# The volatility risk premium: an empirical study on the S&P 500 index

Ivan Guo Gregoire Loeper

Centre for Quantitative Finance and Investment Strategies,*
School of Mathematical Sciences
Clayton Campus, Monash University, VIC, 3800, Australia

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**Abstract**

We perform an empirical analysis of trading strategies based on the systematic selling of delta hedged options, aiming at capturing the so-called *volatility risk premium*. We compare the performance across different strikes and maturities, and perform a breakdown of the drivers of performance. We also examine how such strategies can be combined to extract other premia related to the profile of the volatility surface, e.g. the skew and the term structure. In this first paper we focus on the S&P 500 index over the period 2010–2018.

**Keywords:** systematic strategies, derivatives, volatility risk premium

## 1 Introduction

This work is the first of a series of papers aiming to explore different types of systematic strategies on options, either listed or OTC[1]. Through our study we investigate the existence of market anomalies, or risk premia, i.e., sources of alpha that can be harvested by investors.

There are several ways of building systematic strategies based on options: to state the obvious, one can either go *long* or *short*. A long strategy will systematically buy options, often for the purpose of buying downside protection for a long portfolio. A short strategy will systematically sell options, to harvest the so-called *volatility risk premium* (see, e.g., [1]).

A common example is the covered call strategy (see the companion paper [5] and also [7]), where for each unit of stock (or index) one sells a slightly out of the money call option with short maturity. This combination partly exchanges the returns from equity exposure with volatility exposure, which, in a sense, exchanges some beta with alpha. The performance from volatility is generated via the option premium, while equity return can still be generated up to the strike price. However, the strategy is susceptible to extremely large stock movements in a short period of time, as large stock increases do not provide extra returns, while losses from large stock price decreases are not mitigated.

In this paper, we focus on strategies that consist of selling one option of fixed maturity and moneyness (i.e., strike relative to the spot) every day. The option is then held until maturity and delta hedged every day. These strategies will be studied either individually, or as building blocks of more complex combinations, to exploit some potential anomalies of the volatility surface.

An crucial point is that a short option position presents a very asymmetric risk profile: everything else (in particular the implied volatility) being constant, the upside is limited to earning the decrease in time value of the option (the *theta*), while the downside is potentially unlimited. See Section 4

*The Centre is partially supported by BNP Paribas
[1]Over the counter

and equation (2) for the definition of the various *Greeks*. The positive earning is described by the level of implied volatility, while the negative part depends on the *realised volatility*, multiplied by the level of convexity of the option (the *gamma* effect). The strategy tends to have steady performance in stationary market conditions but can suffer from heavy drawdowns in extreme conditions. This asymmetric, and negatively skewed profile is expected to be rewarded by a positive average expected return, the *volatility risk premium*. The difference between implied and realised volatilities has been well-established [8] and the existence of volatility risk premium is observable across many markets across the globe [4, 11], including the S&P 500 index.

Several rolling call or put strategies can be combined to extract various spreads or term structures offered by the richness of the options. A simple example is the skew arbitrage [9], [10], which can also be realised by trading the difference between two strikes with different implied volatilities. Despite its name, a skew arbitrage is not a true arbitrage. It takes advantage of the shape of the the implied volatility surface by trading a combination of two vanilla options with the same maturity but different strikes. In [10], the options are automatically delta hedged while the weights are chosen so that the portfolio is gamma neutral. The P&L of this strategy consists of the spread between the implied volatilities at the two strikes (thetas), as well as other unhedged Greeks (in particular vega and vanna). The strategy performs as long as the theta outweighs the vega and vanna. A similar strategy can be constructed to be vega neutral instead of gamma neutral.

When executing a systematic options strategy, one has to be mindful of the potential liquidity risks, including transaction costs and market impact, see [2] and the references therein. We do not make any assumptions on the liquidity costs incurred or on the possible market impact. Instead, the performance of each strategy is expressed in terms of equivalent transaction costs: the size of the mid-offer spread that would cancel the performance of the strategy.

Our study is based on data that is made available to us through a partnership with BNP Paribas, obtained directly from settlement prices given by exchanges.

## 2 The building blocks

The building block of our study consists of single strike and maturity strategies. Each strategy involves the systematic selling of constant strike maturity delta-hedged options. Each day, one sells a given *notional*[2] of options, of a given maturity (e.g. 3 months), and given moneyness (e.g. strike equals 90% of the current underlying spot). The options are kept until expiry and *delta-hedged* (in the sense of annihilating the Black-Scholes delta) every day at the close.[3]

Here are the main features of our investment products:

- **Underlyings**: S&P 500

  Our current study will focus on options on the S&P 500 index. We chose this index because it has an extremely liquid options market. Similar analyses will also be carried out for other popular equity indices (e.g., DAX, HSCEI, NKY, SX5E, UKX).

- **Strikes**: 0.8p, 0.9p, 1p, 1c, 1.1c, 1.2c

  We consider out-of-the-money (as well as at-the-money) options: since our strategies area always delta-hedged, there is very little difference between calls and puts. For example, 0.8p corresponds to the put option struck at 80% of the underlying value today.

- **Target Maturity**: 10bd, 21bd, 42bd, 63bd, 126bd, 252bd

  The target maturity of the option is measured in business days, roughly corresponding to time periods of two weeks, one month, two months, three months, six months and one year.

- **Option Type**: Interpolated listed options

[2]The notional can be set as a fixed number of options, or fixed amount of delta, vega, gamma, etc ...
[3]We'll discuss later the the different possible ways of computing the delta

The desired maturities are achieved by interpolating listed maturities. For example, if we want to sell a 3 months option, we will sell a linear combination of the two closest surrounding listed maturities. A forthcoming paper will focus on the comparison between listed and over-the-counter (OTC) strategies.

- **The Black-Scholes formula** The celebrated Black-Scholes formula will be used throughout the paper to express option's prices in terms of certain parameters. The price at time $t$ of an (say) put option with maturity $T$ and strike $\sigma$

$$P = BS_{put}(\sigma, T-t, F_t(T), K, B(t,T)), \tag{1}$$

where $\sigma = \sigma(t,T,K)$ is the Black-Scholes implied volatility, $F_t(T)$ is the forward , $B(t,T)$ the discount factor.

- **Data and interpolation method**: Option prices are settlement prices obtained directly from the exchange (*CBOE* in the present case). Knowing all options prices, one can obtain the forward (by call put parity) and the discount factor (by using deep in the money call-spreads, knows as *boxes*). We parametrize options by their implied Black-Scholes volatility, forward, and discount factor. The interpolation is then done linearly on these three parameters.

Please note that, throughout the paper (especially in the figures), the terminology **spot** will be referring to the current value of the options strategy, rather than the current value of the underlying.

## 2.1 Scaling method

Since different strategies have different amount of exposure (e.g., gamma exposure, vega exposure, etc.), some scaling is required in order to make meaningful comparisons.

In order to illustrate the effect of different scaling methods, we have chosen an extreme example in Figure 1. Each plot contains six strategies with the same strike (1.2c) but different maturities. The various scaling methods involve normalising the following features of the daily returns: standard deviation, mean absolute deviation, median absolute deviation, inter-quartile range and standard deviation without outliers (defined to be more than 3 standard deviations away from the mean). Some methods are better for demonstrating the variability within the data sets, while other are more robust in the presence of extreme events.

As seen in Figure 1, the short maturity (10bd and 21bd), out-of-the-money strategies are very sensitive to extreme events, while have very little movement otherwise. From looking at the non-scaled version, it is clear that some scaling is needed to compare these extremely heavy tailed strategies against others. The median absolute deviation and inter-quartile range methods, despite being robust measures of spread, greatly distort the heavy tail strategies when used for scaling. The three other methods, standard deviation, mean absolute deviation and standard deviation with clipped outliers, all perform somewhat similarly and are more reasonable scaling approaches for our purposes.

**In this paper, unless stated otherwise, all strategies are scaled by the standard deviation of the corresponding daily returns.** We chose the standard deviation due to its simplicity and allows for straightforward performance comparisons. **In particular, when normalising by the standard deviation, the annualised performance of the strategy is equal to its Sharpe ratio.**

## 2.2 Testing period

Even though we have data for the period 2007–2018, we have chosen focus our current study only on the period 2010–2018. This is done to exclude the effects of the global financial crisis (GFC) from our analysis. This is because the aim of the study is to provide an empirical study of options strategies under more typical economic conditions, without the various distortion from the GFC.

In order to properly study and model the effects of the GFC, one would require data from much longer periods (i.e., many decades) and that is beyond the scope of the current work. Note that,

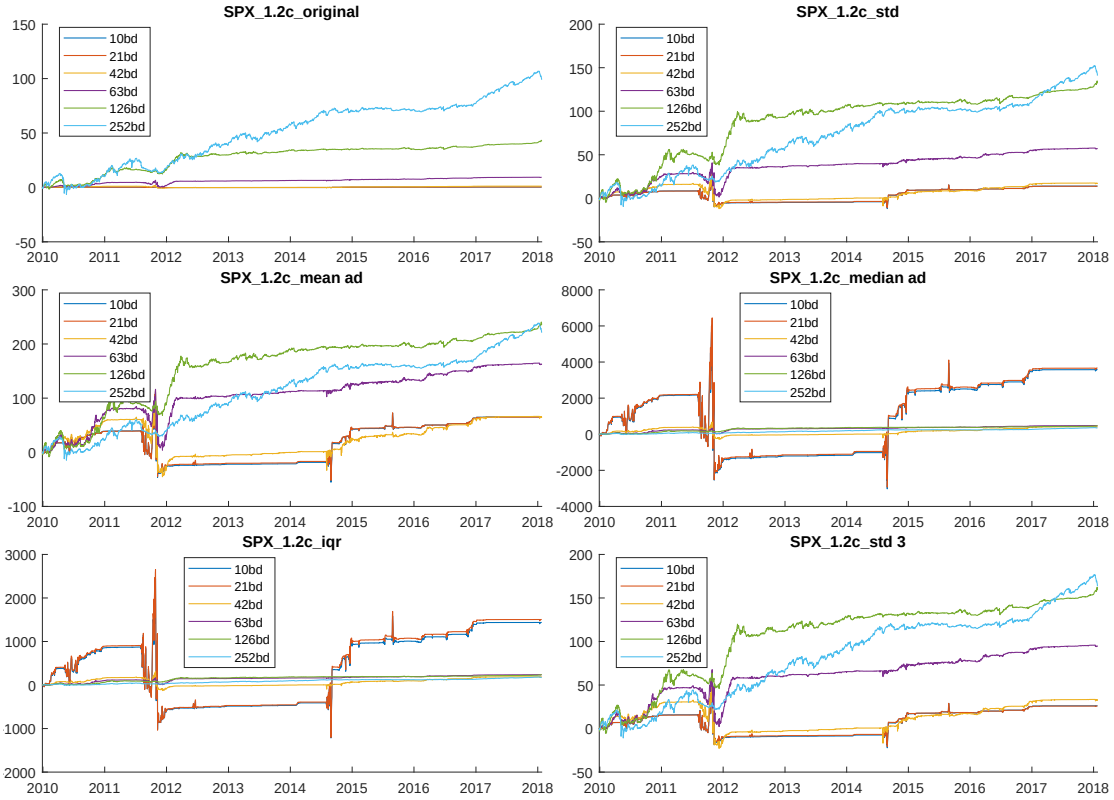


Figure 1: Comparing different methods of scaling by normalising some features of the daily returns. The six plots correspond to: no scaling, scaled by standard deviation, scaled by mean absolute deviation, scaled by median absolute deviation, scaled by inter-quartile range, scaled by standard deviation with clipped outliers (more than 3 st.dev. away from mean).

despite excluding the GFC, the time period we have chosen still includes some extreme events of smaller magnitudes, such as the European debt crisis at the end of 2011.

Figure 2 shows the performance of a few strategies from 2007 onwards versus from 2010 onwards, with appropriate scaling. We can see that GFC and its after-effects accounts for almost all of the performance of the four shorter maturity strategies (10bd, 21bd, 42bd and 63bd). On the other hand, when the GFC is excluded, we can better evaluate the typical performance of those strategies. A particular telling difference can be seen by comparing how the 2011 mini-crisis is depicted in the two graphs.

Therefore for most of the paper, we will be focusing our analysis on the period 2010–2018. In some cases, the analogous result from the period 2007–2018 will also be partially reported for comparison.

## 2.3 Benchmarking

As a benchmark, we have chosen the BNPIFUS index. The BNPIFUS is a synthetic index, available via Bloomberg, whose returns are the same as the shortest maturity future contract on the S&P 500. Since in the options strategies we study, the gains are not reinvested, this benchmark appears more relevant than the usual total return version of the S&P 500.

We also perform the same scaling on the BNPIFUS index, i.e., normalising the standard deviation of daily returns. Figure 3 shows the performance of some typical options strategies against the BNPIFUS.

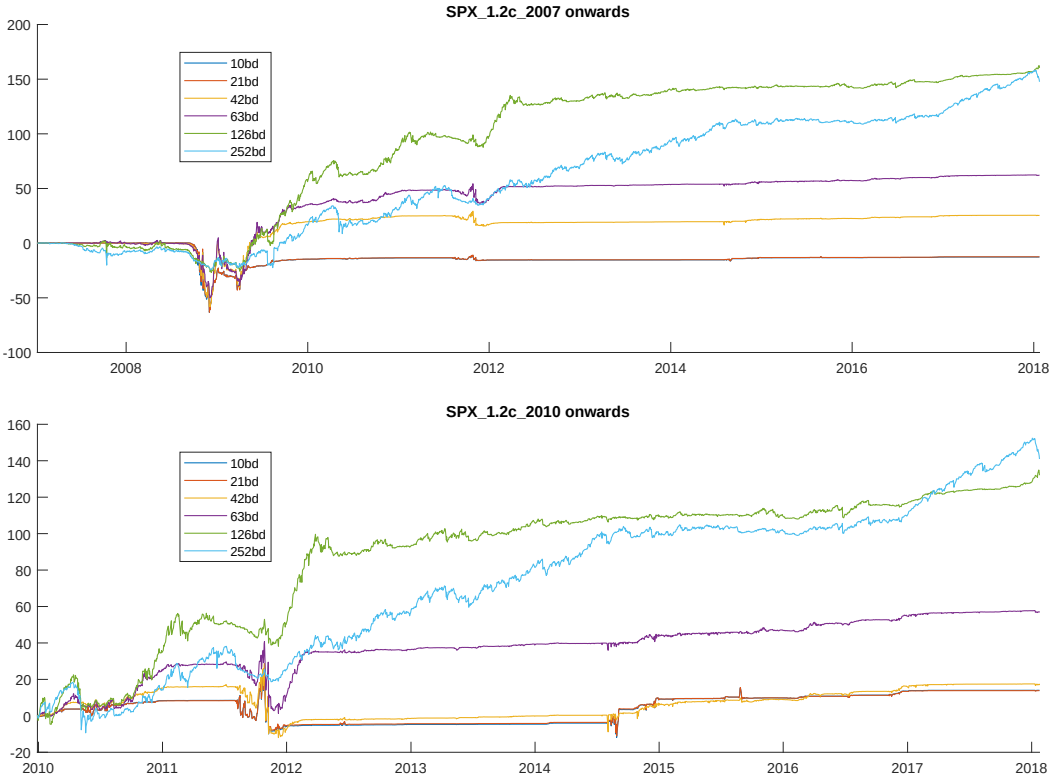


Figure 2: Showing the effect of including data from the GFC

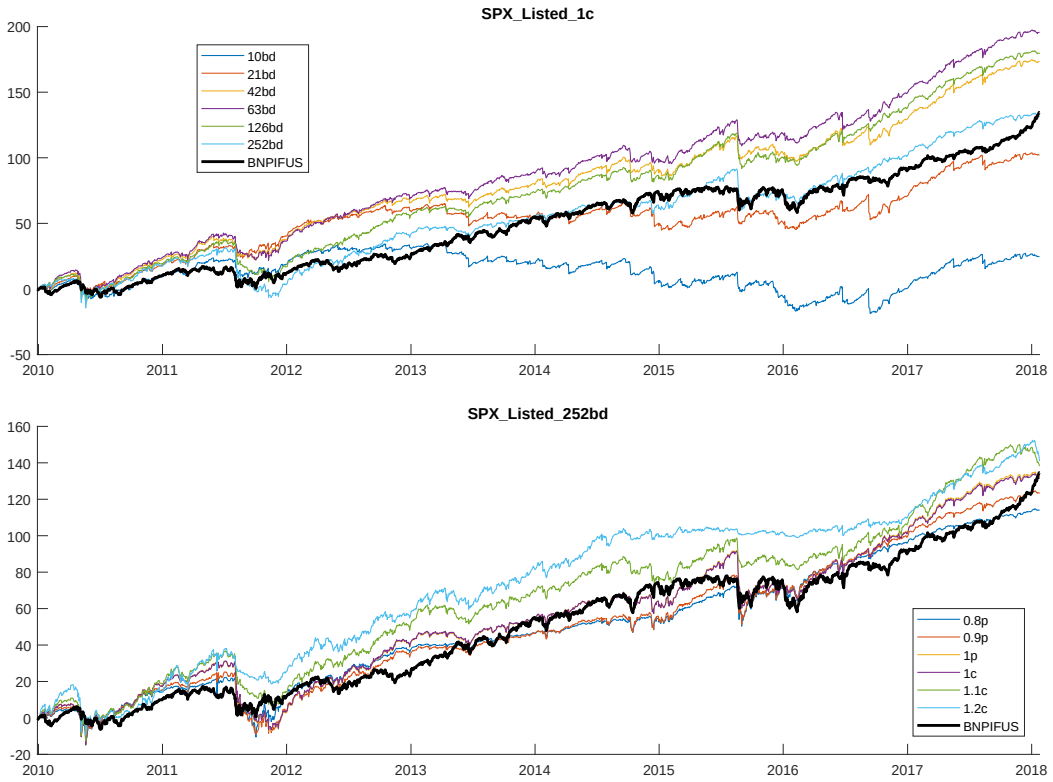


Figure 3: The top graph shows strategies that sell one ATM call for different maturities. The bottom graph shows strategies that sell one 1 Year maturity call for different strikes. The performances are benchmarked against the BNPIFUS index, shown by the black bold line.

# 3 Performance of the strategy v.s. other indicators

One natural question is how do these strategies perform compared to the market, or how do they perform relative to the level of implied volatility. In this section we will detect and measure the level of linear dependence between the returns of the strategies and various indicators.

## 3.1 Against implied volatility

Figure 4 shows the joint evolution of the six at-the-money strategies (with varying maturities) against their respective Black-Scholes implied volatilities. Figure 5 contains the corresponding scatter plots of the daily changes. The six implied volatility plots have similar shapes, indicating that the at-the-money implied volatility have similar movements across different maturities. Since the strategies involve selling options, the correlation between spot returns and changes in implied volatility are negative.

Generally speaking, options with longer maturities have greater vega exposure (due to the coupled term $\sigma\sqrt{T}$ the Black-Scholes model). This is indeed observed in Figures 4 and 5, where the correlation between spot returns and changes in implied volatility has greater magnitude for longer maturities.

## 3.2 Against forward

Next, Figure 6 shows the joint evolution of the same six strategies against their respective forwards. Figures 7 contains the corresponding scatter plots of the daily changes. Understandably the six forward curves are essentially identical, modulo changes in discount factors.

A basic property of vanilla option prices is that the convexity of prices at stock prices close to the strike increases with shorter maturities. In other words, options with shorter maturities have greater gamma exposure. This is visible from Figures 6 and 7, as larger absolute changes in forward values leads to greater spot losses in cases with short maturities. The shapes of the scatter plot is a direct consequence of the volatility smile, keeping in mind that each strategy has a short position on vanilla options.

Recall that each strategy is already delta hedged to eliminate the Black-Scholes delta. If the volatility and the underlying are uncorrelated, then the strategy and the forward should also be uncorrelated. However, from Figures 6 and 7, we see that the strategy and the forward are positively correlated, particularly for longer maturity cases. This suggests there is some correlation between the volatility and the underlying, a well-known stylised fact in equity market known as the leverage effect (see, e.g., [3]). Hence there exists some "residual delta" beyond the Black-Scholes delta. We will extract and further study this "residual delta" (as well as other Greeks) via regression techniques in the next section.

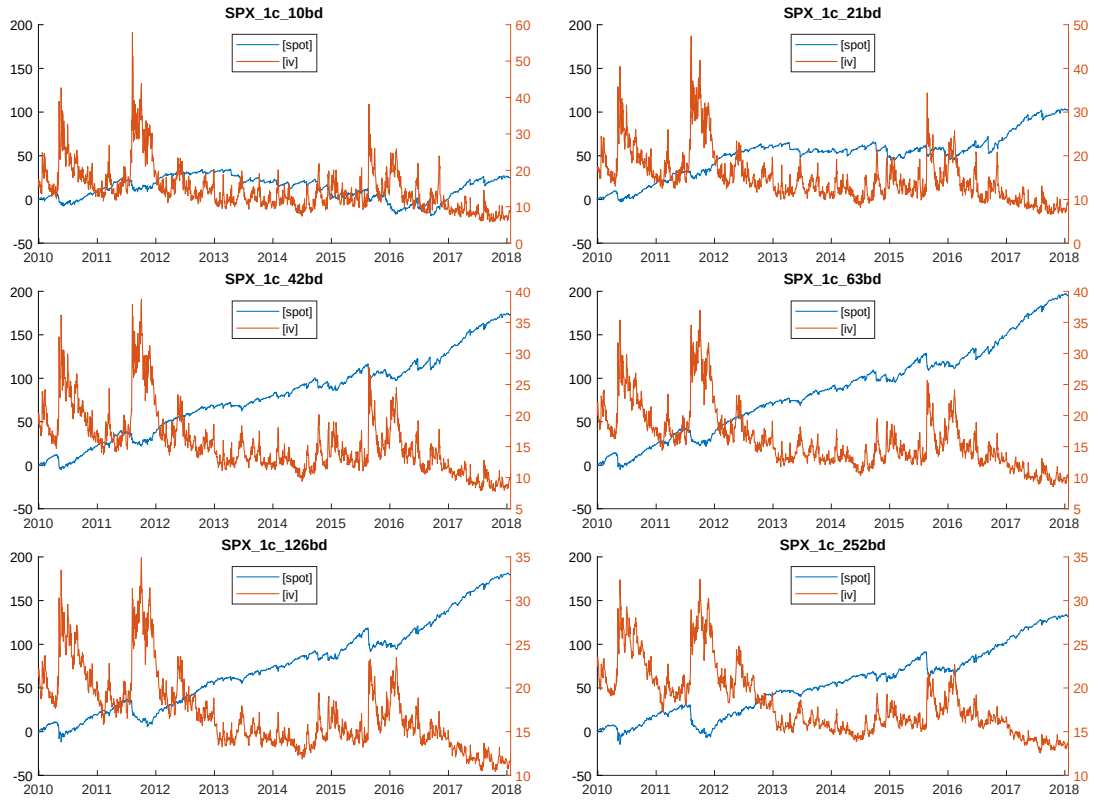


Figure 4: Joint evolution of the strategy and the implied volatility level (for the same maturity)

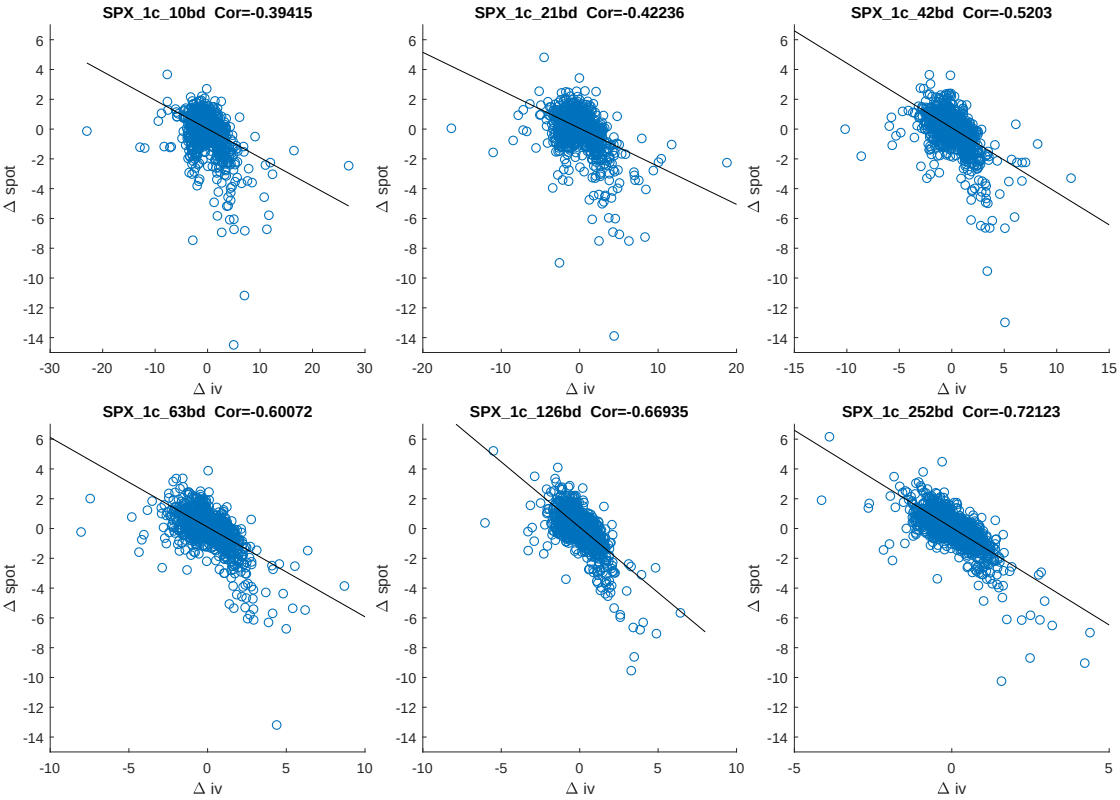


Figure 5: Scatter plot of the spot return against the change in implied volatility. The regression line and correlation coefficient are included for each subplot.

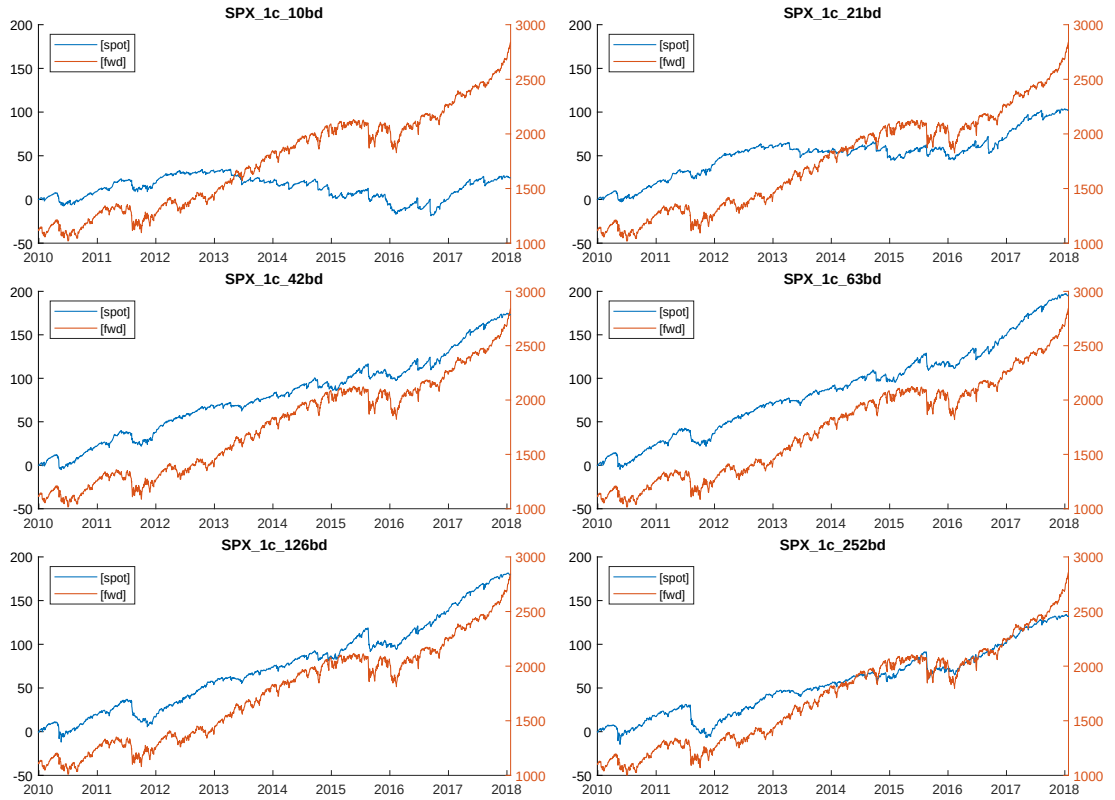


Figure 6: Joint evolution of the strategy and the forward level (for the same maturity)

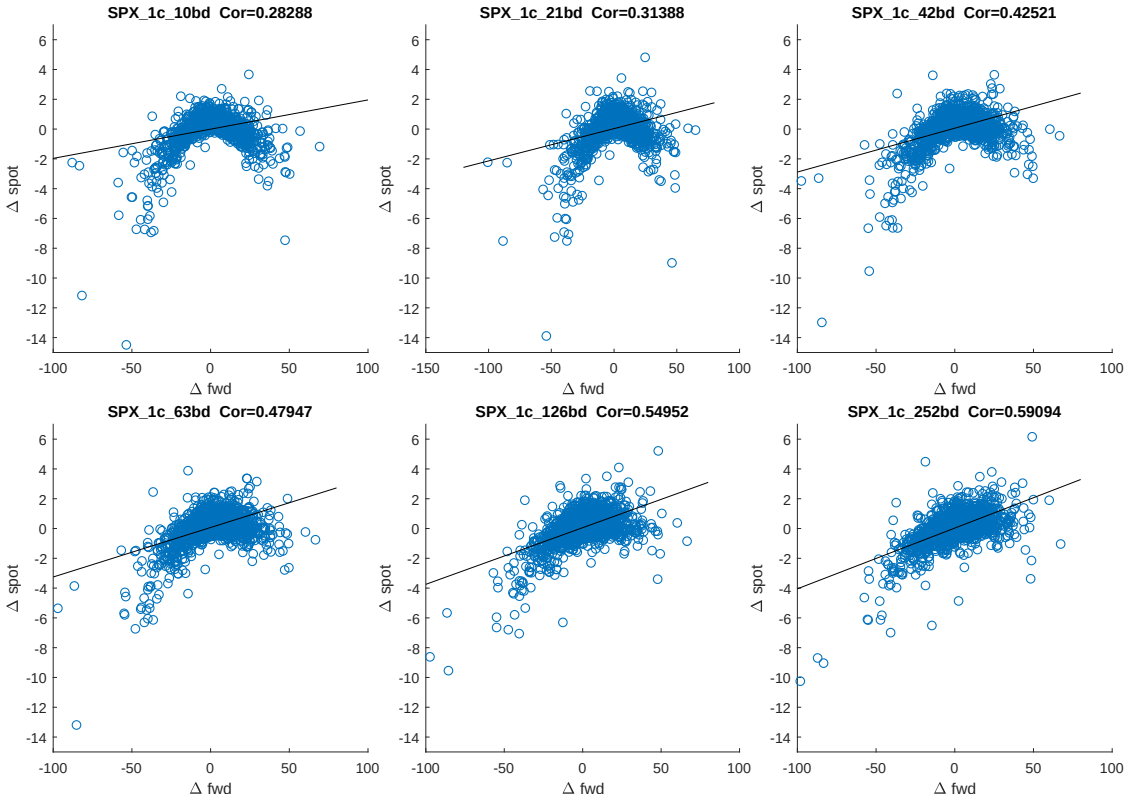


Figure 7: Scatter plot of the spot return against the forward return. The regression line and correlation coefficient are included for each subplot.

# 4 Empirical Greeks via regression

Let us denote the current value of the strategy by $u(t, s, \sigma)$, where $t$ is time, $s$ is the forward price of the underlying (at a fixed maturity) and $\sigma$ is the current implied volatility (at a fixed relative strike). Neglecting the effect of interest rates, the evolution of the strategy is given by

$$du = \partial_t u dt + \partial_s u dS_t + \partial_\sigma u d\sigma + \frac{1}{2}\partial_{ss} u (dS_t)^2 + \partial_{s\sigma} u (dS_t)(d\sigma) + \frac{1}{2}\partial_{\sigma\sigma} u (d\sigma)^2 + \cdots \tag{2}$$

$$= Theta + Delta + Vega + Gamma + Vanna + Volga + \cdots . \tag{3}$$

In other words, the performance of the strategy can be broken down into its sensitivity or Greeks with respect to the different indicators.

Since the portfolio composition of each strategy is dynamic, with old options expiring and new options entering every day at different absolute strike levels, it is difficult to analytically compute the contribution of each term. Instead, we aim to empirically estimate the Greeks via regression.

Note that, modulo the effect of interest rate and dividends, it is equivalent to substitute the underlying value by its forward price when analysing the contributions from Greeks. The notations $\Delta$fwd and $\Delta$iv are used to denote the daily changes in the forward and the implied volatility, respectively.

- Delta, $\partial_s u dS_t$ — sensitivity with respect to the forward price of the underlying.

  Since the strategies are already delta-hedged (in terms of Black-Scholes delta), we normally would expect the delta to be zero. However, this is not the only way to compute delta in models with non-constant volatility (see, e.g., [6]). As discussed in the previous section, there may be exist some residual delta due to correlations between the underlying and the volatility. We will measure this by regressing the returns against $\Delta$fwd.

- Vega, $\partial_\sigma u d\sigma$ — sensitivity with respect to the implied volatility.

  We will measure this by regressing the returns against $\Delta$iv. Note that the implied volatility is only taken daily at a particular relative strike determined by the strategy. The dynamic nature of the portfolio composition means that the strategy is actually sensitive to multiple parts of the volatility surface at any time. Hence this term will only be an estimate of the full effect.

- Gamma, $\frac{1}{2}\partial_{ss} u (dS_t)^2$ — second order sensitivity with respect to the forward price of the underlying.

  We will measure this by regressing the returns against $(\Delta\text{fwd})^2$. Since $(dS_t)^2 = (\sigma_{\text{realised}} S_t)^2 dt$ also corresponds to the quadratic variation of the underlying and the *realised volatility*, we can interpret gamma as a measure of the performance generated by the realised volatility.

- Theta, $\partial_t u dt$ — sensitivity with respect to time.

  Theta corresponds to the change of the value of the strategy over time, assuming all other parameters (forward, implied volatility) are held constant. We cannot measure this directly by regressing with respect to time, since that would give us the total (not partial) derivative. In order to properly measure this, let us first recall that, neglecting the interest rate, the Black-Scholes equation stipulates that $\partial_t u + \frac{1}{2}\sigma^2 S_t^2 \partial_{ss} u = 0$ where $\sigma$ is the implied volatility. Hence we will measure this by regressing the returns against $(\text{iv})^2$. Thus theta can also be interpreted as a measure of the performance of the strategy generated by the *implied volatility*.

- Vanna, $\partial_{s\sigma} u (dS_t)(d\sigma)$ — sensitivity with respect to the leverage between underlying and implied volatility.

  We will measure this by regressing the returns against $(\Delta\text{fwd})(\Delta\text{iv})$. This effectively corresponds to the correlation between the underlying and the implied volatility.

- Volga $\frac{1}{2}\partial_{\sigma\sigma} u (d\sigma)^2$ — second order sensitivity with respect to the implied volatility.

  This term is measureed by regressing the returns against $(\Delta\text{iv})^2$.

Since the various indicators may be correlated, we expect to see some overlap between the contributions of different Greeks. Even though the overlap cannot be explicitly assigned to any individual term, the regression will naturally take this into account when computing the total variance contributed by the Greeks.

The complete regression is of the form

$$\Delta u = A\Delta\text{fwd} + B\Delta\text{iv} + C(\Delta\text{fwd})^2 + D(\text{iv})^2 + E(\Delta\text{fwd})(\Delta\text{iv}) + F(\Delta\text{iv})^2 + \text{residuals}.$$

Note that $A, B, C, D, E$ and $F$ are **not constants**, instead they are **functions** of the form $\sum_{i=-2}^{5} \alpha_i(\text{fwd})^i$ fitted by the regression. These are used to allow the Greeks some local dependence on the forward price of the underlying.

In the following tests, various terms of the regression are switched on and off, depending on the Greeks to be measured. The results are presented in Figures 8 to 13, where the evolution of the strategy return is plotted against the regression estimate for various combinations of Greeks. The $R^2$ value for each regression is also provided. Figures 14 and 15 shows the detailed composition of the regression estimate in the full regression. For reference, Figure 16 shows the analogous breakdown for the period including 2008.

## 4.1 Delta and Vega

Figure 8 shows that there is some residual delta detected for longer maturities, but overall the sensitivity against the underlying is very low. This is to be expected as the strategies are already delta-hedged and we are essentially detecting parts of the vega (and correlations between the underlying and the implied volatility) at longer maturities. This is confirmed by Figures 9 and 10, since the addition of delta to vega makes very little difference.

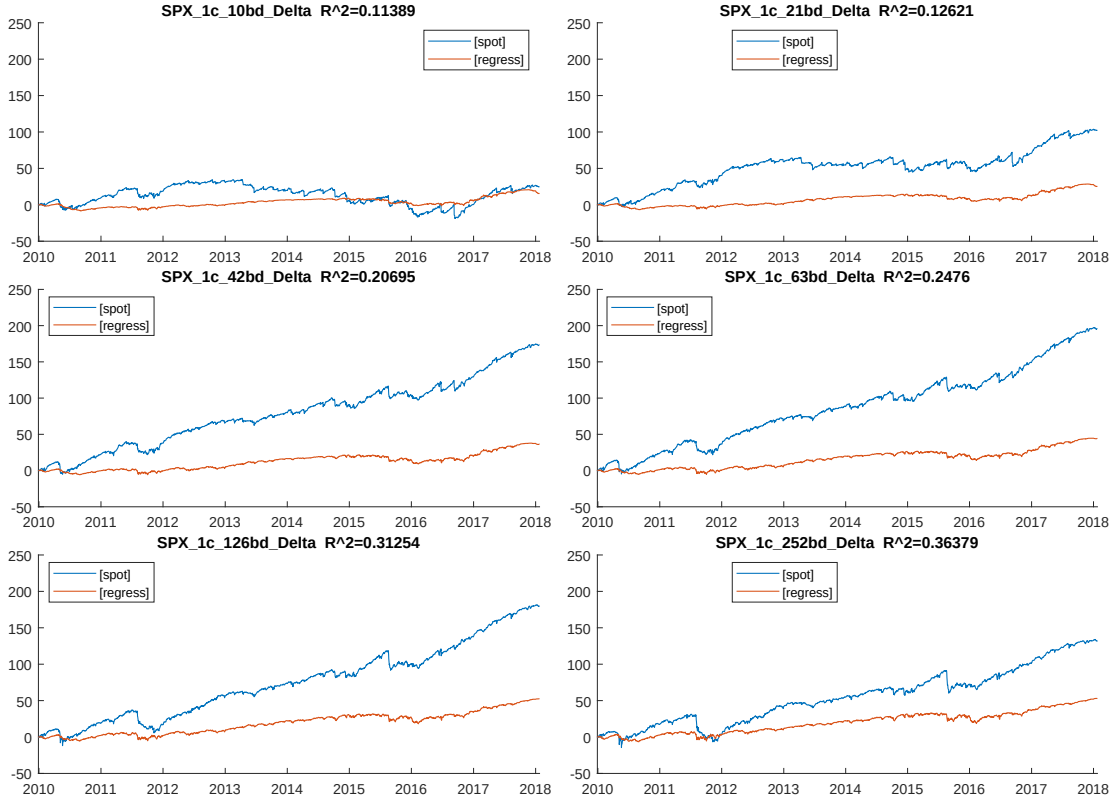


Figure 8: Regression against delta.

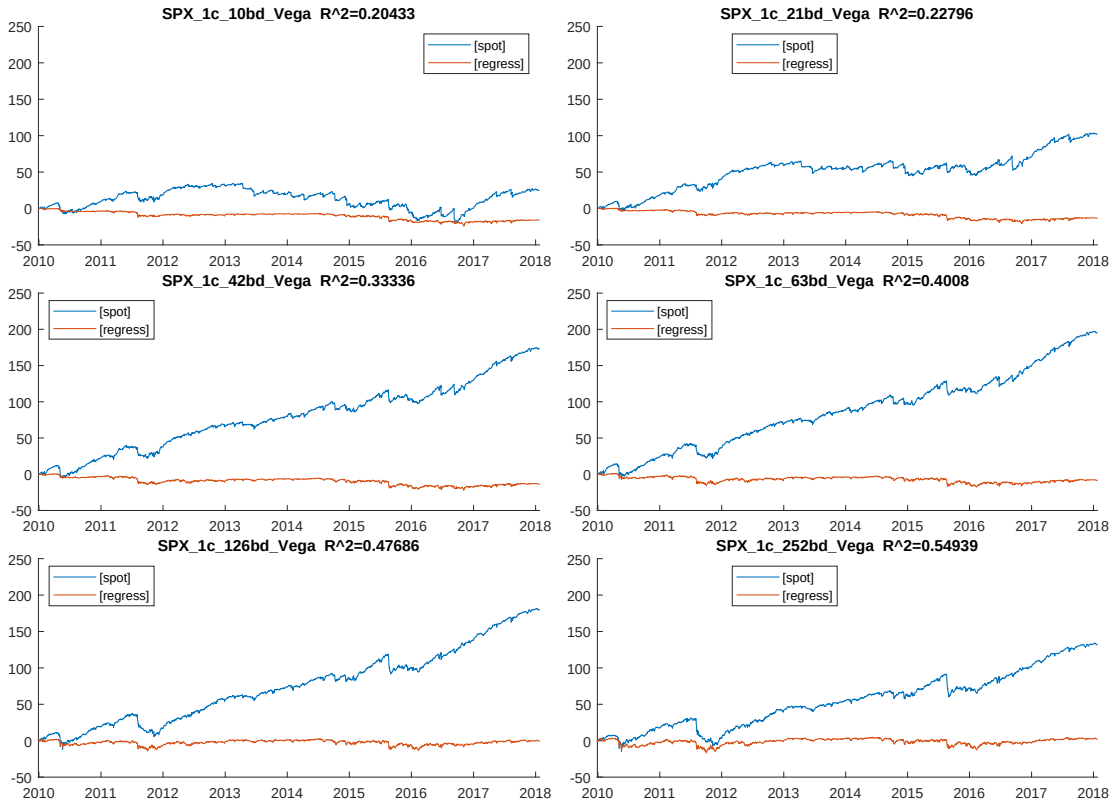


Figure 9: Regression against vega.

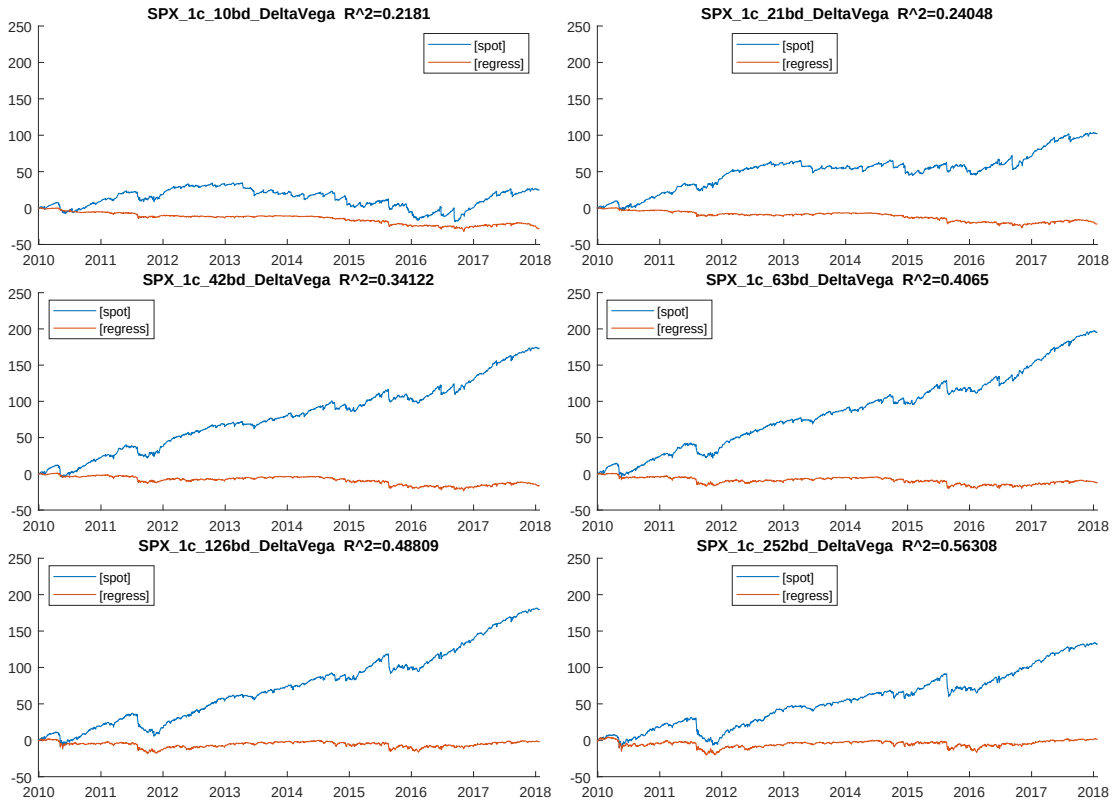


Figure 10: Regression against delta and vega.

## 4.2 Gamma, Theta, Vanna and Volga

Next, we progressively add the "second order terms", i.e., gamma, theta, vanna and volga to the regression in Figures 11, 12 and 13. These terms captures the spread between realised and implied volatilities. We observe the opposite phenomenon to before: shorter maturities have higher exposure to these second order terms. Overall, most of these contributions come from gamma and theta, while the impact of vanna and volga is negligible.

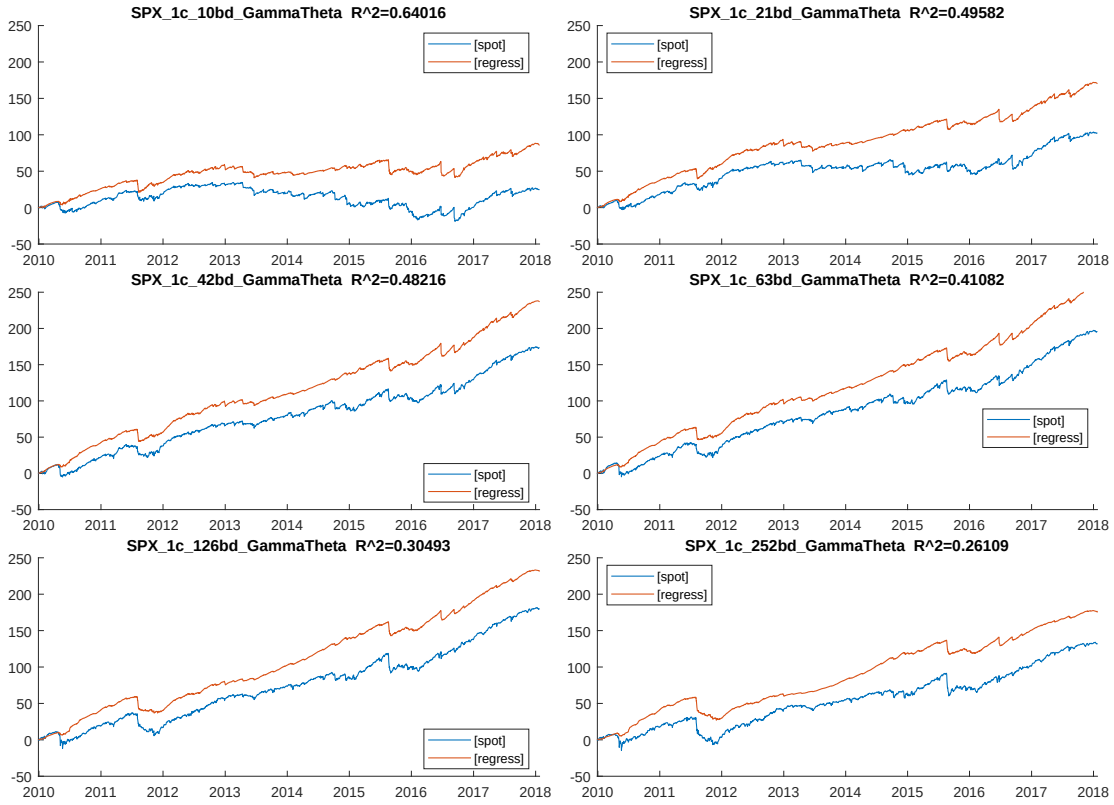


Figure 11: Regression against gamma and theta.

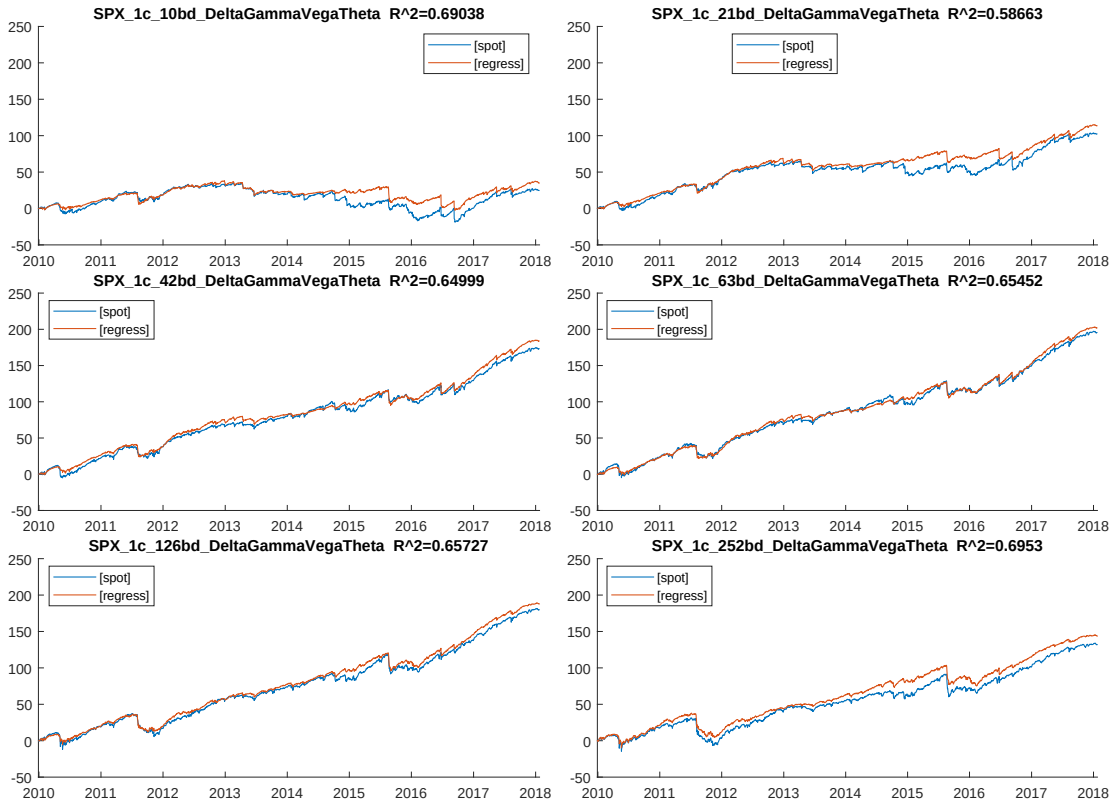


Figure 12: Regression against delta, gamma, vega and theta.

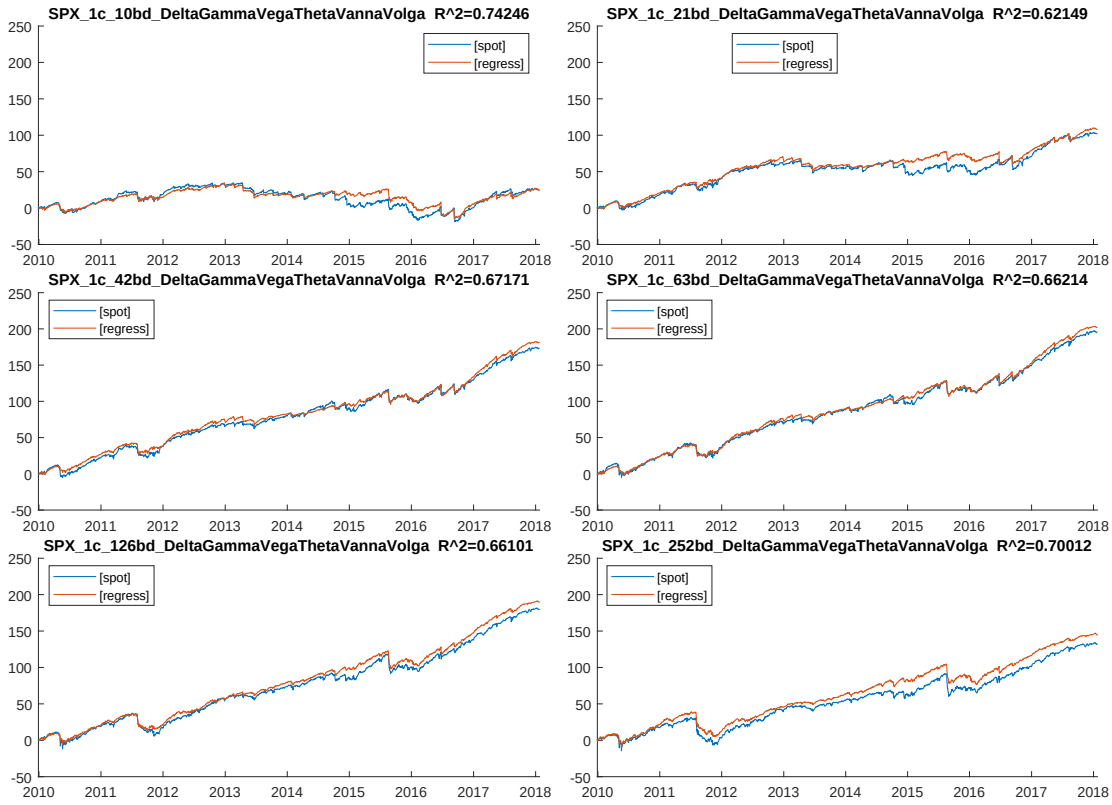


Figure 13: Regression against delta, gamma, vega, theta, vanna and volga.

## 4.3 Realised versus implied

Figures 14 and 15 shows the detailed composition of the regression estimate into delta, gamma, vega, theta and vanna. In particular, we see that gamma and theta have much greater magnitude than the rest. For reference, a similar breakdown for the period including 2008 is shown in Figure 16. We can clearly see the impact of the GFC on the second order component, particularly in the very short and very long maturities.

To summarise, shorter maturity strategies have lower delta and vega exposure than longer maturities, but higher exposure to second order terms. Medium maturity strategies (between 42bd and 126bd) are able to generate most consistent performances via the spread between realised and implied volatilities. This is because, for very short maturities, the gamma (realised) and theta (implied) are very close in magnitude and mostly cancel out, offering very little volatility premium. On the other hand, for very long maturities, both gamma (realised) and theta (implied) decrease in magnitude, reducing the resulting spread.

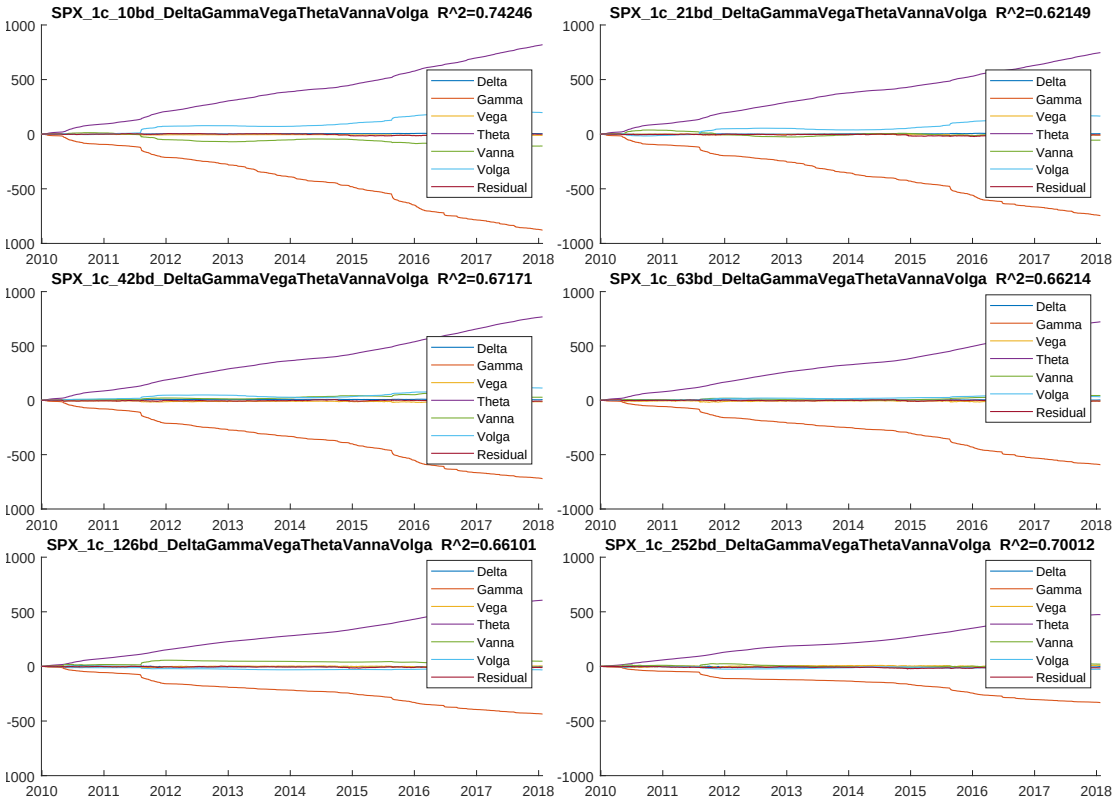


Figure 14: Regression result broken down into delta, gamma, vega, theta, vanna and volga

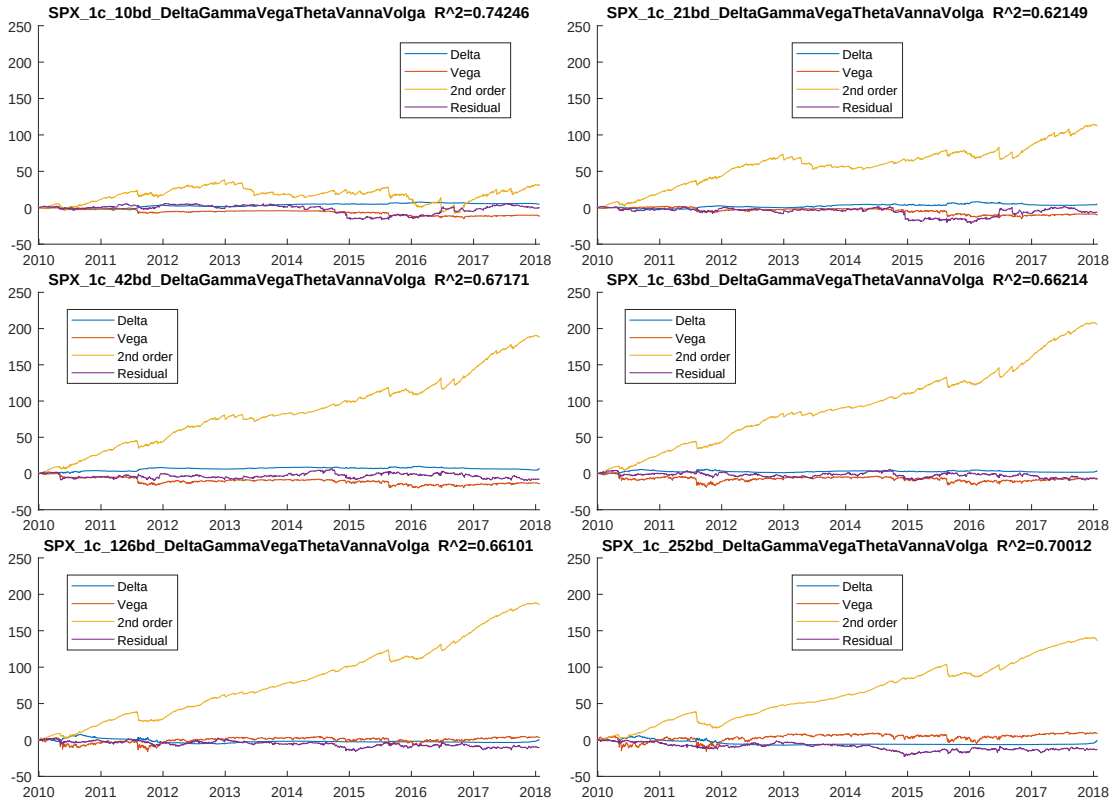


Figure 15: Delta, vega and 2nd order terms (gamma+theta+vanna+volga)

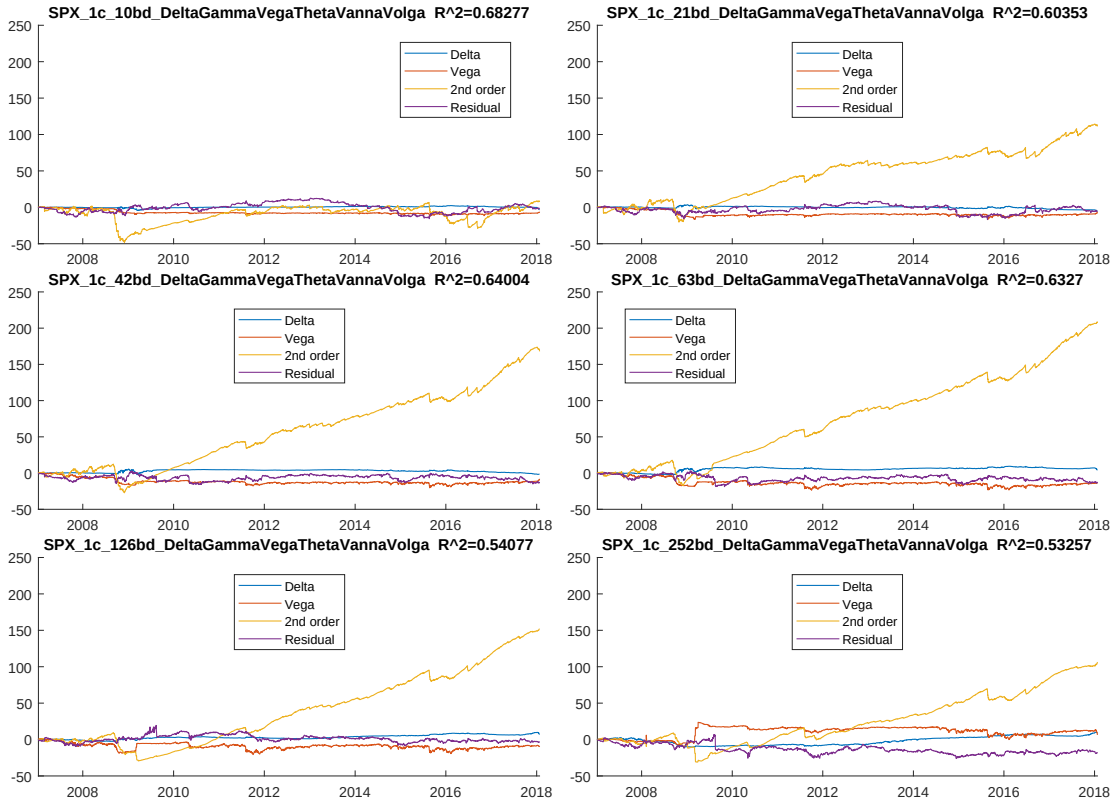


Figure 16: Delta, vega and 2nd order terms (gamma+theta+vanna+volga) for 2008 onwards

### 4.4 Summary of findings

We have empirically decomposed the return of the options strategy in to various sensitivities via a regression technique with respect to the underlying and the implied volatility. We find that shorter maturity strategies have lower delta and vega exposure but higher second order exposures, i.e., to gamma, theta, vanna and volga. The second order terms are mostly dominated by theta and gamma, which correspond to the effects of the implied and realised volatilities, respectively. The combined second order term can be interpreted as the spread between implied and realised volatility, also known as the volatility risk premium.

Overall, the best performing strategies are those with medium maturities (from 42bd to 126bd), as they can generate the most consistent returns via the spread between realised and implied volatilities. This can be attributed to the fact that, implied and realised volatilities mostly cancel out at very short maturities but have decreasing magnitudes for very long maturities.

# 5 Single and pair strategies

In this section, we examine and rank the performance of the strategies across all available strikes and maturities. We begin by simply comparing the single strategies, i.e., strategies involving options of a single strike and maturity, before moving on to looking at pair strategies traded at a 1:1 ratio (e.g., strike strangles). **As before, all single and pairs strategies are scaled so that the daily returns have unit standard deviation.** The strategies are ranked according to their annualised Sharpe ratio.

In the result tables, we display the following properties.

- The annualised **Sharpe ratio** is the principle performance criteria used to judge the performance of various strategies. As usual, it is calculated by dividing the return of the strategy by the standard deviation.
- We also include the metric **max transaction cost** or Max TC. It is a conservative estimate of the maximum daily transaction cost (in points of implied volatility) the strategy can incur while still maintaining a positive return. This is calculated by dividing the total return by the total vega of the strategy, which is given by he individual vegas multiplied by the absolutes value of the corresponding weights, summed across all days and constituents of the strategy.
- The **options** traded in the strategy and their corresponding **weights**. Note that, since each strategy by default is selling an option every day, **a positive weight corresponds to a short position in the option while a negative weight correspond to a long position in the option**.
- The **alpha and beta** of the strategy. These are computed via a simple regression against the BNPIFUS index. It provides some clues to whether the strategy is moving with or against the market.

  Since all strategies (and combination of strategies) as well as the BNPIFUS benchmark are scaled to have unit standard deviation, **the regression beta is in fact equal to the correlation between the strategy and the benchmark.** Using this fact, we can easily compute the **beta-adjusted Sharpe ratio** (i.e., Sharpe ratio of the strategy minus beta times the BNPIFUS index) in the following way:

$$\text{beta-adjusted Sharpe ratio} = \text{alpha} \times \sqrt{1 - \text{beta}^2} \times \sqrt{252}.$$

- The **99% CVaR** of daily returns is also included. For reference, note that the 99% CVaR of a standard normal is approximately -2.6652.
- The **Sharpe ratio** of the same strategy, but including 2008. This provides some indication of the robustness of the strategy in the event of an extreme market crash.

Note that these properties will be used throughout the remainder of the paper, from Section 5 to Section 8.

Since in all cases the two ATM options (1p and 1c) have almost identical trajectories, only 1c will be consider while 1p will be omitted, reducing repetition in the results. For reference, the benchmark, BNPIFUS index, has a Sharpe ratio of 1.0505 and a 99% CVaR of -3.6557 over the same period of testing, from 2010 to 2018. If we include the GFC, the BNPIFUS index has a Sharpe ratio of 0.6021 from 2007 to 2018.

## 5.1 Single strategies

Table 17 shows the top 20 strategies involving vanilla options of a single strike and maturity, while the top 5 plots are given by Figure 17.

The best performing strategies are the close to the money calls (1c or 1.1c) at a medium maturity (42bd to 126bd). At-the-money and upside strike strategies (1c, 1.1c and 1.2c) tend to outperform downside strike strategies (0.8p and 0.9p). This could be explained by the fact that the market rallied during the period of analysis.

| | SR | MaxTC | Option | Weight | Alpha | Beta | CVar | SR08 |
|---|---|---|---|---|---|---|---|---|
| 1 | 1.5261 | 1.9706 | 1c,63bd | 0.6070 | 0.0653 | 0.4660 | -5.6225 | 1.0577 |
| 2 | 1.4979 | 3.1628 | 1.1c,42bd | 2.8124 | 0.0694 | 0.3780 | -5.3961 | 1.1285 |
| 3 | 1.4734 | 3.4738 | 1.1c,126bd | 1.3098 | 0.0577 | 0.5309 | -4.9735 | 1.0659 |
| 4 | 1.4333 | 3.0291 | 1.1c,63bd | 1.4218 | 0.0618 | 0.4305 | -5.4106 | 1.0053 |
| 5 | 1.3999 | 2.6133 | 1c,126bd | 0.8840 | 0.0530 | 0.5322 | -5.3932 | 0.8337 |
| 6 | 1.3527 | 1.5140 | 1c,42bd | 0.8397 | 0.0578 | 0.4150 | -5.7682 | 0.8358 |
| 7 | 1.1484 | 2.3670 | 1.1c,21bd | 4.8042 | 0.0490 | 0.3522 | -6.1557 | 0.5231 |
| 8 | 1.1358 | 2.3364 | 1.1c,10bd | 5.8607 | 0.0481 | 0.3547 | -6.2018 | 0.4149 |
| 9 | 1.1006 | 5.2916 | 1.2c,252bd | 1.4251 | 0.0320 | 0.5639 | -5.0403 | 0.8423 |
| 10 | 1.0788 | 3.4162 | 1.1c,252bd | 0.9269 | 0.0329 | 0.5303 | -4.9452 | 0.7724 |
| 11 | 1.0312 | 4.3810 | 1.2c,126bd | 3.1152 | 0.0330 | 0.4836 | -5.7445 | 0.9145 |
| 12 | 1.0249 | 2.9170 | 1c,252bd | 0.8174 | 0.0263 | 0.5789 | -5.4265 | 0.5962 |
| 13 | 0.9619 | 3.1684 | 0.9p,252bd | 0.8420 | 0.0173 | 0.6543 | -5.7630 | 0.4431 |
| 14 | 0.9044 | 2.4999 | 0.9p,126bd | 0.8330 | 0.0181 | 0.5871 | -6.1594 | 0.4462 |
| 15 | 0.9019 | 2.1435 | 0.9p,63bd | 0.6028 | 0.0233 | 0.5060 | -6.5254 | 0.6705 |
| 16 | 0.8888 | 3.5493 | 0.8p,252bd | 0.9973 | 0.0128 | 0.6525 | -5.8490 | 0.3341 |
| 17 | 0.8560 | 3.2471 | 0.8p,126bd | 1.0779 | 0.0140 | 0.6026 | -6.1642 | 0.3226 |
| 18 | 0.8054 | 3.1936 | 0.8p,63bd | 0.8845 | 0.0184 | 0.4892 | -6.0672 | 0.1873 |
| 19 | 0.7966 | 0.7768 | 1c,21bd | 0.6978 | 0.0301 | 0.3042 | -6.0877 | 0.5398 |
| 20 | 0.7862 | 1.9425 | 0.9p,42bd | 0.8908 | 0.0204 | 0.4404 | -6.6043 | 0.5839 |

Table 1: Top 20 strategies.

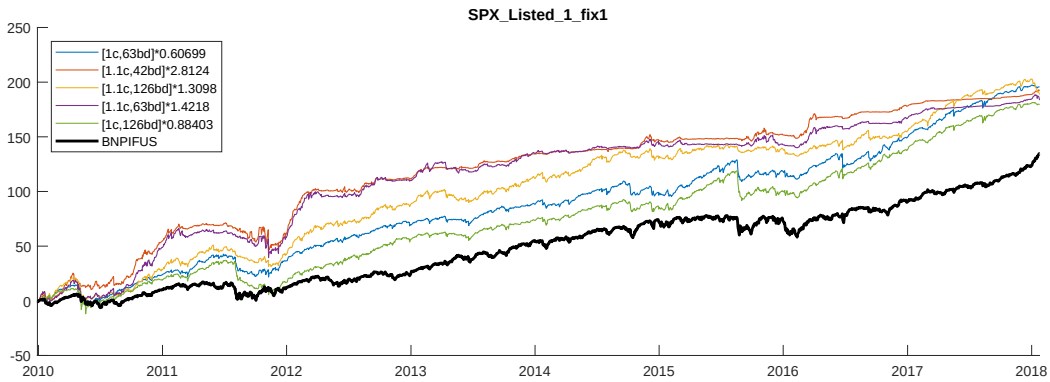


Figure 17: Plots of the top 5 strategies. The performances are benchmarked against the BNPIFUS index, shown by the black bold line.

## 5.2 Strike pairs

Table 2 shows the top 20 strike pairs, i.e., two options with different strikes but the same maturity, traded at a 1:1 ratio. This shows the effect of diversification across the volatility skew. The top 5 plots are given by Figure 18.

All of the top 20 only involve options of medium maturity (42bd to 126bd). Overall, the top strike pairs outperform the top single strategies and have a slightly improved beta.

| | SR | MaxTC | Option | Weight | Option | Weight | Alpha | Beta | CVar | SR08 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.6619 | 2.1896 | 1c,63bd | 0.4718 | 1.1c,63bd | 0.4718 | 0.0713 | 0.5051 | -5.2120 | 1.1440 |
| 2 | 1.5560 | 1.7393 | 1c,42bd | 0.7259 | 1.1c,42bd | 0.7259 | 0.0678 | 0.4563 | -5.4849 | 1.0140 |
| 3 | 1.5209 | 1.9723 | 1c,63bd | 0.5881 | 1.2c,63bd | 0.5881 | 0.0640 | 0.4804 | -5.5992 | 1.0394 |
| 4 | 1.5018 | 2.9130 | 1c,126bd | 0.5545 | 1.1c,126bd | 0.5545 | 0.0576 | 0.5586 | -5.1568 | 0.9881 |
| 5 | 1.4486 | 2.8093 | 1c,126bd | 0.7566 | 1.2c,126bd | 0.7566 | 0.0533 | 0.5730 | -5.1425 | 0.9599 |
| 6 | 1.4343 | 3.6454 | 1.1c,126bd | 0.9851 | 1.2c,126bd | 0.9851 | 0.0538 | 0.5522 | -5.1323 | 1.0751 |
| 7 | 1.4312 | 2.1940 | 0.8p,63bd | 0.4179 | 1c,63bd | 0.4179 | 0.0536 | 0.5520 | -6.1408 | 0.7944 |
| 8 | 1.3720 | 1.7383 | 0.8p,42bd | 0.6486 | 1c,42bd | 0.6486 | 0.0535 | 0.4980 | -6.1828 | 0.6839 |
| 9 | 1.3495 | 3.0094 | 1.1c,42bd | 2.4923 | 1.2c,42bd | 2.4923 | 0.0602 | 0.3744 | -5.4687 | 0.9014 |
| 10 | 1.3440 | 1.5060 | 1c,42bd | 0.8297 | 1.2c,42bd | 0.8297 | 0.0567 | 0.4232 | -5.7604 | 0.8141 |
| 11 | 1.3360 | 3.3762 | 0.8p,126bd | 0.6962 | 1.1c,126bd | 0.6962 | 0.0397 | 0.6714 | -5.6183 | 0.7807 |
| 12 | 1.3220 | 2.0317 | 0.9p,63bd | 0.3296 | 1c,63bd | 0.3296 | 0.0482 | 0.5298 | -6.1640 | 0.9344 |
| 13 | 1.3177 | 2.9334 | 1.1c,63bd | 1.2201 | 1.2c,63bd | 1.2201 | 0.0546 | 0.4294 | -5.4145 | 0.8797 |
| 14 | 1.3070 | 3.1050 | 0.8p,63bd | 0.6812 | 1.1c,63bd | 0.6812 | 0.0438 | 0.5831 | -5.9477 | 0.6199 |
| 15 | 1.2765 | 3.2276 | 0.8p,42bd | 1.2308 | 1.1c,42bd | 1.2308 | 0.0472 | 0.5021 | -6.1136 | 0.6097 |
| 16 | 1.2593 | 2.9159 | 0.9p,126bd | 0.5697 | 1.1c,126bd | 0.5697 | 0.0375 | 0.6324 | -5.6612 | 0.7749 |
| 17 | 1.2564 | 2.7956 | 0.8p,126bd | 0.5284 | 1c,126bd | 0.5284 | 0.0385 | 0.6135 | -5.9299 | 0.6728 |
| 18 | 1.2463 | 2.4294 | 0.9p,63bd | 0.4977 | 1.1c,63bd | 0.4977 | 0.0409 | 0.5685 | -6.2038 | 0.9155 |
| 19 | 1.2270 | 1.6422 | 0.9p,42bd | 0.4920 | 1c,42bd | 0.4920 | 0.0451 | 0.4864 | -6.3312 | 0.8024 |
| 20 | 1.1898 | 2.5660 | 0.9p,126bd | 0.4457 | 1c,126bd | 0.4457 | 0.0364 | 0.5825 | -5.8603 | 0.6702 |

Table 2: Top 20 strike pairs, traded at the 1:1 ratio.

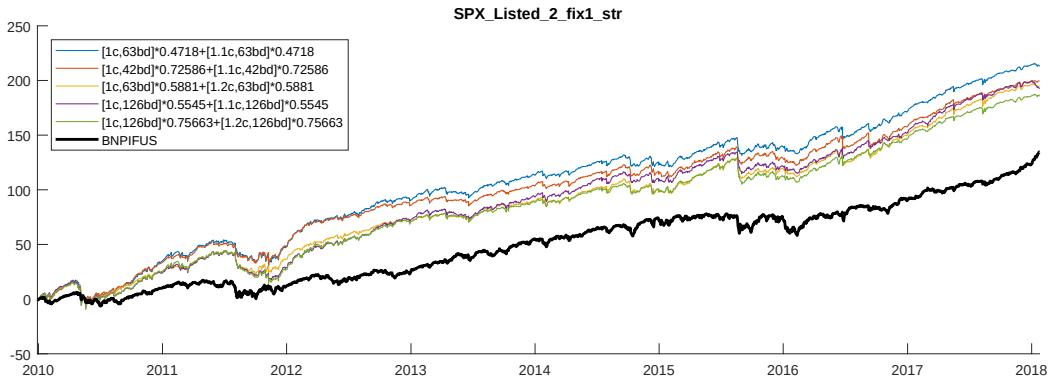


Figure 18: Plots of the top 5 strike pairs. The performances are benchmarked against the BNPIFUS index, shown by the black bold line.

## 5.3 Strike strangles

Table 3 shows the top 20 strike strangles, i.e., one out of the money call option and one out of money put option with the same maturity, traded at a 1:1 ratio. The top 5 plots are given by Figure 18.

The strategies considered here is a subset of the strike pairs considered earlier. Compared to single strategies, the top strike strangles have increased beta but decreased alpha.

| | SR | MaxTC | Option | Weight | Option | Weight | Alpha | Beta | CVar | SR08 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.3360 | 3.3762 | 0.8p,126bd | 0.6962 | 1.1c,126bd | 0.6962 | 0.0397 | 0.6714 | -5.6183 | 0.7807 |
| 2 | 1.3070 | 3.1050 | 0.8p,63bd | 0.6812 | 1.1c,63bd | 0.6812 | 0.0438 | 0.5831 | -5.9477 | 0.6199 |
| 3 | 1.2765 | 3.2276 | 0.8p,42bd | 1.2308 | 1.1c,42bd | 1.2308 | 0.0472 | 0.5021 | -6.1136 | 0.6097 |
| 4 | 1.2593 | 2.9159 | 0.9p,126bd | 0.5697 | 1.1c,126bd | 0.5697 | 0.0375 | 0.6324 | -5.6612 | 0.7749 |
| 5 | 1.2463 | 2.4294 | 0.9p,63bd | 0.4977 | 1.1c,63bd | 0.4977 | 0.0409 | 0.5685 | -6.2038 | 0.9155 |
| 6 | 1.1181 | 2.2725 | 0.9p,42bd | 0.7900 | 1.1c,42bd | 0.7900 | 0.0376 | 0.4967 | -6.5890 | 0.8630 |
| 7 | 1.1116 | 3.7802 | 0.9p,252bd | 0.5806 | 1.2c,252bd | 0.5806 | 0.0250 | 0.6809 | -5.3656 | 0.6941 |
| 8 | 1.0899 | 4.1898 | 0.8p,252bd | 0.6552 | 1.2c,252bd | 0.6552 | 0.0231 | 0.6879 | -5.3389 | 0.6359 |
| 9 | 1.0753 | 3.2888 | 0.9p,252bd | 0.4663 | 1.1c,252bd | 0.4663 | 0.0261 | 0.6291 | -5.4314 | 0.6599 |
| 10 | 1.0682 | 3.4727 | 0.8p,252bd | 0.5198 | 1.1c,252bd | 0.5198 | 0.0251 | 0.6375 | -5.4406 | 0.6202 |
| 11 | 1.0499 | 3.5147 | 0.8p,126bd | 0.9332 | 1.2c,126bd | 0.9332 | 0.0220 | 0.6665 | -5.9103 | 0.5956 |
| 12 | 1.0433 | 2.7787 | 0.9p,126bd | 0.7364 | 1.2c,126bd | 0.7364 | 0.0238 | 0.6333 | -5.9621 | 0.6435 |
| 13 | 0.9216 | 2.1381 | 0.9p,63bd | 0.5878 | 1.2c,63bd | 0.5878 | 0.0235 | 0.5223 | -6.5623 | 0.6932 |
| 14 | 0.8516 | 1.9154 | 0.9p,21bd | 0.9418 | 1.1c,21bd | 0.9418 | 0.0291 | 0.3706 | -6.9362 | 0.5829 |
| 15 | 0.8502 | 1.9072 | 0.9p,10bd | 1.1426 | 1.1c,10bd | 1.1426 | 0.0292 | 0.3678 | -6.9369 | 0.5303 |
| 16 | 0.8376 | 3.0655 | 0.8p,63bd | 0.8527 | 1.2c,63bd | 0.8527 | 0.0188 | 0.5135 | -6.2220 | 0.2515 |
| 17 | 0.8018 | 2.6631 | 0.8p,21bd | 1.5989 | 1.1c,21bd | 1.5989 | 0.0258 | 0.3732 | -5.8085 | 0.0901 |
| 18 | 0.7988 | 2.6514 | 0.8p,10bd | 1.9518 | 1.1c,10bd | 1.9518 | 0.0256 | 0.3730 | -5.8096 | 0.0636 |
| 19 | 0.7854 | 1.9135 | 0.9p,42bd | 0.8811 | 1.2c,42bd | 0.8811 | 0.0197 | 0.4495 | -6.6409 | 0.5763 |
| 20 | 0.7598 | 3.1214 | 0.8p,42bd | 1.4802 | 1.2c,42bd | 1.4802 | 0.0195 | 0.4283 | -5.8771 | 0.1682 |

Table 3: Top 20 strike strangles, traded at the 1:1 ratio.

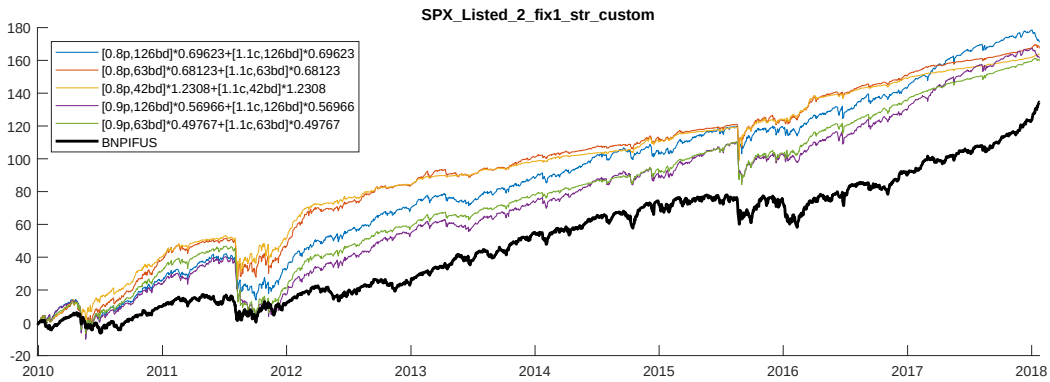


Figure 19: Plots of the top 5 strike strangles. The performances are benchmarked against the BNPIFUS index, shown by the black bold line.

## 5.4 Calendar pairs

Table 4 shows the top 20 calendar pairs, i.e., two options with different maturities but the same strike, traded at a 1:1 ratio. This shows the effect of diversification across the term structure. The top 5 plots are given by Figure 20.

Once again, the best strategies are the upside strike options with medium maturities.

| | SR | MaxTC | Option | Weight | Option | Weight | Alpha | Beta | CVar | SR08 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.6616 | 3.3674 | 1.1c,42bd | 1.0024 | 1.1c,126bd | 1.0024 | 0.0689 | 0.5410 | -4.9715 | 1.2290 |
| 2 | 1.5838 | 3.2107 | 1.1c,21bd | 1.1612 | 1.1c,126bd | 1.1612 | 0.0630 | 0.5558 | -4.9770 | 1.0549 |
| 3 | 1.5715 | 3.2484 | 1.1c,63bd | 0.7367 | 1.1c,126bd | 0.7367 | 0.0645 | 0.5217 | -5.1284 | 1.1161 |
| 4 | 1.5690 | 3.2418 | 1.1c,10bd | 1.1898 | 1.1c,126bd | 1.1898 | 0.0622 | 0.5542 | -4.9862 | 1.0268 |
| 5 | 1.5023 | 2.1776 | 1c,63bd | 0.3666 | 1c,126bd | 0.3666 | 0.0614 | 0.5022 | -5.5907 | 0.9973 |
| 6 | 1.4936 | 3.0740 | 1.1c,42bd | 0.9694 | 1.1c,63bd | 0.9694 | 0.0660 | 0.4238 | -5.2232 | 1.0786 |
| 7 | 1.4694 | 1.7630 | 1c,42bd | 0.3562 | 1c,63bd | 0.3562 | 0.0628 | 0.4495 | -5.6647 | 0.9770 |
| 8 | 1.4579 | 2.8750 | 1.1c,21bd | 1.1690 | 1.1c,63bd | 1.1690 | 0.0627 | 0.4397 | -5.1957 | 0.9350 |
| 9 | 1.4554 | 2.8909 | 1.1c,10bd | 1.2109 | 1.1c,63bd | 1.2109 | 0.0626 | 0.4400 | -5.2150 | 0.9137 |
| 10 | 1.4347 | 1.9129 | 1c,42bd | 0.4491 | 1c,126bd | 0.4491 | 0.0578 | 0.4924 | -5.6476 | 0.8890 |
| 11 | 1.4345 | 2.8901 | 1.1c,10bd | 1.9747 | 1.1c,42bd | 1.9747 | 0.0649 | 0.3849 | -5.4329 | 0.8658 |
| 12 | 1.4247 | 2.8645 | 1.1c,21bd | 1.8463 | 1.1c,42bd | 1.8463 | 0.0644 | 0.3835 | -5.4688 | 0.8921 |
| 13 | 1.3717 | 3.2249 | 1.1c,63bd | 0.6316 | 1.1c,252bd | 0.6316 | 0.0498 | 0.5525 | -4.9035 | 0.9942 |
| 14 | 1.3713 | 2.2091 | 1c,63bd | 0.3639 | 1c,252bd | 0.3639 | 0.0508 | 0.5372 | -5.5486 | 0.9202 |
| 15 | 1.3343 | 3.3324 | 1.1c,42bd | 0.7865 | 1.1c,252bd | 0.7865 | 0.0473 | 0.5557 | -4.9024 | 1.0269 |
| 16 | 1.2817 | 3.4443 | 1.1c,126bd | 0.5600 | 1.1c,252bd | 0.5600 | 0.0445 | 0.5473 | -4.9093 | 0.9385 |
| 17 | 1.2768 | 1.9177 | 1c,42bd | 0.4457 | 1c,252bd | 0.4457 | 0.0450 | 0.5359 | -5.6064 | 0.7985 |
| 18 | 1.2474 | 1.3316 | 1c,21bd | 0.3412 | 1c,63bd | 0.3412 | 0.0514 | 0.4107 | -5.8511 | 0.8569 |
| 19 | 1.2226 | 2.7393 | 1c,126bd | 0.4309 | 1c,252bd | 0.4309 | 0.0397 | 0.5646 | -5.3472 | 0.7446 |
| 20 | 1.2041 | 3.1763 | 1.1c,21bd | 0.8583 | 1.1c,252bd | 0.8583 | 0.0392 | 0.5539 | -5.0154 | 0.8440 |

Table 4: Top 20 calendar pairs, traded at the 1:1 ratio.

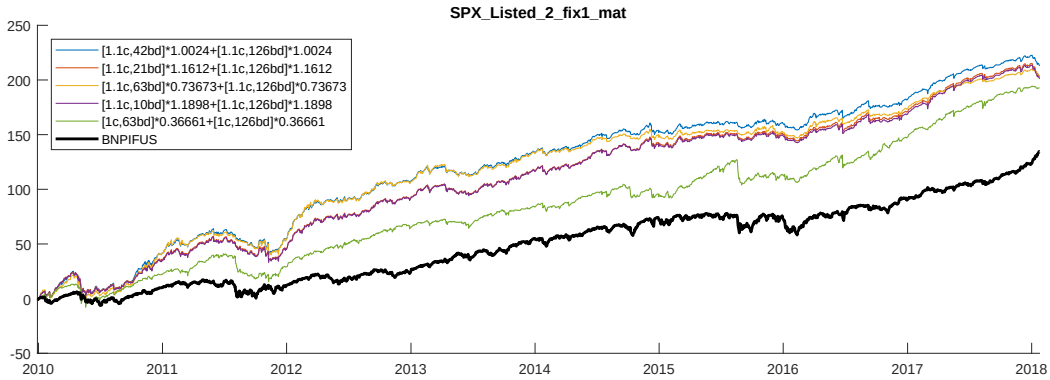


Figure 20: Plots of the top 5 calendar pairs. The performances are benchmarked against the BNPIFUS index, shown by the black bold line.

## 5.5 All pairs

Table 5 shows the top 20 pairs of all possible options, traded at a 1:1 ratio. The top 5 plots are given by Figure 21.

| | SR | MaxTC | Option | Weight | Option | Weight | Alpha | Beta | CVar | SR08 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.6619 | 2.1896 | 1c,63bd | 0.4718 | 1.1c,63bd | 0.4718 | 0.0713 | 0.5051 | -5.2120 | 1.1440 |
| 2 | 1.6616 | 3.3674 | 1.1c,42bd | 1.0024 | 1.1c,126bd | 1.0024 | 0.0689 | 0.5410 | -4.9715 | 1.2290 |
| 3 | 1.6597 | 2.1096 | 1.1c,42bd | 0.5447 | 1c,63bd | 0.5447 | 0.0720 | 0.4914 | -5.4045 | 1.1701 |
| 4 | 1.5886 | 2.7607 | 1.1c,63bd | 0.6130 | 1c,126bd | 0.6130 | 0.0634 | 0.5546 | -5.0692 | 1.0207 |
| 5 | 1.5876 | 2.7328 | 1.1c,42bd | 0.7502 | 1c,126bd | 0.7502 | 0.0635 | 0.5525 | -5.2559 | 1.0337 |
| 6 | 1.5838 | 3.2107 | 1.1c,21bd | 1.1612 | 1.1c,126bd | 1.1612 | 0.0630 | 0.5558 | -4.9770 | 1.0549 |
| 7 | 1.5834 | 1.9997 | 1.1c,21bd | 0.5751 | 1c,63bd | 0.5751 | 0.0677 | 0.4837 | -5.5151 | 1.0566 |
| 8 | 1.5750 | 2.2749 | 1c,63bd | 0.4328 | 1.1c,126bd | 0.4328 | 0.0656 | 0.5077 | -5.4429 | 1.1288 |
| 9 | 1.5736 | 1.9929 | 1.1c,10bd | 0.5811 | 1c,63bd | 0.5811 | 0.0673 | 0.4813 | -5.5365 | 1.0403 |
| 10 | 1.5715 | 3.2484 | 1.1c,63bd | 0.7367 | 1.1c,126bd | 0.7367 | 0.0645 | 0.5217 | -5.1284 | 1.1161 |
| 11 | 1.5690 | 3.2418 | 1.1c,10bd | 1.1898 | 1.1c,126bd | 1.1898 | 0.0622 | 0.5542 | -4.9862 | 1.0268 |
| 12 | 1.5631 | 1.8750 | 1c,42bd | 0.5968 | 1.1c,63bd | 0.5968 | 0.0670 | 0.4757 | -5.3653 | 1.0055 |
| 13 | 1.5561 | 2.1054 | 1c,63bd | 0.5469 | 1.2c,126bd | 0.5469 | 0.0646 | 0.5048 | -5.4559 | 1.1356 |
| 14 | 1.5560 | 1.7393 | 1c,42bd | 0.7259 | 1.1c,42bd | 0.7259 | 0.0678 | 0.4563 | -5.4849 | 1.0140 |
| 15 | 1.5232 | 1.9676 | 1.2c,10bd | 0.6053 | 1c,63bd | 0.6053 | 0.0650 | 0.4679 | -5.6207 | 1.0291 |
| 16 | 1.5224 | 1.9669 | 1.2c,21bd | 0.6048 | 1c,63bd | 0.6048 | 0.0649 | 0.4684 | -5.6203 | 1.0228 |
| 17 | 1.5209 | 1.9723 | 1c,63bd | 0.5881 | 1.2c,63bd | 0.5881 | 0.0640 | 0.4804 | -5.5992 | 1.0394 |
| 18 | 1.5172 | 1.9598 | 1.2c,42bd | 0.6013 | 1c,63bd | 0.6013 | 0.0644 | 0.4712 | -5.6156 | 1.0347 |
| 19 | 1.5081 | 2.1127 | 0.8p,42bd | 0.4996 | 1c,63bd | 0.4996 | 0.0606 | 0.5202 | -6.0646 | 0.8961 |
| 20 | 1.5040 | 2.0261 | 0.8p,10bd | 0.5510 | 1c,63bd | 0.5510 | 0.0620 | 0.4950 | -5.9827 | 0.8656 |

Table 5: Top 20 pairs, traded at the 1:1 ratio.

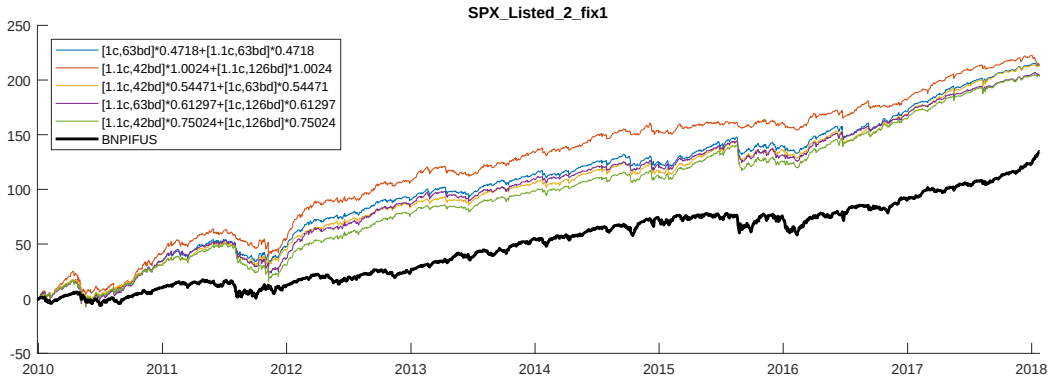


Figure 21: Plots of the top 5 pairs. The performances are benchmarked against the BNPIFUS index, shown by the black bold line.

## 5.6 Summary of findings

We have examined the top single and pairs strategies, ranked according to Sharpe ratio. The top performing single strategies have upside strikes (1c, 1.1c) with medium maturity (42bd, 63bd and 126bd).

For pairs strategies, we focused on strategies which traded at a 1:1 ratio, capturing the effect of diversification. Both strike pairs and calendar pairs offer better performance than the single strategies, offering a slightly alpha, but also a greater beta to the market.

# 6 Fixed weights spread and convexity strategies

Strategies based on options spreads are popular among hedge funds. They play on the relative *richness* of one option against the other. A classical example is the skew arbitrage: in equity markets, one commonly observes that lower strike options have a higher implied volatility. When this volatility is overpriced, it makes sense to sell low strike puts and hedge them with options closer to the money .

In this section, we examine the strategies that involve either two or three options with fixed weights. There are three main cases: strike spreads (options with the same maturity but various strikes), calendar spreads (options with the same strike but various maturities) and all options. For the pair strategies, we consider spread strategy where the weights are in a 1:-1 ratio. For the triplet strategies, we consider convexity strategies with weights in the ratio of 1:-2:1. As before, all pairs and triplet strategies are also scaled so that the daily returns have unit standard deviation. These weights ratio are applied to all possible pairs and triples, and then the top results are reported for each case. Note that, since the weights are applied to the systematic strategies themselves, positive weights actually correspond to short positions in the options.

For a detailed breakdown of the different columns of the result tables, as well as their calculations and interpretations, please refer to the beginning of Section 5.

It is worth reiterating that this section only considers strategies with predetermined weight ratios. Strategies with Greeks adjusted or optimised weights will be explored in the next sections.

## 6.1 Fixed weights strike spreads

We first examine combinations of options from the same maturity (aka strike spreads) with predetermined weight ratios. Table 6 contains the top strike spread pairs, traded at the 1:-1 ratio. The plots of the top 5 strategies can then be found in Figure 22.

The top strategies consist of shorting ATM calls or 1.1 calls while longing 1.2 calls, at various maturities. This is somewhat counterintuitive since the positions are opposite to what one might expect in a skew arbitrage strategy. This is due to the fact that proper skew arbitrage strategies require weights adjusted for the gamma of the options, which is explored in the later sections. The top 8 strategies are all quite robust with respect to transaction costs. Beyond the top 10, all other combinations fail to beat the BNPIFUS benchmark.

| | SR | MaxTC | Option | Weight | Option | Weight | Alpha | Beta | CVar | SR08 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.6492 | 2.9112 | 1.1c,42bd | 3.1486 | 1.2c,42bd | -3.1486 | 0.0792 | 0.3733 | -5.3276 | 1.4011 |
| 2 | 1.5390 | 2.5431 | 1.1c,63bd | 1.6437 | 1.2c,63bd | -1.6437 | 0.0694 | 0.4170 | -5.2894 | 1.1110 |
| 3 | 1.5178 | 1.8627 | 1c,63bd | 0.6214 | 1.2c,63bd | -0.6214 | 0.0661 | 0.4466 | -5.6110 | 1.0435 |
| 4 | 1.3585 | 1.9877 | 1.1c,126bd | 1.7112 | 1.2c,126bd | -1.7112 | 0.0573 | 0.4279 | -4.9855 | 0.8550 |
| 5 | 1.3575 | 1.4896 | 1c,42bd | 0.8473 | 1.2c,42bd | -0.8473 | 0.0587 | 0.4054 | -5.7584 | 0.8429 |
| 6 | 1.2411 | 1.8379 | 1c,126bd | 0.9909 | 1.2c,126bd | -0.9909 | 0.0489 | 0.4427 | -5.5371 | 0.5931 |
| 7 | 1.2394 | 2.2506 | 1.1c,21bd | 5.2502 | 1.2c,21bd | -5.2502 | 0.0549 | 0.3499 | -6.0954 | 0.7598 |
| 8 | 1.2224 | 2.2228 | 1.1c,10bd | 6.3858 | 1.2c,10bd | -6.3858 | 0.0537 | 0.3527 | -6.1430 | 0.6088 |
| 9 | 1.0855 | 0.9363 | 1c,63bd | 0.7207 | 1.1c,63bd | -0.7207 | 0.0462 | 0.3351 | -5.9430 | 0.6994 |
| 10 | 1.0672 | 1.0274 | 0.8p,63bd | -0.6654 | 1c,63bd | 0.6654 | 0.0578 | 0.1429 | -5.3376 | 0.9304 |
| 11 | 0.9879 | 0.8750 | 1c,42bd | 0.9161 | 1.1c,42bd | -0.9161 | 0.0404 | 0.3297 | -6.0089 | 0.4881 |
| 12 | 0.9573 | 0.9093 | 0.8p,42bd | -0.8651 | 1c,42bd | 0.8651 | 0.0477 | 0.1909 | -5.6147 | 0.7493 |
| 13 | 0.9442 | 0.9282 | 0.8p,126bd | -1.1961 | 1c,126bd | 1.1961 | 0.0561 | 0.0514 | -4.8379 | 0.7103 |
| 14 | 0.8730 | 1.9422 | 0.9p,63bd | 0.6129 | 1.2c,63bd | -0.6129 | 0.0229 | 0.4844 | -6.5127 | 0.6226 |
| 15 | 0.7970 | 0.7727 | 1c,21bd | 0.7000 | 1.2c,21bd | -0.7000 | 0.0303 | 0.3005 | -6.0855 | 0.5605 |
| 16 | 0.7843 | 1.8755 | 0.9p,42bd | 0.8976 | 1.2c,42bd | -0.8976 | 0.0210 | 0.4295 | -6.5727 | 0.5773 |
| 17 | 0.7791 | 0.4786 | 0.9p,126bd | -1.5649 | 1c,126bd | 1.5649 | 0.0597 | -0.1608 | -3.7971 | 0.5432 |
| 18 | 0.7742 | 0.5158 | 0.9p,63bd | -0.7604 | 1c,63bd | 0.7604 | 0.0524 | -0.0545 | -4.1337 | 0.5051 |
| 19 | 0.7562 | 3.0140 | 0.8p,42bd | 1.5257 | 1.2c,42bd | -1.5257 | 0.0216 | 0.3932 | -5.6419 | 0.1247 |
| 20 | 0.7552 | 2.6171 | 0.8p,63bd | 0.9005 | 1.2c,63bd | -0.9005 | 0.0175 | 0.4538 | -5.9707 | 0.1056 |

Table 6: Top 20 strike spread strategies, traded at the 1:-1 ratio.

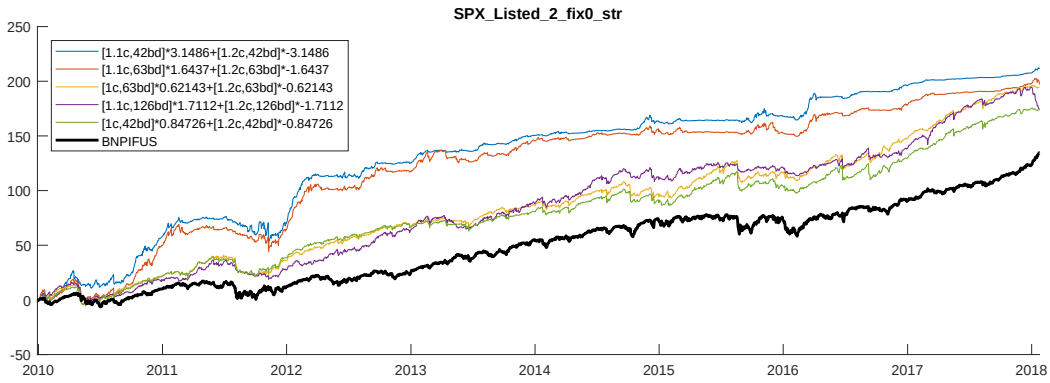


Figure 22: Plots of the top 5 strike spread strategies. The performances are benchmarked against the BNPIFUS index, shown by the black bold line.

## 6.2 Fixed weights strike convexity triplets

Next, Table 7 contains the top strike convexity triplets, i.e, traded at the 1:-2:1 ratio. Note that the strike of the option corresponding to the weight ratio of -2 is always between the other two. Figure 23 contains the plots of the top 5.

| | SR | MaxTC | Option | Weight | Option | Weight | Option | Weight | Alpha | Beta | CVar | SR08 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.4157 | 1.0534 | 1c,63bd | 0.8182 | 0.9p,63bd | -0.4091 | 1.2c,63bd | -0.4091 | 0.0717 | 0.2647 | -5.3291 | 0.9675 |
| 2 | 1.3909 | 1.4087 | 1c,63bd | 0.6875 | 0.8p,63bd | -0.3438 | 1.2c,63bd | -0.3438 | 0.0664 | 0.3208 | -5.4708 | 1.0763 |
| 3 | 1.3798 | 1.0166 | 1c,126bd | 1.5766 | 0.9p,126bd | -0.7883 | 1.2c,126bd | -0.7883 | 0.0690 | 0.2712 | -4.8832 | 0.7326 |
| 4 | 1.2530 | 1.3329 | 1c,126bd | 1.2272 | 0.8p,126bd | -0.6136 | 1.2c,126bd | -0.6136 | 0.0590 | 0.3005 | -5.0722 | 0.7303 |
| 5 | 1.2079 | 1.1815 | 1c,42bd | 0.8918 | 0.8p,42bd | -0.4459 | 1.2c,42bd | -0.4459 | 0.0555 | 0.3117 | -5.7259 | 0.8384 |
| 6 | 1.1774 | 0.9812 | 1c,63bd | 0.7572 | 0.8p,63bd | -0.3786 | 1.1c,63bd | -0.3786 | 0.0571 | 0.2573 | -5.6600 | 0.9241 |
| 7 | 1.1655 | 0.8984 | 1c,42bd | 1.0002 | 0.9p,42bd | -0.5001 | 1.2c,42bd | -0.5001 | 0.0576 | 0.2392 | -5.6154 | 0.7039 |
| 8 | 1.1458 | 0.7046 | 1c,63bd | 0.9079 | 0.9p,63bd | -0.4539 | 1.1c,63bd | -0.4539 | 0.0604 | 0.1785 | -5.4012 | 0.7522 |
| 9 | 1.0725 | 0.4855 | 1c,126bd | 2.2427 | 0.9p,126bd | -1.1213 | 1.1c,126bd | -1.1213 | 0.0606 | 0.1055 | -4.4482 | 0.5310 |
| 10 | 1.0203 | 0.8920 | 1c,42bd | 0.9340 | 0.8p,42bd | -0.4670 | 1.1c,42bd | -0.4670 | 0.0463 | 0.2711 | -5.8578 | 0.6656 |
| 11 | 1.0127 | 0.7011 | 1c,126bd | 1.6228 | 0.8p,126bd | -0.8114 | 1.1c,126bd | -0.8114 | 0.0509 | 0.1945 | -4.9820 | 0.5852 |
| 12 | 0.9438 | 0.6568 | 1c,42bd | 1.0447 | 0.9p,42bd | -0.5223 | 1.1c,42bd | -0.5223 | 0.0470 | 0.1879 | -5.7332 | 0.4899 |
| 13 | 0.9027 | 1.1621 | 1.1c,126bd | 1.6053 | 0.8p,126bd | -0.8026 | 1.2c,126bd | -0.8026 | 0.0517 | 0.0773 | -4.6412 | 0.7554 |
| 14 | 0.8404 | 1.1710 | 0.9p,63bd | 0.8362 | 0.8p,63bd | -0.4181 | 1.2c,63bd | -0.4181 | 0.0232 | 0.4502 | -6.5357 | 0.7648 |
| 15 | 0.7235 | 1.1666 | 0.9p,42bd | 1.1558 | 0.8p,42bd | -0.5779 | 1.2c,42bd | -0.5779 | 0.0188 | 0.4041 | -6.8241 | 0.6819 |
| 16 | 0.7158 | 0.6658 | 1c,21bd | 0.7095 | 0.8p,21bd | -0.3548 | 1.2c,21bd | -0.3548 | 0.0285 | 0.2501 | -6.1090 | 0.6002 |
| 17 | 0.7113 | 0.7197 | 1.1c,126bd | 1.7076 | 0.9p,126bd | -0.8538 | 1.2c,126bd | -0.8538 | 0.0476 | -0.0422 | -3.9505 | 0.5539 |
| 18 | 0.6991 | 0.8802 | 0.9p,126bd | 1.3364 | 0.8p,126bd | -0.6682 | 1.2c,126bd | -0.6682 | 0.0133 | 0.4646 | -6.0212 | 0.2770 |
| 19 | 0.6835 | 1.0485 | 1.1c,63bd | 1.3221 | 0.8p,63bd | -0.6610 | 1.2c,63bd | -0.6610 | 0.0429 | 0.0022 | -4.7354 | 0.7667 |
| 20 | 0.6630 | 1.2812 | 0.9p,10bd | 1.5021 | 0.8p,10bd | -0.7510 | 1.2c,10bd | -0.7510 | 0.0225 | 0.2906 | -7.0509 | 0.6757 |

Table 7: Top 20 strike convexity strategies, traded at the 1:-2:1 ratio.

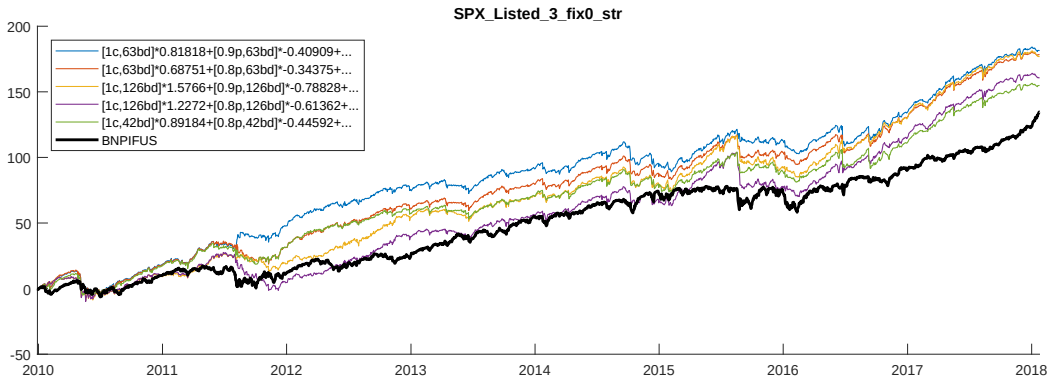


Figure 23: Plots of the top 5 strike convexity strategies. The performances are benchmarked against the BNPIFUS index, shown by the black bold line.

## 6.3 Fixed weights calendar spreads

In this subsection, we examine combinations of options with the same strike but different maturities (aka calendar spreads) with predetermined weight ratios. Table 8 contains the top calendar spread pairs, traded at the 1:-1 ratio. The plots of the top 5 strategies can then be found in Figure 24.

Overall, calendar spreads performs much better than strike spreads in both Sharpe ratio and 99% CVaR. Most of the top strategies consist of buying options with shorter maturities while selling options with longer maturities. The top 4 strategies all involve only ATM calls. All 20 top strategies outperform the benchmark. However, note that many of these calendar spreads are much more sensitive to transaction costs in comparison to strike spreads. The main exceptions are the strategies involving 1.2 calls, which are quite robust with respect to transaction costs but have inferior performance otherwise.

| | SR | MaxTC | Option | Weight | Option | Weight | Alpha | Beta | CVar | SR08 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2.1619 | 0.7312 | 1c,10bd | -0.9800 | 1c,63bd | 0.9800 | 0.1150 | 0.3199 | -3.3102 | 1.7419 |
| 2 | 2.0731 | 0.2595 | 1c,10bd | -2.4879 | 1c,21bd | 2.4879 | 0.1315 | -0.0134 | -2.1413 | 2.1194 |
| 3 | 1.6697 | 0.4644 | 1c,10bd | -1.2817 | 1c,42bd | 1.2817 | 0.1007 | 0.0678 | -2.3359 | 1.4069 |
| 4 | 1.4601 | 0.3860 | 1c,42bd | -1.6164 | 1c,63bd | 1.6164 | 0.0627 | 0.4422 | -5.0494 | 1.0961 |
| 5 | 1.4495 | 1.3480 | 1.1c,10bd | -4.2780 | 1.1c,42bd | 4.2780 | 0.0704 | 0.3160 | -5.5243 | 1.2720 |
| 6 | 1.4111 | 0.5000 | 1c,21bd | -1.0280 | 1c,63bd | 1.0280 | 0.0663 | 0.3411 | -4.2020 | 0.9931 |
| 7 | 1.3666 | 1.0899 | 1.1c,21bd | -4.6546 | 1.1c,42bd | 4.6546 | 0.0673 | 0.2843 | -5.5052 | 1.0931 |
| 8 | 1.3505 | 0.7390 | 1c,63bd | 1.0714 | 1c,252bd | -1.0714 | 0.0809 | 0.0638 | -5.1666 | 0.9370 |
| 9 | 1.3468 | 1.9586 | 1.1c,10bd | -1.6539 | 1.1c,63bd | 1.6539 | 0.0583 | 0.4007 | -5.5270 | 0.9916 |
| 10 | 1.3107 | 1.7729 | 1.1c,21bd | -1.7043 | 1.1c,63bd | 1.7043 | 0.0567 | 0.3912 | -5.5213 | 0.9231 |
| 11 | 1.2941 | 2.2890 | 1.1c,10bd | -1.3899 | 1.1c,126bd | 1.3899 | 0.0498 | 0.4792 | -5.0453 | 0.8947 |
| 12 | 1.2582 | 0.4946 | 1c,63bd | 1.3518 | 1c,126bd | -1.3518 | 0.0644 | 0.2241 | -5.3899 | 0.8865 |
| 13 | 1.2394 | 2.0853 | 1.1c,21bd | -1.3991 | 1.1c,126bd | 1.3991 | 0.0473 | 0.4645 | -5.0579 | 0.8047 |
| 14 | 1.1660 | 0.2459 | 1.1c,10bd | -25.7800 | 1.1c,21bd | 25.7800 | 0.0516 | 0.3296 | -6.1412 | 0.8699 |
| 15 | 1.1266 | 0.9487 | 1.1c,42bd | -2.3694 | 1.1c,63bd | 2.3694 | 0.0446 | 0.3991 | -5.7025 | 0.6601 |
| 16 | 1.1016 | 5.1339 | 1.2c,21bd | -1.4319 | 1.2c,252bd | 1.4319 | 0.0325 | 0.5571 | -5.0554 | 0.8490 |
| 17 | 1.1015 | 5.1632 | 1.2c,10bd | -1.4307 | 1.2c,252bd | 1.4307 | 0.0324 | 0.5586 | -5.0526 | 0.8512 |
| 18 | 1.1002 | 4.8483 | 1.2c,42bd | -1.4410 | 1.2c,252bd | 1.4410 | 0.0331 | 0.5474 | -5.0693 | 0.8113 |
| 19 | 1.0784 | 0.5434 | 1c,10bd | -0.8456 | 1c,126bd | 0.8456 | 0.0589 | 0.1359 | -2.4502 | 0.8058 |
| 20 | 1.0573 | 3.6999 | 1.2c,42bd | -3.2815 | 1.2c,126bd | 3.2815 | 0.0363 | 0.4575 | -5.6916 | 0.8862 |

Table 8: Top 20 calendar spread strategies, traded at the 1:-1 ratio.

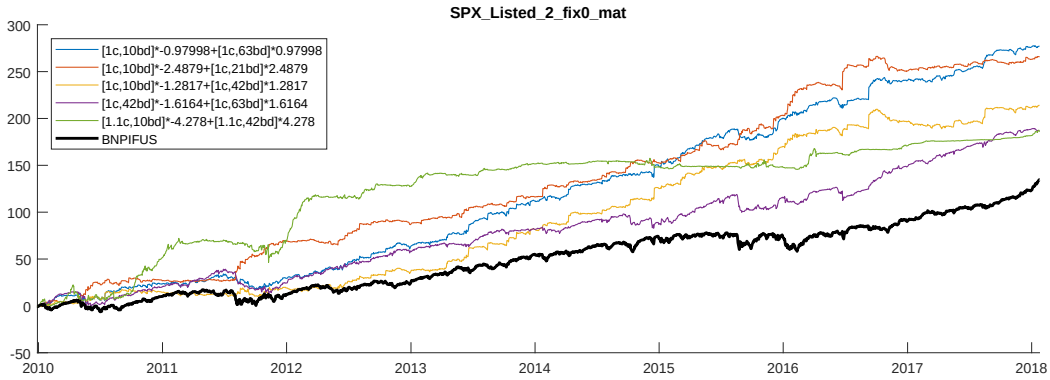


Figure 24: Plots of the top 5 calendar spread strategies. The performances are benchmarked against the BNPIFUS index, shown by the black bold line.

## 6.4 Fixed weights calendar convexity triplets

Next, Table 9 contains the top calendar convexity triplets, i.e, traded at the 1:-2:1 ratio. Note that the maturity of the option corresponding to the weight ratio of -2 is always between the other two. Figure 25 contains the plots of the top five.

| | SR | MaxTC | Option | Weight | Option | Weight | Option | Weight | Alpha | Beta | CVar | SR08 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2.4738 | 0.7340 | 1c,63bd | 1.4272 | 1c,10bd | -0.7136 | 1c,252bd | -0.7136 | 0.1376 | 0.2755 | -4.3155 | 1.8228 |
| 2 | 2.3521 | 0.6403 | 1c,63bd | 1.4997 | 1c,10bd | -0.7498 | 1c,126bd | -0.7498 | 0.1237 | 0.3691 | -4.2682 | 1.7986 |
| 3 | 1.8264 | 0.5916 | 1c,63bd | 1.3872 | 1c,21bd | -0.6936 | 1c,252bd | -0.6936 | 0.0971 | 0.2715 | -4.8654 | 1.2794 |
| 4 | 1.7063 | 0.3861 | 1c,42bd | 2.0559 | 1c,10bd | -1.0280 | 1c,252bd | -1.0280 | 0.1184 | -0.1656 | -3.6289 | 1.1813 |
| 5 | 1.6815 | 0.4978 | 1c,63bd | 1.4600 | 1c,21bd | -0.7300 | 1c,126bd | -0.7300 | 0.0819 | 0.3633 | -4.8375 | 1.2055 |
| 6 | 1.6534 | 0.5348 | 1c,63bd | 1.5283 | 1c,42bd | -0.7641 | 1c,252bd | -0.7641 | 0.0873 | 0.2546 | -5.3534 | 1.2071 |
| 7 | 1.5952 | 0.2972 | 1c,42bd | 2.3985 | 1c,10bd | -1.1992 | 1c,126bd | -1.1992 | 0.1048 | -0.0657 | -3.4248 | 1.1383 |
| 8 | 1.5382 | 0.4344 | 1c,63bd | 1.6774 | 1c,42bd | -0.8387 | 1c,126bd | -0.8387 | 0.0725 | 0.3684 | -5.3928 | 1.1555 |
| 9 | 1.4255 | 0.4745 | 1c,126bd | 1.7763 | 1c,10bd | -0.8882 | 1c,252bd | -0.8882 | 0.0866 | 0.0484 | -2.4116 | 0.9016 |
| 10 | 1.2678 | 1.1718 | 1.1c,21bd | -3.5028 | 1.1c,10bd | 1.7514 | 1.1c,63bd | 1.7514 | 0.0547 | 0.3796 | -5.5057 | 0.8414 |
| 11 | 1.2314 | 0.5479 | 1.1c,21bd | -9.9159 | 1.1c,10bd | 4.9580 | 1.1c,42bd | 4.9580 | 0.0617 | 0.2395 | -5.4389 | 0.8445 |
| 12 | 1.1804 | 1.3809 | 1.1c,21bd | -2.8085 | 1.1c,10bd | 1.4042 | 1.1c,126bd | 1.4042 | 0.0447 | 0.4482 | -5.0570 | 0.7087 |
| 13 | 1.1759 | 0.8328 | 1.1c,126bd | 2.6360 | 1.1c,10bd | -1.3180 | 1.1c,252bd | -1.3180 | 0.0585 | 0.2346 | -4.8975 | 0.8170 |
| 14 | 1.1174 | 0.7775 | 1.1c,126bd | 2.6388 | 1.1c,21bd | -1.3194 | 1.1c,252bd | -1.3194 | 0.0560 | 0.2180 | -4.8847 | 0.7411 |
| 15 | 1.1017 | 4.8986 | 1.2c,21bd | -2.8661 | 1.2c,10bd | 1.4330 | 1.2c,252bd | 1.4330 | 0.0326 | 0.5556 | -5.0579 | 0.8455 |
| 16 | 1.0939 | 4.3817 | 1.2c,42bd | -2.8877 | 1.2c,21bd | 1.4439 | 1.2c,252bd | 1.4439 | 0.0335 | 0.5353 | -5.0611 | 0.7557 |
| 17 | 1.0931 | 4.3971 | 1.2c,42bd | -2.8879 | 1.2c,10bd | 1.4439 | 1.2c,252bd | 1.4439 | 0.0336 | 0.5333 | -5.0599 | 0.7536 |
| 18 | 1.0496 | 3.0858 | 1.2c,42bd | -6.6362 | 1.2c,21bd | 3.3181 | 1.2c,126bd | 3.3181 | 0.0375 | 0.4322 | -5.7106 | 0.7805 |
| 19 | 1.0473 | 3.1024 | 1.2c,42bd | -6.6339 | 1.2c,10bd | 3.3169 | 1.2c,126bd | 3.3169 | 0.0377 | 0.4275 | -5.7080 | 0.7704 |
| 20 | 1.0434 | 3.7840 | 1.2c,21bd | -6.3722 | 1.2c,10bd | 3.1861 | 1.2c,126bd | 3.1861 | 0.0347 | 0.4690 | -5.6670 | 0.9092 |

Table 9: Top 20 maturity convexity strategies, traded at the 1:-2:1 ratio.

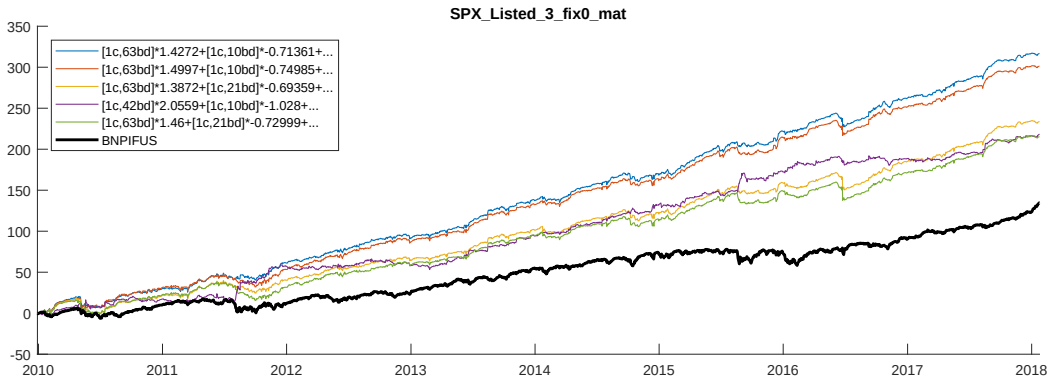


Figure 25: Plots of the top 5 maturity convexity strategies. The performances are benchmarked against the BNPIFUS index, shown by the black bold line.

## 6.5 Fixed weights spreads and convexity strategies over all options

Now, we look at fixed weight ratio strategies over all options, without restrictions on strikes or maturities. Table 10 contains the top pairs, traded at the 1:-1 ratio. Then the top 5 plots can be found in Figure 26.

At the top of the table we once again find the top calendar spreads we have seen previously, followed by the top strike spreads. There are a few new additions too, consist of selling a 1.1 call at short to mid-maturities (10bd to 42bd) and buying a 1.2 call at a slightly longer maturity (21bd to 63bd). These new strategies all perform quite well and are robust with respect to transaction costs.

| | SR | MaxTC | Option | Weight | Option | Weight | Alpha | Beta | CVar | SR08 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2.1619 | 0.7312 | 1c,10bd | -0.9800 | 1c,63bd | 0.9800 | 0.1150 | 0.3199 | -3.3102 | 1.7419 |
| 2 | 2.0731 | 0.2595 | 1c,10bd | -2.4879 | 1c,21bd | 2.4879 | 0.1315 | -0.0134 | -2.1413 | 2.1194 |
| 3 | 1.8346 | 2.2615 | 1.1c,42bd | 3.9816 | 1.2c,63bd | -3.9816 | 0.0931 | 0.3394 | -5.2183 | 1.4795 |
| 4 | 1.6697 | 0.4644 | 1c,10bd | -1.2817 | 1c,42bd | 1.2817 | 0.1007 | 0.0678 | -2.3359 | 1.4069 |
| 5 | 1.6492 | 2.9112 | 1.1c,42bd | 3.1486 | 1.2c,42bd | -3.1486 | 0.0792 | 0.3733 | -5.3276 | 1.4011 |
| 6 | 1.5538 | 3.0739 | 1.2c,21bd | -2.9336 | 1.1c,42bd | 2.9336 | 0.0731 | 0.3747 | -5.3694 | 1.3592 |
| 7 | 1.5417 | 3.0906 | 1.2c,10bd | -2.9075 | 1.1c,42bd | 2.9075 | 0.0723 | 0.3754 | -5.3737 | 1.3267 |
| 8 | 1.5390 | 2.5431 | 1.1c,63bd | 1.6437 | 1.2c,63bd | -1.6437 | 0.0694 | 0.4170 | -5.2894 | 1.1110 |
| 9 | 1.5329 | 1.9461 | 1.2c,42bd | -0.6118 | 1c,63bd | 0.6118 | 0.0661 | 0.4601 | -5.6206 | 1.0703 |
| 10 | 1.5295 | 1.9623 | 1.2c,21bd | -0.6090 | 1c,63bd | 0.6090 | 0.0657 | 0.4636 | -5.6232 | 1.0839 |
| 11 | 1.5288 | 1.9639 | 1.2c,10bd | -0.6086 | 1c,63bd | 0.6086 | 0.0656 | 0.4641 | -5.6234 | 1.0802 |
| 12 | 1.5178 | 1.8627 | 1c,63bd | 0.6214 | 1.2c,63bd | -0.6214 | 0.0661 | 0.4466 | -5.6110 | 1.0435 |
| 13 | 1.4861 | 2.9021 | 1.2c,42bd | -1.4871 | 1.1c,63bd | 1.4871 | 0.0654 | 0.4268 | -5.3581 | 1.0764 |
| 14 | 1.4858 | 3.3281 | 1.2c,42bd | -1.3313 | 1.1c,126bd | 1.3313 | 0.0593 | 0.5185 | -4.9550 | 1.0641 |
| 15 | 1.4784 | 3.4233 | 1.2c,21bd | -1.3177 | 1.1c,126bd | 1.3177 | 0.0584 | 0.5253 | -4.9669 | 1.0861 |
| 16 | 1.4774 | 3.4328 | 1.2c,10bd | -1.3162 | 1.1c,126bd | 1.3162 | 0.0582 | 0.5265 | -4.9693 | 1.0864 |
| 17 | 1.4603 | 1.7077 | 1.1c,10bd | -0.6293 | 1c,63bd | 0.6293 | 0.0625 | 0.4451 | -5.6652 | 1.0344 |
| 18 | 1.4601 | 0.3860 | 1c,42bd | -1.6164 | 1c,63bd | 1.6164 | 0.0627 | 0.4422 | -5.0494 | 1.0961 |
| 19 | 1.4529 | 2.9319 | 1.2c,63bd | -1.3797 | 1.1c,126bd | 1.3797 | 0.0590 | 0.4914 | -4.9829 | 0.9944 |
| 20 | 1.4523 | 2.9851 | 1.2c,21bd | -1.4449 | 1.1c,63bd | 1.4449 | 0.0632 | 0.4279 | -5.3867 | 1.0762 |

Table 10: Top 20 spread strategies over all options, traded at the 1:-1 ratio.

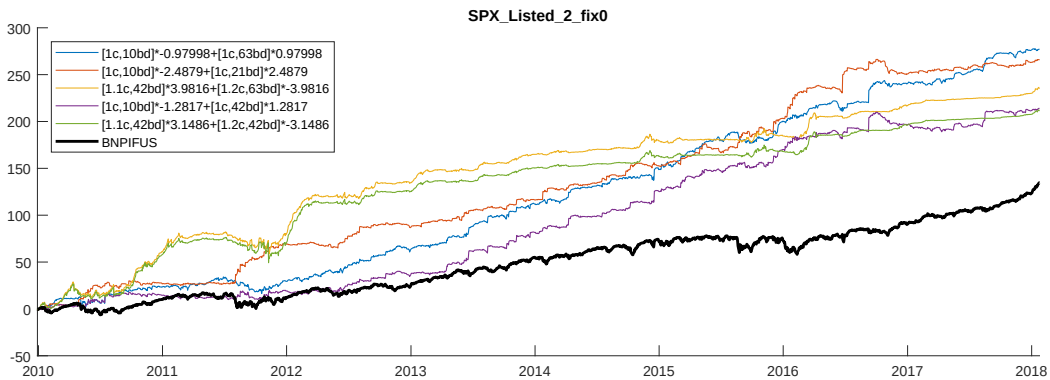


Figure 26: Plots of the top 5 spread strategies over all options. The performances are benchmarked against the BNPIFUS index, shown by the black bold line.

## 6.6 Summary of findings

In this section, we looked at the performance of spreads and convexity triplets. The spreads are two strategies traded at a 1:-1 ratio while convexity triplets are three strategies traded at a 1:-2:1 ratio. Strike spreads perform slightly better than single strategies via improved alphas, while strike convexity triplets perform worse. Also, the top strike spreads and convexity triplets do not have positions consistent with typical skew or smile arbitrage strategies, indicating that these types of arbitrage strategies are somewhat inconsistent in their performances.

We also notice calendar spreads and calendar convexity strategies perform significantly better than single, pairs and strike spread strategies, with significantly better alphas. As before, most top strategies (singles, spreads or convexity) play on the richness of medium maturity, upside strike options comparatively to shorter or longer dated options. However, their performances are more susceptible to transaction costs.

# 7 Sensitivity adjusted spread strategies

A natural way to detect skew arbitrage opportunities is to adjust the weights so that the portfolio is either gamma or vega neutral, so that the corresponding effects are neutralised. Instead of fixing the weights at the ratio of 1:-1 for spreads, or 1:-2:1 for convexity triples, we will inversely scale the weight of each options strategy by the median of the gamma/vega throughout the lifetime of the strategy. This constructs a portfolio that is approximately gamma/vega neutral over the entire testing period, while restricting to options strategies that sells a fixed amount of vanilla options each day.

A more accurate way of constructing a gamma/vega neutral strategy is to inversely scale, every day, the amount of options by the gamma/vega on that day. However, this may cause some instabilities in cases if the gamma/vega ever becomes very close to zero on any of the trading days. This idea will be explored in a future study.

For a detailed breakdown of the different columns of the result tables, as well as their calculations and interpretations, please refer to the beginning of Section 5.

## 7.1 Gamma adjusted strike spreads

Table 11 shows the top strike spreads where the weights are inversely scaled by the median gamma. Overall, the performance of the strategies are quite poor, with only the top strategy outperforming the benchmark. However, we do see weights which are more in line with what is typically observed in skew arbitrages.

| | SR | MaxTC | Option | Weight | Option | Weight | Alpha | Beta | CVar | SR08 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.1490 | 1.9358 | 1.1c,10bd | 6.0151 | 1c,10bd | -0.0543 | 0.0499 | 0.3400 | -6.1841 | 0.4225 |
| 2 | 0.9823 | 1.1916 | 1.1c,21bd | 5.1109 | 1c,21bd | -0.2097 | 0.0431 | 0.2832 | -5.9955 | 0.4687 |
| 3 | 0.8857 | 0.8713 | 1.1c,42bd | 3.0787 | 1c,42bd | -0.4681 | 0.0437 | 0.1824 | -5.0075 | 0.9087 |
| 4 | 0.6876 | 1.5438 | 0.9p,10bd | 1.2675 | 1c,10bd | -0.0322 | 0.0223 | 0.3171 | -6.7423 | 0.4962 |
| 5 | 0.6677 | 0.8043 | 0.8p,126bd | 1.9518 | 0.9p,126bd | -0.8125 | 0.0078 | 0.5185 | -5.7009 | 0.1331 |
| 6 | 0.6420 | 1.1908 | 0.8p,42bd | 2.1077 | 0.9p,42bd | -0.4772 | 0.0179 | 0.3407 | -4.8152 | -0.0388 |
| 7 | 0.6391 | 0.7700 | 1.2c,252bd | 2.7337 | 1.1c,252bd | -1.2650 | 0.0166 | 0.3581 | -4.6273 | 0.5870 |
| 8 | 0.6295 | 1.8405 | 0.8p,42bd | 1.5589 | 1c,42bd | -0.0974 | 0.0146 | 0.3783 | -5.5781 | 0.0725 |
| 9 | 0.6165 | 0.9344 | 0.8p,63bd | 1.3347 | 0.9p,63bd | -0.4001 | 0.0122 | 0.4023 | -5.2366 | -0.0876 |
| 10 | 0.5968 | 1.4793 | 0.8p,63bd | 0.9453 | 1c,63bd | -0.1050 | 0.0083 | 0.4423 | -5.9077 | 0.0415 |
| 11 | 0.5867 | 0.5281 | 0.8p,252bd | 2.4345 | 0.9p,252bd | -1.3857 | 0.0028 | 0.5161 | -5.4869 | 0.0422 |
| 12 | 0.5852 | 1.1798 | 1.2c,126bd | 3.5602 | 1c,126bd | -0.3747 | 0.0152 | 0.3271 | -5.5983 | 0.7071 |
| 13 | 0.5841 | 0.8670 | 0.8p,252bd | 1.5701 | 1c,252bd | -0.6502 | -0.0007 | 0.5668 | -5.5652 | 0.0759 |
| 14 | 0.5773 | 0.9961 | 0.9p,21bd | 1.0777 | 1c,21bd | -0.1223 | 0.0171 | 0.2918 | -6.6511 | 0.4403 |
| 15 | 0.5676 | 1.1604 | 0.8p,126bd | 1.2880 | 1c,126bd | -0.2875 | -0.0004 | 0.5470 | -6.1267 | 0.1064 |
| 16 | 0.5513 | 1.0325 | 1.2c,126bd | 4.0541 | 1.1c,126bd | -0.7029 | 0.0119 | 0.3445 | -5.6714 | 0.7133 |
| 17 | 0.5039 | 0.9340 | 0.8p,252bd | 1.3420 | 1.1c,252bd | -0.5947 | -0.0038 | 0.5379 | -5.7611 | 0.0306 |
| 18 | 0.5026 | 1.3670 | 0.8p,63bd | 0.9111 | 1.1c,63bd | -0.3243 | 0.0048 | 0.4057 | -5.8858 | -0.0370 |
| 19 | 0.4909 | 0.3419 | 0.9p,252bd | 2.1326 | 1c,252bd | -1.5516 | -0.0060 | 0.5583 | -5.0664 | 0.0839 |
| 20 | 0.4797 | 2.7797 | 0.8p,10bd | 2.2370 | 1c,10bd | -0.0074 | 0.0111 | 0.2889 | -4.8062 | -0.1334 |

Table 11: Top 20 gamma adjusted strike spread strategies.

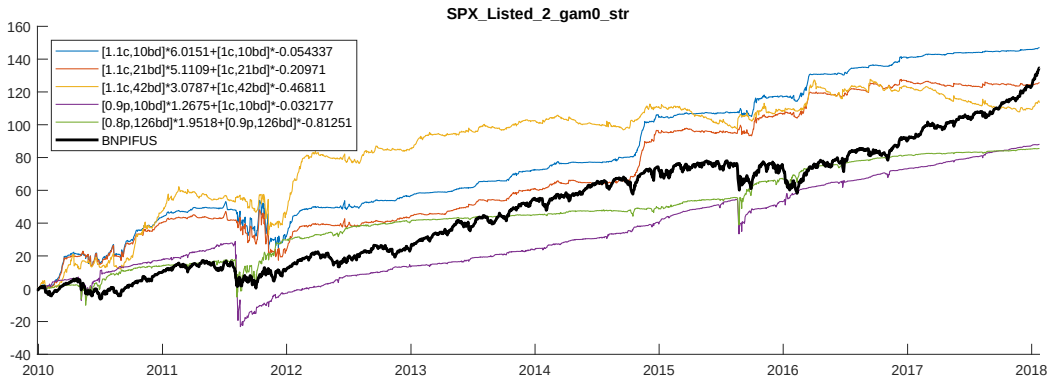


Figure 27: Plots of the top 5 gamma adjusted strike spread strategies. The performances are benchmarked against the BNPIFUS index, shown by the black bold line.

## 7.2 Gamma adjusted strike convexity triplets

Table 12 shows the top strike convexity triplets where the weights are inversely scaled by the median gamma.

| | SR | MaxTC | Option | Weight | Option | Weight | Option | Weight | Alpha | Beta | CVar | SR08 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.0501 | 1.7167 | 1.1c,10bd | 2.7342 | 0.9p,10bd | 0.9729 | 1c,10bd | -0.0494 | 0.0398 | 0.3980 | -6.9634 | 0.5412 |
| 2 | 0.9068 | 1.0847 | 1.1c,21bd | 2.3494 | 0.9p,21bd | 0.8498 | 1c,21bd | -0.1928 | 0.0333 | 0.3603 | -6.8840 | 0.5577 |
| 3 | 0.8465 | 0.7726 | 1.2c,126bd | 3.0315 | 0.8p,126bd | 1.4295 | 0.9p,126bd | -1.1902 | 0.0248 | 0.4309 | -4.9156 | 0.6617 |
| 4 | 0.8280 | 0.7127 | 1.1c,42bd | 1.5650 | 0.9p,42bd | 0.8623 | 1c,42bd | -0.4759 | 0.0256 | 0.4014 | -6.4263 | 0.8045 |
| 5 | 0.8059 | 0.7563 | 0.8p,42bd | 2.4903 | 0.9p,42bd | -1.1277 | 1.1c,42bd | 1.0233 | 0.0335 | 0.2613 | -3.8606 | 0.0666 |
| 6 | 0.7962 | 1.1688 | 1.2c,126bd | 2.1393 | 0.8p,126bd | 1.0088 | 1c,126bd | -0.4503 | 0.0088 | 0.6250 | -5.5383 | 0.5555 |
| 7 | 0.7931 | 0.4188 | 0.8p,126bd | 2.6111 | 0.9p,126bd | -2.1740 | 1.1c,126bd | 0.9601 | 0.0290 | 0.3167 | -3.5708 | 0.2674 |
| 8 | 0.7888 | 1.4701 | 0.8p,42bd | 1.5112 | 1.1c,42bd | 0.6209 | 1c,42bd | -0.1888 | 0.0230 | 0.4035 | -5.7524 | 0.2450 |
| 9 | 0.7758 | 0.7932 | 0.8p,252bd | 1.3096 | 1.2c,252bd | 1.2542 | 1c,252bd | -1.0847 | 0.0102 | 0.5849 | -4.4670 | 0.3316 |
| 10 | 0.7187 | 0.8652 | 0.8p,252bd | 1.1990 | 1.2c,252bd | 1.1482 | 1.1c,252bd | -1.0627 | 0.0035 | 0.6309 | -5.2692 | 0.2626 |
| 11 | 0.7083 | 0.5013 | 1.2c,252bd | 1.9304 | 1c,252bd | -1.6696 | 0.9p,252bd | 1.1474 | 0.0133 | 0.4731 | -4.3402 | 0.4518 |
| 12 | 0.7043 | 0.3285 | 0.8p,126bd | 2.8920 | 0.9p,126bd | -2.4078 | 1c,126bd | 0.6454 | 0.0240 | 0.3084 | -4.2057 | 0.1364 |
| 13 | 0.6943 | 0.5540 | 1.2c,252bd | 1.8552 | 1.1c,252bd | -1.7169 | 0.9p,252bd | 1.1027 | 0.0035 | 0.6088 | -4.6498 | 0.3555 |
| 14 | 0.6749 | 0.7899 | 0.8p,126bd | 1.3475 | 1c,126bd | -0.6014 | 1.1c,126bd | 0.4954 | 0.0033 | 0.5920 | -5.8252 | 0.2009 |
| 15 | 0.6706 | 1.0351 | 0.8p,63bd | 0.9432 | 1.1c,63bd | 0.3357 | 1c,63bd | -0.2094 | 0.0116 | 0.4625 | -5.9096 | 0.1185 |
| 16 | 0.6604 | 0.7312 | 1.2c,126bd | 3.1318 | 1c,126bd | -0.6592 | 0.9p,126bd | 0.6148 | 0.0070 | 0.5226 | -5.5030 | 0.6631 |
| 17 | 0.6565 | 1.0404 | 1.2c,126bd | 2.0812 | 0.8p,126bd | 0.9814 | 1.1c,126bd | -0.7217 | 0.0030 | 0.5792 | -5.8207 | 0.4331 |
| 18 | 0.6296 | 0.3669 | 1.2c,252bd | 2.6516 | 1.1c,252bd | -2.4540 | 1c,252bd | 1.1467 | 0.0094 | 0.4576 | -4.0970 | 0.5024 |
| 19 | 0.6167 | 0.4861 | 0.8p,252bd | 1.6735 | 1c,252bd | -1.3861 | 1.1c,252bd | 0.7416 | 0.0033 | 0.5375 | -4.8623 | 0.1169 |
| 20 | 0.6136 | 2.5112 | 0.8p,10bd | 2.1595 | 1.1c,10bd | 0.7878 | 1c,10bd | -0.0142 | 0.0173 | 0.3234 | -5.1710 | -0.0505 |

Table 12: Top 20 strike gamma adjusted strike convexity triplets.

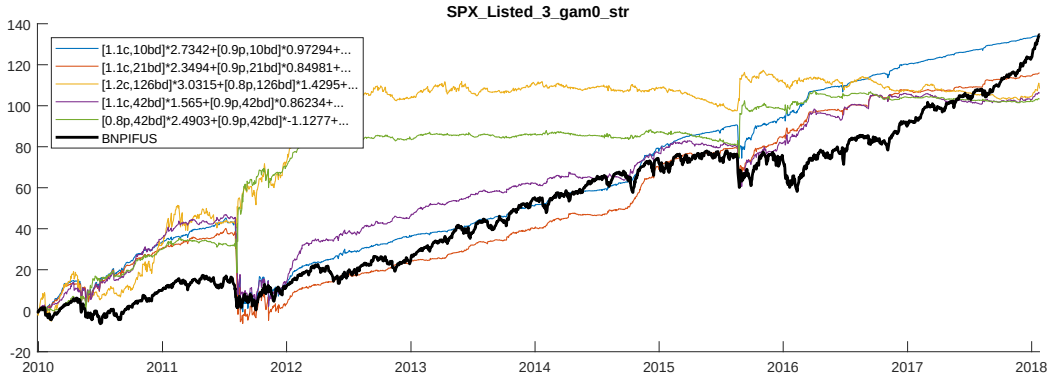


Figure 28: Plots of the top 5 gamma adjusted strike convexity triplets. The performances are benchmarked against the BNPIFUS index, shown by the black bold line.

## 7.3 Vega adjusted strike spreads

Table 13 shows the top strike spreads where the weights are inversely scaled by the median vega. Similar to the case with gamma, the performance of the strategies are again quite poor.

| | SR | MaxTC | Option | Weight | Option | Weight | Alpha | Beta | CVar | SR08 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.1372 | 1.2372 | 1.1c,10bd | 6.2351 | 1c,10bd | -0.2309 | 0.0534 | 0.2754 | -5.8380 | 0.4434 |
| 2 | 1.0480 | 1.1425 | 1.1c,42bd | 3.0941 | 1c,42bd | -0.3724 | 0.0507 | 0.2318 | -5.2043 | 0.9760 |
| 3 | 0.9074 | 0.9810 | 1.1c,21bd | 5.1212 | 1c,21bd | -0.2775 | 0.0403 | 0.2544 | -5.8413 | 0.4478 |
| 4 | 0.7496 | 1.3252 | 1.2c,252bd | 2.0030 | 1c,252bd | -0.6359 | 0.0246 | 0.3423 | -4.7315 | 0.5786 |
| 5 | 0.7400 | 0.6942 | 1.1c,63bd | 1.7071 | 1c,63bd | -0.3902 | 0.0322 | 0.2174 | -5.0436 | 0.6407 |
| 6 | 0.7366 | 0.9686 | 1.2c,252bd | 2.6239 | 1.1c,252bd | -1.1083 | 0.0196 | 0.4043 | -4.8133 | 0.6389 |
| 7 | 0.7077 | 0.5104 | 1.1c,126bd | 2.1972 | 1c,126bd | -1.1139 | 0.0300 | 0.2199 | -4.2973 | 0.5749 |
| 8 | 0.6765 | 0.8423 | 0.9p,10bd | 1.3540 | 1c,10bd | -0.2224 | 0.0257 | 0.2558 | -6.3135 | 0.5524 |
| 9 | 0.6363 | 0.8741 | 1.1c,42bd | 2.3850 | 0.9p,42bd | -0.7183 | 0.0424 | -0.0346 | -4.4732 | 0.7034 |
| 10 | 0.6184 | 1.1750 | 1.2c,252bd | 1.8513 | 0.9p,252bd | -0.7103 | 0.0270 | 0.1807 | -4.4750 | 0.5049 |
| 11 | 0.6164 | 1.2807 | 1.2c,126bd | 3.5465 | 1c,126bd | -0.3521 | 0.0164 | 0.3386 | -5.6235 | 0.7223 |
| 12 | 0.5726 | 1.2342 | 1.2c,126bd | 3.3322 | 0.9p,126bd | -0.4886 | 0.0246 | 0.1730 | -5.2243 | 0.7164 |
| 13 | 0.5364 | 0.7588 | 0.8p,42bd | 2.3826 | 0.9p,42bd | -0.7540 | 0.0153 | 0.2791 | -4.1943 | -0.1404 |
| 14 | 0.4782 | 1.0460 | 0.8p,42bd | 1.5904 | 1c,42bd | -0.2011 | 0.0079 | 0.3357 | -5.4135 | -0.0132 |
| 15 | 0.4670 | 0.5383 | 0.8p,63bd | 1.5290 | 0.9p,63bd | -0.6183 | 0.0078 | 0.3266 | -4.6555 | -0.2202 |
| 16 | 0.4629 | 1.5148 | 0.8p,10bd | 2.2666 | 1c,10bd | -0.0826 | 0.0120 | 0.2596 | -4.6198 | -0.1340 |
| 17 | 0.4626 | 0.3336 | 1.1c,252bd | 2.0895 | 1c,252bd | -1.5705 | 0.0236 | 0.0831 | -4.0217 | 0.3030 |
| 18 | 0.4600 | 0.7968 | 1.2c,126bd | 4.1543 | 1.1c,126bd | -0.8136 | 0.0081 | 0.3152 | -5.5990 | 0.6728 |
| 19 | 0.4533 | 0.3873 | 0.8p,126bd | 2.4046 | 0.9p,126bd | -1.3412 | 0.0022 | 0.3990 | -5.0170 | -0.0095 |
| 20 | 0.4336 | 0.5442 | 0.9p,21bd | 1.1039 | 1c,21bd | -0.2634 | 0.0115 | 0.2387 | -6.3055 | 0.3497 |

Table 13: Top 20 vega adjusted strike spread strategies.

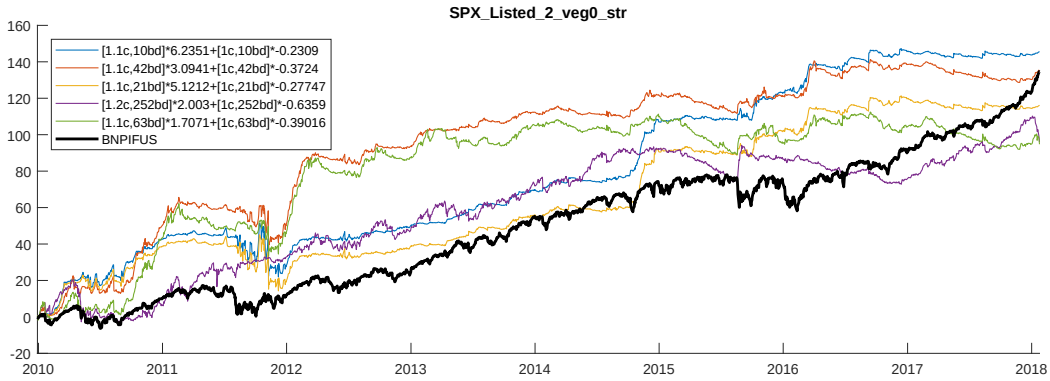


Figure 29: Plots of the top 5 vega adjusted strike spread strategies. The performances are benchmarked against the BNPIFUS index, shown by the black bold line.

## 7.4 Vega adjusted strike convexity triplets

Table 14 shows the top strike convexity triplets where the weights are inversely scaled by the median vega.

| | SR | MaxTC | Option | Weight | Option | Weight | Option | Weight | Alpha | Beta | CVar | SR08 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.1562 | 1.0494 | 1.1c,10bd | 3.9191 | 0.9p,10bd | 0.8835 | 1c,10bd | -0.2903 | 0.0503 | 0.3400 | -6.5355 | 0.5722 |
| 2 | 0.9868 | 0.8187 | 1.1c,42bd | 2.0512 | 0.8p,42bd | 1.9522 | 0.9p,42bd | -1.2355 | 0.0490 | 0.1989 | -4.0065 | 0.3484 |
| 3 | 0.9239 | 0.7315 | 1.1c,42bd | 2.2499 | 0.9p,42bd | 0.6776 | 1c,42bd | -0.5416 | 0.0337 | 0.3697 | -6.0378 | 0.8995 |
| 4 | 0.8909 | 1.0962 | 1.1c,42bd | 1.4256 | 0.8p,42bd | 1.3568 | 1c,42bd | -0.3432 | 0.0301 | 0.3931 | -5.8384 | 0.4230 |
| 5 | 0.8503 | 0.7732 | 1.1c,21bd | 3.1923 | 0.9p,21bd | 0.7248 | 1c,21bd | -0.3459 | 0.0327 | 0.3153 | -6.5325 | 0.5039 |
| 6 | 0.8216 | 0.8863 | 1.2c,252bd | 1.6344 | 1c,252bd | -1.0378 | 0.8p,252bd | 0.9655 | 0.0158 | 0.5435 | -4.5120 | 0.4401 |
| 7 | 0.8132 | 0.7439 | 1.2c,252bd | 1.9620 | 1c,252bd | -1.2458 | 0.9p,252bd | 0.7527 | 0.0195 | 0.4790 | -4.6992 | 0.5596 |
| 8 | 0.7844 | 0.4339 | 0.9p,126bd | -2.0785 | 0.8p,126bd | 1.8632 | 1.1c,126bd | 1.3881 | 0.0402 | 0.1393 | -3.2749 | 0.3996 |
| 9 | 0.7792 | 1.3709 | 0.8p,10bd | 2.0310 | 1.1c,10bd | 1.9980 | 1c,10bd | -0.1480 | 0.0279 | 0.3209 | -5.4204 | 0.0660 |
| 10 | 0.7483 | 0.7396 | 1.2c,252bd | 1.7722 | 0.9p,252bd | -1.3598 | 0.8p,252bd | 1.0469 | 0.0253 | 0.3296 | -4.6566 | 0.4200 |
| 11 | 0.7463 | 0.5211 | 0.8p,63bd | 1.2499 | 1.1c,63bd | 1.1117 | 0.9p,63bd | -1.0109 | 0.0351 | 0.1793 | -3.6664 | 0.0725 |
| 12 | 0.7378 | 0.8286 | 1.2c,126bd | 3.3326 | 0.9p,126bd | -0.9773 | 0.8p,126bd | 0.8760 | 0.0254 | 0.3183 | -5.2277 | 0.6994 |
| 13 | 0.6914 | 0.7137 | 0.8p,63bd | 0.8808 | 1.1c,63bd | 0.7835 | 1c,63bd | -0.3581 | 0.0138 | 0.4495 | -5.5882 | 0.1948 |
| 14 | 0.6809 | 0.4371 | 1.1c,63bd | 1.2631 | 1c,63bd | -0.5774 | 0.9p,63bd | 0.5743 | 0.0150 | 0.4213 | -5.6097 | 0.6218 |
| 15 | 0.6675 | 0.8662 | 1.2c,126bd | 2.9621 | 0.8p,126bd | 0.7786 | 1c,126bd | -0.5881 | 0.0062 | 0.5410 | -5.3760 | 0.5920 |
| 16 | 0.6268 | 1.0882 | 0.8p,21bd | 1.6603 | 1.1c,21bd | 1.6561 | 1c,21bd | -0.1795 | 0.0190 | 0.3090 | -5.4492 | 0.0081 |
| 17 | 0.6136 | 0.4621 | 0.8p,126bd | 1.4073 | 1c,126bd | -1.0630 | 1.1c,126bd | 1.0484 | 0.0008 | 0.5717 | -5.1341 | 0.1926 |
| 18 | 0.6129 | 0.2409 | 1c,252bd | -2.9096 | 1.1c,252bd | 1.9355 | 0.9p,252bd | 1.7580 | 0.0113 | 0.4127 | -3.6961 | 0.2504 |
| 19 | 0.6085 | 0.3928 | 1c,252bd | -1.7406 | 0.8p,252bd | 1.6195 | 1.1c,252bd | 1.1579 | 0.0060 | 0.4893 | -4.3074 | 0.1371 |
| 20 | 0.5898 | 0.6773 | 1.2c,126bd | 3.3554 | 1c,126bd | -0.6662 | 0.9p,126bd | 0.4920 | 0.0063 | 0.4665 | -5.4248 | 0.6535 |

Table 14: Top 20 vega adjusted strike convexity triplets.

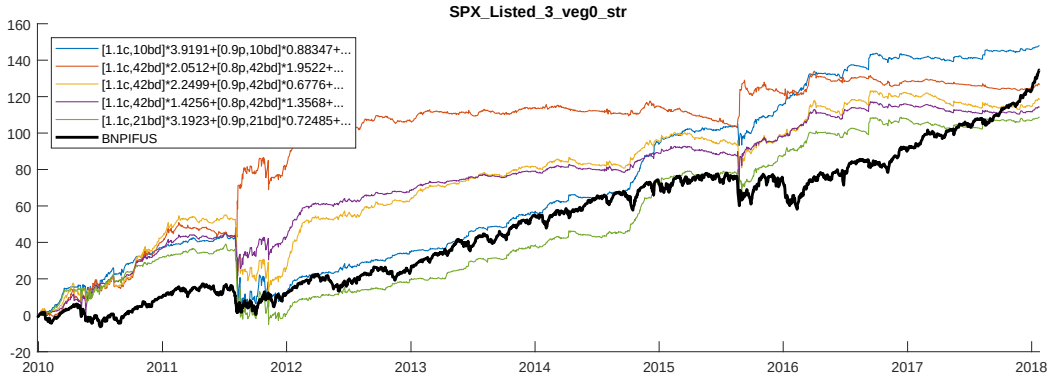


Figure 30: Plots of the top 5 vega adjusted strike convexity triplets. The performances are benchmarked against the BNPIFUS index, shown by the black bold line.

## 7.5 Gamma adjusted calendar spreads

Table 15 shows the top calendar spreads where the weights are inversely scaled by the median gamma. The performances are quite good, but slightly worse than the fixed weights counterparts. Similar to the fixed weights case, the top strategies involve buying short maturity, at the money options while selling long maturity, at the money options.

| | SR | MaxTC | Option | Weight | Option | Weight | Alpha | Beta | CVar | SR08 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.6174 | 1.3830 | 1c,63bd | 0.7205 | 1c,21bd | -0.1702 | 0.0702 | 0.4790 | -5.5222 | 1.1251 |
| 2 | 1.5917 | 1.8001 | 1c,63bd | 0.6380 | 1c,10bd | -0.0403 | 0.0690 | 0.4721 | -5.5634 | 1.1121 |
| 3 | 1.5580 | 0.6020 | 1c,63bd | 1.2318 | 1c,42bd | -0.9553 | 0.0668 | 0.4736 | -5.3688 | 1.1527 |
| 4 | 1.5311 | 0.8357 | 1c,42bd | 1.2119 | 1c,21bd | -0.3691 | 0.0675 | 0.4381 | -5.6206 | 0.9232 |
| 5 | 1.4721 | 1.3151 | 1c,42bd | 0.9283 | 1c,10bd | -0.0757 | 0.0646 | 0.4254 | -5.7004 | 0.9209 |
| 6 | 1.4157 | 2.4996 | 1c,126bd | 0.8966 | 1c,10bd | -0.0133 | 0.0538 | 0.5339 | -5.3604 | 0.8444 |
| 7 | 1.4099 | 2.2127 | 1c,126bd | 0.9273 | 1c,21bd | -0.0512 | 0.0534 | 0.5359 | -5.3272 | 0.8317 |
| 8 | 1.3490 | 1.6162 | 1c,126bd | 1.0447 | 1c,42bd | -0.1895 | 0.0496 | 0.5353 | -5.2288 | 0.7808 |
| 9 | 1.2956 | 2.2951 | 1.1c,126bd | 1.3896 | 1.1c,10bd | -1.3805 | 0.0499 | 0.4797 | -5.0447 | 0.8965 |
| 10 | 1.2920 | 2.2875 | 1.1c,126bd | 1.3900 | 1.1c,21bd | -1.1361 | 0.0496 | 0.4801 | -5.0462 | 0.8735 |
| 11 | 1.2895 | 0.9453 | 1.1c,10bd | -6.6308 | 1.1c,42bd | 4.8337 | 0.0648 | 0.2483 | -5.4304 | 0.9823 |
| 12 | 1.2808 | 0.9309 | 1.1c,21bd | -5.4988 | 1.1c,42bd | 4.8724 | 0.0640 | 0.2517 | -5.4515 | 0.9571 |
| 13 | 1.2623 | 1.1007 | 1c,126bd | 1.2681 | 1c,63bd | -0.2966 | 0.0441 | 0.5357 | -5.1566 | 0.6630 |
| 14 | 1.1468 | 1.4013 | 1.1c,126bd | 1.7218 | 1.1c,63bd | -0.7837 | 0.0418 | 0.4606 | -5.2038 | 0.7688 |
| 15 | 1.1115 | 1.6581 | 1.1c,126bd | 1.5037 | 1.1c,42bd | -1.0890 | 0.0394 | 0.4631 | -5.1536 | 0.6687 |
| 16 | 1.0902 | 1.1386 | 1.1c,10bd | -4.0671 | 1.1c,63bd | 1.8633 | 0.0476 | 0.3181 | -5.4135 | 0.6795 |
| 17 | 1.0815 | 1.1259 | 1.1c,21bd | -3.3548 | 1.1c,63bd | 1.8682 | 0.0470 | 0.3198 | -5.4196 | 0.6393 |
| 18 | 1.0359 | 0.5472 | 1c,21bd | 0.9782 | 1c,10bd | -0.2618 | 0.0447 | 0.3109 | -6.0214 | 0.7583 |
| 19 | 1.0273 | 2.8592 | 1c,252bd | 0.8203 | 1c,10bd | -0.0039 | 0.0264 | 0.5792 | -5.4196 | 0.5981 |
| 20 | 1.0203 | 2.7077 | 1c,252bd | 0.8270 | 1c,21bd | -0.0145 | 0.0259 | 0.5794 | -5.4132 | 0.5908 |

Table 15: Top 20 gamma adjusted calendar spread strategies.

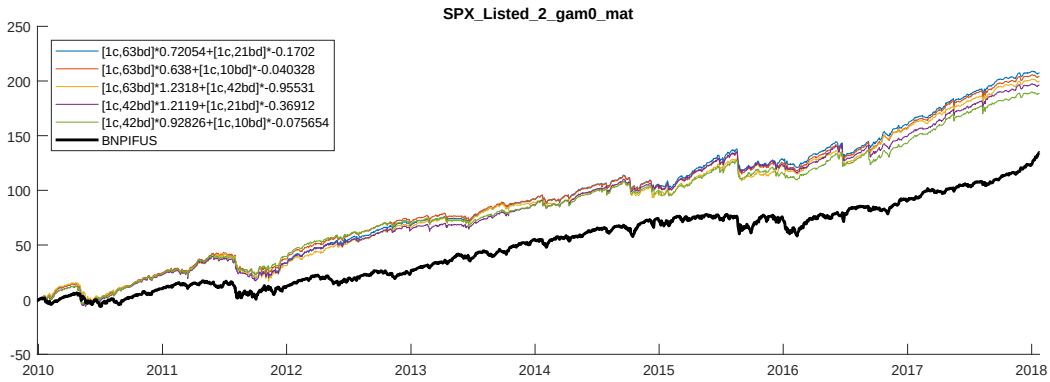


Figure 31: Plots of the top 5 gamma adjusted calendar spread strategies. The performances are benchmarked against the BNPIFUS index, shown by the black bold line.

## 7.6 Gamma adjusted calendar convexity triplets

Table 16 shows the top calendar convexity triplets where the weights are inversely scaled by the median gamma.

| | SR | MaxTC | Option | Weight | Option | Weight | Option | Weight | Alpha | Beta | CVar | SR08 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.6220 | 0.9588 | 1c,63bd | 0.8132 | 1c,21bd | -0.3842 | 1c,10bd | 0.0514 | 0.0704 | 0.4796 | -5.4574 | 1.1217 |
| 2 | 1.4969 | 0.4478 | 1c,42bd | 1.5911 | 1c,21bd | -0.9692 | 1c,10bd | 0.1297 | 0.0664 | 0.4211 | -5.2832 | 0.8573 |
| 3 | 1.4013 | 1.8411 | 1c,126bd | 0.9585 | 1c,21bd | -0.1059 | 1c,10bd | 0.0142 | 0.0527 | 0.5371 | -5.3015 | 0.8168 |
| 4 | 1.2883 | 1.6221 | 1.1c,21bd | -2.2725 | 1.1c,126bd | 1.3902 | 1.1c,10bd | 1.3811 | 0.0494 | 0.4805 | -5.0469 | 0.8446 |
| 5 | 1.2674 | 0.5076 | 1.1c,21bd | -11.0472 | 1.1c,10bd | 6.7140 | 1.1c,42bd | 4.8943 | 0.0630 | 0.2543 | -5.4575 | 0.8645 |
| 6 | 1.2566 | 0.9896 | 1c,126bd | 1.1831 | 1c,42bd | -0.4293 | 1c,21bd | 0.0654 | 0.0442 | 0.5286 | -5.1305 | 0.7068 |
| 7 | 1.2307 | 0.9861 | 1c,126bd | 1.2263 | 1c,42bd | -0.4449 | 1c,10bd | 0.0181 | 0.0427 | 0.5264 | -5.0829 | 0.6776 |
| 8 | 1.1248 | 0.7919 | 1.1c,126bd | 1.8969 | 1.1c,63bd | -1.7267 | 1.1c,42bd | 1.3738 | 0.0424 | 0.4306 | -5.1506 | 0.8189 |
| 9 | 1.1067 | 0.5012 | 1c,126bd | 1.5819 | 1c,63bd | -0.7401 | 1c,42bd | 0.2870 | 0.0349 | 0.5260 | -5.0501 | 0.4825 |
| 10 | 1.0712 | 0.6533 | 1.1c,21bd | -6.7184 | 1.1c,10bd | 4.0831 | 1.1c,63bd | 1.8706 | 0.0462 | 0.3211 | -5.4185 | 0.5768 |
| 11 | 1.0131 | 2.4817 | 1c,252bd | 0.8337 | 1c,21bd | -0.0293 | 1c,10bd | 0.0039 | 0.0255 | 0.5794 | -5.4128 | 0.5833 |
| 12 | 1.0121 | 2.3099 | 1.1c,252bd | 0.9488 | 1.1c,21bd | -0.7592 | 1.1c,10bd | 0.4614 | 0.0297 | 0.5150 | -5.0023 | 0.6922 |
| 13 | 0.9551 | 2.1966 | 0.9p,252bd | 0.8955 | 0.9p,21bd | -0.2018 | 0.9p,10bd | 0.1207 | 0.0163 | 0.6628 | -5.5316 | 0.4171 |
| 14 | 0.9541 | 1.8811 | 1c,252bd | 0.8769 | 1c,42bd | -0.1012 | 1c,21bd | 0.0154 | 0.0219 | 0.5778 | -5.3977 | 0.5408 |
| 15 | 0.9471 | 0.6293 | 0.9p,21bd | -1.4942 | 0.9p,63bd | 0.9687 | 0.9p,10bd | 0.8934 | 0.0221 | 0.5683 | -5.7887 | 0.6679 |
| 16 | 0.9460 | 1.7551 | 0.9p,252bd | 0.9979 | 0.9p,42bd | -0.3035 | 0.9p,10bd | 0.1345 | 0.0159 | 0.6605 | -5.4565 | 0.3763 |
| 17 | 0.9457 | 1.7552 | 0.9p,252bd | 0.9967 | 0.9p,42bd | -0.3032 | 0.9p,21bd | 0.1123 | 0.0159 | 0.6606 | -5.4617 | 0.3776 |
| 18 | 0.9454 | 1.9070 | 1c,252bd | 0.8841 | 1c,42bd | -0.1020 | 1c,10bd | 0.0042 | 0.0213 | 0.5776 | -5.4005 | 0.5321 |
| 19 | 0.9291 | 1.6465 | 1.1c,252bd | 1.0284 | 1.1c,63bd | -0.4583 | 1.1c,42bd | 0.3646 | 0.0255 | 0.4985 | -5.1541 | 0.6612 |
| 20 | 0.9087 | 1.3248 | 0.9p,252bd | 1.0825 | 0.9p,63bd | -0.3163 | 0.9p,42bd | 0.1646 | 0.0138 | 0.6570 | -5.4042 | 0.3252 |

Table 16: Top 20 strike gamma adjusted calendar convexity triplets.

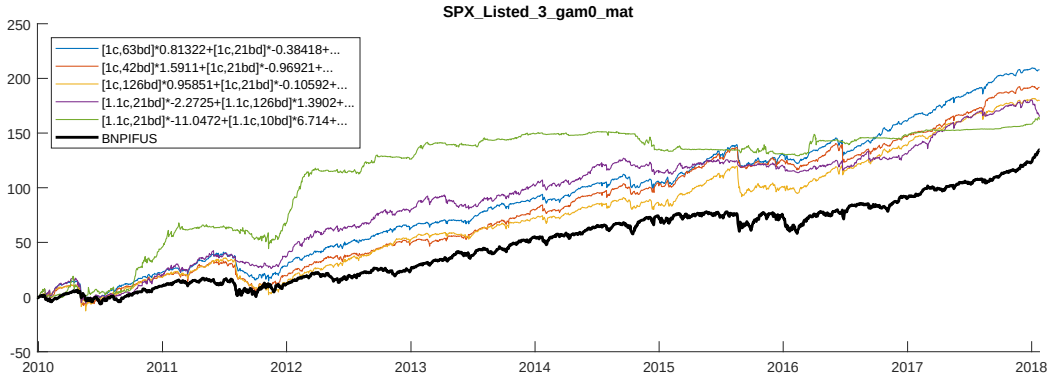


Figure 32: Plots of the top 5 gamma adjusted calendar convexity triplets. The performances are benchmarked against the BNPIFUS index, shown by the black bold line.

## 7.7 Vega adjusted calendar spreads

Table 17 shows the top calendar spreads where the weights are inversely scaled by the median vega. Interestingly, the top three strategies all have excellent performances, even outperforming the fixed weights versions. However, the performance drops off drastically as we go down the list, with only the top 9 manage to outperform the benchmark.

| | SR | MaxTC | Option | Weight | Option | Weight | Alpha | Beta | CVar | SR08 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2.5522 | 0.2988 | 1c,21bd | 2.8998 | 1c,10bd | -2.4592 | 0.1489 | 0.1789 | -3.5494 | 2.3351 |
| 2 | 2.5391 | 0.6481 | 1c,42bd | 1.7966 | 1c,10bd | -1.1523 | 0.1348 | 0.3794 | -3.8238 | 1.7977 |
| 3 | 2.3091 | 0.8698 | 1c,63bd | 1.0142 | 1c,10bd | -0.7815 | 0.1168 | 0.4338 | -4.0928 | 1.7850 |
| 4 | 1.7223 | 1.1872 | 1c,126bd | 1.1705 | 1c,10bd | -0.4259 | 0.0743 | 0.5167 | -4.5098 | 1.0621 |
| 5 | 1.5256 | 0.5656 | 1c,63bd | 1.0329 | 1c,21bd | -0.9385 | 0.0707 | 0.3839 | -4.4733 | 1.0731 |
| 6 | 1.3918 | 0.3437 | 1c,42bd | 1.8616 | 1c,21bd | -1.4079 | 0.0674 | 0.3063 | -4.3946 | 0.8273 |
| 7 | 1.3036 | 0.8846 | 1c,126bd | 1.1918 | 1c,21bd | -0.5113 | 0.0494 | 0.4946 | -4.7368 | 0.7042 |
| 8 | 1.1431 | 0.2266 | 1c,42bd | -2.3732 | 1c,63bd | 1.9753 | 0.0493 | 0.3437 | -4.4528 | 0.8731 |
| 9 | 1.1196 | 1.3341 | 1c,252bd | 0.9536 | 1c,10bd | -0.2466 | 0.0330 | 0.5666 | -5.0669 | 0.6800 |
| 10 | 0.9937 | 0.5536 | 1c,126bd | 1.4840 | 1c,42bd | -0.8418 | 0.0310 | 0.4774 | -4.7234 | 0.4998 |
| 11 | 0.8717 | 1.0315 | 1c,252bd | 0.9625 | 1c,21bd | -0.2935 | 0.0183 | 0.5537 | -5.2480 | 0.4531 |
| 12 | 0.7814 | 0.3281 | 1c,126bd | 1.9713 | 1c,63bd | -0.9307 | 0.0180 | 0.4722 | -4.7704 | 0.2477 |
| 13 | 0.6469 | 0.7035 | 1c,252bd | 1.0707 | 1c,42bd | -0.4317 | 0.0047 | 0.5449 | -5.2843 | 0.3080 |
| 14 | 0.6466 | 0.3876 | 1.1c,21bd | -8.2903 | 1.1c,42bd | 4.9346 | 0.0371 | 0.0555 | -4.9614 | 0.3912 |
| 15 | 0.6232 | 0.3769 | 1.1c,10bd | -10.0992 | 1.1c,42bd | 4.8448 | 0.0366 | 0.0399 | -4.9051 | 0.4304 |
| 16 | 0.6230 | 0.0486 | 0.8p,10bd | 90.9488 | 0.8p,21bd | -72.2903 | 0.0193 | 0.3016 | -4.9049 | 0.0580 |
| 17 | 0.5493 | 0.2888 | 1.1c,42bd | 6.0527 | 1.1c,63bd | -2.6530 | 0.0339 | 0.0100 | -4.5908 | 0.7038 |
| 18 | 0.5478 | 0.5479 | 0.9p,252bd | 1.3511 | 0.9p,42bd | -1.1281 | 0.0019 | 0.4922 | -3.9966 | 0.0476 |
| 19 | 0.5436 | 0.0169 | 0.9p,10bd | 52.8077 | 0.9p,21bd | -42.8686 | 0.0292 | 0.0762 | -5.2609 | -0.0242 |
| 20 | 0.4966 | 0.4778 | 1c,252bd | 1.2114 | 1c,63bd | -0.4066 | -0.0048 | 0.5458 | -5.2828 | 0.1561 |

Table 17: Top 20 vega adjusted calendar spread strategies.

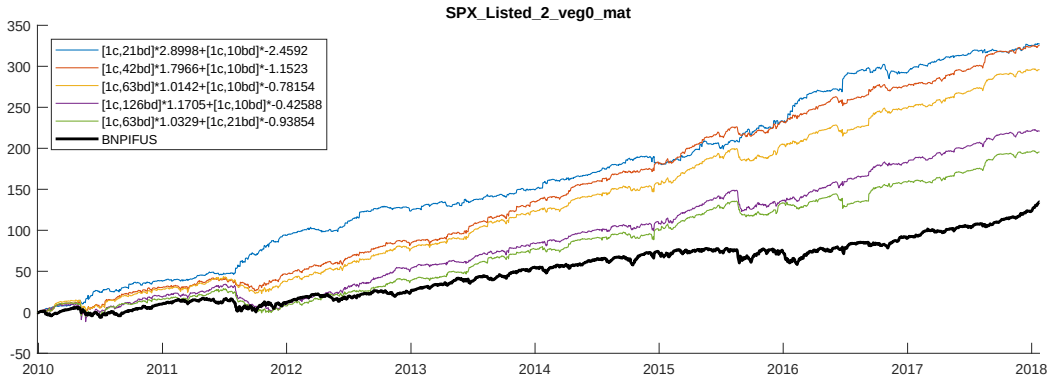


Figure 33: Plots of the top 5 vega adjusted calendar spread strategies. The performances are benchmarked against the BNPIFUS index, shown by the black bold line.

## 7.8 Vega adjusted calendar convexity triplets

Table 18 shows the top calendar convexity triplets where the weights are inversely scaled by the median vega.

| | SR | MaxTC | Option | Weight | Option | Weight | Option | Weight | Alpha | Beta | CVar | SR08 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.9872 | 0.5270 | 1c,126bd | 3.0785 | 1c,252bd | -2.1655 | 1c,10bd | -0.5600 | 0.1204 | 0.0725 | -3.7495 | 0.8246 |
| 2 | 1.6974 | 0.2163 | 1c,42bd | 3.6439 | 1c,63bd | -1.5164 | 1c,10bd | -1.1686 | 0.0989 | 0.1210 | -3.9983 | 0.9453 |
| 3 | 1.4525 | 0.2776 | 1c,126bd | -2.1405 | 1c,63bd | 2.0213 | 1c,10bd | -0.7788 | 0.0968 | -0.0804 | -3.4826 | 1.1092 |
| 4 | 1.4262 | 0.3733 | 1c,126bd | 3.1227 | 1c,252bd | -2.1966 | 1c,21bd | -0.6699 | 0.0877 | 0.0323 | -4.0465 | 0.5350 |
| 5 | 0.8903 | 0.3400 | 1.1c,42bd | 8.0285 | 1.1c,21bd | -6.7441 | 1.1c,63bd | -1.7595 | 0.0527 | 0.0518 | -5.1199 | 0.8288 |
| 6 | 0.8753 | 0.3346 | 1.1c,10bd | -8.3167 | 1.1c,42bd | 7.9794 | 1.1c,63bd | -1.7487 | 0.0525 | 0.0395 | -5.0791 | 0.8325 |
| 7 | 0.8645 | 0.2022 | 1c,126bd | 3.5337 | 1c,252bd | -2.4857 | 1c,42bd | -1.0023 | 0.0630 | -0.1284 | -4.2147 | 0.3276 |
| 8 | 0.8429 | 0.2864 | 1c,126bd | 1.1770 | 1c,21bd | -1.0099 | 1c,10bd | 0.4282 | 0.0228 | 0.4573 | -5.0033 | 0.3251 |
| 9 | 0.6649 | 0.1961 | 1.1c,21bd | -16.7418 | 1.1c,10bd | 10.3864 | 1.1c,42bd | 4.9825 | 0.0372 | 0.0709 | -4.9747 | 0.3108 |
| 10 | 0.6461 | 0.1281 | 1c,21bd | -1.7336 | 1c,63bd | 0.9539 | 1c,10bd | 0.7351 | 0.0208 | 0.3011 | -4.7283 | 0.2927 |
| 11 | 0.6294 | 0.1233 | 1c,126bd | -2.0914 | 1c,63bd | 1.9749 | 1c,21bd | -0.8973 | 0.0485 | -0.1339 | -3.6807 | 0.5377 |
| 12 | 0.6128 | 0.3587 | 1c,252bd | 0.9616 | 1c,21bd | -0.5865 | 1c,10bd | 0.2487 | 0.0032 | 0.5351 | -5.4168 | 0.2222 |
| 13 | 0.5670 | 0.2536 | 0.9p,42bd | -2.4567 | 0.9p,10bd | 1.9166 | 0.9p,252bd | 1.4712 | 0.0073 | 0.4297 | -4.5858 | 0.0271 |
| 14 | 0.5444 | 0.2461 | 0.9p,42bd | -2.4439 | 0.9p,21bd | 1.5477 | 0.9p,252bd | 1.4634 | 0.0062 | 0.4246 | -4.5850 | 0.0290 |
| 15 | 0.5100 | 0.2039 | 1c,252bd | -1.4400 | 1c,63bd | 0.9666 | 1c,10bd | -0.3724 | 0.0614 | -0.4421 | -3.4055 | 0.5857 |
| 16 | 0.5001 | 0.0618 | 1c,42bd | 3.7604 | 1c,63bd | -1.5649 | 1c,21bd | -1.4220 | 0.0290 | 0.0371 | -4.5312 | 0.0538 |
| 17 | 0.4927 | 0.2926 | 0.9p,21bd | -2.3042 | 0.9p,10bd | 1.4192 | 0.9p,252bd | 1.0894 | -0.0007 | 0.4796 | -4.0212 | 0.0567 |
| 18 | 0.4911 | 0.2009 | 1.1c,42bd | 4.1040 | 1.1c,63bd | -3.5977 | 1.1c,126bd | 1.7176 | 0.0205 | 0.1582 | -4.1890 | 0.4467 |
| 19 | 0.4873 | 0.3004 | 1.1c,42bd | 5.0525 | 1.1c,21bd | -4.2442 | 1.1c,126bd | -1.0573 | 0.0347 | -0.0606 | -4.6068 | 0.7241 |
| 20 | 0.4811 | 0.2952 | 1.1c,10bd | -5.2647 | 1.1c,42bd | 5.0511 | 1.1c,126bd | -1.0570 | 0.0348 | -0.0682 | -4.5849 | 0.7668 |

Table 18: Top 20 strike vega adjusted calendar convexity triplets.

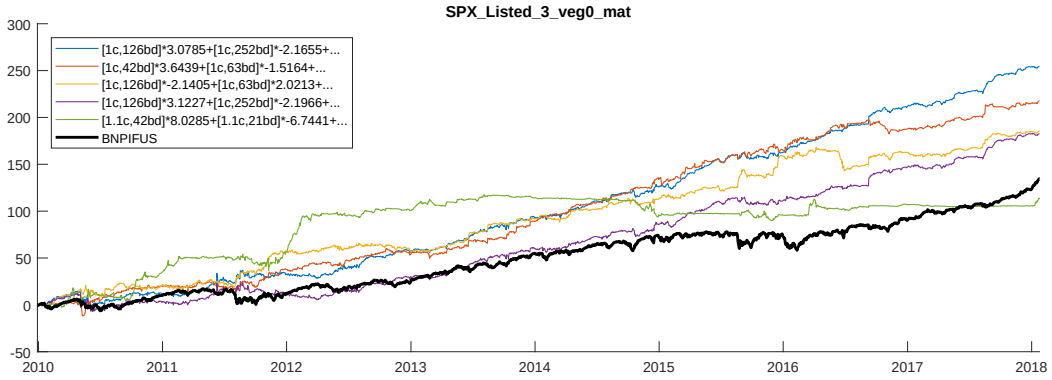


Figure 34: Plots of the top 5 vega adjusted calendar convexity triplets. The performances are benchmarked against the BNPIFUS index, shown by the black bold line.

## 7.9 Gamma adjusted spreads over all options

Table 19 shows the top gamma adjusted strategies over all option pairs.

| | SR | MaxTC | Option | Weight | Option | Weight | Alpha | Beta | CVar | SR08 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.6174 | 1.3830 | 1c,63bd | 0.7205 | 1c,21bd | -0.1702 | 0.0702 | 0.4790 | -5.5222 | 1.1251 |
| 2 | 1.5917 | 1.8001 | 1c,63bd | 0.6380 | 1c,10bd | -0.0403 | 0.0690 | 0.4721 | -5.5634 | 1.1121 |
| 3 | 1.5580 | 0.6020 | 1c,63bd | 1.2318 | 1c,42bd | -0.9553 | 0.0668 | 0.4736 | -5.3688 | 1.1527 |
| 4 | 1.5311 | 0.8357 | 1c,42bd | 1.2119 | 1c,21bd | -0.3691 | 0.0675 | 0.4381 | -5.6206 | 0.9232 |
| 5 | 1.5177 | 2.7836 | 1.1c,42bd | 2.8700 | 1c,10bd | -0.0356 | 0.0711 | 0.3700 | -5.3992 | 1.1523 |
| 6 | 1.4875 | 3.3057 | 1.1c,126bd | 1.3256 | 1c,10bd | -0.0119 | 0.0585 | 0.5320 | -4.9462 | 1.0784 |
| 7 | 1.4831 | 2.9123 | 1.1c,126bd | 1.3649 | 1c,21bd | -0.0458 | 0.0581 | 0.5332 | -4.9022 | 1.0713 |
| 8 | 1.4721 | 1.3151 | 1c,42bd | 0.9283 | 1c,10bd | -0.0757 | 0.0646 | 0.4254 | -5.7004 | 0.9209 |
| 9 | 1.4537 | 2.7293 | 1.1c,63bd | 1.4507 | 1c,10bd | -0.0286 | 0.0633 | 0.4267 | -5.4222 | 1.0263 |
| 10 | 1.4351 | 2.0290 | 1.1c,42bd | 2.9912 | 1c,21bd | -0.1385 | 0.0678 | 0.3416 | -5.3940 | 1.1211 |
| 11 | 1.4194 | 2.1486 | 1.1c,126bd | 1.4981 | 1c,42bd | -0.1650 | 0.0546 | 0.5257 | -4.7782 | 1.0309 |
| 12 | 1.4157 | 2.4996 | 1c,126bd | 0.8966 | 1c,10bd | -0.0133 | 0.0538 | 0.5339 | -5.3604 | 0.8444 |
| 13 | 1.4099 | 2.2127 | 1c,126bd | 0.9273 | 1c,21bd | -0.0512 | 0.0534 | 0.5359 | -5.3272 | 0.8317 |
| 14 | 1.4047 | 2.0948 | 1.1c,63bd | 1.5203 | 1c,21bd | -0.1120 | 0.0613 | 0.4115 | -5.4174 | 1.0008 |
| 15 | 1.3490 | 1.6162 | 1c,126bd | 1.0447 | 1c,42bd | -0.1895 | 0.0496 | 0.5353 | -5.2288 | 0.7808 |
| 16 | 1.3350 | 1.6092 | 1c,126bd | 1.0570 | 0.9p,10bd | -0.6156 | 0.0526 | 0.4754 | -4.6478 | 0.7252 |
| 17 | 1.3340 | 1.6011 | 1c,126bd | 1.0591 | 0.9p,21bd | -0.5159 | 0.0528 | 0.4724 | -4.6322 | 0.7213 |
| 18 | 1.3022 | 1.5207 | 1.1c,126bd | 1.6958 | 1c,63bd | -0.2408 | 0.0488 | 0.5025 | -4.6889 | 0.9127 |
| 19 | 1.2956 | 2.2951 | 1.1c,126bd | 1.3896 | 1.1c,10bd | -1.3805 | 0.0499 | 0.4797 | -5.0447 | 0.8965 |
| 20 | 1.2920 | 2.2875 | 1.1c,126bd | 1.3900 | 1.1c,21bd | -1.1361 | 0.0496 | 0.4801 | -5.0462 | 0.8735 |

Table 19: Top 20 gamma adjusted spread strategies over all options.

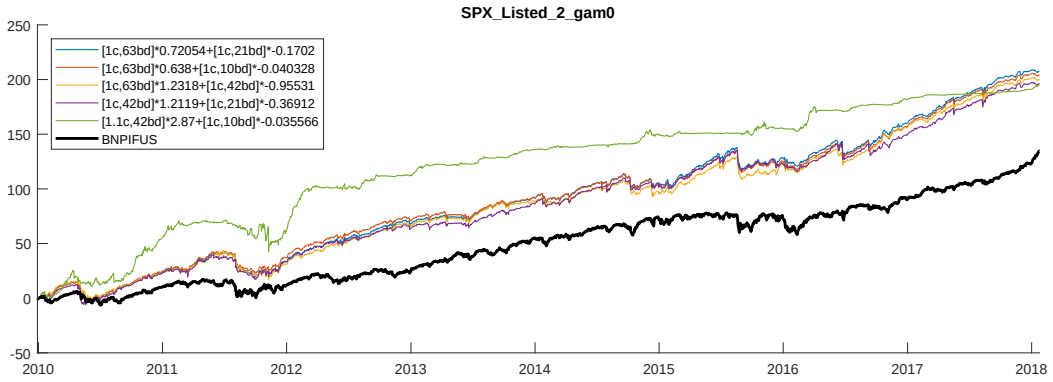


Figure 35: Plots of the top 5 gamma adjusted spread strategies over all options. The performances are benchmarked against the BNPIFUS index, shown by the black bold line.

## 7.10 Vega adjusted spreads over all options

Table 20 shows the top vega adjusted strategies over all option pairs.

| | SR | MaxTC | Option | Weight | Option | Weight | Alpha | Beta | CVar | SR08 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2.5522 | 0.2988 | 1c,21bd | 2.8998 | 1c,10bd | -2.4592 | 0.1489 | 0.1789 | -3.5494 | 2.3351 |
| 2 | 2.5391 | 0.6481 | 1c,42bd | 1.7966 | 1c,10bd | -1.1523 | 0.1348 | 0.3794 | -3.8238 | 1.7977 |
| 3 | 2.3091 | 0.8698 | 1c,63bd | 1.0142 | 1c,10bd | -0.7815 | 0.1168 | 0.4338 | -4.0928 | 1.7850 |
| 4 | 1.7298 | 1.6560 | 1.1c,126bd | 1.6197 | 1c,10bd | -0.2987 | 0.0743 | 0.5246 | -4.2498 | 1.2921 |
| 5 | 1.7223 | 1.1872 | 1c,126bd | 1.1705 | 1c,10bd | -0.4259 | 0.0743 | 0.5167 | -4.5098 | 1.0621 |
| 6 | 1.5489 | 1.6804 | 1.1c,42bd | 3.0441 | 1c,10bd | -0.2350 | 0.0774 | 0.3054 | -5.2129 | 1.2560 |
| 7 | 1.5263 | 1.4917 | 1.1c,63bd | 1.6002 | 1c,10bd | -0.2818 | 0.0723 | 0.3602 | -5.1401 | 1.1663 |
| 8 | 1.5256 | 0.5656 | 1c,63bd | 1.0329 | 1c,21bd | -0.9385 | 0.0707 | 0.3839 | -4.4733 | 1.0731 |
| 9 | 1.4495 | 1.3624 | 1.1c,126bd | 1.6535 | 1c,21bd | -0.3596 | 0.0573 | 0.5134 | -4.3586 | 1.0226 |
| 10 | 1.3918 | 0.3437 | 1c,42bd | 1.8616 | 1c,21bd | -1.4079 | 0.0674 | 0.3063 | -4.3946 | 0.8273 |
| 11 | 1.3170 | 1.4188 | 1.1c,42bd | 3.0721 | 1c,21bd | -0.2796 | 0.0637 | 0.2909 | -5.2449 | 1.0894 |
| 12 | 1.3036 | 0.8846 | 1c,126bd | 1.1918 | 1c,21bd | -0.5113 | 0.0494 | 0.4946 | -4.7368 | 0.7042 |
| 13 | 1.2557 | 1.2085 | 1.1c,63bd | 1.6286 | 1c,21bd | -0.3382 | 0.0562 | 0.3457 | -5.1010 | 0.9455 |
| 14 | 1.2146 | 1.0504 | 1.1c,126bd | 1.8359 | 1c,42bd | -0.5280 | 0.0445 | 0.4831 | -4.1982 | 0.8745 |
| 15 | 1.1914 | 1.6255 | 1.1c,252bd | 1.0793 | 1c,10bd | -0.2098 | 0.0403 | 0.5248 | -4.3395 | 0.8943 |
| 16 | 1.1506 | 2.5857 | 1.2c,252bd | 1.5403 | 1c,10bd | -0.1265 | 0.0358 | 0.5537 | -4.7425 | 0.8980 |
| 17 | 1.1490 | 1.2425 | 1.1c,21bd | 5.1099 | 1c,10bd | -0.2348 | 0.0545 | 0.2710 | -5.7928 | 0.5613 |
| 18 | 1.1431 | 0.2266 | 1c,42bd | -2.3732 | 1c,63bd | 1.9753 | 0.0493 | 0.3437 | -4.4528 | 0.8731 |
| 19 | 1.1372 | 1.2372 | 1.1c,10bd | 6.2351 | 1c,10bd | -0.2309 | 0.0534 | 0.2754 | -5.8380 | 0.4434 |
| 20 | 1.1196 | 1.3341 | 1c,252bd | 0.9536 | 1c,10bd | -0.2466 | 0.0330 | 0.5666 | -5.0669 | 0.6800 |

Table 20: Top 20 vega adjusted spread strategies over all options.

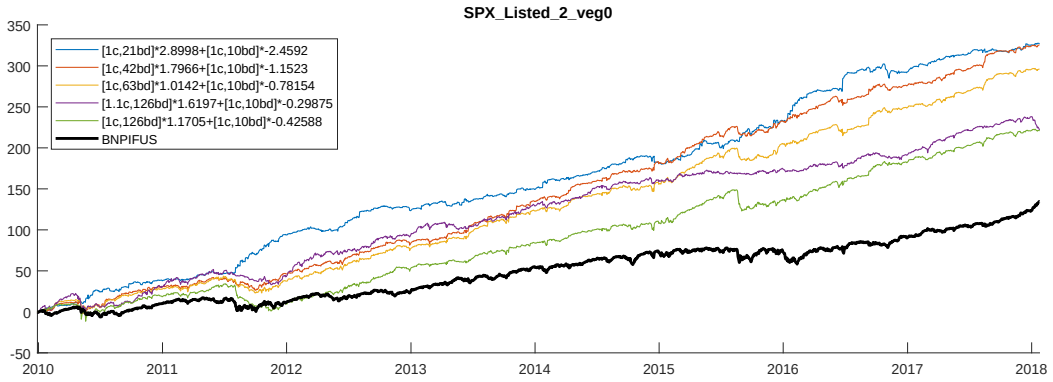


Figure 36: Plots of the top 5 vega adjusted spread strategies over all options. The performances are benchmarked against the BNPIFUS index, shown by the black bold line.

## 7.11 Summary of findings

We studied spread and convexity strategies in which the weights are inversely scaled according to the median gamma or vega of the option. The intention is to create static weight portfolios which are approximately gamma or vega neutral. By doing so we indeed recover strategies resembling the more typical skew and smile arbitrages. However, the performance of most of these strategies are subpar, as they are extremely sensitive to any downward shocks to the market. The main exception here are the vega adjusted calendar spreads, which were able to generate very high alphas.

For future studies, it would be interesting to see the effect of dynamically adjusting option weights according to their evolving gammas or vegas.

# 8 Optimised spread strategies

So far, we have only studied combinations of systematic options strategies with fixed weight ratios. In this section, we focus on strategies whose weights are chosen to maximise the (annualised) Sharpe ratio.

The data is divided into two periods of equal size. The weights are optimised with respect to the first period (in-sample) then tested on the second period (out-of-sample). The strategies are sorted based on the optimised performance in the in-sample period. The metrics Max TC and 99% CVaR (see previous section for more details) are computed for the out-of-sample period.

Suppose that the vector of available strategies has mean $\mu$ and a positive-definite covariance matrix $\Sigma$. If $w$ is the portfolio weights, then the problem of maximising the Sharpe ratio is as follows

$$\max_w \frac{w^T\mu}{\sqrt{w^T\Sigma w}}.$$

We claim that the maximum Sharpe ratio is given by, $\sqrt{\mu^T\Sigma^{-1}\mu}$, achieved by $w = \lambda\Sigma^{-1}\mu$ where $\lambda > 0$ is any positive constant. Indeed this is easy to verify by the Cauchy-Schwarz inequality

$$(w^T\Sigma w)(\mu^T\Sigma^{-1}\mu) \geq (w^T\Sigma^{\frac{1}{2}}\Sigma^{-\frac{1}{2}}\mu)^2 = (w^T\mu)^2.$$

Note that the optimal portfolio $w = \lambda\Sigma^{-1}\mu$ coincides with the Markowitz portfolio in mean-variance optimisation, since the problem is equivalent to maximising return $w^T\mu$ given a fixed variance budget $w^T\Sigma w$.

For a detailed breakdown of the different columns of the result tables, as well as their calculations and interpretations, please refer to the beginning of Section 5.

## 8.1 Optimised strike spreads

Table 21 contains the top optimised strike spread pairs, and the top five plots can be found in Figure 37.

| | SR(in) | SR(out) | MaxTC | Option | Weight | Option | Weight | Alpha | Beta | CVar |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2.2140 | 1.2769 | 1.6247 | 1.2c,42bd | -19.0397 | 1.1c,42bd | 5.1067 | 0.1353 | 0.3850 | -5.2224 |
| 2 | 1.9382 | 1.1424 | 1.3453 | 1.2c,63bd | -5.9205 | 1.1c,63bd | 2.7283 | 0.1110 | 0.5352 | -5.2873 |
| 3 | 1.7878 | 0.9038 | 1.2262 | 1.1c,126bd | 1.6575 | 0.9p,126bd | -0.3646 | 0.0651 | 0.4148 | -4.9194 |
| 4 | 1.7655 | 1.6258 | 1.7868 | 1.1c,63bd | 0.9712 | 1c,63bd | 0.4201 | 0.0874 | 0.6167 | -5.1160 |
| 5 | 1.7566 | 1.6383 | 1.5533 | 1.1c,42bd | 1.8131 | 1c,42bd | 0.6356 | 0.0884 | 0.5578 | -5.1906 |
| 6 | 1.7332 | 1.3272 | 1.0173 | 1c,63bd | 0.6960 | 0.9p,63bd | -0.2679 | 0.0645 | 0.3095 | -5.4689 |
| 7 | 1.7223 | 0.9633 | 1.1246 | 1.1c,126bd | 1.8231 | 1c,126bd | -0.3404 | 0.0599 | 0.5340 | -4.9683 |
| 8 | 1.7023 | 1.2278 | 2.4336 | 1.1c,126bd | 1.4652 | 1.2c,126bd | -0.3760 | 0.0606 | 0.5355 | -4.9818 |
| 9 | 1.7015 | 1.1755 | 2.2818 | 1.1c,126bd | 1.4656 | 0.8p,126bd | -0.1066 | 0.0632 | 0.5345 | -4.9860 |
| 10 | 1.6813 | 1.4066 | 2.1014 | 1.1c,63bd | 2.2944 | 0.8p,63bd | 0.2315 | 0.1045 | 0.8228 | -5.3151 |
| 11 | 1.6674 | 1.1868 | 1.6991 | 1.1c,63bd | 2.5159 | 0.9p,63bd | -0.0530 | 0.1073 | 0.7174 | -5.4330 |
| 12 | 1.6305 | 1.8718 | 2.6224 | 1.1c,42bd | 4.9555 | 0.8p,42bd | 0.6060 | 0.1305 | 0.8317 | -5.4936 |
| 13 | 1.6019 | 1.7539 | 2.5365 | 1.1c,42bd | 5.4761 | 0.9p,42bd | 0.0196 | 0.1355 | 0.7456 | -5.3962 |
| 14 | 1.5806 | 1.4899 | 1.6654 | 1c,63bd | 0.5644 | 1.2c,63bd | -0.5310 | 0.0600 | 0.4072 | -5.6132 |
| 15 | 1.5744 | 1.2121 | 1.2054 | 1.2c,42bd | -2.8590 | 1c,42bd | 0.7535 | 0.0532 | 0.3272 | -5.7424 |
| 16 | 1.5735 | 1.4826 | 1.6050 | 1c,63bd | 0.5776 | 0.8p,63bd | -0.0674 | 0.0607 | 0.4062 | -5.5594 |
| 17 | 1.5546 | 1.1541 | 0.9740 | 1c,42bd | 0.8288 | 0.9p,42bd | -0.2024 | 0.0524 | 0.3096 | -5.7891 |
| 18 | 1.5428 | 1.4113 | 0.9372 | 1c,126bd | 1.4469 | 0.9p,126bd | -0.8381 | 0.0685 | 0.2805 | -4.7486 |
| 19 | 1.4988 | 1.2912 | 1.3119 | 1c,42bd | 0.7436 | 0.8p,42bd | 0.0897 | 0.0524 | 0.3921 | -5.8003 |
| 20 | 1.3896 | 1.5500 | 2.4688 | 1.2c,126bd | 2.0108 | 1c,126bd | 0.7081 | 0.0637 | 0.7384 | -5.1739 |

Table 21: Top 20 strike spread pair strategies, optimised and sorted based on the in-sample period.

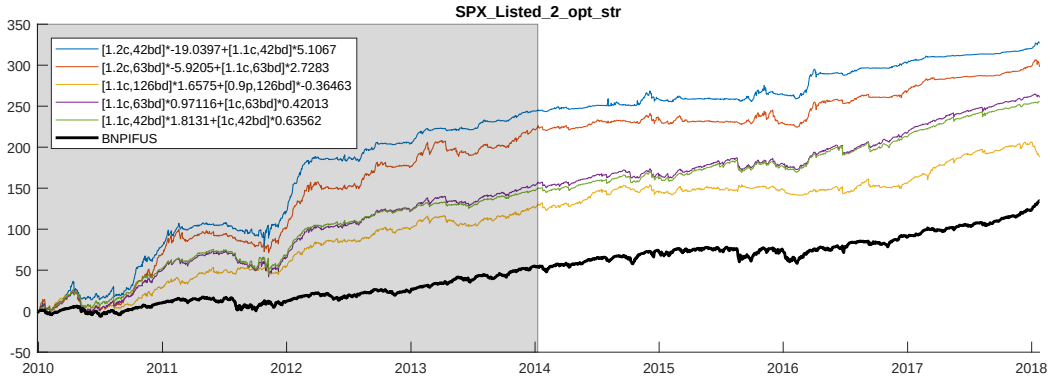


Figure 37: Plots of the top 5 strike spread pairs. The in-sample period is shaded grey. The performances are benchmarked against the BNPIFUS index, shown by the black bold line.

Table 22 contains the top optimised strike spread triplets, and the top five plots can be found in Figure 38.

| | SR(in) | SR(out) | MaxTC | Option | Weight | Option | Weight | Option | Weight | Alpha | Beta | CVar |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2.2799 | 1.5186 | 1.4172 | 1.2c,42bd | -14.6150 | 1.1c,42bd | 3.5157 | 1c,42bd | 0.3976 | 0.1212 | 0.4377 | -5.0199 |
| 2 | 2.2469 | 1.3879 | 1.7152 | 1.2c,42bd | -18.6428 | 1.1c,42bd | 4.8276 | 0.8p,42bd | 0.3816 | 0.1334 | 0.4582 | -5.2138 |
| 3 | 2.2141 | 1.2898 | 1.6297 | 1.2c,42bd | -19.0050 | 1.1c,42bd | 5.0839 | 0.9p,42bd | 0.0168 | 0.1351 | 0.3908 | -5.2167 |
| 4 | 1.9919 | 1.5030 | 1.4757 | 1.2c,63bd | -4.2625 | 1.1c,63bd | 1.6981 | 1c,63bd | 0.3048 | 0.1012 | 0.5388 | -5.0576 |
| 5 | 1.9599 | 1.2773 | 1.4612 | 1.2c,63bd | -5.8417 | 1.1c,63bd | 2.5817 | 0.8p,63bd | 0.2108 | 0.1092 | 0.6113 | -5.1790 |
| 6 | 1.9434 | 0.7390 | 0.4802 | 1.1c,126bd | 1.7987 | 0.9p,126bd | -1.2452 | 0.8p,126bd | 1.0804 | 0.0662 | 0.4554 | -4.5765 |
| 7 | 1.9389 | 1.0968 | 1.2402 | 1.2c,63bd | -5.9471 | 1.1c,63bd | 2.7641 | 0.9p,63bd | -0.0312 | 0.1114 | 0.5185 | -5.3028 |
| 8 | 1.8873 | 1.2993 | 0.6975 | 1c,63bd | 0.7702 | 0.9p,63bd | -0.6244 | 0.8p,63bd | 0.5001 | 0.0691 | 0.3438 | -5.2803 |
| 9 | 1.8610 | 1.3972 | 1.0189 | 1.1c,63bd | 0.6491 | 1c,63bd | 0.6171 | 0.9p,63bd | -0.2938 | 0.0832 | 0.4237 | -5.2254 |
| 10 | 1.8051 | 1.0620 | 1.0527 | 1.1c,126bd | 1.3025 | 0.9p,126bd | -0.5442 | 1c,126bd | 0.4000 | 0.0695 | 0.3852 | -4.8772 |
| 11 | 1.8023 | 1.4489 | 1.1054 | 1.1c,42bd | 1.5901 | 1c,42bd | 0.7487 | 0.9p,42bd | -0.2658 | 0.0846 | 0.4523 | -5.2287 |
| 12 | 1.7993 | 0.9210 | 1.2085 | 1.1c,126bd | 1.7015 | 1.2c,126bd | -0.4525 | 0.9p,126bd | -0.3403 | 0.0627 | 0.3796 | -4.9288 |
| 13 | 1.7928 | 1.3918 | 0.5274 | 0.9p,126bd | -1.9700 | 1c,126bd | 1.7412 | 0.8p,126bd | 1.1877 | 0.0769 | 0.3238 | -4.3585 |
| 14 | 1.7660 | 1.6151 | 1.7368 | 1.1c,63bd | 0.9533 | 1c,63bd | 0.4302 | 0.8p,63bd | -0.0291 | 0.0871 | 0.6029 | -5.1234 |
| 15 | 1.7594 | 1.6689 | 1.5871 | 1.1c,42bd | 1.8350 | 1c,42bd | 0.6182 | 0.8p,42bd | 0.1050 | 0.0892 | 0.5808 | -5.1830 |
| 16 | 1.7485 | 1.3202 | 1.0128 | 1c,63bd | 0.6935 | 1.2c,63bd | -0.5021 | 0.9p,63bd | -0.2570 | 0.0640 | 0.2920 | -5.4604 |
| 17 | 1.7419 | 0.9129 | 0.6105 | 1.1c,63bd | 2.5503 | 0.8p,63bd | 0.9548 | 0.9p,63bd | -0.6633 | 0.1050 | 0.7435 | -5.1914 |
| 18 | 1.7359 | 0.9491 | 1.0438 | 1.1c,126bd | 1.8919 | 1.2c,126bd | -0.5257 | 1c,126bd | -0.3447 | 0.0571 | 0.4777 | -4.9604 |
| 19 | 1.7272 | 1.1434 | 0.7112 | 1c,42bd | 0.8858 | 0.8p,42bd | 0.7874 | 0.9p,42bd | -0.6358 | 0.0571 | 0.3389 | -5.4631 |
| 20 | 1.7232 | 0.9509 | 1.0298 | 1.1c,126bd | 1.8616 | 1c,126bd | -0.3931 | 0.8p,126bd | 0.0538 | 0.0591 | 0.5480 | -4.9549 |

Table 22: Top 20 strike spread triplet strategies, optimised and sorted based on the in-sample period.

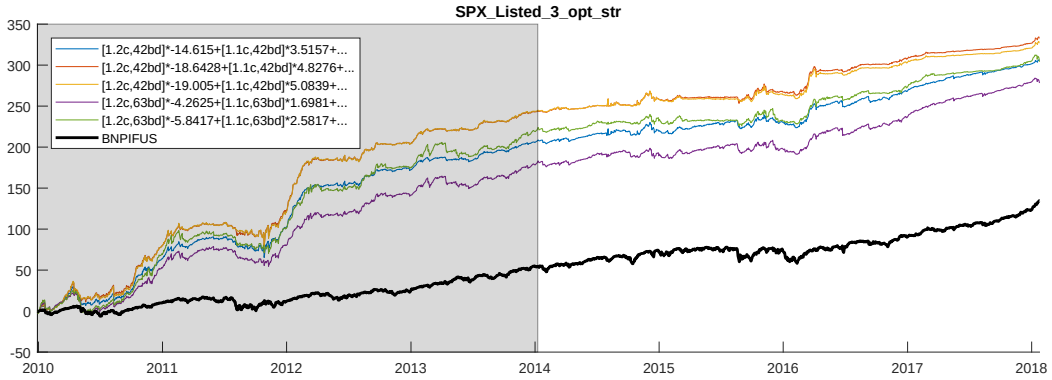


Figure 38: Plots of the top 5 strike spread triplets. The in-sample period is shaded grey. The performances are benchmarked against the BNPIFUS index, shown by the black bold line.

## 8.2 Optimised calendar spreads

Table 23 contains the top optimised calendar spread pairs, and the top five plots can be found in Figure 39.

| | SR(in) | SR(out) | MaxTC | Option | Weight | Option | Weight | Alpha | Beta | CVar |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2.7682 | 2.4290 | 0.2619 | 1c,21bd | 2.5716 | 1c,10bd | -2.1557 | 0.1318 | 0.1697 | -3.6806 |
| 2 | 2.1883 | 2.8351 | 0.6437 | 1c,42bd | 1.7240 | 1c,10bd | -1.0039 | 0.1284 | 0.4090 | -4.1794 |
| 3 | 1.9926 | 2.5892 | 0.8540 | 1c,63bd | 0.9797 | 1c,10bd | -0.6988 | 0.1122 | 0.4438 | -4.2815 |
| 4 | 1.8877 | 0.5553 | 0.3751 | 1.1c,21bd | -7.4684 | 1.1c,42bd | 7.4464 | 0.1074 | 0.4533 | -5.5036 |
| 5 | 1.8809 | 0.6023 | 0.4109 | 1.1c,10bd | -8.9651 | 1.1c,42bd | 7.4140 | 0.1093 | 0.4538 | -5.4952 |
| 6 | 1.8517 | 1.5387 | 2.5974 | 1.1c,42bd | 1.6404 | 1.1c,126bd | 1.1721 | 0.0921 | 0.6955 | -4.9799 |
| 7 | 1.8241 | 1.1929 | 1.0795 | 1.1c,126bd | 2.3884 | 1.1c,252bd | -0.7937 | 0.0770 | 0.5140 | -5.1296 |
| 8 | 1.7791 | 1.3549 | 2.3782 | 1.1c,126bd | 1.0657 | 1.1c,63bd | 0.8315 | 0.0831 | 0.6837 | -5.1035 |
| 9 | 1.7207 | 1.4831 | 0.9767 | 1c,63bd | 0.8093 | 1c,252bd | -0.4815 | 0.0716 | 0.2803 | -5.6445 |
| 10 | 1.7142 | 0.8236 | 0.9349 | 1.1c,10bd | -2.9129 | 1.1c,63bd | 2.7027 | 0.0936 | 0.6421 | -5.5274 |
| 11 | 1.7142 | 0.8131 | 0.9207 | 1.1c,63bd | 2.7056 | 1.1c,21bd | -2.3993 | 0.0931 | 0.6434 | -5.5263 |
| 12 | 1.7031 | 1.3269 | 2.6696 | 1.1c,126bd | 1.3979 | 1.1c,21bd | 0.4579 | 0.0662 | 0.6001 | -4.9945 |
| 13 | 1.7026 | 1.3223 | 2.6654 | 1.1c,126bd | 1.3988 | 1.1c,10bd | 0.5404 | 0.0660 | 0.5996 | -4.9936 |
| 14 | 1.6857 | 1.4494 | 2.1490 | 1.1c,63bd | 1.7841 | 1.1c,42bd | 1.6838 | 0.1191 | 0.7665 | -5.2275 |
| 15 | 1.6682 | 1.3074 | 2.0351 | 1.1c,63bd | 2.2834 | 1.1c,252bd | 0.1197 | 0.1035 | 0.7599 | -5.3538 |
| 16 | 1.6609 | 1.3339 | 0.7052 | 1c,63bd | 0.9179 | 1c,126bd | -0.6156 | 0.0619 | 0.3341 | -5.5932 |
| 17 | 1.6436 | 1.7416 | 2.6197 | 1.1c,42bd | 3.8367 | 1.1c,252bd | 0.4738 | 0.1114 | 0.7867 | -5.1319 |
| 18 | 1.5852 | 1.6734 | 1.1894 | 1c,63bd | 0.6909 | 1c,21bd | -0.1810 | 0.0665 | 0.4516 | -5.5149 |
| 19 | 1.5673 | 1.4914 | 1.6753 | 1c,63bd | 0.5167 | 1c,42bd | 0.0632 | 0.0599 | 0.4280 | -5.6322 |
| 20 | 1.5540 | 1.5648 | 0.7031 | 1c,42bd | 1.1647 | 1c,21bd | -0.3910 | 0.0633 | 0.4052 | -5.5823 |

Table 23: Top 20 calendar spread pair strategies, optimised and sorted based on the in-sample period.

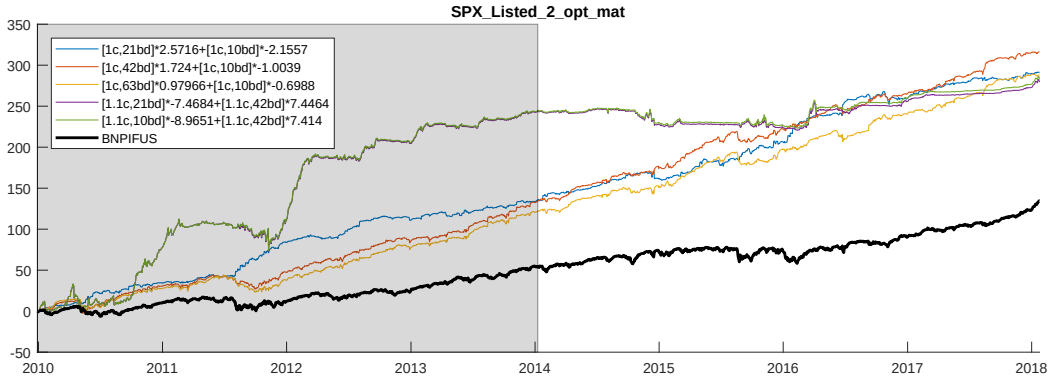


Figure 39: Plots of the top 5 calendar spread pairs. The in-sample period is shaded grey. The performances are benchmarked against the BNPIFUS index, shown by the black bold line.

Table 24 contains the top optimised calendar spread triplets, and the top five plots can be found in Figure 40.

| | SR(in) | SR(out) | MaxTC | Option | Weight | Option | Weight | Option | Weight | Alpha | Beta | CVar |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 3.0500 | 3.1023 | 0.3339 | 1c,21bd | 2.2195 | 1c,10bd | -2.1823 | 1c,63bd | 0.3816 | 0.1580 | 0.2975 | -3.3742 |
| 2 | 3.0384 | 3.1193 | 0.3294 | 1c,10bd | -2.2141 | 1c,21bd | 2.1200 | 1c,42bd | 0.6805 | 0.1598 | 0.2834 | -3.3705 |
| 3 | 2.8646 | 2.7629 | 0.2980 | 1c,21bd | 2.4384 | 1c,10bd | -2.1793 | 1c,126bd | 0.2550 | 0.1416 | 0.2548 | -3.4722 |
| 4 | 2.8036 | 2.5565 | 0.2750 | 1c,21bd | 2.5230 | 1c,10bd | -2.1776 | 1c,252bd | 0.1239 | 0.1339 | 0.2266 | -3.5282 |
| 5 | 2.2346 | 2.8684 | 0.5892 | 1c,42bd | 1.9434 | 1c,10bd | -1.0296 | 1c,252bd | -0.2334 | 0.1362 | 0.3408 | -4.2906 |
| 6 | 2.2181 | 2.7636 | 0.6798 | 1c,63bd | 1.3789 | 1c,10bd | -0.7356 | 1c,252bd | -0.6270 | 0.1354 | 0.2900 | -4.2433 |
| 7 | 2.1968 | 2.8380 | 0.6681 | 1c,42bd | 1.4480 | 1c,10bd | -0.9726 | 1c,63bd | 0.1730 | 0.1277 | 0.4192 | -4.1821 |
| 8 | 2.1957 | 2.8174 | 0.5995 | 1c,42bd | 1.8526 | 1c,10bd | -1.0194 | 1c,126bd | -0.1269 | 0.1298 | 0.3894 | -4.1835 |
| 9 | 2.1404 | 2.5306 | 0.5563 | 1c,63bd | 1.5192 | 1c,126bd | -0.8211 | 1c,10bd | -0.7288 | 0.1214 | 0.3505 | -4.2750 |
| 10 | 2.0255 | 0.9865 | 0.8403 | 1.1c,21bd | -4.7108 | 1.1c,42bd | 4.1922 | 1.1c,126bd | 0.9143 | 0.0956 | 0.5886 | -5.0049 |
| 11 | 2.0219 | 1.0173 | 0.8782 | 1.1c,10bd | -5.6055 | 1.1c,42bd | 4.1195 | 1.1c,126bd | 0.9236 | 0.0963 | 0.5887 | -4.9972 |
| 12 | 1.9547 | 1.4623 | 1.1371 | 1.1c,126bd | 2.2435 | 1.1c,42bd | 1.5611 | 1.1c,252bd | -0.8814 | 0.1060 | 0.6149 | -5.1128 |
| 13 | 1.9150 | 0.7502 | 0.5323 | 1.1c,21bd | -6.6743 | 1.1c,42bd | 6.3459 | 1.1c,252bd | 0.3150 | 0.0995 | 0.5438 | -5.3460 |
| 14 | 1.9100 | 0.7952 | 0.5713 | 1.1c,10bd | -7.9749 | 1.1c,42bd | 6.2659 | 1.1c,252bd | 0.3257 | 0.1006 | 0.5458 | -5.3290 |
| 15 | 1.9001 | 0.5765 | 0.4052 | 1.1c,21bd | -6.8723 | 1.1c,42bd | 6.0956 | 1.1c,63bd | 0.5926 | 0.1059 | 0.4949 | -5.4662 |
| 16 | 1.8946 | 0.6201 | 0.4411 | 1.1c,10bd | -8.2088 | 1.1c,42bd | 5.9882 | 1.1c,63bd | 0.6255 | 0.1075 | 0.4974 | -5.4573 |
| 17 | 1.8891 | 0.5327 | 0.2358 | 1.1c,21bd | -12.7261 | 1.1c,42bd | 7.4595 | 1.1c,10bd | 6.3808 | 0.1064 | 0.4558 | -5.5047 |
| 18 | 1.8739 | 1.2741 | 1.0474 | 1.1c,126bd | 2.1560 | 1.1c,252bd | -0.8402 | 1.1c,63bd | 0.6522 | 0.0935 | 0.5907 | -5.2349 |
| 19 | 1.8616 | 1.5582 | 1.8640 | 1.1c,42bd | 2.1642 | 1.1c,126bd | 1.3073 | 1.1c,63bd | -0.5128 | 0.0887 | 0.6654 | -4.9202 |
| 20 | 1.8357 | 1.3074 | 1.1118 | 1.1c,126bd | 2.4137 | 1.1c,252bd | -0.8397 | 1.1c,21bd | 0.5971 | 0.0826 | 0.5417 | -5.1576 |

Table 24: Top 20 calendar spread triplet strategies, optimised and sorted based on the in-sample period.

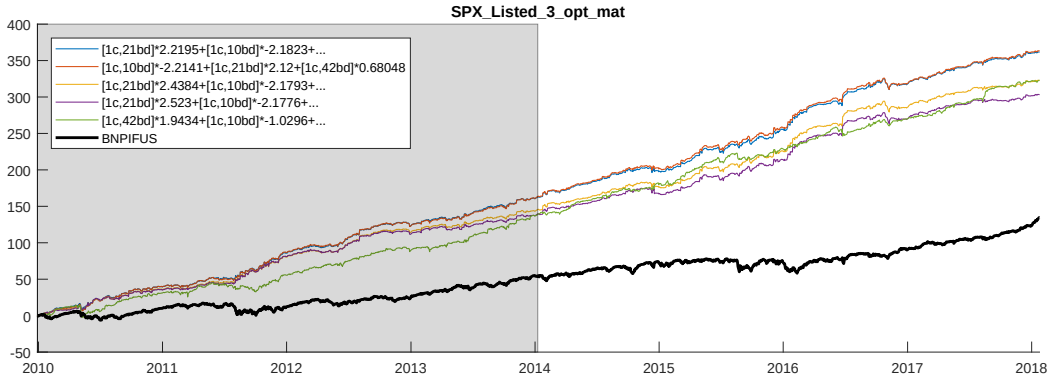


Figure 40: Plots of the top 5 calendar spread triplets. The in-sample period is shaded grey. The performances are benchmarked against the BNPIFUS index, shown by the black bold line.

## 8.3 Optimised pairs and triples

Table 25 contains the top optimised pairs over all options, and the top five plots can be found in Figure 41.

| | SR(in) | SR(out) | MaxTC | Option | Weight | Option | Weight | Alpha | Beta | CVar |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2.7682 | 2.4290 | 0.2619 | 1c,21bd | 2.5716 | 1c,10bd | -2.1557 | 0.1318 | 0.1697 | -3.6806 |
| 2 | 2.2140 | 1.2769 | 1.6247 | 1.2c,42bd | -19.0397 | 1.1c,42bd | 5.1067 | 0.1353 | 0.3850 | -5.2224 |
| 3 | 2.1883 | 2.8351 | 0.6437 | 1c,42bd | 1.7240 | 1c,10bd | -1.0039 | 0.1284 | 0.4090 | -4.1794 |
| 4 | 2.1787 | 1.5649 | 1.5803 | 1.2c,63bd | -9.2131 | 1.1c,42bd | 6.2310 | 0.1419 | 0.3846 | -5.0650 |
| 5 | 2.1515 | 1.1068 | 1.6939 | 1.2c,21bd | -40.1693 | 1.1c,42bd | 4.4820 | 0.1207 | 0.3350 | -5.0922 |
| 6 | 2.1464 | 1.0873 | 1.6711 | 1.2c,10bd | -50.3821 | 1.1c,42bd | 4.4445 | 0.1196 | 0.3314 | -5.1014 |
| 7 | 2.0007 | 0.9878 | 1.3433 | 1.2c,42bd | -13.8926 | 1.1c,63bd | 2.4005 | 0.1112 | 0.5071 | -5.2937 |
| 8 | 1.9926 | 2.5892 | 0.8540 | 1c,63bd | 0.9797 | 1c,10bd | -0.6988 | 0.1122 | 0.4438 | -4.2815 |
| 9 | 1.9596 | 0.9018 | 1.3932 | 1.2c,21bd | -30.1986 | 1.1c,63bd | 2.2076 | 0.1036 | 0.4675 | -5.0609 |
| 10 | 1.9596 | 0.8888 | 1.3753 | 1.2c,10bd | -38.1131 | 1.1c,63bd | 2.1964 | 0.1031 | 0.4639 | -5.0686 |
| 11 | 1.9382 | 1.1424 | 1.3453 | 1.2c,63bd | -5.9205 | 1.1c,63bd | 2.7283 | 0.1110 | 0.5352 | -5.2873 |
| 12 | 1.9270 | 1.1097 | 1.3211 | 1.1c,126bd | 2.0852 | 1.2c,252bd | -1.1396 | 0.0662 | 0.3942 | -5.1968 |
| 13 | 1.8877 | 0.5553 | 0.3751 | 1.1c,21bd | -7.4684 | 1.1c,42bd | 7.4464 | 0.1074 | 0.4533 | -5.5036 |
| 14 | 1.8809 | 0.6023 | 0.4109 | 1.1c,10bd | -8.9651 | 1.1c,42bd | 7.4140 | 0.1093 | 0.4538 | -5.4952 |
| 15 | 1.8802 | 1.2336 | 0.8554 | 1c,63bd | 0.7494 | 0.9p,126bd | -0.5250 | 0.0692 | 0.2054 | -5.4513 |
| 16 | 1.8683 | 1.6775 | 0.9515 | 1.1c,126bd | 1.9595 | 1c,10bd | -0.5187 | 0.0914 | 0.5653 | -3.8463 |
| 17 | 1.8517 | 1.5387 | 2.5974 | 1.1c,42bd | 1.6404 | 1.1c,126bd | 1.1721 | 0.0921 | 0.6955 | -4.9799 |
| 18 | 1.8241 | 1.1929 | 1.0795 | 1.1c,126bd | 2.3884 | 1.1c,252bd | -0.7937 | 0.0770 | 0.5140 | -5.1296 |
| 19 | 1.8182 | 0.9869 | 0.5790 | 1.1c,63bd | 1.9775 | 1c,10bd | -0.5538 | 0.0914 | 0.3544 | -4.5535 |
| 20 | 1.7945 | 0.9840 | 1.0973 | 1.1c,126bd | 1.9311 | 1c,252bd | -0.4885 | 0.0694 | 0.4367 | -5.1171 |

Table 25: Top 20 pair strategies over all options, optimised and sorted based on the in-sample period.

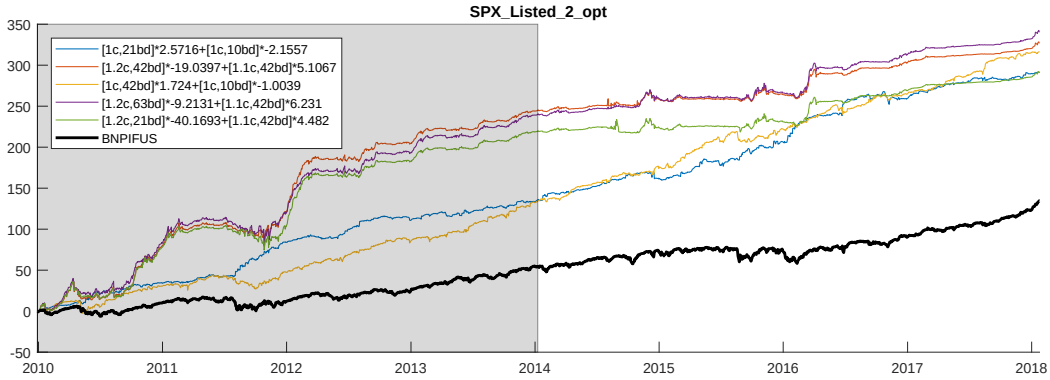


Figure 41: Plots of the top 5 pairs over all options. The in-sample period is shaded grey. The performances are benchmarked against the BNPIFUS index, shown by the black bold line.

Table 26 contains the top optimised triplets over all options, and the top five plots can be found in Figure 42.

An issue with optimising three or more options is that the top results all have extremely similar compositions: buy 10bd ATM calls, sell 21bd ATM calls, and trade a small amount of a third option. This indicates that the performances are all generated by essentially the top pair (10bd ATM call and 21 bd ATM call), while other triples are being obscured by the sorting process. In order obtain a better view of other types of strategies, we present an alternative table where overly similar strategies are filtered out. This can be found in Subsection 8.5.

| | SR(in) | SR(out) | MaxTC | Option | Weight | Option | Weight | Option | Weight | Alpha | Beta | CVar |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 3.0500 | 3.1023 | 0.3339 | 1c,21bd | 2.2195 | 1c,10bd | -2.1823 | 1c,63bd | 0.3816 | 0.1580 | 0.2975 | -3.3742 |
| 2 | 3.0384 | 3.1193 | 0.3294 | 1c,10bd | -2.2141 | 1c,21bd | 2.1200 | 1c,42bd | 0.6805 | 0.1598 | 0.2834 | -3.3705 |
| 3 | 2.9876 | 2.6292 | 0.2775 | 1c,21bd | 2.5073 | 1c,10bd | -2.2546 | 1.1c,42bd | 0.8512 | 0.1510 | 0.2124 | -3.3753 |
| 4 | 2.9539 | 2.7155 | 0.2924 | 1c,21bd | 2.4224 | 1c,10bd | -2.2096 | 1.1c,126bd | 0.5020 | 0.1480 | 0.2844 | -3.3344 |
| 5 | 2.9242 | 2.5736 | 0.2712 | 1c,21bd | 2.5042 | 1c,10bd | -2.2619 | 1.1c,63bd | 0.4221 | 0.1483 | 0.2213 | -3.3793 |
| 6 | 2.8646 | 2.7629 | 0.2980 | 1c,21bd | 2.4384 | 1c,10bd | -2.1793 | 1c,126bd | 0.2550 | 0.1416 | 0.2548 | -3.4722 |
| 7 | 2.8503 | 2.5908 | 0.2722 | 1c,21bd | 2.5725 | 1c,10bd | -2.2363 | 0.9p,10bd | 0.2109 | 0.1363 | 0.1897 | -3.5273 |
| 8 | 2.8496 | 2.5900 | 0.2722 | 1c,21bd | 2.5718 | 1c,10bd | -2.2358 | 0.9p,21bd | 0.1732 | 0.1362 | 0.1899 | -3.5246 |
| 9 | 2.8480 | 2.5259 | 0.2709 | 1c,21bd | 2.5446 | 1c,10bd | -2.1834 | 0.8p,42bd | 0.2482 | 0.1343 | 0.2136 | -3.6192 |
| 10 | 2.8405 | 2.5356 | 0.2722 | 1c,21bd | 2.5379 | 1c,10bd | -2.1835 | 0.8p,63bd | 0.1441 | 0.1336 | 0.2225 | -3.5894 |
| 11 | 2.8332 | 2.5864 | 0.2736 | 1c,21bd | 2.5522 | 1c,10bd | -2.2185 | 0.9p,42bd | 0.1550 | 0.1352 | 0.2102 | -3.5698 |
| 12 | 2.8314 | 2.6153 | 0.2776 | 1c,21bd | 2.5363 | 1c,10bd | -2.2127 | 0.9p,63bd | 0.1115 | 0.1352 | 0.2227 | -3.5539 |
| 13 | 2.8206 | 2.5413 | 0.2750 | 1c,21bd | 2.5016 | 1c,10bd | -2.1699 | 1.1c,252bd | 0.1633 | 0.1347 | 0.2264 | -3.5422 |
| 14 | 2.8146 | 2.3707 | 0.2603 | 1c,21bd | 2.5391 | 1c,10bd | -2.1000 | 1.2c,42bd | -1.6513 | 0.1307 | 0.1540 | -3.7590 |
| 15 | 2.8087 | 2.5267 | 0.2717 | 1c,21bd | 2.5386 | 1c,10bd | -2.1772 | 0.8p,126bd | 0.1402 | 0.1324 | 0.2242 | -3.5398 |
| 16 | 2.8077 | 2.4906 | 0.2631 | 1c,21bd | 2.5898 | 1c,10bd | -2.2212 | 1.2c,126bd | 0.3726 | 0.1372 | 0.2066 | -3.4997 |
| 17 | 2.8074 | 2.3744 | 0.2604 | 1.2c,21bd | -3.8204 | 1c,21bd | 2.5433 | 1c,10bd | -2.1027 | 0.1311 | 0.1554 | -3.7529 |
| 18 | 2.8074 | 2.3733 | 0.2603 | 1.2c,10bd | -4.8457 | 1c,21bd | 2.5432 | 1c,10bd | -2.1026 | 0.1311 | 0.1552 | -3.7518 |
| 19 | 2.8036 | 2.5565 | 0.2750 | 1c,21bd | 2.5230 | 1c,10bd | -2.1776 | 1c,252bd | 0.1239 | 0.1339 | 0.2266 | -3.5282 |
| 20 | 2.8028 | 2.5235 | 0.2692 | 1c,21bd | 2.5607 | 1c,10bd | -2.2010 | 1.2c,252bd | 0.1739 | 0.1357 | 0.2138 | -3.5047 |

Table 26: Top 20 triplet strategies over all options, optimised and sorted based on the in-sample period.

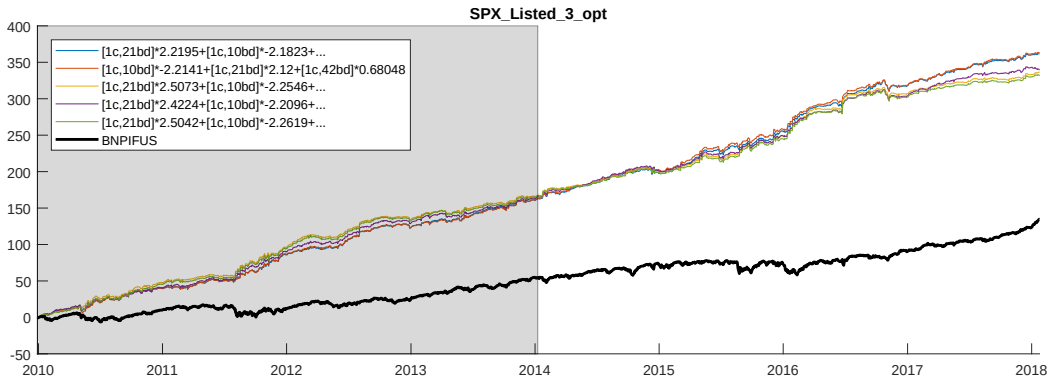


Figure 42: Plots of the top 5 triplets over all options. The in-sample period is shaded grey. The performances are benchmarked against the BNPIFUS index, shown by the black bold line.

## 8.4 Optimised combinations

In this part, we look at the optimal combination of all options with the same maturity. Interestingly, the shorter maturities generally have better out-of-sample performances than longer maturities.

| | SR(in) | SR(out) | MaxTC | Option | Weight | Option | Weight | Option | Weight | Option | Weight | Option | Weight | Alpha | Beta | CVar |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2.4180 | 1.4145 | 0.7957 | 1.2c,42bd | -13.3221 | 1.1c,42bd | 3.0943 | 0.8p,42bd | 0.9580 | 0.9p,42bd | -0.6964 | 1c,42bd | 0.5955 | 0.1210 | 0.4172 | -4.8333 |
| 2 | 2.1865 | 1.3254 | 0.6246 | 1.2c,63bd | -3.4618 | 1.1c,63bd | 1.2574 | 0.9p,63bd | -0.7093 | 0.8p,63bd | 0.6692 | 1c,63bd | 0.5974 | 0.1010 | 0.4440 | -4.7719 |
| 3 | 2.0117 | 1.0450 | 0.5039 | 0.9p,126bd | -1.5786 | 1.1c,126bd | 1.2643 | 0.8p,126bd | 1.1151 | 1c,126bd | 0.6934 | 1.2c,126bd | -0.3065 | 0.0742 | 0.3931 | -4.5187 |
| 4 | 1.5376 | 1.1134 | 0.7906 | 1.2c,21bd | -30.5106 | 1.1c,21bd | 4.1728 | 1c,21bd | 0.4869 | 0.8p,21bd | 0.3864 | 0.9p,21bd | -0.2693 | 0.0680 | 0.2907 | -5.3565 |
| 5 | 1.3954 | 1.5859 | 1.4100 | 1.2c,10bd | -62.8531 | 1.1c,10bd | 9.9120 | 0.8p,10bd | 0.3473 | 1c,10bd | -0.1702 | 0.9p,10bd | -0.0273 | 0.0968 | 0.2312 | -5.4360 |
| 6 | 1.1320 | 0.8846 | 0.9422 | 1.1c,252bd | 1.1883 | 0.9p,252bd | -0.6584 | 1.2c,252bd | 0.4213 | 0.8p,252bd | 0.3093 | 1c,252bd | -0.0706 | 0.0398 | 0.4872 | -4.8403 |

Table 27: Optimised combinations of options of all strikes at a fixed maturity, over an the in-sample period.

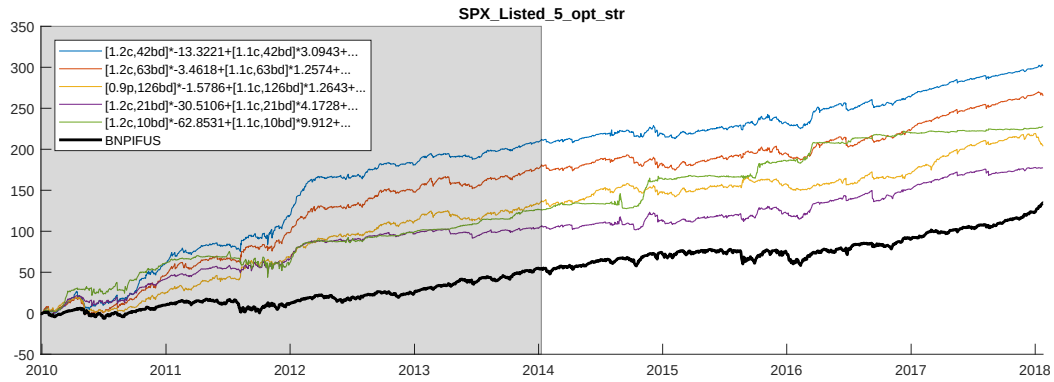


Figure 43: Plots of the optimised strike combinations. The in-sample period is shaded grey. The performances are benchmarked against the BNPIFUS index, shown by the black bold line.

Next, we look at the optimal combination of all options with the same strike. The combination involving ATM calls have, by far, the best performance.

| | SR(in) | SR(out) | MaxTC | Option | Weight | Option | Weight | Option | Weight | Option | Weight | Option | Weight | Option | Weight | Alpha | Beta | CVar |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 3.1443 | 3.2231 | 0.3291 | 1c,10bd | -2.1708 | 1c,21bd | 2.1179 | 1c,63bd | 0.6154 | 1c,252bd | -0.2295 | 1c,126bd | -0.1315 | 1c,42bd | 0.0966 | 0.1700 | 0.2438 | -3.5032 |
| 2 | 2.1410 | 1.0838 | 0.3695 | 1.1c,21bd | -8.7699 | 1.1c,10bd | 5.7269 | 1.1c,42bd | 5.1392 | 1.1c,126bd | 2.1260 | 1.1c,63bd | -1.2605 | 1.1c,252bd | -0.6661 | 0.0994 | 0.4933 | -5.0271 |
| 3 | 1.5069 | 0.5206 | 0.3137 | 1.2c,10bd | -242.0598 | 1.2c,21bd | 197.0388 | 1.2c,42bd | -34.2624 | 1.2c,63bd | 11.5780 | 1.2c,126bd | 1.9845 | 1.2c,252bd | 0.6060 | 0.0644 | 0.6086 | -5.2074 |
| 4 | 1.3815 | 0.7183 | 0.0392 | 0.8p,10bd | 97.7148 | 0.8p,21bd | -84.1153 | 0.8p,42bd | 8.1787 | 0.8p,63bd | -1.6336 | 0.8p,126bd | -1.3607 | 0.8p,252bd | 0.7440 | 0.0522 | 0.3517 | -5.3728 |
| 5 | 1.0669 | 0.9266 | 0.0399 | 0.9p,10bd | 30.5265 | 0.9p,21bd | -25.9308 | 0.9p,126bd | -1.5835 | 0.9p,63bd | 1.3173 | 0.9p,252bd | 1.2153 | 0.9p,42bd | -0.2323 | 0.0292 | 0.4952 | -5.1712 |

Table 28: Optimised combinations of options of all maturities at a fixed strike, over an the in-sample period.

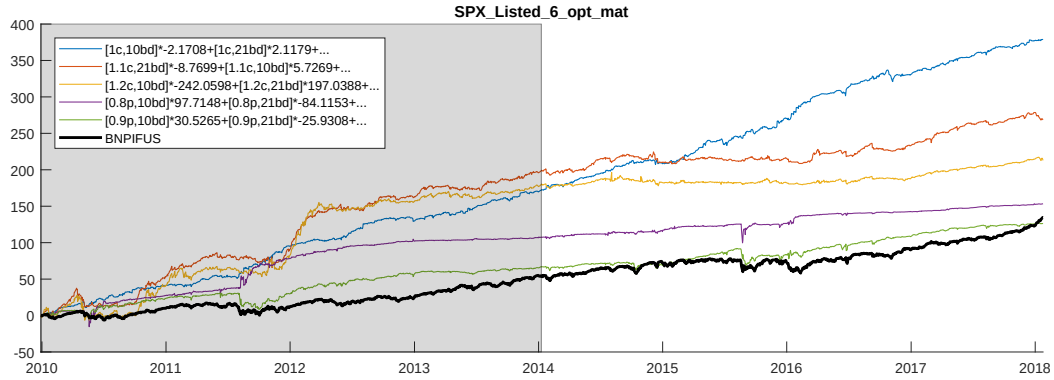


Figure 44: Plots of the optimised maturity combinations. The in-sample period is shaded grey. The performances are benchmarked against the BNPIFUS index, shown by the black bold line.

## 8.5 Optimised and filtered

As mentioned earlier, when optimising over three or more options, the top results are all variations of the same strategy involving the same top pair, while other types of strategies are obscured during the sorting process. In order to get a better view of all strategies, in this section, we filter out strategies that are "overly similar" to an existing (and better) strategy. The criteria for two strategies to be "overly similar" is that their out-of-sample correlation must be at least 80%. Tables 29 to 32 show the top combinations of three, four, five and six options, after the filtering has been applied. Figures 45 to 48 contains the corresponding plots.

| | SR(in) | SR(out) | MaxTC | Option | Weight | Option | Weight | Option | Weight | Alpha | Beta | CVar |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 3.0500 | 3.1023 | 0.3339 | 1c,21bd | 2.2195 | 1c,10bd | -2.1823 | 1c,63bd | 0.3816 | 0.1580 | 0.2975 | -3.3742 |
| 2 | 2.4758 | 3.1190 | 0.6394 | 1c,42bd | 1.6293 | 1.1c,42bd | 1.4334 | 1c,10bd | -1.1990 | 0.1591 | 0.4688 | -3.8585 |
| 3 | 2.3330 | 1.3939 | 0.7722 | 1.2c,63bd | -7.5451 | 1.1c,42bd | 5.5326 | 1c,10bd | -0.4380 | 0.1311 | 0.1794 | -4.4808 |
| 4 | 2.3109 | 1.6201 | 1.6107 | 1.2c,42bd | -13.8464 | 1.1c,42bd | 3.1937 | 1c,63bd | 0.3192 | 0.1199 | 0.4552 | -4.9022 |
| 5 | 2.2870 | 2.6435 | 0.5695 | 1c,42bd | 1.8754 | 1c,10bd | -0.9498 | 0.9p,126bd | -0.3033 | 0.1317 | 0.2940 | -4.3528 |
| 6 | 2.2731 | 1.1345 | 0.8327 | 1.2c,63bd | -7.8101 | 1.1c,42bd | 7.3728 | 1.1c,21bd | -3.5422 | 0.1357 | 0.3474 | -5.2178 |
| 7 | 2.2458 | 1.3199 | 1.9195 | 1.2c,21bd | -32.3834 | 1.1c,42bd | 3.1498 | 1.1c,126bd | 0.6597 | 0.1149 | 0.4751 | -4.7312 |
| 8 | 2.2326 | 1.1175 | 0.8113 | 1.2c,21bd | -35.8726 | 1.1c,42bd | 4.4690 | 1c,10bd | -0.3427 | 0.1227 | 0.2106 | -4.7544 |
| 9 | 2.1832 | 1.1666 | 0.5958 | 0.9p,126bd | -1.0069 | 1c,63bd | 0.8369 | 0.8p,42bd | 0.8144 | 0.0793 | 0.1558 | -5.1555 |
| 10 | 2.1203 | 1.5660 | 0.7670 | 1.1c,126bd | 2.7713 | 1.2c,252bd | -1.3287 | 1c,10bd | -0.4423 | 0.0965 | 0.4022 | -4.1418 |
| 11 | 2.1164 | 1.7982 | 0.8339 | 1.1c,42bd | 1.8149 | 1.1c,126bd | 1.4487 | 1c,10bd | -0.6756 | 0.1152 | 0.5329 | -3.9946 |
| 12 | 2.0461 | 0.7059 | 0.5265 | 1.1c,126bd | 1.8616 | 0.8p,42bd | 1.2048 | 0.9p,126bd | -1.0296 | 0.0761 | 0.3585 | -4.6301 |
| 13 | 2.0320 | 1.0552 | 0.6742 | 1.1c,21bd | -4.6424 | 1.1c,42bd | 3.8718 | 1c,42bd | 0.5665 | 0.0871 | 0.4600 | -5.1761 |
| 14 | 2.0161 | 0.8735 | 1.0388 | 1.2c,42bd | -14.4709 | 1.1c,63bd | 2.6139 | 1.2c,252bd | -0.3456 | 0.1130 | 0.4258 | -5.3186 |
| 15 | 1.9783 | 0.7472 | 0.4837 | 1.1c,10bd | -9.3445 | 1.1c,42bd | 6.8292 | 0.8p,42bd | 0.8008 | 0.1027 | 0.5713 | -5.5303 |
| 16 | 1.9242 | 0.6832 | 0.3266 | 1.1c,42bd | 7.2031 | 1.1c,21bd | -6.5445 | 1c,10bd | -0.3306 | 0.1141 | 0.3424 | -5.2551 |
| 17 | 1.9095 | 1.3384 | 0.9470 | 1.1c,126bd | 2.6376 | 1.1c,252bd | -1.1246 | 0.8p,42bd | 0.5913 | 0.0844 | 0.5874 | -5.1623 |
| 18 | 1.8996 | 1.1109 | 0.6235 | 1.1c,63bd | 1.7102 | 1c,10bd | -0.5832 | 0.8p,42bd | 0.5222 | 0.0872 | 0.4034 | -4.3326 |
| 19 | 1.8981 | 0.3201 | 0.1951 | 1.1c,63bd | 2.5590 | 0.8p,42bd | 1.8642 | 0.9p,126bd | -1.3094 | 0.1082 | 0.3621 | -5.0414 |
| 20 | 1.8792 | 1.0345 | 1.5646 | 1.2c,42bd | -15.8795 | 1.2c,63bd | 5.2688 | 1.1c,126bd | 1.2461 | 0.0694 | 0.5127 | -4.8674 |

Table 29: Top 20 triplet strategies over all options, optimised and sorted based on the in-sample period. Strategies overly similar (> 80% correlation) to a better strategy are filtered out.

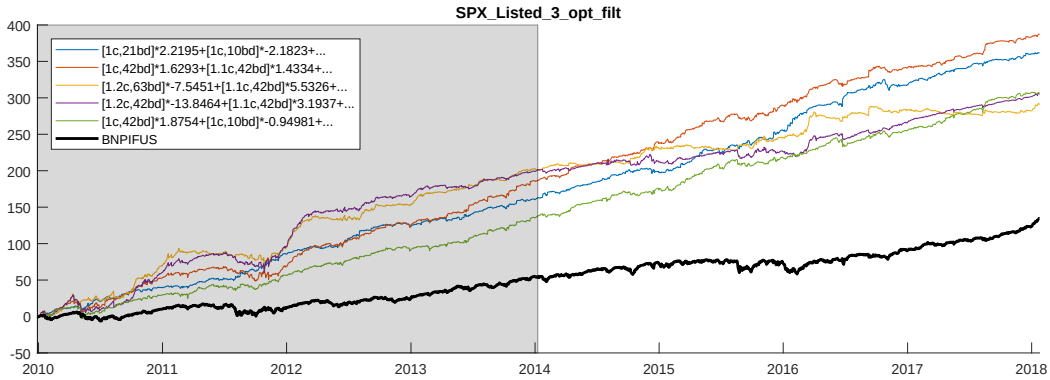


Figure 45: Plots of the top 5 triplets over all options, after filtering out similar strategies. The in-sample period is shaded grey. The performances are benchmarked against the BNPIFUS index, shown by the black bold line.

| | SR(in) | SR(out) | MaxTC | Option | Weight | Option | Weight | Option | Weight | Option | Weight | Alpha | Beta | CVar |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 3.4989 | 2.6772 | 0.2974 | 1.2c,63bd | -3.6921 | 1c,21bd | 2.2640 | 1.1c,42bd | 2.2545 | 1c,10bd | -2.1169 | 0.1691 | 0.1743 | -3.1579 |
| 2 | 3.1314 | 3.1006 | 0.3332 | 1c,10bd | -2.0986 | 1c,21bd | 2.0982 | 1c,63bd | 0.5633 | 0.9p,126bd | -0.2527 | 0.1660 | 0.2429 | -3.5192 |
| 3 | 2.9652 | 3.0406 | 0.6462 | 1.2c,63bd | -5.1727 | 1.1c,42bd | 3.3480 | 1c,42bd | 1.3544 | 1c,10bd | -1.1184 | 0.1801 | 0.3716 | -3.5925 |
| 4 | 2.6076 | 1.8068 | 0.7864 | 1.2c,63bd | -6.1034 | 1.1c,42bd | 3.9722 | 1.1c,126bd | 0.9798 | 1c,10bd | -0.6224 | 0.1394 | 0.3563 | -3.7754 |
| 5 | 2.5995 | 2.2782 | 0.5598 | 1c,63bd | 1.3084 | 0.9p,126bd | -1.1286 | 0.8p,42bd | 0.9326 | 1c,10bd | -0.6020 | 0.1348 | 0.1986 | -4.1229 |
| 6 | 2.5266 | 3.0966 | 0.6341 | 1.1c,42bd | 1.5631 | 1c,63bd | 1.4161 | 1c,10bd | -0.9563 | 1c,252bd | -0.8015 | 0.1745 | 0.3077 | -4.0401 |
| 7 | 2.4811 | 2.6234 | 0.6371 | 1.1c,126bd | 1.4613 | 1c,42bd | 1.2116 | 1.2c,252bd | -1.0194 | 1c,10bd | -0.8940 | 0.1335 | 0.3932 | -4.0421 |
| 8 | 2.4623 | 1.2975 | 1.1945 | 1.2c,42bd | -11.6490 | 1.1c,42bd | 2.4826 | 1.1c,126bd | 1.6247 | 1.2c,252bd | -1.1230 | 0.1133 | 0.3634 | -4.8597 |
| 9 | 2.4303 | 1.2117 | 0.5267 | 0.9p,126bd | -1.3141 | 1.1c,42bd | 1.1851 | 0.8p,42bd | 1.0826 | 1c,63bd | 0.8329 | 0.1050 | 0.1688 | -4.8161 |
| 10 | 2.3806 | 1.9172 | 0.8380 | 1.2c,42bd | -14.2896 | 1.1c,42bd | 3.4312 | 1c,63bd | 0.5776 | 1c,21bd | -0.4679 | 0.1336 | 0.4746 | -4.8680 |
| 11 | 2.3764 | 1.8221 | 0.8984 | 1.2c,21bd | -29.7018 | 1.1c,42bd | 3.3590 | 1c,126bd | 0.5558 | 1c,10bd | -0.5248 | 0.1288 | 0.3567 | -4.2775 |
| 12 | 2.3518 | 1.5824 | 0.9909 | 1.2c,21bd | -26.2916 | 1.1c,42bd | 2.5650 | 1c,63bd | 0.6169 | 1c,252bd | -0.4765 | 0.1210 | 0.3059 | -4.8129 |
| 13 | 2.3333 | 1.2582 | 1.2637 | 1.2c,21bd | -36.4947 | 1.1c,42bd | 3.3656 | 0.8p,42bd | 2.0056 | 0.8p,21bd | -1.7357 | 0.1094 | 0.4801 | -5.0147 |
| 14 | 2.3259 | 1.0160 | 0.7392 | 1.2c,42bd | -19.1405 | 1.1c,42bd | 5.6277 | 0.8p,42bd | 1.1157 | 0.9p,126bd | -0.6673 | 0.1489 | 0.2884 | -5.2056 |
| 15 | 2.3254 | 1.2273 | 0.8482 | 1.2c,63bd | -7.2737 | 1.1c,42bd | 7.1644 | 1.1c,10bd | -5.0169 | 0.8p,42bd | 0.5092 | 0.1332 | 0.4411 | -5.2351 |
| 16 | 2.3171 | 0.8925 | 0.7360 | 1.2c,21bd | -40.9169 | 1.1c,42bd | 4.7879 | 0.8p,42bd | 1.2284 | 0.9p,126bd | -0.6255 | 0.1316 | 0.2664 | -5.1240 |
| 17 | 2.2958 | 0.6609 | 0.2984 | 0.8p,42bd | 3.9754 | 0.8p,21bd | -2.7265 | 1.1c,126bd | 1.7923 | 0.9p,126bd | -1.3956 | 0.0867 | 0.3938 | -4.3524 |
| 18 | 2.2800 | 1.8073 | 0.7322 | 1.1c,126bd | 2.5185 | 1.1c,42bd | 1.3346 | 1.2c,252bd | -1.2507 | 1c,10bd | -0.6073 | 0.1217 | 0.4372 | -4.0324 |
| 19 | 2.1428 | 0.2304 | 0.0874 | 0.8p,42bd | 5.5600 | 0.8p,21bd | -3.9536 | 1.1c,63bd | 2.0201 | 0.9p,126bd | -1.6187 | 0.1051 | 0.3588 | -4.4559 |
| 20 | 2.1405 | 1.3473 | 0.5375 | 1.1c,126bd | 2.3986 | 0.8p,42bd | 1.2017 | 0.9p,252bd | -0.9403 | 1c,10bd | -0.5176 | 0.1078 | 0.3418 | -4.1727 |

Table 30: Top 20 quadruplet strategies over all options, optimised and sorted based on the in-sample period. Strategies overly similar ($> 80\%$ correlation) to a better strategy are filtered out.



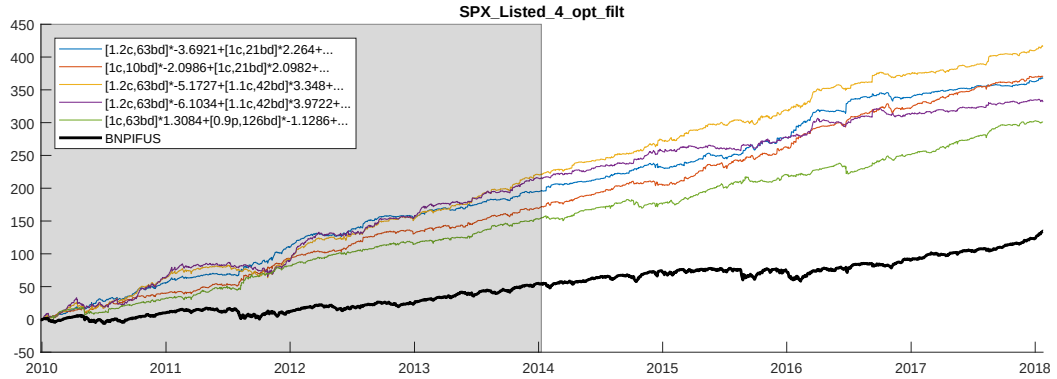


Figure 46: Plots of the top 5 quadruplets over all options, after filtering out similar strategies. The in-sample period is shaded grey. The performances are benchmarked against the BNPIFUS index, shown by the black bold line.

| | SR(in) | SR(out) | MaxTC | Option | Weight | Option | Weight | Option | Weight | Option | Weight | Option | Weight | Alpha | Beta | CVar |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 3.6481 | 3.2104 | 0.3468 | 1.2c,63bd | -3.6738 | 1c,10bd | -2.1790 | 1.1c,42bd | 2.1302 | 1c,21bd | 1.9660 | 1c,42bd | 0.5191 | 0.1895 | 0.2578 | -3.0062 |
| 2 | 3.0322 | 3.1045 | 0.5828 | 1.2c,63bd | -5.0386 | 1.1c,42bd | 3.4516 | 1c,42bd | 1.6295 | 1c,10bd | -1.1657 | 0.9p,126bd | -0.2932 | 0.1958 | 0.3006 | -3.7803 |
| 3 | 2.9359 | 2.5390 | 0.5167 | 1.1c,42bd | 1.4582 | 0.9p,126bd | -1.4157 | 1c,63bd | 1.3464 | 0.8p,42bd | 1.1955 | 1c,10bd | -0.7946 | 0.1741 | 0.2084 | -3.6532 |
| 4 | 2.8553 | 2.7072 | 0.6935 | 1.2c,21bd | -22.7047 | 1.1c,42bd | 2.2794 | 1c,63bd | 1.0620 | 1c,10bd | -0.7812 | 1.1c,252bd | -0.5769 | 0.1634 | 0.2959 | -3.7659 |
| 5 | 2.7795 | 2.8283 | 0.4593 | 1c,42bd | 1.9126 | 1c,10bd | -1.3082 | 0.8p,42bd | 1.0642 | 1.1c,63bd | 1.0055 | 0.9p,252bd | -0.9701 | 0.1827 | 0.2098 | -3.9343 |
| 6 | 2.7756 | 2.5052 | 0.4364 | 0.8p,42bd | 2.8068 | 1c,42bd | 1.9410 | 0.8p,21bd | -1.9382 | 0.9p,126bd | -1.0244 | 1c,10bd | -0.9513 | 0.1474 | 0.2750 | -4.1276 |
| 7 | 2.7682 | 1.9116 | 0.7129 | 1.2c,63bd | -5.8153 | 1.1c,42bd | 3.5468 | 1.1c,126bd | 2.0198 | 1.2c,252bd | -1.0984 | 1c,10bd | -0.6014 | 0.1502 | 0.3094 | -3.8057 |
| 8 | 2.6951 | 1.3481 | 0.5922 | 1.2c,42bd | -8.3938 | 1.1c,42bd | 2.3434 | 0.9p,126bd | -1.1731 | 0.8p,42bd | 1.0334 | 1c,63bd | 0.7343 | 0.1295 | 0.2018 | -4.7799 |
| 9 | 2.5935 | 2.4550 | 0.6943 | 1.2c,63bd | -6.1606 | 1.1c,42bd | 4.3194 | 1c,126bd | 1.3714 | 1c,252bd | -0.7819 | 1c,10bd | -0.6638 | 0.1622 | 0.2566 | -3.9931 |
| 10 | 2.5679 | 3.0649 | 0.6913 | 1.2c,42bd | -8.0449 | 1.1c,10bd | 3.5846 | 1c,63bd | 1.2925 | 1c,10bd | -0.8368 | 1c,252bd | -0.6376 | 0.1601 | 0.2612 | -4.0011 |
| 11 | 2.5653 | 2.4854 | 0.6377 | 1.2c,42bd | -2.9492 | 1.1c,126bd | 1.4836 | 1c,42bd | 1.1520 | 1.2c,252bd | -1.0350 | 1c,10bd | -0.8125 | 0.1307 | 0.3559 | -4.2007 |
| 12 | 2.5506 | 1.1169 | 0.6034 | 1.2c,63bd | -5.8778 | 1.1c,42bd | 4.0651 | 1.1c,126bd | 1.2825 | 0.8p,42bd | 1.2651 | 0.9p,126bd | -1.1258 | 0.1420 | 0.3299 | -4.6035 |
| 13 | 2.5494 | 0.7678 | 0.3732 | 1.2c,21bd | -37.0636 | 0.8p,42bd | 4.1885 | 1.1c,42bd | 4.0920 | 0.8p,21bd | -3.0442 | 0.9p,126bd | -0.9560 | 0.1287 | 0.2879 | -4.7697 |
| 14 | 2.5285 | 1.2751 | 0.6762 | 1.2c,21bd | -30.5444 | 1.1c,42bd | 3.2188 | 0.8p,42bd | 2.0989 | 0.8p,21bd | -1.7582 | 1c,10bd | -0.4217 | 0.1095 | 0.3358 | -4.5437 |
| 15 | 2.5261 | 1.9635 | 0.5970 | 1.2c,63bd | -5.3069 | 1.1c,42bd | 3.5939 | 1c,63bd | 0.9358 | 0.9p,126bd | -0.6366 | 1c,21bd | -0.5133 | 0.1466 | 0.2683 | -4.6720 |
| 16 | 2.5215 | 1.4467 | 0.5524 | 1.2c,21bd | -26.9485 | 1.1c,42bd | 3.2591 | 0.9p,126bd | -1.7799 | 1c,126bd | 1.2529 | 0.8p,63bd | 0.9789 | 0.1439 | 0.2999 | -4.5221 |
| 17 | 2.5045 | 1.3569 | 1.2073 | 1.2c,42bd | -11.5188 | 1.1c,42bd | 2.3569 | 1.1c,126bd | 1.6126 | 1.2c,252bd | -1.1967 | 0.8p,42bd | 0.3036 | 0.1121 | 0.3976 | -4.8809 |
| 18 | 2.4993 | 0.8703 | 0.3970 | 1.2c,42bd | -16.6921 | 1.1c,42bd | 4.8380 | 0.8p,42bd | 4.0535 | 0.8p,21bd | -2.9894 | 0.9p,126bd | -1.0049 | 0.1433 | 0.3081 | -4.9114 |
| 19 | 2.4878 | 1.2080 | 0.4417 | 1.1c,42bd | 1.8078 | 1.1c,126bd | 1.7404 | 0.8p,42bd | 1.2127 | 0.9p,252bd | -1.0395 | 1c,10bd | -0.6000 | 0.1223 | 0.2075 | -3.8669 |
| 20 | 2.4839 | 2.0707 | 0.6566 | 1.2c,42bd | -13.1658 | 1.1c,42bd | 3.1186 | 1c,63bd | 0.9337 | 1.1c,252bd | -0.5857 | 1c,21bd | -0.5372 | 0.1399 | 0.3584 | -4.8000 |

Table 31: Top 20 quintuplet strategies over all options, optimised and sorted based on the in-sample period. Strategies overly similar ($> 80\%$ correlation) to a better strategy are filtered out.

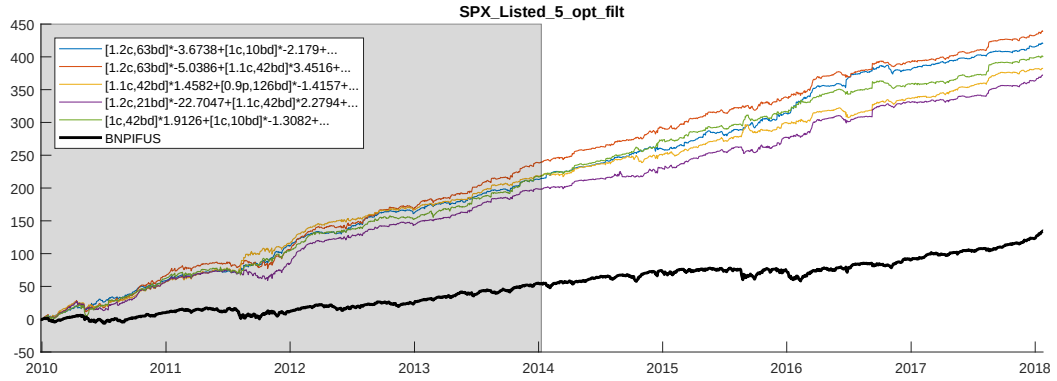


Figure 47: Plots of the top 5 quintuplet over all options, after filtering out similar strategies. The in-sample period is shaded grey. The performances are benchmarked against the BNPIFUS index, shown by the black bold line.

| | SR(in) | SR(out) | MaxTC | Option | Weight | Option | Weight | Option | Weight | Option | Weight | Option | Weight | Option | Weight | Alpha | Beta | CVar |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 3.7414 | 3.2430 | 0.3616 | 1.2c,63bd | -3.5909 | 1c,10bd | -2.0559 | 1.1c,42bd | 2.0098 | 1c,21bd | 1.9238 | 1c,63bd | 0.4459 | 1.2c,252bd | -0.3613 | 0.1878 | 0.2244 | -3.0665 |
| 2 | 3.3776 | 2.5618 | 0.2641 | 1c,21bd | 2.2682 | 1c,10bd | -2.1537 | 0.8p,42bd | 2.0409 | 0.8p,21bd | -1.4345 | 1.1c,126bd | 0.7750 | 0.9p,252bd | -0.5908 | 0.1601 | 0.2220 | -3.2144 |
| 3 | 3.2524 | 3.0977 | 0.5045 | 1.2c,63bd | -4.4963 | 1.1c,42bd | 3.3808 | 1c,42bd | 1.8483 | 1c,10bd | -1.2112 | 0.8p,42bd | 0.9588 | 0.9p,126bd | -0.8722 | 0.2107 | 0.2600 | -3.7066 |
| 4 | 3.0591 | 2.8497 | 0.5963 | 1.2c,21bd | -23.0356 | 1.1c,42bd | 2.5436 | 1c,63bd | 1.2190 | 1c,252bd | -0.9547 | 1c,10bd | -0.8774 | 0.8p,42bd | 0.7217 | 0.1875 | 0.2585 | -3.8113 |
| 5 | 3.0443 | 2.6252 | 0.6604 | 1.2c,42bd | -9.0838 | 1.1c,42bd | 2.3605 | 1c,63bd | 1.1077 | 1c,10bd | -0.7506 | 1.2c,252bd | -0.5211 | 0.9p,126bd | -0.3839 | 0.1691 | 0.2160 | -3.9791 |
| 6 | 3.0139 | 2.6409 | 0.4235 | 0.8p,42bd | 4.4568 | 0.8p,21bd | -2.4066 | 0.8p,126bd | -1.9493 | 1c,42bd | 1.5583 | 1c,10bd | -1.1645 | 1.1c,63bd | 0.8101 | 0.1750 | 0.2457 | -3.8973 |
| 7 | 3.0129 | 2.7261 | 0.6265 | 1.2c,42bd | -9.9239 | 1.1c,42bd | 2.4418 | 1c,42bd | 1.1679 | 1c,10bd | -1.0244 | 0.8p,42bd | 0.9979 | 0.8p,21bd | -0.8920 | 0.1638 | 0.4265 | -3.7857 |
| 8 | 2.9269 | 2.4681 | 0.4224 | 0.8p,42bd | 3.2585 | 0.8p,21bd | -2.1748 | 1c,42bd | 1.6437 | 0.9p,126bd | -1.2578 | 1c,10bd | -0.9635 | 1.1c,126bd | 0.7492 | 0.1598 | 0.3476 | -3.8879 |
| 9 | 2.8984 | 2.6341 | 0.5276 | 1.1c,42bd | 1.6521 | 0.9p,252bd | -1.3573 | 0.8p,42bd | 1.1565 | 1c,63bd | 1.0901 | 1c,10bd | -0.9758 | 1.1c,126bd | 0.6446 | 0.1839 | 0.1512 | -3.7630 |
| 10 | 2.8800 | 1.2966 | 0.4480 | 1.2c,21bd | -21.0435 | 0.8p,42bd | 3.2900 | 1.1c,42bd | 2.1897 | 0.8p,21bd | -2.1101 | 0.9p,126bd | -1.4378 | 1c,63bd | 0.7024 | 0.1361 | 0.2424 | -4.4756 |
| 11 | 2.8778 | 2.3867 | 0.1339 | 0.9p,10bd | 20.5438 | 0.9p,21bd | -16.3575 | 1.1c,42bd | 1.4992 | 1c,63bd | 1.1860 | 0.9p,126bd | -1.1624 | 1c,10bd | -0.7852 | 0.1649 | 0.0784 | -4.0089 |
| 12 | 2.8554 | 1.9901 | 0.7152 | 1.2c,63bd | -5.5410 | 1.1c,42bd | 3.2989 | 1.1c,126bd | 2.0114 | 1.2c,252bd | -1.1887 | 1c,10bd | -0.6306 | 0.8p,42bd | 0.4159 | 0.1480 | 0.3513 | -3.7357 |
| 13 | 2.8164 | 1.7592 | 0.4424 | 1.2c,42bd | -9.0735 | 1.1c,42bd | 2.7115 | 0.9p,126bd | -1.3406 | 0.8p,42bd | 1.1659 | 1c,63bd | 1.1108 | 1c,21bd | -0.5277 | 0.1549 | 0.2178 | -4.5688 |
| 14 | 2.7761 | 2.2522 | 0.5720 | 1.2c,63bd | -5.6478 | 1.1c,42bd | 4.3428 | 1c,126bd | 1.4453 | 0.9p,126bd | -1.3744 | 0.8p,42bd | 1.0806 | 1c,10bd | -0.6104 | 0.1773 | 0.2338 | -4.0803 |
| 15 | 2.7664 | 2.8873 | 0.6643 | 1.2c,21bd | -19.2710 | 1.1c,10bd | 3.0797 | 1.1c,126bd | 1.3079 | 1c,42bd | 1.1651 | 1.2c,252bd | -0.9730 | 1c,10bd | -0.9470 | 0.1553 | 0.3611 | -3.6351 |
| 16 | 2.7523 | 1.4415 | 0.5290 | 1.2c,42bd | -9.8396 | 1.1c,42bd | 3.1038 | 1.1c,126bd | 1.3561 | 0.8p,42bd | 1.0058 | 0.9p,252bd | -0.7269 | 1c,10bd | -0.6220 | 0.1460 | 0.2469 | -4.0378 |
| 17 | 2.7494 | 0.8623 | 0.3434 | 1.2c,21bd | -27.6912 | 0.8p,42bd | 4.3802 | 0.8p,21bd | -2.9690 | 1.1c,42bd | 2.9119 | 0.9p,126bd | -1.4149 | 1.1c,126bd | 1.0957 | 0.1386 | 0.3768 | -4.2558 |
| 18 | 2.6787 | 2.6442 | 0.5203 | 0.8p,42bd | 2.2689 | 0.8p,21bd | -1.5546 | 1c,252bd | -1.3440 | 1.1c,126bd | 1.2705 | 1c,63bd | 1.0576 | 1c,10bd | -0.8319 | 0.1565 | 0.3797 | -4.0372 |
| 19 | 2.6656 | 2.2809 | 0.6086 | 1.1c,126bd | 1.2264 | 1c,63bd | 0.9798 | 1.2c,252bd | -0.9504 | 0.9p,126bd | -0.6426 | 0.8p,10bd | 0.6214 | 1c,10bd | -0.6076 | 0.1331 | 0.2334 | -4.1777 |
| 20 | 2.6648 | 1.0935 | 0.6082 | 1.2c,42bd | -10.7933 | 1.1c,42bd | 2.6441 | 1.1c,126bd | 2.0016 | 1.2c,252bd | -1.0873 | 0.8p,42bd | 1.0767 | 0.9p,126bd | -0.7869 | 0.1318 | 0.3055 | -4.7537 |

Table 32: Top 20 sextuplet strategies over all options, optimised and sorted based on the in-sample period. Strategies overly similar ($> 80\%$ correlation) to a better strategy are filtered out.

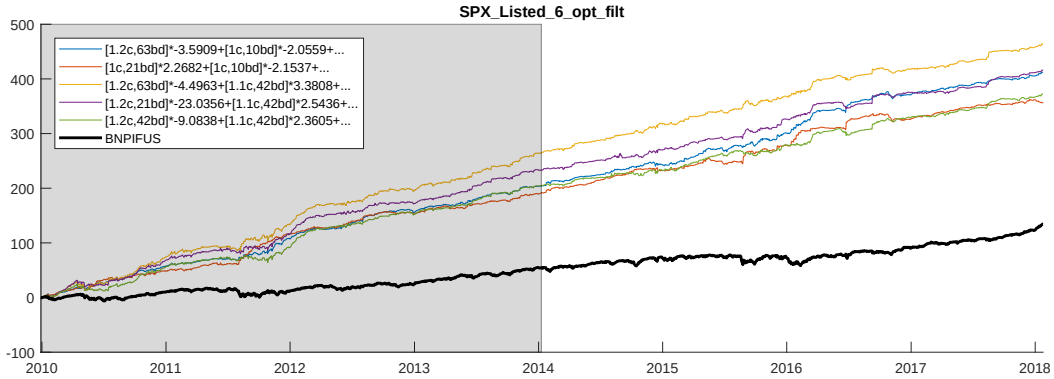


Figure 48: Plots of the top 5 sextuplet over all options, after filtering out similar strategies. The in-sample period is shaded grey. The performances are benchmarked against the BNPIFUS index, shown by the black bold line.

## 8.6 Summary of findings

We examined strategies in which the weights chosen to optimise the Sharpe ratio for an in-sample period. Even though many of these produce excellent in-sample performances, their out-of-sample results are somewhat inconsistent due to overfitting. Optimised calendar spreads exhibit much more consistent out-of-sample performances than optimised strike spreads. With a few exceptions (see the best triplets and quadruplets), these optimised strategies often have an out of sample Sharpe ratio largely inconsistent with the in sample Sharpe ratio. Optimised combinations of up to six options are provided, displaying the asymptotic optimal performance in a theoretical setting. However, their usage as practical strategies may be difficult to justify.

# 9 Conclusion

In this study, we studied a class of systematic vanilla options strategies on the S&P 500 and their ability to generate performances via the volatility risk premium. Each strategy involves selling a single vanilla option every day, at a predetermined relative strike and maturity. The individual options are then delta hedged and held to maturity.

By empirically analysing the sensitivities of these strategies with respect to the underlying and the implied volatility, we observe that shorter maturity strategies have lower delta and vega exposure but higher gamma, theta, vanna and volga exposures, when compared against longer maturity strategies. Overall, the best performing strategies are those with medium maturities (from 42bd to 126bd), as they can generate the most consistent returns via the spread between realised and implied volatilities. This can be attributed to the fact that, implied and realised volatilities mostly cancel out at very short maturities but have decreasing magnitudes for very long maturities. At-the-money and upside strike strategies (1c, 1.1c and 1.2c) tend to outperform downside strike strategies (0.8p and 0.9p).

We then studied the efficacy of combining several options strategy in order to generate additional performance. Overall, the best performing combinations are the calendar spreads and calendar convexity strategies, followed by the pairs strategies and the strike spreads and convexity strategies. Generally speaking, pairs strategies (i.e., diversification) and spread strategies have improved Sharpe ratio over the single strategies. The pairs strategies tends to have slightly increased alphas and betas, while spread strategies have lower betas but significantly higher alphas.

Fixed weights strategies tend to outperform gamma or vega adjusted strategies in most cases. The inconsistency in the gamma and vega adjusted strategies could be attributed to the fact that the adjustment are based on a static, median gamma or vega value across the period, rather than being rebalanced daily. Interestingly, top fixed weights strategies have opposite positions to the typical skew arbitrage strategies whose performances are hindered by extreme events in the market. The top strategies here include the fixed weight calendar spreads and calendar convexity triplets, as well as the vega adjusted calendar spreads, which are able to generate very high levels of alpha.

Finally, combination strategies with optimised weights can generate excellent performances when they mostly consist of at-the-money options, but can otherwise perform poorly in out-of-sample tests due to overfitting. The stand-out optimised pairs are the calendar spreads, which have the most consistent out-of-sample performances.

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