

# Quantum applications of ion trapping

*Hartmut Häffner, UC Berkeley*

1. Introduction to ion trapping
2. Quantum computing
3. Sources of decoherence
4. Scaling and quantum simulation
5. Applications of QIP to precision measurements

# Plan

## Lecture #1: Introduction

- Paul traps
- Laser ion-interaction

## Lecture #2: Quantum computing

- Quantum gates
- Quantum state tomography

## Lecture #3: Decoherence

- Qubit decoherence
- Scaling

## Lecture #4: Scaling and quantum simulation

- Scaling and anomalous heating
- Quantum simulation

## Lecture #5: Applications

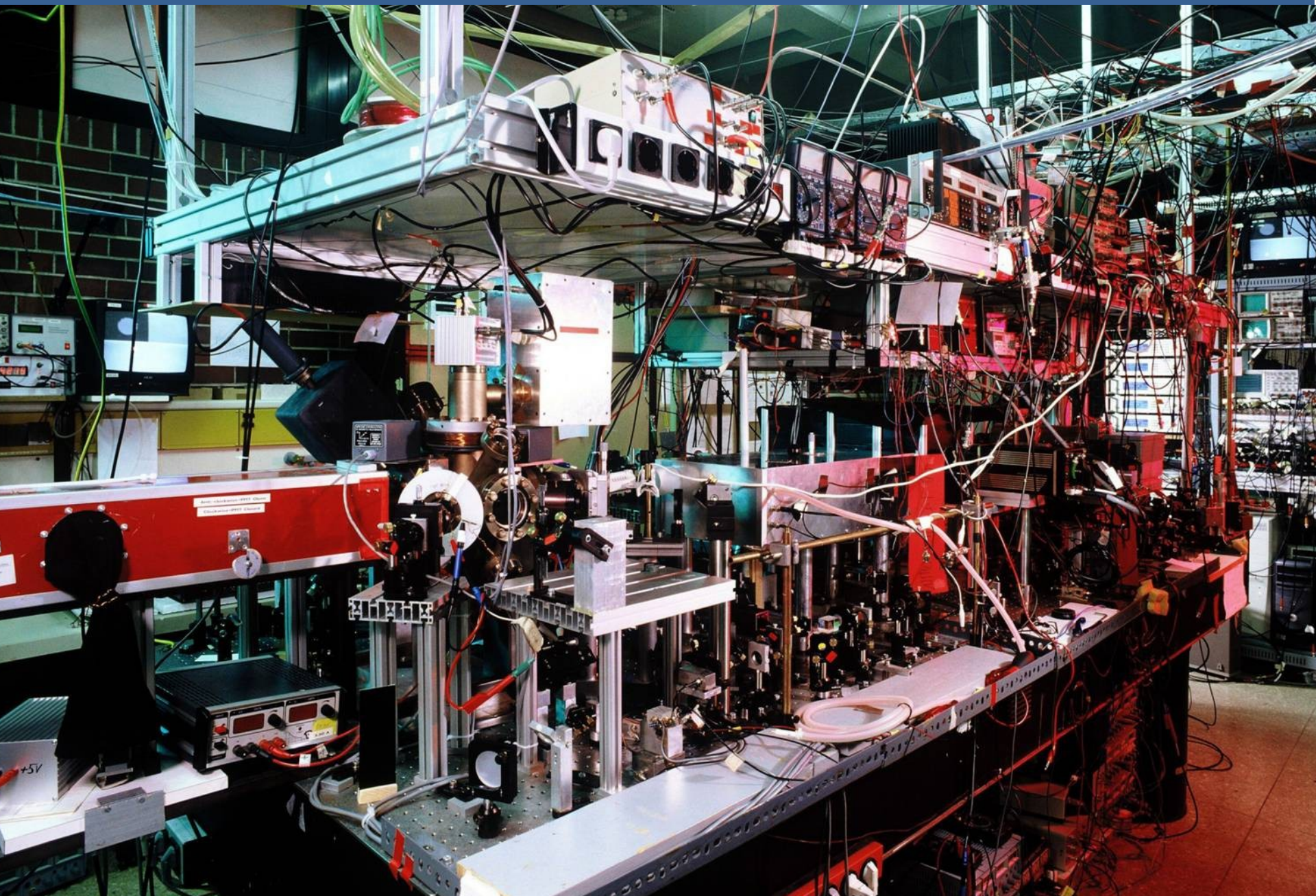
- Decoherence-free subspaces
- Michelson-Morley experiment

# The DiVincenzo criteria for quantum computing

- I. Scalable physical system, well characterized qubits ✓
- II. Ability to initialize the state of the qubits with sufficient fidelity ✓
- III. Long relevant coherence times, much longer than gate operation time ✓
- IV. "Universal" set of quantum gates with sufficient fidelity ( ✓ )
- V. Qubit-specific measurement capability with sufficient fidelity ✓

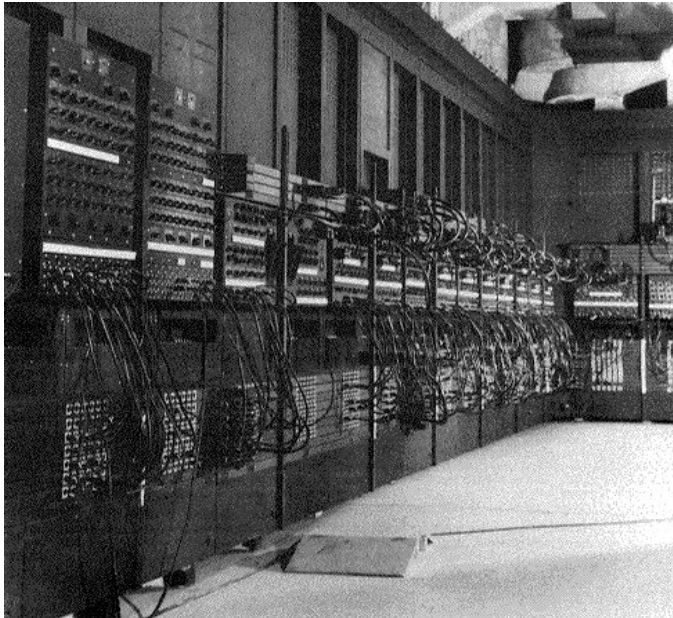


# Scalability ?

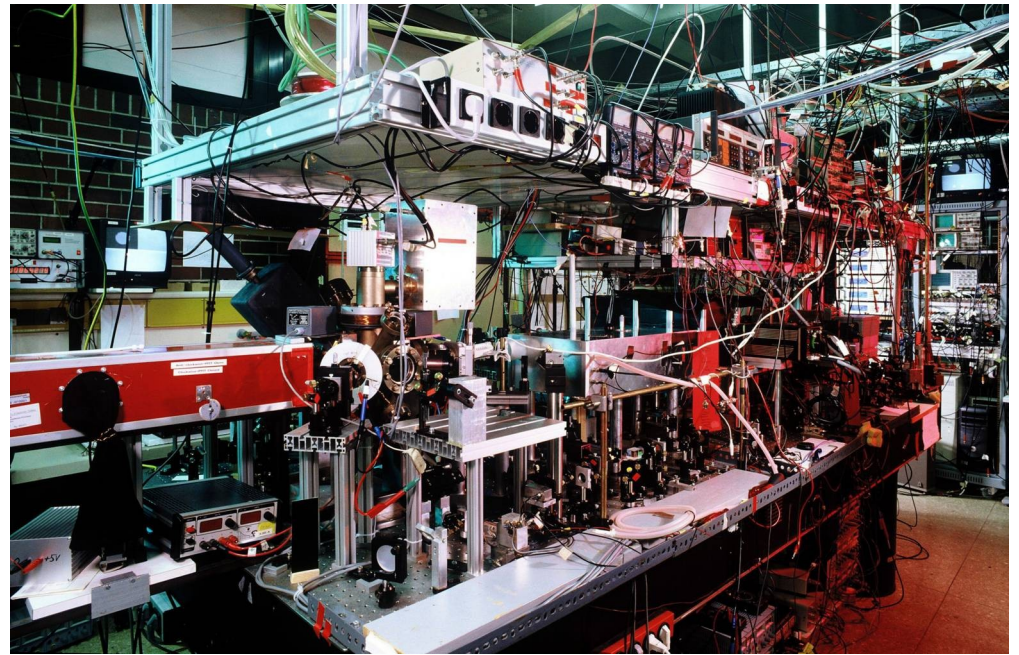




# Scalability ?

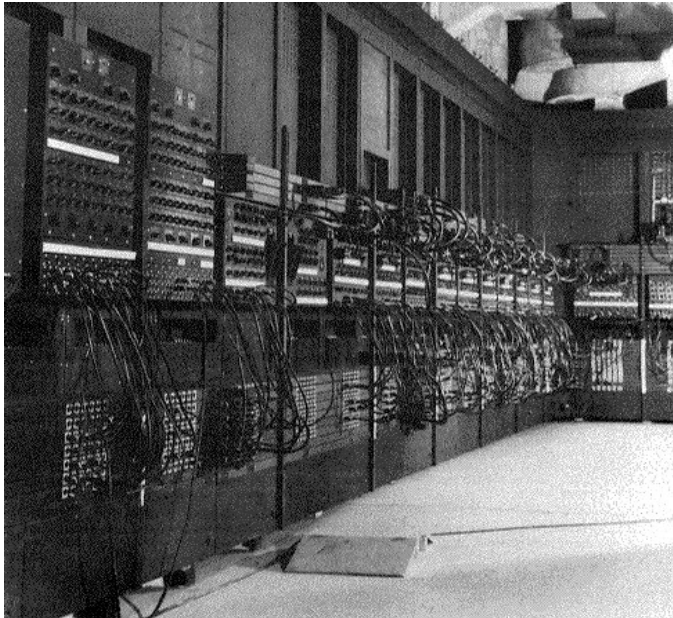


ENIAC, 1950

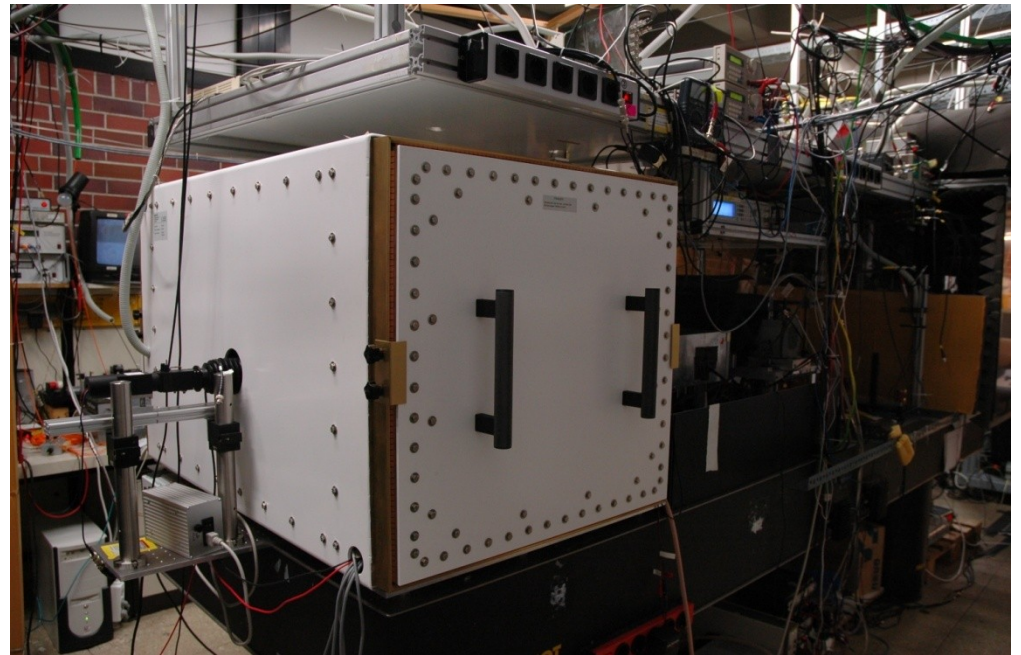


Quantum computer, 2005

# Scalability ?



ENIAC, 1950

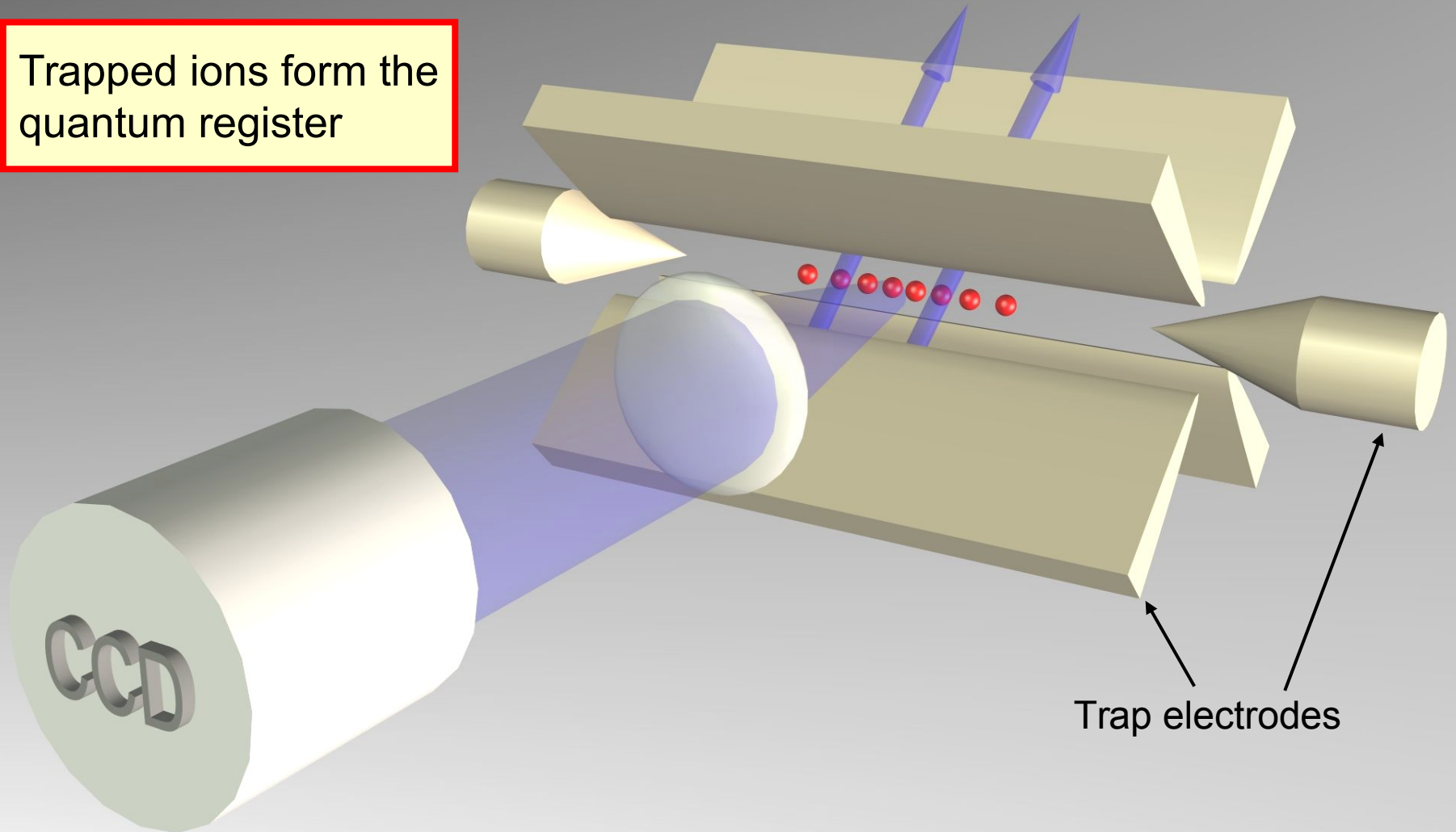


Quantum computer, 2009



# Scalability ?

Trapped ions form the  
quantum register



# Scalability ?

## Problems :

- Coupling strength between internal and motional states of a N-ion string decreases as

$$\eta \propto \frac{1}{\sqrt{N}}$$

(momentum transfer from photon to ion string becomes more difficult)

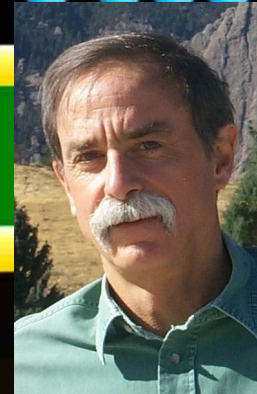
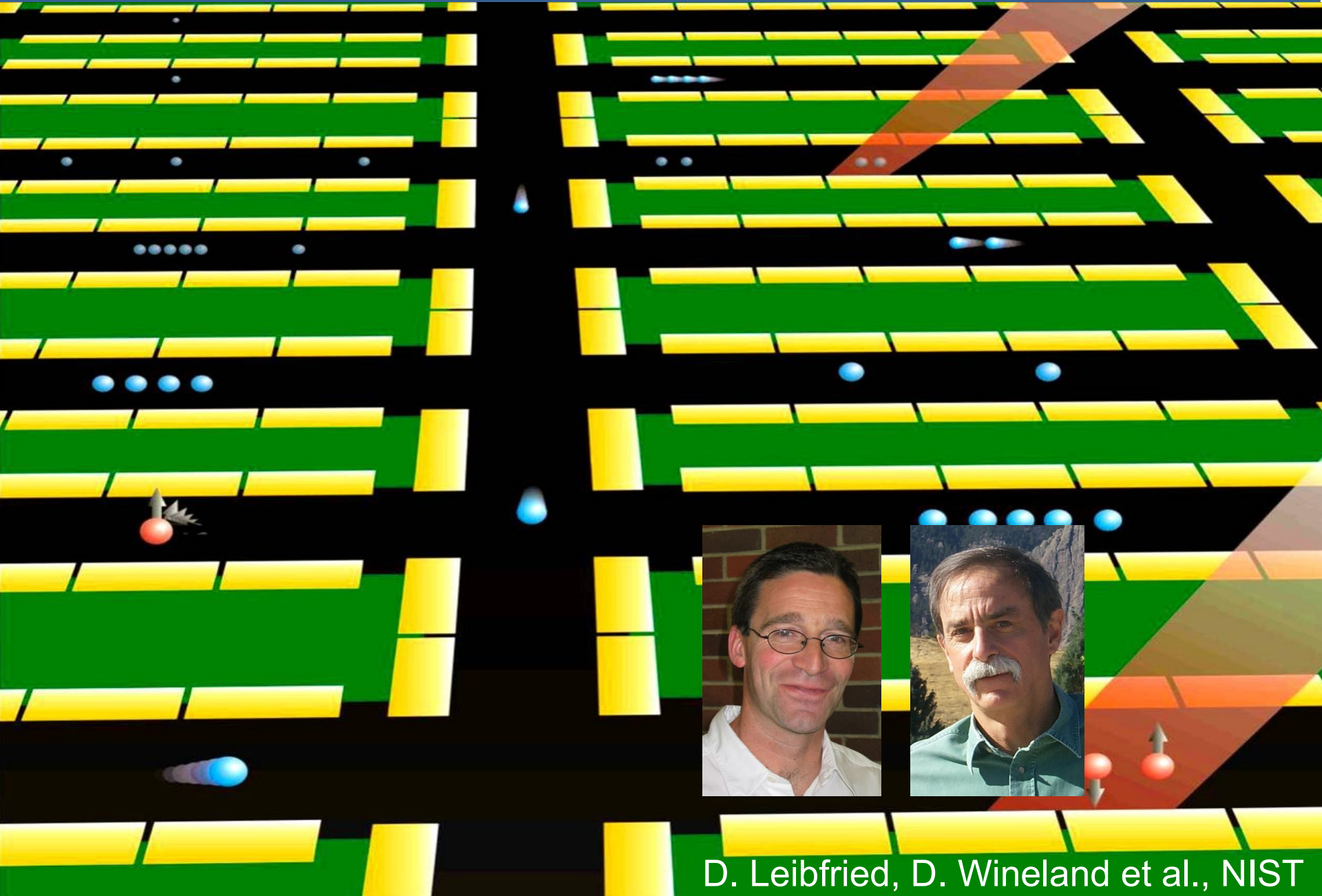
-> Gate operation speed slows down

- More vibrational modes increase risk of spurious excitation of unwanted modes
- Distance between neighbouring ions decreases -> addressing more difficult

-> Use flexible trap potentials to split long ion string into smaller segments and perform operations on these smaller strings

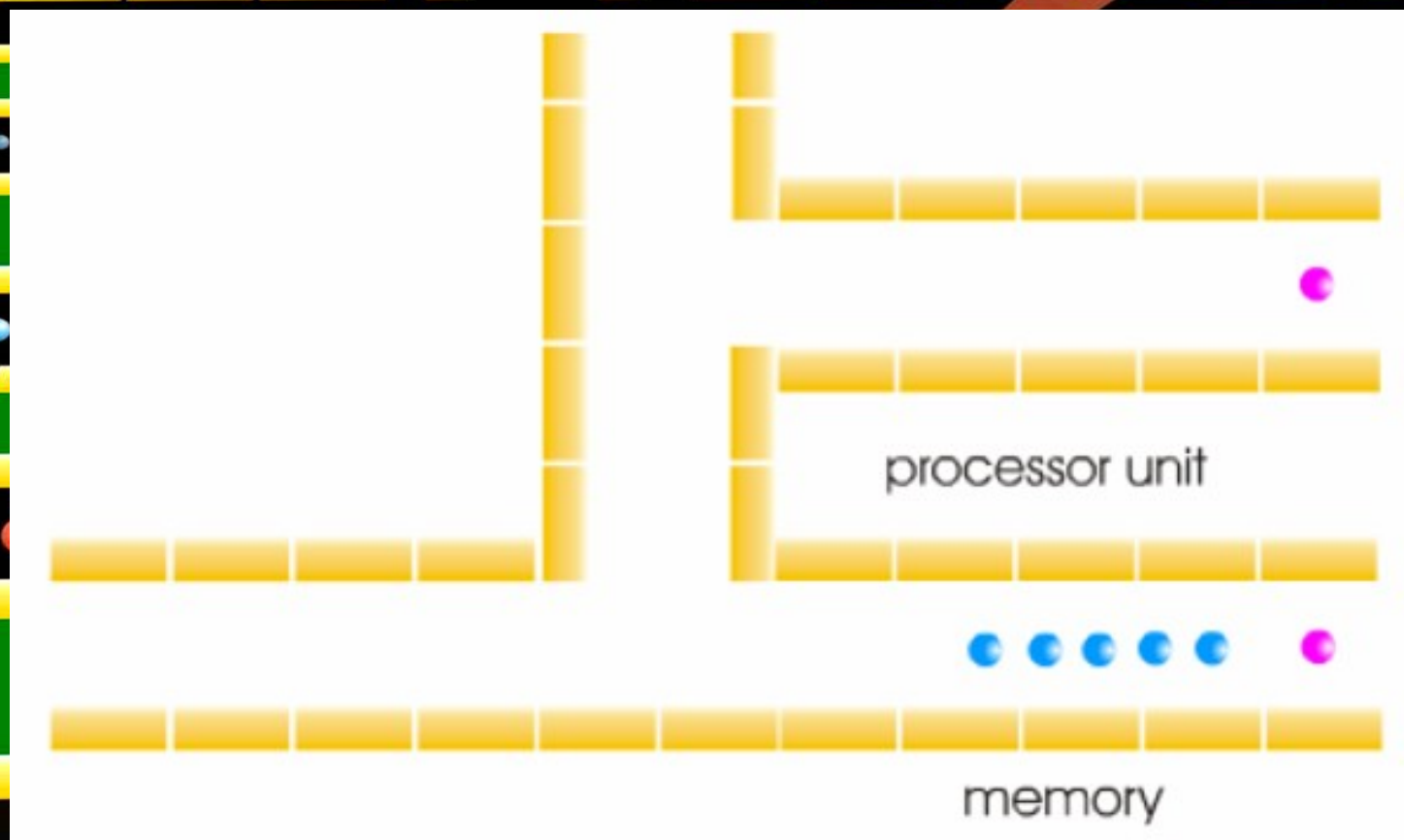


# Scaling ion trap quantum computers

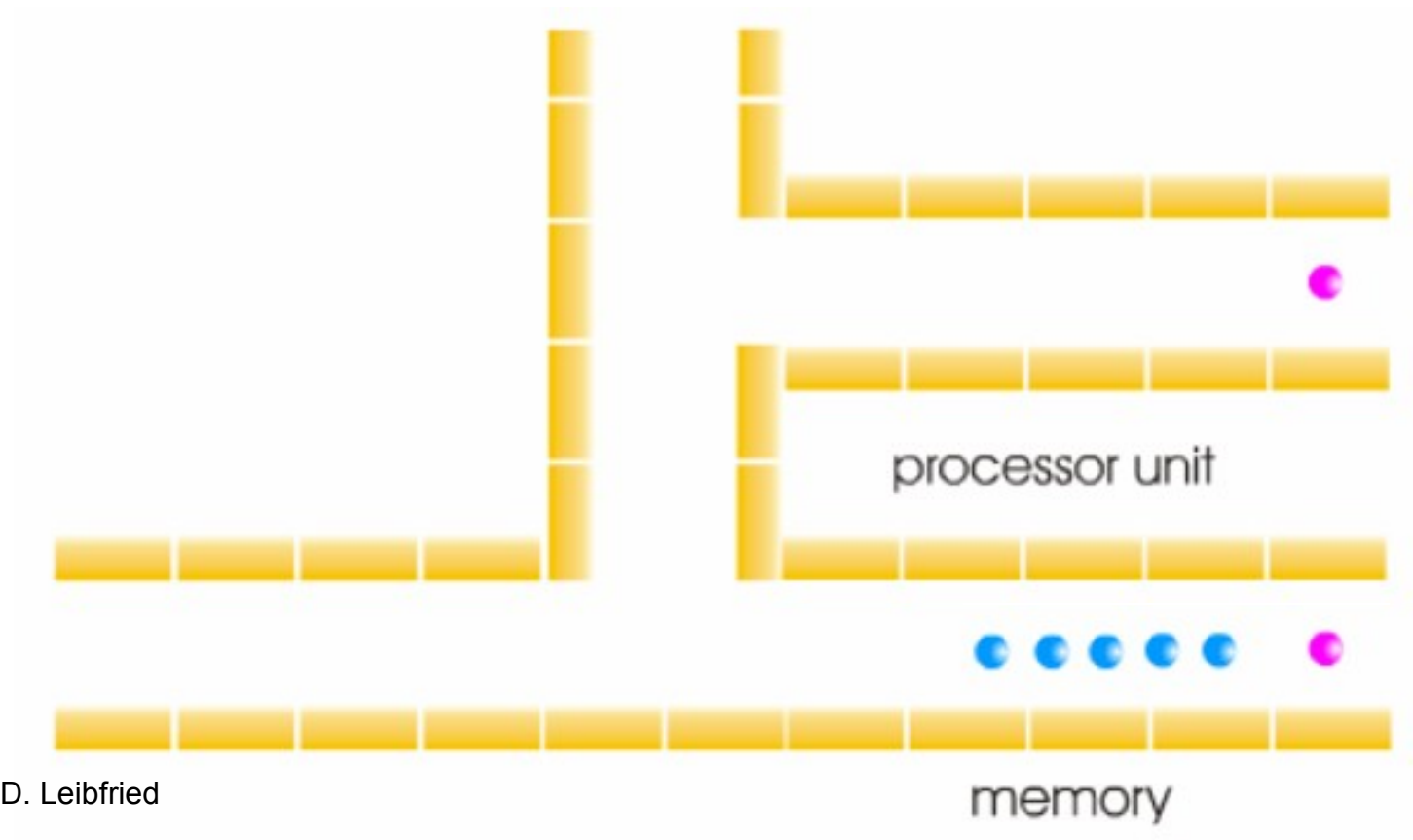


D. Leibfried, D. Wineland et al., NIST

# Scaling ion trap quantum computers

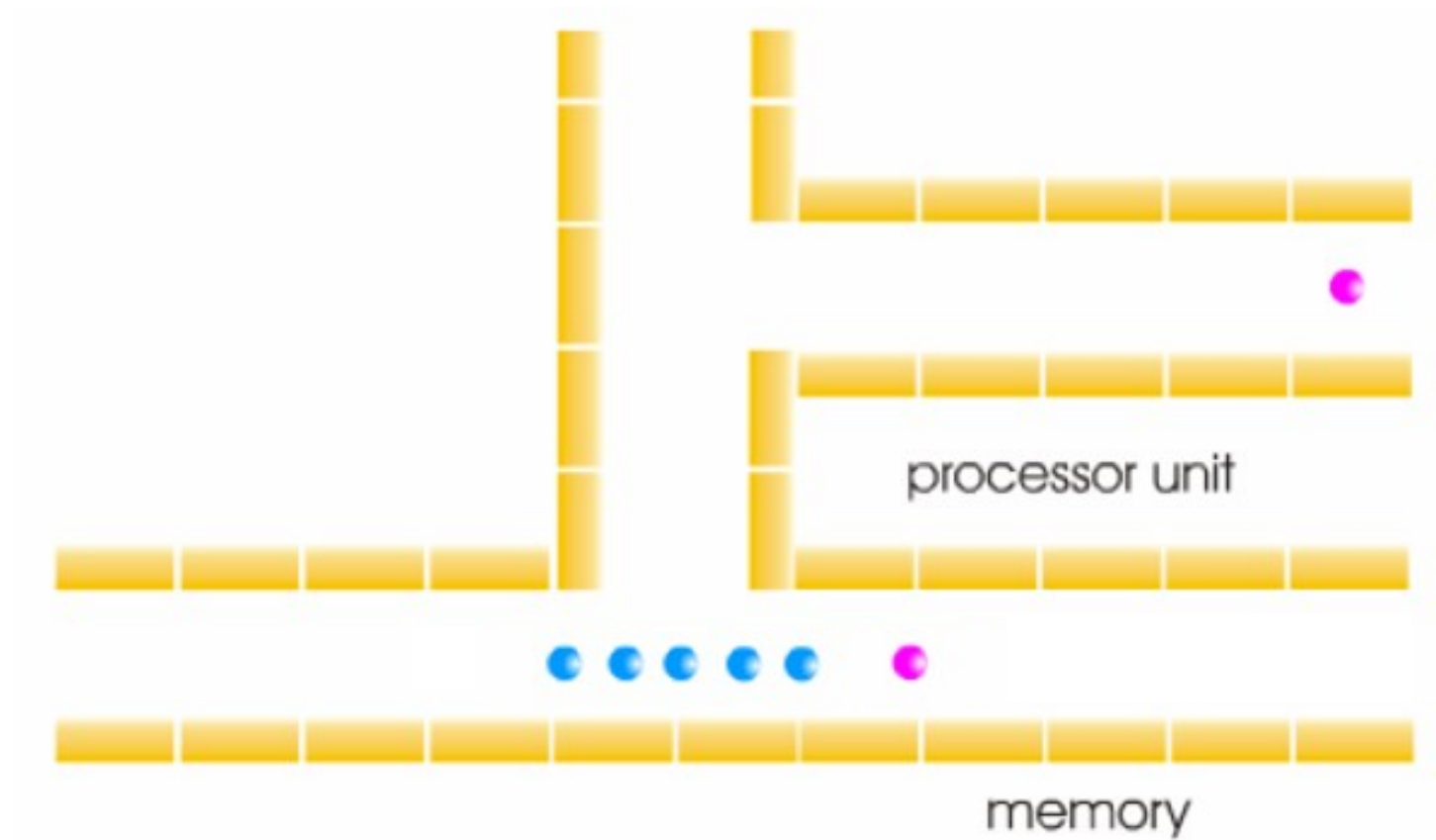


# Scaling ion trap quantum computers

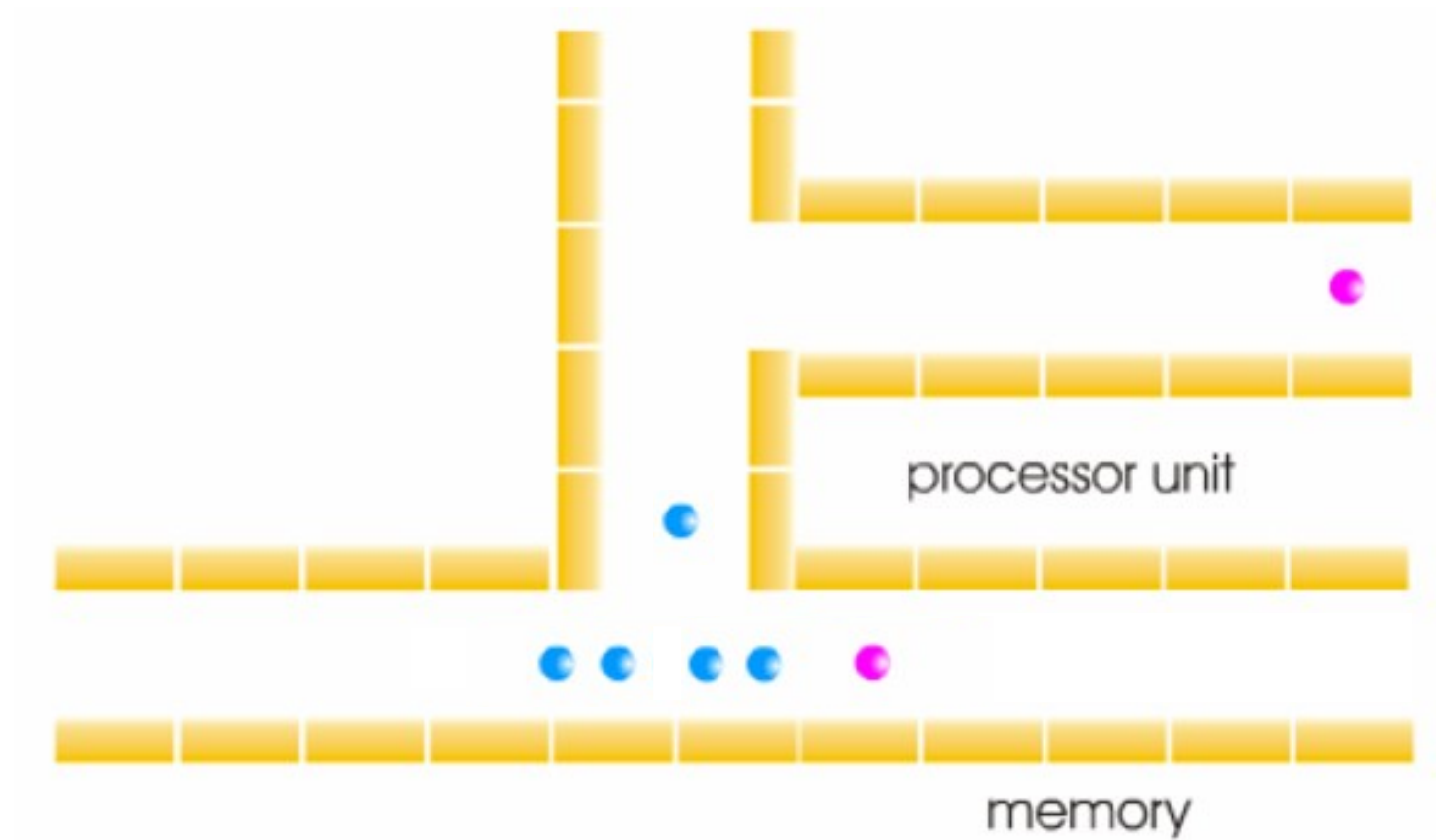




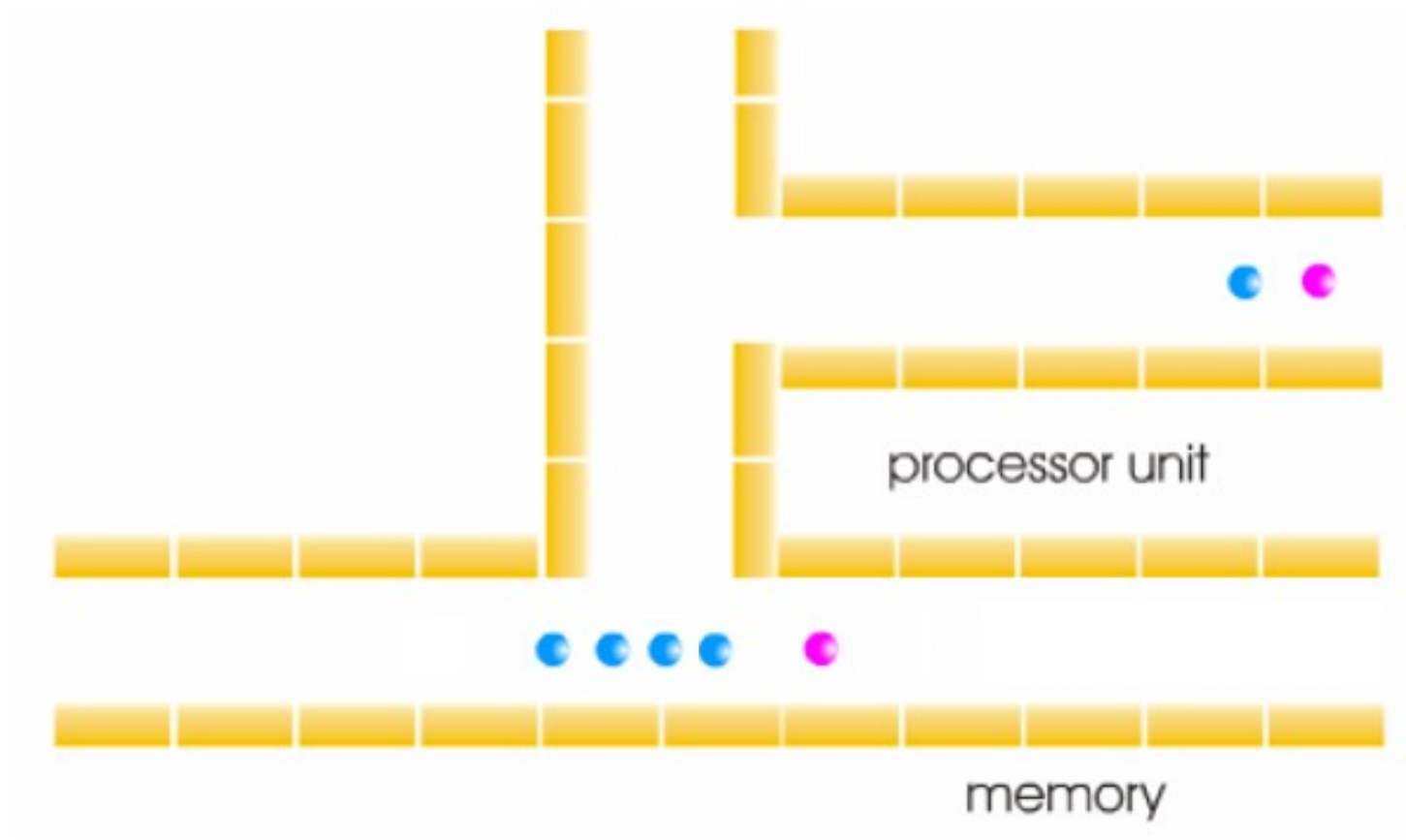
# Scaling ion trap quantum computers



# Scaling ion trap quantum computers

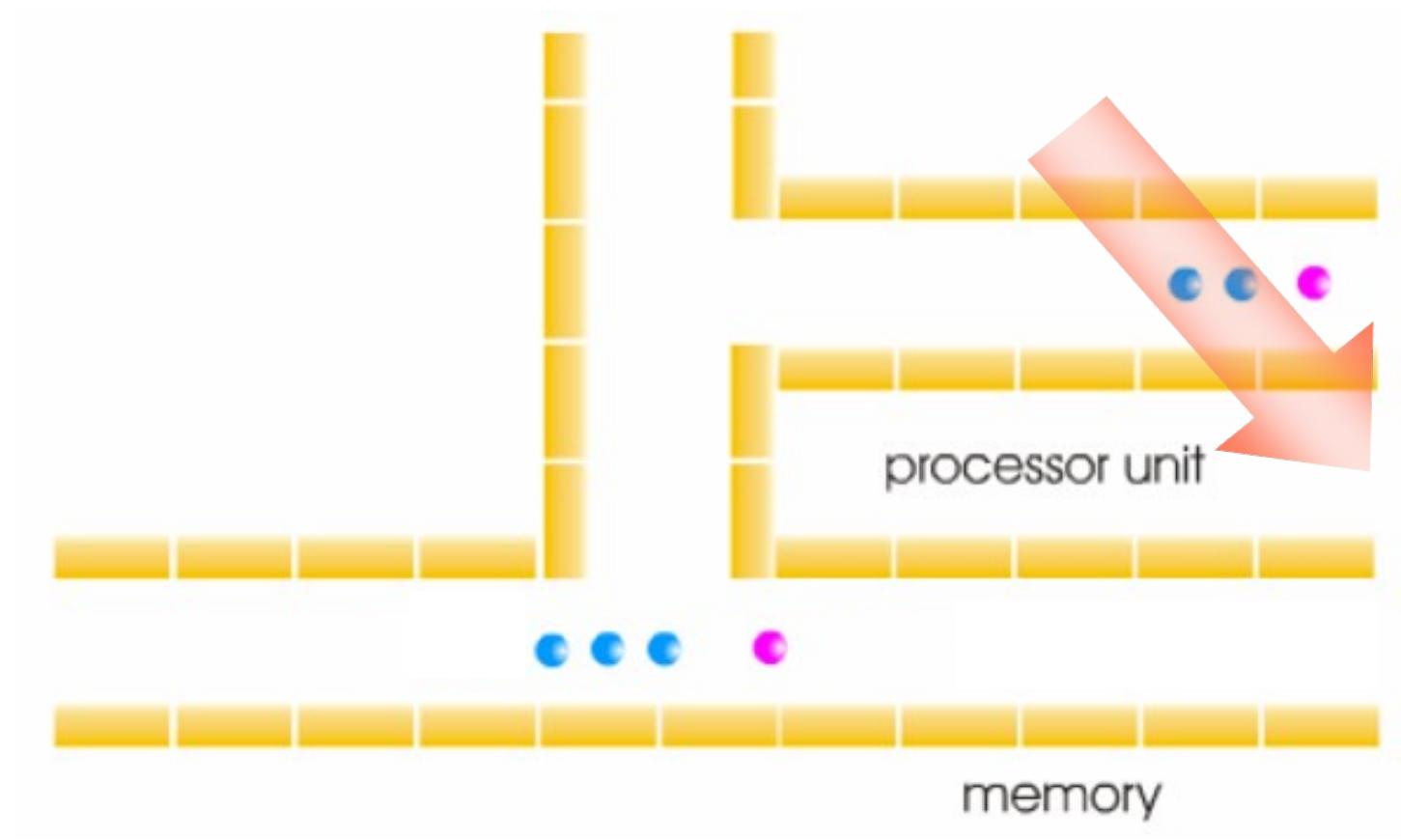


# Scaling ion trap quantum computers



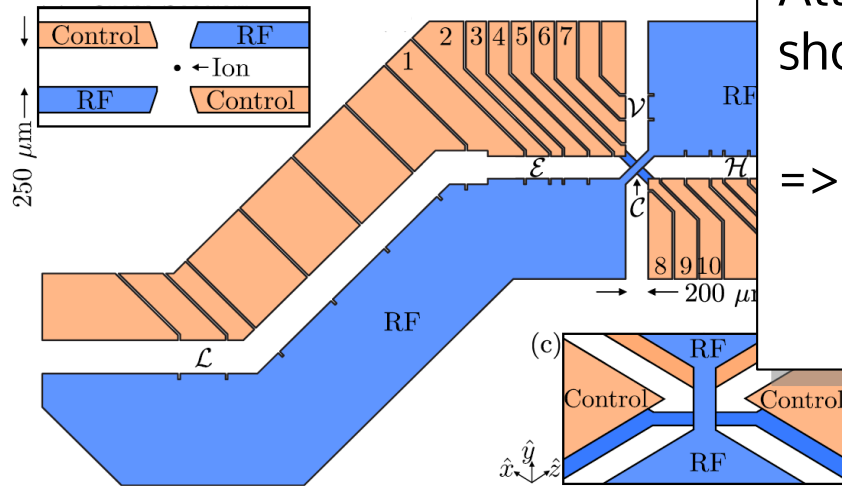


# Scaling ion trap quantum computers



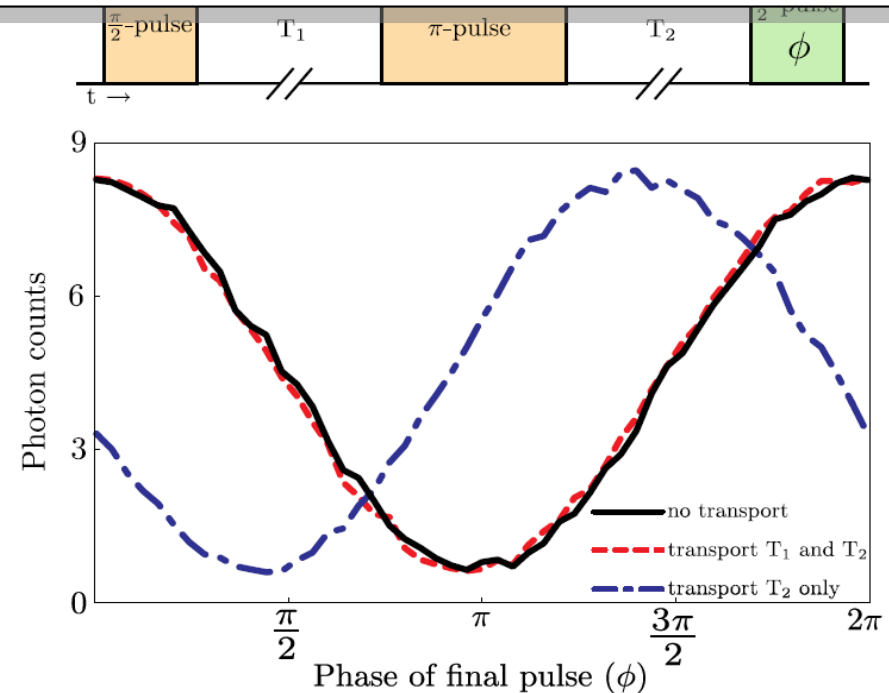
„Architecture for a large-scale ion-trap quantum computer“,  
D. Kielpinski et al., Nature **417**, 709 (2002).

# Coherent transport through junctions



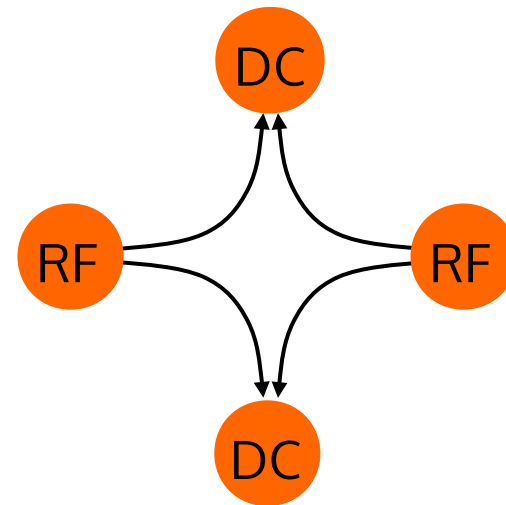
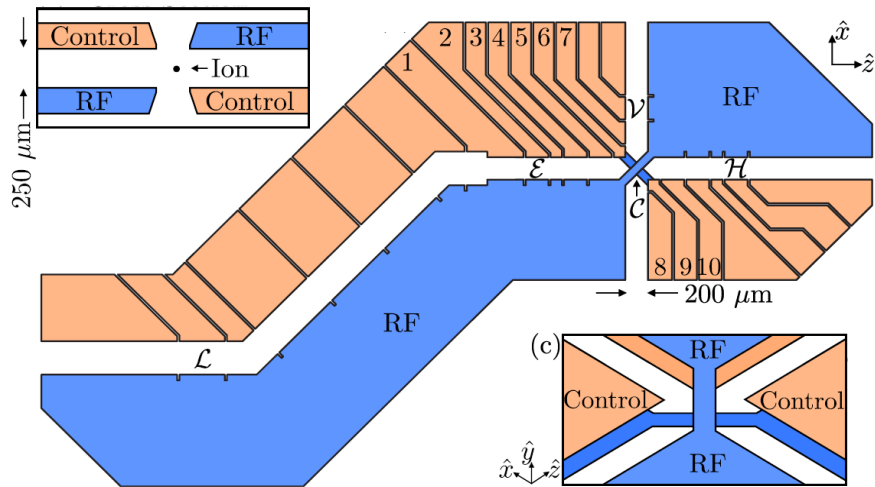
All ingredients for this approach have been shown to work together!

=> Home *et al.*, "Complete methods set for scalable ion trap quantum information processing," Science 325, 1227 (2009).



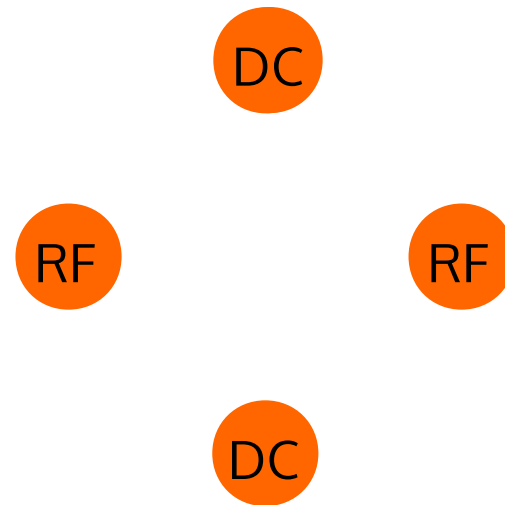
Transport		Energy Gain (recooling method) quanta/trip
$\mathcal{E}$ - $\mathcal{C}$ - $\mathcal{E}$	1 ion	$3.2 \pm 1.8$
$\mathcal{E}$ - $\mathcal{C}$ - $\mathcal{H}$ - $\mathcal{C}$ - $\mathcal{E}$	1 ion	$7.9 \pm 1.5$
$\mathcal{E}$ - $\mathcal{C}$ - $\mathcal{V}$ - $\mathcal{C}$ - $\mathcal{E}$	1 ion	$14.5 \pm 2.0$
$\mathcal{E}$ - $\mathcal{C}$ - $\mathcal{E}$	2 ions	$5.4 \pm 1.2$
$\mathcal{E}$ - $\mathcal{C}$ - $\mathcal{H}$ - $\mathcal{C}$ - $\mathcal{E}$	2 ions	$16.6 \pm 1.8$
$\mathcal{E}$ - $\mathcal{C}$ - $\mathcal{V}$ - $\mathcal{C}$ - $\mathcal{E}$	2 ions	$53.0 \pm 1.2$

# Scalable traps

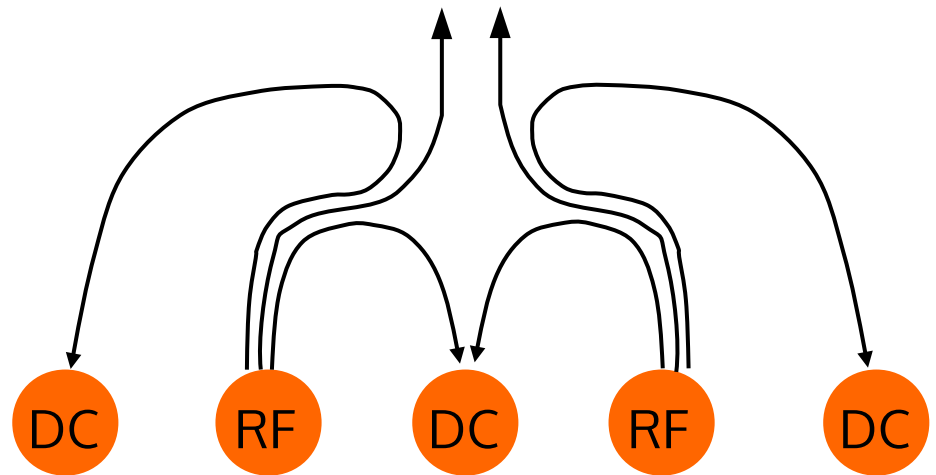
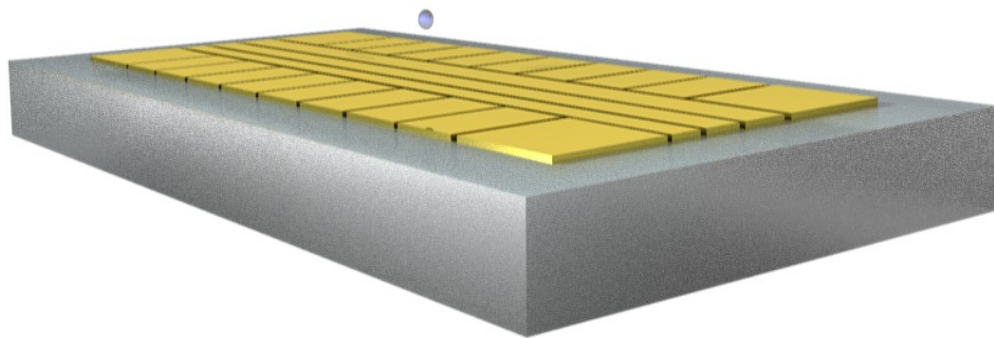




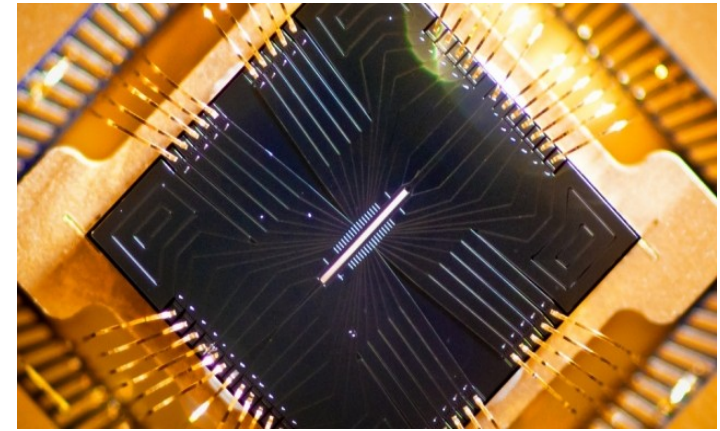
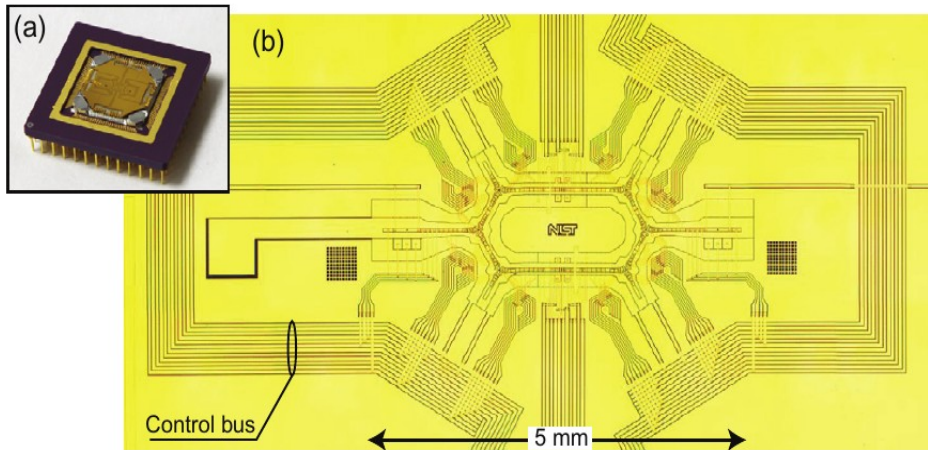
# Surface traps



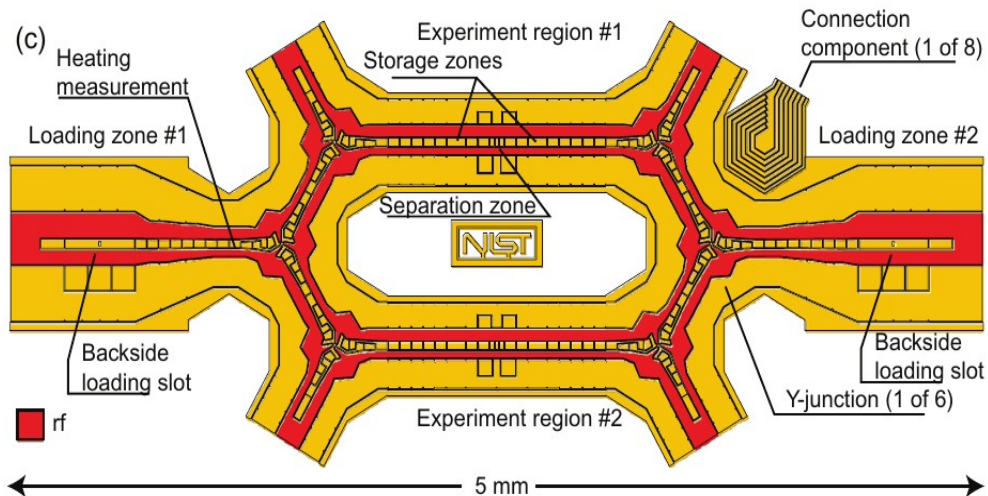
# Microfabricated surface traps



# Microfabricated surface traps



Sandia National Labs



Georgia Tech  
Research Institute

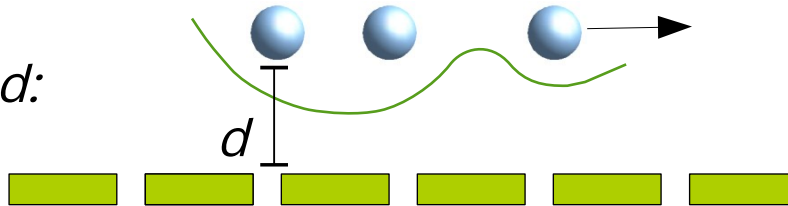
NIST, Amini *et al.*, NJP 2010



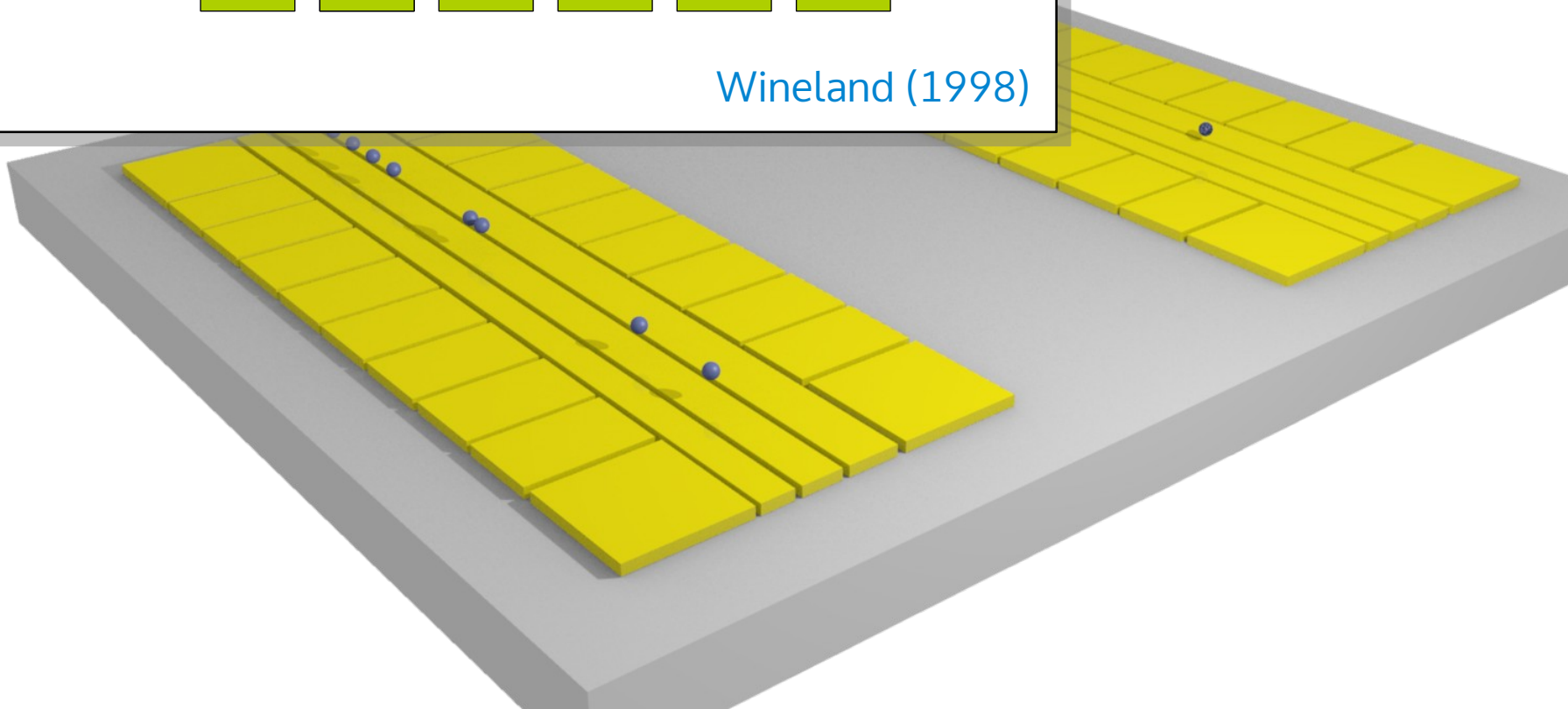
# Ion trap quantum computing

Need to split and merge ion strings fast for multi-qubit gates.

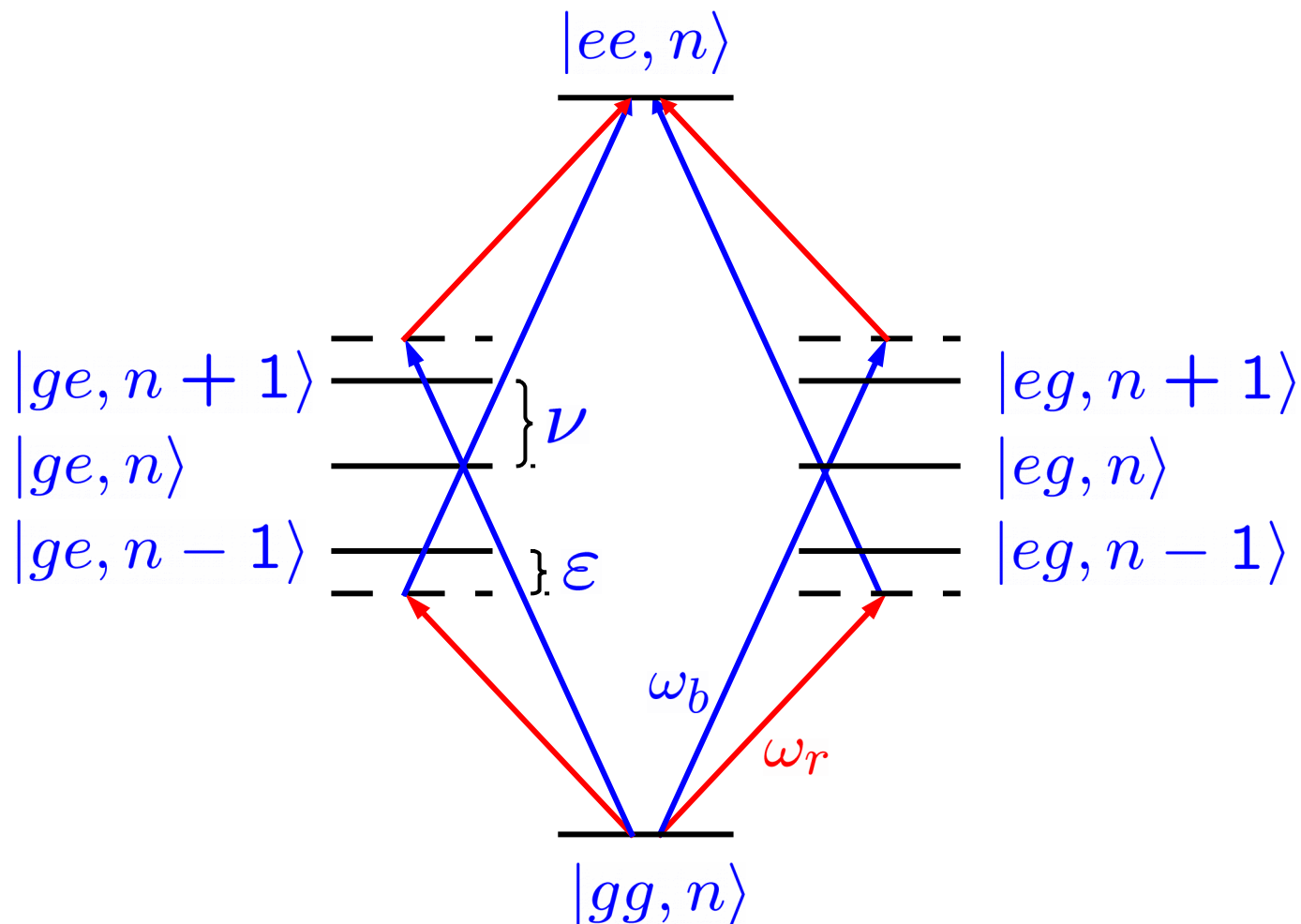
Speed  $\sim 1/d$ :



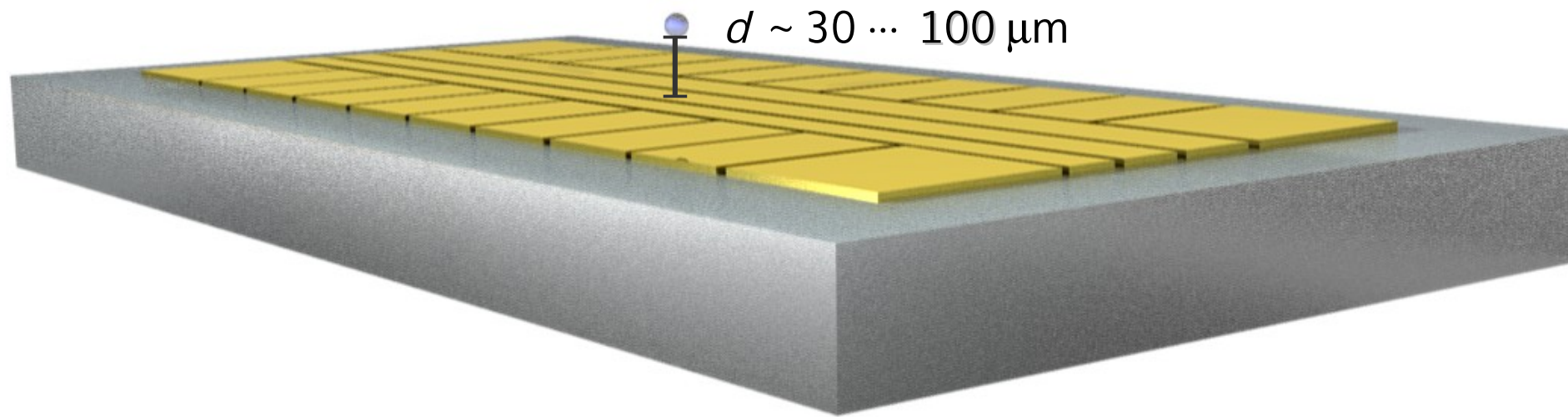
Wineland (1998)



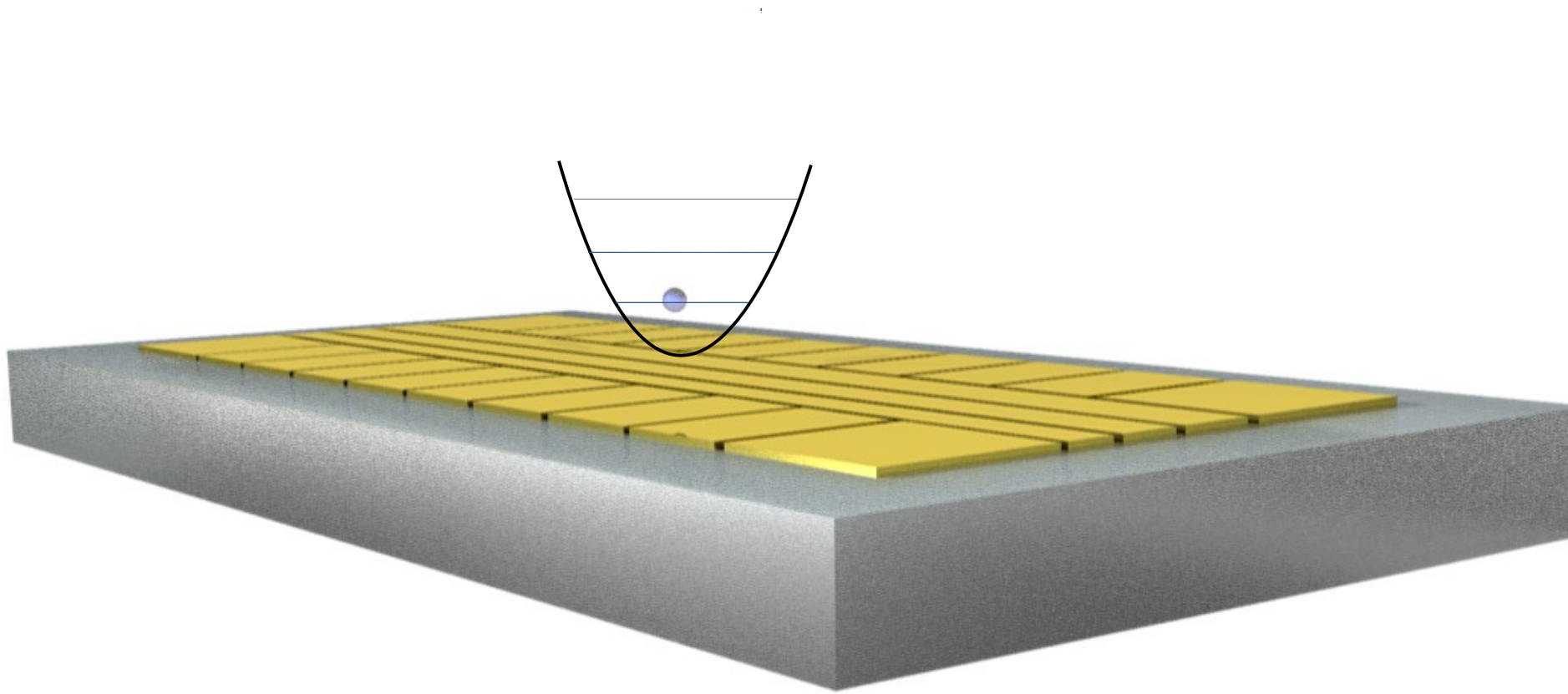
# Decoherence due to ion motion



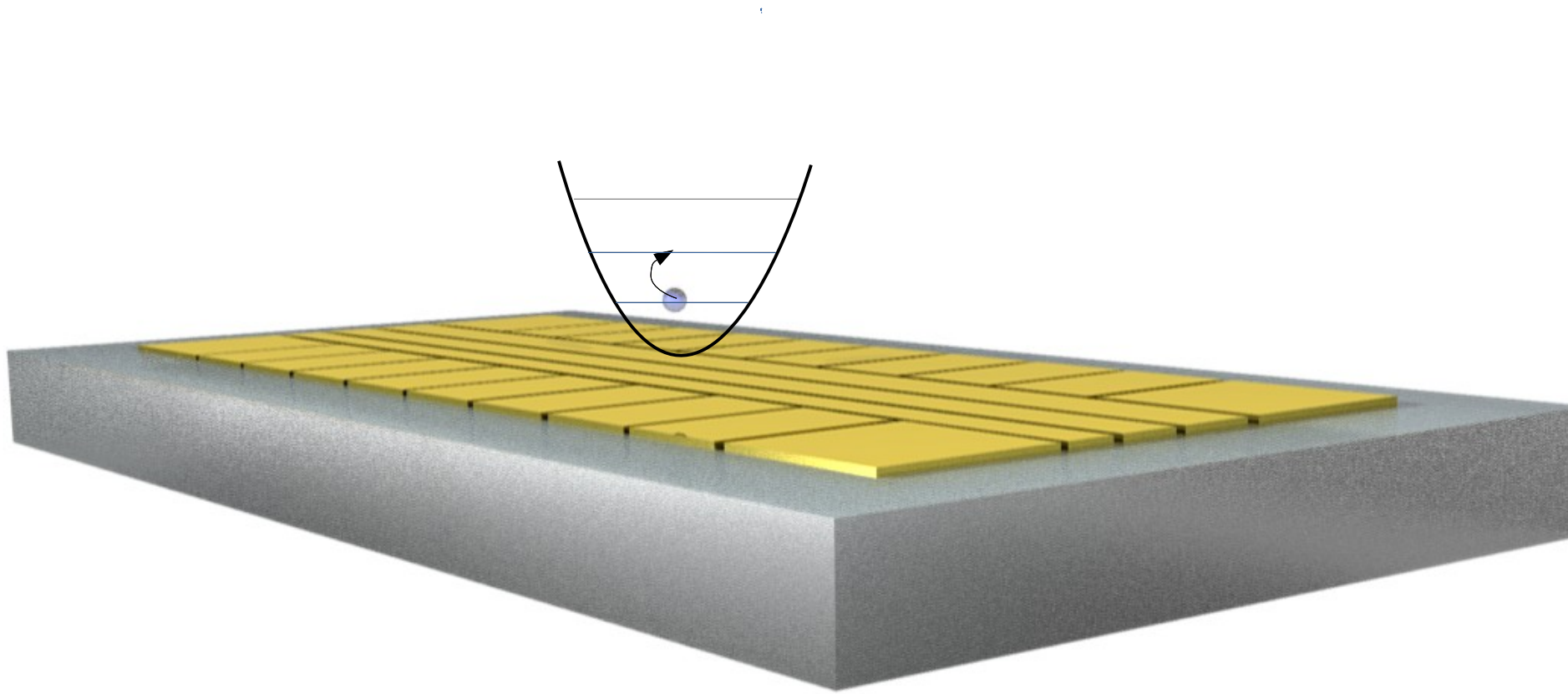
# Motional decoherence



# Motional decoherence

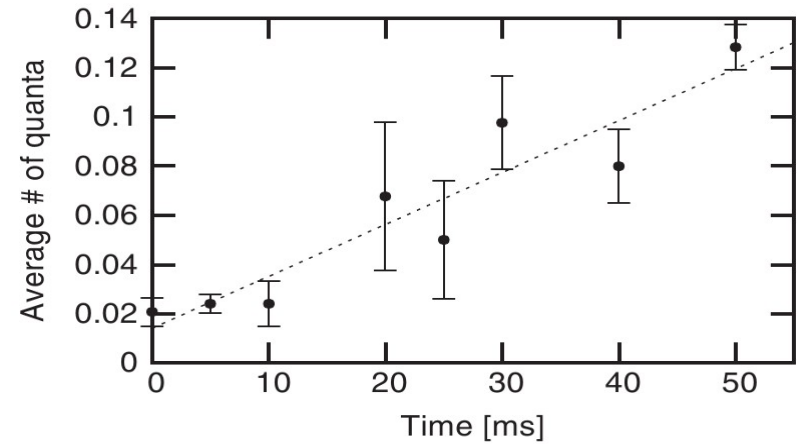
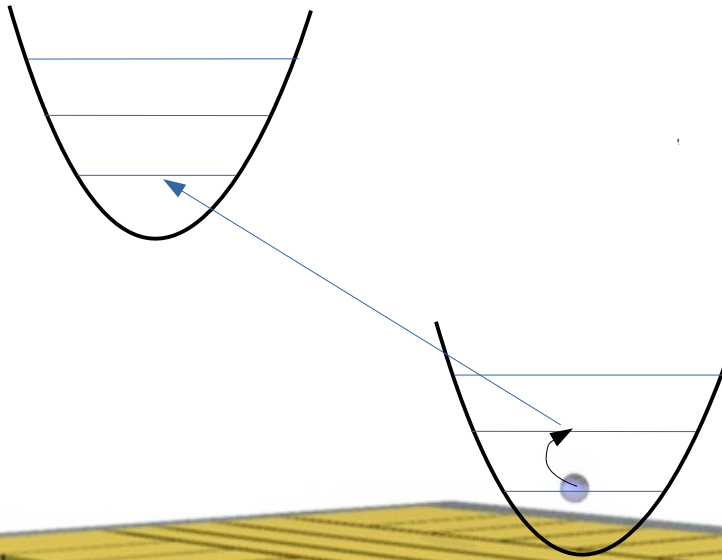


# Motional decoherence





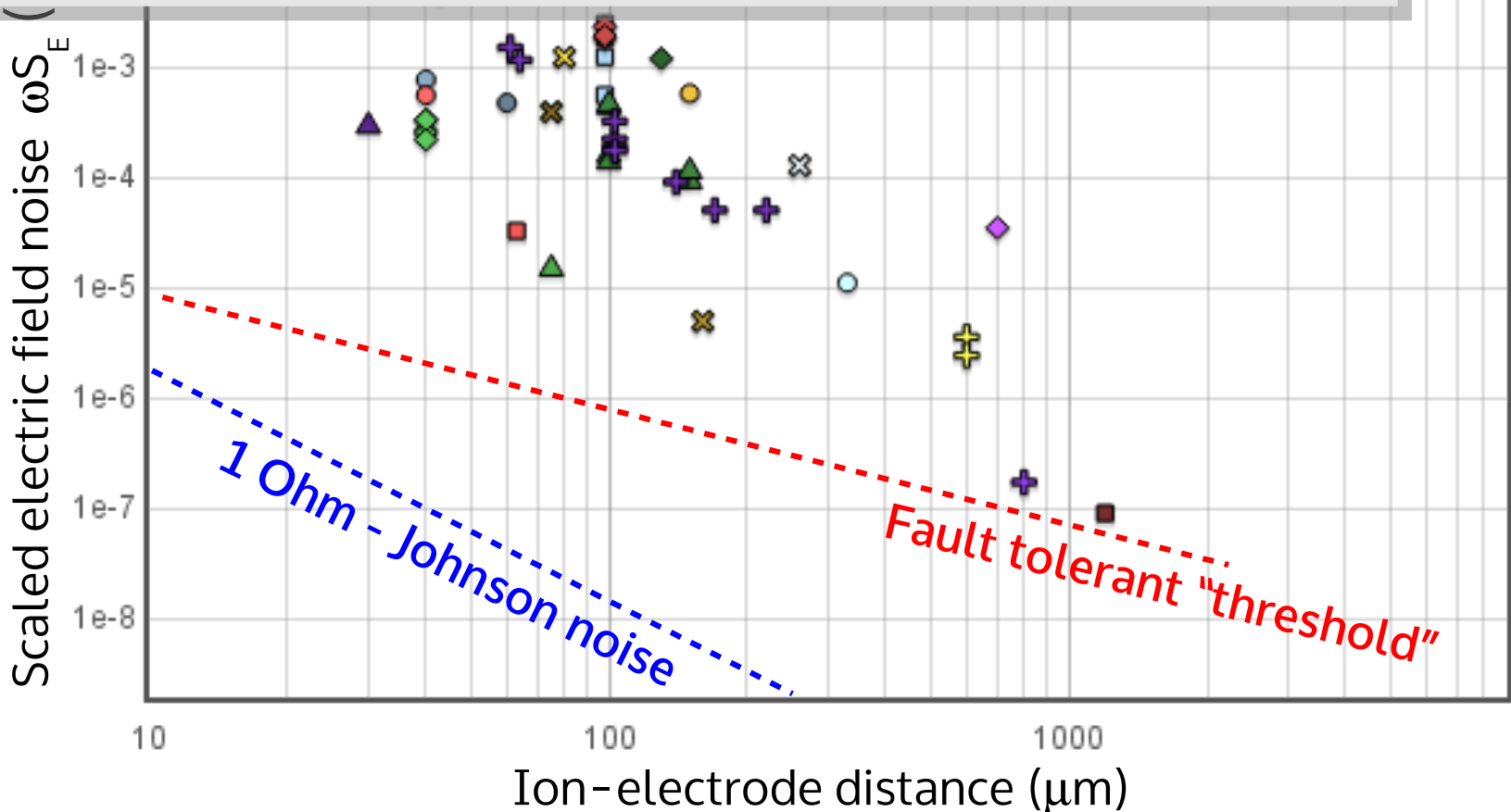
# Motional decoherence



Labaziewicz et al., PRL **100**, 013001 (2008)

# Excessive heating in ion traps

- Need to know what is causing it to deal with it
- Other technologies have similar problems:  
scanning probe microscopy, Casimir force measurements,  
free fall of charged particles, tests of general relativity

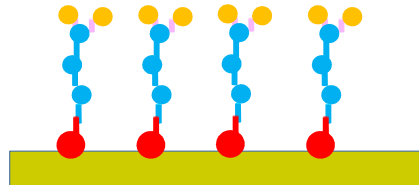


# What is causing “the” anomalous heating ?

- fluctuating patch potentials, ad-atom diffusion (Wineland 1998)



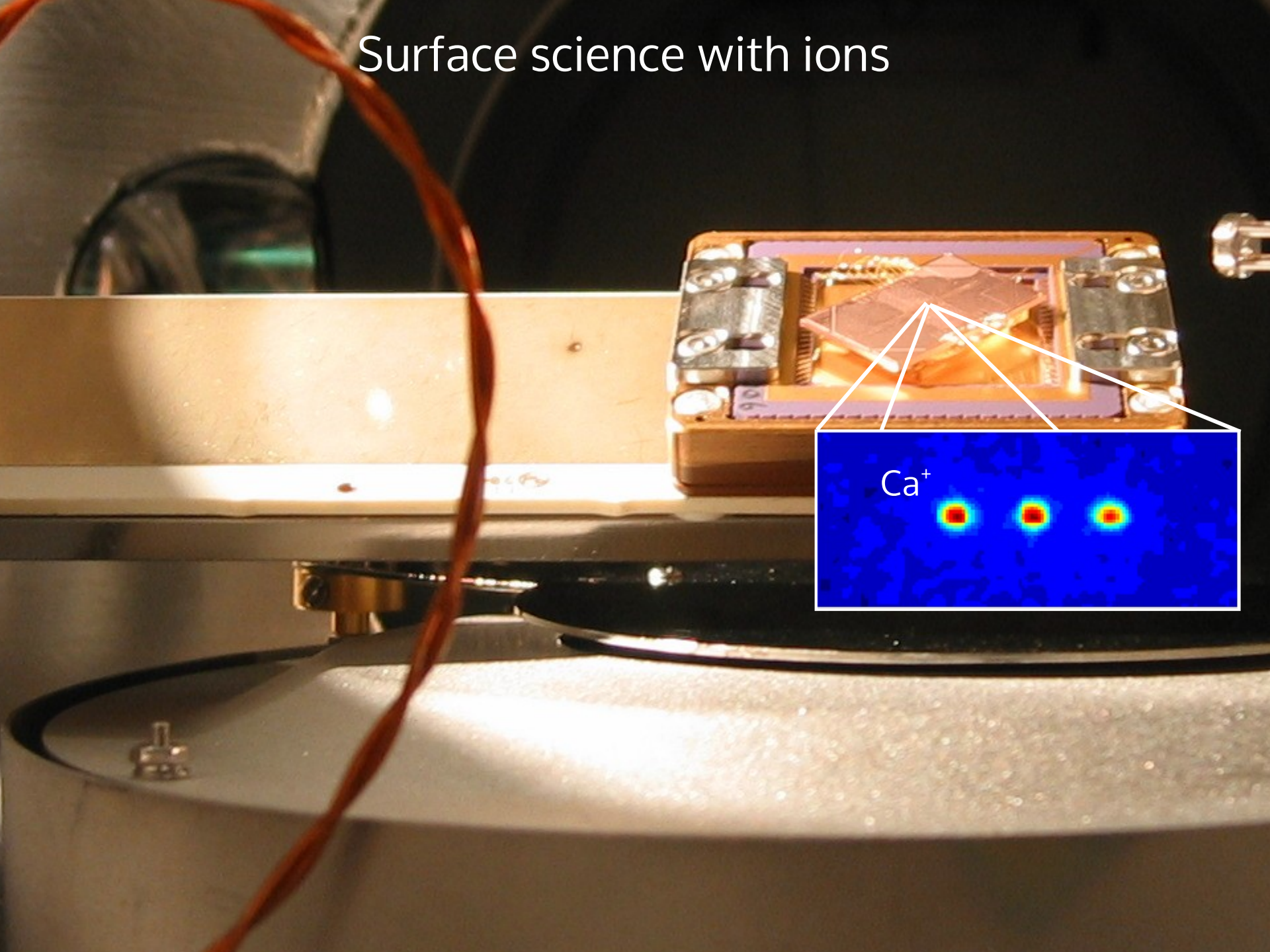
- independently fluctuating dipoles (Daniilidis 2010)



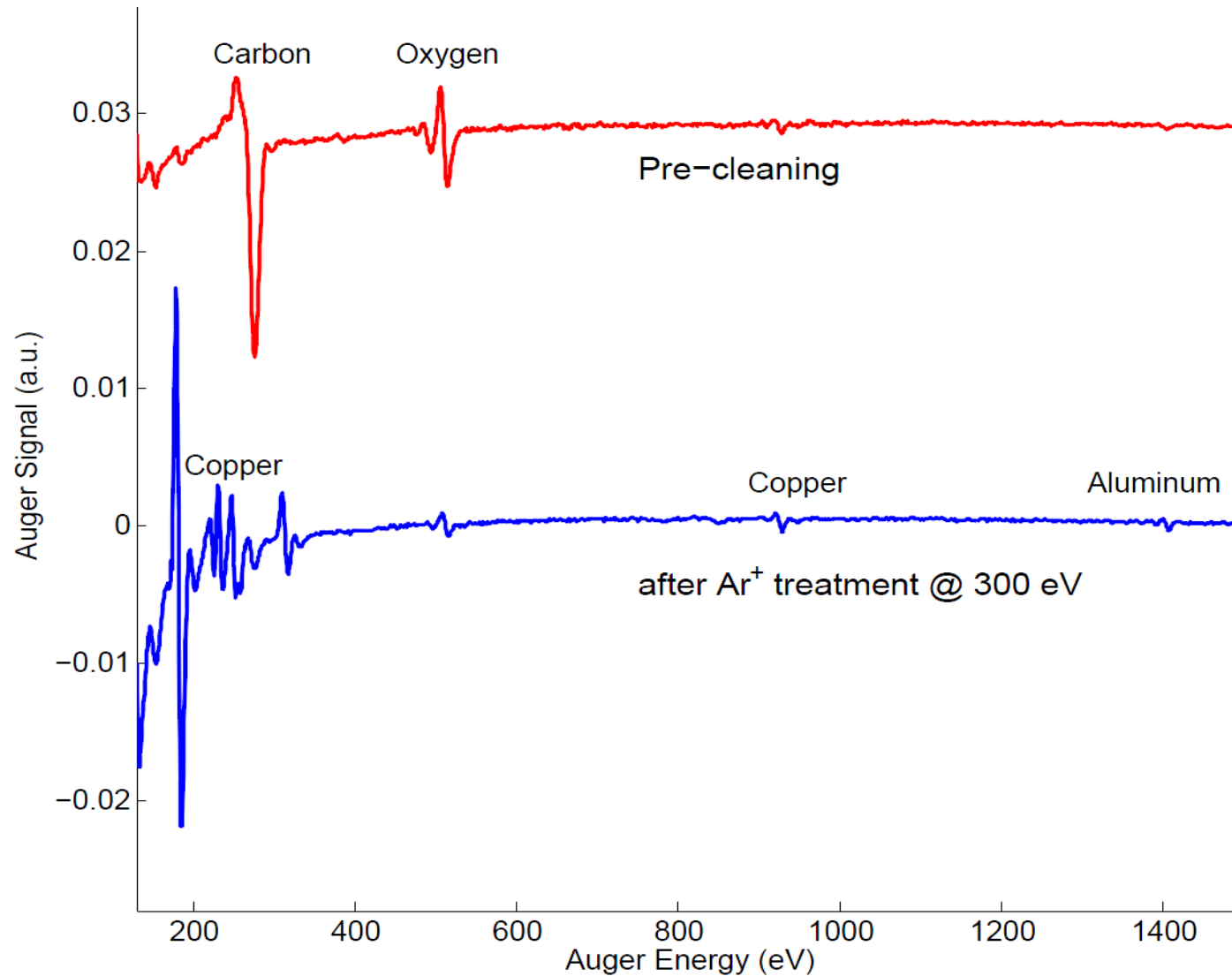
- fluctuating strength of dipoles (Safavi-Naini 2011, 2013)



# Surface science with ions

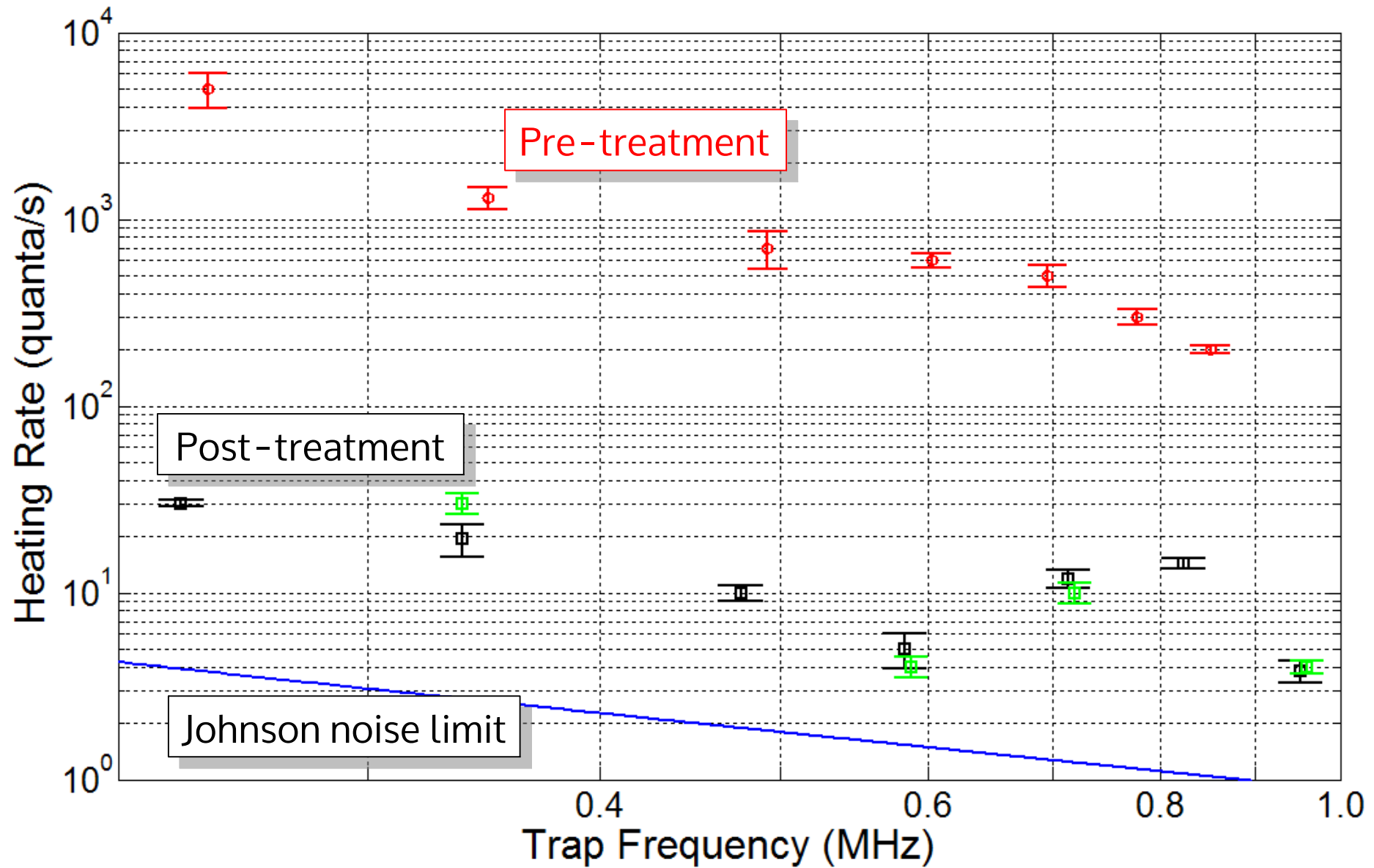


# Auger spectra of a Cu-Al surface



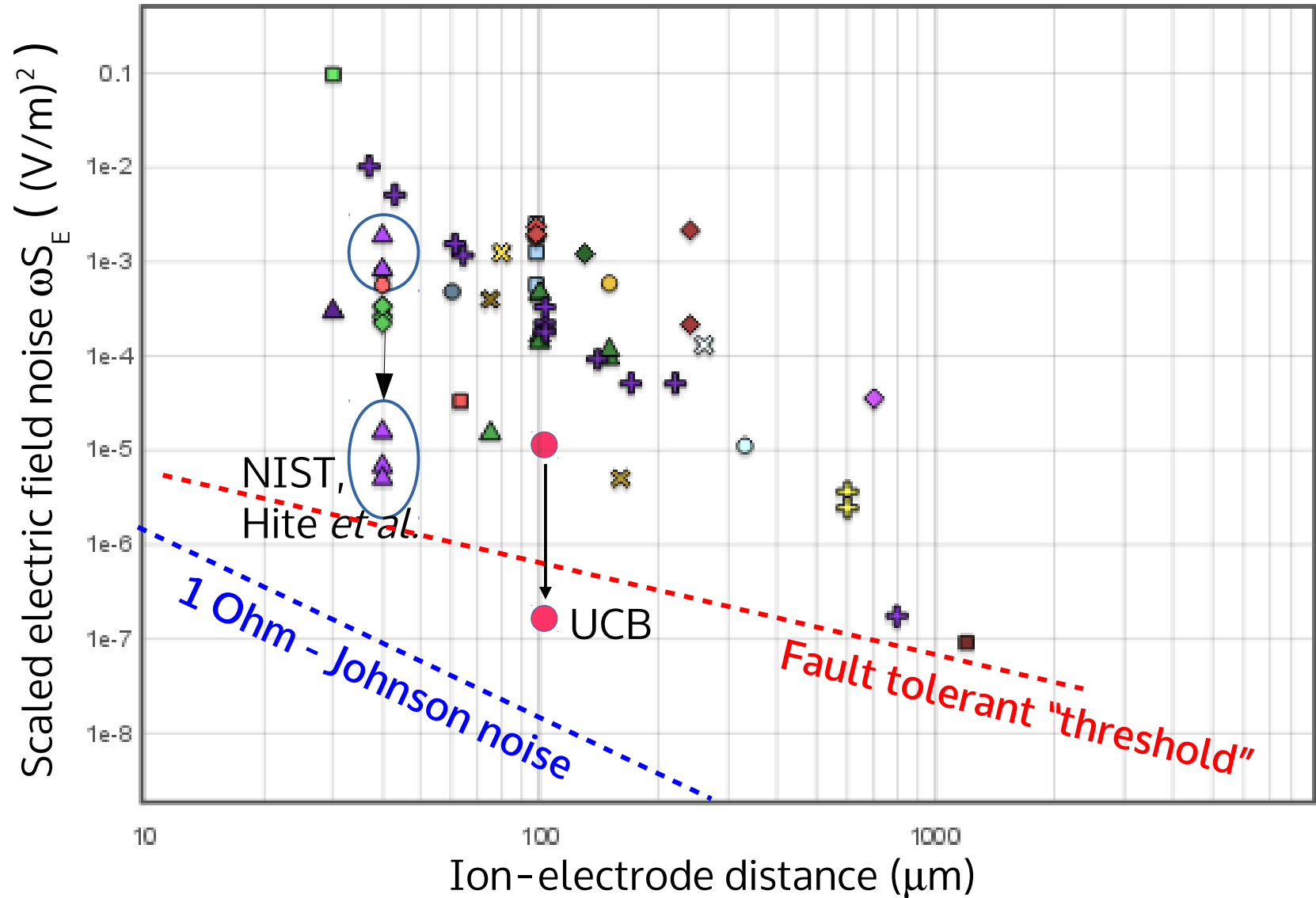


# Heating rates



# Excessive heating in ion traps

From: [http://www.quantum.gatech.edu/heating\\_rate\\_plot.shtml](http://www.quantum.gatech.edu/heating_rate_plot.shtml)



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- Qubit decoherence
- Scaling

## Lecture #4: Scaling and quantum simulation

- Scaling and anomalous heating
- Quantum simulation

## Lecture #5: Applications

- Decoherence-free subspaces
- Michelson-Morley experiment

# Information content

$$|\Psi\rangle_{\text{reg}} = \alpha_0 |000\rangle + \alpha_1 |001\rangle + \alpha_2 |010\rangle + \alpha_3 |011\rangle + \alpha_4 |100\rangle + \alpha_5 |101\rangle + \alpha_6 |110\rangle + \alpha_7 |111\rangle$$

# bits	classical	quantum mechanical
1	1	0.5208 + 0.7059i, 0.3014 + 0.3736i
2	01	0.2044 + 0.4911i, 0.1732 + 0.3855i, 0.2040 + 0.4890i, 0.3193 + 0.3947i
3	001	0.2583 + 0.2704i, 0.2310 + 0.1150i, 0.2956 + 0.3118i, 0.3558 + 0.2113i, 0.1943 + 0.1377i, 0.3273 + 0.2613i, 0.0643 + 0.2033i, 0.3643 + 0.1654i
4	1010	0.1691 + 0.0891i, 0.1096 + 0.0828i, 0.1420 + 0.2873i, 0.0741 + 0.2419i, 0.1902 + 0.0448i, 0.2495 + 0.0039i, 0.1738 + 0.2933i, 0.2102 + 0.0653i, 0.0686 + 0.0980i, 0.1246 + 0.2170i, 0.2570 + 0.0933i, 0.2234 + 0.1540i, 0.1513 + 0.0213i, 0.1863 + 0.3243i, 0.2606 + 0.1912i, 0.0194 + 0.1390i
5	10001	0.1060 + 0.1416i, 0.0103 + 0.0118i, 0.0064 + 0.0976i, 0.0734 + 0.0716i, 0.0030 + 0.2054i, 0.0902 + 0.0035i, 0.1605 + 0.1804i, 0.0218 + 0.2280i, 0.0083 + 0.2326i, 0.1438 + 0.1853i, 0.1429 + 0.1030i, 0.0037 + 0.1171i, 0.0038 + 0.0503i, 0.0446 + 0.1512i, 0.1379 + 0.0752i, 0.0135 + 0.2255i, 0.0863 + 0.1707i, 0.1483 + 0.0968i, 0.1686 + 0.1749i, 0.1627 + 0.0629i, 0.0197 + 0.1033i, 0.1067 + 0.2192i, 0.1038 + 0.1605i, 0.0830 + 0.0499i, 0.0361 + 0.1971i, 0.1587 + 0.1477i, 0.1642 + 0.0314i, 0.1709 + 0.0487i, 0.1124 + 0.1426i, 0.1303 + 0.1480i, 0.0284 + 0.0870i, 0.1059 + 0.1351i
6	110101	0.0595 + 0.1064i, 0.0295 + 0.1327i, 0.0929 + 0.0406i, 0.1090 + 0.0379i, 0.0559 + 0.1286i, 0.0015 + 0.0345i, 0.0624 + 0.1196i, 0.1120 + 0.1350i, 0.1180 + 0.0345i, 0.1367 + 0.0356i, 0.1255 + 0.0074i, 0.0547 + 0.0116i, 0.0923 + 0.0952i, 0.1087 + 0.0284i, 0.0288 + 0.1254i, 0.1345 + 0.0258i, 0.0846 + 0.0254i, 0.0939 + 0.1478i, 0.0348 + 0.0654i, 0.0816 + 0.0505i, 0.1384 + 0.0467i, 0.0498 + 0.0543i, 0.0974 + 0.0584i, 0.0582 + 0.0879i, 0.0932 + 0.0178i, 0.1039 + 0.0057i, 0.0590 + 0.0682i, 0.0615 + 0.1293i, 0.0974 + 0.1388i, 0.1245 + 0.0393i, 0.0552 + 0.0238i, 0.0632 + 0.1297i, 0.0884 + 0.0354i, 0.0841 + 0.0900i, 0.1065 + 0.1437i, 0.0760 + 0.0988i, 0.1154 + 0.1293i, 0.0727 + 0.0015i, 0.0276 + 0.0204i, 0.1041 + 0.1217i, 0.1460 + 0.0639i, 0.1199 + 0.1323i, 0.1046 + 0.1092i, 0.0721 + 0.1021i, 0.0170 + 0.0514i, 0.0988 + 0.0247i, 0.0543 + 0.0231i, 0.0208 + 0.0284i, 0.0842 + 0.0628i, 0.1223 + 0.1272i, 0.1002 + 0.0729i, 0.1485 + 0.1213i, 0.1429 + 0.0685i, 0.0087 + 0.0680i, 0.0535 + 0.0670i, 0.0815 + 0.0613i, 0.0389 + 0.1340i, 0.0888 + 0.0008i, 0.0073 + 0.0442i, 0.0849 + 0.0073i, 0.1042 + 0.1030i, 0.1430 + 0.0966i, 0.1115 + 0.1461i, 0.1100 + 0.0821i
7	1001010	0.0880 + 0.0466i, 0.1054 + 0.0684i, 0.0239 + 0.0866i, 0.0759 + 0.0090i, 0.0563 + 0.1020i, 0.1006 + 0.0988i, 0.0769 + 0.0649i, 0.0246 + 0.0273i, 0.0485 + 0.0942i, 0.0186 + 0.0554i, 0.1045 + 0.0790i, 0.0384 + 0.0455i, 0.0053 + 0.1037i, 0.0815 + 0.0078i, 0.0965 + 0.0597i, 0.0309 + 0.0315i, 0.0271 + 0.0925i, 0.1006 + 0.0362i, 0.0141 + 0.0734i, 0.1015 + 0.0058i, 0.0757 + 0.0385i, 0.0914 + 0.0537i, 0.0226 + 0.0468i, 0.0491 + 0.0607i, 0.0087 + 0.0665i, 0.0918 + 0.0122i, 0.0606 + 0.0969i, 0.0344 + 0.0814i, 0.0404 + 0.0853i, 0.0936 + 0.0879i, 0.0401 + 0.0723i, 0.0079 + 0.0217i, 0.0216 + 0.0294i, 0.0053 + 0.0675i, 0.0611 + 0.0579i, 0.0131 + 0.0064i, 0.0563 + 0.0096i, 0.0126 + 0.0293i, 0.0830 + 0.0441i, 0.0404 + 0.0511i, 0.0888 + 0.0980i, 0.0050 + 0.0643i, 0.0645 + 0.0355i, 0.1024 + 0.0516i, 0.0311 + 0.0644i, 0.0959 + 0.0174i, 0.0110 + 0.0894i, 0.0070 + 0.1031i, 0.0253 + 0.0642i, 0.1006 + 0.0031i, 0.0068 + 0.0876i, 0.0285 + 0.0658i, 0.1078 + 0.0756i, 0.0229 + 0.0099i, 0.0537 + 0.0458i, 0.0313 + 0.0405i, 0.0725 + 0.0179i, 0.1033 + 0.0898i, 0.0827 + 0.0904i, 0.0718 + 0.0487i, 0.0141 + 0.1032i, 0.0103 + 0.0159i, 0.0016 + 0.0938i, 0.0311 + 0.0830i, 0.0881 + 0.0479i, 0.1063 + 0.0669i, 0.0019 + 0.1026i, 0.0884 + 0.0690i, 0.0670 + 0.0267i, 0.0604 + 0.0380i, 0.0263 + 0.0203i, 0.0886 + 0.0529i, 0.0284 + 0.0441i, 0.0813 + 0.0500i, 0.0711 + 0.0189i, 0.0198 + 0.0670i, 0.0686 + 0.0265i, 0.0184 + 0.0633i, 0.0582 + 0.0546i, 0.0672 + 0.0501i, 0.0740 + 0.0584i, 0.0730 + 0.1016i, 0.0946 + 0.0369i, 0.0014 + 0.0433i, 0.0335 + 0.0332i, 0.0840 + 0.0444i, 0.0331 + 0.0308i, 0.0999 + 0.0425i, 0.0732 + 0.0542i, 0.0080 + 0.0779i, 0.0076 + 0.0330i, 0.0013 + 0.0121i, 0.0245 + 0.0478i, 0.0557 + 0.0503i, 0.0494 + 0.0016i, 0.0758 + 0.0716i, 0.0628 + 0.0781i, 0.0549 + 0.0304i, 0.0080 + 0.0282i, 0.0208 + 0.0764i, 0.0409 + 0.0845i, 0.0893 + 0.0425i, 0.0989 + 0.0562i, 0.0122 + 0.0774i, 0.0876 + 0.0614i, 0.0979 + 0.0497i, 0.0169 + 0.0480i, 0.0132 + 0.0095i, 0.0822 + 0.0478i, 0.0778 + 0.0395i, 0.0703 + 0.0326i, 0.0813 + 0.0919i, 0.0715 + 0.0819i, 0.0953 + 0.1024i, 0.0293 + 0.0602i, 0.0452 + 0.0015i, 0.0230 + 0.0643i
8	10101011	0.0199 + 0.0027i, 0.0033 + 0.0063i, 0.0005 + 0.0656i, 0.0443 + 0.0262i, 0.0573 + 0.0359i, 0.0622 + 0.0704i, 0.0491 + 0.0176i, 0.0194 + 0.0664i, 0.0111 + 0.0506i, 0.0502 + 0.0687i, 0.0729 + 0.0376i, 0.0629 + 0.0765i, 0.0717 + 0.0288i, 0.0239 + 0.0410i, 0.0207 + 0.0140i, 0.0413 + 0.0387i, 0.0126 + 0.0325i, 0.0163 + 0.0509i, 0.0167 + 0.0519i, 0.0502 + 0.0738i, 0.0041 + 0.0148i, 0.0177 + 0.0088i, 0.0514 + 0.0436i, 0.0240 + 0.0747i, 0.0236 + 0.0018i, 0.0555 + 0.0671i, 0.0736 + 0.0021i, 0.0101 + 0.0400i, 0.0053 + 0.0148i, 0.0097 + 0.0552i, 0.0128 + 0.0193i, 0.0702 + 0.0720i, 0.0105 + 0.0106i, 0.0476 + 0.0402i, 0.0207 + 0.0690i, 0.0170 + 0.0726i, 0.0549 + 0.0258i, 0.0423 + 0.0337i, 0.0728 + 0.0383i, 0.0254 + 0.0115i, 0.0543 + 0.0105i, 0.0727 + 0.0410i, 0.0448 + 0.0559i, 0.0678 + 0.0307i, 0.0578 + 0.0276i, 0.0293 + 0.0220i, 0.0559 + 0.0670i, 0.0125 + 0.0483i, 0.0737 + 0.0186i, 0.0151 + 0.0754i, 0.0598 + 0.0494i, 0.0473 + 0.0177i, 0.0125 + 0.0525i, 0.0024 + 0.0513i, 0.0222 + 0.0104i, 0.0748 + 0.0017i, 0.0733 + 0.0202i, 0.0176 + 0.0090i, 0.0739 + 0.0053i, 0.0524 + 0.0657i, 0.0042 + 0.0139i, 0.0462 + 0.0025i, 0.0303 + 0.0566i, 0.0166 + 0.0414i, 0.0141 + 0.0213i

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1	1	0.5208 + 0.7059i, 0.3014 + 0.3736i
2	01	0.2044 + 0.4911i, 0.1732 + 0.3855i, 0.2040 + 0.4890i, 0.3193 + 0.3947i
3	001	0.2583 + 0.2704i, 0.2310 + 0.1150i, 0.2956 + 0.3118i, 0.3558 + 0.2113i, 0.1943 + 0.1377i, 0.3273 + 0.2613i, 0.0643 + 0.2033i, 0.3643 + 0.1654i
4	1010	0.1691 + 0.0891i, 0.1096 + 0.0828i, 0.1420 + 0.2873i, 0.0741 + 0.2419i, 0.1902 + 0.0448i, 0.2495 + 0.0039i, 0.1738 + 0.2933i, 0.2102 + 0.0653i, 0.0686 + 0.0980i, 0.1246 + 0.2170i, 0.2570 + 0.0933i, 0.2234 + 0.1540i, 0.1513 + 0.0213i, 0.1863 + 0.3243i, 0.2606 + 0.1912i, 0.0194 + 0.1390i
5	10001	0.1060 + 0.1416i, 0.0103 + 0.0118i, 0.0064 + 0.0976i, 0.0734 + 0.0716i, 0.0030 + 0.2054i, 0.0902 + 0.0035i, 0.1605 + 0.1804i, 0.0218 + 0.2280i, 0.0083 + 0.2326i, 0.1438 + 0.1853i, 0.1429 + 0.1030i, 0.0037 + 0.1171i, 0.0038 + 0.0503i, 0.0446 + 0.1512i, 0.1379 + 0.0752i, 0.0135 + 0.2255i, 0.0863 + 0.1707i, 0.1483 + 0.0968i, 0.1686 + 0.1749i, 0.1627 + 0.0629i, 0.0197 + 0.1033i, 0.1067 + 0.2192i, 0.1038 + 0.1605i, 0.0830 + 0.0499i, 0.0361 + 0.1971i, 0.1587 + 0.1477i, 0.1642 + 0.0314i, 0.1709 + 0.0487i, 0.1124 + 0.1426i, 0.1303 + 0.1480i, 0.0284 + 0.0870i, 0.1059 + 0.1351i
6	110101	0.0595 + 0.1064i, 0.0295 + 0.1327i, 0.0929 + 0.0406i, 0.1090 + 0.0379i, 0.0559 + 0.1286i, 0.0015 + 0.0345i, 0.0624 + 0.1196i, 0.1120 + 0.1350i, 0.1180 + 0.0345i, 0.1367 + 0.0356i, 0.1255 + 0.0074i, 0.0547 + 0.0116i, 0.0923 + 0.0952i, 0.1087 + 0.0284i, 0.0288 + 0.1254i, 0.1345 + 0.0258i, 0.0846 + 0.0254i, 0.0939 + 0.1478i, 0.0348 + 0.0654i, 0.0816 + 0.0505i, 0.1384 + 0.0467i, 0.0498 + 0.0543i, 0.0974 + 0.0584i, 0.0582 + 0.0879i, 0.0932 + 0.0178i, 0.1039 + 0.0057i, 0.0590 + 0.0682i, 0.0615 + 0.1293i, 0.0974 + 0.1388i, 0.1245 + 0.0393i, 0.0552 + 0.0238i, 0.0632 + 0.1297i, 0.0884 + 0.0354i, 0.0841 + 0.0960i, 0.1065 + 0.1437i, 0.0760 + 0.0988i, 0.1154 + 0.1293i, 0.0727 + 0.0015i, 0.0276 + 0.0204i, 0.1041 + 0.1217i, 0.1460 + 0.0639i, 0.1199 + 0.1323i, 0.1046 + 0.1092i, 0.0721 + 0.1021i, 0.0170 + 0.0514i, 0.0988 + 0.0247i, 0.0543 + 0.0231i, 0.0208 + 0.0284i, 0.0842 + 0.0628i, 0.1223 + 0.1272i, 0.1002 + 0.0729i, 0.1485 + 0.1213i, 0.1429 + 0.0685i, 0.0087 + 0.0680i, 0.0535 + 0.0670i, 0.0815 + 0.0613i, 0.0389 + 0.1340i, 0.0888 + 0.0008i, 0.0073 + 0.0442i, 0.0849 + 0.0073i, 0.1042 + 0.1030i, 0.1430 + 0.0966i, 0.1115 + 0.1461i, 0.1100 + 0.0821i
7	1001010	0.0880 + 0.0466i, 0.1054 + 0.0684i, 0.0239 + 0.0866i, 0.0759 + 0.0090i, 0.0583 + 0.1020i, 0.1006 + 0.0988i, 0.0769 + 0.0649i, 0.0246 + 0.0273i, 0.0485 + 0.0942i, 0.0186 + 0.0554i, 0.1045 + 0.0790i, 0.0384 + 0.0455i, 0.0053 + 0.1037i, 0.0815 + 0.0078i, 0.0965 + 0.0597i, 0.0309 + 0.0315i, 0.0271 + 0.0925i, 0.1006 + 0.0362i, 0.0141 + 0.0734i, 0.0216 + 0.0294i, 0.0757 + 0.0385i, 0.0914 + 0.0537i, 0.0226 + 0.0468i, 0.0491 + 0.0607i, 0.0087 + 0.0665i, 0.0606 + 0.0969i, 0.0344 + 0.0814i, 0.0404 + 0.0853i, 0.0936 + 0.0879i, 0.0401 + 0.0723i, 0.0079 + 0.0217i, 0.0216 + 0.0294i, 0.0053 + 0.0675i, 0.0611 + 0.0579i, 0.0131 + 0.0064i, 0.0563 + 0.0096i, 0.0126 + 0.0293i, 0.0830 + 0.0441i, 0.0404 + 0.0511i, 0.0888 + 0.0980i, 0.0050 + 0.0643i, 0.0645 + 0.0355i, 0.1024 + 0.0516i, 0.0311 + 0.0644i, 0.0959 + 0.0174i, 0.0110 + 0.0894i, 0.0070 + 0.1031i, 0.0253 + 0.0642i, 0.1006 + 0.0031i, 0.0068 + 0.0876i, 0.0285 + 0.0658i, 0.1078 + 0.0756i, 0.0229 + 0.0099i, 0.0537 + 0.0458i, 0.0313 + 0.0405i, 0.0725 + 0.0179i, 0.1033 + 0.0898i, 0.0827 + 0.0904i, 0.0718 + 0.0487i, 0.0141 + 0.1032i, 0.0103 + 0.0159i, 0.0016 + 0.0938i, 0.0311 + 0.0830i, 0.0881 + 0.0479i, 0.1063 + 0.0669i, 0.0019 + 0.1026i, 0.0884 + 0.0690i, 0.0670 + 0.0267i, 0.0604 + 0.0380i, 0.0263 + 0.0203i, 0.0886 + 0.0529i, 0.0284 + 0.0441i, 0.0813 + 0.0500i, 0.0231 + 0.0077i, 0.0649 + 0.0339i, 0.0652 + 0.0656i, 0.0711 + 0.0189i, 0.0198 + 0.0670i, 0.0686 + 0.0265i, 0.0184 + 0.0633i, 0.0582 + 0.0546i, 0.0672 + 0.0501i, 0.0740 + 0.0584i, 0.0730 + 0.1016i, 0.0946 + 0.0369i, 0.0014 + 0.0433i, 0.0335 + 0.0332i, 0.0840 + 0.0444i, 0.0331 + 0.0308i, 0.0999 + 0.0425i, 0.0732 + 0.0542i, 0.0080 + 0.0779i, 0.0076 + 0.0330i, 0.0013 + 0.0121i, 0.0245 + 0.0478i, 0.0557 + 0.0503i, 0.0494 + 0.0016i, 0.0758 + 0.0716i, 0.0628 + 0.0781i, 0.0549 + 0.0304i, 0.0080 + 0.0282i, 0.0208 + 0.0764i, 0.0409 + 0.0845i, 0.0893 + 0.0425i, 0.0989 + 0.0562i, 0.0122 + 0.0774i, 0.0876 + 0.0614i, 0.0979 + 0.0497i, 0.0169 + 0.0480i, 0.0132 + 0.0095i, 0.0822 + 0.0478i, 0.0778 + 0.0395i, 0.0723 + 0.0326i, 0.0813 + 0.0919i, 0.0715 + 0.0819i, 0.0953 + 0.1024i, 0.0293 + 0.0602i, 0.0452 + 0.0015i, 0.0230 + 0.0643i
8	10101011	0.0199 + 0.0027i, 0.0033 + 0.0063i, 0.0005 + 0.0656i, 0.0443 + 0.0262i, 0.0573 + 0.0359i, 0.0622 + 0.0704i, 0.0491 + 0.0176i, 0.0194 + 0.0664i, 0.0111 + 0.0506i, 0.0502 + 0.0687i, 0.0729 + 0.0376i, 0.0629 + 0.0765i, 0.0717 + 0.0288i, 0.0239 + 0.0410i, 0.0207 + 0.0140i, 0.0413 + 0.0387i, 0.0126 + 0.0325i, 0.0163 + 0.0509i, 0.0167 + 0.0519i, 0.0502 + 0.0738i, 0.0041 + 0.0148i, 0.0177 + 0.0086i, 0.0514 + 0.0436i, 0.0240 + 0.0747i, 0.0236 + 0.0018i, 0.0555 + 0.0671i, 0.0736 + 0.0021i, 0.0101 + 0.0400i, 0.0053 + 0.0148i, 0.0097 + 0.0552i, 0.0128 + 0.0193i, 0.0702 + 0.0720i, 0.0105 + 0.0106i, 0.0476 + 0.0402i, 0.0207 + 0.0690i, 0.0170 + 0.0726i, 0.0207 + 0.0367i, 0.0254 + 0.0115i, 0.0543 + 0.0105i, 0.0727 + 0.0410i, 0.0448 + 0.0559i, 0.0678 + 0.0307i, 0.0578 + 0.0276i, 0.0293 + 0.0220i, 0.0559 + 0.0670i, 0.0125 + 0.0483i, 0.0737 + 0.0186i, 0.0151 + 0.0754i, 0.0598 + 0.0494i, 0.0473 + 0.0177i, 0.0125 + 0.0525i, 0.0024 + 0.0513i, 0.0222 + 0.0104i, 0.0748 + 0.0017i, 0.0733 + 0.0202i, 0.0176 + 0.0090i, 0.0739 + 0.0053i, 0.0524 + 0.0657i, 0.0042 + 0.0139i, 0.0462 + 0.0025i, 0.0303 + 0.0566i, 0.0166 + 0.0414i, 0.0141 + 0.0213i, 0.0059 + 0.0284i, 0.0006 + 0.0010i, 0.0608 + 0.0685i, 0.0014 + 0.0667i, 0.0077 + 0.0196i, 0.0272 + 0.0439i, 0.0557 + 0.0123i, 0.0746 + 0.0458i, 0.0120 + 0.0255i, 0.0126 + 0.0508i, 0.0242 + 0.0666i, 0.0023 + 0.0437i, 0.0276 + 0.0756i, 0.0021 + 0.0610i, 0.0612 + 0.0118i, 0.0770 + 0.0642i, 0.0085 + 0.0148i, 0.0480 + 0.0493i, 0.0102 + 0.0516i, 0.0239 + 0.0595i, 0.0104 + 0.0293i, 0.0172 + 0.0340i, 0.0306 + 0.0372i, 0.0104 + 0.0469i, 0.0186 + 0.0136i, 0.0715 + 0.0002i, 0.0301 + 0.0609i, 0.0394 + 0.0396i, 0.0072 + 0.0164i, 0.0017 + 0.0080i, 0.0123 + 0.0121i, 0.0651 + 0.0314i, 0.0678 + 0.0314i, 0.0144 + 0.0041i, 0.0764 + 0.0726i, 0.0549 + 0.0116i, 0.0672 + 0.0296i, 0.0370 + 0.0240i, 0.0382 + 0.0130i, 0.0222 + 0.0691i, 0.0047 + 0.0249i, 0.0202 + 0.0566i, 0.0144 + 0.0317i, 0.0707 + 0.0308i, 0.0085 + 0.0390i, 0.0010 + 0.0130i, 0.0285 + 0.0404i, 0.0538 + 0.0494i, 0.0685 + 0.0012i, 0.0458 + 0.0645i, 0.0121 + 0.0619i, 0.0244 + 0.0538i, 0.0180 + 0.0356i, 0.0006 + 0.0064i, 0.0306 + 0.0633i, 0.0501 + 0.0149i, 0.0066 + 0.0343i, 0.0593 + 0.0010i, 0.0747 + 0.0238i, 0.0551 + 0.0675i, 0.0603 + 0.0644i, 0.0183 + 0.0257i, 0.0151 + 0.0679i, 0.0203 + 0.0370i, 0.0550 + 0.0432i, 0.0753 + 0.0475i, 0.0491 + 0.0510i, 0.0421 + 0.0475i, 0.0654 + 0.0528i, 0.0618 + 0.0393i, 0.0515 + 0.0550i, 0.0517 + 0.0397i, 0.0633 + 0.0671i, 0.0748 + 0.0745i, 0.0375 + 0.0634i, 0.0630 + 0.0463i, 0.0494 + 0.0453i, 0.0236 + 0.0100i, 0.0509 + 0.0196i, 0.0276 + 0.0619i, 0.0723 + 0.0515i, 0.0376 + 0.0011i, 0.0070 + 0.0433i, 0.0519 + 0.0350i, 0.0397 + 0.0697i, 0.0171 + 0.0217i, 0.0559 + 0.0050i, 0.0053 + 0.0367i, 0.0743 + 0.0758i, 0.0160 + 0.0711i, 0.0124 + 0.0433i, 0.0492 + 0.0503i, 0.0000 + 0.0596i, 0.0259 + 0.0082i, 0.0212 + 0.0001i, 0.0034 + 0.0418i, 0.0072 + 0.0005i, 0.0316 + 0.0348i, 0.0630 + 0.0151i, 0.0671 + 0.0607i, 0.0017 + 0.0477i, 0.0560 + 0.0012i, 0.0654 + 0.0687i, 0.0562 + 0.0587i, 0.0736 + 0.0699i, 0.0506 + 0.0585i, 0.0572 + 0.0293i, 0.0266 + 0.0255i, 0.0681 + 0.0389i, 0.0268 + 0.0435i, 0.0670 + 0.0514i, 0.0302 + 0.0522i, 0.0195 + 0.0726i, 0.0273 + 0.0594i, 0.0573 + 0.0568i, 0.0502 + 0.0668i, 0.0724 + 0.0764i, 0.0642 + 0.0388i, 0.0362 + 0.0485i, 0.0485 + 0.0611i, 0.0045 + 0.0346i, 0.0418 + 0.0440i, 0.0351 + 0.0132i, 0.0665 + 0.0101i, 0.0659 + 0.0169i, 0.0364 + 0.0081i, 0.0607 + 0.0109i, 0.0506 + 0.0352i, 0.0000 + 0.0607i, 0.0101 + 0.0217i, 0.0381 + 0.0173i, 0.0030 + 0.0701i, 0.0175 + 0.0006i, 0.0253 + 0.0454i, 0.0693 + 0.0341i, 0.0242 + 0.0504i, 0.0194 + 0.0242i, 0.0334 + 0.0178i, 0.0649 + 0.0321i, 0.0142 + 0.0230i, 0.0392 + 0.0518i, 0.0349 + 0.0321i, 0.0251 + 0.0264i, 0.0293 + 0.0434i, 0.0683 + 0.0092i, 0.0587 + 0.0130i, 0.0681 + 0.0215i, 0.0353 + 0.0429i, 0.0616 + 0.0374i, 0.0103 + 0.0734i, 0.0050 + 0.0179i, 0.0289 + 0.0369i, 0.0288 + 0.0406i, 0.0373 + 0.0611i, 0.0747 + 0.0149i, 0.0264 + 0.0701i, 0.0195 + 0.0711i, 0.0451 + 0.0010i, 0.0404 + 0.0592i, 0.0126 + 0.0730i, 0.0375 + 0.0627i, 0.0382 + 0.0131i, 0.0650 + 0.0510i, 0.0470 + 0.0715i, 0.0470 + 0.0661i, 0.0436 + 0.0458i, 0.0472 + 0.0474i, 0.0079 + 0.0003i, 0.0122 + 0.0757i, 0.0319 + 0.0693i, 0.0432 + 0.0534i, 0.0207 + 0.0339i, 0.0604 + 0.0540i, 0.0299 + 0.0470i, 0.0024 + 0.0231i, 0.0451 + 0.0660i, 0.0431 + 0.0086i, 0.0155 + 0.0225i, 0.0067 + 0.0075i, 0.0719 + 0.0306i, 0.0200 + 0.0257i, 0.0157 + 0.0728i, 0.0038 + 0.0646i

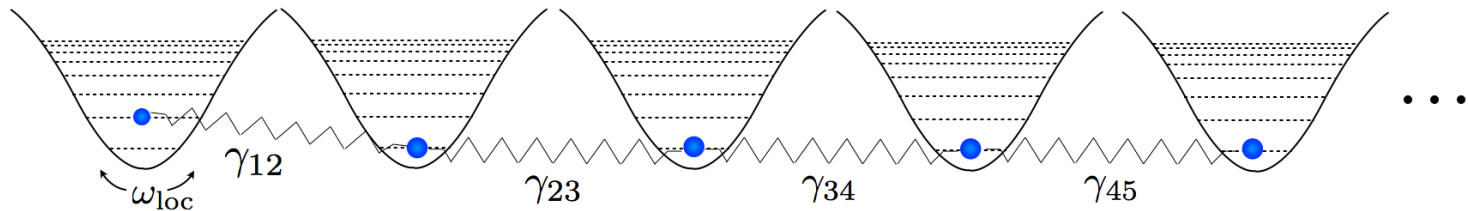


# Many-body physics

Oscillator chain of ions

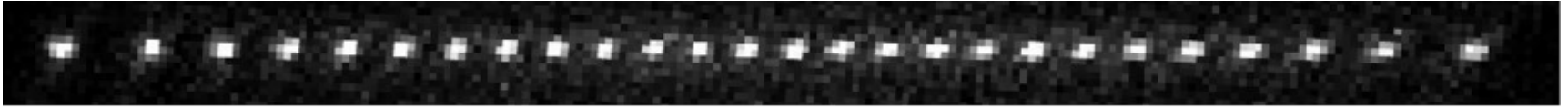


Oscillator chain of ions



# Many-body physics

## Oscillator chain of ions



$$\begin{aligned}
 H = & \hbar \sum \Delta_{\text{ion}} \sigma_i^z + \hbar \sum \omega_{r,i} a_i^\dagger a_i + \sum t_{ij} (a_i^\dagger a_j + a_i a_j^\dagger) \\
 & + \sum \lambda_i (\sigma_i^- a_i e^{-i\delta t} + \sigma_i^+ a_i^\dagger e^{i\delta t}) + \sum \mu_i (\sigma_i^- e^{-i\varphi} + \sigma_i^+ e^{i\varphi})
 \end{aligned}$$

Choose detuning such that the motion is not excited:

$$\begin{aligned}
 H = & \hbar \sum_{ij} \lambda_i \lambda_j \sum_m \underbrace{\frac{b_{i,m} b_{j,m}}{\nu^2 - \omega_m^2}}_{\text{detuning from normal mode } m} \sigma_i^x \sigma_j^x + \sum \mu_i (\sigma_i^- e^{-i\varphi} + \sigma_i^+ e^{i\varphi})
 \end{aligned}$$

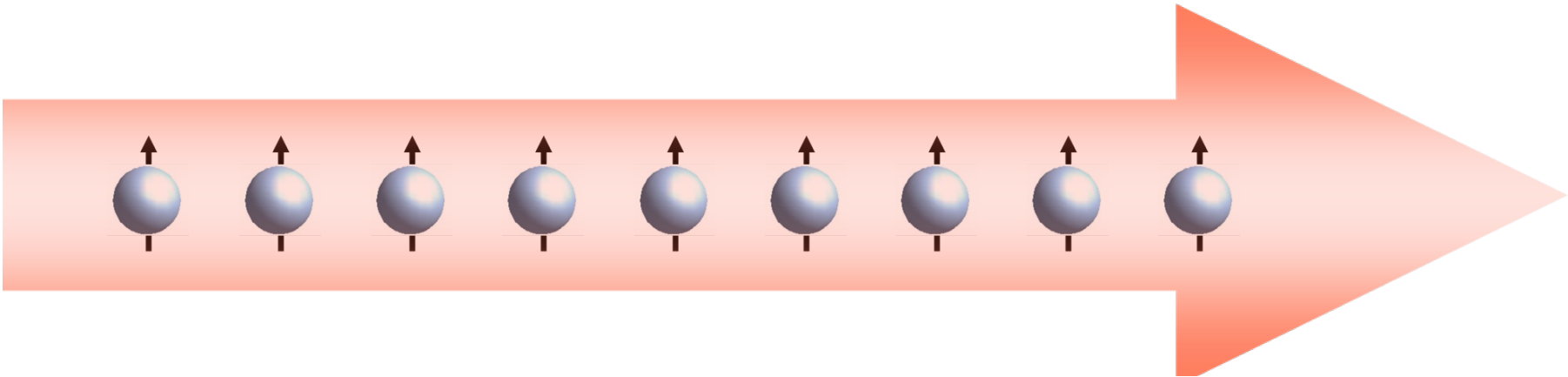
projection of the local mode  $j$   
on normal mode  $m$

$$= \hbar \sum_{ij} J_{ij} \sigma_i^x \sigma_j^x + \sum_i B_i^x \sigma_i^x + B_i^y \sigma_i^y$$

Transverse Ising model

# Adiabatic quantum simulation

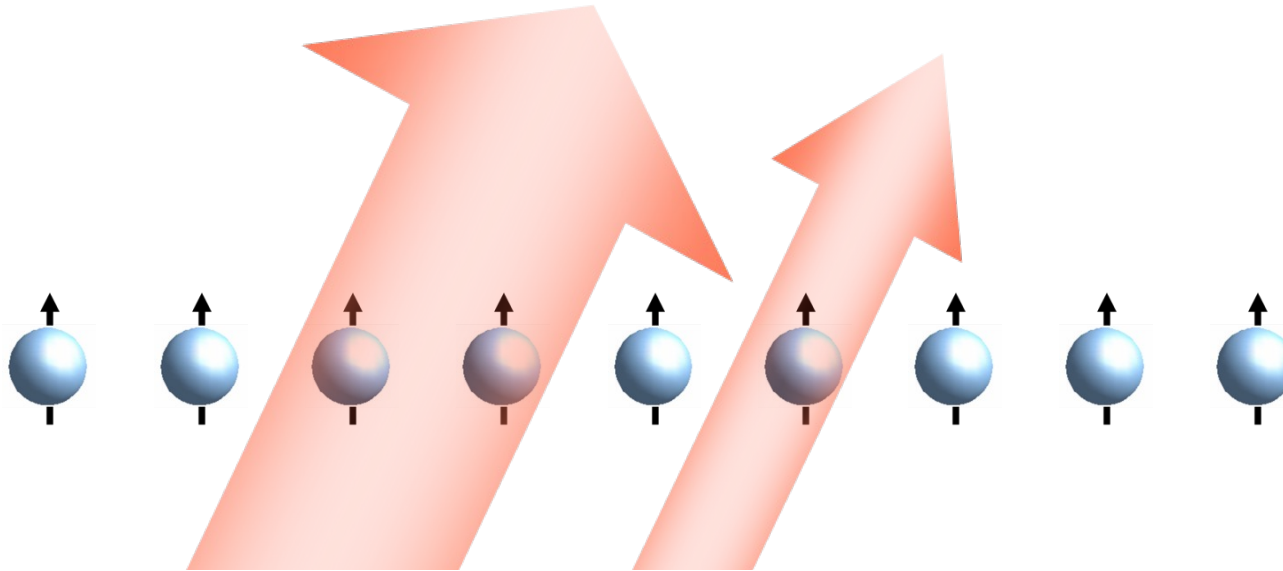
Magnetic field controlled by carrier excitations



$$\sum_i B_i^x \sigma_i^x + B_i^y \sigma_i^y$$

# Adiabatic quantum simulation

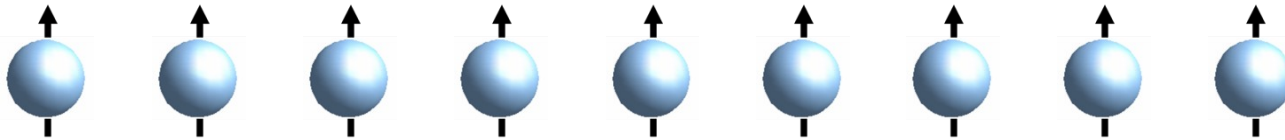
Adjustable, long range spin-spin coupling via quantum gates:



$$\hbar \sum_{ij} J_{ij} \sigma_i^x \sigma_j^x + \sum_i B_i^x \sigma_i^x + B_i^y \sigma_i^y$$

# Adiabatic quantum simulation

Adjustable, long range spin-spin coupling via quantum gates:



Procedure:

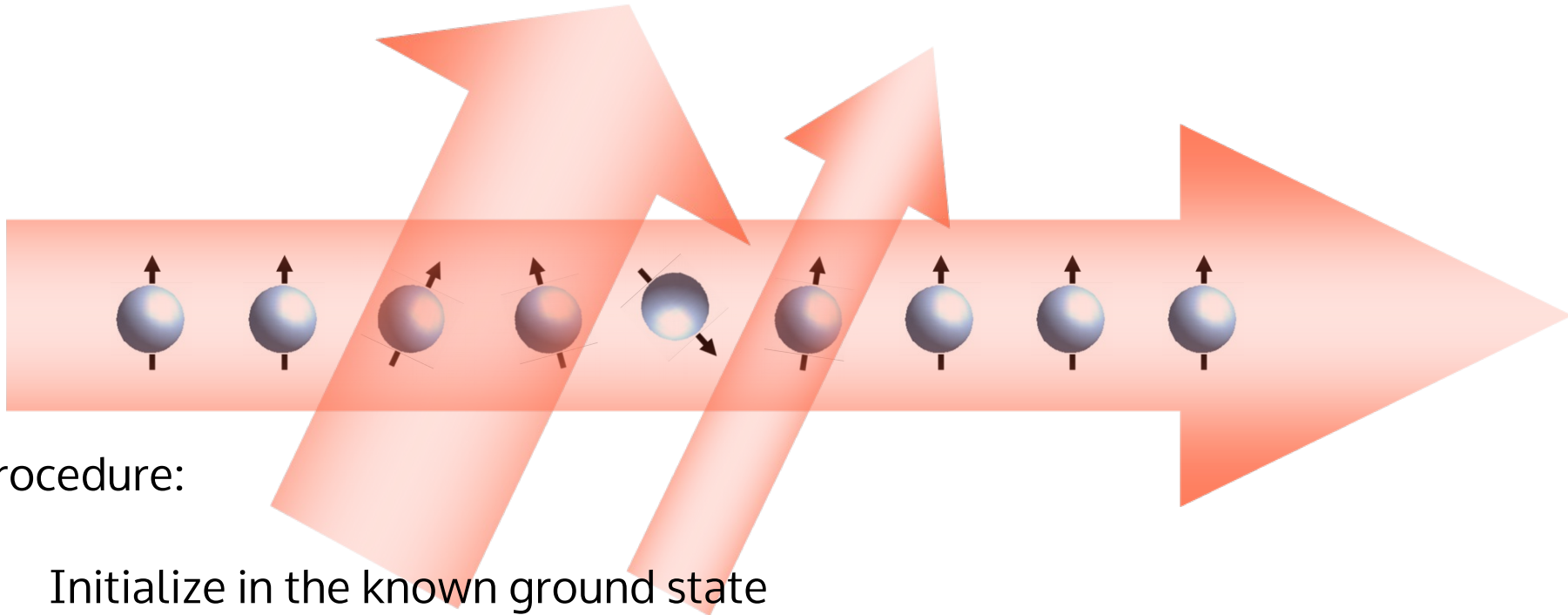
- Initialize in the known ground state

- Switch on the Hamiltonian adiabatically

- Measure desired expectation value



# Adiabatic quantum simulation



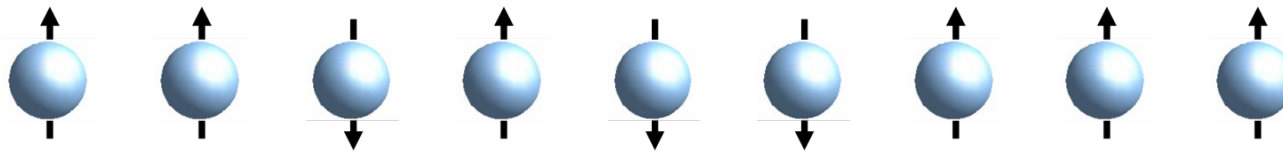
Procedure:

Initialize in the known ground state

Switch on the Hamiltonian adiabatically

Measure desired expectation value

# Adiabatic quantum simulation



Procedure:

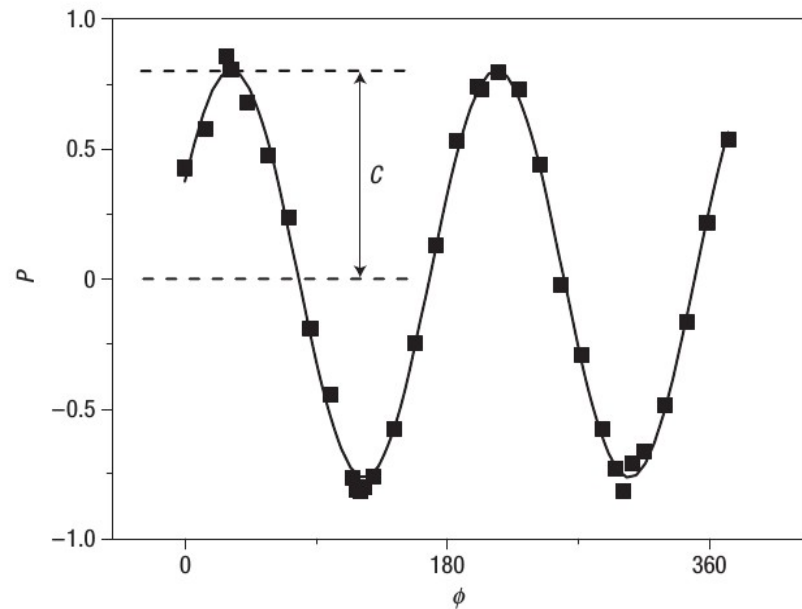
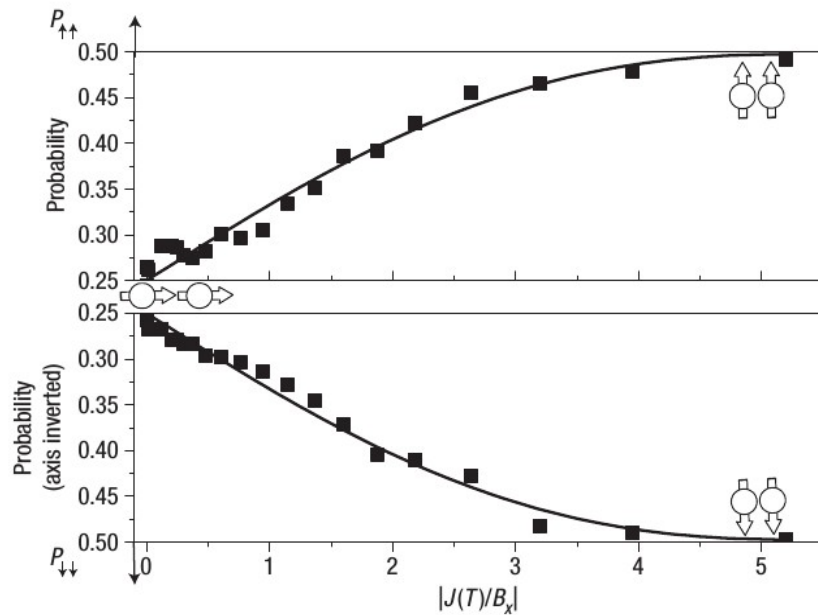
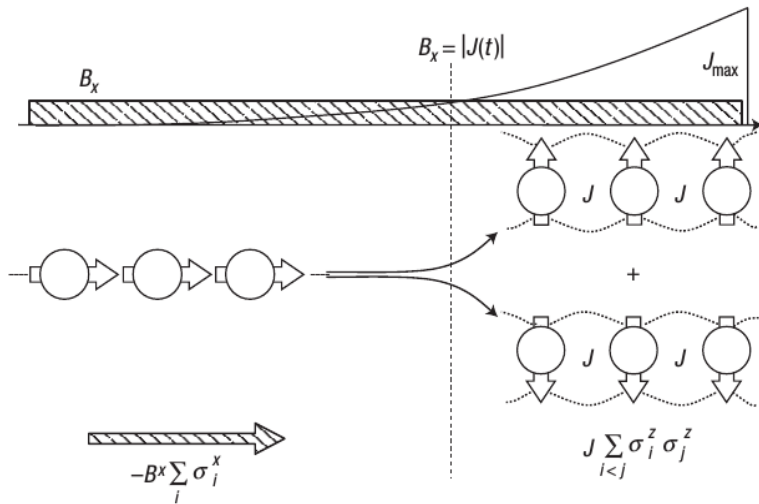
Initialize in the known ground state

Switch on the Hamiltonian adiabatically

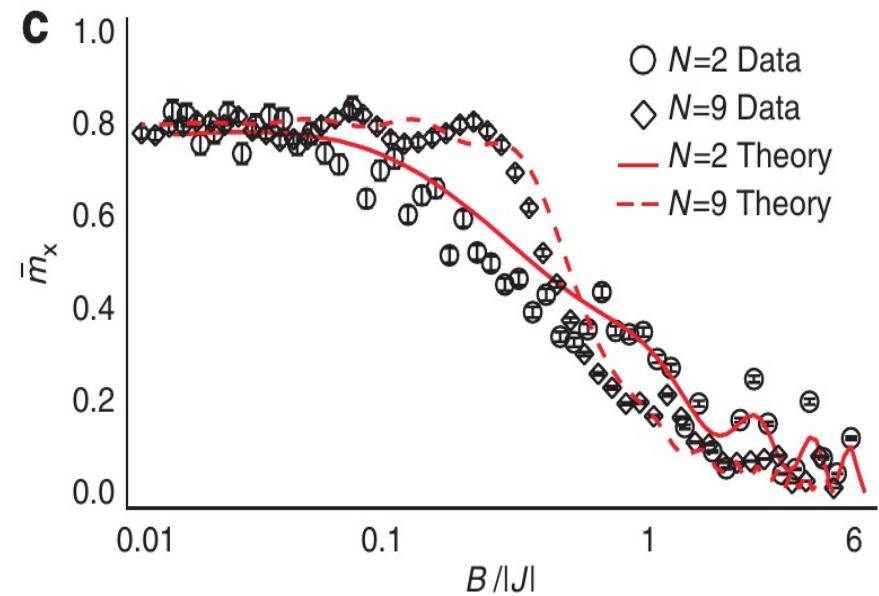
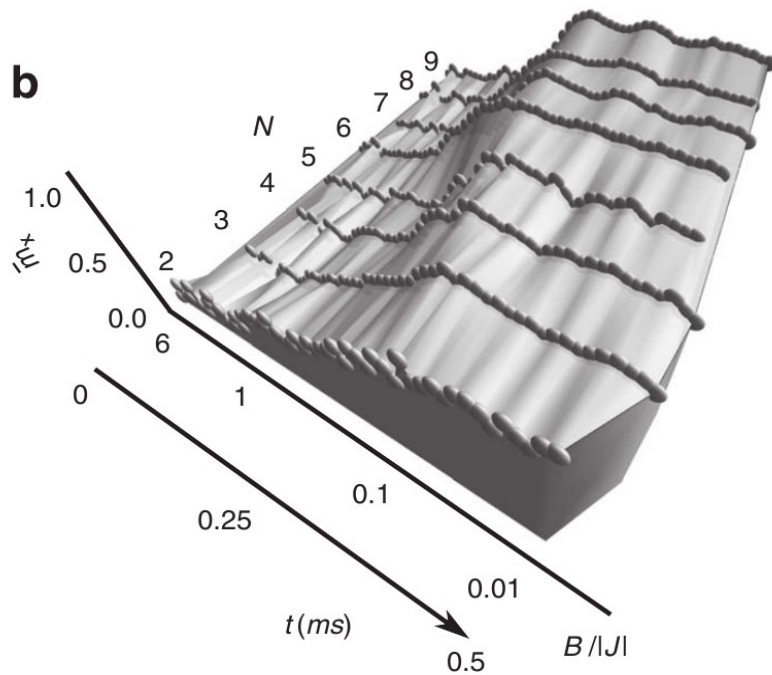
Measure desired expectation value

# Simulation of a quantum magnet

A. Friedenauer, et al.,  
Nature Physics 4, 757 (2008)

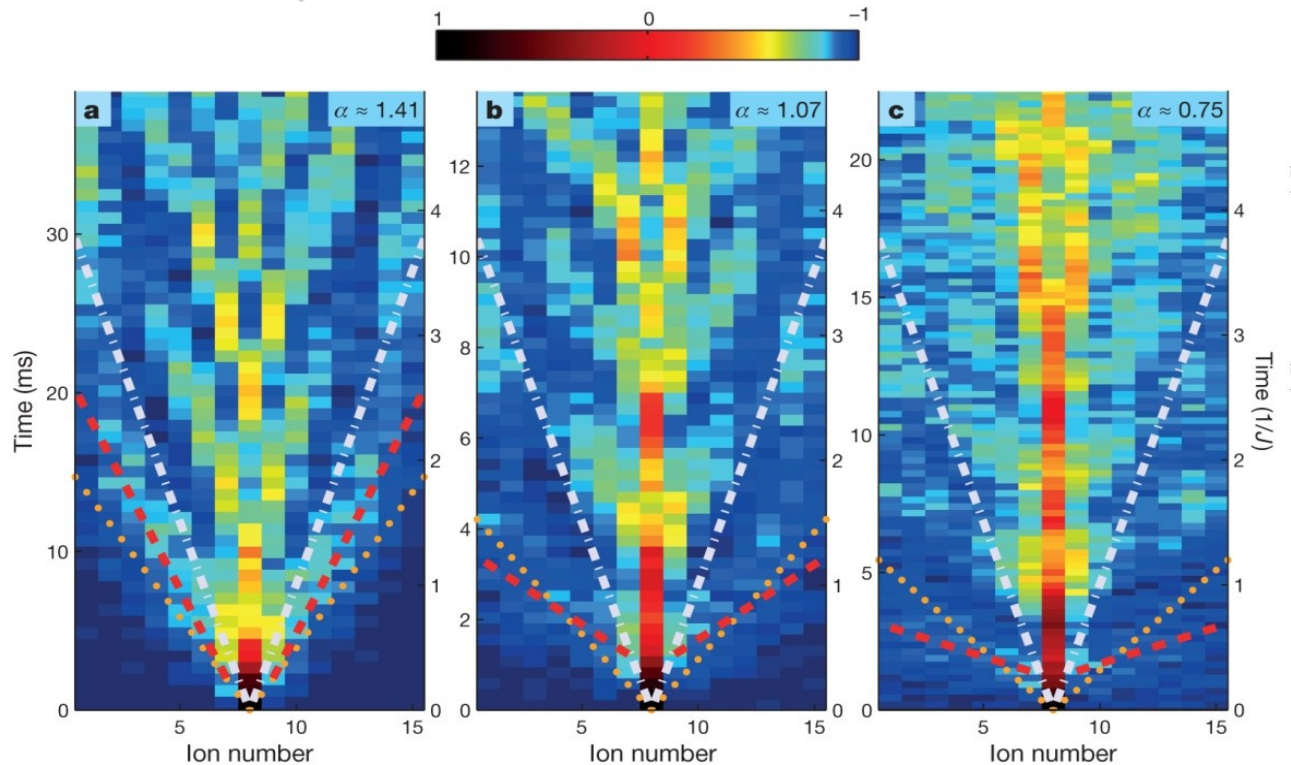


# Onset of a quantum phase transition



# Quasiparticle engineering

$$H_{\text{Ising}} = \hbar \sum_{i < j} J_{ij} \sigma_i^x \sigma_j^x + \hbar B \sum_i \sigma_i^z$$



- Watching quasi particles (spin excitations) propagate
- Testing Lieb-Robinson bounds

# Plan

## Lecture #1: Introduction

- Paul traps
- Laser ion-interaction

## Lecture #2: Quantum computing

- Quantum gates
- Quantum state tomography

## Lecture #3: Decoherence

- Qubit decoherence
- Scaling

## Lecture #4: Scaling and quantum simulation

- Scaling and anomalous heating
- Quantum simulation

## Lecture #5: Applications

- Decoherence-free subspaces
- Michelson-Morley experiment