Exercice sheet #1

Elias Hühn & Max Bechthold

Exercise #1

a) $z = \frac{\lambda_{obs} - \lambda_{rest}}{\lambda_{rest}}$

Lyman α: ~~earth: 1215,67 Å~~

~~spectrum λ_{rest} =~~

rest: $\lambda_{rest} = 1215,67$ Å

obs: $\lambda_{obs} \approx 1213,5$ Å

$\Rightarrow z = -1,78 \cdot 10^{-3}$

$$z = \sqrt{\frac{1 + v_r/c}{1 - v_r/c}} - 1 \quad \Rightarrow \quad v_r = \frac{cz^2 + 2cz}{(z+1)^2 + 1} = -534 \frac{km}{s}$$

b) $d = 780 \, kpc \cong 2,4 \cdot 10^{19} \, km$

$$v_{theory} = H_0 d = 71 \frac{km/s}{Mpc} \cdot 0,78 \, Mpc = 55,38 \frac{km}{s}$$

$v_{theory} > v_{measured}$

$\Rightarrow$ Andromeda would be expected to move away from an observer on earth, but it is actually moving towards him

$\Rightarrow$ M31 and MW interact gravitationally (?) and will collide.

c) $z = 0,05$

$$v = c \cdot \frac{(z+1)^2 - 1}{(z+1)^2 + 1} = 14615 \frac{km}{s} = \cancel{4,7 \ldots \frac{pc}{s}} \; 4,7 \cdot 10^{-10} \frac{pc}{s}$$

$$d = \frac{v}{H_0} = \frac{\cancel{4,7 \cdot 10^{-16} \frac{Mpc}{s}} \; 14615 \frac{km}{s}}{71 \frac{km/s}{Mpc}} \cong 206 \, Mpc$$

Exercice 2

$E \sim k_B T$

$T/K \sim 1{,}5 \cdot 10^{10} \cdot (t/s)^{-1/2}$

a) $\gamma + \gamma \rightarrow e^- + e^+$

$E = mc^2$ $\qquad m_e = 9{,}1 \cdot 10^{-31}\ kg$

$E_e = 8{,}18 \cdot 10^{-14}\ J$

$\approx 510555{,}5\ eV$

$\approx 5{,}1 \cdot 10^5\ eV$

$E_{ges} = 2 E_e$

$\approx 10{,}2 \cdot 10^5\ eV \approx 1{,}63 \cdot 10^{-13}\ J$

$\Rightarrow E_{ges} = \frac{f}{2} k_B T$

$\Leftrightarrow \frac{E_{ges}}{k_B} = T$

$\Rightarrow T \approx 1{,}18 \cdot 10^{10}\ K$

$\Rightarrow$ increasing expansion leads to smaller photon energy

b) $t \approx 1s \Rightarrow T = 1{,}5 \cdot 10^{10}\,K$

$$E \sim k_B \cdot T = 8{,}617 \cdot 10^{-5} \frac{eV}{K} \cdot 1{,}5 \cdot 10^{10}\,K$$
$$= 1{,}29\ MeV$$

Imbalance for $10^9\ \bar{x}$ (anti-part.) $10^9 + 1\ x$ (part.)

$$\underbrace{\gamma+\gamma}_{10^9} \rightarrow \cancel{p+\bar{p}}\ \underbrace{\bar{x}}_{10^9} + \underbrace{x}_{10^9+1} \rightarrow \underbrace{\bar{x}+x}_{10^{18}} + \cancel{1x} \rightarrow \underbrace{\gamma+\gamma}_{10^9}$$

but one left $\Rightarrow 10^9\,\gamma$ for each baryon

c) $p + n \rightleftharpoons D + \gamma$

$m_p = 938{,}272\ MeV$
$m_n = 939{,}565\ MeV$
$m_D = 1875{,}612\ MeV$
source: wikipedia.org

$$E_p + E_n \rightleftharpoons E_D + \overset{?}{E_\gamma}$$

$$\Rightarrow E_\gamma = E_p + E_n - E_D$$
$$= 2{,}225\ MeV$$

$$E_\gamma = h \cdot \nu \Leftrightarrow \nu = \frac{E_\gamma}{h}$$

$$\Rightarrow \nu \approx 5{,}33 \cdot 10^{20}\ Hz$$

There are several reactions that can form ^4He that mainly involve D $\Rightarrow$ only a small amount is left.