Assignment 3

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1a) $\dot{x} = f(x, u, t)$

$$e_k = \underbrace{x_{k+1}}_{(1)} - \underbrace{x(t_{k+1})}_{(2)}$$

(1): $x(t_k) + \Delta t(b_1 k_1 + b_2 k_2)$

where

$$k_1 = \overbrace{f(x(t_k), u(t_k))}^{f}$$

$$k_2 = f(x(t_k) + a\Delta t \cdot k_1, u(t_k + c\Delta t))$$

$$= f + a\Delta t \frac{\partial f}{\partial x}\Big|_{x_k} f + c\Delta t \frac{\partial f}{\partial u}\Big|_{u_k} + O(\Delta t^2)$$

$$x_{k+1} = x(t_k) + \Delta t\, b_1 f + \Delta t\, b_2 \left(f + a\Delta t \frac{\partial f}{\partial x}\Big|_{x_k} f + c\Delta t \frac{\partial f}{\partial u}\Big|_{u_k} + O(\Delta t^2)\right)$$

(2): $x(t_{k+1}) = x(t_k) + \Delta t f + \frac{\Delta t^2}{2} \dot{f} + O(\Delta t^3)$

$$\dot{f} \rightarrow \frac{\partial f}{\partial x} f + \frac{\partial f}{\partial u}$$

$$\Rightarrow x(t_{k+1}) = x(t_k) + \Delta t f + \frac{\Delta t^2}{2}\left(\frac{\partial f}{\partial x} f + \frac{\partial f}{\partial u}\right) + O(\Delta t^3)$$

$$\Rightarrow e = x_{k+1} - x(t_{k+1}) =$$

$$x(t_k) + \Delta t\, b_1 f + \Delta t\, b_2 \left(f + a\Delta t \frac{\partial f}{\partial x} f + c\Delta t \frac{\partial f}{\partial u} + O(\Delta t^2)\right) -$$

$$-\left(x(t_k) + \Delta t f + \frac{\Delta t^2}{2}\left(\frac{\partial f}{\partial x} f + \frac{\partial f}{\partial u}\right) + O(\Delta t^3)\right) =$$

$$= \Delta t f \underbrace{\left(b_1 + b_2 - 1\right)}_{(1)} + \Delta t^2 \frac{\partial f}{\partial x} \underbrace{\left(a b_2 - \frac{1}{2}\right)}_{(2)} + \Delta t^2 \frac{\partial f}{\partial u} \underbrace{\left(c b_2 - \frac{1}{2}\right)}_{(3)} + O(\Delta t^3)$$

Thus we need the following conditions

$$\boxed{\begin{array}{ll} (1) & b_1 + b_2 = 1 \\ (2) & a b_2 = \frac{1}{2} \\ (3) & c b_2 = \frac{1}{2} \end{array}}$$