Test 1
Semester 2

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1 Introduction

Here are Multi Notes to prepare for the first test

2 Double Integrals

Double integrals are integrals that represent the volumes under a surface, rather than **Definite Integrals**, which compute the area under a curve.
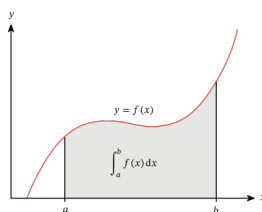
2.1 Definite Integrals Recap

Figure 1: Example of a definite Integral

Integrals are limits of a Riemann sum, which can be represented by the following equation:

$$\int_a^b f(x)\delta x = f(x_1)\Delta x_1 + f(x_2)\Delta x_2 + ... = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i)\Delta x_i \text{ ,where } \max \Delta x_i \to 0$$

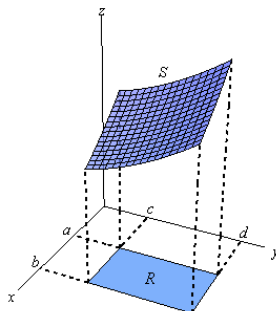## 2.2 Applying Definite Integrals to double integrals

Figure 2: Example of a double Integral

Given that we have to find the area underneath the curve of a function $z = f(x, y)$, we will need to find **volume**, not area. We're going to define the rectangle area by:

$$R = \{(x, y)|a \leq x \leq b, c \leq y \leq d\} = [a, b] \times [c, d] \text{ (Cartesian Sum)}$$

And thus we can define a solid S using the equation:

$$S = \{(x, y, z) | (x, y) \in \mathbb{R}, 0 \leq z \leq f(x, y)\}$$

If one is to use equal subintervals to calculate, then you can use a double Riemann sum, with the area of a rectangle equal to $\Delta x \times \Delta y \times f(x, y)$. You can represent that sum, with

$$\lim_{m,n \to \infty} \sum_{j=1}^{m} \sum_{i=1}^{n} f(x_i^*, y_i^*) \Delta A$$

(**NOTE** the *'s mean that the values are at a specific point) Where m and n represent the number if subintervals. Double Riemann sums give volume, which when "limited" simplifies to

$$\iint_R f(x, y) \delta A = \iint_{[a,b] \times [c,d]} f(x, y) \delta A$$

2.3   Notes

1. If $f(x, y) \geq 0$ and $f(x, y) \in \mathbb{R}$ then $\iint_R f(x, yA) =$ is the volume of the solid bounded by x, y, and the surface

2. $\delta A = \delta x \delta y$ or $\delta y \delta x$. Order will impact evaluation but not result.

3. If $f(x, y)$ is continious in R, then the double integral exists. (**NOTE**: You can get away with some discontinuity, such as with steps and the like, but not with stuff like infinite behavior)