

# Proving Shell Theorem the Shell

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## 1 Intro

Just setting out to prove the shell theorem inside the shell.

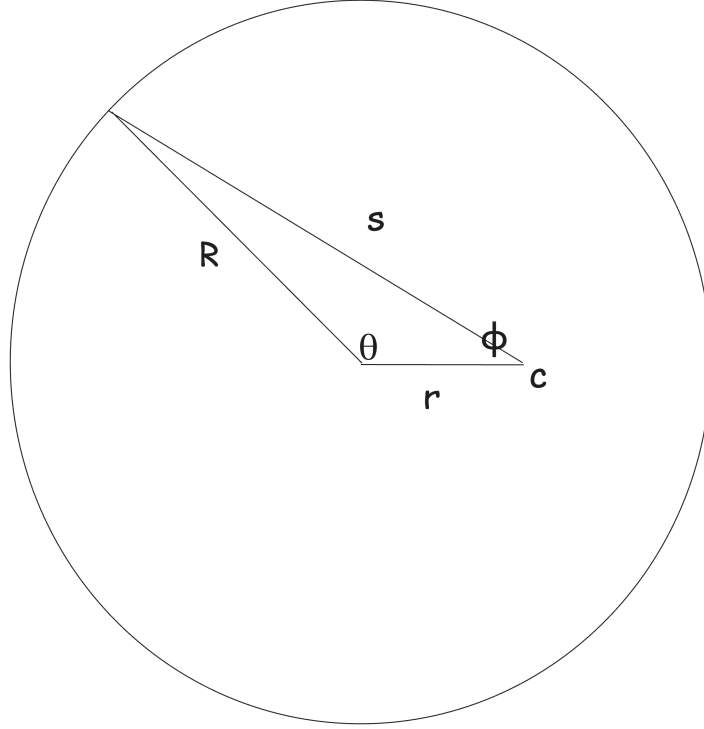


Figure 1: Reference

We first start with the formula for gravity.

$$F = \frac{GmM}{r^2}$$

We then have to represent it for a single point on the sphere. The cosine because we have to remember stuff cancels out.

$$\delta F = \frac{Gm\delta M}{s^2} \cos(\phi)$$

Then we have to convert the terms and the integrand.

$$\begin{aligned} \delta M &= \sigma \delta A \\ &= \sigma R \sin(\theta) \delta s = \sigma 2\pi R^2 \sin(\theta) \delta \theta \\ \delta F &= \frac{Gm 2\pi R^2 \sigma \sin(\theta) \delta \theta}{s^2} \end{aligned}$$

We then need to do additional conversions given the number of terms that we are dealing with here. Let's rewrite some different variables.

$$R^2 = s^2 \sin^2 \phi + (s \cos \phi - r)^2 = s^2 + r^2 - 2sr \cos \phi$$

$$\cos\phi = \frac{-R^2 + s^2 + r^2}{2sr}$$

Also convert  $\delta s$  and  $\delta\theta$ , the negative cos is because the theta is the wrong side

$$\begin{aligned} s^2 &= R^2 \sin^2 \theta + (r - R \cos \theta)^2 = R^2 + r^2 - 2rR \cos \theta \\ 2s\delta s &= 2rR \sin \theta \delta \theta \\ \delta \theta &= \frac{s\delta s}{rR \sin \theta} \end{aligned}$$

Substituting all the Substitutions:

$$\begin{aligned} \delta F &= \frac{Gm2\pi R^2 \sigma \sin(\theta)}{s^2} \left( \frac{s\delta s}{rR \sin \theta} \right) \left( \frac{-R^2 + s^2 + r^2}{2sr} \right) \\ \delta F &= \frac{GmM}{4\pi Rr^2} \left( \frac{-R^2 + s^2 + r^2}{s^2} \right) = \frac{GmM}{4\pi Rr^2} \left( 1 + \frac{r^2 - R^2}{s^2} \right) \\ F &= \frac{GmM}{4\pi Rr^2} \int_{R-r}^{R+r} 1 + \frac{r^2 - R^2}{s^2} \delta s = 0 \end{aligned}$$

## 2 Gravastar

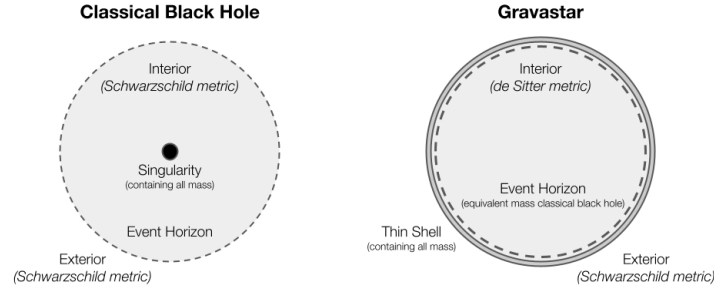


Figure 2: Reference

1. Alternative to Black holes
2. Massive Stars -> supernova -> either black hole or neutron star
3. Gravastar -> when it collapses, creates energy from core which pushes out and which fights collapsing rest of the star, creating new matter ultrathin bubble
4. Empty but full of energy vacuum , 1e44 more energy than a black hole
5. Give idea of big bang, because u get new dimensions with this
6. We get stuff from 2 collapsing gravastars / black holes