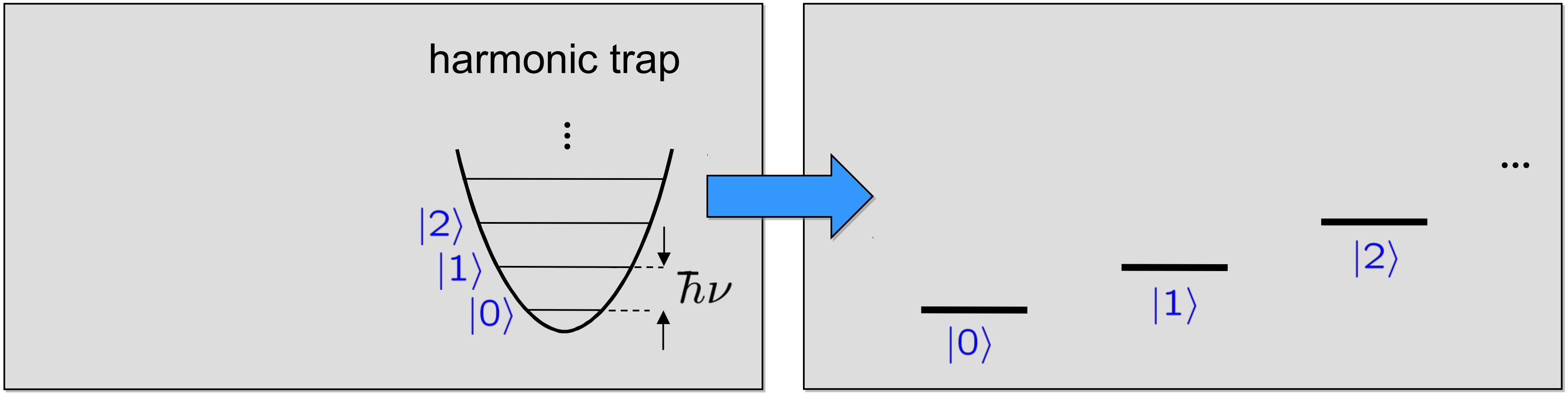


In the rest frame of the ion,
the laser appears to be
frequency-modulated.

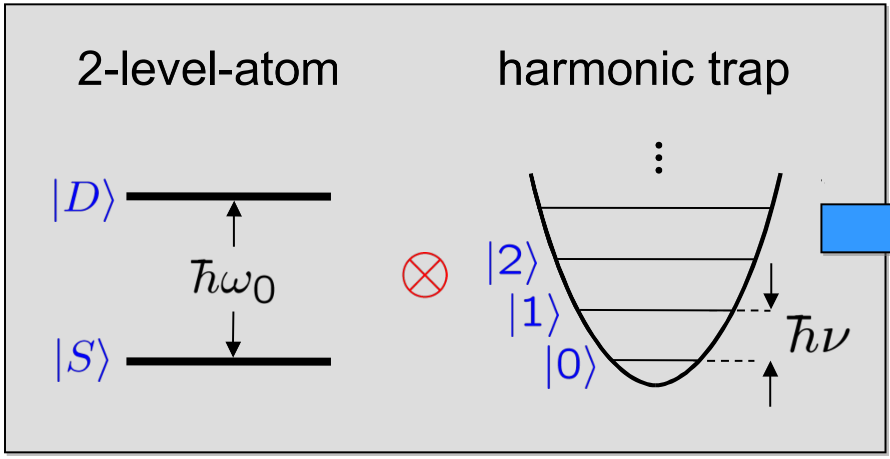
→ Absorption on the sideband possible



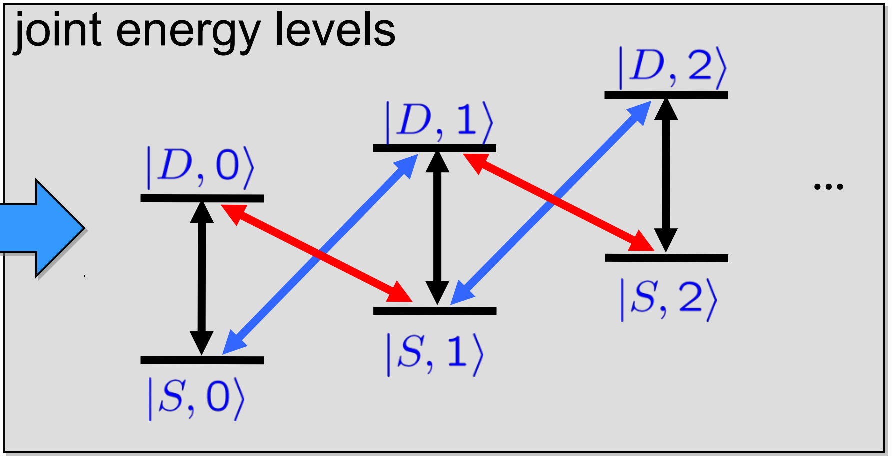
Ion motion



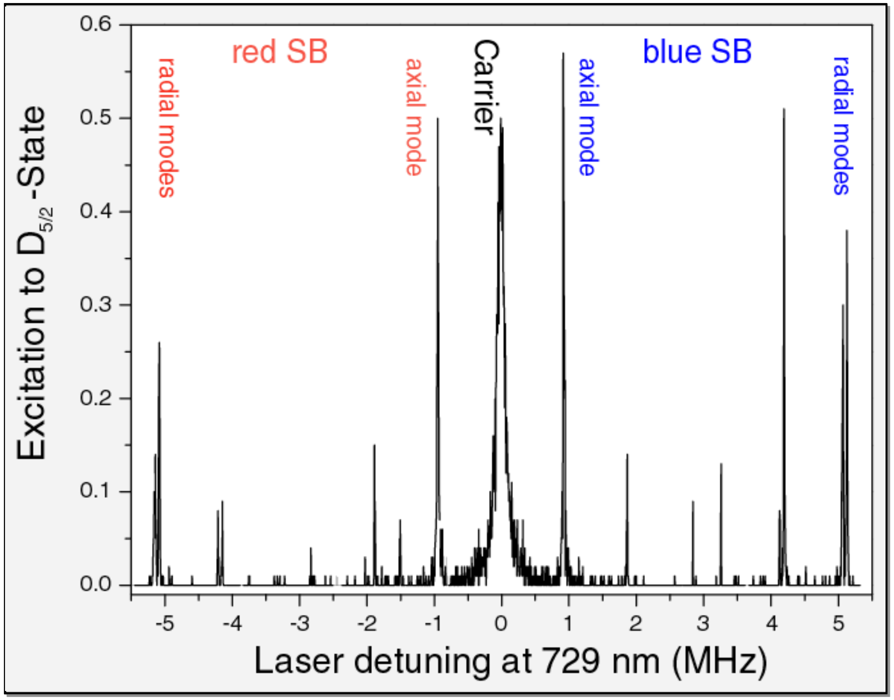
Ion motion



2-level-atom
harmonic trap
$|D\rangle$
$\hbar\omega_0$
$|S\rangle$
$\otimes$
$|2\rangle$
$|1\rangle$
$|0\rangle$
$\hbar\nu$

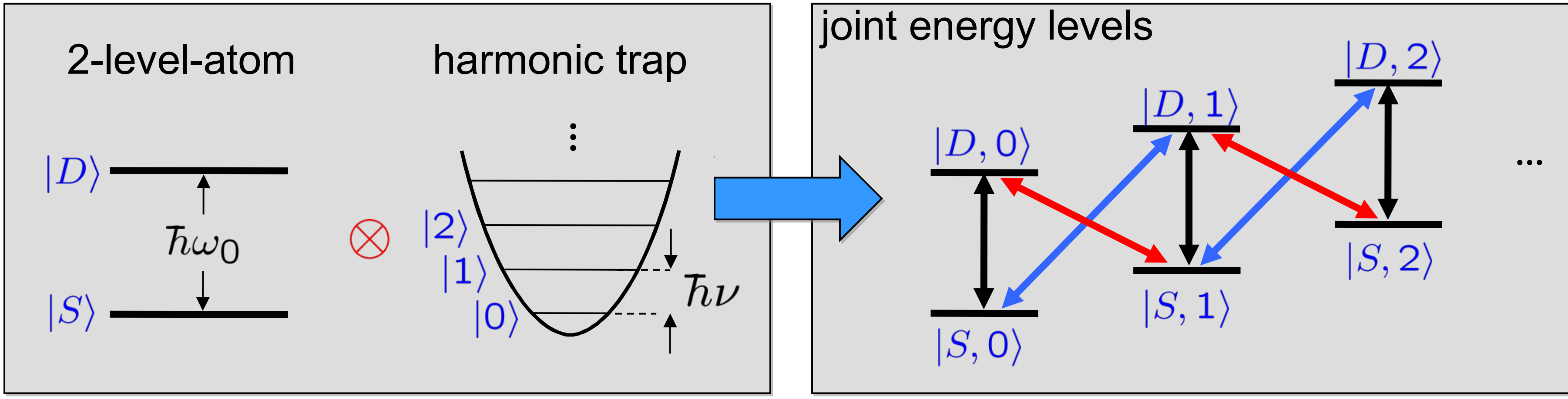


joint energy levels
$|D,0\rangle$
$|D,1\rangle$
$|D,2\rangle$
$|S,0\rangle$
$|S,1\rangle$
$|S,2\rangle$
...



red SB
blue SB
radial modes
axial mode
Carrier
axial mode
radial modes
Excitation to $D_{5/2}$ -State
Laser detuning at 729 nm (MHz)
0.6
0.5
0.4
0.3
0.2
0.1
0.0
-5
-4
-3
-2
-1
0
1
2
3
4
5

Ion motion



$H^{(el)}$ $\qquad$ $H^{(m)}$

Approximations:

Ion: Electronic structure of the ion approximated by two-level system
(laser is (near-) resonant and couples only two levels)

$$H^{(el)} = \hbar\frac{\omega_0}{2}(|e\rangle\langle e| - |g\rangle\langle g|)$$

Trap: Only a single harmonic oscillator taken into account

$$H^{(m)} = \hbar\nu a^\dagger a$$

Laser-ion interaction

General description for the interaction between the ion and a running-wave laser beam

Hamiltonian: $H = H^{(el)} + H^{(m)} + H^{(i)}$

$$H^{(el)} = \hbar \frac{\omega_0}{2} (|e\rangle\langle e| - |g\rangle\langle g|)$$

$$H^{(m)} = \hbar \nu a^\dagger a$$

$$H^{(i)} = \hbar \Omega (|g\rangle\langle e| + |e\rangle\langle g|) \cos(k\hat{x} - \omega t + \phi)$$

laser frequency

laser phase

Ω : Rabi frequency
(coupling strength between laser and ion)

Laser-ion interaction

$$H_{int} = \frac{\hbar\Omega}{2}\sigma_+ \exp\{i\eta(e^{-i\nu t}a + e^{i\nu t}a^\dagger)\}e^{i(\omega_0-\omega)t+i\phi} + h.c.$$

with the Lamb-Dicke parameter: $\eta = kx_0$

For $\Psi = |n\rangle$: $\eta\sqrt{2n+1} \ll 1$ Taylor expansion of the exponential up to first order:

$$H_{int} = \frac{\hbar\Omega}{2}\sigma_+\{\mathbb{1} + i\eta(e^{-i\nu t}a + e^{i\nu t}a^\dagger)\}e^{-i\delta t+i\phi} + h.c.$$

Control parameters:

Ω	Rabi frequency (coupling strength)
$\delta = \omega - \omega_0$	Detuning of laser with respect to atomic transition
ϕ	Phase of laser

Lamb-Dicke regime

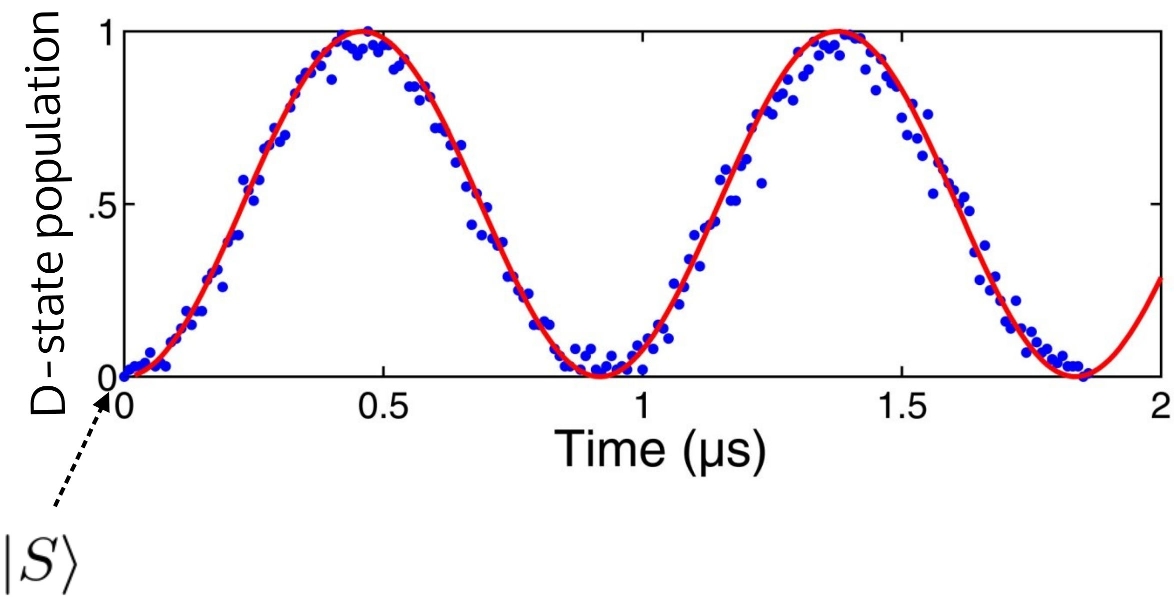
$$H_{int} = \frac{\hbar\Omega}{2}\sigma_+\{\mathbb{1}+i\eta(e^{-i\nu t}a+e^{i\nu t}a^\dagger)\}e^{-i\delta t+i\phi} + c.c.$$

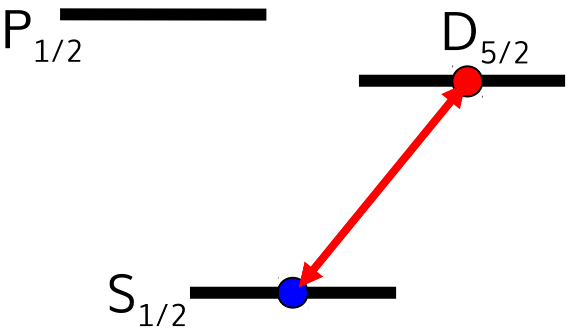
1. Carrier resonance: $\delta = 0$ $$H_{int} = \frac{\hbar\Omega}{2}\{\sigma_+ e^{+i\phi} + \sigma_- e^{-i\phi}\}$$

The laser couples ground and excited state without affecting the motional state.

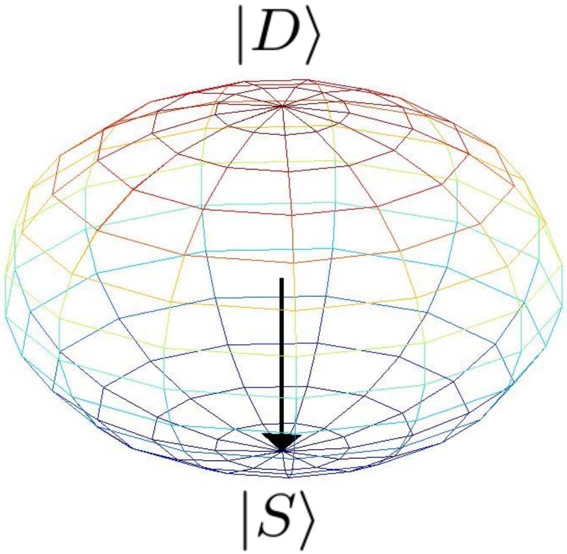
The coupling does not depend on |*n*>. $|g, n\rangle \longleftrightarrow |e, n\rangle$

Single qubit gates



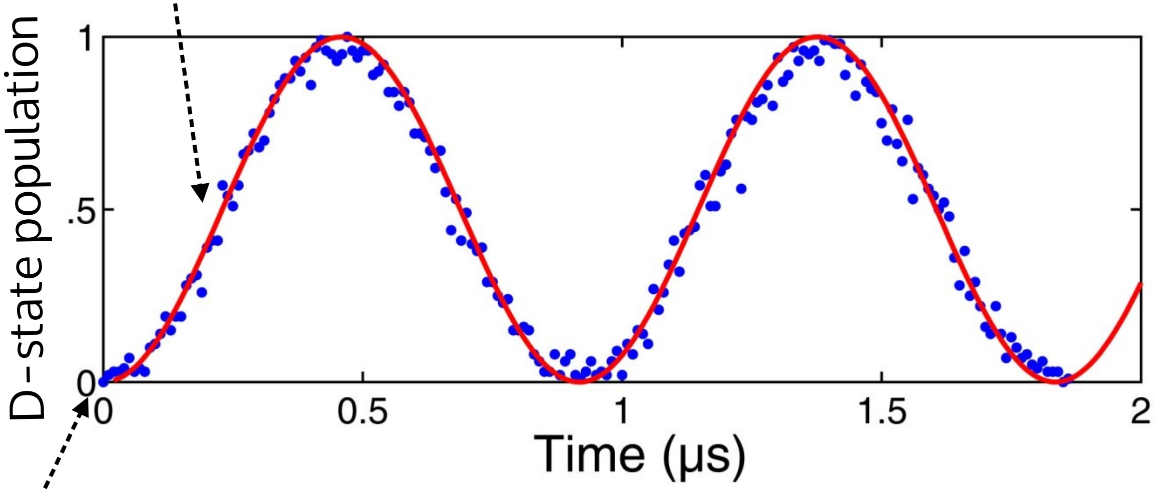


$$H_{int} = \frac{\hbar\Omega}{2}\{\sigma_+ e^{+i\phi} + \sigma_- e^{-i\phi}\}$$

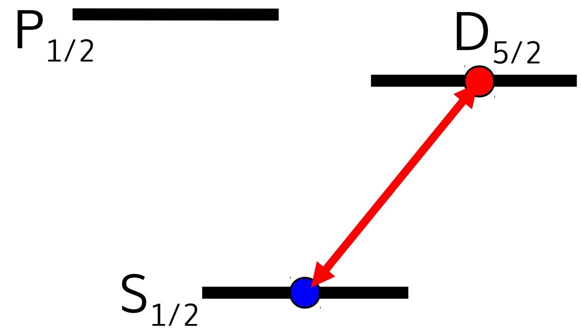


Single qubit gates

$(|S\rangle + |D\rangle)/\sqrt{2}$

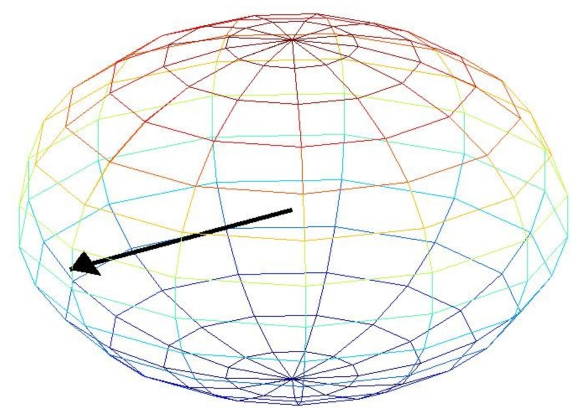


$|S\rangle$



$$H_{int} = \frac{\hbar\Omega}{2}\{\sigma_+ e^{+i\phi} + \sigma_- e^{-i\phi}\}$$

$|D\rangle$



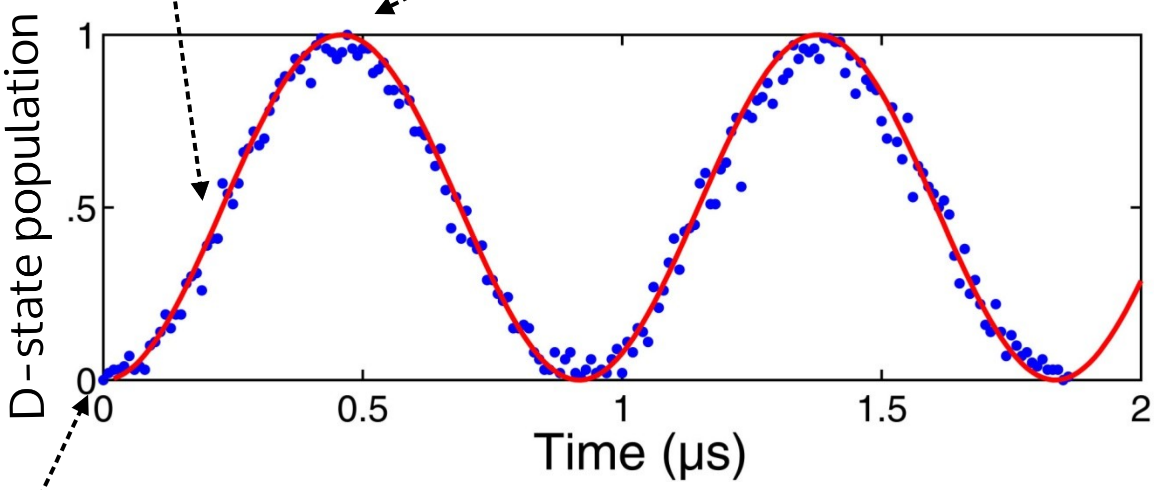
$(|S\rangle + |D\rangle)/\sqrt{2}$

$|S\rangle$

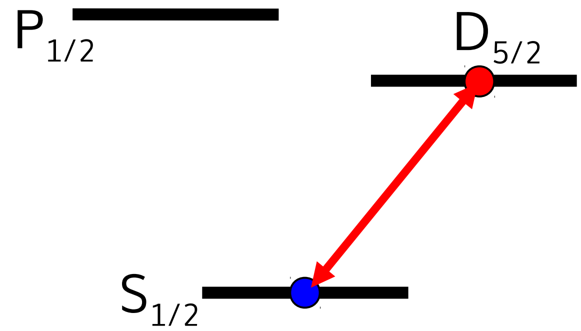
Single qubit gates

$(|S\rangle + |D\rangle)/\sqrt{2}$

$|D\rangle$



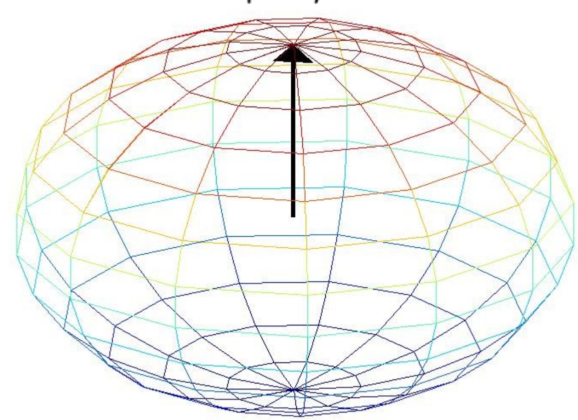
$|S\rangle$



$$H_{int} = \frac{\hbar\Omega}{2}\{\sigma_+ e^{+i\phi} + \sigma_- e^{-i\phi}\}$$

$|D\rangle$

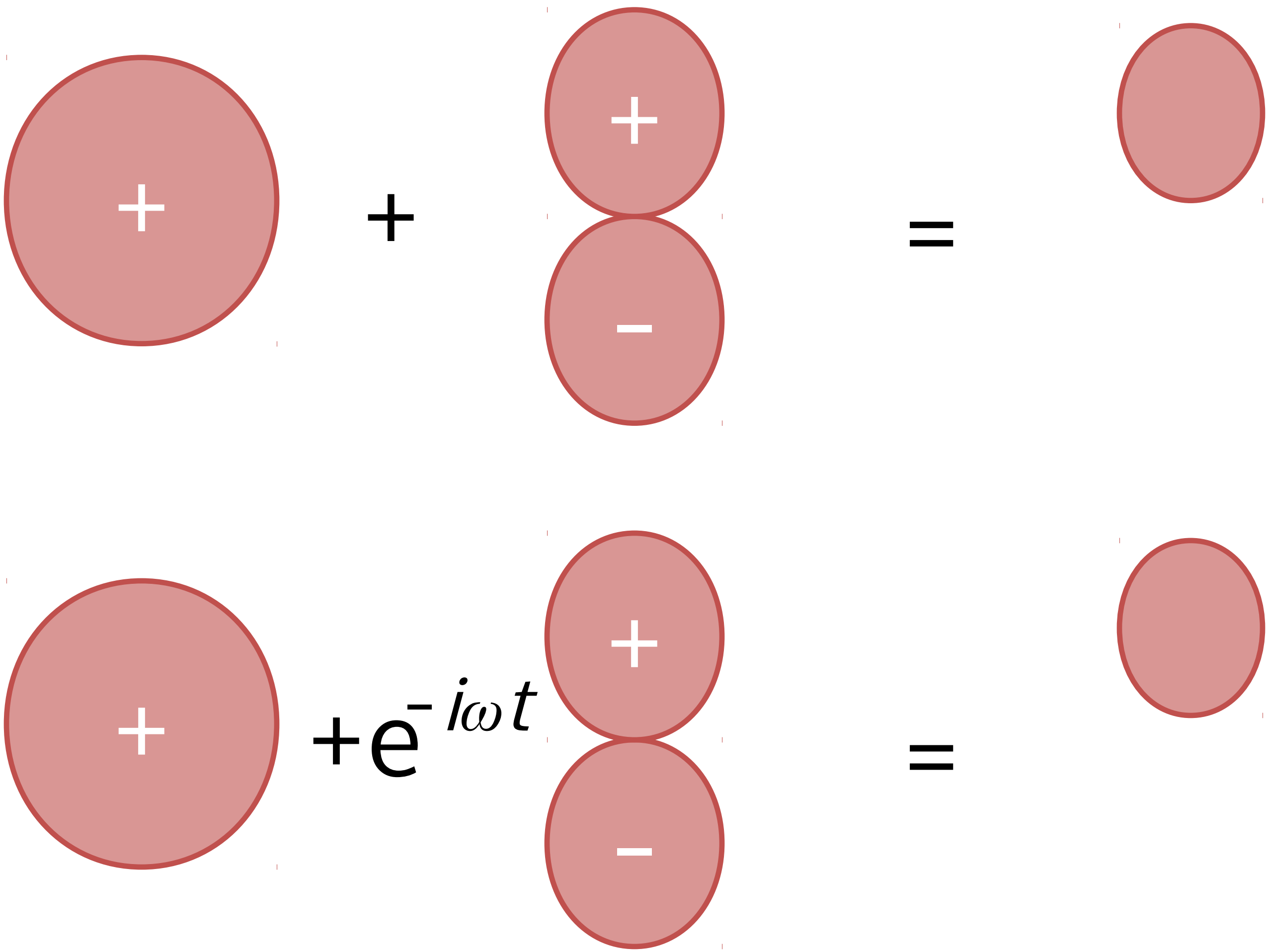
$(|S\rangle + |D\rangle)/\sqrt{2}$



$|S\rangle$

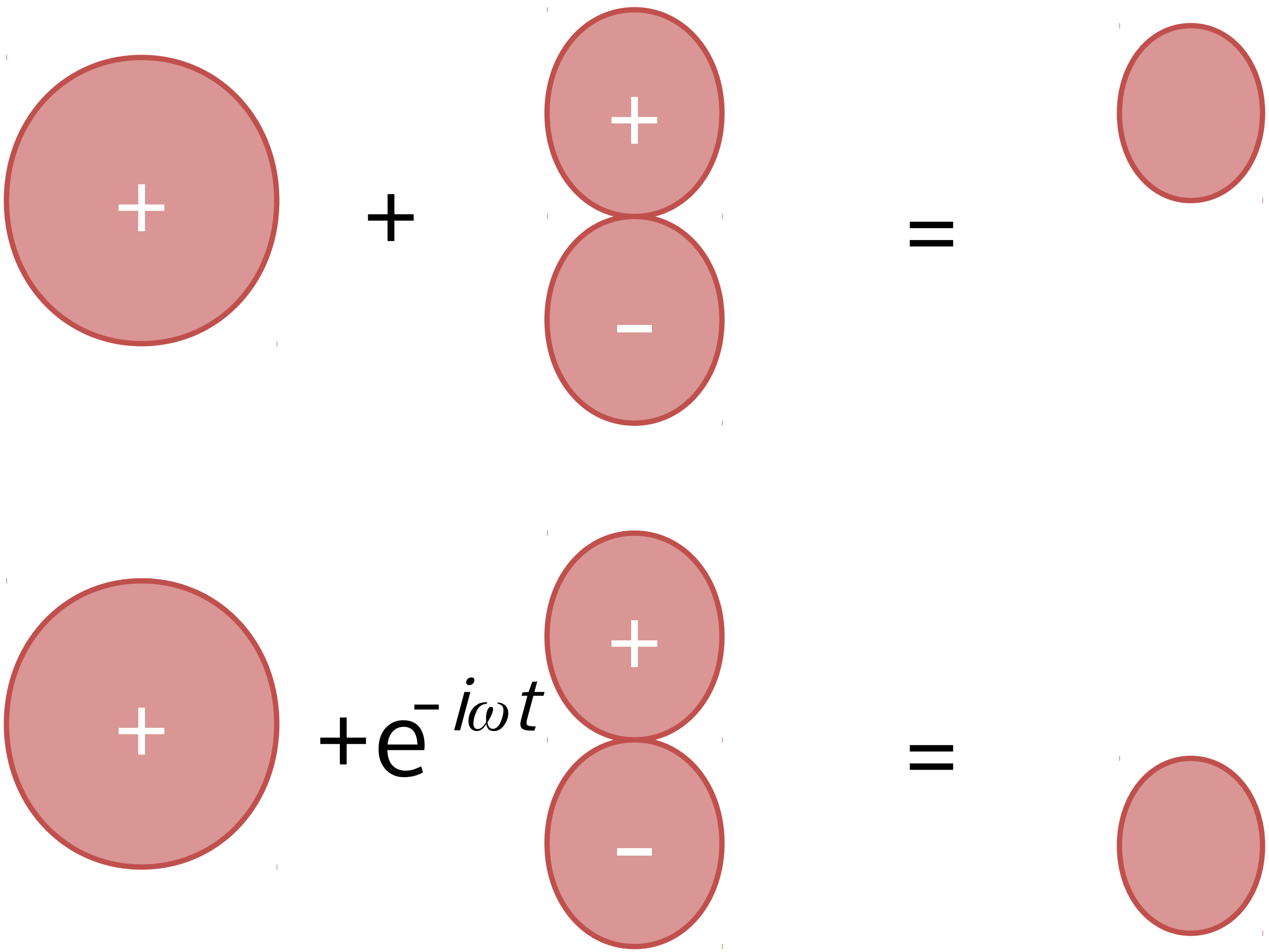
The phase ...

$$H_{int} = \frac{\hbar\Omega}{2}\{\sigma_+ e^{+i\phi} + \sigma_- e^{-i\phi}\}$$



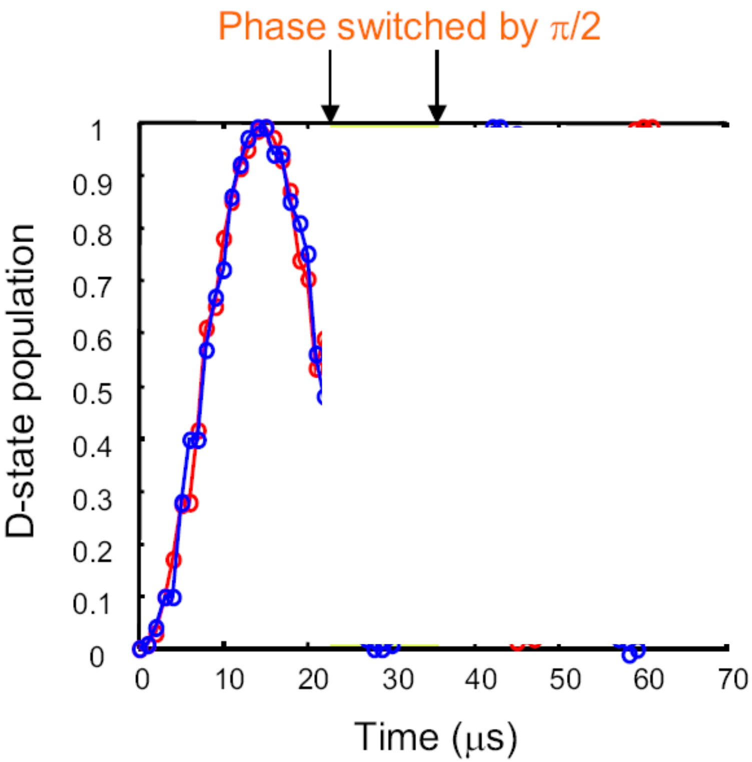
The phase ...

$$H_{int} = \frac{\hbar\Omega}{2}\{\sigma_+ e^{+i\phi} + \sigma_- e^{-i\phi}\}$$



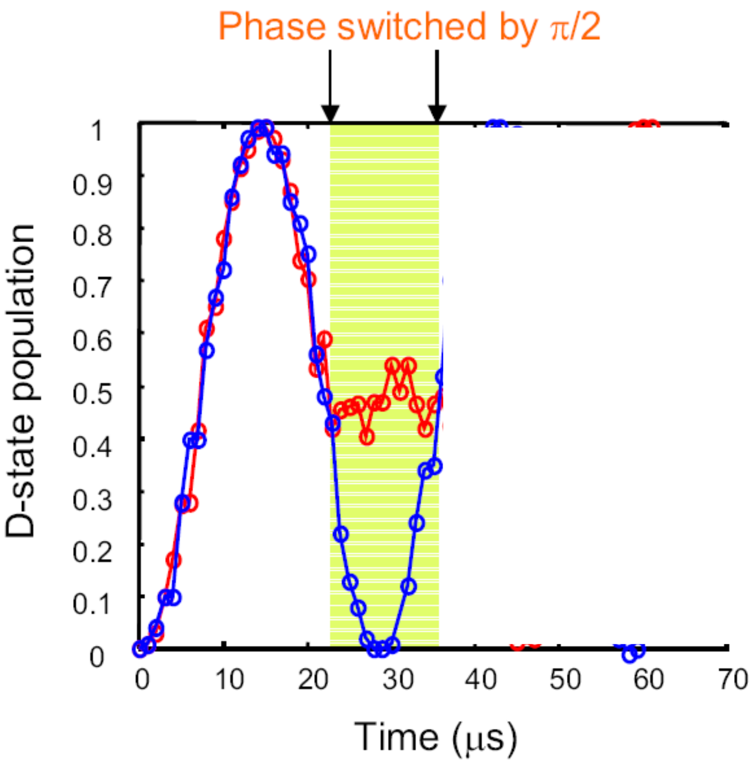
The phase ...

$$H_{int} = \frac{\hbar\Omega}{2}\{\sigma_+ e^{+i\phi} + \sigma_- e^{-i\phi}\}$$



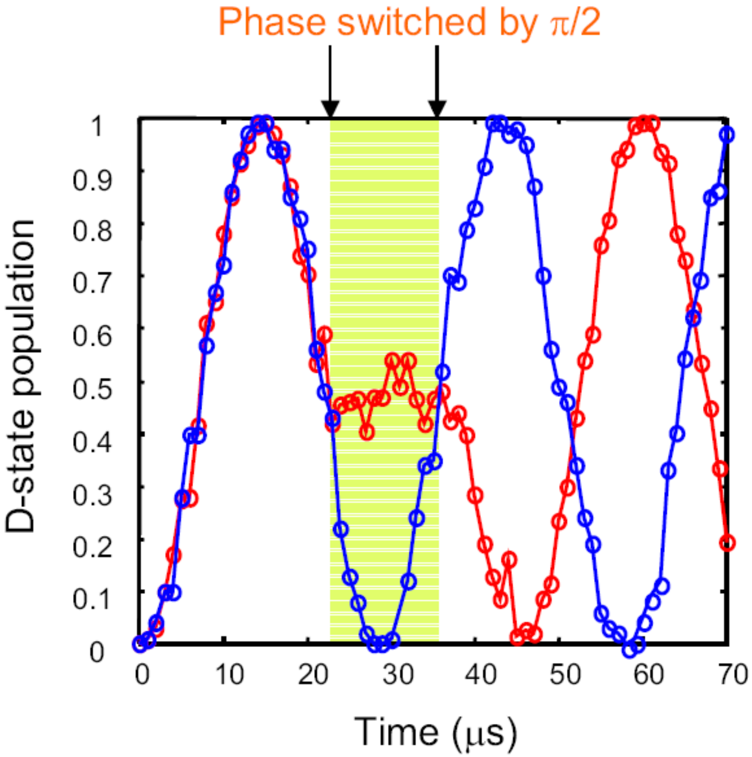
The phase ...

$$H_{int} = \frac{\hbar\Omega}{2}\{\sigma_+ e^{+i\phi} + \sigma_- e^{-i\phi}\}$$



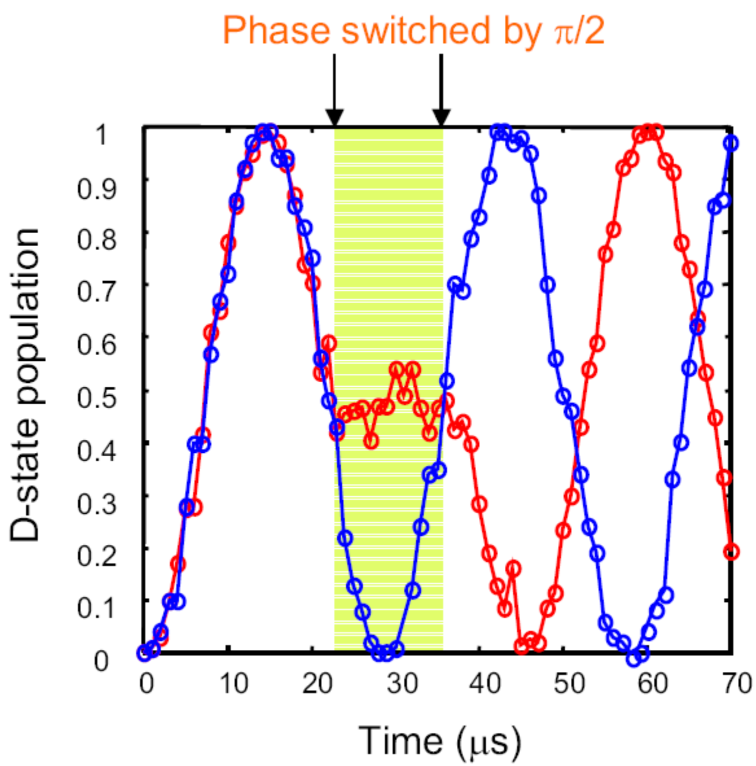
The phase ...

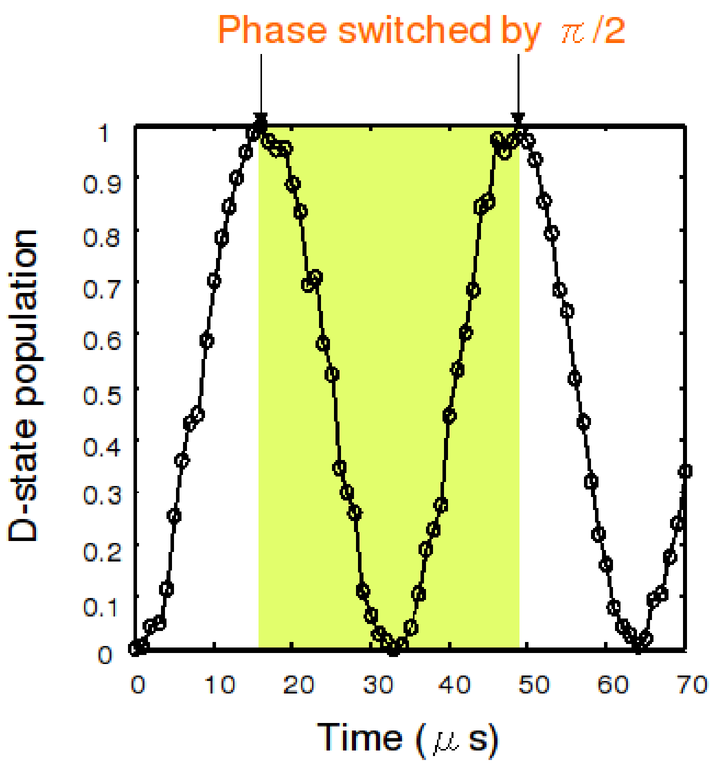
$$H_{int} = \frac{\hbar\Omega}{2}\{\sigma_+ e^{+i\phi} + \sigma_- e^{-i\phi}\}$$



The phase …

$$H_{int} = \frac{\hbar\Omega}{2}\{\sigma_+ e^{+i\phi} + \sigma_- e^{-i\phi}\}$$





Lamb-Dicke regime

$$H_{int} = \frac{\hbar\Omega}{2}\sigma_+\{\mathbb{1}+i\eta(e^{-i\nu t}a+e^{i\nu t}a^\dagger)\}e^{-i\delta t+i\phi} + c.c.$$

1. Carrier resonance: $\delta = 0$ $$H_{int} = \frac{\hbar\Omega}{2}\{\sigma_+ e^{+i\phi} + \sigma_- e^{-i\phi}\}$$

The laser couples ground and excited state without affecting the motional state.

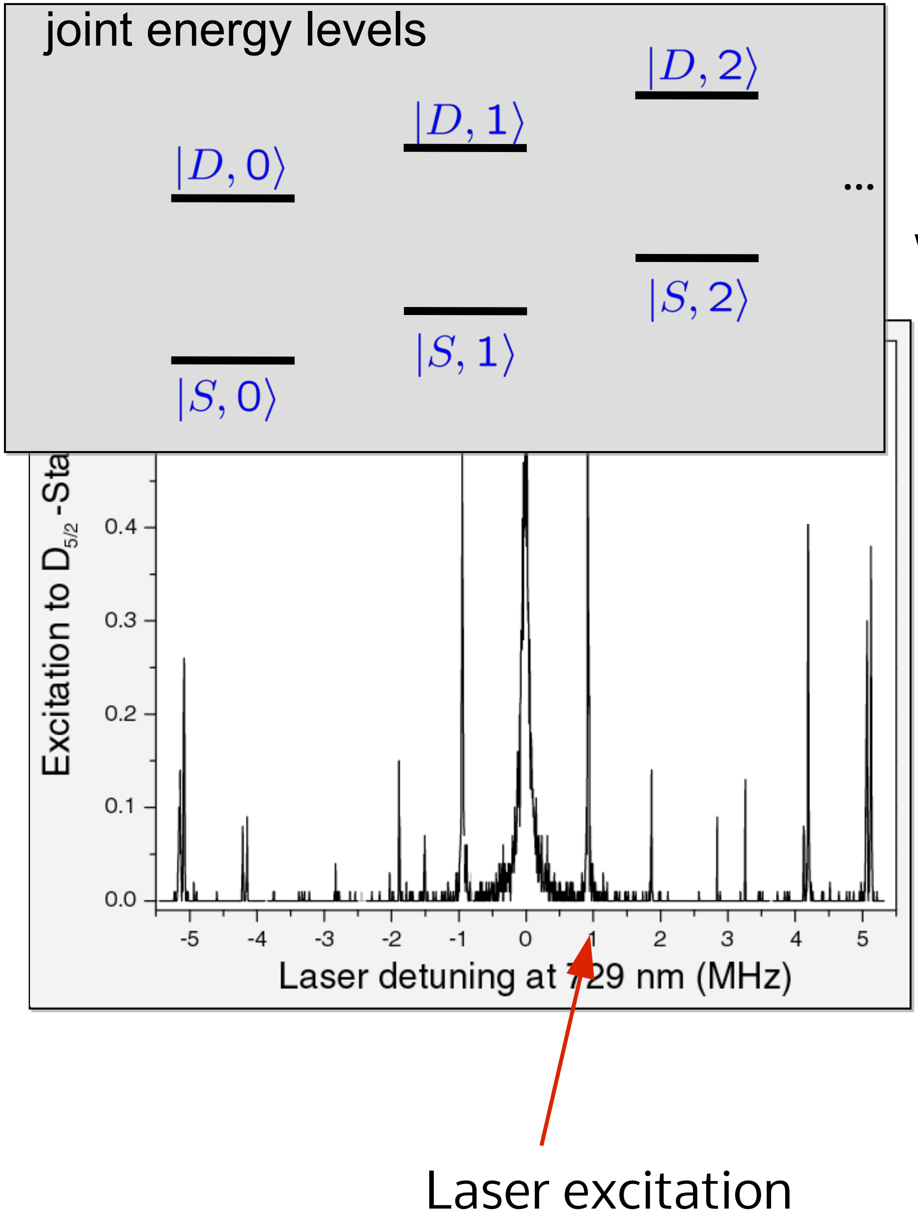
The coupling does not depend on |*n*>. $|g,n\rangle \longleftrightarrow |e,n\rangle$

2. Red sideband: $\delta = -\nu$ $$H_{int} = \frac{\hbar\Omega}{2} i\eta\{\sigma_+ a\, e^{+i\phi} - \sigma_- a^\dagger e^{-i\phi}\}$$

$|g,n\rangle \longleftrightarrow |e,n-1\rangle$

As compared to the carrier resonance, the coupling strength on the sidebands is reduced.
The coupling strength depends on the motional quantum number.

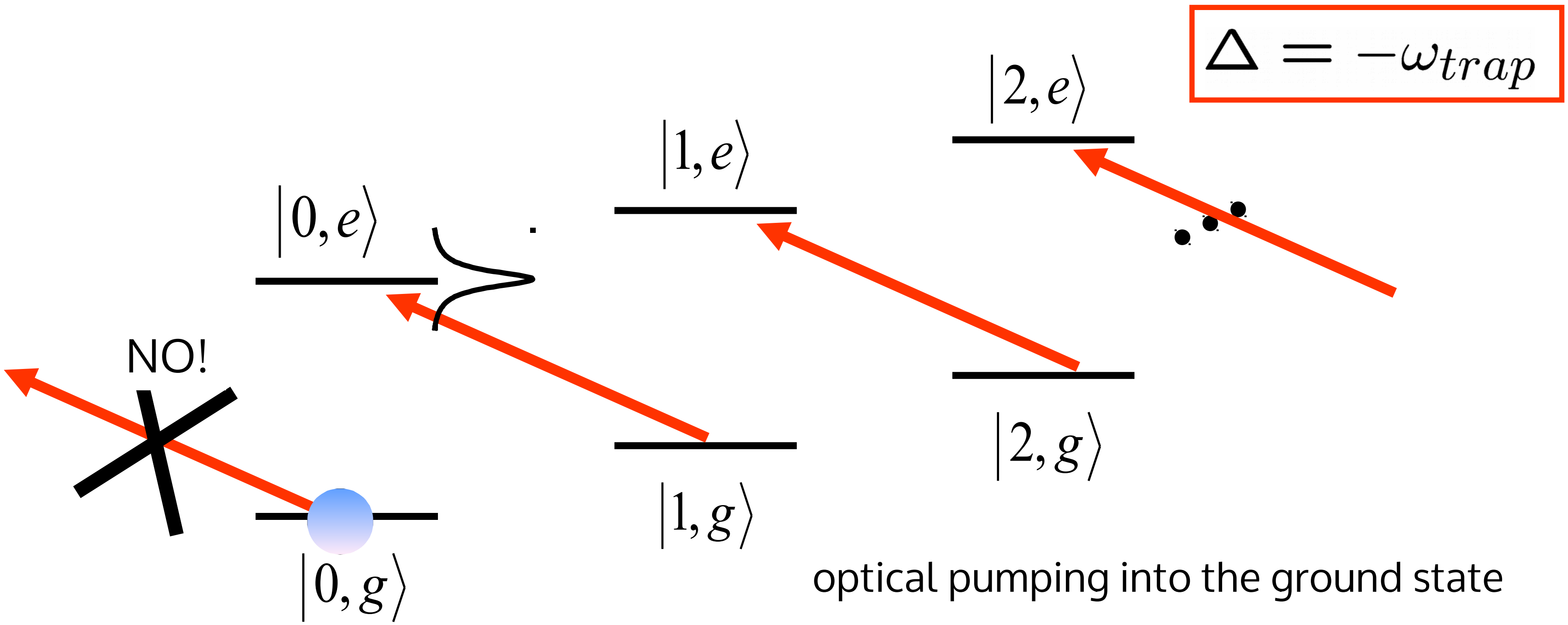
Lamb-Dicke regime



Which statement is true?

1) the ion motion stays unaffected.
2) the motional quantum number is reduced.
3) the ion gets micromotion
4) the motional quantum number increases by 1.

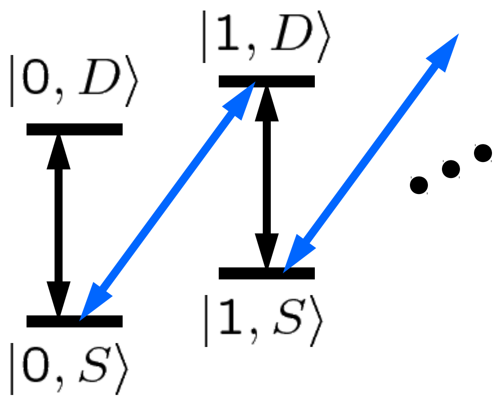
Sideband cooling



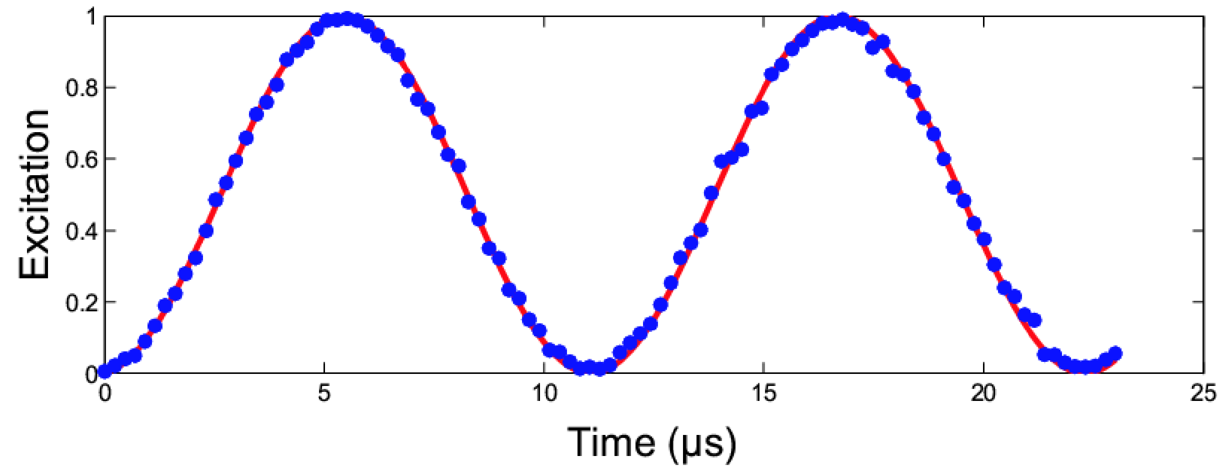
optical pumping into the ground state

Signature: no further excitation possible
„dark state" |0>

Ion motion

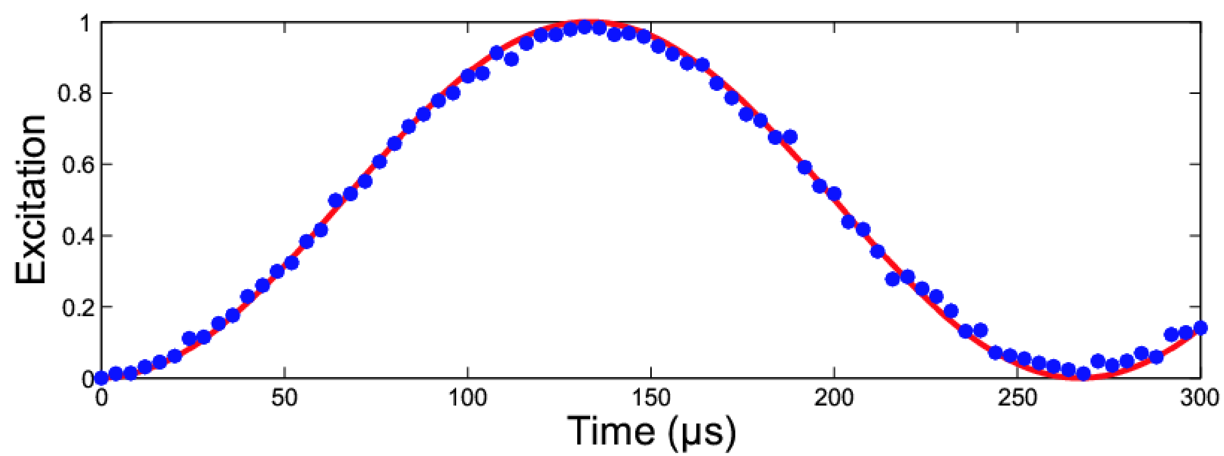


carrier



carrier and sideband
Rabi oscillations
with Rabi frequencies

$$\Omega, \quad \eta\Omega$$



$\eta = kx_0$ Lamb-Dicke parameter

Plan

Lecture #1: Introduction
- Paul traps
- Laser ion-interaction

Lecture #2: Quantum computing
- Quantum gates
- Quantum state engineering

Lecture #3: Decoherence I/Scaling
- Qubit decoherence
- Scaling

Lecture #4: Decoherence II/Quantum emulation
- Anomalous heating
- Quantum emulation

Lecture #5: Applications
- Atomic clocks
- Fundamental symmetry tests

Plan

Lecture #1: Introduction
- Paul traps
- Laser ion-interaction

Lecture #2: Quantum computing
- Quantum gates
- Quantum state engineering

Lecture #3: Decoherence I/Scaling
- Qubit decoherence
- Scaling

Lecture #4: Decoherence II/Quantum emulation
- Anomalous heating
- Quantum emulation

Lecture #5: Applications
- Atomic clocks
- Fundamental symmetry tests