Quantum computing with trapped ions

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- Introduction to ion trap quantum computing
- Single ion addressing approach
- Coherent operations with global interactions
- Conclusions

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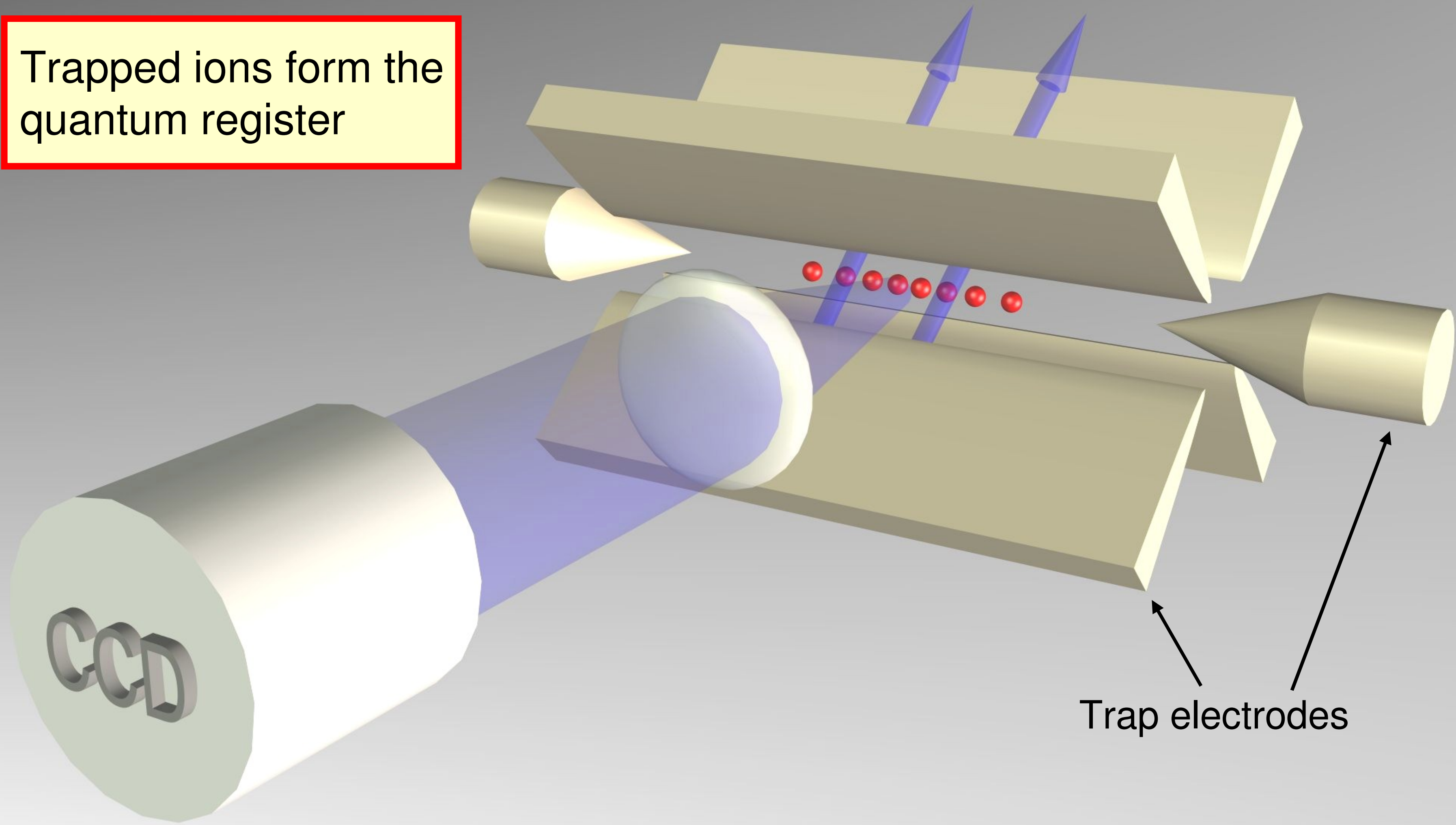
Baltimore, June 5th 2009

Ion trap quantum computing

Trapped ions form the quantum register



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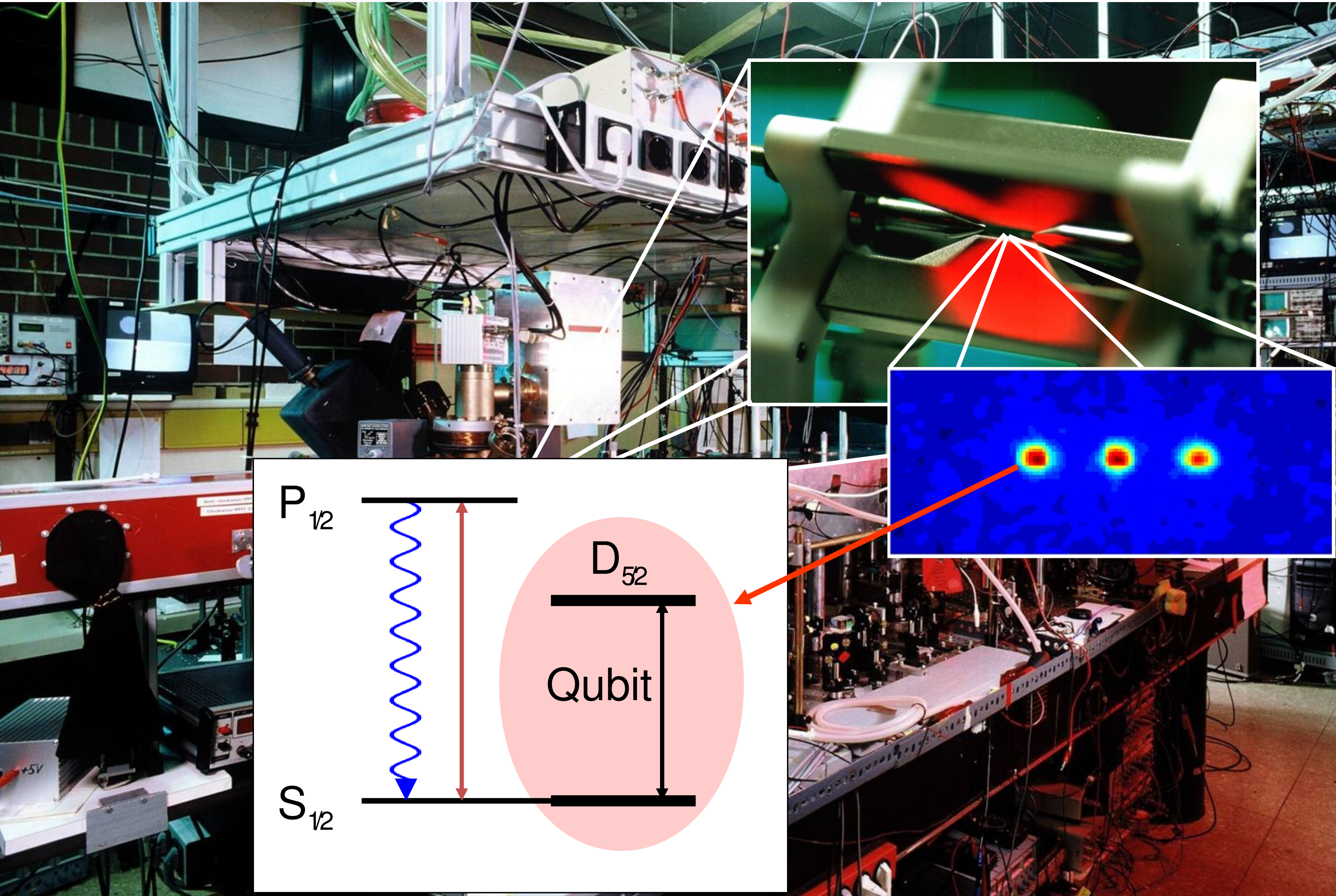
The hardware



Requirements for quantum computing

Classical computer

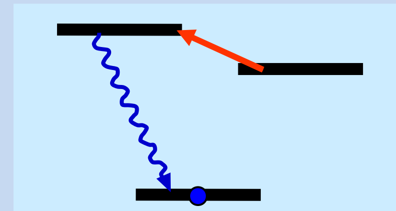
- Initialization
- 1-bit operations (NOT)
- 2-bit gates (e.g. NAND)

Computational space:

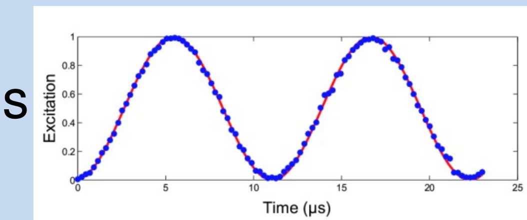
00
01
10
11

- Read out
  → result

Quantum computer



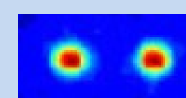
- Initialization
- 1-qubit rotations
  → superpositions



- 2-qubit gates (CNOT gate)
  → entanglement

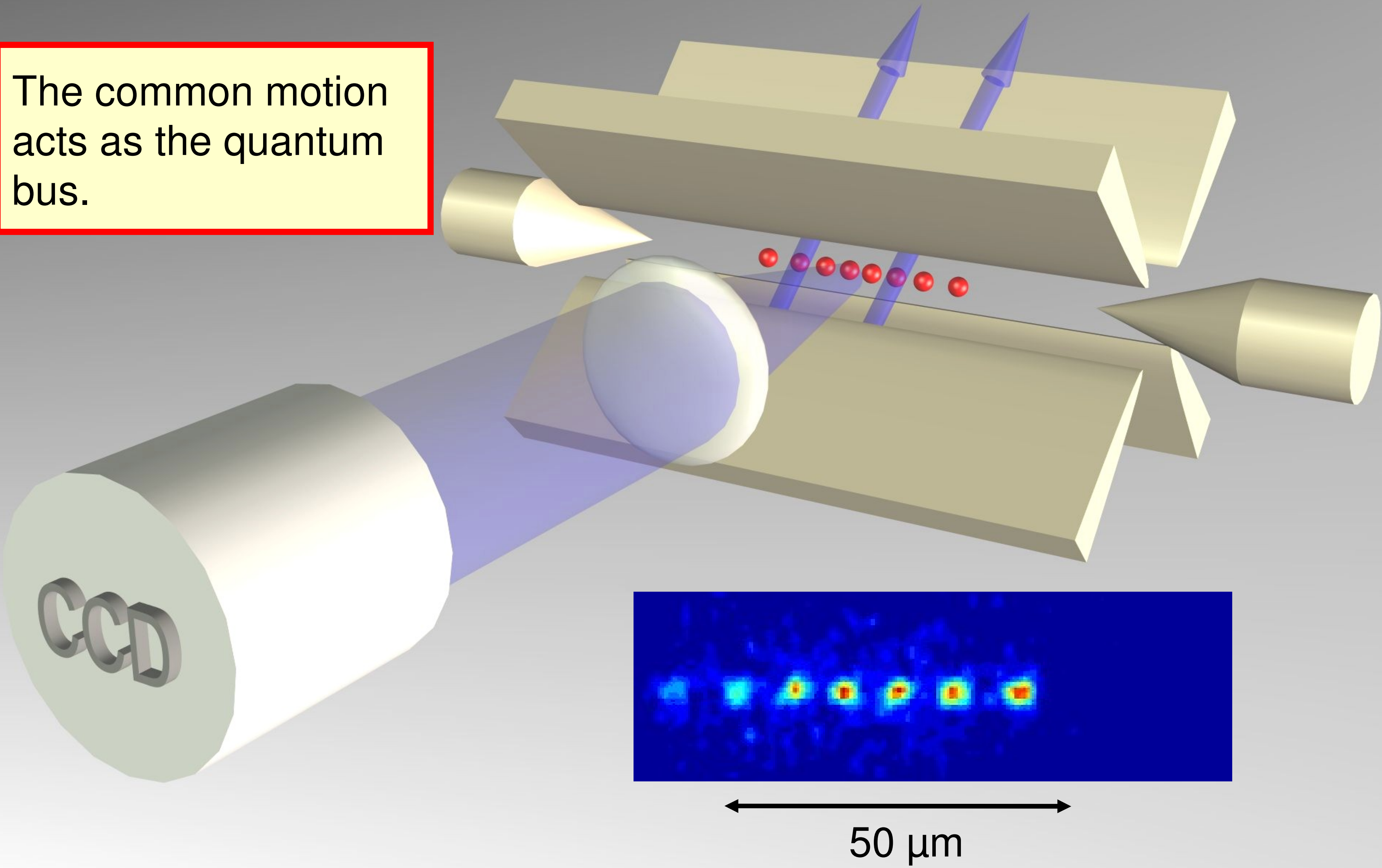
Computational space:

- Read out of qubits
  → gain of classical information

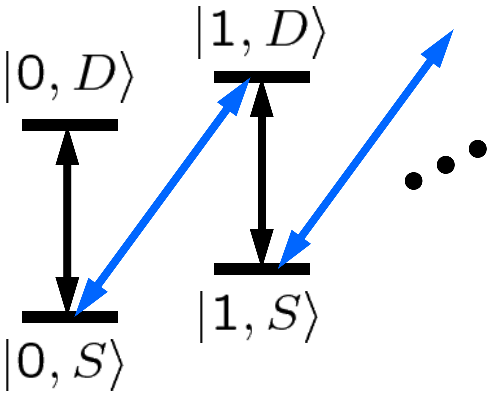


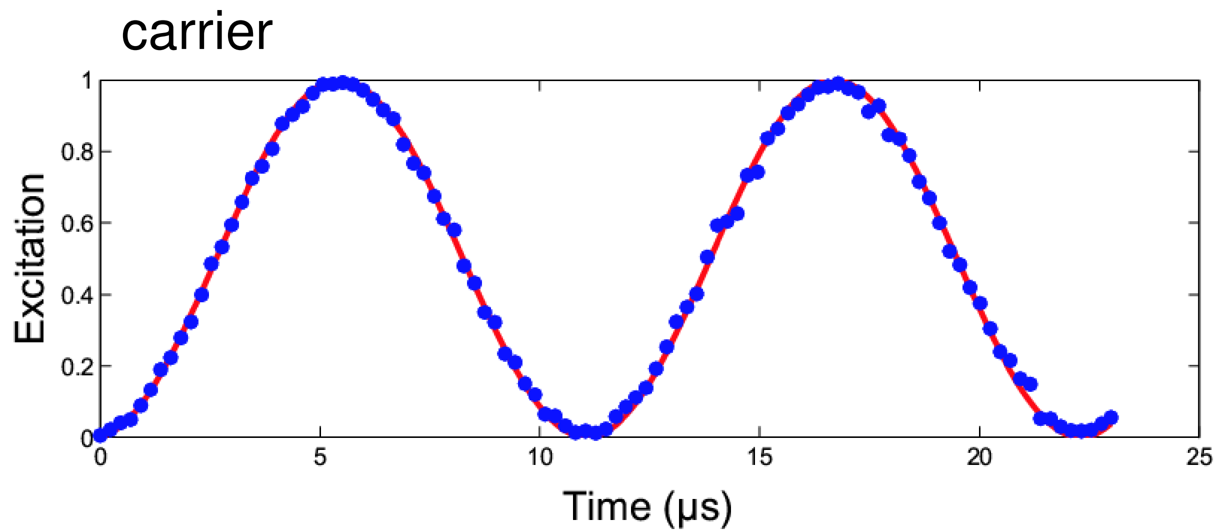
Having the qubits interact

The common motion acts as the quantum bus.



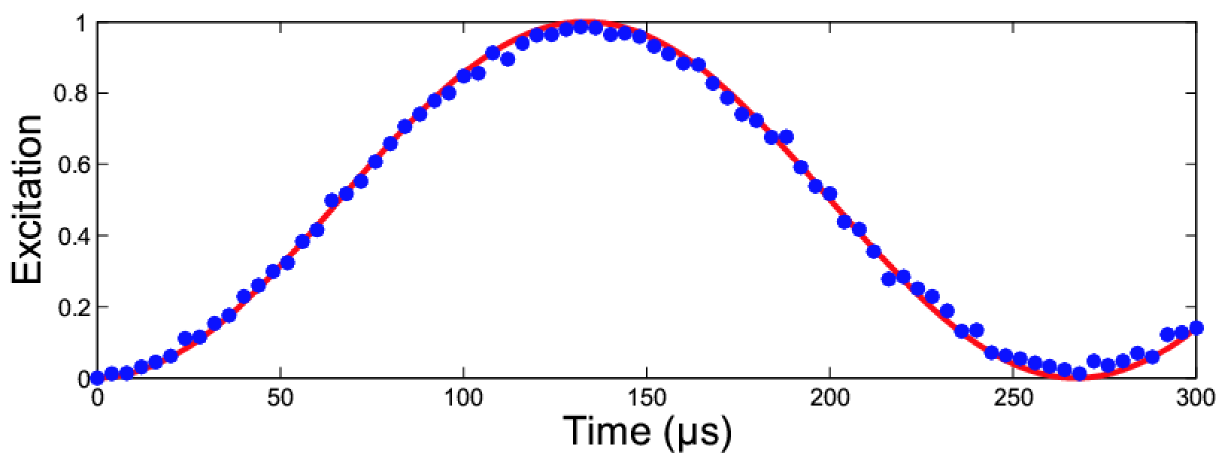
Coherent manipulation





carrier and sideband
Rabi oscillations
with Rabi frequencies

$$\Omega, \quad \eta\Omega$$



$\eta = kx_0$ Lamb-Dicke parameter

- Introduction to ion trap quantum information
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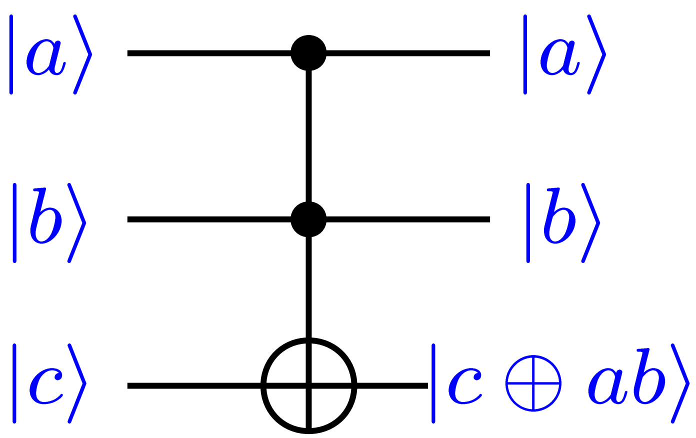
Toffoli gate: controlled-controlled NOT

Toffoli gate (Tommaso Toffoli, 1980):

…… is a universal reversible logic gate, i.e. any reversible circuit can be constructed from Toffoli gates.

also known as the controlled-controlled-NOT or CCNOT-gate operation

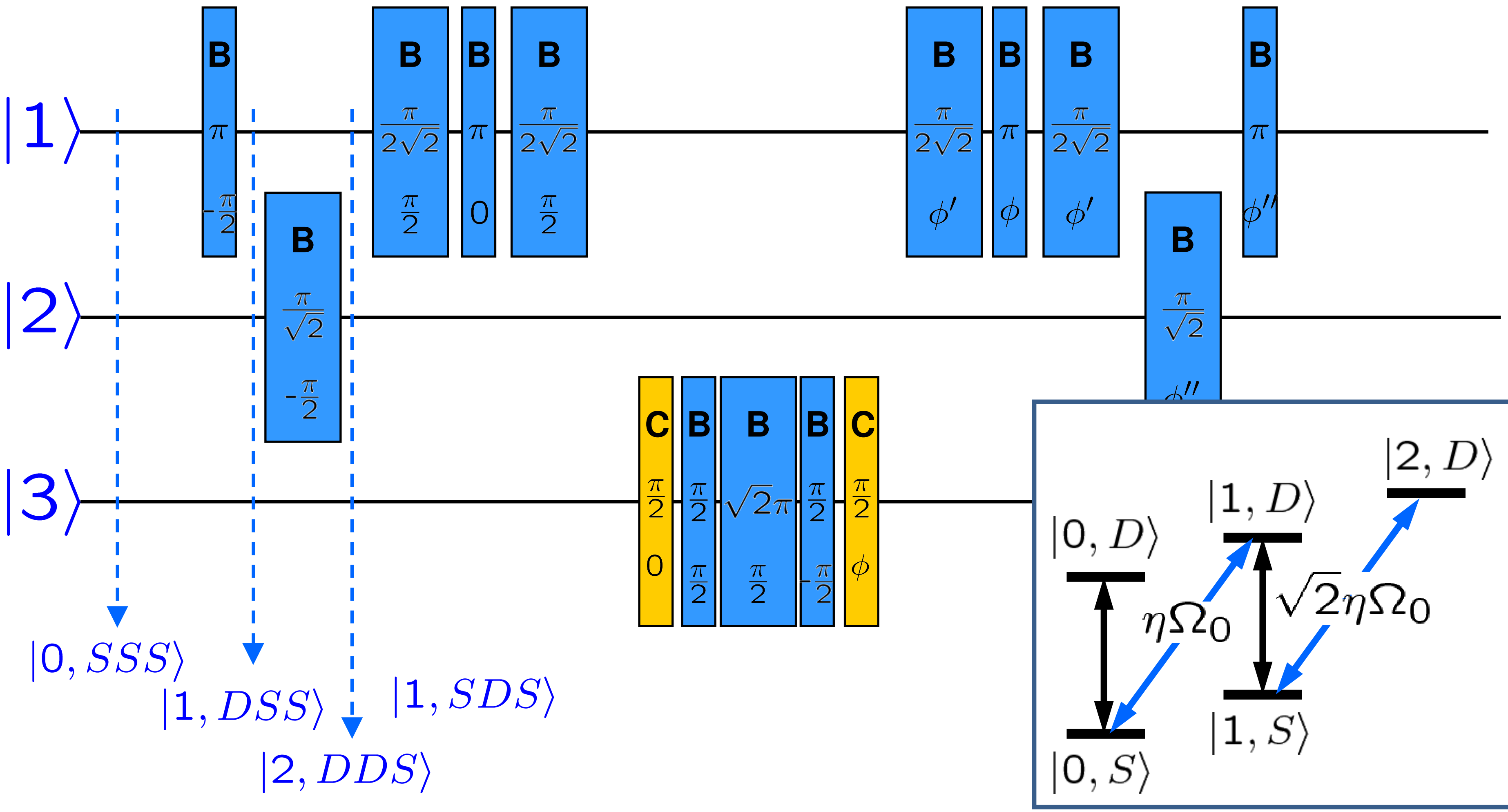


$$= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & i \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -i & 0 \end{pmatrix}$$

useful, e.g. for error correction

use 2-phonon excitation

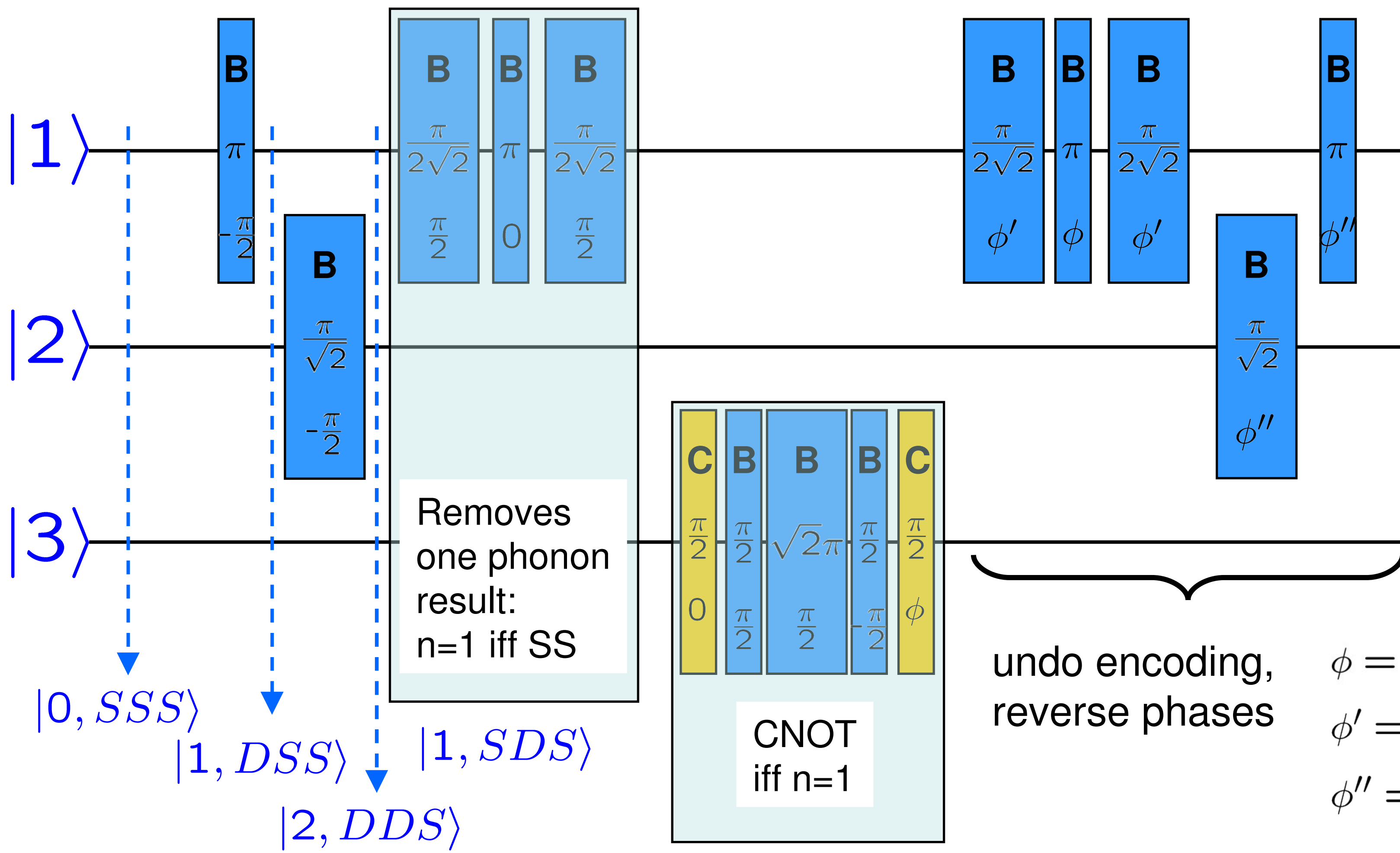
Th. Monz, K. Kim et al., Innsbruck 2008



Toffoli gate: pulse sequence

use 2-phonon excitation

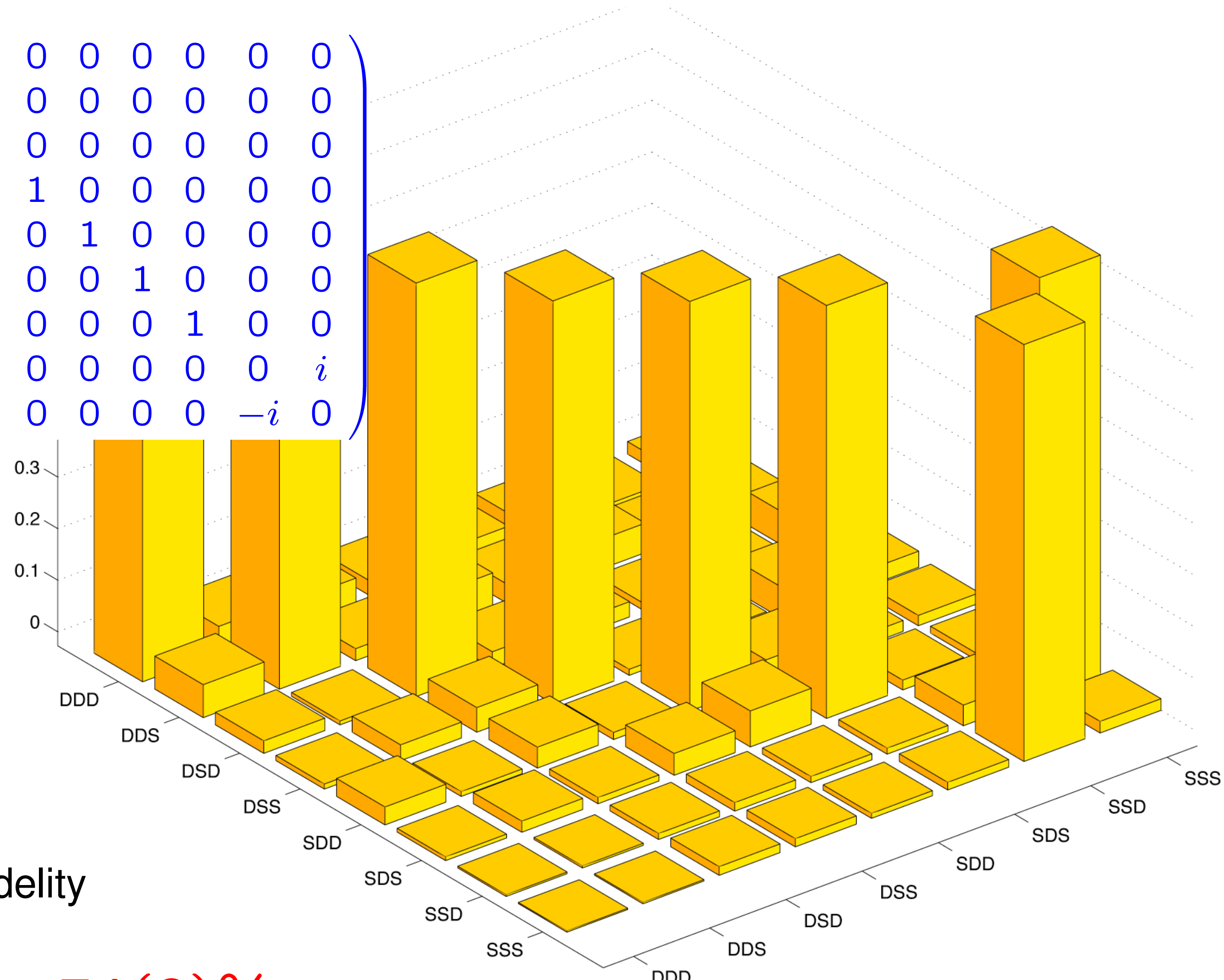
Th. Monz, K. Kim et al., Innsbruck 2008



$$\phi = (\tfrac{1}{\sqrt{2}} - 1)\pi$$
$$\phi' = (-\tfrac{1}{2} + \tfrac{1}{\sqrt{2}})\pi$$
$$\phi'' = (\tfrac{1}{2} + \tfrac{1}{\sqrt{2}})\pi$$

Toffoli gate: experimental truth table

$$
\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & i \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -i & 0
\end{pmatrix}
$$

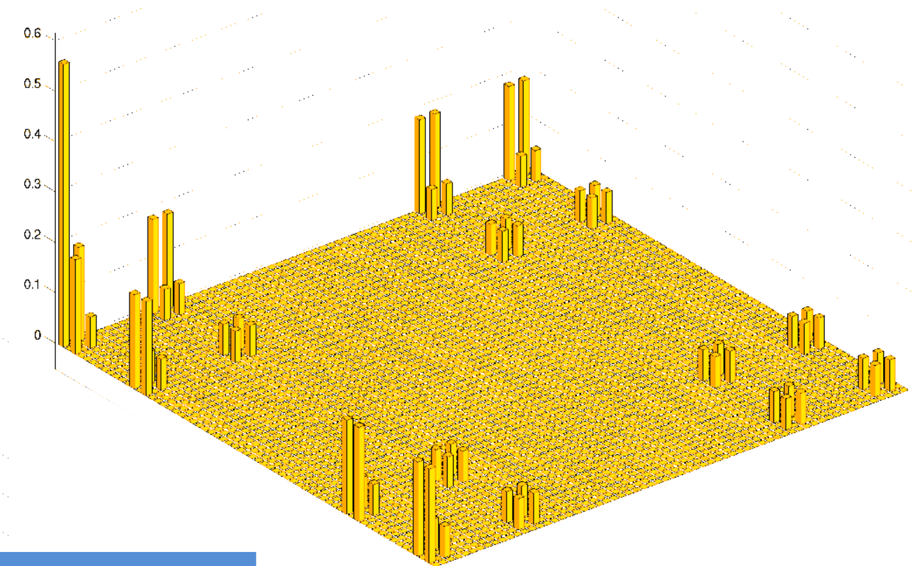


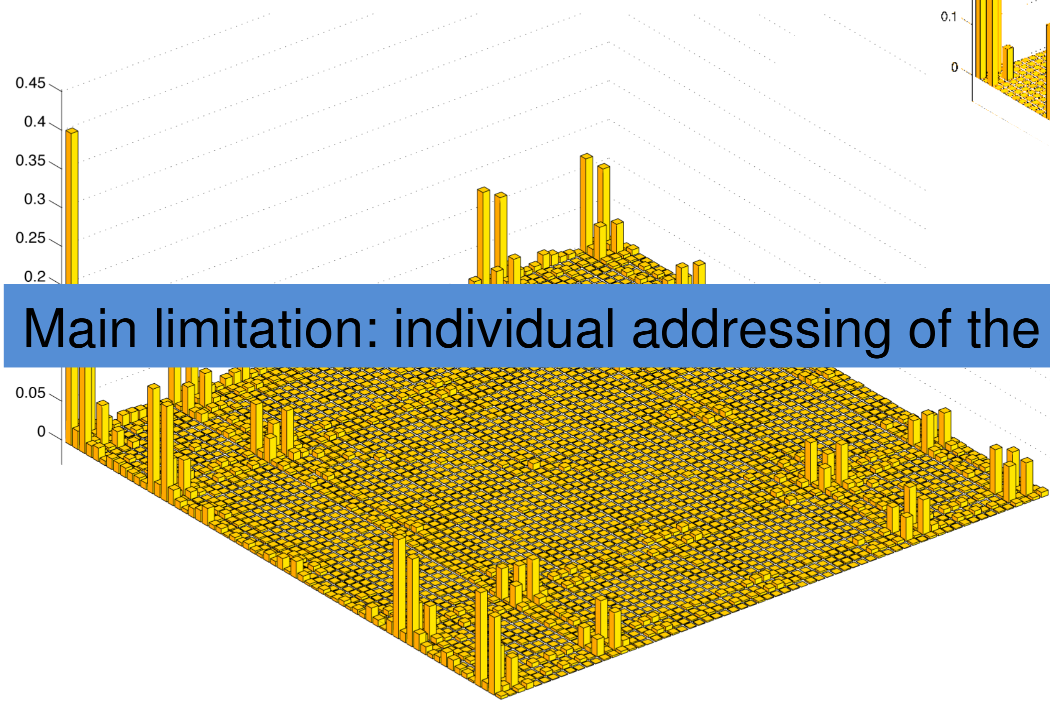
State Fidelity

$$F_{\mathrm{Toff}} = 74(3)\%$$

Toffoli gate: process tomography

$|\chi|$ - matrix for ideal TOFFOLI gate operation





Main limitation: individual addressing of the ions

$$F_{\mathrm{Toff}} = 71(3)\%$$

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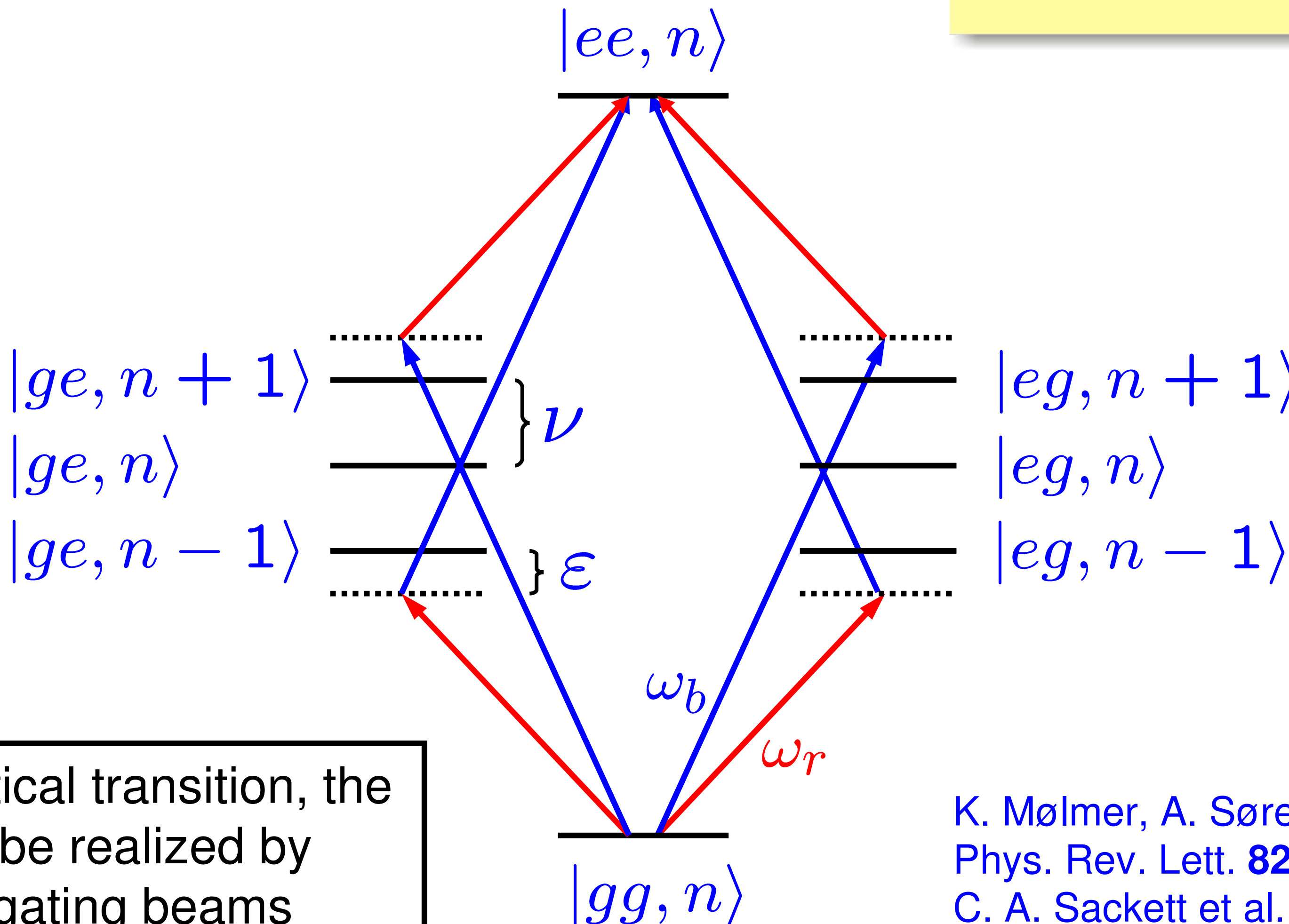
Mølmer - Sørensen gate operation

$$|gg\rangle \rightarrow |ee\rangle, \;\; |ge\rangle \rightarrow |eg\rangle$$

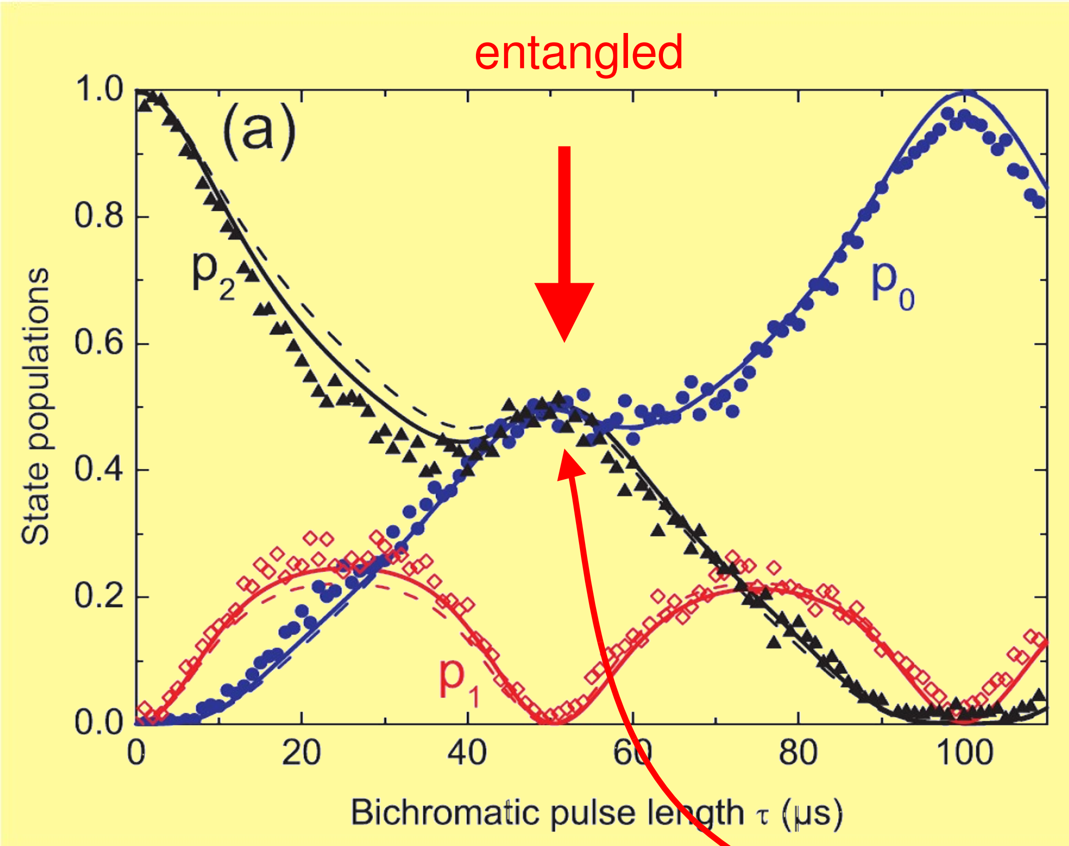
$$\omega_b = \omega_0 + (\nu - \varepsilon)$$
$$\omega_r = \omega_0 - (\nu - \varepsilon)$$



on an optical transition, the gate can be realized by co-propagating beams

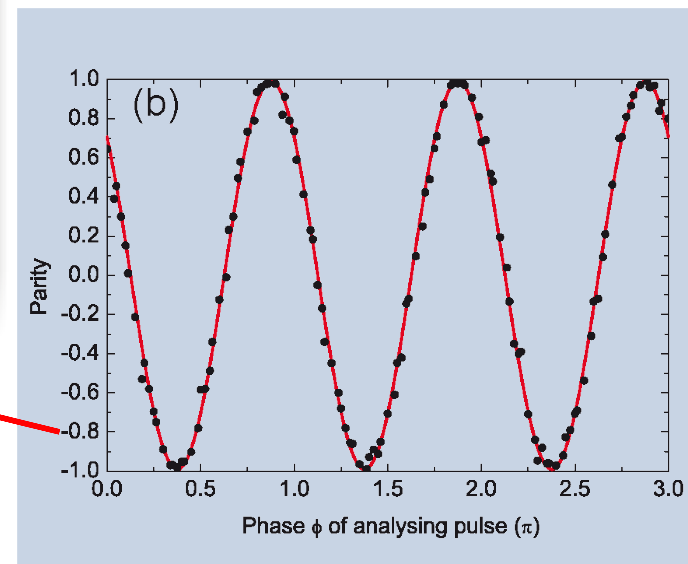
K. Mølmer, A. Sørensen, Phys. Rev. Lett. **82**, 1971 (1999)
C. A. Sackett et al., Nature **404**, 256 (2000)

Bell states with Mølmer-Sørensen gate



J. Benhelm, G. Kirchmair, C. Roos, Nature Physics **4** 463 (2008).

Theory: C. Roos, New Journal of Physics **10,** 013002 (2008).

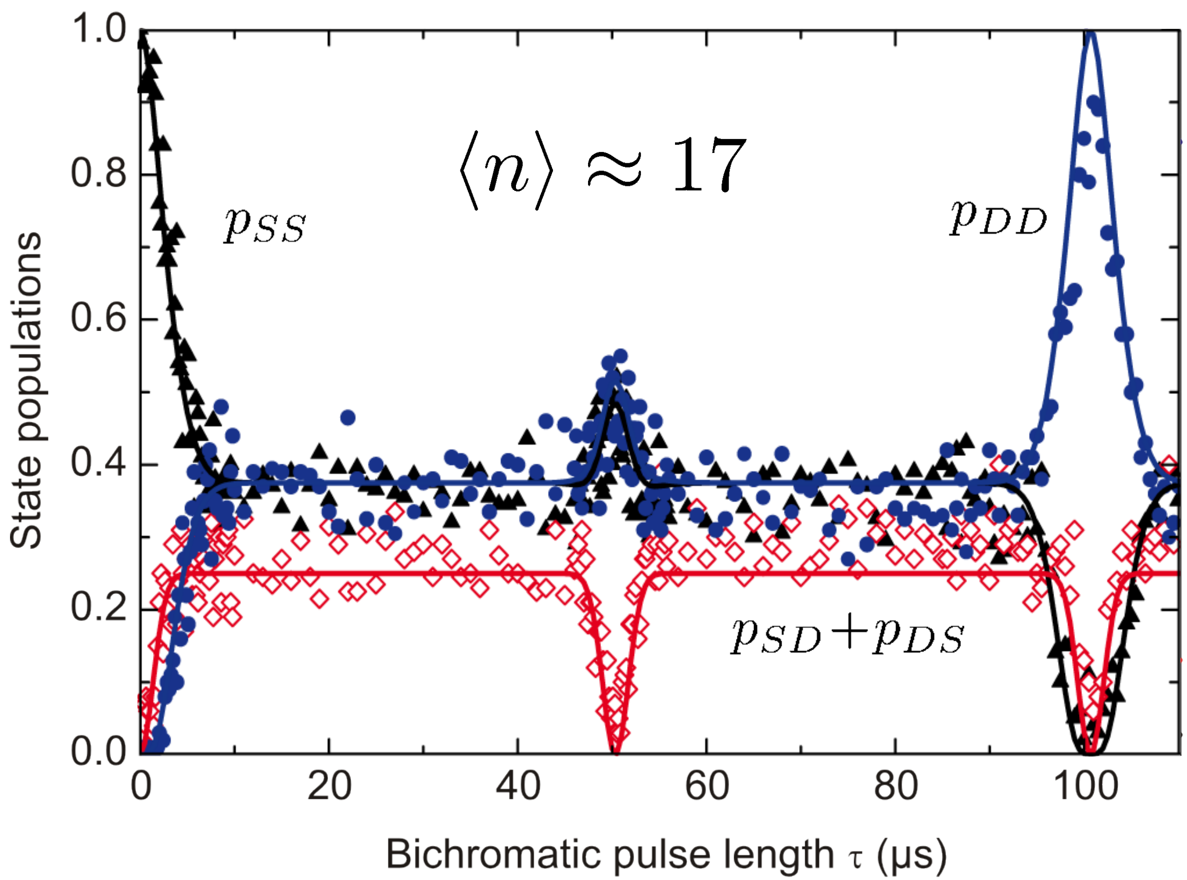


gate duration $51\mu s$

average fidelity

$$\boxed{F_{\mathrm{MS}} = 99.3(0.2)\%}$$

Gate operation after Doppler cooling



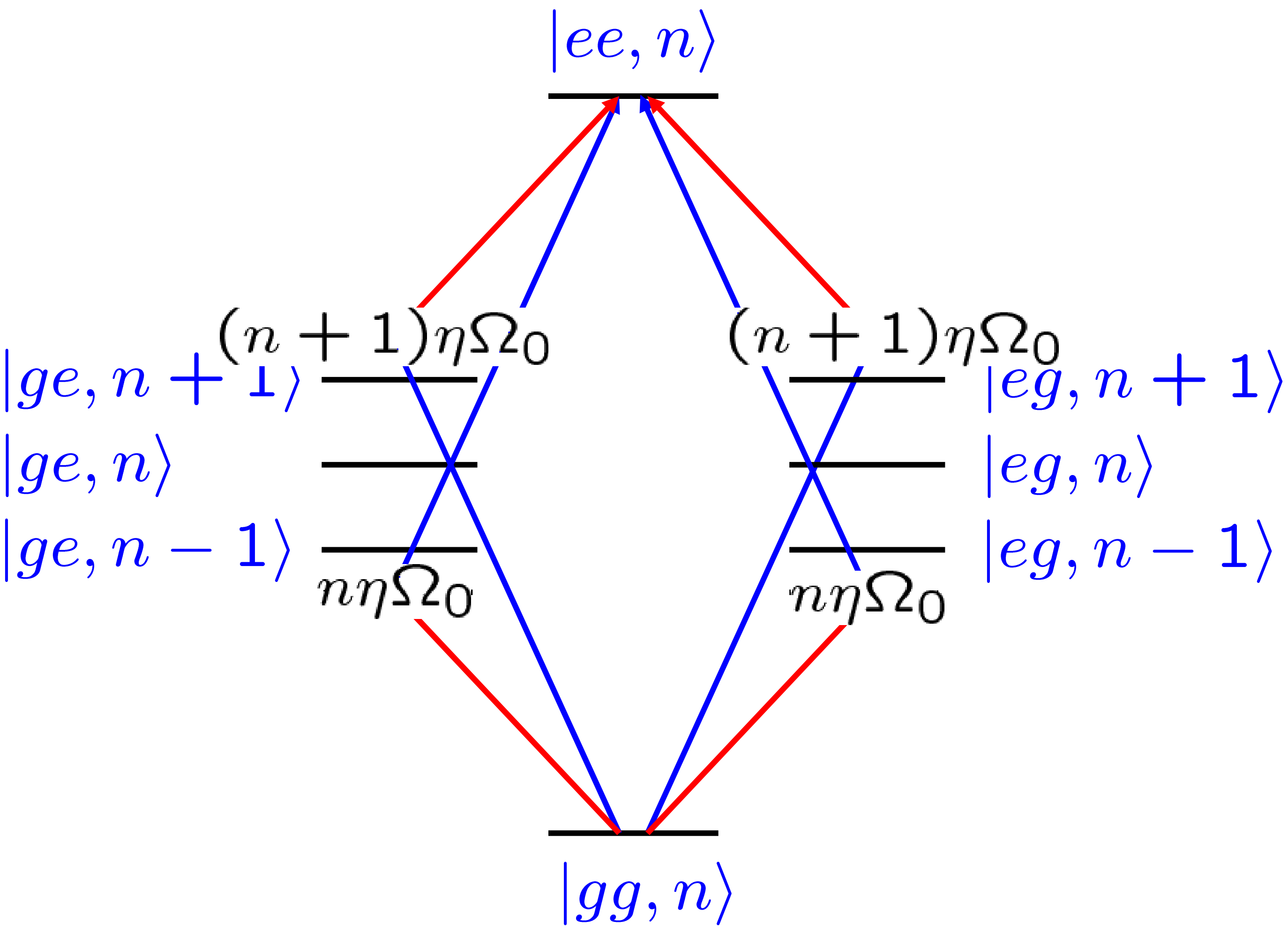
Bell state:
$$\Psi = |SS\rangle + i|DD\rangle$$

Fidelity :

F = 96.1(5) %

Gate operation ≈ independent of motional state !

Mølmer - Sørensen gate operation



Arbitrary quantum gates

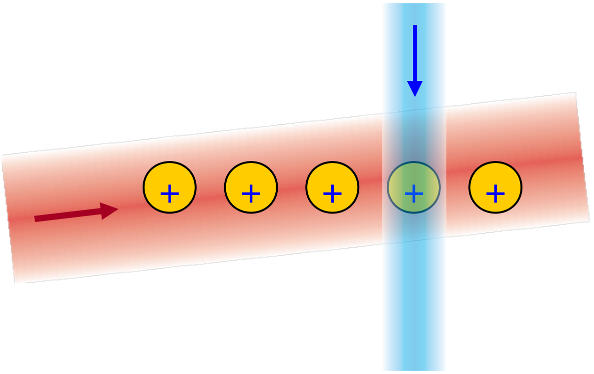
N ions

Basic set of operations:

$$H_i \in \{ {S_y}^2, S_y, \underbrace{\sigma_z^{(1)}, \sigma_z^{(2)} \ldots \sigma_z^{(N)}}_{\text{individual light shift gates}} \}$$

Mølmer-Sørensen gate

global single atom spin flips



- favorable ion addressing by light shifts (~ Ω^2)
- no interferometric stability between beams required

$$H_i, H_j \rightarrow [H_i, H_j] \text{ generate Lie algebra } \mathcal{L} \text{ with dim } \mathcal{L} = 4^N$$

Arbitrary unitary operations can be achieved !

...but how ?

Arbitrary quantum gates

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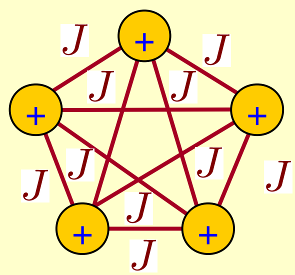
IQI

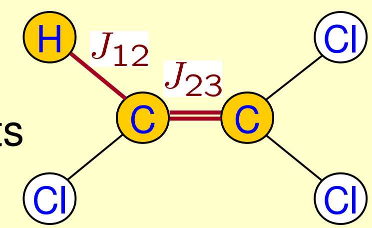
Similarity with NMR systems:

Ion qubits



all ions coupled with equal strength

NMR qubits

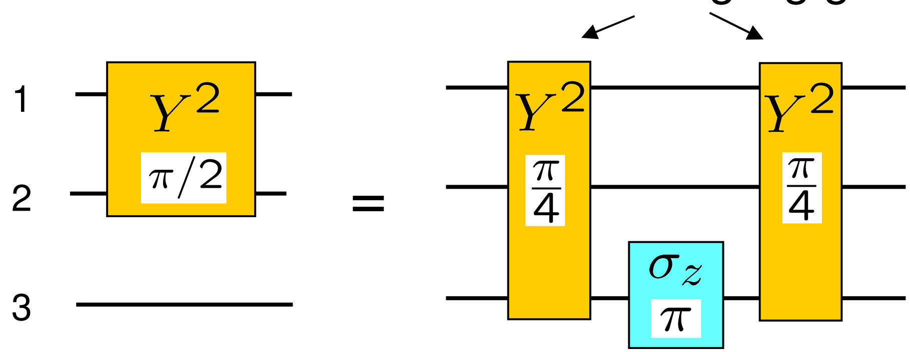


spin-spin couplings (unequal strength)

Refocussing of unwanted interactions:

Entangling gate
between ion 1 and 2

half-entangling gates



light shift pulse : phase shift by π

Arbitrary quantum gates

Quantum optimal control:

$$H(t) = \sum_{k=1}^{n} \alpha_k(t) H_k$$

Find $\{\alpha_k(t), k = 1 \ldots n\}$ such that $U_{gate} \overset{!}{=} \mathcal{T} \int_0^{\mathcal{T}} dt\, e^{-\frac{i}{\hbar} \sum_k \alpha_k(t) H_k}$

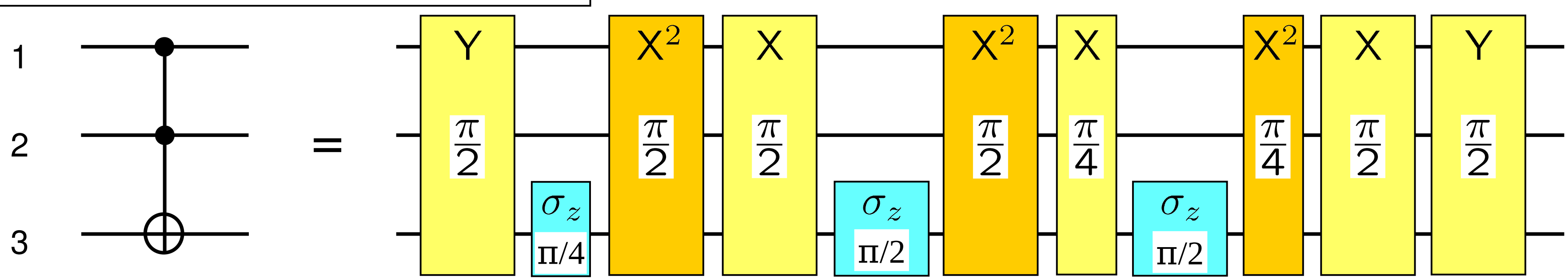
Gradient ascent algorithm: N. Khaneja *et al.*, J. Magn. Res. **172**, 296 (2005).

Modification of search algorithm: V. Nebendahl *et al.*, PRA (2009)

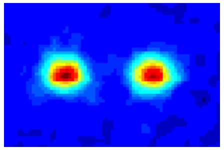
No simultaneous application of several Hamiltonians !

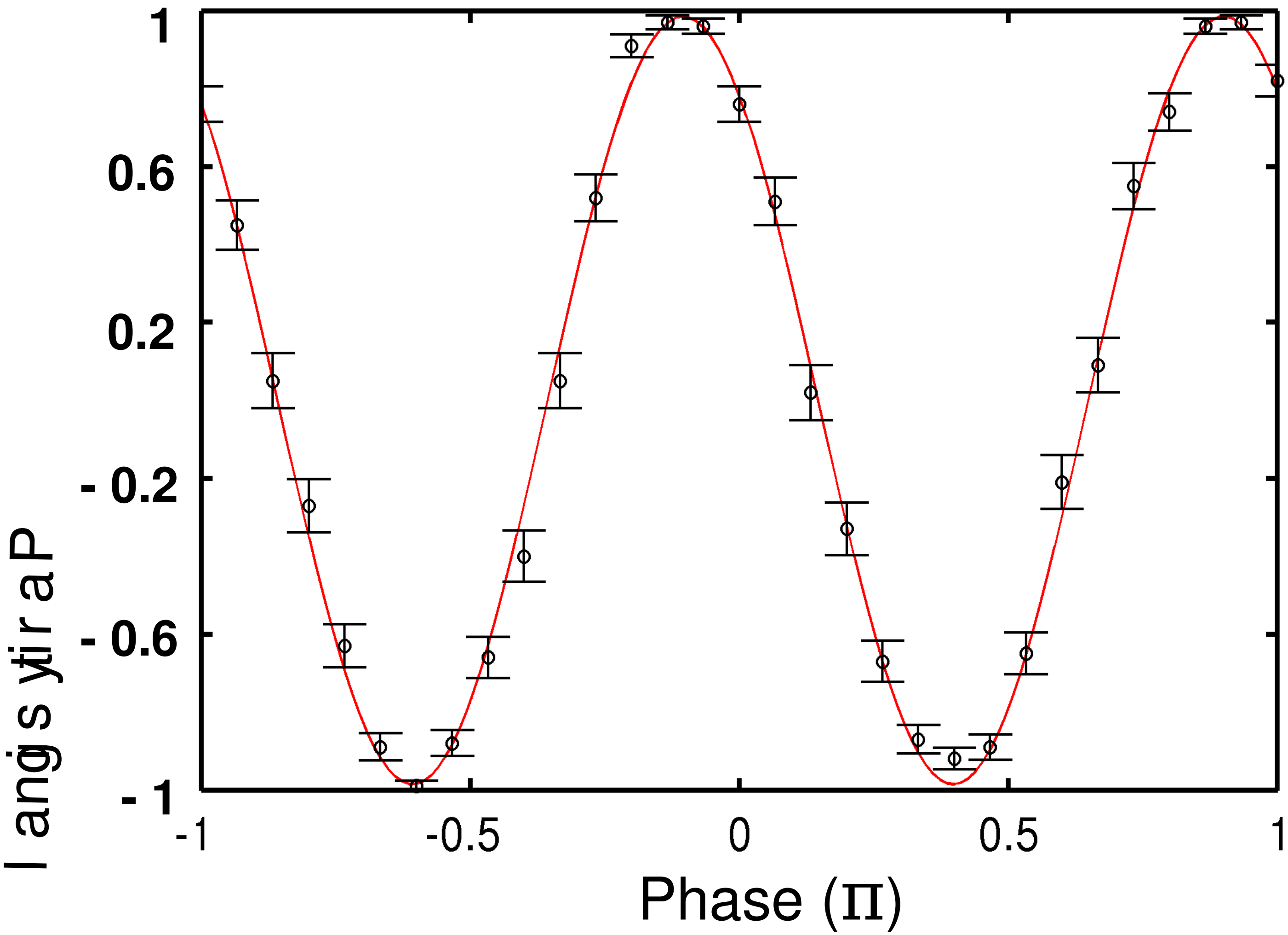
→ Sequence of pulses with variable length

Example: quantum Toffoli gate

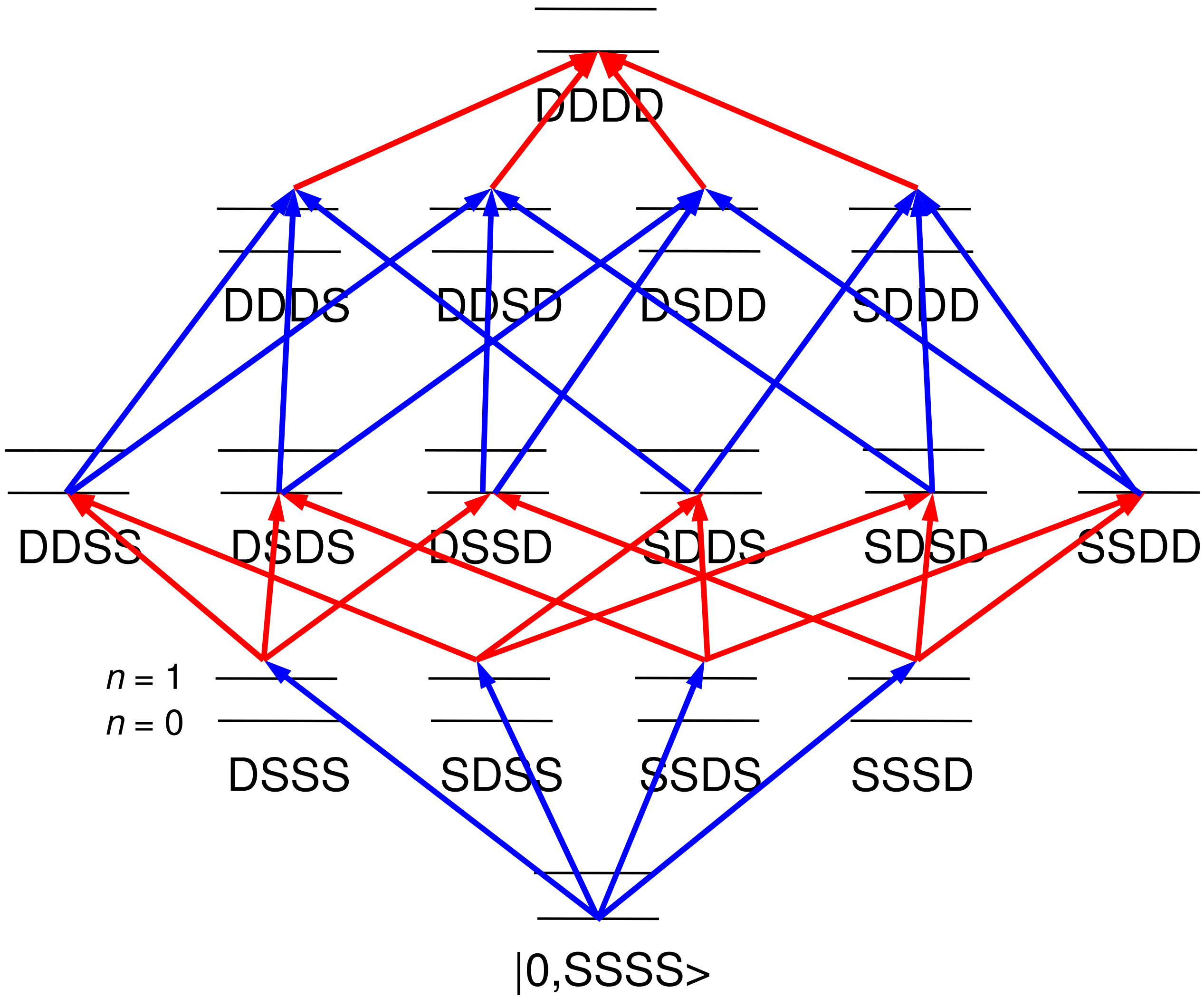


Two-ion GHZ-state



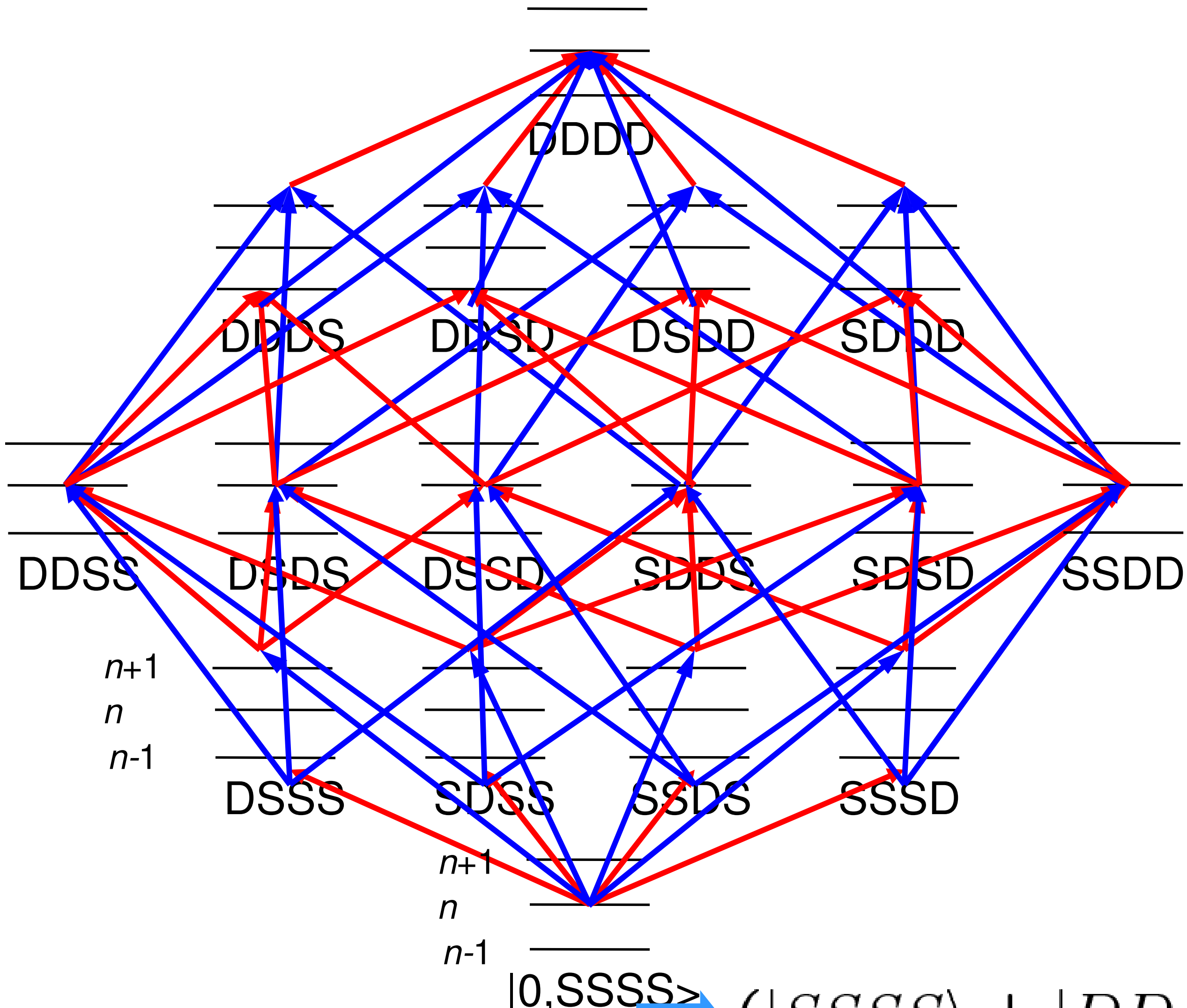


Entangling four ions



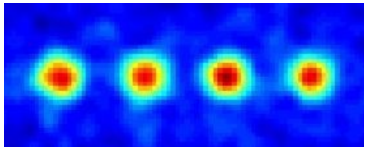
$$\Rightarrow (|SSSS\rangle + |DDDD\rangle)/\sqrt{2}$$

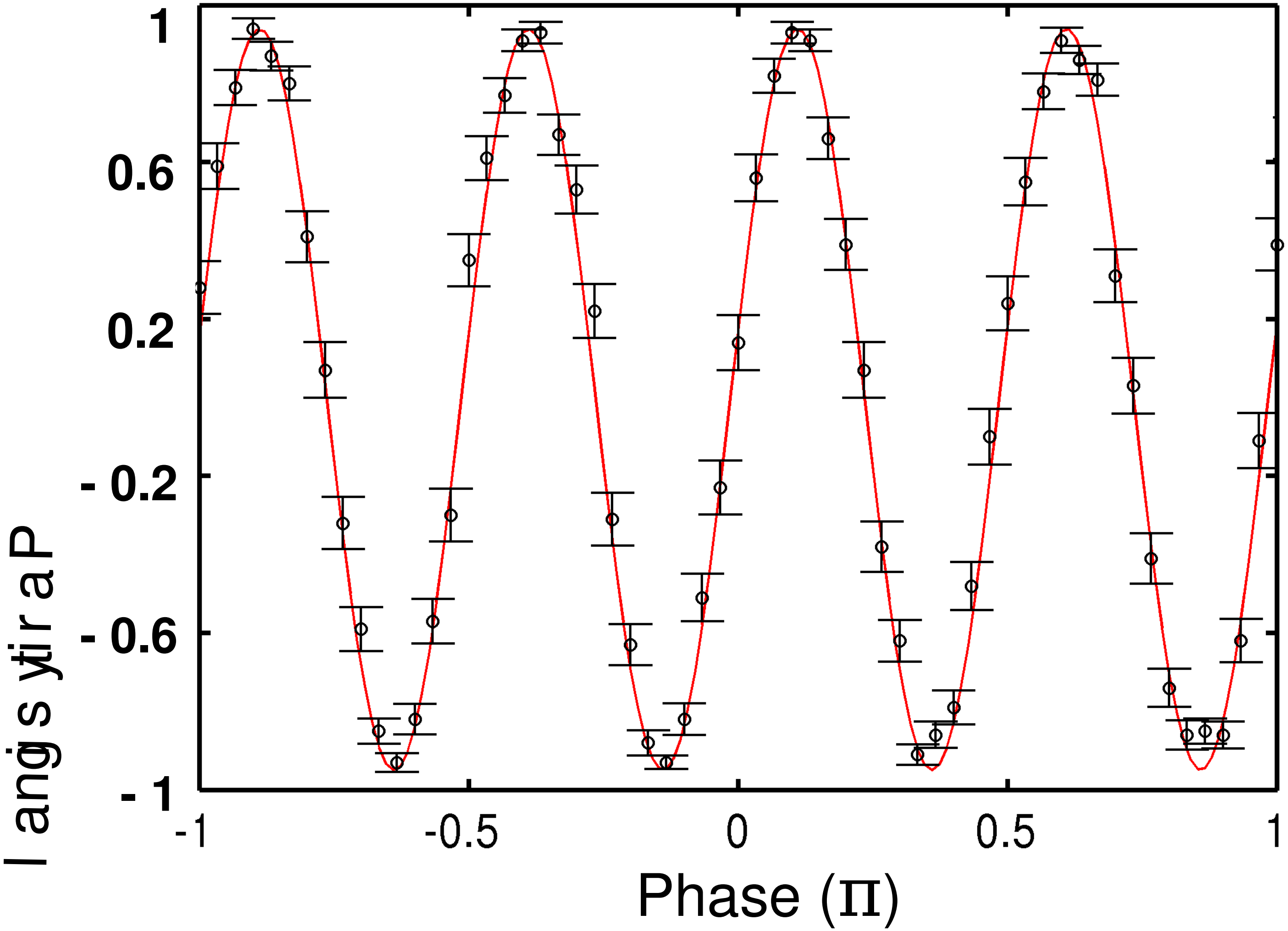
A complex interferometer …

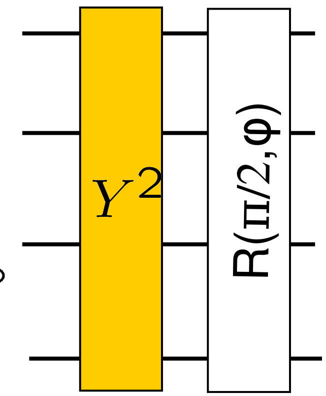


$$(|SSSS\rangle + |DDDD\rangle)/\sqrt{2}$$

Four-ion GHZ state

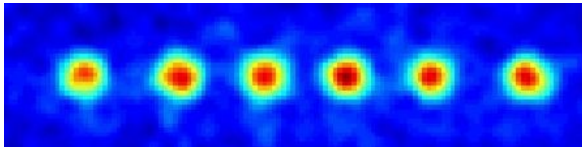


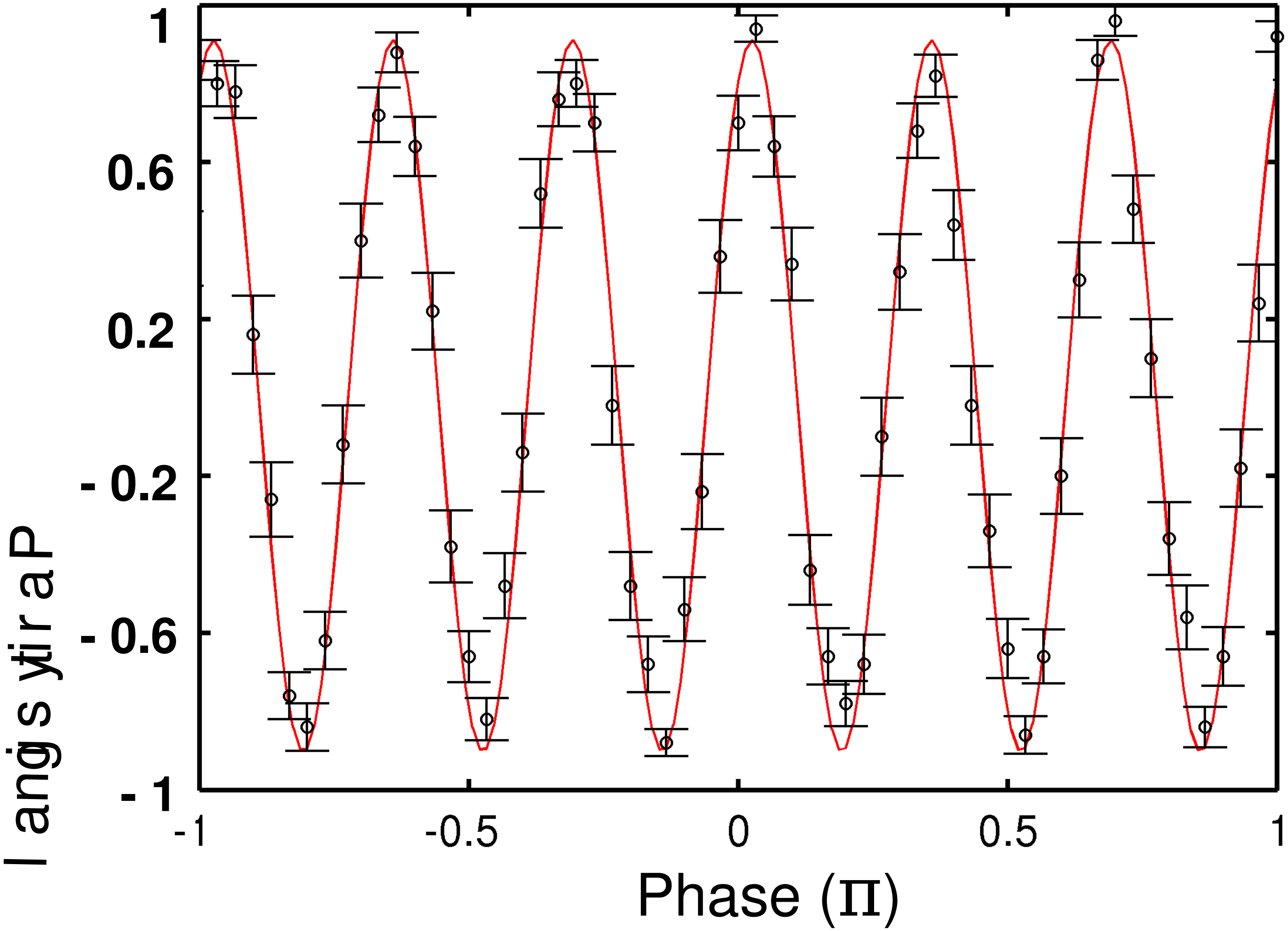




See also: Leibfried *et al.*, Nature **438**, 639 (2005)

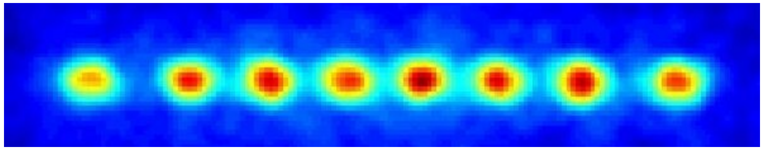
Six-ion GHZ-state



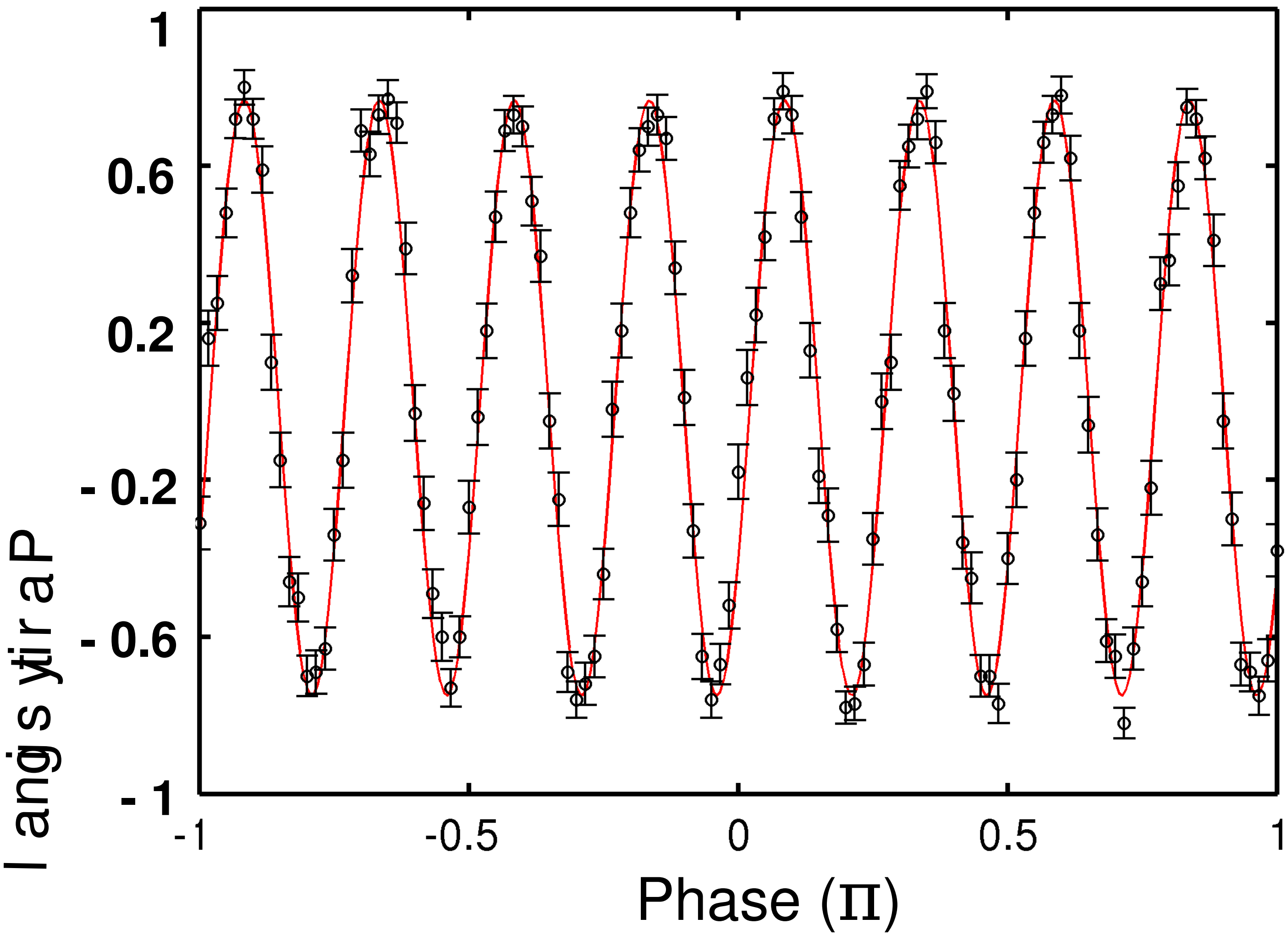


See also: Leibfried *et al.*, Nature **438**, 639 (2005)

Eight-ion GHZ state



T. Monz, P. Schindler, J. Barreiro, M. Hennrich



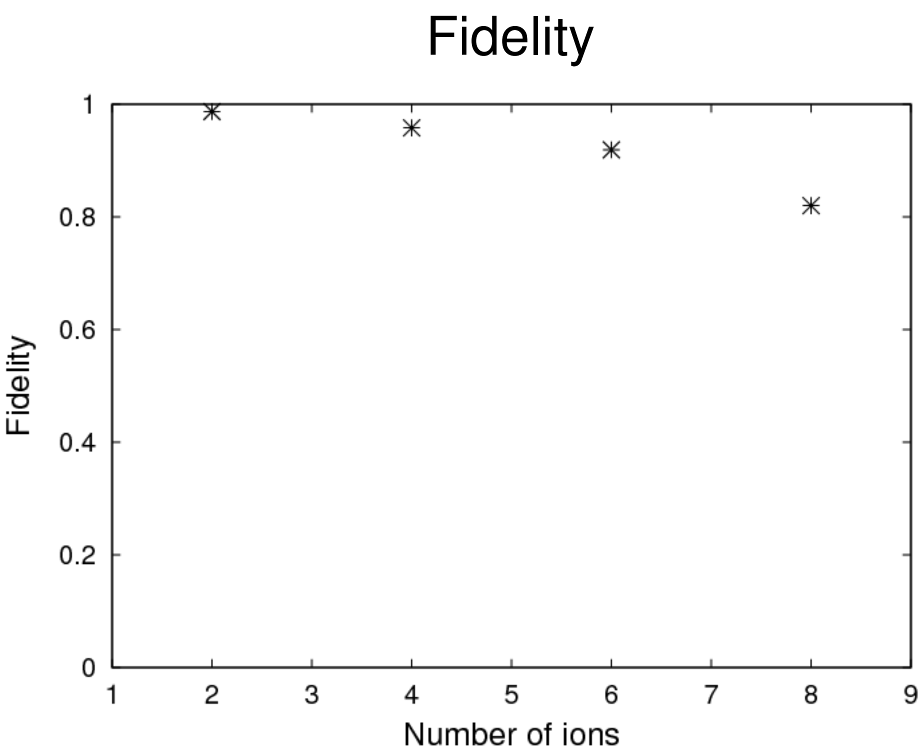
GHZ-state fidelities

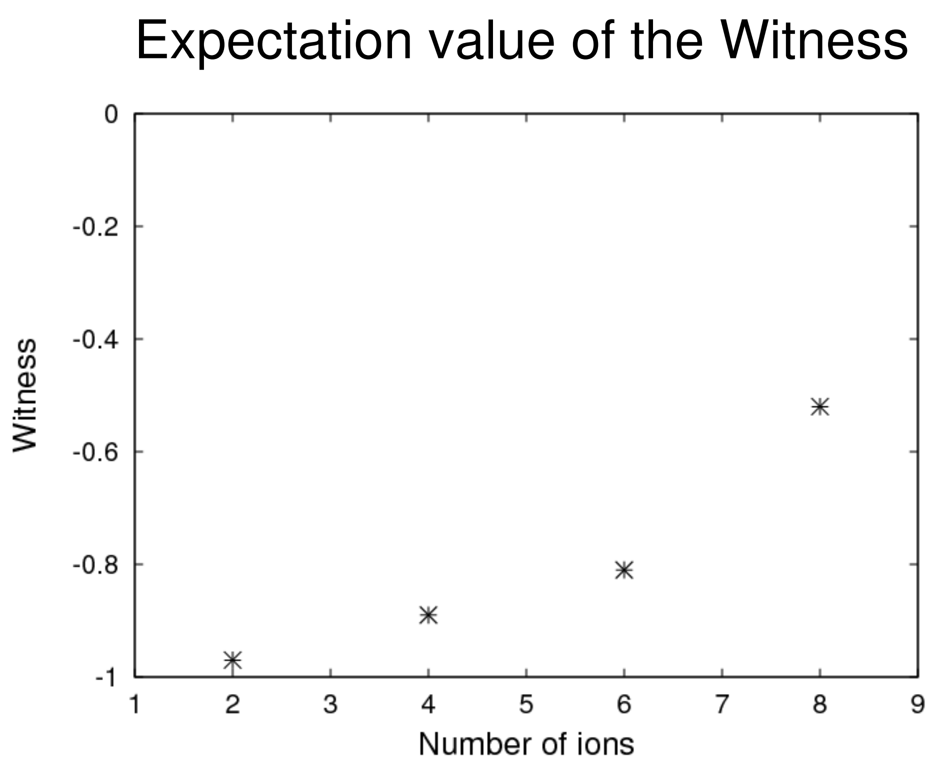
# ions	Fidelity	Witness
2	0.99 (1)	-0.97 (1)
4	0.96 (2)	-0.89 (1)
6	0.92 (3)	-0.81 (3)
8	0.82 (3)	-0.52 (3)

$$F = (P_{SS\ldots S} + P_{DD\ldots D} + 2\,\mathrm{Contrast})/2$$

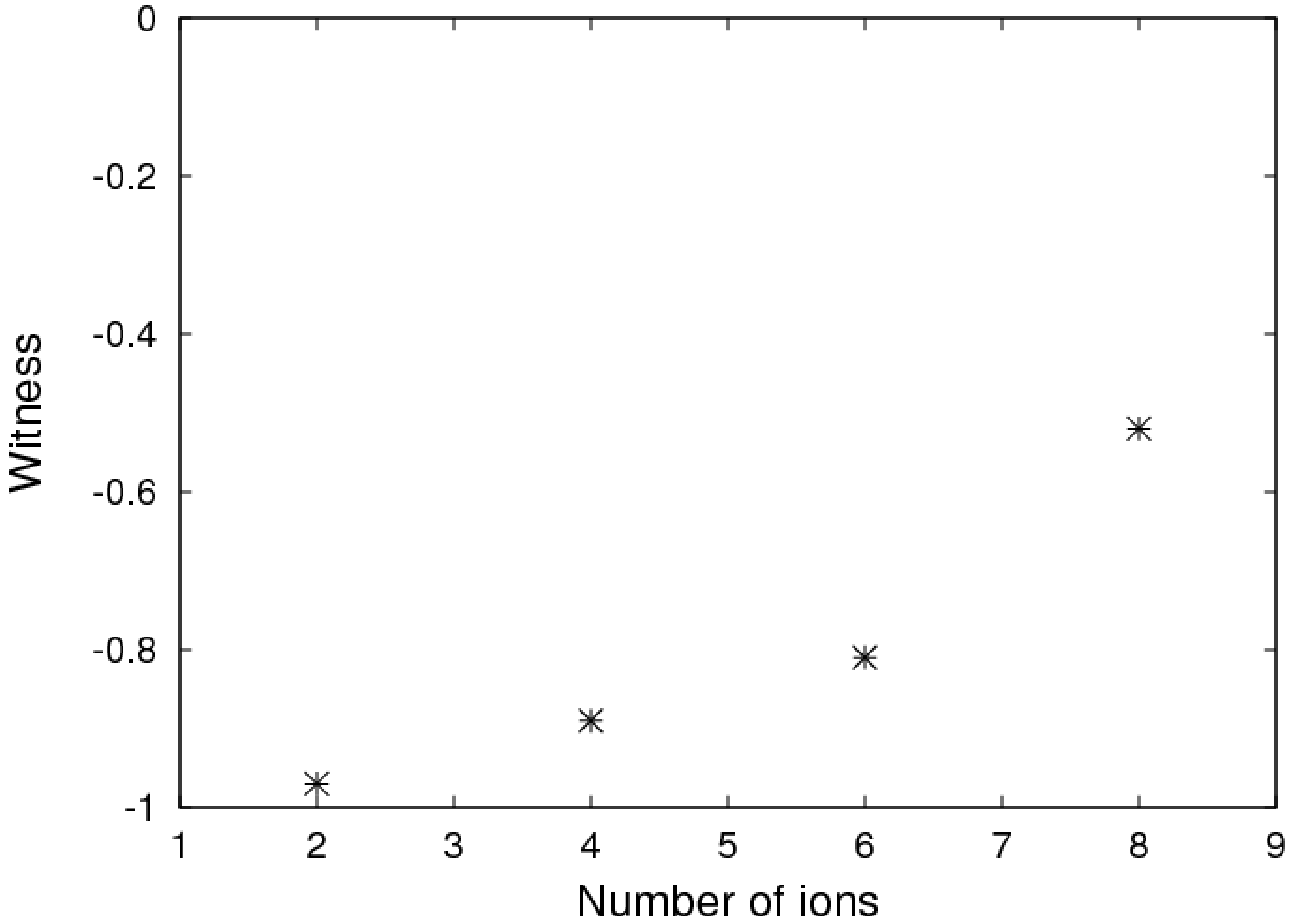
$$W = 1 - 4\,\mathrm{Contrast}$$

GHZ-state fidelities



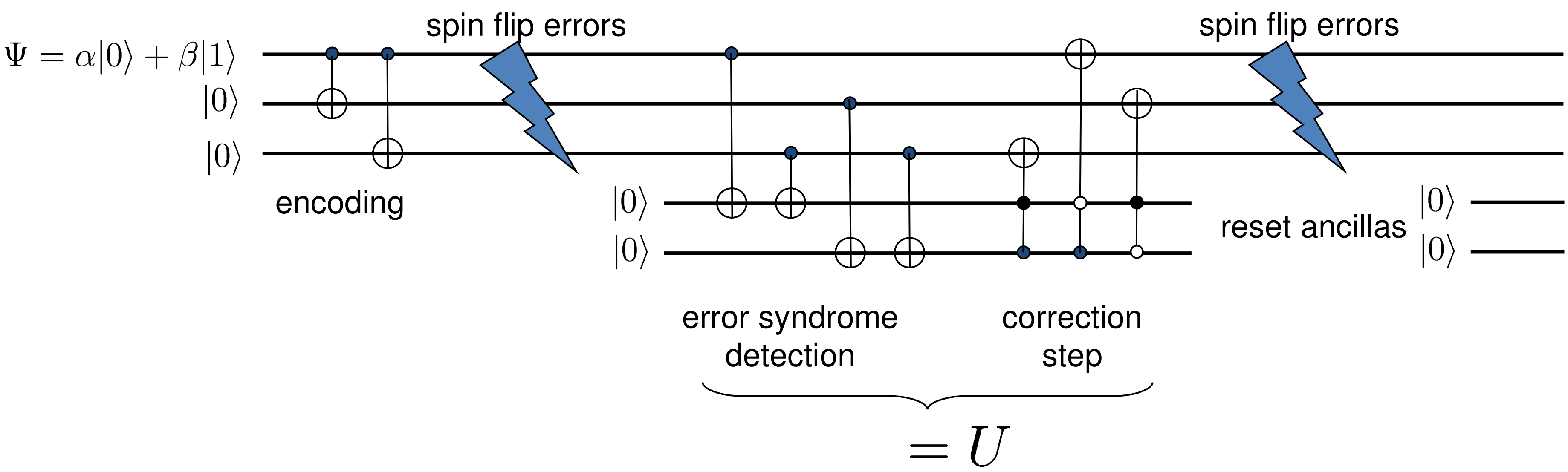


Entanglement

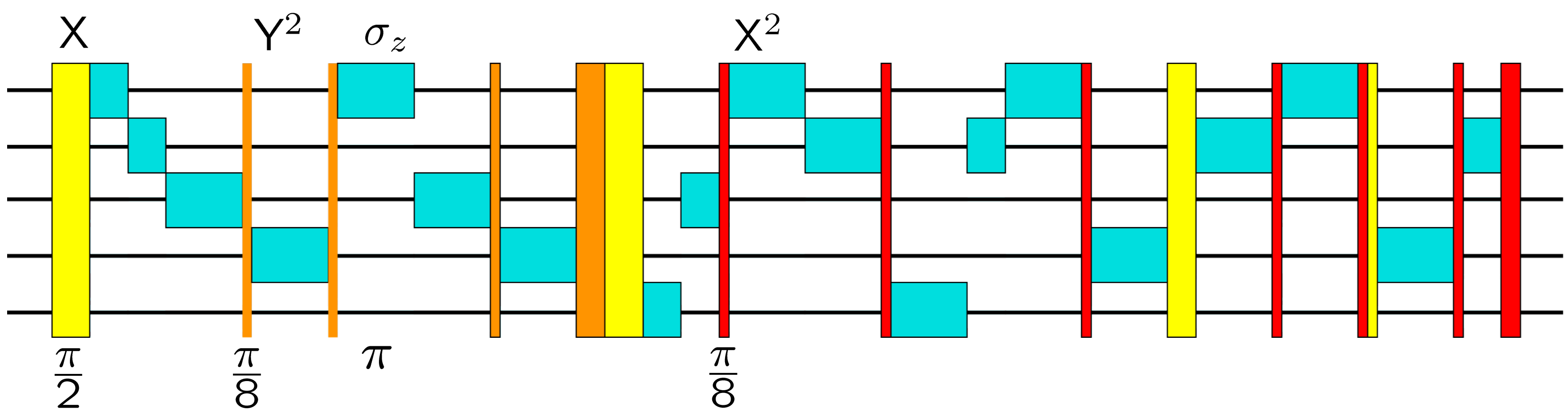


Quantum error correction

Example: quantum error correction: 3 qubits encode a logical qubit (protection against spin flips)



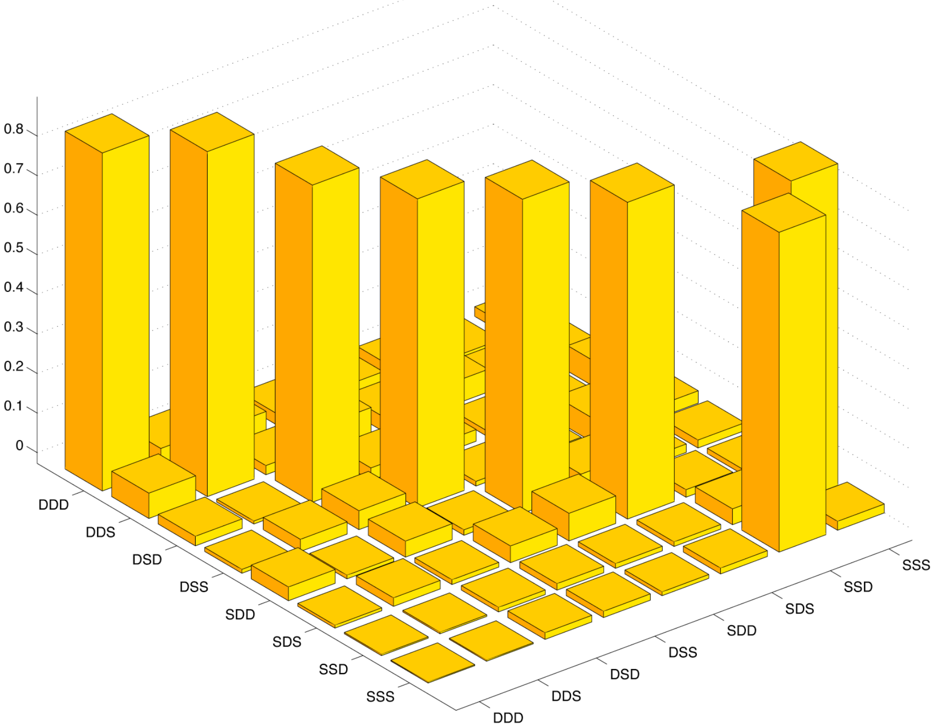
Implementation : 34 laser pulses (11 entangling pulses)

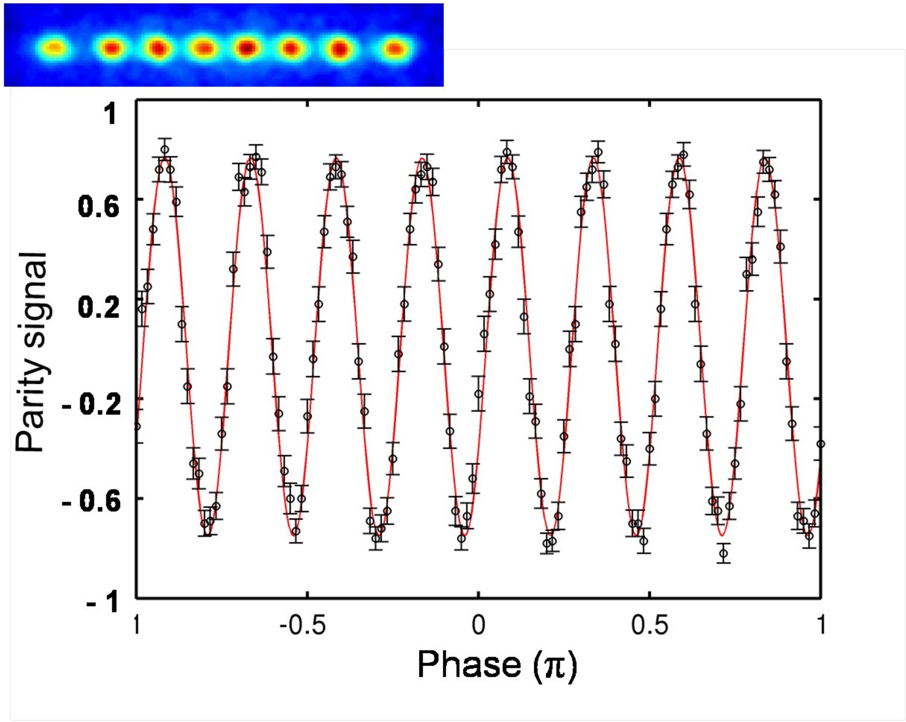
$U =$ 

V. Nebendahl *et al.*, Phys. Rev. A **79**, 012312 (2009).

Conclusions

Toffoli gate





GHZ-states

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The Innsbruck ion trap group

universität innsbruck

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