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Vol 49 no. 4 (December 1996) pp. 535-52

MEASURING THE GROWTH AND VARIABILITY OF TAX BASES OVER THE BUSINESS CYCLE

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Abstract *- The income elasticity of taxes is frequently used as an indicator of both short-run variability and long-run growth. This paper shows that the standard method for estimating elasticities results in asymptotic bias and inconsistent standard errors. Additionally, the standard estimates only provide information about the long-run growth potential of the tax. Thus, it is possible for a tax with a small estimated elasticity to fluctuate highly over the business cycle. This paper uses time-series econometric techniques to provide unbiased estimates of the long-run growth potential (the long-run elasticity) and cyclical variability (the short-run elasticity) of all major state tax bases.*

The income elasticity of state and local government taxes has been used as an indicator of both the short-run cyclical stability and long-run growth potential of various tax bases for decades. In their seminal contribution, Groves and Kahn (1952) argued that more income-elastic tax bases had more growth potential over time because, as income grew, income-elastic tax bases would grow more rapidly than income. At the same time, fluctuations in income would cause more income-elastic tax bases to be more unstable over the business cycle. Following this logic, there is an inherent trade-off between growth and stability in tax bases, with more income-elastic bases both growing faster and being less stable over the business cycle. While some authors have questioned whether this trade-off automatically exists, it is often accepted in the literature without much question.[1]

Estimates of the income elasticity of various taxes have commonly been obtained by regressing the log level of tax revenues on the log level of income.[2] There is reason, however, to question the adequacy of these estimates, in terms of their ability to measure either long-run growth potential or short-run cyclical variability, because of the

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econometric properties of the regression. First, because both income and tax revenues drift systematically upward through time (i.e., they are nonstationary), regressing tax revenues on income provides information only about the long-run relationship between the tax and income.[3] Beginning with Groves and Kahn (1952), many authors have assumed that this elasticity estimate, which is a measure of the long-run growth potential, also measures short-run variability.[4] This may not be correct because two taxes that have the same long-run growth rate will not necessarily vary by the same amount around that growth rate. Thus, one number cannot be used to proxy both the growth and variability of a tax. To avoid this problem, some researchers have proxied variability by measuring the deviation around trend growth, but this method also has problems. A second problem with the standard regression is that the elasticity estimates from these regressions will be asymptotically biased, and the estimates will have inconsistent standard errors which make standard hypothesis testing inaccurate. These regressions will also produce artificially high *R* squares.[5]

This paper begins with a brief discussion of the model traditionally used to estimate the income elasticity of taxes. It then analyzes the time-series properties of income and taxes to identify the potential problems with the traditional methods of estimation. After finding that these problems can be important, better time-series econometric techniques are used to provide asymptotically unbiased estimates of the long-run growth potential (the long-run elasticity) of all major state tax bases. Then a different model is used to measure the cyclical variability (the short-run elasticity) of the tax bases. These results are then compared with several other measures that are found in the literature. The paper concludes by examining the stability of the estimated long-run and short-run elasticities over the past four decades, to determine whether the current estimates might be useful for designing future tax policy aimed at reducing revenue variability and increasing long-run revenue growth.

STANDARD MODEL OF TAX REVENUE

The standard model that has been widely used to estimate the income elasticity of tax revenue is

1

$$\ln(B_t) = \alpha + \beta \ln(Y_t) + \varepsilon_t$$

where B_t represents the level of the tax base during period t and Y_t is the level of aggregate income during the period.[6] The coefficient β from this regression represents the income elasticity of revenues from this tax base.[7] Following Groves and Kahn (1952), the income elasticity, β, serves as a proxy for both the long-run growth and the short-run variability of the tax. This paper examines some of the properties of the model of variability of tax revenues implied in equation 1, using data on aggregate national tax bases and national income for estimation. Using the actual tax base eliminates any problems with measuring tax rates and their changes, and, because the data are aggregated across states, it eliminates any problems of state business cycles differing from the national cycle. This provides a reasonable and tractable data set on which to explore the econometric

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issues involved in estimating tax revenue elasticity.

The data used here represent all of the major tax bases used by state governments. Two proxies are examined for the state personal income tax base. Personal taxable income from the Internal Revenue Service's *Statistics of Income* series and adjusted gross income from the same source will be examined. Most states use something close to federal adjusted gross income as the base for state income taxes, so this measure closely approximates the state income tax base. Empirical results presented later in the paper do show some interesting differences between these two measures. Corporate taxable income, also from *Statistics of Income*, will be used as the measure of the corporate income tax base. Total retail sales and nonfood retail sales, both from the *Statistical Abstract of the United States*, will be used as the measure of the sales tax base, without and with a food exemption, respectively.[8] Total motor fuel consumption, in gallons, from the *Statistical Abstract*, will be used as a measure of the motor fuels tax base. Finally, total liquor store sales, also from the *Statistical Abstract*, will be used as the measure of the alcohol "sin" tax base. While this is perhaps the least accurate of all of the tax base measures, it at least provides a consistent measure of the tax base through time. Gross domestic product (GDP) will be used to measure aggregate income. All of the variables are converted to constant dollars using the GDP deflator, and their natural logs are used in the regressions. The data set covers the period from 1951 to 1991, with the exceptions that corporate taxable income was only available from 1960 to 1989 and that personal taxable income and fuel consumption were only available up through 1990.

THE STATIONARITY OF TAX BASES AND INCOME

Depending on the econometric properties of the variables in equation 1, the estimate of β may be subject to several problems. The important characteristic of the variables is whether they are stationary. A stationary variable is one that tends to return to some mean value through time. Any variable that has a trend will not be stationary (i.e., it will be nonstationary), as it will continue to move upward through time, with no inherent tendency to return downward. The test that is commonly used to determine whether a variable is stationary is the Augmented Dickey–Fuller (ADF) test.[9] The results of this test for all of the tax bases and GDP are given in Part A of Table 1. A significant *t*-statistic indicates that the variable is stationary in its level form, while an insignificant *t*-statistic indicates that the variable is nonstationary.[10]

In Part A of Table 1, all of the *t*-statistics are insignificant, which means that the variables are all nonstationary in their regular or levels form. Because both income and the tax base in equation 1 are nonstationary, two problems will result. The first, and most important, of these problems is that it becomes necessary to separately estimate the long-run and short-run relationships between the variables by using two different regression equations. The levels regression given in equation 1 between the two nonstationary variables provides the long-run relationship, while a second regression, using stationary versions of the variables, must be used to obtain the short-run relationship. To

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TABLE 1
STATIONARITY TESTS

Variable	*t*-Statistic
Part A: Test for Stationarity in Levels	
GDP	–1.01
PINC	–1.07
AGI	–0.83
CINC	–2.21
SALES	–0.92
NFSALES	–0.79
FUEL	–2.24
LIQUOR	–1.93
Part B: Test for Trend Stationarity	
GDP	–1.63
PINC	–1.65
AGI	–1.99
CINC	–2.46
SALES	–2.62
NFSALES	–3.01
FUEL	–0.75
LIQUOR	–0.16
Part C: Test for Difference Stationarity	
Δ*GDP*	–4.20**
Δ*PINC*	–4.14**
Δ*AGI*	–4.44**
Δ*CINC*	–3.06*
Δ*SALES*	–4.26**
Δ*NFSALES*	–4.50**
Δ*FUEL*	–3.00*
Δ*LIQUOR*	–3.04*

Notes: For definitions of the variables, see Table 2. *Indicates rejection of the null hypothesis (that the variable is nonstationary) at a five percent level and **at a one percent level. Thus a significant value indicates that the variable is stationary.

gain a better understanding of why this is the case, consider the data plotted in Figure 1a.

Figure 1a shows two hypothetical tax bases and national income. Both tax bases have the same long-run rate of growth, which in this example is two times the growth rate of GDP. The two tax bases differ, however, in how much they vary over the business cycle. Tax base 1 has more variability over the business cycle (a larger decline during recessions and more growth during booms) than does tax base 2. Figure 1b shows the regression plots of both tax bases. The dashed line shows that the least-squares regression lines are identical for the two tax bases. The slope of this dashed line is the elasticity estimate that would be obtained from equation 1. It is easy to see that equation 1 would produce identical elasticity estimates for both tax bases. The estimate for both is 2.0, which shows that over the long run a one percent growth in national income is associated with a two percent growth in both of the tax bases. The most important point is that this estimate is insensitive to changes in how much the tax base fluctuates around that trend growth. In fact, a tax base that grew in a straight line (or even one that moved counter to the business cycle) would still receive an estimate of 2.0 from equation 1 as long as it trended upward at the same long-run rate of growth as the tax bases given in Figure 1a. Thus, the estimate from equation 1 measures only the long-run relationship between the tax base and income.

To obtain an estimate of the short-run variability of the tax, the variables must be transformed into stationary form. The exact transformation necessary to make the variables stationary depends upon the nature of the variable. A nonstationary variable may be either trend stationary or difference stationary. A trend-stationary variable can be rendered stationary by removing a linear time trend from the variable, while a difference-stationary variable may only be rendered stationary by taking the first difference of, or change in, the variable. The familiar "random walk" is an example of a nonstationary variable that is difference stationary. Within the present context, the change form is simply the annual growth rate of the variable. Which of these types of nonstationary, if any, is present here is of great importance not only because it will have to be used to determine which transformation needs to be done to

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FIGURE 1a. Hypothetical GDP and Tax Bases in Levels

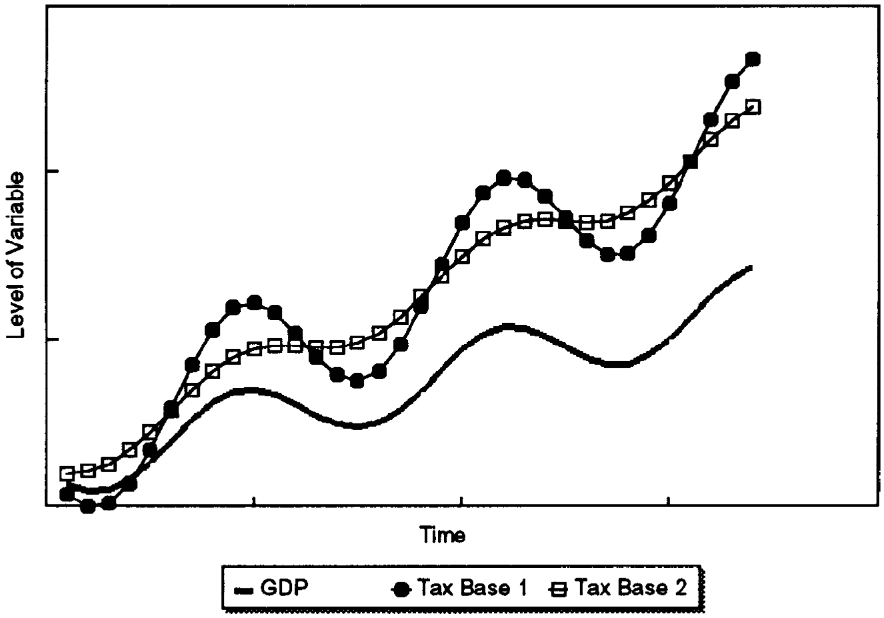
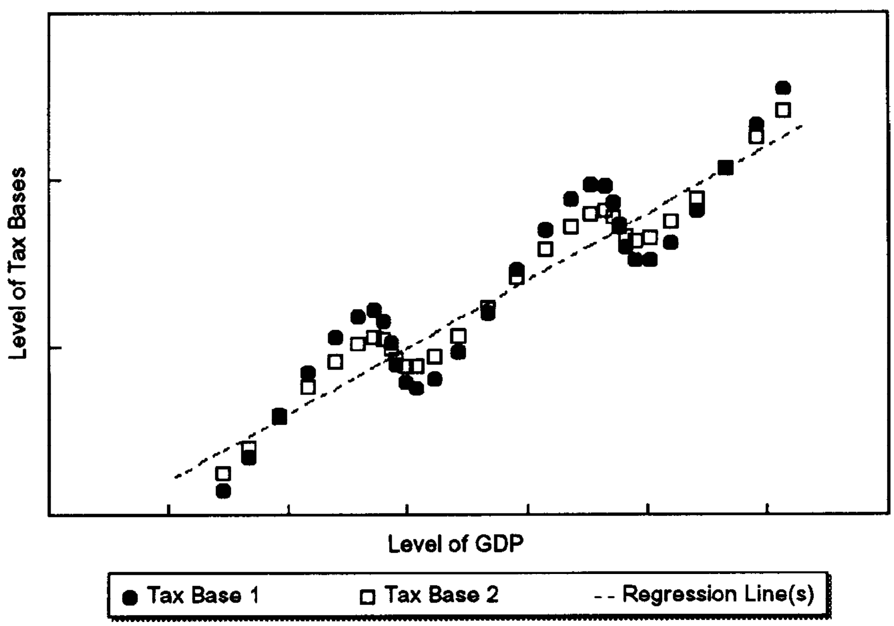


FIGURE 1b. Regression Plot of GDP and Tax Bases in Levels



Note that the same regression line fits both tax bases. This common regression line has a slope of 2.0, which is the long-run relationship. The short-run relationships (which are the vertical distances between the regression line and the observations) do not get reflected in the estimated slope coefficient (the income elasticity estimates).

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estimate the short-run elasticity of the tax, but also because several previous authors attempted to measure cyclical variability by regressing the log level of the tax on a trend line and then computing the deviation of the regression residuals. If, in fact, the variables are difference stationary rather than trend stationary, then this alternative regression will also have econometric problems because it is misspecified.[11]

The results in Part B of Table 1 show the tests of whether the variables are stationary after adjusting for a constant trend. All of the *t*-statistics suggest that the variables are still nonstationary, implying that the trend does not remove the nonstationarity. The results in Part C of the table test whether the variables are stationary in their change, or first difference form, denoted by a delta (Δ) before the variable. All of the *t*-statistics are now significant, suggesting that all of the variables are stationary in their change form. Thus, the correct version of the regression to obtain the short-run income elasticity is given by

$$\Delta \ln (B_t) = \alpha + \beta \, \Delta \ln (Y_t) + \varepsilon. \quad (2)$$

To gain a better understanding of what is being estimated by equation 2, consider the data given in Figure 2a. The tax bases in Figure 2a are the same data from Figure 1a, but in their change (or difference) form. In Figure 2a it becomes easier to see that tax base 1 has more pronounced ups and downs over the business cycle than does GDP, but that tax base 2 has a smoother cycle than GDP. Figure 2b shows the regression plot, and the slope of the dashed lines is the β coefficients that would be estimated by equation 2. Because both variables in the regression are stationary, the coefficient estimates from the regression are not subject to any bias or inconsistency.

The estimates for the data in Figure 2b are 1.5 for tax base 1 and 0.5 for tax base 2. These short-run income elasticities are the percent change in the tax base associated with a one percent change in national income. They measure how pronounced the cyclical ups and downs in the tax base are with respect to GDP. Just like the β estimates from equation 1 measure the long-run relationship (and are insensitive to the short-run relationship), these β estimates measure the short-run relationship (and are insensitive to the long-run relationship). Thus, to identify both the long-run growth potential and short-run cyclical variability of a tax base, it is necessary to estimate both equations. This is done in the next section of this paper.

The second problem that the nonstationarity creates with the estimation of equation 1 is that the coefficient estimate obtained from the regression (which is the long-run elasticity) will be asymptotically biased and its standard error will be inconsistently estimated. To better understand why this problem exists, return to the data in Figure 1b which showed the regression plot for a hypothetical example of the relationship between taxes and income over the long run. For tax base 1, the vertical distance between the data points (the black circles) and the regression line (the dashed line) is the error term in the regression. The error term is clearly correlated through time. This serial correlation in the error term results in the coefficient estimate being biased (the direction of the bias depending on whether there are more upward or downward dips) and also results in an inconsistent estimation of

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FIGURE 2a. GDP and Tax Bases in Change (or Difference) Form

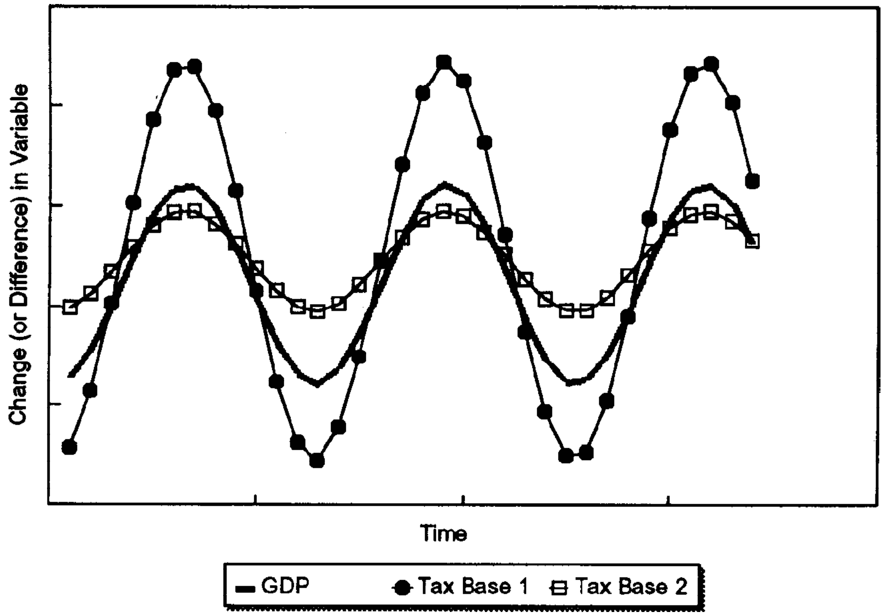
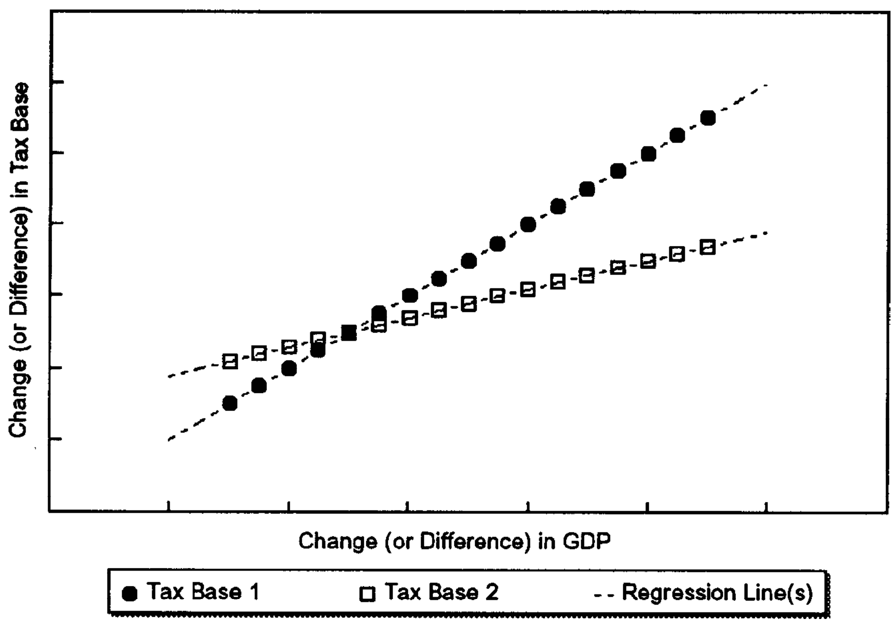


FIGURE 2b. Regression Plot of GDP and Tax Bases in Change (or Difference) Form



Note that the slope of the regression line that fits tax base 1 now differs from the slope of the regression line that fits tax base 2. The estimated slope coefficients (the income elasticity estimates) are 1.5 for tax base 1 and 0.5 for tax base 2. These reflect the short-run relationships between the tax bases and GDP. The long-run relationships are not, however, picked up in this regression.

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the standard error of the coefficient. This inconsistent standard error could result in faulty conclusions, for example, regarding the difference between the income elasticity of the sales and income taxes. There are two econometric techniques that can be used to correct for these problems. Stock and Watson (1993) show that adding leads and lags of the change in the independent variable to the regression will correct for the coefficient bias. This procedure is called dynamic ordinary least squares (DOLS) and is typically performed using five leads and lags.[12] While DOLS eliminates the coefficient bias, the inconsistency of the standard error must be corrected by using a second procedure, the Newey–West (1987) correction. This technique corrects for serial correlation of the standard error by modeling it as a moving average process. The combined use of DOLS and Newey–West can produce better estimates of the long-run elasticity (and its standard error) than have been obtained in the past from the standard levels regression.[13]

Before proceeding to estimate the long- and short-run income elasticities of the tax bases, it is important to discuss one last potential problem in the estimate of the short-run elasticity from equation 2. Two nonstationary variables that have a long-run relationship with one another will tend to move back together whenever they get too far apart. Thus, one may observe one variable moving down in the same period another is moving up simply because the variables deviated from the levels implied by their long-run relationship. This error correction, as it is often called, can cause a bias in the estimate of the short-run elasticity. Engle and Granger (1987) show that this bias can be removed by the inclusion of another variable in equation 2 that shows how far apart the two variables were from their long-run relationship during the prior time period. This is often called the error correction term, and it is simply the estimated error from equation 1 in the prior time period. Including the estimated error (lagged once) from the estimation of equation 1 as an independent variable in equation 2 adjusts for this bias. This revised form of equation 2 is called the error correction model.[14]

The next section of this paper will use all four models discussed above to obtain estimates of both the short- and long-run elasticities of the tax bases discussed earlier. Both regular OLS and DOLS will be used to obtain estimates of the long-run income elasticity of the tax bases from equation 1, and the regular change and error correction models will be used to obtain estimates of the short-run income elasticity of the tax bases from equation 2.

ESTIMATES OF TAX BASE ELASTICITY

In equation 1, where the log of income is regressed against the log of a tax base, the β coefficient shows how rapidly a tax base grows when compared to income in the long run. A β greater than one indicates that the tax base grows faster than income, whereas a β less than one indicates a tax base that grows slower than income. Equation 2 uses the change in the logs of the variables rather than the variables themselves, thus the coefficient β in equation 2 directly measures the cyclical component of tax base variability because it shows the percent change in the tax base that occurs as a result of a one percent change in income. A β greater than one indicates that the tax base fluctuates more than income over the business cycle, whereas a β less than one indicates that a tax base fluctuates

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less than income over the business cycle. While unusual, a negative β would indicate that the tax base moves counter to the business cycle, rising during recessions and falling during booms. The specification in equation 2 also has the advantage that the variables are stationary, as was shown in the previous section, so is not subject to the problems associated with the standard levels regression. The bigger advantage to using the specification in equation 2 rather than in equation 1, however, is that equation 2 measures the short-run relationship between income and the tax base, which is of importance when talking about how a tax fluctuates over the business cycle.

Table 2 gives the elasticity estimates from both the long-run models of equation 1 and short-run models of equation 2. The levels OLS estimates in the first column are asymptotically biased, and the next column corrects for any bias by using DOLS with the Newey–West correction. A comparison of those first two columns shows that there are only slight differences between the corrected and uncorrected estimates, indicating that these more sophisticated time-series techniques have a relatively small effect on the results with these data. This is an important finding because it implies that the estimates found in previous works are not severely biased and are still of value as measures of long-run growth potential. The third column estimates the change form of the model, as represented in equation 2, and the fourth column shows the estimate from this model when an error correction term is added to this change form. There are only small differences in the coefficients when comparing the regular change model to the error correction

TABLE 2
TAX BASE ELASTICITY ESTIMATES

	Estimates of Long-Run Elasticity				Estimates of Short-Run Elasticity			
	Levels — OLS		Levels — DOLS with Newey–West Correction		Regular Change Model		Error Correction Model	
Variable	β	R^2	β	R^2	β	R^2	β	R^2
Personal Taxable Income (*PINC*)	1.235 (0.018)	0.992	1.215 (0.014)	0.997	1.195 (0.171)	0.569	1.164 (0.161)	0.629
Adjusted Gross Income (*AGI*)	0.977 (0.009)	0.997	0.945 (0.007)	0.998	1.015 (0.098)	0.740	0.970 (0.100)	0.757
Corporate Taxable Income (*CINC*)	0.691 (0.110)	0.586	0.670 (0.094)	0.635	3.562 (0.655)	0.523	3.369 (0.685)	0.539
Retail Sales (*SALES*)	0.691 (0.008)	0.995	0.660 (0.008)	0.997	1.084 (0.096)	0.772	1.039 (0.094)	0.796
Nonfood Retail Sales (*NFSALES*)	0.732 (0.012)	0.990	0.701 (0.015)	0.994	1.431 (0.114)	0.804	1.377 (0.108)	0.836
Motor Fuel Usage (*FUEL*)	1.098 (0.036)	0.960	0.996 (0.044)	0.969	0.731 (0.182)	0.303	0.729 (0.175)	0.373
Liquor Store Sales (*LIQUOR*)	0.259 (0.051)	0.399	0.254 (0.043)	0.752	–0.024 (0.216)	0.001	–0.011 (0.219)	0.008

Note: Standard errors of elasticity estimates are given in parentheses.

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model. Thus, again, the policy implications of the regressions are affected relatively little when using the more sophisticated times-series procedure with this data. Again, this is important because it suggests that studies estimating short-run income elasticities may use the simpler regular change form of equation 2 to obtain rough estimates of the actual income elasticities. The big difference is between the first and second columns, which are the levels regressions typically used to estimate tax base variability, and the third and fourth columns, which report the change specifications of the model.

THE DIFFERENCE BETWEEN SHORT-RUN AND LONG-RUN ELASTICITY

From Table 2, looking at the first two columns, the standard elasticity estimates for corporate taxable income and retail sales are practically identical. Following the line of reasoning begun by Groves and Kahn (1952) regarding tax base elasticity, this would imply that these two taxes have both the same long-run growth potential and the same short-run cyclical variability. The third and fourth columns of Table 2 show that this conclusion would be partially incorrect. While it is true that the similarity of the long-run elasticities would lead to the conclusion that the two taxes have similar long-run growth potentials, it is incorrect to extrapolate this to conclusions about short-run variability. The estimates of the short-run elasticity from Table 2 show that corporate taxable income is much more variable over the business cycle than is retail sales. This was the problem with the estimates from equation 1 explained earlier with Figures 1 and 2. To gain a better applied understanding of the different things being measured by the standard elasticity and the elasticity measured in first-difference form, a graph of the log change in GDP, retail sales, and corporate income is given in Figure 3.

From Figure 3, it is easier to see what the different elasticity estimates are measuring in this applied context. Because both corporate taxable income and retail sales have approximately the same long-run growth rate relative to the growth rate of GDP (as would be shown by a horizontal line in the figure corresponding to each tax's mean growth), they both are estimated to have the same long-run elasticity from the levels regression. But, over the business cycle, corporate taxable income varies significantly more than retail sales. This difference in short-run volatility is captured in the estimates from the change specifications in the form of equation 2, but is not reflected in the estimates from the levels specifications like equation 1. These results show that a tax with a higher elasticity estimated from a levels regression need not fluctuate more over the business cycle. While the long-run elasticity estimate from a levels regression is invariant to the cyclical variability of a tax base, the short-run elasticity is invariant to differences in long-run growth. Not only is this true in principle, the results in Table 2 show that in fact there is little relationship between the long-run income elasticity of a tax base and the short-run elasticity.

The long-run elasticity shows how a tax base will tend to grow over time as income grows, whereas the short-run elasticity shows how a tax base will fluctuate over a business cycle as income fluctuates. Based upon the error correction model estimates of the short-run elasticities, the most cyclically variable of the tax bases is corporate income, followed by nonfood retail sales, personal taxable income, retail

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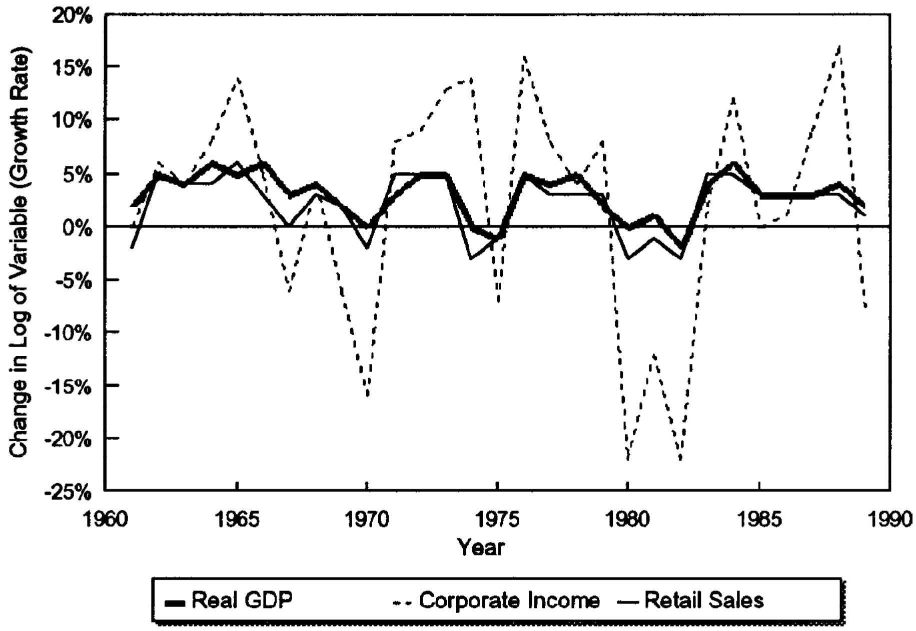
FIGURE 3. Log Change in GDP, Retail Sales, and Corporate Taxable Income



sales, adjusted gross income, motor fuels, and liquor sales. It is worth noting, however, that the estimates for personal income, retail sales, adjusted gross income, and motor fuels are not statistically different from one. The elasticities of both corporate income and nonfood retail sales are significantly greater than one, implying that they fluctuate more than income over the business cycle. While the estimated short-run elasticity of taxable personal income is larger than that of total retail sales, and retail sales is higher than adjusted gross income, the differences are not statistically significant. The difference between the estimated short-run elasticities of the sales tax base with and without food is, however, significant, implying that the food exemption does significantly increase the cyclical variability of a state's retail sales tax. The least cyclic of the tax bases is liquor, which has a negative estimated coefficient (which would imply that it moves countercyclically), but it is not significantly different from zero. The *R*-squared from this regression is so low that one may conclude that liquor sales are almost unaffected by the business cycle.

The large differences between the estimated short-run and long-run elasticities also have implications for the trade-off between the growth and variability of tax bases. Contrary to the implication of the Groves and Kahn model, these estimates show that it is possible for a state to reduce revenue variability without sacrificing long-run growth. For example, both the personal income tax and motor fuels tax have a higher long-run growth rate and also a lower cyclical variability than the corporate income tax. Additionally,

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while the personal income tax has about the same cyclical variability as the retail sales tax, it has a significantly higher long-run growth rate. While several other comparisons could be made with these estimates, it is evident that the trade-off between growth and variability is not automatic, as has been assumed in the past.

ALTERNATIVE MEASURES OF REVENUE VARIABILITY

Several authors (White 1983; Dye and McGuire 1991) have used a different method to calculate cyclical (or short-run) variability. Their measure is based upon the residual variance from a levels regression including only a constant and a time trend. This specification would be correct if tax revenues were trend stationary, but, as the results in Part B of Table 1 show, they are not. Even if tax revenues were trend stationary, however, this variance measure would still not capture cyclical variability because the deviations from the trend include fluctuations not related to changes in income. Two tax bases that have the same expected downturn during a recession, but one having more year-to-year variance, would be estimated to have different variances. Additionally, a tax which does not respond very much to the business cycle (such as the liquor tax) but has non-cycle-related variation may receive a variance calculation similar to a tax which does respond to the business cycle. The change specification represented in equation 2 precisely measures the business cycle-related variance, as opposed to the simple deviation around the trend. For policy purposes, cycle-related variability is more important because states tend to run into fiscal problems because of recession-related reductions in revenue growth rather than simply because revenues tend to fluctuate around the long-run trend.

Table 3 compares these three different ways of measuring the variability of tax bases. Following the procedure of these other authors, the standard deviation of the tax around the trend line in the levels regression was computed for each of the tax bases. The results of this calculation are presented in Table 3 along with the corresponding DOLS estimates of the long-run income elasticity and the error correction model estimates of the short-run income elasticity. The associated standard errors of the elasticity estimates are also given in parentheses.

Table 3 shows that the variance measure does a poor job of isolating the revenue variability due to the business cycle. While it finds that the corporate income tax is the most variable, it also finds that the motor fuels tax and the sin tax have high variability. However, according to the estimates from the error correction model, the motor fuels tax and the sin tax are the least responsive to the business cycle. Clearly, these two taxes have a significant amount of non-cycle-related variance that is being picked up by this variance measure. State policymakers trying to design tax structures that will be less sensitive to business cycle downturns would be severely misguided by these variance estimates.

State policymakers might be interested in both a measure of the cyclical variability of a tax and a separate measure of the random variance of the tax. Fortunately, both of these measures can be obtained from a single model. While the short-run elasticity estimates serve as the first measure, the standard errors of the short-run coefficient

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TABLE 3
COMPARISON OF MEASURES OF REVENUE VARIABILITY

Variable	Dynamic OLS Estimate of Long-Run Elasticity	Error Correction Model Estimate of Short-Run Elasticity	Standard Deviation of Residuals from Trend Regression
Personal taxabe income (*PINC*)	1.215 (0.014)	1.164 (0.161)	7.024
Adjusted gross income (*AGI*)	0.945 (0.007)	0.970 (0.100)	4.241
Corporate taxable income (*CINC*)	0.670 (0.094)	3.369 (0.685)	16.178
Retail sales (*SALES*)	0.660 (0.008)	1.039 (0.094)	3.514
Nonfood retail sales (*NFSALES*)	0.701 (0.015)	1.377 (0.108)	4.420
Motor fuel usage (*FUEL*)	0.996 (0.044)	0.729 (0.175)	10.220
Liquor store sales (*LIQUOR*)	0.254 (0.043)	–0.011 (0.218)	11.794

estimates serve as the second measure. More precisely, the magnitude of the coefficient indicates how much, on average, a tax base will be affected by a change in income, while the standard error of the short-run elasticity coefficient estimate shows the predictability of this relationship, that is, how closely the tax follows the business cycle. For example, because the short-run elasticity estimates for the personal income tax and the retail sales tax are fairly similar, but the standard error of the elasticity estimate for the personal income tax is higher, these two taxes have similar cyclical variability, but the personal income tax is subject to more non-cycle-related variance than the sales tax. Thus, from a practitioner's standpoint, both the short-run elasticity estimate and its standard error provide useful information. Together they serve as a decomposition of the cycle-related and non-cycle-related variability of the tax and are both important to consider in state tax policy decisions.

STABILITY OF THE ELASTICITY ESTIMATES

If these estimates are to be a useful basis for state policy decisions, they must remain fairly stable through time. If, for example, the short-run elasticity of a tax changes frequently, it would make it more difficult to use current estimates to design future tax policy. This section explores the structural stability of both the long-run and short-run elasticities over the past four decades to see whether the elasticity of these taxes changed at some midpoint during the sample period.

If there were some reason to expect that a change occurred in a particular year, a standard *F*-test could be performed to see if there is a statistically significant difference between the coefficient in the years before and after the break. When the break point is unknown, however, and a search is performed over the entire sample to see which break

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point yields the highest value of the *F*-statistic, standard critical values become inappropriate. To correct for this, Andrews (1993) calculated the distribution of the highest value of the *F*-statistic when the test is performed for many alternative points in the sample period. His test, called the sup-Wald test, is performed by finding the highest value of the *F*-statistic over the sample, and then comparing it to the critical values given in his paper. This section uses this test to see if there was a change in income elasticities for these tax bases during the sample.

The *F*-statistic for a change in the DOLS estimates of the long-run elasticity, and also the *F*-statistic for a change in the error correction model estimate of the short-run elasticity, was obtained for each year in the sample for each of the tax bases.[15] The largest *F*-statistic value over these years, along with the estimated change in the elasticity estimate at the implied break point, is given in Table 4. The row labeled *F*-statistic gives the highest *F*-statistic over the entire sample period, and the row labeled year gives the year in which the highest value occurred. The final rows give the estimated change in the elasticity estimate. The estimates show a permanent change in the long-run elasticity for every tax base during the sample period. While these changes are all statistically significant, they are quantitatively insignificant. The largest of the changes in the long-run elasticity occurs for corporate taxable income in 1974, but it is estimated that the elasticity rose by only 0.027 (recall that the original estimate was 3.369). The estimates indicate that the long-run growth potential of every tax base except corporate taxable income fell somewhat during the past few decades, although the decline was very small.

To explore whether there were changes in the short-run elasticities, this procedure was repeated for the error correction model. In this specification, the long-run elasticity was allowed to vary based upon the findings just reported,

TABLE 4
TESTS FOR THE STABILITY OF THE ESTIMATED INCOME ELASTICITIES

	Personal Taxable Income	Adjusted Gross Income	Corporate Taxable Income	Retail Sales	Nonfood Retail Sales	Motor Fuel Usage	Liquor Store Sales
			Long-Run Elasticity from DOLS Model				
F-statistic	32.71**	63.10**	8.19*	10.94*	29.15**	113.01**	84.46**
Year	1975	1975	1974	1977	1977	1981	1981
Change in long-run elasticity	–0.007	–0.004	0.027	–0.002	–0.005	–0.024	–0.022
			Short-Run Elasticity from Error Correction Model				
F-statistic	3.78	5.26	1.79	12.45**	14.80**	7.40	18.50**
Year	1970–1	1967	1970–1	1969	1970–1	1979	1972
Change in short-run elasticity	–0.351	0.342	1.020	0.379	0.453	–0.672	–0.968

Notes: *Indicates significance at a five percent level and **at a one percent level. Critical values for the *F*-statistic using the sup-Wald test are 11.48 (one percent) and 7.93 (five percent).

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and the results are shown in the bottom half of Table 4.[16] The estimated changes in the short-run elasticities are of greater magnitude than the changes in the long-run elasticities, but four of the seven are statistically insignificant. For the three major state tax bases—personal taxable income (or adjusted gross income), corporate taxable income, and retail sales—only the retail sales tax base shows a statistically significant change in terms of its short-run variability over the past few decades. This suggests that the variability of taxes stays relatively stable through time, so these estimates can be helpful in the design of tax policy. Even for the two tax bases where there was a change, retail sales and liquor store sales, the changes occurred more than 20 years ago.

In an attempt to provide the estimates most relevant for current policy decisions, Table 5 shows the estimates of the current short-run elasticities of all of the major state tax bases, taking into account the changes just reported.

There are two noticeable changes in the conclusions reached earlier. First, nonfood retail sales is now significantly more variable than either adjusted gross income or personal taxable income. This would imply that the state personal income tax is more stable over the business cycle than the retail sales tax when food is exempted.[17] There is, however, no significant difference between the variability of the personal income tax and the retail sales tax when food is included. Second, the short-run elasticity for liquor store sales is now significantly negative. This would imply that it moves countercyclically, with liquor purchases rising during recessions and falling during booms.

Taken as a whole, these results suggest that the short-run elasticity of all major state tax bases remains fairly constant through time. While it appears that the short-run elasticity of the retail sales and sin taxes changed around 1970, there have been no significant changes in any of the tax bases since 1970. This is important with respect to state fiscal

TABLE 5
ESTIMATES OF THE CURRENT SHORT-RUN ELASTICITIES OF THE TAX BASES

Tax Base	Sample	Short-Run Elasticity
Personal Taxable Income (*PINC*)	All	1.164 (0.161)
Adjusted Gross Income (*AGI*)	All	0.970 (0.100)
Corporate Taxable Income (*CINC*)	All	3.369 (0.685)
Retail Sales (*SALES*)	1969+	1.229 (0.098)
Nonfood Retail Sales (*NFSALES*)	1970–1+	1.612 (0.111)
Motor Fuel Usage (*FUEL*)	All	0.729 (0.175)
Liquor Store Sales (*LIQUOR*)	1972+	–0.586 (0.225)

Notes: Standard errors of elasticity estimates are given in parentheses.

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policy because it suggests that current estimates can be used as an accurate guide for designing future tax policy aimed at reducing cyclical revenue variability.

Conclusions about Tax Elasticity Estimation

The income elasticity of tax bases is important in the long run as an indicator of tax revenue growth and in the short run as a measure of the cyclical variability of tax revenues. Previous studies of the income elasticity of taxes have used several different methodologies to estimate the income elasticities of tax bases, but these methodologies were not always adequate for finding consistent and unbiased estimates. In addition, the important distinction between the long-run income elasticity of a tax base, which indicates how revenues from that base will grow over time as income grows, and the short-run elasticity of a tax base, which indicates how much revenues from that base will fluctuate over the business cycle, has not always been clearly made. For example, corporate taxable income and retail sales have approximately the same long-run income elasticity, but corporate income has a much greater short-run elasticity. Thus, the two tax bases show approximately the same rate of growth as income grows over the long run, even though corporate income taxes will fluctuate much more in response to short-run fluctuations in income. As another example, some studies have concluded that taxes on alcoholic beverages are unstable over the business cycle, but, when the model is correctly specified, fluctuations in the alcoholic beverage tax base are, if anything, counter to the business cycle and actually have a stabilizing impact on state revenues.

From a policy perspective, it is important not only to generate consistent and unbiased estimates, but also to have some confidence that those estimates will be stable over time. An examination of the tax base data since the early 1950s suggests that the estimates of income elasticity of the tax bases are relatively stable, so they can be useful guides for designing future tax policy. This paper uses aggregate data for the United States, so it is useful for seeing overall state trends. For state policy purposes, a similar type of analysis could easily be done for each individual state. The methodological issues discussed here could be even more significant at the state level, where changes in economic and population trends could have more substantial effects on the time-series properties of the data.

ENDNOTES

The authors wish to thank Stefan Norrbin, Kevin Reffett, Stratford Douglas, John Barkoulas, seminar participants at West Virginia University, and anonymous referees of this journal for helpful comments and suggestions. Any shortcomings remain the responsibility of the authors.

[1] This is not only mentioned in the papers which estimate the elasticities, but also in papers about state fiscal crises. See, for example, Gold (1991) and Mattoon and Testa (1992).

[2] See, for example, Groves and Kahn (1952), Wilford (1965), Vickrey (1949), Legler and Shapiro (1968), Friedlaender, Swanson, and Due (1973), Fox and Campbell (1984), Williams et al. (1973), and Mikesell (1977).

[3] For a good overview of the literature on nonstationary time-series estimation, see Ogaki (1993), Hamilton (1994), or Mills (1990). See also Granger and Newbold (1974), Phillips (1986), Stock (1987), and Nelson and Plosser (1982).

[4] Williams et al. (1973) did recognize this problem within the context of elasticity estimation and attempted to correctly estimate both short-run and long-run elasticity. Their contribution has largely been ignored in the literature, however, as the more recent studies have used only the long-run measure provided by the levels regression. Fox and Campbell (1984) also attempted to allow for differing elasticities over different phases of the business cycle, but their approach still suffers from

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not being able to capture the short-run dynamics needed to infer about revenue variability over the business cycle because of the use of levels.

[5] Groves and Kahn (1952) find *R*-squares greater than 0.80 in 21 of their 24 regressions; Wilford (1965) finds *R*-squares greater than 0.97 in 15 of 20 regressions (with 9 being larger than 0.99); Legler and Shapiro (1968) find *R*-squares greater than 0.97 in all 8 of their regressions (with 5 being larger than 0.99); Friedlaender, Swanson, and Due (1973) find *R*-squares above 0.99 in 23 of their 24 regressions (with the other being 0.97); and Fox and Campbell (1984) find *R*-squares greater than 0.97 in 7 of their 10 regressions (the lowest of the 10 was 0.82).

[6] Tax revenues can be used in place of the tax base as the dependent variable if another independent variable is included to measure changes in the tax rate. Also, per capita income and population are sometimes substituted for aggregate income in this equation. For ease of exposition, aggregate income will be used here, but the results also apply to the alternative specification.

[7] If one did not have explicit data on tax bases and had to use revenues as the dependent variable instead, Park (1992) has shown that, asymptotically, the estimate of β will be correct if an independent variable denoting the change in the tax rate is included in the regression (and it is stationary) and the variables B_t and Y_t are transformed in a manner consistent with canonical cointegrating regressions.

[8] Of the 46 states that currently have a general sales tax, 26 exempt food purchases.

[9] See Dickey and Fuller (1979), Said and Dickey (1984), and Fuller (1976).

[10] The ADF critical values used for this paper are calculated using MacKinnon's (1990) tables and sample size adjustment formula.

[11] For examples of this methodology, see White (1983) and Dye and McGuire (1991). Their measure will be discussed in more detail later in the paper.

[12] See also Saikkonen (1991) and Phillips and Loretan (1991).

[13] See Hamilton (1994) for a good discussion of these techniques.

[14] This is only a partial error correction because there is not a similar term for ΔY_t. Johansen (1992) notes that this is sufficient if Y_t is weakly exogenous, and a Geweke–Meese–Dent (1983) test shows that the partial error correction model is adequate. The *F*-statistics for the test (and significance levels) were *PINC* 1.325 (0.31), *CINC* 1.844 (0.26), *SALES* 2.071 (0.13), *NFSALES* 1.875 (0.16), *FUEL* 1.266 (0.33), and *LIQUOR* 1.901 (0.16).

[15] This test can only be performed for midpoints in the sample, because it is impossible to get accurate estimates of the elasticities using less than ten observations.

[16] Because of the small magnitude of the changes in the long-run elasticities, however, almost identical results are found when the error correction model is searched without allowing the long-run elasticity to change.

[17] This implication will hold if the sales tax taxes the entire retail sales tax base, but nothing more. Ring (1989) notes that in the average state only 59 percent of the retail sales tax base is retail sales because nonretail transactions are a part of the sales tax base.

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