Deflected Subgradient convergence analysis

First of all a quick recap of the different aspects of the analysis.
Current formulation of the problem

$$\begin{aligned}\max_{\beta_i} &- \frac{1}{2}\sum_i\sum_j \beta_i\beta_j K(x_i, x_j)\\ &- \epsilon\sum_i|\beta_i|\\ &+ \sum_i y_i\beta_i \\ &With\ the\ constraints \qquad \begin{cases}\sum_i \beta_i = 0\\ \beta_i \in [-C,\ C]\end{cases}\end{aligned} \tag{1}$$

Pseudocode of the algorithm

Algorithm 1: Compute β

while *optima not found* **do**
  $v \longleftarrow f(\beta)$
  $g \longleftarrow \nabla f(\beta)$
  $d \longleftarrow \gamma g + (1-\gamma) dprev$
  `project` d `as in` (2)
  $stepsize \longleftarrow \frac{\psi(v - f_{ref} + \delta)}{\|d\|^2}$
  $\beta \longleftarrow \beta - stepsize \cdot d$
  `project` β `as in` (3)
end

Direction projection formulation

$$\begin{aligned}\min_s \quad & \frac{1}{2}\|d - s\|^2\\ & With\ the\ constraints \qquad \left\{\ (\beta_i - s_i) \in [-C,\ C]\right.\end{aligned} \tag{2}$$

β projection formulation

$$\min_{\beta_{proj}} \quad \frac{1}{2} \|\beta - \beta_{proj}\|^2$$
$$With\ the\ constraints \qquad \begin{cases} \sum_i \beta^i_{proj} = 0 \\ \beta^i_{proj} \in [-C,\ C] \end{cases} \tag{3}$$

Projection algorithms

In this section the focus will focus on how the two projection problems will be solved.
(2) $\longrightarrow$ *Projected Gradient method for BCQP*
(3) $\longrightarrow$ *Convex Knapsack Separable Problem algorithm*

Deflected Subgradient algorithm

(1) $\longrightarrow$ *Target level Polyak stepsize with nonvanishing threshold*

The approach that we are going to analyze is target value stepsize. With this method f^* is approximated by an estimate that is updated as the algorithm proceeds. The estimate will be the *target level* which is defined wrt two values: f^k_{ref} which is the *reference value*, and δ_k which is the *threshold*. This two values will be used to approximate f^* in the formulation of the stepsize.

$$0 \leq \nu_k = \beta_k \frac{f_k - f^k_{ref} - \delta_k}{\|d_k\|^2} \qquad 0 \leq \beta_k \leq \alpha_k \leq 1 \tag{4}$$