Actividad 2: PCA

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```r
M <- read.csv('paises_mundo.csv')
M
```

```
##    CrecPobl MortInf PorcMujeres   PNB95 ProdElec LinTelf ConsAgua PropBosq
## 1       1.0      30          41    2199     3903      12       94       53
## 2       3.0     124          46    4422      955       6       57       19
## 3       4.3      21          13  133540    91019      96      497        1
## 4       2.5      34          24   44609    19883      42      180        2
## 5       1.3      22          31  278431    65962     160     1043       22
## 6       1.4       6          43  337909   167155     510      933       19
## 7       0.6       6          41  216547    53259     465      304       47
## 8       2.0      79          42   28599     9891       2      220        6
## 9       0.3       8          40  250710    72236     457      917       20
## 10      3.0      95          48    2034        6       5       26       45
## 11      0.4      13          49   21356    31397     190      295       31
## 12      2.3      69          37    5905     2824      35      201       45
## 13      1.6      44          35  579787   260682      75      246       66
## 14     -0.6      15          48   11225   381333     335     1544       33
## 15      2.9      56          38    8615     2740       4       38       44
## 16      1.3       6          45  573695   554227     590     1602       49
## 17      1.8      26          37   70263    43354     100      174       52
## 18      3.1      90          43    1784      435       8       20       58
## 19      1.8      26          45   12870    38000      47      687       74
## 20      0.9      10          40  435137   164993     415      632       66
## 21      3.4      86          33    9248     2305       8       66       34
## 22      2.5      13          30    8884     4772     164      780       28
## 23      0.9       9          38    7150    10982      32      870       16
## 24      1.6      12          32   59151    25276     132     1626       12
## 25      1.3      34          45  744890   928083      34      461       13
## 26      0.2       6          46  156027    40097     613      233       12
## 27      2.0      37          29   11390     6182      79      446       22
## 28      2.3      36          26   15997     8256      61      581       43
## 29      2.2      56          29   45507    51947      46      956        0
## 30      1.8      36          34    9057     3211      53      245        6
## 31      5.8      16          13   42806    18870     283      884        0
## 32      0.3      11          48   15848    24740     208      337       38
## 33      0.2       7          36  532347   161654     385      781       51
## 34      2.6     112          41    5722     1293       2       51       13
## 35      2.3      39          37   71865    27062      21      686       26
## 36      0.4       5          48  105174    65546     550      440       77
## 37      0.5       6          44 1451051   476200     558      665       25
```

```
## 38        2.9      89            44    3759     933      30      57      71
## 39        3.0      73            51    6719    6115       4      35      42
## 40        0.5       8            36   85885   40623     493     523      47
## 41        2.9      44            26   14255    3161      27     139      39
## 42        2.0      72            43    1777     362       8       7       1
## 43        0.6       6            40  371039   79647     525     518      10
## 44        3.0      45            30    3566    2672      29     294      41
## 45       -0.3      11            44   42129   33486     185     661      18
## 46        1.9      68            32  319660  386500      13     612      17
## 47        1.7      51            40  190105   53414      17      96      60
## 48        2.7     108            18   24600   27060      33    4575       4
## 49        3.2      45            24  113400   79128      79    1362      11
## 50        0.1       6            33   52765   17105     365     233       6
## 51        1.1       4            44    6686    4780     555     636       1
## 52        2.7       8            40   87875   32781     418     408       6
## 53        4.7      31            21    6354    5076      73     173       1
## 54        2.9      58            46    7583    3539       9      87       2
## 55       -0.3      11            28   28941   22798     230     525       0
## 56        2.3      32            28   10673    5184      82     271       8
## 57        3.6      61            21   23400   17800      59     880       0
## 58        2.5      12            37   78321   39093     166     768      54
## 59        2.0      55            35   29545   11100      43     427      20
## 60        2.1      33            31  304596  147926      96     899      25
## 61        1.8     113            48    1353     490       3      55      22
## 62        1.8      83            43   35840    3500       3     101      44
## 63        2.5      91            40    4391     927       4     150      37
## 64        3.1      46            36    1659    1688      23     367      50
## 65        2.9      80            36   28411   15530       4      41      17
## 66        0.5       5            46  136077  113488     556     488      31
## 67        1.0       7            44   51655   35135     479     589      28
## 68        4.5      18            15   10578    6187      77     564      19
## 69        3.0      90            26   59991   58529      16    2053       2
## 70        1.9      23            34    7253    3380     114     754      42
## 71        2.7      41            29    8158   36415      31     109      32
## 72        2.1      47            29   55019   15563      47     300      53
## 73        0.4      14            46  107829  135347     148     321      28
## 74       -0.1       7            43   96829   31380     361     739      34
## 75        0.3       6            43 1094734  325383     502     205      10
## 76        0.0       8            47   39990   58705     236     266      34
## 77        0.0      23            44   33488   55136     131    1134      27
## 78        2.8      62            42    5070    1002      10     202      39
## 79        1.8       4            38   79831   20046     478      84       7
## 80        3.1      32            26   15780   15186      63     435       4
## 81        1.3      16            35   12616    4387      11     503      27
## 82        2.2      77            28    7510    1333       3     633      18
## 83        0.6       4            48  209720  142895     681     341      68
## 84        0.8       6            40  286014   65724     613     173      30
## 85        2.3      50            37  130918  189316      95     359       4
## 86        1.3      35            46  159630   71177      59     602      25
## 87        3.1      82            49    3703    1913       3      40      38
## 88        2.1      39            30   16369    6714      58     381       4
## 89        1.9      48            35  169452   78322     212     585      26
## 90        0.1      15            49   84084  202995     157     673      16
## 91        0.6      18            40   16458    7617     196     241       4
```

```
## 92       2.4      23           33   65382    73116      111       382        52
## 93       2.2      41           49   17634    12270       11       414        26
## 94       4.2     100           29    4044     2159       12       335         8
## 95       2.6     109           45    3605     7785        8       186        43
## 96       2.8      55           44    5933     7334       14       136        23
##    PropDefor ConsEner EmisCO2
## 1        0.0      341     1.2
## 2        0.7       89     0.5
## 3        0.0     4566    13.1
## 4        0.8      906     3.0
## 5        0.1     1504     3.5
## 6        0.0     5341    15.3
## 7       -0.4     3301     7.2
## 8        4.1       64     0.2
## 9       -0.3     5120    10.1
## 10       1.3       20     0.1
## 11      -0.4     2392     9.9
## 12       1.2      373     1.0
## 13       0.6      718     1.4
## 14      -0.2     2438     6.4
## 15       0.6      103     0.2
## 16      -1.1     7854    14.4
## 17       0.7      622     1.8
## 18       0.2      331     1.6
## 19       0.0     1129    11.2
## 20       0.1     2982     6.6
## 21       1.0      103     0.5
## 22       3.0      558     1.2
## 23       1.0      923     2.6
## 24      -0.1     1012     2.6
## 25       0.7      664     2.3
## 26       0.0     3977    10.4
## 27       2.9      337     1.4
## 28       1.8      565     1.8
## 29       0.0      600     1.5
## 30       2.3      370     0.7
## 31       0.0    10531    33.9
## 32       0.1     3243     7.0
## 33       0.0     2458     5.7
## 34       0.3       22     0.1
## 35       3.4      316     0.8
## 36       0.0     5997     8.2
## 37      -0.1     4042     6.3
## 38       0.6      652     5.5
## 39       1.4       93     0.2
## 40       0.0     2260     7.2
## 41       1.8      210     0.6
## 42       5.1       29     0.1
## 43      -0.3     4580     9.2
## 44       2.2      204     0.6
## 45      -0.5     2383     5.8
## 46       0.6      248     0.9
## 47       1.1      366     1.0
## 48       0.1     1213     3.4
```

```
## 49      0.0    1505    4.0
## 50     -1.2    3137    8.7
## 51      0.0    7932    6.8
## 52     -0.3    2717    8.1
## 53     -1.0    1067    3.0
## 54      0.6     110    0.2
## 55      0.0    8622   11.2
## 56      0.6     964    2.9
## 57     -1.4    2499    8.1
## 58      2.1    1699    3.8
## 59     -1.4     327    1.1
## 60      1.3    1561    3.8
## 61      0.8      40    0.1
## 62      1.3      49    0.1
## 63      1.0      28    0.1
## 64      1.9     300    0.6
## 65      0.7     162    0.9
## 66     -1.4    5318   14.1
## 67      0.0    4245    7.6
## 68      0.0    2392    5.3
## 69      3.5     254    0.6
## 70      1.9     618    1.7
## 71      2.8     299    0.6
## 72      0.4     367    1.0
## 73     -0.1    2401    8.9
## 74     -0.5    1827    4.8
## 75     -1.1    3732    9.8
## 76      0.0    3868   13.1
## 77      0.0    1733    5.4
## 78      0.7      97    0.4
## 79      2.3    8103   17.7
## 80     -4.3     997    3.3
## 81      1.4      97    0.3
## 82      1.1      66    0.1
## 83      0.0    5723    6.6
## 84     -0.6    3629    6.4
## 85     -0.8    2146    7.5
## 86      3.5     769    2.0
## 87      1.2      34    0.1
## 88     -1.9     595    1.6
## 89      0.0     957    2.5
## 90     -0.3    3180   11.7
## 91     -0.6     629    1.6
## 92      1.2    2186    5.7
## 93      1.5     101    0.3
## 94      0.0     206    0.7
## 95      1.1     149    0.3
## 96      0.7     438    1.8
```

Parte 1

Generando las matrices de covarianza y de correlación

```r
MC <- cov(M)
MCR <- cor(M)
```

Calculando descomposición espectral de las matrices

```r
EMC <- eigen(MC)
EMC
```

```
## eigen() decomposition
## $values
##  [1] 6.163576e+10 6.581612e+09 4.636256e+06 3.107232e+05 1.216015e+04
##  [6] 5.137767e+02 3.627885e+02 4.542082e+01 5.800868e+00 1.438020e+00
## [11] 4.768083e-01
##
## $vectors
##                [,1]          [,2]          [,3]          [,4]          [,5]
##  [1,] -1.658168e-06  4.706785e-07  0.0001263736 -1.928408e-05 -0.0055373971
##  [2,] -4.048139e-05 -1.774254e-05  0.0082253821 -2.493257e-03 -0.0944030203
##  [3,]  5.739096e-06 -1.084543e-05  0.0001318149  5.538307e-03  0.0314036410
##  [4,]  8.880376e-01  4.597632e-01  0.0026022071 -3.893588e-04 -0.0003327409
##  [5,]  4.597636e-01 -8.880405e-01  0.0005694896  1.096305e-03  0.0002207819
##  [6,]  3.504341e-04  4.016179e-04 -0.0619424889  7.641174e-03  0.9921404486
##  [7,]  2.625508e-04 -1.122118e-03 -0.0401453227 -9.991411e-01  0.0057795144
##  [8,]  4.089564e-06  7.790843e-06  0.0012719918  6.435797e-03  0.0419331615
##  [9,] -1.073825e-06  2.350808e-07  0.0001916177  4.043796e-05 -0.0018090751
## [10,]  2.547156e-03  7.126782e-04 -0.9972315499  3.973568e-02 -0.0625729475
## [11,]  4.643724e-06 -1.315731e-06 -0.0020679047 -5.626049e-05 -0.0042367120
##                [,6]          [,7]          [,8]          [,9]         [,10]
##  [1,]  1.243456e-02  5.359089e-03 -8.390810e-02 -6.778358e-02 -1.158091e-01
##  [2,]  9.917515e-01  2.258020e-02 -7.891128e-02 -1.637836e-02  4.264872e-04
##  [3,]  8.552992e-02 -1.136481e-01  9.856498e-01 -1.468464e-02  8.241465e-03
##  [4,] -8.621005e-06 -7.566477e-06  1.217248e-05 -3.971469e-07  4.274451e-07
##  [5,]  1.955408e-05  1.544658e-05 -2.558998e-05  1.059471e-06 -1.353881e-06
##  [6,]  9.109622e-02  4.748682e-02 -3.416812e-02 -5.379549e-03 -3.409423e-03
##  [7,] -1.087229e-03 -6.863294e-03  4.698731e-03  7.965261e-05  3.621425e-05
##  [8,]  1.721948e-02 -9.920538e-01 -1.169638e-01  1.416566e-03  5.891758e-03
##  [9,]  1.758667e-03 -7.455427e-03  1.811443e-02  1.283039e-01 -9.859317e-01
## [10,]  2.639673e-03 -3.764707e-03  1.267052e-03  2.262931e-03  2.672618e-04
## [11,] -1.877994e-02 -1.709137e-03 -5.204823e-03 -9.891529e-01 -1.200519e-01
##               [,11]
##  [1,]  9.872887e-01
##  [2,] -2.092491e-02
##  [3,]  8.344324e-02
##  [4,]  2.723996e-07
##  [5,] -2.086857e-07
##  [6,]  4.944397e-04
##  [7,]  4.780416e-04
##  [8,] -3.748976e-03
##  [9,] -1.052934e-01
## [10,]  5.906241e-05
## [11,] -8.221371e-02
```

```r
EMCR <- eigen(MCR)
EMCR
```

```
## eigen() decomposition
## $values
##  [1] 4.02987902 1.92999195 1.37041115 0.86451597 0.79414057 0.72919997
##  [7] 0.57130511 0.32680096 0.16806846 0.14632819 0.06935866
##
## $vectors
##              [,1]        [,2]        [,3]        [,4]        [,5]        [,6]
##  [1,] -0.314119414  0.34835747 -0.07352541  0.44028717  0.32972147 -0.18392437
##  [2,] -0.392395442 -0.04136238 -0.17759254  0.13398483 -0.08340489 -0.08656390
##  [3,]  0.116546319 -0.58283641  0.16686305 -0.05865031 -0.18654100  0.16835650
##  [4,]  0.295393771 -0.17690839 -0.53343025  0.26248209  0.14110658  0.04653378
##  [5,]  0.258964724 -0.17356372 -0.61438847  0.17389644  0.07521971  0.02821905
##  [6,]  0.446082934 -0.02719077  0.15177250 -0.04959796  0.05416498  0.02442175
##  [7,]  0.092410503  0.32060987 -0.37024258 -0.73603097 -0.02671021 -0.30940890
##  [8,]  0.005692925 -0.45742697  0.16480339 -0.04024882  0.41531702 -0.75356463
##  [9,] -0.243652293 -0.15408201 -0.02961449 -0.33650345  0.73261463  0.50894232
## [10,]  0.415029554  0.23286257  0.20608749  0.06730166  0.23100421  0.05806466
## [11,]  0.374531032  0.29168698  0.20631751  0.14843513  0.24028756 -0.02809233
##               [,7]        [,8]        [,9]       [,10]       [,11]
##  [1,]  0.1628974320 -0.09481963  0.52181220  0.34674573 -0.10062784
##  [2,]  0.6398040762 -0.32307802 -0.29031618 -0.38959240  0.17487096
##  [3,]  0.5310867107  0.05209889  0.23599758  0.42854658 -0.16786800
##  [4,] -0.1490207046 -0.44913216 -0.36995675  0.34911534 -0.15247432
##  [5,]  0.1082745817  0.50343911  0.30681318 -0.33770404  0.12366382
##  [6,] -0.0008501608 -0.56975094  0.44733110 -0.20997673  0.44992596
##  [7,]  0.2357666690 -0.05962470  0.08358225  0.20561803 -0.07067780
##  [8,] -0.0806036686  0.04275404 -0.07438520 -0.08671232 -0.01493710
##  [9,]  0.0112333588 -0.01607505 -0.01868615 -0.03209758  0.07259619
## [10,]  0.2711228006 -0.05023582 -0.04339752 -0.36147417 -0.67912543
## [11,]  0.3352822144  0.30978009 -0.37666244  0.28779437  0.46737561
```

Ahora la proporción de varianza explicada por cada vector:

```r
PC <- c()
for(i in 1:11){
PC <- append(PC,EMC[[1]][i] / sum(diag(MC)))
}
PC
```

```
##  [1] 9.034543e-01 9.647298e-02 6.795804e-05 4.554567e-06 1.782429e-07
##  [6] 7.530917e-09 5.317738e-09 6.657763e-10 8.502887e-11 2.107843e-11
## [11] 6.989035e-12
```

```r
PCR <- c()
for(i in 1:11){
PCR <- append(PC,EMCR[[1]][i] / sum(diag(MCR)))
}
PCR
```

```
##  [1] 9.034543e-01 9.647298e-02 6.795804e-05 4.554567e-06 1.782429e-07
##  [6] 7.530917e-09 5.317738e-09 6.657763e-10 8.502887e-11 2.107843e-11
## [11] 6.989035e-12 6.305332e-03
```

Y los resultados acumulados:

```
PC <- cumsum(PC)
PC
```

```
##  [1] 0.9034543 0.9999273 0.9999953 0.9999998 1.0000000 1.0000000 1.0000000
##  [8] 1.0000000 1.0000000 1.0000000 1.0000000
```

```
pca <- prcomp(M, scale = TRUE)
pca$rotation
```

```
##                      PC1         PC2         PC3         PC4         PC5
## CrecPobl     0.314119414 -0.34835747  0.07352541 -0.44028717  0.32972147
## MortInf      0.392395442  0.04136238  0.17759254 -0.13398483 -0.08340489
## PorcMujeres -0.116546319  0.58283641 -0.16686305  0.05865031 -0.18654100
## PNB95       -0.295393771  0.17690839  0.53343025 -0.26248209  0.14110658
## ProdElec    -0.258964724  0.17356372  0.61438847 -0.17389644  0.07521971
## LinTelf     -0.446082934  0.02719077 -0.15177250  0.04959796  0.05416498
## ConsAgua    -0.092410503 -0.32060987  0.37024258  0.73603097 -0.02671021
## PropBosq    -0.005692925  0.45742697 -0.16480339  0.04024882  0.41531702
## PropDefor    0.243652293  0.15408201  0.02961449  0.33650345  0.73261463
## ConsEner    -0.415029554 -0.23286257 -0.20608749 -0.06730166  0.23100421
## EmisCO2     -0.374531032 -0.29168698 -0.20631751 -0.14843513  0.24028756
##                     PC6           PC7         PC8         PC9        PC10
## CrecPobl    -0.18392437  0.1628974320 -0.09481963  0.52181220 -0.34674573
## MortInf     -0.08656390  0.6398040762 -0.32307802 -0.29031618  0.38959240
## PorcMujeres  0.16835650  0.5310867107  0.05209889  0.23599758 -0.42854658
## PNB95        0.04653378 -0.1490207046 -0.44913216 -0.36995675 -0.34911534
## ProdElec     0.02821905  0.1082745817  0.50343911  0.30681318  0.33770404
## LinTelf      0.02442175 -0.0008501608 -0.56975094  0.44733110  0.20997673
## ConsAgua    -0.30940890  0.2357666690 -0.05962470  0.08358225 -0.20561803
## PropBosq    -0.75356463 -0.0806036686  0.04275404 -0.07438520  0.08671232
## PropDefor    0.50894232  0.0112333588 -0.01607505 -0.01868615  0.03209758
## ConsEner     0.05806466  0.2711228006 -0.05023582 -0.04339752  0.36147417
## EmisCO2     -0.02809233  0.3352822144  0.30978009 -0.37666244 -0.28779437
##                    PC11
## CrecPobl    -0.10062784
## MortInf      0.17487096
## PorcMujeres -0.16786800
## PNB95       -0.15247432
## ProdElec     0.12366382
## LinTelf      0.44992596
## ConsAgua    -0.07067780
## PropBosq    -0.01493710
## PropDefor    0.07259619
## ConsEner    -0.67912543
## EmisCO2      0.46737561
```

Podemos ver que con tan solo el primer componente ya tenemos el 90% de la varianza explicada lo cual ya es muchisimo. En particular notemos que si tomamos los primeros 2 ya tenemos el 99% lo cual es más que suficiente. Asimismo, sucede lo mismo con los vectores que vienen de la matriz de correlación por lo que es un resultado en el cual podemos confiar.

Parte 2

```
library(stats)
library(factoextra)
```

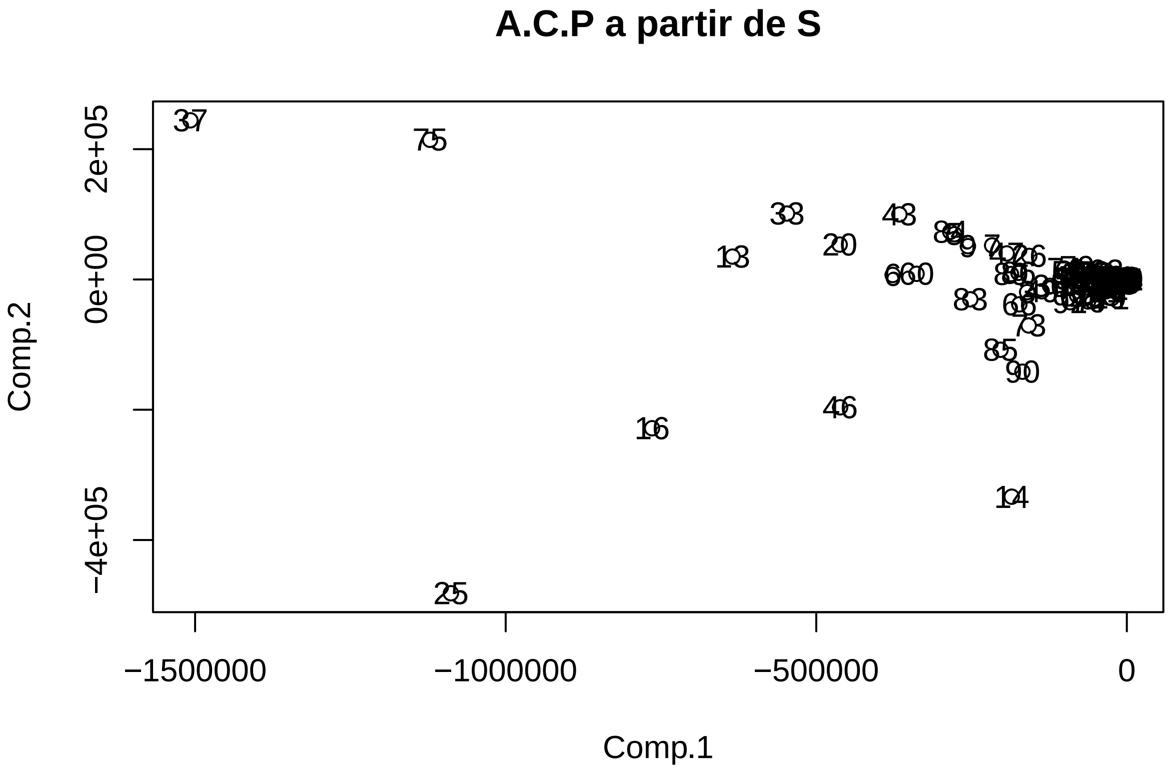
```
## Warning: package 'factoextra' was built under R version 4.1.3

## Loading required package: ggplot2

## Warning: package 'ggplot2' was built under R version 4.1.3

## Welcome! Want to learn more? See two factoextra-related books at https://goo.gl/ve3WBa
```

```
library(ggplot2)
X=read.csv("paises_mundo.csv")
resS=princomp(X)
compS=as.matrix(X)%*%resS$loadings
plot(compS[,1:2],type="p", main = "A.C.P a partir de S")
text(compS[,1],compS[,2],1:nrow(compS))
```



```r
biplot(resS)
```

```
## Warning in arrows(0, 0, y[, 1L] * 0.8, y[, 2L] * 0.8, col = col[2L], length =
## arrow.len): zero-length arrow is of indeterminate angle and so skipped

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```
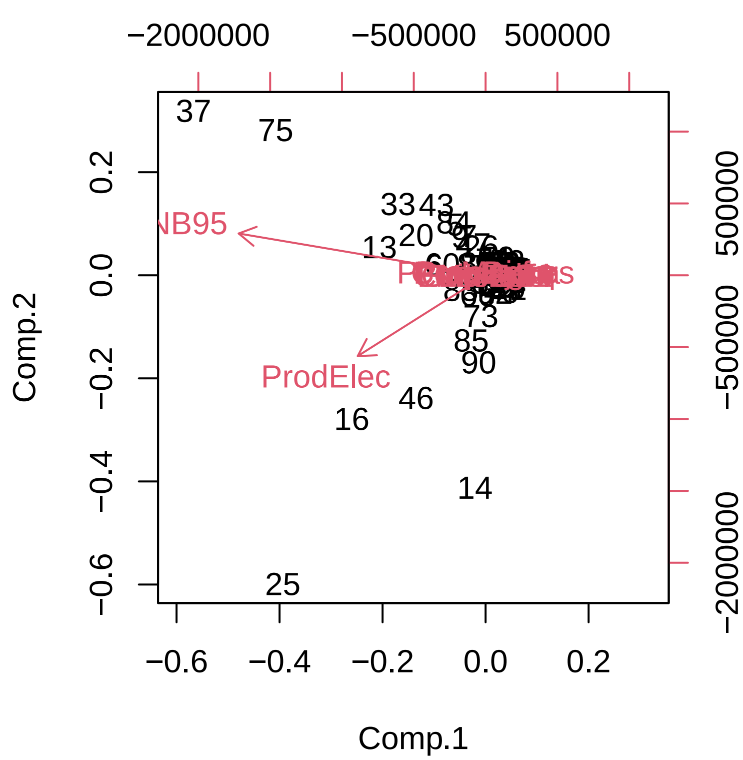



las variables de ProdElec y PNB95 son las que muestran mayor influencie en la tabla de varianzas y covarianzas. Esto debido a que como se observa en l abase de datos, estas 2 variables tienen valores de hasta 100,000, que comparado con el resto, son valores muy grandes.

Parte 3

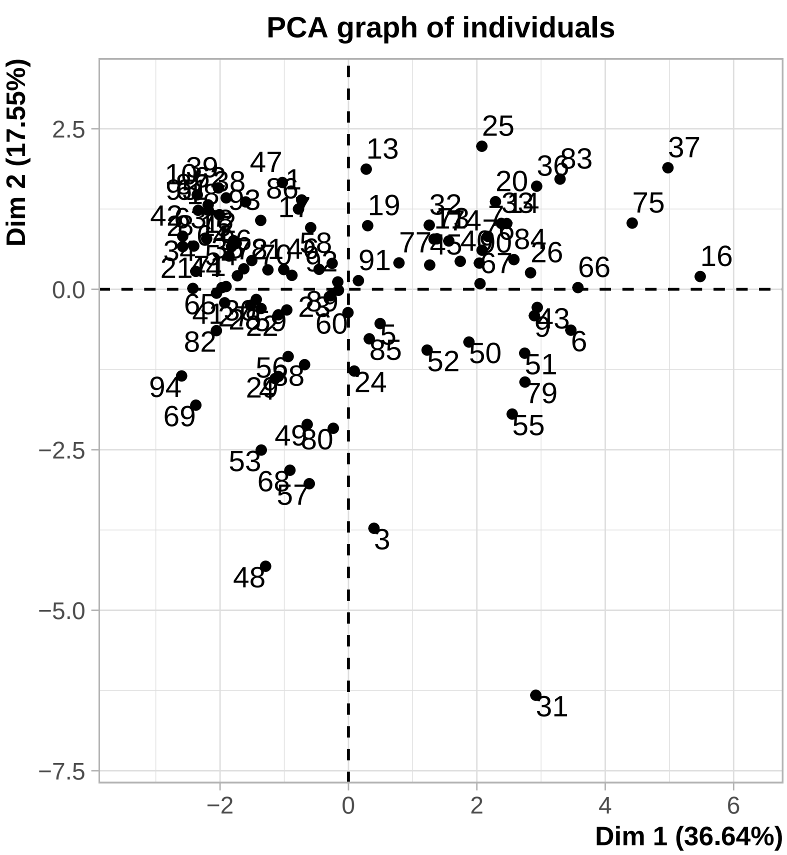
```
library(FactoMineR) # Instalar FactoMineR si se usa por primera vez
```

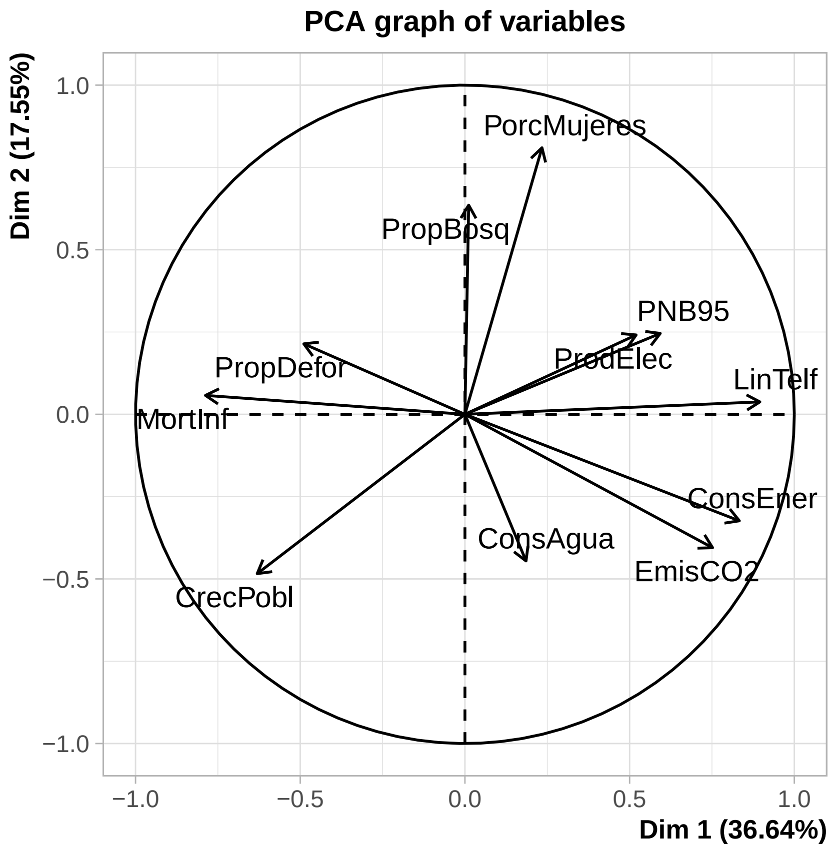
```
## Warning: package 'FactoMineR' was built under R version 4.1.3
```

```
## Warning: package 'FactoMineR' was built under R version 4.1.3
10
```

```
## [1] 10
```

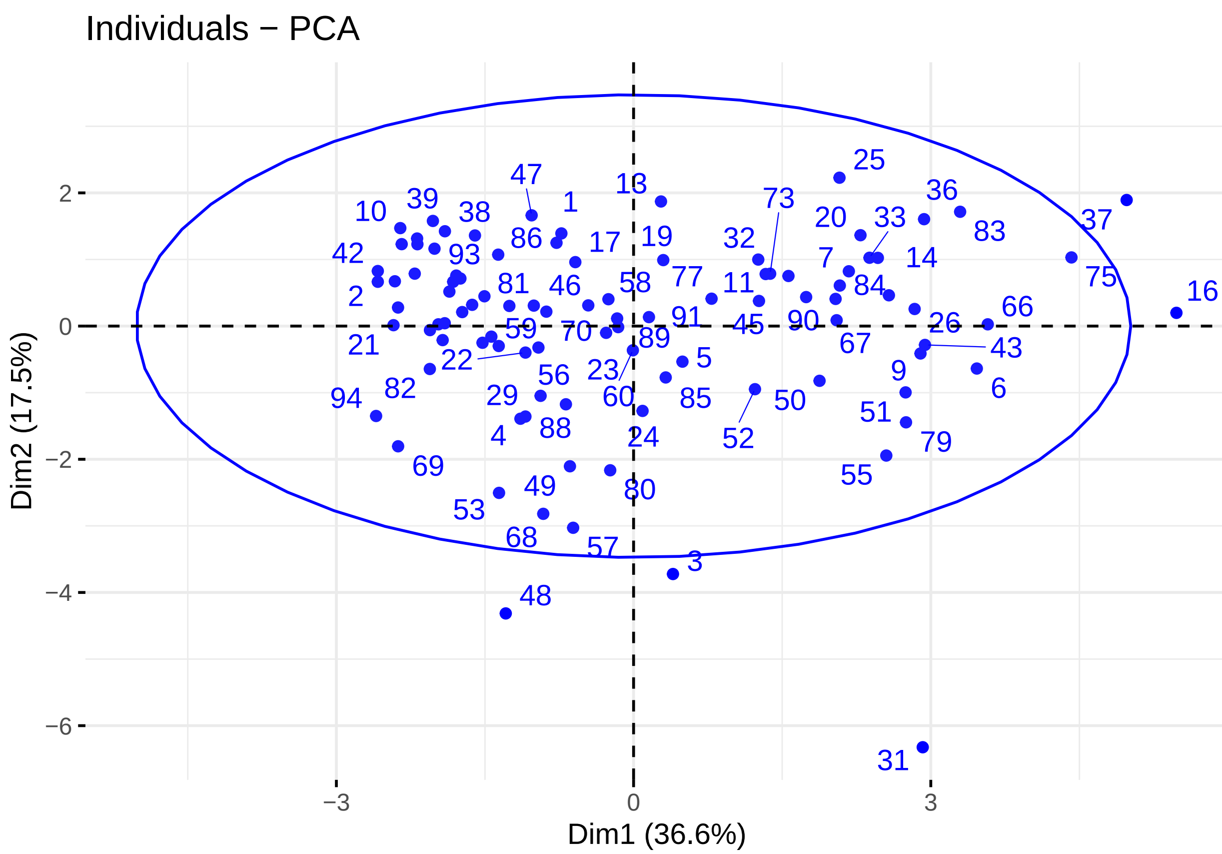
```
library(factoextra) # Instalar factoextra si se usa por primera vez
library(ggplot2) # Instalar ggplot2 si se usa por primera vez
X=read.csv("paises_mundo.csv")
cp3 = PCA(X)
```



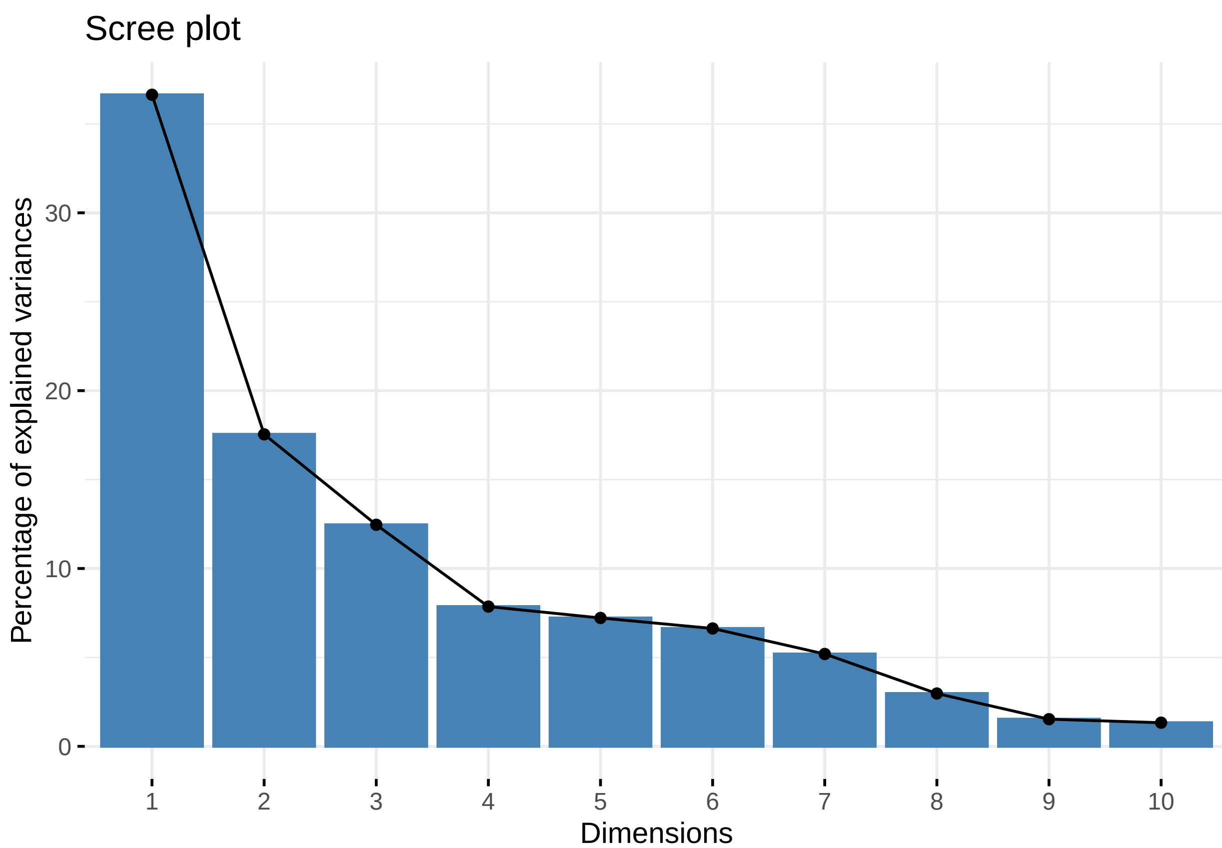



```
fviz_pca_ind(cp3, col.ind = "blue", addEllipses = TRUE, repel = TRUE)
```

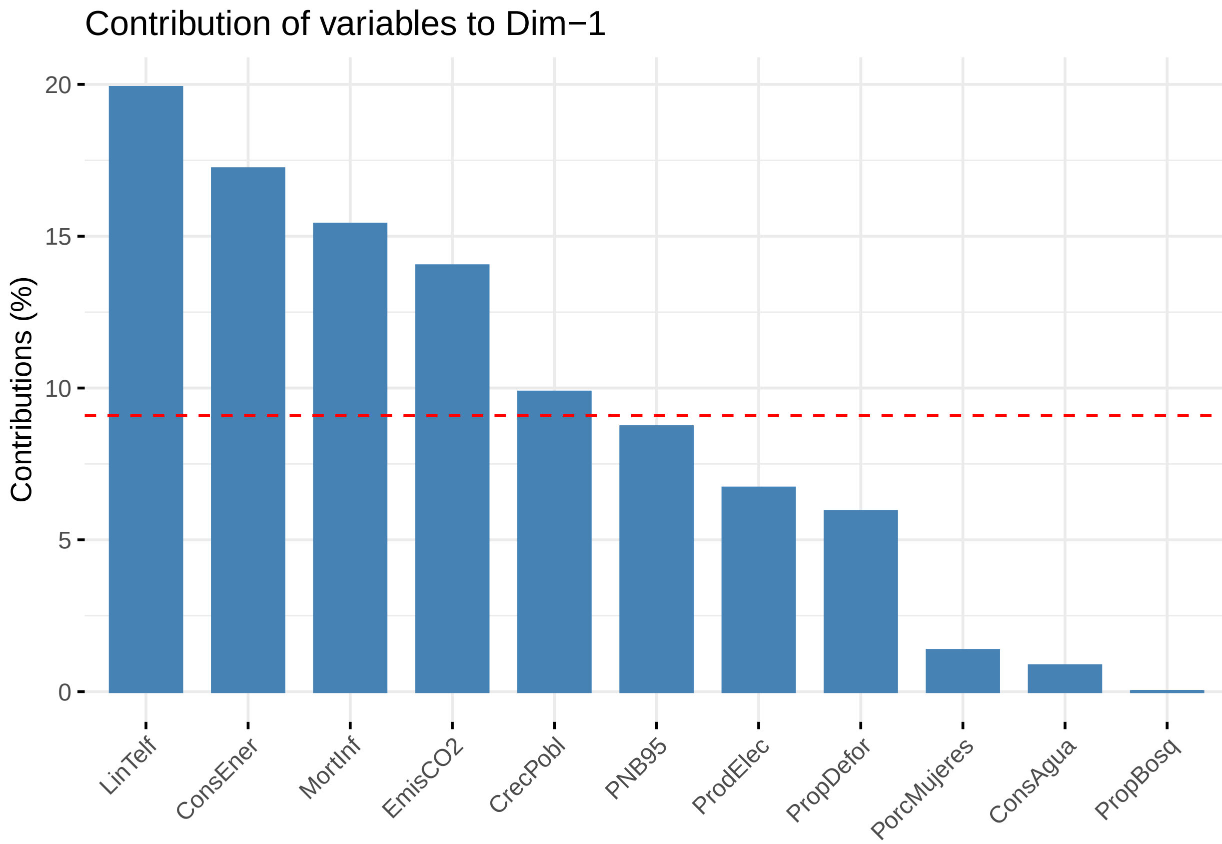
```
## Warning: ggrepel: 27 unlabeled data points (too many overlaps). Consider
## increasing max.overlaps
```




```
fviz_screeplot(cp3)
```




```
fviz_contrib(cp3, choice = c("var"))
```



1. En el primer gráfico, podemos observar el resultado de proyectar los puntos reales sobre el plano definido por los dos componentes principales más relevantes. Se nota una agrupación alrededor de las coordenadas (-1, 1).

2. Al haber seleccionado dos componentes principales, la siguiente gráfica representa los coeficientes de cada variable en estas dos combinaciones lineales. A partir de esto, podemos inferir que cuanto mayor sea la magnitud de estos vectores, mayor será la importancia de esa variable en el comportamiento general del fenómeno.

3. La siguiente gráfica es bastante similar a la primera, pero la escala está ajustada de acuerdo con el valor correspondiente del eigenvalor en cada eje.

4. La cuarta gráfica es conocida como la "gráfica de codo", donde se muestra el avance acumulativo de cada componente. A partir de aquí, podemos comprender por qué es razonable elegir dos vectores como componentes principales.

5. Finalmente, se presenta cómo cada variable afecta al primer componente principal.