$$L_p = \frac{1}{2}\|w\|^2 + C\sum_i(\xi_i + \hat{\xi}_i) + \sum_i \alpha_i(y_i - w\phi_i - b - \varepsilon - \xi_i)$$
$$+ \sum_i \hat{\alpha}_i(-y_i + w\phi_i + b - \varepsilon - \hat{\xi}_i)$$

with variables $w, b, \xi_i, \hat{\xi}_i$

$$- \sum_i \mu_i \xi_i$$
$$- \sum_i \hat{\mu}_i \hat{\xi}_i$$

because constraints $\forall i\ \xi_i, \hat{\xi}_i \geq 0$

we transform $L_p \to L_D$ and variables become $\alpha_i, \hat{\alpha}_i, \mu_i, \hat{\mu}_i$

We want to min $L_p \to \partial L_p = 0$

$$\frac{\partial L_p}{\partial w} = 0 = w + \sum_i \alpha_i(-\phi_i) + \sum_i \hat{\alpha}_i(\phi_i) \qquad w = \sum_i(\alpha_i - \hat{\alpha}_i)\phi_i$$

$$\frac{\partial L_p}{\partial b} = 0 = \sum_i \alpha_i(-1) + \sum_i \hat{\alpha}_i(1) \qquad \sum(\hat{\alpha}_i - \alpha_i) = 0$$

$$\frac{\partial L_p}{\partial \xi_i} = C - \alpha_i - \mu_i = 0 \qquad C = \alpha_i + \mu_i$$

$$\frac{\partial L_p}{\partial \hat{\xi}_i} = C - \hat{\alpha}_i - \hat{\mu}_i = 0 \qquad C = \hat{\alpha}_i + \hat{\mu}_i$$

*

substituting

$$L_D = \frac{1}{2}\sum_{ij}(\alpha_i - \hat{\alpha}_i)(\alpha_j - \hat{\alpha}_j)\phi_i\phi_j - \sum_{ij}(\alpha_i - \hat{\alpha}_i)(\alpha_j - \hat{\alpha}_j)\phi_i\phi_j + \sum_i(\alpha_i - \hat{\alpha}_i)y_i$$
$$+ \cancel{\sum_i(\hat{\alpha}_i - \alpha_i)b} - \sum_i(\alpha_i + \hat{\alpha}_i)\varepsilon + \sum_i \alpha_i(-\xi_i) + \sum_i \hat{\alpha}_i(-\hat{\xi}_i) - \sum_i \mu_i\xi_i - \sum_i \hat{\mu}_i\hat{\xi}_i$$
$$+ C\sum_i(\xi_i + \hat{\xi}_i)$$

$$L_D = -\frac{1}{2}\sum_{ij}(\alpha_i - \hat{\alpha}_i)(\alpha_j - \hat{\alpha}_j)\phi_i\phi_j + \sum_i(\alpha_i - \hat{\alpha}_i)y_i - \sum_i(\alpha_i + \hat{\alpha}_i)\varepsilon$$
$$- \sum_i(\alpha_i + \mu_i)\xi_i - \sum_i(\hat{\alpha}_i + \hat{\mu}_i)\hat{\xi}_i + C\sum_i(\xi_i + \hat{\xi}_i)$$
$$- \sum_i C\xi_i - \sum_i C\hat{\xi}_i \quad *$$

$$L_D = -\frac{1}{2}\sum_{ij}(\alpha_i - \hat{\alpha}_i)(\alpha_j - \hat{\alpha}_j)\phi_i\phi_j + \sum_i(\alpha_i - \hat{\alpha}_i)y_i - \sum_i(\alpha_i + \hat{\alpha}_i)\varepsilon$$

DEPENDING ONLY ON α_is AND $\hat{\alpha}_i$s!

assume $\beta_i = \alpha_i - \hat{\alpha}_i$ and $|\beta_i| = \alpha_i + \hat{\alpha}_i$

$$\boxed{L_D = -\frac{1}{2}\sum_{ij}\beta_i\beta_j\phi_i\phi_j + \sum_i \beta_i y_i - \sum_i |\beta_i|\varepsilon}$$

LITTLE PARENTHESIS

since $\alpha_i, \hat{\alpha}_j, \mu_i, \hat{\mu}_i \geq 0\ \forall i$ (why?)

and $C = \alpha_i + \mu_i$

then $\alpha_i = C - \mu_i$ therefore $\alpha_i \in [0, C]$

equivalently $\hat{\alpha}_i \in [0, C]$