Relaxed Radix Balanced Trees

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Background

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- RRB-Trees are part of the ongoing development of the Scala Programming Language
- The goal is to present a tree datastructure which can be concatenated in O(log n) time at no cost to other operations

Application

- RRB-Trees can be used to implement an efficient vector datastructure
- As it is a purely functional datastructure the operations on it can be easily run in parallel[Stucki et al., 2015]
- An application for efficient vector concatenation can be used to generate SQL queries, or even to dynamically generate XML code or HTML code for websites. [Hull]

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- Multiple variables to control:
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- The main operations are:
  - Indexing
  - Updating
  - Append to the front or back
  - Splitting

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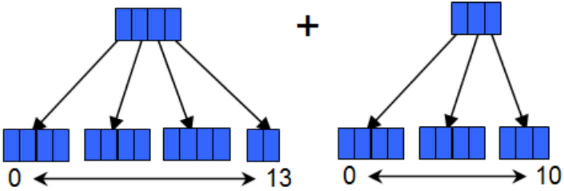
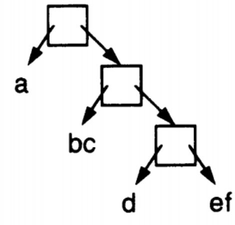


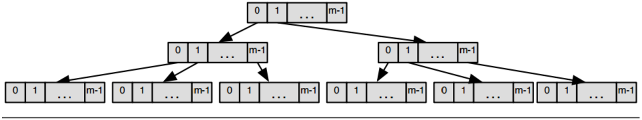
Concatenating Two Vectors



Examples of Ropes

Radix Search

- With a perfectly balanced tree of branching factor m, for any particular sub branch, there are exactly m^{h-1} leaves. Therefore, an index i can be found using $\lfloor i/(m^{h-1}) \rfloor$ [Angle et al., 1999]



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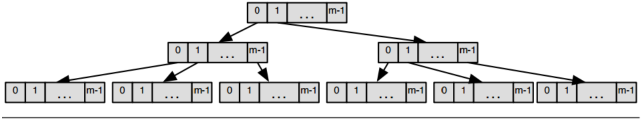

eg. For m = 4. Finding the index 5 using the tree above yields the following calculations:

$$\lfloor 5/(4^{3-1}) \rfloor = 0 \quad \lfloor 5/(4^{1-1}) \rfloor = 5$$
$$\lfloor 5/(4^{2-1}) \rfloor = 1 \quad 5 \ \% \ 4 = 1$$

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- The cost this adds is a slight linear lookup when arriving at a node that is unbalanced.

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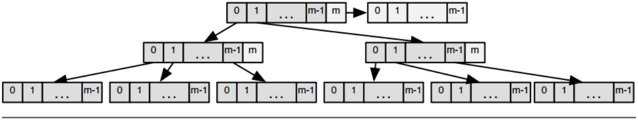
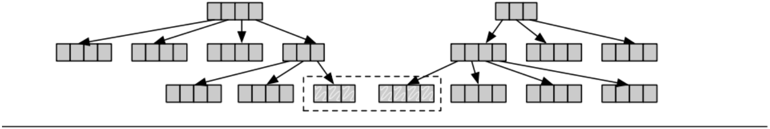
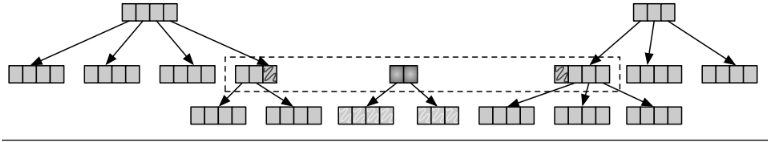


Concatenation Process



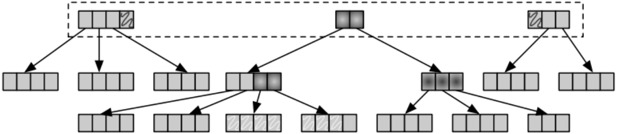
merge the leaf nodes of each tree to create a new tree of height 1

Concatenation Process

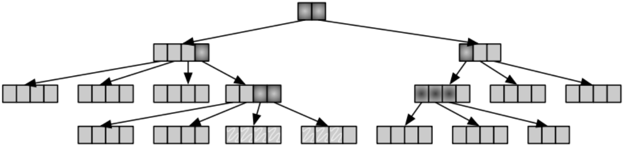


propagate the merge up the tree. Note: At each stage of the merge the tree needs to be rebalanced

Concatenation Process



Only the highlighted nodes need to be modified. The rest are simply shared.



A note on branching factor

- For simplicity of the diagrams a branching factor of 4 was chosen
- In practice the most efficient branching factor is actually 32
- Branching factor controls tree height which affects the runtime of the operations
- Need to minimise branching factor without degenerating into a linked list

	RRB-Vector	With m = 32
indexing	log_m	eC
update	aC	eC
insert ends	$m \times log_m$	aC
concat	$m^2 \times log_m$	L v.s. eC
split	$m \times log_m$	eC

Runtime Complexity In Relation To Similar Structures

	RRB-Vector	Finger Tree	Red-Black Tree
indexing	eC	Log_2	Log_2
update	eC	Log_2	Log_2
insert ends	aC	aC	Log_2
concat	L v.s. eC	Log_2	L
split	eC	Log_2	Log_2

References

All diagrams are sourced from Bagwell and Rompf [2011] and Stucki et al. [2015]

R. L. Angle, E. S. Harriman Jr, and G. B. Ladwig. Radix tree search logic, Feb. 16 1999. US Patent 5,873,078.

P. Bagwell and T. Rompf. Rrb-trees: Efficient immutable vectors. Technical report, 2011.

H.-J. Boehm, R. Atkinson, and M. Plass. Ropes: an alternative to strings. *Software: Practice and Experience*, 25(12):1315–1330, 1995.

R. Hinze and R. Paterson. Finger trees: a simple general-purpose data structure. *Journal of Functional Programming*, 16(02):197–217, 2006.

M. Hull. Balancing simplicity and efficiency in web applications.

N. Stucki, T. Rompf, V. Ureche, and P. Bagwell. Rrb vector: a practical general purpose immutable sequence. In *ACM SIGPLAN Notices*, volume 50, pages 342–354. ACM, 2015.