

1 Write an interpreter

Write an interpreter for the following language.

Syntax:

$$\begin{aligned}
 (AExp) \ a &::= n \mid (\oplus, a_1, a_2) \\
 (BExp) \ b &::= true \mid false \mid (\odot, a_0, a_1) \mid (\oslash, b_0, b_1) \\
 (Com) \ c &::= skip \mid x := a \mid c_0; c_1 \mid if \{b\} else \{c_2\} then \{c_1\} \mid while \{b\} do \{c\} \\
 \oplus &::= + \mid - \mid * \\
 \odot &::= \leq \mid = \\
 \oslash &::= or \mid and
 \end{aligned}$$

n is any integer. x is any variable. *true* and *false* are truth values. *AExp* is arithmetic expression. *BExp* is Boolean expression. *Com* is command.

Semantics:

$$\begin{aligned}
 &\frac{}{(\oplus, n_1, n_2) \rightarrow n_3} \text{ (where } n_1 \oplus n_2 = n_3) \\
 &\frac{a_1 \rightarrow a'_1}{\oplus, a_1, a_2 \rightarrow \oplus, a'_1, a_2} \quad \frac{a_2 \rightarrow a'_2}{\oplus, a_1, a_2 \rightarrow \oplus, a_1, a'_2} \\
 &\frac{}{true \rightarrow true} \quad \frac{}{false \rightarrow false} \\
 &\frac{a_0 \rightarrow n_0 \quad a_1 \rightarrow n_1}{(=, a_0, a_1) \rightarrow true} \text{ (where } n_0 = n_1) \\
 &\frac{a_0 \rightarrow n_0 \quad a_1 \rightarrow n_1}{(=, a_0, a_1) \rightarrow false} \text{ (where } n_0 \neq n_1) \\
 &\frac{a_0 \rightarrow n_0 \quad a_1 \rightarrow n_1}{(\leq, a_0, a_1) \rightarrow true} \text{ (where } n_0 \leq n_1) \\
 &\frac{a_0 \rightarrow n_0 \quad a_1 \rightarrow n_1}{(\leq, a_0, a_1) \rightarrow false} \text{ (where } n_0 > n_1) \\
 &\frac{b_0 \rightarrow t_0 \quad b_1 \rightarrow t_1}{(and, b_0, b_1) \rightarrow t} \text{ (where } t_0 \wedge t_1 \equiv t)
 \end{aligned}$$

$$\frac{b_0 \rightarrow t_0 \quad b_1 \rightarrow t_1}{(or, b_0, b_1) \rightarrow t} \quad (\text{where } t_0 \vee t_1 \equiv t)$$

Let $x \mapsto n$ denote storing n in variable x . That is,

$$\frac{}{x := n \rightarrow x \mapsto n} \quad \frac{a \rightarrow a'}{x := a \rightarrow x := a'}$$

$$\frac{c_0 \rightarrow c'_0}{c_0; c_1 \rightarrow c_0; c'_1} \quad \frac{}{skip; c_1 \rightarrow c_1}$$

$$\frac{b \rightarrow b'}{if \{b\} else \{c_1\} then \{c_0\} \rightarrow if \{b'\} else \{c_1\} then \{c_0\}}$$

$$\frac{}{if \text{ true } else \{c_1\} then \{c_0\} \rightarrow c_0} \quad \frac{}{if \text{ false } else \{c_1\} then \{c_0\} \rightarrow c_1}$$

$$\frac{}{while \{b\} do \{c\} \rightarrow if \{b\} else \{skip\} then \{c; while \{b\} do \{c\}\}}$$

The *skip* command has no effect on the state.