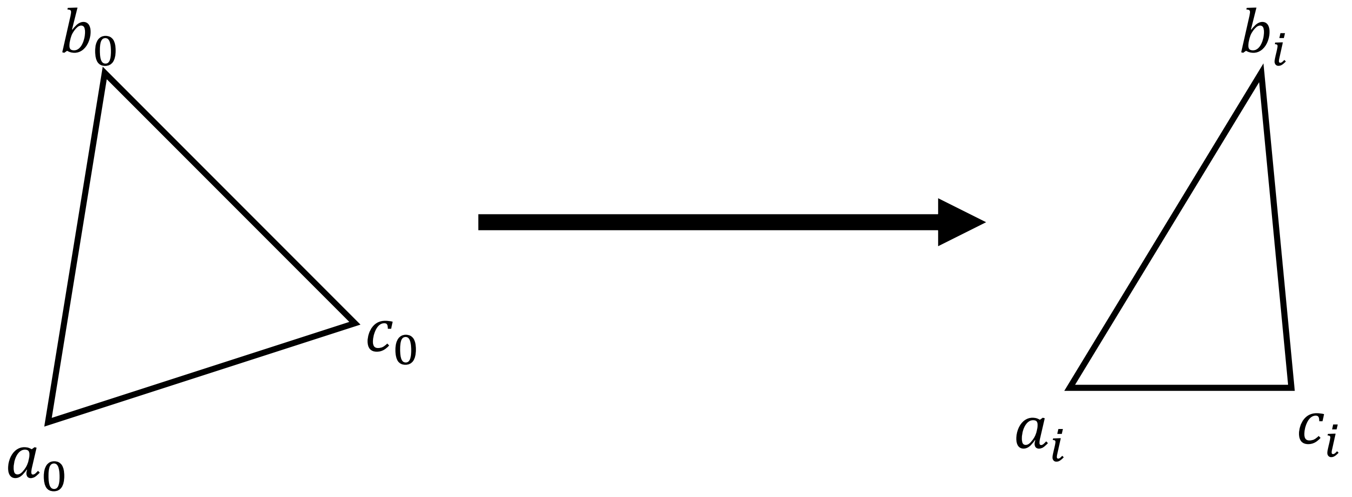
Rectangle Calculation

Our goal



$$J = \begin{pmatrix} a & b \\ c & d \end{pmatrix} = U\Sigma V^T \rightarrow J = U\Sigma I$$

Computation of V

First, let's compute the temporary angle Φ with the following formula:

$$\Phi = \frac{1}{2} atan2(2ab + 2cd, a^2 - b^2 + c^2 - d^2) \tag{9}$$

Let's now compute the following scalars:

$$\begin{aligned} s_{11} &= (a.\, c_\theta + c.\, s_\theta).\, c_\Phi &+& \quad (b.\, c_\theta + d.\, s_\theta).\, s_\Phi \\ s_{22} &= (a.\, s_\theta - c.\, c_\theta).\, s_\Phi &+& \quad (-b.\, s_\theta + d.\, c_\theta).\, c_\Phi \end{aligned} \tag{10}$$

where:

$$\begin{aligned} c_\Phi &= cos(\Phi) \\ s_\Phi &= sin(\Phi) \\ c_\theta &= cos(\theta) \\ s_\theta &= sin(\theta) \end{aligned} \tag{11}$$

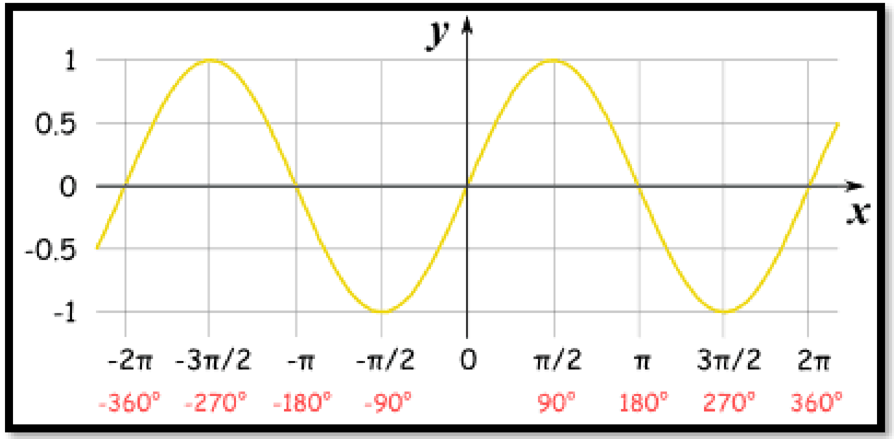
The matrix V is given by:

$$V = \begin{bmatrix} \frac{s_{11}}{|s_{11}|} cos(\Phi) & -\frac{s_{22}}{|s_{22}|} sin(\Phi) \\ \frac{s_{11}}{|s_{11}|} sin(\Phi) & \frac{s_{22}}{|s_{22}|} cos(\Phi) \end{bmatrix} \tag{12}$$

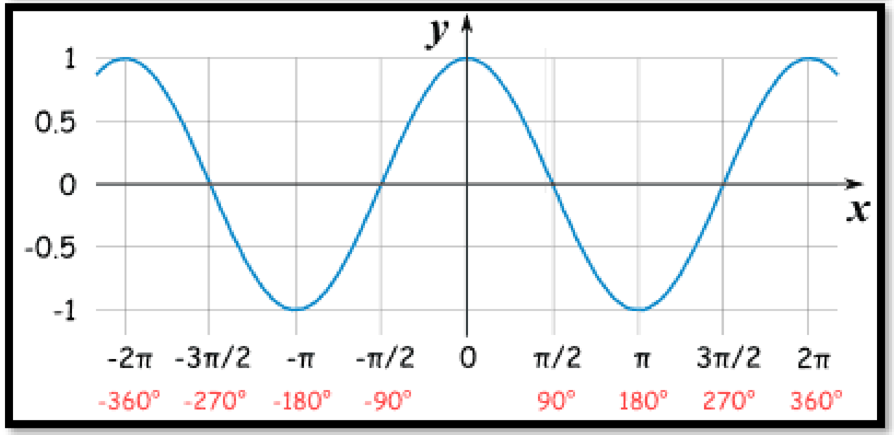
~~$(1)\ \sin(\Phi) = 0$~~

~~$(2)\ cos(\Phi) = \pm 1$~~

~~$(1{+}2)\ \Phi = \pm\pi i$~~



$$\Phi = \pm\frac{\pi i}{2}$$



$$(3)\ \Phi = \frac{1}{2}\operatorname{atan2}(2ab + 2cd, a^2 - b^2 + c^2 - d^2)$$

$$\Rightarrow \pm\frac{\pi i}{2} = \frac{1}{2}\operatorname{atan2}(2ab + 2cd, a^2 - b^2 + c^2 - d^2)$$

$$\Rightarrow \pm\pi i = \operatorname{atan2}(2ab + 2cd, a^2 - b^2 + c^2 - d^2)$$

In terms of the standard arctan function, whose range is $(-\pi/2, \pi/2]$, it can be expressed as follows to define a surface that has no discontinuities except along the semi-infinite line x<0 y=0:

$$\operatorname{atan2}(y, x) = \begin{cases} \arctan\left(\frac{y}{x}\right) & \text{if } x > 0, \\ \arctan\left(\frac{y}{x}\right) + \pi & \text{if } x < 0 \text{ and } y \geq 0, \\ \arctan\left(\frac{y}{x}\right) - \pi & \text{if } x < 0 \text{ and } y < 0, \\ +\frac{\pi}{2} & \text{if } x = 0 \text{ and } y > 0, \\ -\frac{\pi}{2} & \text{if } x = 0 \text{ and } y < 0, \\ \text{undefined} & \text{if } x = 0 \text{ and } y = 0. \end{cases}$$

$\boxed{(-\frac{\pi}{2}, \frac{\pi}{2}]} \rightarrow \boxed{atan2 = 0}$

$\boxed{(+\frac{\pi}{2}, \frac{3\pi}{2}]} \rightarrow \boxed{atan2 = \pi}$

$\boxed{(-\frac{3\pi}{2}, -\frac{\pi}{2}]} \rightarrow \boxed{atan2 = -\pi}$

$\boxed{atan2(.) = \pm\pi i}$

$$\pm\pi i = \operatorname{atan} 2(2ab + 2cd, a^2 - b^2 + c^2 - d^2)$$

➔ *x != 0*

In terms of the standard arctan function, whose range is $(-\pi/2, \pi/2]$, it can be expressed as follows to define a surface that has no discontinuities except along the semi-infinite line x<0 y=0:

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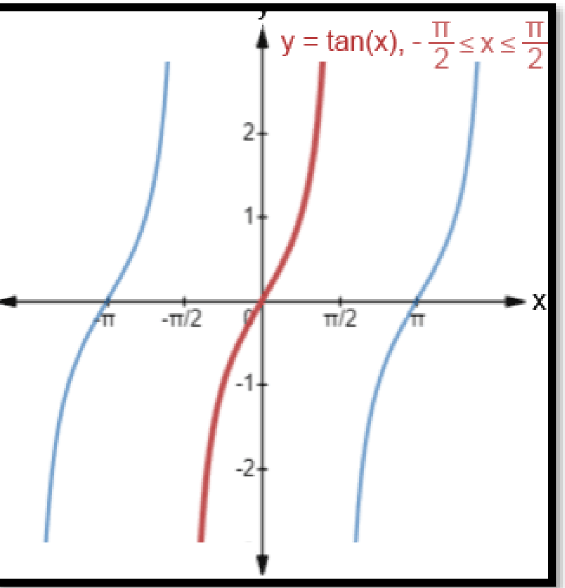
$\boxed{(-\frac{\pi}{2}, \frac{\pi}{2}]} \rightarrow \boxed{atan2 = 0}$

$\boxed{(+\frac{\pi}{2}, \frac{3\pi}{2}]} \rightarrow \boxed{atan2 = \pi}$

$\boxed{(-\frac{3\pi}{2}, -\frac{\pi}{2}]} \rightarrow \boxed{atan2 = -\pi}$

$\boxed{atan2(.) = \pm\pi i}$



Case 1: $\arctan(.) = 0 \quad \Rightarrow (.) = \tan(0) = 0$

Case 2: $\arctan(.) + \pi = \pi \quad \Rightarrow (.) = \tan(-\pi) = 0$

Case 3: $\arctan(.) - \pi = -\pi \quad \Rightarrow (.) = \tan(+\pi) = 0$

$\arctan(.) = 0 \rightarrow (.) = \tan(0) = 0$

(.)=0

In terms of the standard arctan function, whose range is $(-\pi/2, \pi/2]$, it can be expressed as follows to define a surface that has no discontinuities except along the semi-infinite line x<0 y=0:

$$\operatorname{atan2}(y,x) = \begin{cases} \arctan\left(\frac{y}{x}\right) & \text{if } x > 0, \\ \arctan\left(\frac{y}{x}\right) + \pi & \text{if } x < 0 \text{ and } y \geq 0, \\ \text{arcta}\ldots & \\ +\frac{\pi}{2} & \\ -\frac{\pi}{2} & \\ \text{unde}\ldots & \end{cases}$$

$(-\frac{\pi}{2}, \frac{\pi}{2}]$ → $atan2 = 0$

$(+\frac{\pi}{2}, \frac{3\pi}{2}]$ → $atan2 = \pi$

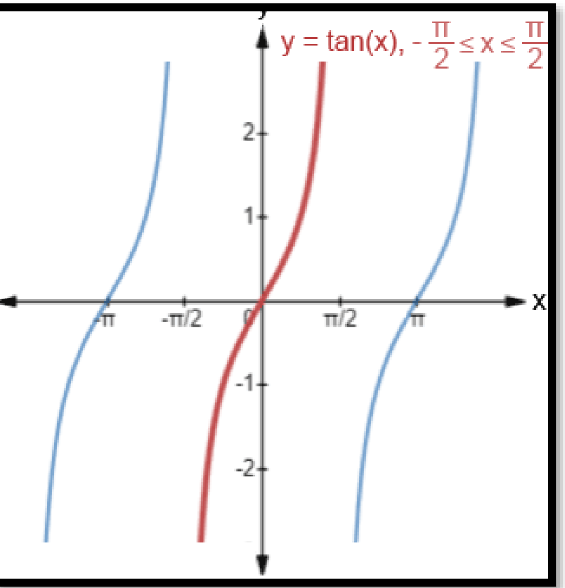
$\ldots an2 = -\pi$

$atan2(.) = \pm\, \pi i$

$= 0$

$\ldots(-\pi) = 0$

$\ldots n(+\pi) = 0$



Finally:

(1) x!=0

(2) $(.) = (\frac{y}{x}) = (\frac{2ab+2cd}{a^2-b^2+c^2-d^2}) = 0$

(1) $a^2 + c^2 - (b^2 + d^2)$!= 0

(2) $2ab + 2cd = 0$

$\arctan(.) = 0$ ➔ $(.) = \tan(0) = 0$

(.)=0