UNIVERSIDAD DE EL SALVADOR
FACULTAD MULTIDICIPLINARIA ORIENTAL
DEPARTAMENTO DE INGENIERIA Y ARQUITECTURA

CARRERA:
INGENIERIA EN SISTEMAS INFORMATICOS

MATERIA:
MATEMATICA IV

DOCENTE:
Lic. Sonia del Carmen Martínez

Actividad:
Tarea II

ESTUDIANTES:

Lizama García, Edwin Daniel LG20012

Barrera Arce, Edwin Eismaeli BA20010

Morales López, Diego Paolo ML20011

Rubio Flores, Henry Vladimir RF20009

Daniel Alessandro, Rosales González RG20034

CICLO I1-2021

CIUDAD UNIVERSITARIA ORIENTAL, 21 DE SEPTIEMBRE DEL 2021

Guia#1

integrales iteradas

15) $\int_0^{\frac{\pi}{2}} \int_0^1 y \cos(x) \, dy \, dx$

$\int_0^1 y \cos(x) \, dy = \cos(x) \int y \, dy = \cos \frac{y^2}{2} \Big|_0^1$

$= \cos(x) \frac{(0)^2}{2} - \cos(x) \frac{(1)^2}{2}$

$= \cos(x) \frac{1}{2}$

$\int_0^{\frac{\pi}{2}} \cos(x) \frac{1}{2} \, dx = \frac{1}{2} \int \cos(x) \, dx$

$= \frac{1}{2} \operatorname{sen}(x) \Big|_0^{\frac{\pi}{2}} = \frac{1}{2} \operatorname{sen}(0) - \frac{1}{2} \operatorname{sen}\left(\frac{\pi}{2}\right)$

$= 0 - 1 = 1 = \frac{1}{2} \cdot 1 = \boxed{\frac{1}{2}}$

20) $\int_{-4}^{4}\int_{0}^{x^2} \sqrt{64-x^3}\, dy\, dx$

$$\int_0^{x^2} \sqrt{64-x^3}\, dy = \sqrt{64-x^3}\,\Big]_0^{x^2}$$

$$= x^2\sqrt{64-x^3}$$

$$\int_{-4}^{4} x^2\sqrt{64-x^3}\, dx$$

$U = 64-x^3 \qquad \frac{du}{dx} = -3x^2$

$dx = \frac{-1}{3x^2}\, du$

$$= -\frac{1}{3}\int \sqrt{u}\, du$$

$$= \frac{-1}{3}\cdot\frac{2u^{\frac{3}{2}}}{3} = \frac{2u^{3/2}}{9} = \boxed{\frac{2(64-x^3)^{3/2}}{9}}$$

3Q) $\int_0^{\pi/4}\int_0^{\cos\theta} 3x^2 \, Sen\,\theta \, dx \, d\theta$

$$\int_0^{\cos\theta} 3x^2 \, Sen(\theta) \, dx = 3\,sen(\theta) \int x^2 \, dx$$

$$= 3\,sen(\theta) \left. \frac{x^3}{3} \right|_0^{\cos(\theta)}$$

$$3\,Sen(\theta) \frac{\cos(\theta)^3}{3} - 3\,Sen(\theta) \frac{0^3}{3}$$

$$= \cos^3(\theta) \, sin(\theta)$$

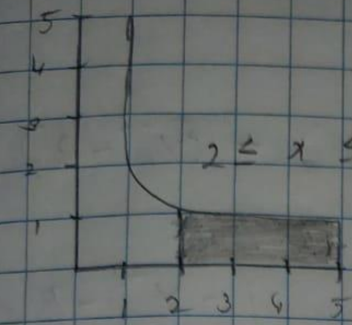
$$\int_0^{\pi/4} \cos^3(\theta) \, sin(\theta) \, d\theta$$

$$u = \cos(\theta) \Rightarrow \frac{du}{d\theta} = -sin(\theta) \Rightarrow d\theta = \frac{-1}{sin(\theta)} du$$

$$= -\int u^3 \, du = \frac{u^4}{4} = \left. -\frac{\cos^4(\theta)}{4} \right|_0^{\pi/4}$$

$$\boxed{I = \frac{3}{16}}$$

38) hallar area de la region



$$y = \frac{1}{\sqrt{x-1}}$$

$$2 \leq x \leq 5$$

$$A = \int_2^5 \int_0^{\frac{1}{\sqrt{x-1}}} dy\, dx$$

$$= \int_0^{\frac{1}{\sqrt{x-1}}} dy = y \Big|_0^{1/\sqrt{x-1}}$$

$$= \int_2^5 \frac{1}{\sqrt{x-1}}\, dx = (x-1)^{-1/2} dx$$

$$= 2(x-1)^{1/2} \Big|_2^5 = 2\left[(5-1)^{1/2} - (2-1)^{1/2}\right] = 2(2-1)$$

$$= 2 \qquad \boxed{A = 2}$$

$$A = \int_0^{1/2} \int_2^5 dx\, dy + \int_{1/2}^{1} \int_2^{\frac{1}{y^2}+1} dx\, dx$$

$$\boxed{\begin{aligned} y = \frac{1}{\sqrt{x-1}} &= y^2(x-1) = 1 \\ x-1 &= \frac{1}{y^2} \\ x &= \frac{1}{y^2} + 1 \end{aligned}}$$

$$\int_2^5 dx + \int_2^{\frac{1}{y^2}+1} dx$$

$$= x \Big|_2^5 + x \Big|_2^{\frac{1}{y^2}+1} = (5-2) + (y^{-2}-2)$$

$$= 3 + y^{-2} - 1 = \int_0^{1/2} 3\,dy + \int_{1/2}^{1} -y^{-2} - 1\,d$$

$$= 3y \Big|_0^{1/2} + \left[-y^{-1} - y\right]\Big|_{1/2}^{1}$$

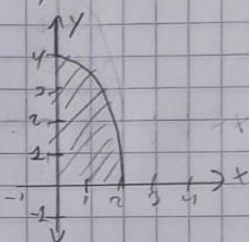
$$= 3\left(\frac{1}{2} - 0\right) + \left(-\frac{1}{y} - y\right)\Big|_{1/2}^{1} = \frac{3}{2} + (-2 - (-2 - \frac{1}{2}$$

$$\frac{3}{2} + (-2 + \frac{5}{2}) = \frac{3}{2} + \frac{1}{2} = \frac{4}{2} = 2$$

$$\boxed{A = 2}$$

44. $y = x$, $y = 2x$, $x = 2$

$$\int_0^2\int_0^{4-x^2} F(x,y)\,dy\,dx, \quad 0 \le y \le 4-x^2, \quad 0 \le x \le 2$$



$$= \int_0^4\int_0^{\sqrt{4-y}} F(x,y)\,dx\,dy$$

$$= \int_0^{\sqrt{4-y}} F(x,y)\,dx$$

$$= F,y \cdot \int_0^{\sqrt{4-y}} x\,dx = F,y \cdot \left[\frac{x^2}{2}\right]\Big|_0^{\sqrt{4-y}}$$

$$= F,y \left[\frac{\sqrt{4-y}^{\,2}}{2} - \frac{0^2}{2}\right]$$

$$= F,y\ \frac{-y+4}{2} = \int_0^4 F,y\ \frac{-y+4}{2}\,dy$$

$$\frac{1}{2} F\left(32 - \frac{64}{3}\right)$$

50) $\int_0^2 \int_0^{4-x^2} f(x,y)\, dy\, dx$

$0 \le x \le 2$ y $0 \le y \le 4-x^2$

$y = 4 - x^2$

$= x = 0 \;,\; y = 4$

$x = 2 \;,\; y = 0$

$x = 1 \;,\; y = 3$

$y = 4 - x^2$

$= x^2 = 4 - y$

$= x = \sqrt{4-y}$

$= \int_0^4 \int_0^{\sqrt{4-y}} f(x,y)\, dx\, dy$

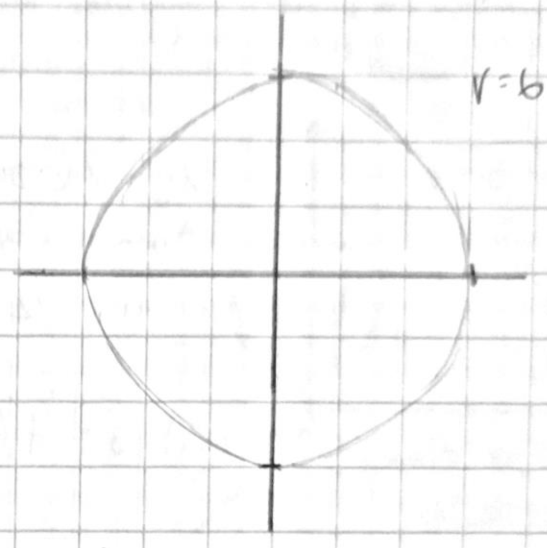
Guia#2

11-

$$\int_0^{2\pi}\int_0^{6} 3r^2 \operatorname{sen}\theta \, dr \, d\theta$$

$0 \leq r \leq 6$

$0 \leq \theta \leq 2\pi$



$$3\cdot\left(\tfrac{1}{3}r^3 \operatorname{sen}\theta \,\Big|_0^6\right)$$

$$3\cdot(72 \operatorname{sen}\theta - 0)$$

$$\int_0^{2\pi} 216 \operatorname{sen}\theta \, d\theta$$

$$216\cdot(-\cos\theta)\Big|_0^{2\pi}$$

$$216\cdot(-\cos 2\pi - (-\cos 0))$$

$$216\cdot(-1-(-1))$$

$$= 0$$

14-

$$\int_0^{\pi/2}\int_0^{3} r e^{-r^2} \, dr \cdot d\theta$$

$u = -r^2$ $\quad du/dr = -2r$

$dr = -1/2r \, du$

$$-1/2 \int_0^3 e^u \, du$$

$$= -e^u/2$$

$$= -e^{-r^2}/2 \,\Big|_0^3$$

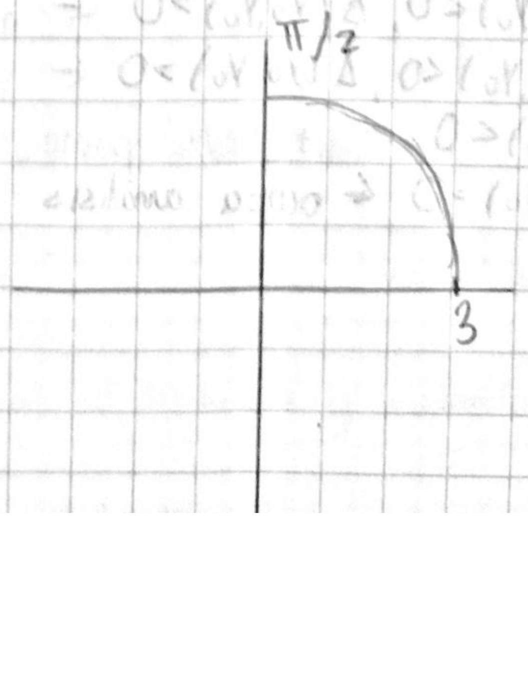
$0 \leq r \leq 3$

$0 \leq \theta \leq \pi/2$

$$= 1/2 - e^{-9}/2 \int_0^{\pi/2} d\theta$$

$$= 1/2 - e^{-9}/2 \cdot \theta \,\Big|_0^{\pi/2}$$

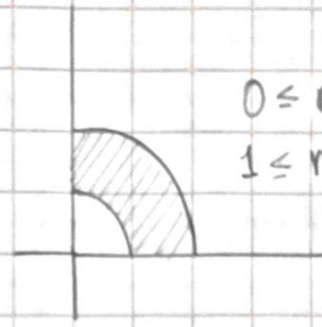
$$= \frac{1/2 - e^{-9}/2\ \pi}{2}$$



29) $\int_0^1 \int_{\sqrt{1-x^2}}^{\sqrt{4-x^2}} x^2/x^2+y^2 \, dy\, dx \quad + \int_1^2 \int_0^{\sqrt{4-x^2}} x^2/x^2+y^2 \, dy\, dx$

$\sqrt{1-x^2} \leq y \leq \sqrt{4-x^2}$

$0 \leq x \leq 1$



$0 \leq \theta \leq \pi/2$

$1 \leq r \leq 2$

$y = \sqrt{1-x^2} \qquad y = \sqrt{4-x^2}$

$y^2 + x^2 = 1 \qquad y^2 + x^2 = 4$

$$\int_0^{\pi/2} \int_1^2 (r\cos\theta)^2/r^2 \; r\, dr\, d\theta$$

$$\int_1^2 \cos^2(\theta)\, r \; dr$$

$$\cos^2(\theta) \cdot r^2/2 \;\Big|_1^2 \quad = \int_0^{\pi/2} 3\cos^2(\theta)/2 \, d\theta$$

$$= 3/2 \int_0^{\pi/2} \cos^2(\theta)\, d\theta$$

$$= 3/2 \cdot \cos(\theta)\,\mathrm{sen}(\theta)/2 + 1/2\,\theta \;\Big|_0^{\pi/2}$$

$$= 3\pi/8$$

$0 \leq y \leq \sqrt{4-x^2} \qquad 0 \leq r \leq 2$

$0 \leq x \leq 1 \qquad 0 \leq \theta \leq \pi/2$

$$\int_0^{\pi/2} \int_0^2 \cos^2(\theta)\, r\, dr\, d\theta$$

$$= \int_0^{\pi/2} 2\cos^2(\theta)\, d\theta$$

$$= 2 \cdot \cos(\theta)\,\mathrm{sen}(\theta)/2 + 1/2\,\theta \;\Big|_0^{\pi/2}$$

$$= \pi/2$$

$$3\pi/8 + \pi/2 = 7/8\,\pi$$

31) $\int_{-5}^{5} \int_{0}^{\sqrt{25-x^2}} (4x + 3y)\, dy\, dx$

$0 \leq y \leq \sqrt{25-x^2}$ $\qquad$ $0 \leq r \leq 5$

$-5 \leq x \leq 5$ $\qquad$ $0 \leq \theta \leq \pi$

$\int_{0}^{\pi} \int_{0}^{5} (4r\cos\theta + 3r\,\text{sen}\,\theta)\, r\, dr\, d\theta$

$(4\cos\theta + 3\,\text{sen}\,\theta) \int_{0}^{5} r^2\, dr$

$= (4\cos\theta + 3\,\text{sen}\,\theta) \quad r^2/2 \Big|_{0}^{5}$

$= \int_{0}^{\pi} 50\cos\theta + 75/2\,\text{sen}\,\theta\, d\theta$

$= 175/2 \int_{0}^{\pi} \cos\theta\, d\theta$

$= 175/2\ \text{sen}\,\theta \Big|_{0}^{\pi}$

$= 0$

Guia#3

1. $\int_{2}^{4}\int_{-2}^{2}\int_{-1}^{1}(x+y+z)\,dx\,dy\,dz$

$\int_{-1}^{1}(x+y+z)\,dx$

$\int_{-1}^{1}y\,dx+\int_{-1}^{1}z\,dx+\int_{-1}^{1}x\,dx$

$=\int_{2}^{4}\int_{-2}^{2}(2y+2z)\,dy\,dz$

$\int_{-2}^{2}(2y+2z)\,dy$

$\int_{-2}^{2}2z\,dy+\int_{-2}^{3}2y\,dy$

$=\int_{2}^{4}8z\,dz$

$=8\cdot\int_{2}^{4}z\,dz$

$=8\cdot\left[\frac{z^{1+1}}{1+1}\right|_{2}^{4}$

$=8\cdot\frac{z^2}{2}\Big|_{2}^{4}=48$

(5) $\int_0^{\pi/2} \int_0^{y^2} \int_0^{y} \cos \, dz\,dx\,dy$

$\int_0^{y} \cos\left(x/y\right) dz$

$\cos\left(x/y\right) z \Big|_0^{y}$

$\int_0^{\pi/2} \int_0^{y^2} y \cos\left(x/y\right) dx\,dy$

$\int_0^{y^2} y \cos\left(x/y\right)$

$y \int_0^{y} \cos(u)\, y\, du$

$y^2 \operatorname{sen} u \Big|_0^{y}$

$= \int_0^{\pi/2} y^2 \operatorname{sen}(y)\, dy$

$= -y^2 \cos y - \left(-2\left(y \operatorname{sen} y + \cos y\right)\right) \Big|_0^{\pi/2}$

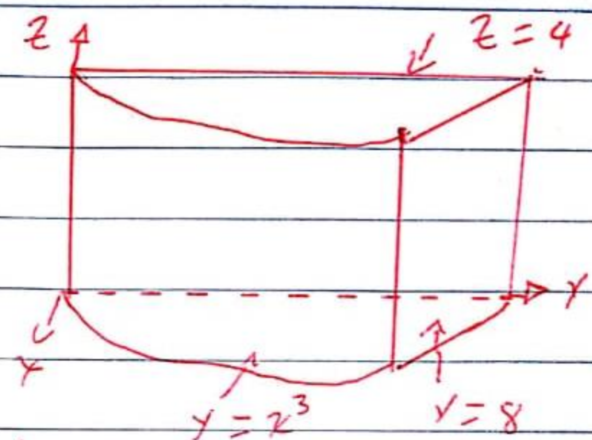
$= -y^2 \cos x + 2\left(y \operatorname{sen} y + \cos y\right) \Big|_0^{\pi/2}$

$= \pi - 2$

13) a) $dz\,dy\,dx$

b) $dx\,dz\,dy$

c) $dy\,dx\,dz$



a) $$\int_0^2 \int_{x^3}^8 \int_0^4 dz\,dy\,dx$$

b) $$\int_0^8 \int_0^4 \int_0^{\sqrt[3]{4}} dx\,dz\,dy$$

c) $$\int_0^4 \int_0^2 \int_{x^3}^8 dy\,dx\,dz$$

41. $x = y^2$, $4 - x = y^2$, $z = 0$, $z = 3$

$$\int_0^3 \int_x^{y^2+4} dy\, dx$$

$$\int_x^{y^2+4} dy$$

$$= \left[1 - y\right]_x^{4+y^2}$$

$$= \left[y\right]_x^{4+y^2}$$

$$= 4 + y^2 - x$$

$$= \int_0^3 (4 + y^2 - x)\, dx$$

$$= \int_0^3 4 + y^2 - x\, dx$$

$$= \int_0^3 4\, dx + \int_0^3 y^2\, dx - \int_0^3 x\, dx$$

$$= 12 + 3y^2 - 9/2$$

Guia#4

(13) El solido en el primer octante acotado por los plano coordenados y el plano $z = 5 - x - y$

$$V = \int_0^5 \int_0^{5-x} \int_0^{5-y} dz\, dy\, dx$$

$$V = \int_0^5 \int_0^{5-x} 5 - x - y \; dy\, dx$$

$$V = \int_0^5 (5y - xy) - y^2/2 \Big|_0^{5-x} dx$$

$$V = \int_0^5 5(y - x) - x(5 - x) - \left(\frac{5 - x}{2}\right)^2 dx$$

$$V = \int_0^5 25 - 5x - 5x + x^2 - \frac{(5 - y)^2}{2}\, dx$$

$$V = \frac{125}{6} v^3$$

(19) Utiliza una integral triple para hallar el volumen del solido mostrado en la figura

$$V=\int_0^2\int_0^{4-y^2}\int_0^{x} dz\,dx\,dy$$

$$V=\int_0^2\int_0^{4-y^2} x\,dx\,dy$$

$$V=\int_0^2 \left(\frac{4-y^2}{2}\right)^2 dy$$

$$V=\frac{1}{2}\int_0^2 16-7y^2\,dy$$

$$V=\frac{1}{2}\int_0^2 16\,dy-\int_0^2 7y^2\,dy$$

$$V=\frac{1}{2}\left(16y-\frac{7y^3}{3}\right)\Big|_0^2$$

$$V=\frac{1}{2}\;16(2)-\frac{7(2)^3}{3}-\left(16(0)-\frac{7(0)^3}{3}\right)$$

$$V=\frac{1}{2}\cdot\frac{40}{3}$$

$$=\frac{20}{3}\,v^3$$

Guia#5

Ejercicio de Tarea hacer : 1, 6

1. $\int_{-1}^{5}\int_{0}^{\frac{\pi}{2}}\int_{0}^{3} r\cos\theta \, dr\, d\theta\, dz$

$\int_{0}^{3} r\cos\theta\, dr = \cos\theta\int_{0}^{3} r\,dr = \cos\theta\,\frac{1}{2}\cdot r^2\Big|_{0}^{3}$

$= \cos\theta\,\frac{1}{2}\left[3^2 - 0\right] = \cos\theta\,\frac{1}{2}(9) = \cos\theta\,\frac{9}{2}$

$= \int_{0}^{\frac{\pi}{2}} \cos\theta\,\frac{9}{2}\, d\theta = \frac{9}{2}\int_{0}^{\frac{\pi}{2}}\cos\theta\, d\theta = \frac{9}{2}\cdot \text{Sen}\,\theta\Big|_{0}^{\frac{\pi}{2}}$

$= \frac{9}{2}\left[\text{Sen}\left(\frac{\pi}{2}\right) - \text{Sen}(0)\right] = \frac{9}{2}\left[1 - 0\right]$

$= \frac{9}{2}(1) = \frac{9}{2}$

$= \int_{-1}^{5} \frac{9}{2}\, dz = \frac{9}{2} z\Big|_{-1}^{5} = \frac{9}{2}(5) - \frac{9}{2}(-1)$

$= 22.5 - (-4.5) = \boxed{27}$

6. $\int_{0}^{\pi/4}\int_{0}^{\pi/4}\int_{0}^{\cos\theta} \rho^2\,\text{sen}\,\phi\cos\phi\, d\rho\, d\theta\, d\phi$

$= \int_{0}^{\cos\theta} \rho^2\,\text{sen}\,\phi\cos\phi\, d\rho = \text{Sen}\,\phi\cos\phi\int_{0}^{\cos\theta}\rho^2\, d\rho$

$= \text{Sen}\,\phi\cos\phi\,\frac{\rho^3}{3}\Big|_{0}^{\cos\theta} = \text{Sen}\,\phi\cos\phi\left[\frac{\cos^3\theta}{3} - \frac{0^3}{3}\right]$

$= \frac{\text{Sen}\,\phi\cos\phi\,(\cos^3\theta)}{3}$

$= \int_{0}^{\pi/4} \frac{\cos^3\theta\,\text{Sen}\,\phi\cos\phi}{3}\, d\theta$

$= \frac{\text{Sen}\,\phi\cos\phi}{3}\int_{0}^{\pi/4}\cos^3\theta\, d\theta$

$= \text{Sen}\,\phi\cos\phi\,\frac{1}{3}\int_{0}^{\pi/4}\cos\theta\,(1 - \text{Sen}^2\theta)\, d\theta$

$$= \text{sen}\,\phi\cos\phi\,\tfrac{1}{3}\int_0^{\pi/4}\frac{\cancel{\cos\theta}}{\cancel{\cos\theta}}(1-u^2)\,du$$

$u = \text{sen}\,\theta$
$du = \cos\theta$
$\frac{du}{\cos\theta} = d\theta$

$$= \text{sen}\,\phi\cos\phi\,\tfrac{1}{3}\int_0^{\pi/4}(1-u^2)\,du$$

$$= \text{sen}\,\phi\cos\phi\,\tfrac{1}{3}\left[\int_0^{\pi/4}du - \int_0^{\pi/4}u^2\,du\right]$$

$$= \text{sen}\,\phi\cos\phi\,\tfrac{1}{3}\left[\left(u-\frac{u^3}{3}\right)\Big|_0^{\pi/4}\right]$$

$u = \text{sen}\,x$

$$= \text{sen}\,\phi\cos\phi\,\tfrac{1}{3}\left[\text{sen}\,x-\frac{\text{sen}^3 x}{3}\right)\Big|_0^{\pi/4}\Big]$$

$$= \text{sen}\,\phi\cos\phi\,\tfrac{1}{3}\left[\text{sen}(\pi/4)-\frac{\text{sen}^3(\pi/4)}{3}-0\right]$$

$$= \text{sen}\,\phi\cos\phi\,\tfrac{1}{3}\left(\frac{\sqrt{2}}{12}-\frac{\sqrt{2}}{12}\right.$$

$$= \text{sen}\,\phi\cos\phi\,\frac{5\sqrt{2}}{36}$$

$$\int_0^{\pi/4}\text{sen}\,\phi\cos\phi\,\frac{5\sqrt{2}}{36}\,d\phi$$

$$= \frac{5\sqrt{2}}{36}\int_0^{\pi/4}\text{sen}\,\phi\cos\phi\,d\phi$$

$u = \text{sen}\,\phi$
$du = \cos\phi\,d\phi$
$\frac{du}{\cos\phi} = d\phi$

$$= \frac{5\sqrt{2}}{36}\int_0^{\pi/4}\frac{\cos\phi}{\cos\phi}\,u\,du$$

$$= \frac{5\sqrt{2}}{36}\int_0^{\pi/4}u\,du$$

$$= \frac{5\sqrt{2}}{36}\left[\frac{u^2}{2}\Big|_0^{\pi/4}\right]$$

$u = \text{sen}\,\phi$

$$= \frac{5\sqrt{2}}{36}\left[\frac{\text{sen}^2(\pi/4)}{2}-0\right]$$

$$\frac{5}{72\sqrt{2}}$$